Supplementary materials for

Baayen, Vasishth, Kliegl, and Bates (2016), The cave of shadows: addressing the human factor with generalized additive mixed models. *Journal of Memory and Language*.

This document describes how data sets discussed in this paper can be obtained, and presents the R-code for the models and corresponding figures and tables.

Required software: R (>= 3.2.2) and R packages devtools (1.9.1 or later), itsadug (2.2 or later), lattice (0.20-33 or later), lme4 (1.1-10 or later), plotfunctions (1.0 or later), mgcv (1.8-12 or later), Rcurl (1.95-4.7 or later), and xtable (1.7-4 or later).

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1 Preliminaries

Once R is started up, the abovementioned packages must be activated. This accomplished in the following code snippet:

```
require(mgcv)
require(lme4)
require(xtable)
require(itsadug)
require(plotfunctions)
require(devtools)
require(RCurl)
require(RePsychLing)
```

The RePsychLing package is currently available on github. It is installed and made available as follows:

```
devtools::install_github("dmbates/RePsychLing",build_vignettes=FALSE)
require(RePsychLing)
```

All data sets analysed in this document are available in the RePsychLing package, with the exception of the baldey dataset, which is available at http://www.mirjamernestus.nl/Ernestus/Baldey/baldey_data.zip. After downloading and unpacking the zip file in a subfolder named data, the database can be loaded into R.

```
baldey = read.table("data/baldey.txt", header=TRUE)
```

One figure requires R code available on github:

```
source(textConnection(getURL(
"https://gist.github.com/mages/5339689/raw/576263b8f0550125b61f4ddba127f5aa00fa2014/add.alpha.R")))
```

Generalized additive models for the larger data sets (baldey, poems) may take up to several hours to fit.

The R code and R output given below make it possible to replicate all results reported in our manuscript, including tables and figures. Technical details on the figures illustrating regression splines in section 2 of the manuscript are not included as they are not part of actual data analysis.

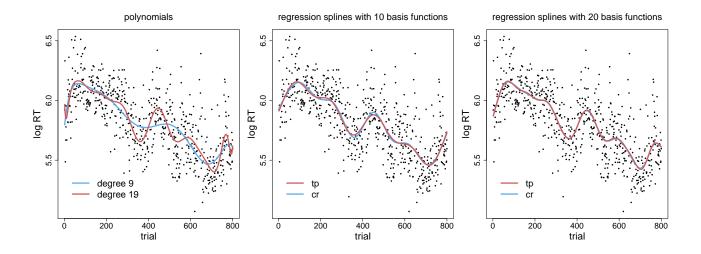
2 Introduction to the generalized additive mixed model

2.1 Figure 1

```
data("KKL")
dfr123 = KKL[KKL$subj==123,]
# set up one row of three plots
par(mfrow=c(1,3), oma=c(2,2,2,2), mar=c(5,5,2,1))
# two regression models with polynomials
poly10.lm = lm(lrt \sim poly(trial, 9), data = dfr123)
poly20.lm = lm(lrt ~ poly(trial,19), data = dfr123)
# make left panel
cx=1.8
plot(dfr123$trial, dfr123$1rt,
     pch=19, cex=0.5, cex.lab=cx+0.3, cex.axis=cx,
     xlab="trial", ylab="log RT")
lines(dfr123$trial, predict(poly10.lm),
     col="steelblue2", lwd=3, lty=1)
lines(dfr123$trial, predict(poly20.lm),
     col="indianred", lwd=3, lty=1)
mtext("polynomials", 3, 1.5, cex=cx-0.5)
legend(0.1, 5.4,
     legend = c("degree 9", "degree 19"),
     lty = rep(1, 2), lwd = rep(3,3),
     col = c("steelblue2", "indianred"), bty = "n", cex = cx+0.2)
# regression splines with 10 basis functions
dfr123.gam = bam(lrt ~s(trial, bs="cr", k=10),
                    data = dfr123, method="fREML")
dfr123$GamCR = predict(dfr123.gam)
dfr123.gamTPRS = bam(lrt ~ s(trial, m=2, bs="tp", k=10),
                    data = dfr123, method="fREML")
dfr123$GamTPRS = predict(dfr123.gamTPRS)
# make center panel
plot(dfr123$trial, dfr123$1rt,
     pch=19, cex=0.5,cex.lab=cx+0.3, cex.axis=cx,
     xlab="trial", ylab="log RT")
lines(dfr123$trial, dfr123$GamCR,
     col="steelblue2", lwd=3)
lines(dfr123$trial, dfr123$GamTPRS,
     col="indianred", lwd=3)
legend(0.1, 5.4,
     legend=rev(c("cr", "tp")),
     lty=rep(1, 2), lwd=rep(3,2),
     col=c("indianred", "steelblue2"), bty="n", cex=cx+0.2)
mtext("regression splines with 10 basis functions", 3, 1.5, cex=cx-0.5)
# regression splines with 20 basis functions
```

```
dfr123.gam
               = bam(lrt ~ s(trial, bs="cr", k=20),
                     data=dfr123, method="fREML")
dfr123$GamCR
               = predict(dfr123.gam)
dfr123.gamTPRS = bam(lrt ~ s(trial, m=2, bs="tp", k=20),
                     data=dfr123, method="fREML")
dfr123$GamTPRS = predict(dfr123.gamTPRS)
# make right panel
plot(dfr123$trial, dfr123$1rt,
     pch=19, cex=0.5, cex.lab=cx+0.3, cex.axis=cx,
     xlab="trial", ylab="log RT")
lines(dfr123$trial, dfr123$GamCR,
     col="steelblue2", lwd=3)
lines(dfr123$trial, dfr123$GamTPRS,
     col="indianred", lwd=3)
mtext("regression splines with 20 basis functions", 3, 1.5, cex=cx-0.5)
legend(0.1, 5.4,
     legend=rev(c("cr", "tp")),
     lty=rep(1, 2), lwd=rep(3,2),
     col=c("indianred", "steelblue2"), bty="n", cex=cx+0.2)
```

Figure 1



AIC values for the models discussed with respect to Figure 1.

```
# a model with a linear effect of trial
AIC(lm(lrt ~ trial, data=dfr123))

[1] -206.4587

# the models with polynomials
AIC(lm(lrt ~ poly(trial, 9), data=dfr123))
```

```
AIC(lm(lrt ~ poly(trial, 19), data=dfr123))

[1] -332.2674

# the models with restricted cubic splines
AIC(bam(lrt ~ s(trial, bs="cr", k=10), data=dfr123, method="fREML"))

[1] -316.748

AIC(bam(lrt ~ s(trial, bs="cr", k=20), data=dfr123, method="fREML"))

[1] -331.5204

# the models with thin plate regression splines
AIC(bam(lrt ~ s(trial, bs="tp", k=10), data=dfr123, method="fREML"))

[1] -311.4005

AIC(bam(lrt ~ s(trial, bs="tp", k=20), data=dfr123, method="fREML"))

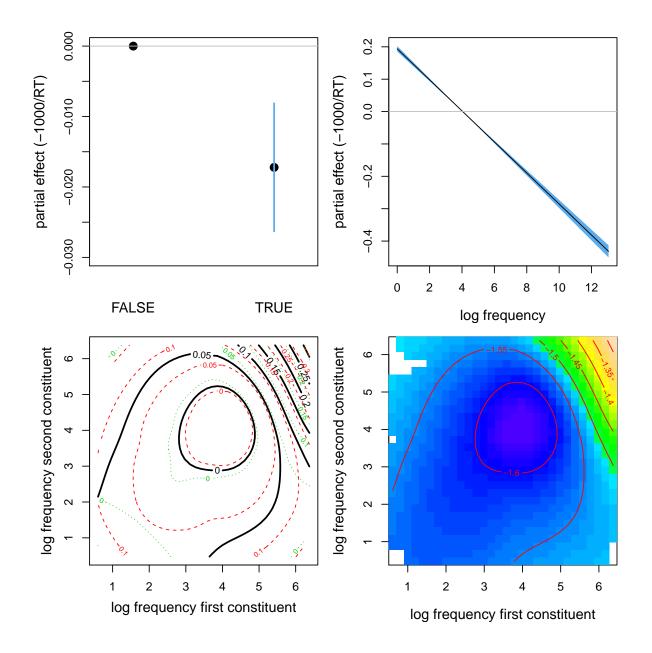
[1] -334.0553
```

Comparison of restricted cubic splines with 10 and 20 basis functions.

2.2 Figure 5 and the model for the vietnamese dataset

```
# the corresponding figure
par(mfrow = c(2,2), mar = c(4,4,1,1))
plot(c(0.5,2.5), c(0, -0.017215), xlim=c(0, 3), ylim=c(-0.03, 0), xaxt="n",
     xlab=" ", ylab="partial effect (-1000/RT)", pch=19, cex=1.5)
abline(h=0, col="gray")
lines(c(2.5, 2.5), c(-0.017215+2*0.004544, -0.017215-2*0.004544), lwd=2, col="steelblue2")
mtext(c("FALSE", "TRUE"), side=1, line=2.5, at=c(0.5, 2.5))
plot(vietnamese.gam, select=1, scheme=1, shade.col="steelblue2", rug=FALSE,
     xlab="log frequency", ylab="partial effect (-1000/RT)")
abline(h=0, col="gray")
plot(vietnamese.gam, select=2, rug=FALSE,
        xlab="log frequency first constituent", ylab="log frequency second constituent",
        main=" ")
vis.gam(vietnamese.gam, view=c("LogFreqSyl1", "LogFreqSyl2"),
        color="topo", plot.type="contour", too.far=0.1,
        xlab="log frequency first constituent", ylab="log frequency second constituent",
        main=" ")
```

Figure 5.



3 The KKL dataset

3.1 Table 1 (main text) and Table 3 (appendix)

```
data(KKL)
# identify starting point for time series of RTs
KKL$FirstTrial = KKL$first==1
# scale trial and soa
KKL$Trial = scale(KKL$trial)
KKL$Soa = scale(KKL$SOA)
# add a quadratic term for trial
KKL$TrialQ = KKL$Trial^2
# define interaction of size by orientation
KKL$Int = interaction(KKL$size, KKL$cardinal)
```

A sequence of linear mixed models and pairwise likelihood ratio tests:

```
kkl.lmer0 = lmer(lrt ~ sze * (spt + obj + grv) * orn +
                 (1|subj)+
                 (0+spt|subj)+
                 (0+grv|subj)+
                 (0+obj|subj)+
                 (0+orn|subj)+
                 (0+spt_orn|subj)+
                 poly(Soa,2,raw=TRUE),
                 data=KKL, REML=FALSE)
fixed-effect model matrix is rank deficient so dropping 3 columns / coefficients
# add in Trial and its interactios with sze and orn
kkl.lmer1 = lmer(lrt ~ sze * (spt + obj + grv) * orn +
                 (1|subj)+
                 (0+spt|subj)+
                 (0+grv|subj)+
                 (O+obj|subj)+
                 (0+orn|subj)+
                 (0+spt_orn|subj)+
                 Trial*(sze+orn)+
                 poly(Soa,2,raw=TRUE),
                 data=KKL, REML=FALSE)
fixed-effect model matrix is rank deficient so dropping 3 columns / coefficients
anova(kkl.lmer0, kkl.lmer1)
Data: KKL
Models:
kkl.lmer0: lrt ~ sze * (spt + obj + grv) * orn + (1 | subj) + (0 + spt |
            subj) + (0 + grv | subj) + (0 + obj | subj) + (0 + orn |
kkl.lmer0:
kkl.lmer0:
             subj) + (0 + spt_orn | subj) + poly(Soa, 2, raw = TRUE)
kkl.lmer1: lrt ~ sze * (spt + obj + grv) * orn + (1 | subj) + (0 + spt |
kkl.lmer1: subj) + (0 + grv | subj) + (0 + obj | subj) + (0 + orn |
               subj) + (0 + spt_orn | subj) + Trial * (sze + orn) + poly(Soa,
kkl.lmer1:
kkl.lmer1: 2, raw = TRUE)
```

```
Df AIC BIC logLik deviance Chisq Chi Df Pr(>Chisq)
kkl.lmer0 25 -25088 -24865 12569
                                   -25138
                                   -25941 802.96
kkl.lmer1 28 -25885 -25636 12970
                                                    3 < 2.2e-16
# add in quadratic effect of Trial
kkl.lmer2 = lmer(lrt ~ sze * (spt + obj + grv) * orn +
                 (1|subj)+
                 (0+spt|subj)+
                 (0+grv|subj)+
                 (0+obj|subj)+
                 (0+orn|subj)+
                 (0+spt_orn|subj)+
                 Trial*(sze+orn)+
                 TrialQ*(sze+orn)+
                 poly(Soa,2,raw=TRUE),
                 data=KKL, REML=FALSE)
fixed-effect model matrix is rank deficient so dropping 3 columns / coefficients
anova(kkl.lmer1, kkl.lmer2)
Data: KKL
Models:
kkl.lmer1: lrt ~ sze * (spt + obj + grv) * orn + (1 | subj) + (0 + spt |
              subj) + (0 + grv | subj) + (0 + obj | subj) + (0 + orn |
kkl.lmer1:
              subj) + (0 + spt_orn | subj) + Trial * (sze + orn) + poly(Soa,
kkl.lmer1:
               2, raw = TRUE)
kkl.lmer1:
kkl.lmer2: lrt ~ sze * (spt + obj + grv) * orn + (1 | subj) + (0 + spt |
             subj) + (0 + grv | subj) + (0 + obj | subj) + (0 + orn |
kkl.lmer2:
kkl.lmer2:
              subj) + (0 + spt_orn | subj) + Trial * (sze + orn) + TrialQ *
kkl.lmer2:
             (sze + orn) + poly(Soa, 2, raw = TRUE)
        Df AIC BIC logLik deviance Chisq Chi Df Pr(>Chisq)
kkl.lmer1 28 -25885 -25636 12970
                                   -25941
kkl.lmer2 31 -26174 -25899 13118
                                   -26236 295.77
                                                    3 < 2.2e-16
# and allow slope of Trial to vary by subject
kkl.lmer3 = lmer(lrt ~ sze * (spt + obj + grv) * orn +
                 (1+Trial|subj)+
                 (0+spt|subj)+
                 (0+grv|subj)+
                 (0+obj|subj)+
                 (0+orn|subj)+
                 (0+spt_orn|subj)+
                Trial*(sze+orn)+
                TrialQ*(sze+orn)+
                 poly(Soa,2,raw=TRUE),
                 data=KKL, REML=FALSE)
fixed-effect model matrix is rank deficient so dropping 3 columns / coefficients
anova(kkl.lmer2, kkl.lmer3)
Data: KKL
Models:
kkl.lmer2: lrt ~ sze * (spt + obj + grv) * orn + (1 | subj) + (0 + spt |
```

```
kkl.lmer2: subj) + (0 + grv | subj) + (0 + obj | subj) + (0 + orn |
kkl.lmer2: subj) + (0 + spt_orn | subj) + Trial *
kkl.lmer2: (sze + orn) + poly(Soa, 2, raw = TRUE)
              subj) + (0 + spt_orn | subj) + Trial * (sze + orn) + TrialQ *
kkl.lmer3: lrt ~ sze * (spt + obj + grv) * orn + (1 + Trial | subj) + (0 +
kkl.lmer3: spt | subj) + (0 + grv | subj) + (0 + obj | subj) + (0 +
kkl.lmer3:
              orn | subj) + (0 + spt_orn | subj) + Trial * (sze + orn) +
kkl.lmer3:
            TrialQ * (sze + orn) + poly(Soa, 2, raw = TRUE)
        Df AIC BIC logLik deviance Chisq Chi Df Pr(>Chisq)
kkl.lmer2 31 -26174 -25899 13118 -26236
kkl.lmer3 33 -26989 -26696 13528
                                 -27055 818.5 2 < 2.2e-16
# show the model (Table 3 in the manuscript)
print(summary(kkl.lmer3),corr=FALSE)
Linear mixed model fit by maximum likelihood ['lmerMod']
Formula: lrt ~ sze * (spt + obj + grv) * orn + (1 + Trial | subj) + (0 +
    spt | subj) + (0 + grv | subj) + (0 + obj | subj) + (0 +
    orn | subj) + (0 + spt_orn | subj) + Trial * (sze + orn) +
   TrialQ * (sze + orn) + poly(Soa, 2, raw = TRUE)
   Data: KKL
     AIC
             BIC logLik deviance df.resid
-26988.9 -26695.5 13527.5 -27054.9
Scaled residuals:
   Min 10 Median
                          30
-5.2071 -0.6203 -0.0982 0.5029 6.4808
Random effects:
 Groups Name
                 Variance Std.Dev. Corr
 subj
         (Intercept) 0.0261682 0.16177
         Trial 0.0026194 0.05118 -0.27
 subj.1 spt
                    0.0044635 0.06681
 subj.2
                    0.0011434 0.03381
         grv
 subj.3
        obj
                     0.0006558 0.02561
                    0.0058135 0.07625
 subj.4 orn
 subj.5 spt_orn
                    0.0011040 0.03323
                     0.0344439 0.18559
 Residual
Number of obs: 53765, groups: subj, 86
Fixed effects:
                             Estimate Std. Error t value
                            5.659e+00 1.753e-02 322.9
(Intercept)
                           1.843e-01 3.501e-02 5.3
sze
                            7.408e-02 7.698e-03
                                                   9.6
spt
                           4.259e-02 4.524e-03
obj
                                                   9.4
                                                   -0.2
                           -8.957e-04 5.122e-03
grv
orn
                           1.375e-02 9.109e-03
                                                   1.5
Trial
                           -4.299e-02 5.755e-03 -7.5
                            1.532e-02 9.031e-04
                                                  17.0
poly(Soa, 2, raw = TRUE)1.0 -1.009e-02 8.036e-04 -12.6
poly(Soa, 2, raw = TRUE)2.0 1.853e-02 8.991e-04
                                                 20.6
                           4.837e-02 1.540e-02
sze:spt
                                                   3.1
sze:obj
                         -1.177e-02 9.048e-03
                                                   -1.3
```

```
-3.750e-02 1.024e-02 -3.7
sze:grv
sze:orn
                            3.931e-02 1.785e-02
                                                    2.2
spt:orn
                            2.028e-02 6.499e-03
                                                    3.1
                           9.050e-03 7.168e-03 1.3
obj:orn
grv:orn
                           1.093e-02 7.192e-03
                                                  1.5
                            1.835e-02 1.145e-02
sze:Trial
                                                    1.6
orn:Trial
                           2.785e-02 3.225e-03
                                                   8.6
sze:TrialQ
                           -9.166e-05 1.790e-03
                                                   -0.1
                           -6.078e-03 4.794e-03
orn:TrialQ
                                                   -1.3
                           -1.425e-02 1.300e-02
sze:spt:orn
                                                   -1.1
sze:obj:orn
                           -3.466e-03 1.434e-02
                                                   -0.2
sze:grv:orn
                           -4.680e-02 1.438e-02
                                                   -3.3
fit warnings:
fixed-effect model matrix is rank deficient so dropping 3 columns / coefficients
```

The summary of kkl.lmer3 is reported as **Table 3** in the Appendix. The above log likelihood ratio tests are summarized in **Table 1**.

3.2 Table 4 (appendix)

Table 4 (in the appendix) reports the summary of the following GAMM:

```
Family: gaussian
Link function: identity
Formula:
lrt ~ sze * (spt + obj + grv) * orn + s(Trial, subj, bs = "fs",
    m = 1) + s(subj, spt, bs = "re") + s(subj, grv, bs = "re") +
    s(subj, obj, bs = "re") + s(subj, orn, bs = "re") + s(subj,
    spt_orn, bs = "re") + s(Trial, by = Int) + s(Soa)
Parametric coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 5.6850584 0.0189221 300.445 < 2e-16
            0.1839522 0.0378441 4.861 1.17e-06
            0.0729251 0.0079223 9.205 < 2e-16
spt
            0.0410758 0.0041140 9.984 < 2e-16
obj
           -0.0004952 0.0049377 -0.100 0.920111
grv
            0.0374653 0.0141694 2.644 0.008193
orn

      sze:spt
      0.0482519
      0.0158446
      3.045
      0.002326

      sze:obj
      -0.0088449
      0.0082283
      -1.075
      0.282407

      sze:grv
      -0.0366134
      0.0098757
      -3.707
      0.000210

            0.0095644 0.0283388 0.338 0.735741
sze:orn
            0.0213368 0.0063789 3.345 0.000824
spt:orn
obj:orn
            0.0082761 0.0068235 1.213 0.225181
             0.0077985 0.0068488
grv:orn
                                       1.139 0.254848
sze:spt:orn -0.0097831 0.0127578 -0.767 0.443181
sze:obj:orn -0.0075086 0.0136470 -0.550 0.582183
sze:grv:orn -0.0482719 0.0136977 -3.524 0.000425
Approximate significance of smooth terms:
                                 edf Ref.df
                                                   F p-value
                             557.867 774.000 661.847 < 2e-16
s(Trial, subj)
s(spt,subj)
                              77.793 86.000 31.591 < 2e-16
                              47.606 84.000 2.164 1.82e-10
s(grv,subj)
s(obj,subj)
                              29.571 84.000 1.340 0.00114
                              44.080 84.000 1.148 < 2e-16
s(orn, subj)
s(spt_orn,subj)
                          47.373 84.000 1.315 1.68e-10
```

```
AIC(kkl.gamD)
[1] -31890
```

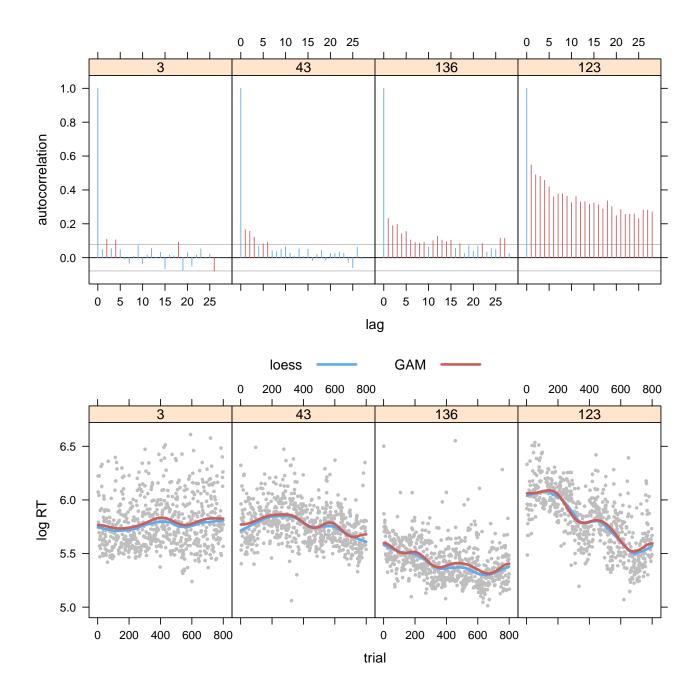
3.3 Figure 9

```
# For Figure 9, we first define a function for acf plots with lattice
acfs = function(m=dat.gamA, s=selectedSubjects, mydata=dat, add_NA=FALSE,
                ci=0.95, rawplot=FALSE) {
 mydata$resids = resid(m)
  res = list()
  n.used = rep(0,length(s))
  for (i in 1:length(s)) {
   tmp = mydata[mydata$subj == s[i],]
   if (!add_NA) {
      x = acf(tmp$resids, plot=rawplot)
    } else {
      v = rep(NA, max(tmp$trial))
     v[tmp$trial] = tmp$resids
      x = acf(v, na.action=na.pass, plot=rawplot)
   res[[i]] = data.frame(
     lag = x lag,
     acf = x$acf,
     subj = factor(s[i]))
   n.used[i] = x$n.used
  Dfr = do.call(rbind, res)
  civec = qnorm((1 + ci)/2)/sqrt(n.used)
  theplot = xyplot(acf ~ lag | subj, type = "h", data = Dfr, #col.line = "black",
    panel = function(x, y, subscripts, ...) {
      panel.abline(h = civec[panel.number()], col.line = "grey")
      panel.abline(h = -civec[panel.number()], col.line = "grey")
      panel.abline(h = 0, col.line = "black")
      d=Dfr[subscripts,]
      panel.xyplot(d$lag, d$acf,
        col=ifelse(abs(d$acf)>civec[panel.number()] & d$lag > 0,
                   "indianred", "steelblue2"),
                   ...)
    #strip = strip.custom(bq = "qrey90"),
    #par.strip.text = list(cex = 0.8),
    xlab="lag", ylab="autocorrelation",
    layout=c(4,1,1))
  return(list(vals=Dfr, plot=theplot))
# a LMM
kkl.lmer = lmer(lrt ~ sze * (spt + obj + grv) * orn +
                 (1|subj)+
                 (0+spt|subj)+
                 (0+grv|subj)+
                 (0+obj|subj)+
                 (0+orn|subj)+
```

Figure 9, lower panels.

```
selectedSubjects = c(3, 43, 136, 123)
dfr = droplevels(KKL[KKL$subj %in% selectedSubjects,])
dfr$subj = ordered(dfr$subj, selectedSubjects)
dfr.gam = bam(lrt~1+s(trial, subj, bs="fs", m=1), data=dfr, method="fREML")
dfr$Gam = predict(dfr.gam)
keys = list(space="top", text=list(c("loess", "GAM")), columns=2,
            lines=list(lty=c(1,1), col=c("steelblue2", "indianred"), lwd=3))
xyplot(lrt ~ trial | subj, data = dfr, #col.line = "indianred",
 panel = function(x, y, subscripts, ...) {
   panel.xyplot(x, y, pch=19, col="gray", cex=0.4, ...)
    panel.loess(x, y, lwd=3, span=0.3, col="steelblue2", ...)
    d = dfr[subscripts,]
   panel.lines(d$trial, d$Gam, col="indianred", lwd=3)
  #strip = strip.custom(bg = "grey90"),
  #par.strip.text = list(cex = 0.8),
  xlab="trial", ylab="log RT",
  layout=c(4,1,1),
  key=keys)
```

Figure 9.

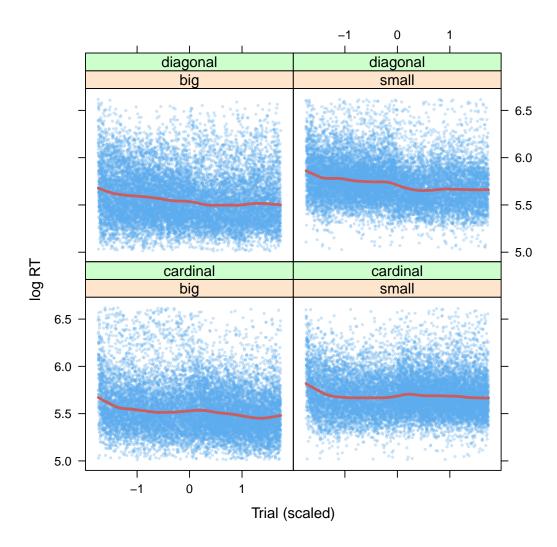


3.4 Figure 10

```
selectedSubjects = c(3, 43, 136, 123)
myColor="steelblue2"
add.alpha <- function(col=NULL, alpha=1){
   if(missing(col)) stop("Please provide a vector of colours.")
   apply(sapply(col, col2rgb)/255, 2,
        function(x) rgb(x[1], x[2], x[3], alpha=alpha))
}

myColorAlpha = add.alpha(myColor, alpha=0.3)

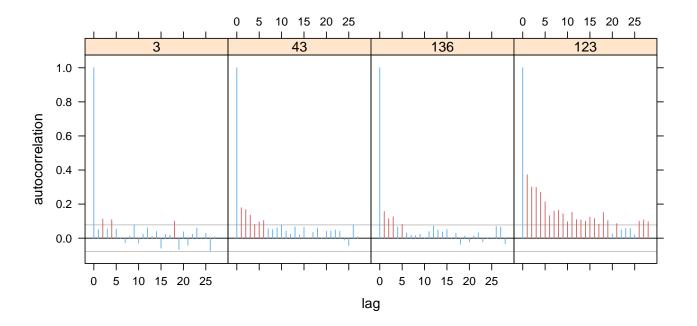
xyplot(lrt~Trial|size+cardinal, data=KKL, type=c("p", "smooth"),span=0.2,
        col.line="indianred", cex=0.3, pch=19, col=myColorAlpha, lwd=3,
        xlab="Trial (scaled)", ylab="log RT")</pre>
```



3.5 Figure 11

```
x = acfs(kkl.lmer3, selectedSubjects, KKL)
print(x[[2]])
```

```
load("models/kkl.lmer3.rda")
x = acfs(kkl.lmer3, selectedSubjects, KKL)
pdf("figs/Figure11.pdf", he=4, wi=8)
print(x[[2]])
dev.off()
```

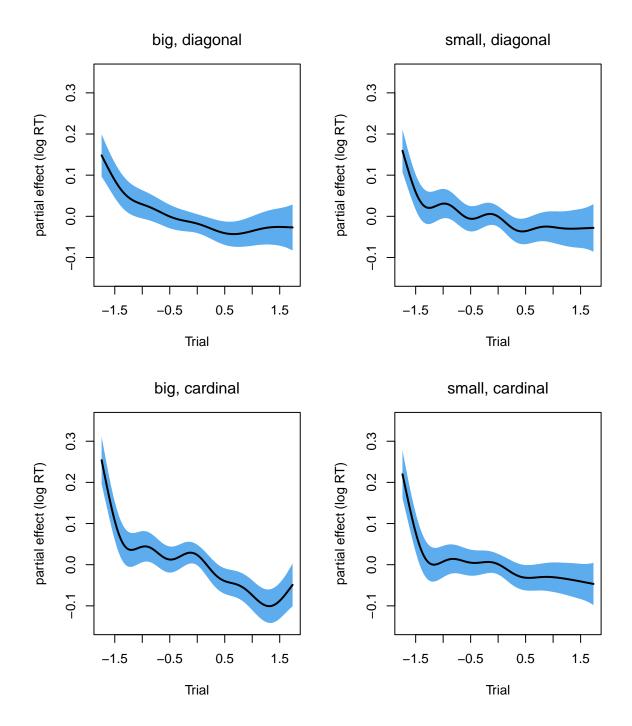


3.6 Figure 12

```
par(mfrow=c(2,2), mar=c(5,5,3,1), oma=rep(2,4))
ylimit=c(-0.15, 0.35)
plot(kkl.gamD, select=7, rug=F, ylim=ylimit, scheme=1,
     xlab="Trial", ylab="partial effect (log RT)",
     shade.col="steelblue2", col="indianred", lwd=2)
mtext("big, cardinal", 3, 1.0)
plot(kkl.gamD, select=8, rug=F, ylim=ylimit, scheme=1,
     xlab="Trial", ylab="partial effect (log RT)",
     shade.col="steelblue2", col="indianred", lwd=2)
mtext("small, cardinal", 3, 1.0)
plot(kkl.gamD, select=9, rug=F, ylim=ylimit, scheme=1,
     xlab="Trial", ylab="partial effect (log RT)",
     shade.col="steelblue2", col="indianred", lwd=2)
mtext("big, diagonal", 3, 1.0)
plot(kkl.gamD, select=10, rug=F, ylim=ylimit, scheme=1,
     xlab="Trial", ylab="partial effect (log RT)",
     shade.col="steelblue2", col="indianred", lwd=2)
mtext("small, diagonal", 3, 1.0)
```

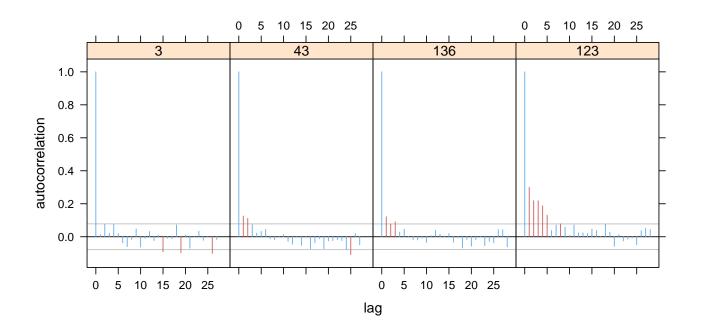
```
pdf("figs/Figure12.pdf", he=8,wi=7)
par(mfrow=c(2,2), mar=c(5,5,3,1), oma=rep(2,4))
ylimit=c(-0.15, 0.35)
plot(kkl.gamD, select=9, rug=F, ylim=ylimit, scheme=1,
     xlab="Trial", ylab="partial effect (log RT)",
     shade.col="steelblue2", col="indianred", lwd=2)
mtext("big, diagonal", 3, 1.0)
plot(kkl.gamD, select=10, rug=F, ylim=ylimit, scheme=1,
     xlab="Trial", ylab="partial effect (log RT)",
     shade.col="steelblue2", col="indianred", lwd=2)
mtext("small, diagonal", 3, 1.0)
plot(kkl.gamD, select=7, rug=F, ylim=ylimit, scheme=1,
     xlab="Trial", ylab="partial effect (log RT)",
     shade.col="steelblue2", col="indianred", lwd=2)
mtext("big, cardinal", 3, 1.0)
plot(kkl.gamD, select=8, rug=F, ylim=ylimit, scheme=1,
     xlab="Trial", ylab="partial effect (log RT)",
     shade.col="steelblue2", col="indianred", lwd=2)
mtext("small, cardinal", 3, 1.0)
dev.off()
```

Figure 12.



3.7 Figure 13

```
x = acfs(kkl.gamD, selectedSubjects, KKL)
print(x[[2]])
```



3.8 Table 2

Table 2 is based on the following models:

```
# reference model
kkl.ref.gam <- bam(lrt ~ sze * (spt + obj + grv) * orn +
                         s(subj, bs="re") +
                         s(subj, spt, bs="re") +
                         s(subj, grv, bs="re") +
                         s(subj, obj, bs="re") +
                         s(subj, orn, bs="re") +
                         s(subj, spt_orn, bs="re") +
                         poly(Soa,2,raw=TRUE),
                   data=KKL, method="fREML", discrete=TRUE)
# 'linear' model
kkl.lin.gam <- bam(lrt ~ sze * (spt + obj + grv) * orn +
                         s(subj, bs="re") +
                         s(subj, Trial, bs="re") +
                         s(subj, spt, bs="re") +
                         s(subj, grv, bs="re") +
                         s(subj, obj, bs="re") +
                         s(subj, orn, bs="re") +
                         s(subj, spt_orn, bs="re") +
                         Trial*(sze+orn)+
                         TrialQ*(sze+orn)+
                         poly(Soa,2,raw=TRUE),
                   data=KKL, method="fREML", discrete=TRUE)
# model with factor smooths
kkl.fs.gam = bam(lrt ~ sze * (spt + obj + grv) * orn +
                         s(Trial, subj, bs="fs", m=1)+
                         s(subj, spt, bs="re") +
                         s(subj, grv, bs="re") +
                         s(subj, obj, bs="re") +
                         s(subj, orn, bs="re") +
                         s(subj, spt_orn, bs="re") +
                         Trial*(sze+orn)+
                         TrialQ*(sze+orn)+
                         s(Soa),
                   data=KKL, method="fREML", discrete=TRUE)
# model with smooths for the interactions with trial
kkl.trial.gam = bam(lrt ~ sze * (spt + obj + grv) * orn +
                         s(Trial, subj, bs="fs", m=1)+
                         s(subj, spt, bs="re") +
                         s(subj, grv, bs="re") +
                         s(subj, obj, bs="re") +
                         s(subj, orn, bs="re") +
                         s(subj, spt_orn, bs="re") +
                         s(Trial, by=Int)+
                         s(Soa),
                    data=KKL, method="fREML", discrete=TRUE)
```

Model comparison was carried out with compareML from the itsadug package.

```
compareML(kkl.ref.gam, kkl.lin.gam)$table
       Model
                 Score Edf
                             Chisq Df p.value Sig.
1 kkl.ref.gam -12495.77 27
2 kkl.lin.gam -13422.25 34 926.484 7.000 < 2e-16 ***
compareML(kkl.lin.gam, kkl.fs.gam)$table
                Score Edf Difference Df p.value Sign.
       Model
1 kkl.lin.gam -13422.25 34
2 kkl.fs.gam -14500.08 31 -1077.822 3.000
                                              <NA>
compareML(kkl.fs.gam, kkl.trial.gam)$table
                   Score Edf Chisq Df p.value Sig.
         Model
  kkl.fs.gam -14500.08 31
2 kkl.trial.gam -14582.64 33 82.565 2.000 < 2e-16 ***
AIC(kkl.ref.gam)
[1] -26009.55
AIC(kkl.lin.gam)
[1] -28047.29
AIC(kkl.fs.gam)
[1] -30876.72
AIC(kkl.trial.gam)
[1] -31040.29
```

These stats are brought together in Table 2.

4 The baldey dataset

4.1 Model and code for Table 5

For the baldey dataset, we restricted ourselves to correct responses to words.

A transform -1000/RT for the reaction times brings a distribution with a strong rightward skew closer to normality. Outlier reaction times (less than 600 ms) are removed.

```
baldey = baldey[baldey$RT > 600,] # 0.4% data loss
baldey$RTinv = -1000/baldey$RT
```

We add information about the initial trials for the subjects, and then scale trial.

```
mintrials = tapply(baldey$trial, baldey$subject, min)
baldey$MinTrial = mintrials[as.character(baldey$subject)]
baldey$AR.start = baldey$MinTrial==baldey$trial
```

We log-transform lemma frequency to avoid outlier effects of high-frequency 'outliers'.

```
baldey$LemmaFreq = log(baldey$CELEX_lemma_freq+1)
```

After scaling predictors,

```
baldey$LemmaFreq = log(baldey$CELEX_lemma_freq+1)
baldey$LemmaFreqSc = as.numeric(scale(baldey$LemmaFreq))
baldey$WordDurSc = as.numeric(scale(baldey$word_duration))
baldey$TrialSc = as.numeric(scale(baldey$trial))
```

we fit a GAMM to the data.

The subtable of the parametric part of the model (see Table 5):

```
summary(baldey.gam)$p.table
```

```
Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.0381929 0.06834953 -15.189467 5.555810e-52
gendermale 0.3732205 0.08909260 4.189131 2.805468e-05
```

The subtable of the smooths of the model (also in Table 5):

summary(baldey.gam)\$s.table

```
edf Ref.df F p-value
s(LemmaFreq):genderfemale
                          3.012133
                                     3.430126 5.511694e+01 1.612847e-38
s(LemmaFreq):gendermale 1.014891
                                     1.021960 1.247278e+02 3.398336e-29
te(word_duration,Trial)
                          8.705616 10.406449 2.271702e+01 4.892898e-44
s(word)
                      1735.807042 2777.000000 2.015884e+00 0.000000e+00
s(gender,word)
                        495.787366 5544.000000 1.108202e-01 1.967590e-07
s(word_duration,subject)
                        18.744049 19.000000 7.988325e+01 1.592307e-293
s(session, subject)
                        154.417238 178.000000 1.263994e+05 0.000000e+00
```

The fREML score,

baldey.gam\$gcv.ubre

```
fREML
-14903.7
```

and the AIC:

AIC(baldey.gam)

[1] -31728.01

4.2 Table 6: the model with linear effects only

The GAMM with linear effects only (Table 6).

The subtable of the parametric part of the model (Table 6).

```
summary(baldey.lin.gam)$p.table
```

```
Estimate
                                    Std. Error
                                                  t value
                                                               Pr(>|t|)
(Intercept)
                      -0.942629376 0.0406017546 -23.216469 1.485951e-118
gendermale
                       0.237397743 0.0574021321 4.135696 3.545059e-05
LemmaFreqSc
                      -0.020886829 0.0016662431 -12.535283 5.478266e-36
WordDurSc
                       0.061461226 0.0070232829 8.751068 2.183294e-18
                       0.006232360 0.0008371858
TrialSc
                                                 7.444417 9.906287e-14
gendermale:LemmaFreqSc 0.002426149 0.0017620096 1.376922 1.685431e-01
WordDurSc:TrialSc
                   -0.005362875 0.0008329795 -6.438183 1.220947e-10
```

The subtable of the smooths of the model (Table 6).

```
summary(baldey.lin.gam)$s.table
```

```
edf Ref.df F p-value
s(word) 1668.12107 2777 2.790862e+00 2.023020e-214
s(gender,word) 451.76914 5544 1.021730e-01 4.817559e-09
s(subject) 17.93030 18 9.009013e+05 0.000000e+00
s(WordDurSc,subject) 18.73591 19 1.213828e+02 6.843180e-178
s(session,subject) 19.89525 20 7.170278e+05 0.000000e+00
```

The fREML and AIC scores:

```
baldey.lin.gam$gcv.ubre
```

```
fREML
-13027.88
```

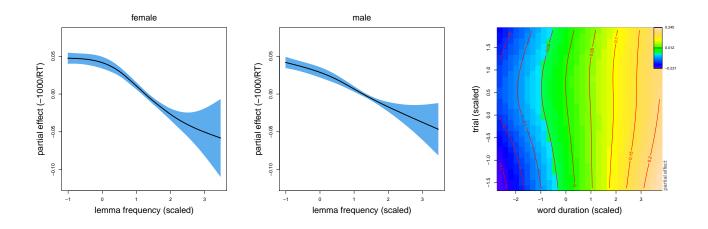
```
AIC(baldey.lin.gam)
```

```
[1] -27582.51
```

Very similar results are obtained with the LMM:

```
baldey.lmer <- lmer(RTinv ~ gender +
                        LemmaFreqSc*gender+
                        WordDurSc*TrialSc +
                        (1+gender|word) +
                        (1+WordDurSc+session|subject),
                        data=baldey)
print(summary(baldey.lmer),corr=F)
Linear mixed model fit by REML ['lmerMod']
Formula: RTinv ~ gender + LemmaFreqSc * gender + WordDurSc * TrialSc +
   (1 + gender | word) + (1 + WordDurSc + session | subject)
  Data: baldey
REML criterion at convergence: -26072.5
Scaled residuals:
   Min 1Q Median 3Q
-5.3489 -0.5975 -0.0130 0.6110 4.3265
Random effects:
Groups Name
                Variance Std.Dev. Corr
        (Intercept) 0.0034549 0.05878
word
         gendermale 0.0009739 0.03121 -0.13
subject (Intercept) 0.0254792 0.15962
         WordDurSc 0.0009483 0.03079 -0.82
         session 0.0003140 0.01772 -0.22 0.06
Residual
                   0.0316786 0.17798
Number of obs: 48385, groups: word, 2780; subject, 20
Fixed effects:
                      Estimate Std. Error t value
(Intercept)
                     -0.8558065 0.0402314 -21.272
gendermale
                     0.0479604 0.0401551 1.194
LemmaFreqSc
                    -0.0208485 0.0016369 -12.737
WordDurSc
                     0.0619134 0.0070152 8.826
TrialSc
                      0.0062278 0.0008371 7.440
gendermale:LemmaFreqSc 0.0023540 0.0017618 1.336
WordDurSc:TrialSc -0.0053851 0.0008328 -6.466
compareML(baldey.gam, baldey.lin)
      Model Score Edf Chisq Df p.value Sig.
1 baldey.lin -13027.88 12
```

4.3 Figure 14



Coefficient of variation for RTs and error rate.

```
library(RePsychLing)
data(baldey)
# analysis 1: error proportions and cv calculated for RTs exceeding 600 ms
baldey = baldey[baldey$RT > 600,]
baldey$RTinv=-1000/baldey$RT
incor = tapply(baldey$response=="incorrect", baldey$subject, sum)
corr = tapply(baldey$response=="correct", baldey$subject, sum)
pincor = incor/(incor+corr)
sds = tapply(baldey$RTinv, baldey$subject, sd)
means = tapply(baldey$RTinv, baldey$subject, mean)
cvs = sds/means
cor.test(pincor, cvs)
Pearson's product-moment correlation
data: pincor and cvs
t = -2.2054, df = 18, p-value = 0.04067
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
-0.75055279 -0.02349328
sample estimates:
       cor
-0.4612191
```

```
# analysis 2: error proportions calculated across all observations,
# whereas cv is calculated over data points with RTs exceeding 600 ms
data(baldey)
incor = tapply(baldey$response=="incorrect", baldey$subject, sum)
corr = tapply(baldey$response=="correct", baldey$subject, sum)
pincor = incor/(incor+corr)
baldey = baldey[baldey$RT > 600,]
baldey$RTinv=-1000/baldey$RT
sds = tapply(baldey$RTinv, baldey$subject, sd)
means = tapply(baldey$RTinv, baldey$subject, mean)
cvs = sds/means
cor.test(pincor, cvs)
Pearson's product-moment correlation
data: pincor and cvs
t = -3.5808, df = 18, p-value = 0.002136
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
-0.8460326 -0.2833349
sample estimates:
       cor
-0.6449863
```

5 The poems dataset

5.1 The full GAMM

```
data(poems) # available in RePsychLing, which was required above
# log-transform and scale frequency
poems$Fre = scale(log(poems$WordFormFrequency+1))
# log-transform reading time
poems$Lrt = log(poems$ReadingTime)
# scale frequency
poems$FreSc = as.numeric(scale(poems$Fre))
# scale trial
poems$TrialSc = as.numeric(scale(poems$Trial))
# order by subject and trial, and mark initial trial as the start
# of a new time series; this is necessary for
poems = poems[order(poems$Subject, poems$Trial),]
pos = tapply(poems$Trial, poems$Subject, min)
poems$MinTrial = pos[as.character(poems$Subject)]
poems$Start = poems$MinTrial==poems$Trial
```

A model with a tensor product smooth of frequency by trial. (Fitting this model takes about 30 minutes, and producing the summary takes another 30 minutes.) This model is summarized in Table 7.

```
Family: gaussian
Link function: identity
Formula:
Lrt ~ te(Fre, TrialSc) + s(Poem, bs = "re") + s(TrialSc, Subject,
   bs = "fs", m = 1) + s(Subject, Fre, bs = "re")
Parametric coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 6.04636 0.01739 347.7 <2e-16
Approximate significance of smooth terms:
                      edf Ref.df
                                 F p-value
te(Fre,TrialSc)
                    10.29 11.4 80.59 <2e-16
                    81.19 87.0 19.45 <2e-16
s(TrialSc,Subject) 2163.53 2934.0 329.42 <2e-16
s(Fre,Subject)
                   304.11 326.0 14.88 <2e-16
R-sq.(adj) = 0.498 Deviance explained = 50.3%
fREML = 49642 Scale est. = 0.08918 n = 275996
```

5.2 A decompositional GAMM

A decompositional model with main effects for frequency and trial and a 'tinsor' product smooth for the interaction of frequency by trial (summarized in Table 8).

```
Family: gaussian
Link function: identity
Formula:
Lrt ~ ti(Fre) + ti(TrialSc) + ti(Fre, TrialSc) + s(Poem, bs = "re") +
    s(TrialSc, Subject, bs = "fs", m = 1) + s(Subject, Fre, bs = "re")
Parametric coefficients:
           Estimate Std. Error t value Pr(>|t|)
                      0.0174 347.4 <2e-16
(Intercept) 6.0461
Approximate significance of smooth terms:
                      edf Ref.df
                                       F p-value
                    1.566 1.880 247.865 <2e-16
ti(Fre)
ti(TrialSc)
                            3.915 90.924 <2e-16
                    3.899
ti(Fre,TrialSc)
                   8.051 10.187 9.931 <2e-16
s(Poem)
                  81.180 87.000 19.405 <2e-16
s(TrialSc,Subject) 2163.630 2934.000 323.821 <2e-16
s(Fre,Subject)
               304.082 326.000 14.874 <2e-16
R-sq.(adj) = 0.498 Deviance explained = 50.3%
fREML = 49636 Scale est. = 0.089175 n = 275996
```

5.3 A model with linear effects only

Table 9 summarizes a LMM for the poems data.

```
poems.lmer = lmer(Lrt ~ FreSc * TrialSc +
                      (1|Poem) +
                      (1+FreSc+TrialSc|Subject),
                      data = poems, REML=TRUE)
print(summary(poems.lmer),corr=FALSE)
Linear mixed model fit by REML ['lmerMod']
Formula: Lrt ~ FreSc * TrialSc + (1 | Poem) + (1 + FreSc + TrialSc | Subject)
  Data: poems
REML criterion at convergence: 150493.3
Scaled residuals:
  Min 1Q Median 3Q
                                 Max
-5.9258 -0.6089 -0.1266 0.4605 5.6861
Random effects:
 Groups Name
                   Variance Std.Dev. Corr
 Subject (Intercept) 0.059568 0.24407
         FreSc 0.001176 0.03430 -0.61
         TrialSc 0.009619 0.09808 0.01 0.04
         (Intercept) 0.002534 0.05034
 Poem
 Residual
                    0.099396 0.31527
Number of obs: 275996, groups: Subject, 326; Poem, 87
Fixed effects:
              Estimate Std. Error t value
(Intercept) 6.0389852 0.0145774 414.3
FreSc
            -0.0525598 0.0019991 -26.3
TrialSc
             -0.0782900 0.0054865 -14.3
FreSc:TrialSc 0.0038955 0.0006195 6.3
```

Informal comparison of poems.gam and poems.lmer:

```
# AIC scores
AIC(poems.gam)

[1] 92688.55

AIC(poems.lmer)

[1] 150517.3

# REML scores
poems.gam$gcv.ubre

fREML
49642.31

# 150493.3 for lmer, see summary above
cor(fitted(poems.gam), poems$Lrt)^2

[1] 0.5030827

cor(fitted(poems.lmer), poems$Lrt)^2

[1] 0.4292628
```

5.4 Figure 15

```
# an ancillary function for acfs with lattice
acfs = function(dat=poems, model = poems.gam) {
  dat$Resid = resid_gam(model,AR_start=poems$Start,incl_na=TRUE)
  res=list()
  for (i in 1:nlevels(dat$Subject)) {
    tmp = dat[dat$Subject==levels(dat$Subject)[i],]
    acfmod = acf(tmp$Resid, na.action=na.pass, plot=F)
    acfobs = acf(tmp$Lrt, plot=F)
   res[[i]] = data.frame(lag
                                 = acfmod$lag,
                          acf
                                 = acfmod$acf,
                          acfObs = acfobs\$acf)
   res[[i]]$subj = levels(dat$Subject)[i]
   res[[i]]$n.used.mod = acfmod$n.used
    res[[i]]$n.used.obs = acfobs$n.used
   res[[i]]$ci95obs = qnorm((1+0.95)/2)/sqrt(acfobs$n.used)
    res[[i]]$ci95mod = qnorm((1+0.95)/2)/sqrt(acfmod$n.used)
  res = do.call(rbind, res)
  return(res)
acfdat = acfs()
```

```
load("models/acfdat.poems.rda")
```

The top panel of Figure 15 is produced by the following code snippet.

```
selectedSubjects = c("Subj265", "Subj176", "Subj19", "Subj183")
tmp = acfdat[acfdat$subj %in% selectedSubjects,]
tmp$subj = ordered(tmp$subj, selectedSubjects)
ci = unname(tapply(tmp$ci95obs, tmp$subj, mean))
theplotObs = xyplot(acfObs ~ lag | subj, type = "h", dat = tmp, ylim=c(0,1),
    panel = function(x, y, subscripts, ...) {
      panel.abline(h = ci[panel.number()], col.line = "grey")
     panel.abline(h = -ci[panel.number()], col.line = "grey")
      panel.abline(h = 0, col.line = "black")
      d=tmp[subscripts,]
      panel.xyplot(d$lag, d$acf0bs,
        col=ifelse(abs(d$acfObs)> ci[panel.number()] & d$lag > 0,
                   "indianred", "steelblue2"),
                   ...)
    xlab="lag", ylab="autocorrelation",
    layout=c(4,1,1)
print(theplotObs) # show plot
```

The center panel of Figure 15 has a smooth based on a simple GAM with only a factor smooth:

```
Lrt ~ s(Trial, Subject, bs="fs", m=1)
```

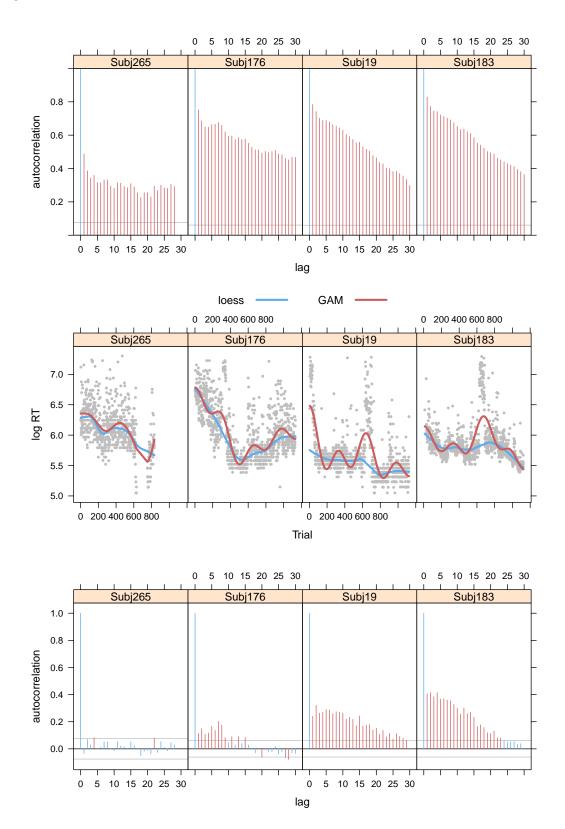
The following code produces the center panel:

```
selectedSubjects = c("Subj265", "Subj176", "Subj19", "Subj183")
dfr = droplevels(poems[poems$Subject %in% selectedSubjects,])
dfr$Subject = ordered(dfr$Subject, selectedSubjects)
dfr.gam = bam(Lrt~1+s(Trial, Subject, bs="fs", m=1), data=dfr, method="fREML")
dfr$Gam = predict(dfr.gam)
keys = list(space="top", text=list(c("loess", "GAM")), columns=2,
           lines=list(lty=c(1,1), col=c("steelblue2", "indianred"), lwd=3))
xyplot(Lrt ~ Trial | Subject, data = dfr,
 panel = function(x, y, subscripts, ...) {
    panel.xyplot(x, y, pch=19, col="gray", cex=0.4, ...)
   panel.loess(x, y, lwd=3, span=0.3, col="steelblue2", ...)
    d = dfr[subscripts,]
   panel.lines(d$Trial, d$Gam, col="indianred", lwd=3)
  xlab="Trial", ylab="log RT",
  layout=c(4,1,1),
 key=keys)
```

The bottom panel of Figure 15 is obtained in a similar way, but now using the model predictions.

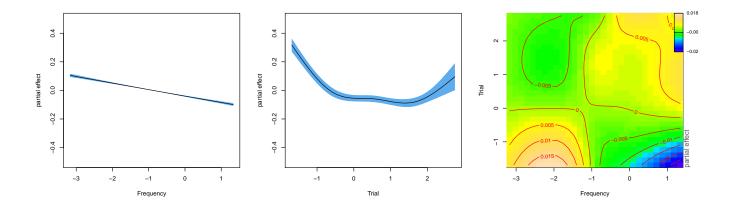
```
tmp = acfdat[acfdat$subj %in% selectedSubjects,]
tmp$subj = ordered(tmp$subj, selectedSubjects)
ci = unname(tapply(tmp$ci95mod, tmp$subj, mean))
theplot = xyplot(acf ~ lag | subj, type = "h", dat = tmp,
    panel = function(x, y, subscripts, ...) {
      panel.abline(h = ci[panel.number()], col.line = "grey")
      panel.abline(h = -ci[panel.number()], col.line = "grey")
     panel.abline(h = 0, col.line = "black")
      d=tmp[subscripts,]
     panel.xyplot(d$lag, d$acf,
        col=ifelse(abs(d$acf)> ci[panel.number()] & d$lag > 0,
                   "indianred", "steelblue2"),
    },
    xlab="lag", ylab="autocorrelation",
    layout=c(4,1,1)
print(theplot)
```

Figure 15.



5.5 Figure 16

This takes about 10 minutes to complete.



5.6 Figure 17 and corresponding LMMs

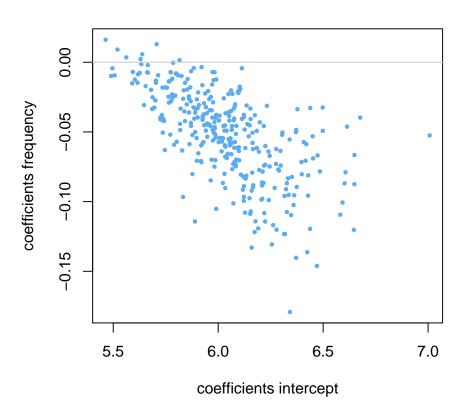
First, we fit a sequence of models with more complex by-subject random effects structure, and show that by-subject random slopes for frequency receive good support.

```
# by-subject random intercepts
poems.lmer1 = lmer(Lrt ~ FreSc * TrialSc +
                        (1|Poem) +
                        (1|Subject),
                        data = poems, REML=FALSE)
# by-subject random intercepts and by-subject random slopes for trial
poems.lmer2 = lmer(Lrt ~ FreSc * TrialSc +
                        (1|Poem) +
                        (1+TrialSc|Subject),
                        data = poems, REML=FALSE)
# by-subject random intercepts and by-subject random slopes for trial
# and in addition by-subject random slopes for frequency
poems.lmer3 = lmer(Lrt ~ FreSc * TrialSc +
                        (1|Poem) +
                        (1+FreSc+TrialSc|Subject),
                        data = poems, REML=FALSE)
# a simpler model without correlation parameters for frequency
poems.lmer4 = lmer(Lrt ~ FreSc * TrialSc +
                        (1 | Poem) +
                        (1+TrialSc|Subject) +
                        (0+FreSc|Subject),
                        data = poems, REML=FALSE)
```

```
# compare the first three models
anova(poems.lmer1, poems.lmer2, poems.lmer3)
Data: poems
Models:
poems.lmer1: Lrt ~ FreSc * TrialSc + (1 | Poem) + (1 | Subject)
poems.lmer2: Lrt ~ FreSc * TrialSc + (1 | Poem) + (1 + TrialSc | Subject)
poems.lmer3: Lrt ~ FreSc * TrialSc + (1 | Poem) + (1 + FreSc + TrialSc | Subject)
                AIC
                      BIC logLik deviance
                                             Chisq Chi Df Pr(>Chisq)
           Df
poems.lmer1 7 172200 172274 -86093 172186
poems.lmer2 9 152934 153029 -76458 152916 19270.2
                                                        2 < 2.2e-16
poems.lmer3 12 150478 150605 -75227 150454 2461.7
                                                      3 < 2.2e-16
```

```
# and test for the correlation parameters for frequency
anova(poems.lmer4, poems.lmer3)
Data: poems
Models:
poems.lmer4: Lrt ~ FreSc * TrialSc + (1 | Poem) + (1 + TrialSc | Subject) +
poems.lmer4: (0 + FreSc | Subject)
poems.lmer3: Lrt ~ FreSc * TrialSc + (1 | Poem) + (1 + FreSc + TrialSc | Subject)
          Df AIC BIC logLik deviance Chisq Chi Df Pr(>Chisq)
poems.lmer4 10 150608 150713 -75294 150588
poems.lmer3 12 150478 150605 -75227 150454 133.66
                                                  2 < 2.2e-16
# note that the summary of poems.lmer3 indicates the frequency
# correlational structure is carried almost exclusively by
# the frequency by intercept correlation
print(summary(poems.lmer3),corr=FALSE)
Linear mixed model fit by maximum likelihood ['lmerMod']
Formula: Lrt ~ FreSc * TrialSc + (1 | Poem) + (1 + FreSc + TrialSc | Subject)
  Data: poems
             BIC logLik deviance df.resid
    AIC
150478.3 150604.6 -75227.1 150454.3 275984
Scaled residuals:
   Min 1Q Median
                          3Q
-5.9261 -0.6089 -0.1266 0.4605 5.6860
Random effects:
Groups Name
                 Variance Std.Dev. Corr
Subject (Intercept) 0.059408 0.24374
         FreSc
                   0.001172 0.03424 -0.61
         TrialSc
                    0.009589 0.09792 0.01 0.04
Poem
         (Intercept) 0.002528 0.05028
Residual
                    0.099396 0.31527
Number of obs: 275996, groups: Subject, 326; Poem, 87
Fixed effects:
              Estimate Std. Error t value
(Intercept) 6.0389850 0.0145582 414.8
FreSc
             -0.0525595 0.0019960
                                   -26.3
TrialSc
             -0.0782901 0.0054780 -14.3
FreSc:TrialSc 0.0038961 0.0006195 6.3
```

Figure 17 is obtained with the following code, extracting the coefficients from poems.lmer3:



5.7 Figure 18

