

Verifiable C

*Applying the Verified Software Toolchain
to C programs*

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1 Overview

Verifiable C is a language and program logic for reasoning about the functional correctness of C programs. The *language* is a subset of CompCert C light; it is a dialect of C in which side-effects and loads have been factored out of expressions. The *program logic* is a higher-order separation logic, a kind of Hoare logic with better support for reasoning about pointer data structures, function pointers, and data abstraction.

Verifiable C is *foundationally sound*. That is, it is proved (with a machine-checked proof in the Coq proof assistant) that,

Whatever observable property about a C program you prove using the Verifiable C program logic, that property will actually hold on the assembly-language program that comes out of the C compiler.

This soundness proof comes in two parts: The program logic is proved sound with respect to the semantics of CompCert C, by a team of researchers primarily at Princeton University; and the C compiler is proved correct with respect to those same semantics, by a team of researchers primarily at INRIA. This chain of proofs from top to bottom, connected in Coq at specification interfaces, is part of the *Verified Software Toolchain*.



To use Verifiable C, one must have had some experience using Coq, and some familiarity with the basic principles of Hoare logic. These can be obtained by studying Pierce’s *Software Foundations* interactive textbook, and doing the exercises all the way to chapter “Hoare2.”

It is also useful to read the brief introductions to Hoare Logic and Separation Logic, covered in Appel’s *Program Logics for Certified Compilers*, [Chapters 2 and 3](#).

PROGRAM LOGICS FOR CERTIFIED COMPILERS (Cambridge University Press, 2014) describes *Verifiable C* version 1.1. If you are interested in the semantic model, soundness proof, or memory model of VST, the book is well worth reading. But it is not a reference manual.

More recent VST versions differ in several ways from what the PLCC book describes.

- In the LOCAL component of an assertion, one writes `temp i v` instead of ``(eq v) (eval_id i)`.
- In the SEP component of an assertion, backticks are not used (predicates are not lifted).
- In general, the backtick notation is rarely needed.
- The type-checker now has a more refined view of char and short types.
- `field_mapsto` is now called `field_at`, and it is dependently typed.
- `typed_mapsto` is renamed to `data_at`, and last two arguments are swapped.
- `umapsto` (“untyped mapsto”) no longer exists.
- `mapsto sh t v w` now permits either ($w = \text{Vundef}$) or the value w belongs to type t . This permits describing uninitialized locations, i.e., `mapsto_sh t v = mapsto_sh t v Vundef`. For function calls, one uses `forward.call` instead of `forward`.
- C functions may fall through the end of the function body, and this is (per the C semantics) equivalent to a `return; statement`.

2 Installation

The Verified Software Toolchain runs on Linux, Mac, or Windows. You will need to install:

1. Coq 8.5pl2, from coq.inria.fr. Follow the standard installation instructions.
2. CompCert 2.7.1, from compcert.inria.fr. You will want to build the *clightgen* tool, using these commands: `./configure ia32-linux`; `make clightgen`. You might replace `ia32-linux` with `ia32-macosx` or `ia32-cygwin`. Verifiable C should work on other 32-bit architectures as well, but has not been extensively tested.
3. VST 1.7, from vst.cs.princeton.edu, or else an appropriate version from <https://github.com/ucsb-cs262>. After unpacking, read the `BUILD_ORGANIZATION` file (or simply `make -j`).

WORKFLOW. Within `vst`, the `progs` directory contains some sample C programs with their verifications. The workflow is:

- Write a C program *F.c*.
- Run `clightgen F.c` to translate it into a Coq file *F.v*.
- Write a verification of *F.v* in a file such as `verif_F.v`. That latter file must import both *F.v* and the VST *Floyd*¹ program verification system, `floyd.proofauto`.

LOAD PATHS. Interactive development environments (CoqIDE or Proof General) will need their load paths properly initialized through command-line arguments. Running `make` in `vst` creates a file `.loadpath` with the right arguments. You can then do (for example),

```
coqide `cat .loadpath` progs/verif_reverse.v
```

See the heading `USING PROOF GENERAL AND COQIDE` in the file `BUILD_ORGANIZATION` for more information.

¹Named after Robert W. Floyd (1936–2001), a pioneer in program verification.

3 Verifiable C programming

Chapter 22

⁸
See PLCC

Verifiable C is a *language* (subset of C) and a *program logic* (higher-order impredicative concurrent separation logic).

In writing Verifiable C programs you must:

- Make each memory dereference into a top level expression (PLCC page 143)
- Avoid casting between integers and pointers.
- Avoid goto and switch statements.
- * Avoid nesting function calls and assignments inside subexpressions.
- * Factor && and || operators into if statements (to capture short circuiting behavior).

The items marked * are accomplished automatically by CompCert's clightgen tool. That is, if you have function calls or assignments inside expressions, clightgen will factor the your program adding extra assignments to temporary variables.

There's a special treatment of malloc/free; see [Chapter 47](#).

4 *Clightgen and ASTs*

We will introduce Verifiable C by explaining the proof of a simple C program: adding up the elements of an array.

```
#include <stddef.h>
```

```
int sumarray(int a[], int n) {
  int i,s,x;
  i=0;
  s=0;
  while (i<n) {
    x=a[i];
    s+=x;
    i++;
  }
  return s;
}
```

```
int four[4] = {1,2,3,4};
```

```
int main(void) {
  int s;
  s = sumarray(four,4);
  return s;
}
```

You can examine this program in `VST/progs/sumarray.c`. Then look at `progs/sumarray.v` to find the output of CompCert's *clightgen* utility: it is the abstract syntax tree (AST) of the C program, expressed in Coq. In `sumarray.v` there are definitions such as,

...

Definition `_main` : ident := 54%positive.

...

Definition `_s` : ident := 50%positive.

...

Definition $f_sumarray := \{ |$ $fn_return := tint; \dots$ $fn_params := ((_a, (tptr\ tint)) :: (_n, tint) :: nil);$ $fn_temps := ((_i, tint) :: (_s, tint) :: (_x, tint) :: nil);$ $fn_body :=$ $(Ssequence$ $(Sset\ _i\ (Econst_int\ (Int.repr\ 0)\ tint))$ $(Ssequence$ $(Sset\ _s\ (Econst_int\ (Int.repr\ 0)\ tint))$ $(Ssequence\ \dots$ $)))$ $| \}$

...

Definition $prog : Clight.program := \{ | \dots | \}$

In general it's never necessary to read the AST file such as `sumarray.v`. But it's useful to know what kind of thing is in there. C-language identifiers such as `main` and `s` are represented in ASTs as positive numbers; the definitions `_main` and `_s` are abbreviations for these. The AST for `sumarray` is in the function-definition `f_sumarray`.

There you can see that `sumarray`'s return type is `int`. To represent the syntax of C type-expressions, CompCert defines,

Inductive $type : Type :=$ $| Tvoid: type$ $| Tint: intsize \rightarrow signedness \rightarrow attr \rightarrow type$ $| Tpointer: type \rightarrow attr \rightarrow type$ $| Tstruct: ident \rightarrow attr \rightarrow type$ $| \dots$

and we abbreviate $tint := Tint\ l32\ Signed\ noattr$.

5 *Use the IDE*

[Chapter 6](#) through [Chapter 16](#) are meant to be read while you have the file `progs/verif_sumarray.v` open in a window of your interactive development environment for Coq. You can use Proof General, CoqIDE, or any other IDE that supports Coq.

Reading these chapters will be much less informative if you cannot see the proof state as each chapter discusses it.

Before starting the IDE, read about load paths, at the heading `USING PROOF GENERAL AND COQIDE` in the file `VST/BUILD_ORGANIZATION`.

6 *Functional spec, API spec*

A program without a specification cannot be incorrect, it can only be surprising.
(Paraphrase of J. J. Horning, 1982)

The file `progs/verif_sumarray.v` contains the specification of `sumarray.c`, and the proof of correctness of the C program with respect to that specification. For larger programs, one would typically break this down into three or more files:

1. Functional specification
2. API specification
3. Function-body correctness proofs, one per file.

To prove correctness of `sumarray.c`, we start by writing a *functional spec* of adding-up-a-sequence, then an *API spec* of adding-up-an-array-in-C.

FUNCTIONAL SPEC. A *mathematical model* of this program is the sum of a sequence of integers: $\sum_{i=0}^{n-1} x_i$. It's conventional in Coq to use `list` to represent a sequence; we can represent the sum with a list-fold:

Definition `sum_Z` : `list Z` → `Z` := `fold_right Z.add 0`.

A functional spec contains not only definitions; it's also useful to include theorems about this mathematical domain:

Lemma `sum_Z_app`: $\forall a\ b, \text{sum_Z } (a++b) = \text{sum_Z } a + \text{sum_Z } b$.

Proof.

intros. induction a; simpl; omega.

Qed.

The data types used in a functional spec can be any kind of mathematics at all, as long as we have a way to relate them to the integers, tuples, and sequences used in a C program. But the mathematical integers `Z` and the 32-bit modular integers `Int.int` are often relevant. Notice that this functional spec does not depend on `sumarray.v` or even on anything in the

Verifiable C libraries. This is typical, and desirable: the functional spec is about mathematics, not about C programming.

THE APPLICATION PROGRAMMER INTERFACE of a C program is expressed in its header file: function prototypes and data-structure definitions that explain how to call upon the modules' functionality. In *Verifiable C*, an *API specification* is written as a series of *function specifications* (funspecs) corresponding to the function prototypes.

We start `verif_sumarray.v` with some standard boilerplate:

Require Import floyd.proofauto.

Require Import progs.sumarray.

Instance CompSpecs : compspecs. make_compspecs prog. **Defined.**

Definition Vprog : varspecs. mk_varspecs prog. **Defined.**

The first line imports Verifiable C and its *Floyd* proof-automation library. The second line imports the AST of the program to be proved. Lines 3 and 4 are identical in any verification: see [Chapter 23](#) and [Chapter 42](#).

After the boilerplate (and the functional spec), we have the function specifications for each function in the API spec:

Definition sumarray_spec :=

DECLARE _sumarray

WITH a: val, sh : share, contents : list Z, size: Z

PRE [_a OF (tptr tint), _n OF tint]

PROP(readable_share sh;

0 ≤ size ≤ Int.max_signed;

Forall (fun x ⇒ Int.min_signed ≤ x ≤ Int.max_signed) contents)

LOCAL(temp _a a; temp _n (Vint (Int.repr size)))

SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a)

POST [tint]

PROP()

LOCAL(temp ret_temp (Vint (Int.repr (sum_Z contents))))

SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a).

The funspec begins, **Definition** $f_spec := \text{DECLARE } id_f \dots$ where f is the name of the C function.

A function is specified by its *precondition* and its *postcondition*. The **WITH** clause quantifies over Coq values that may appear in both the precondition and the postcondition. The precondition is parameterized by the C-language function parameters, and the postcondition is parameterized by a identifier `ret_temp`, which is short for, “the temporary variable holding the return value.” But really, the Coq variable `_a` does not have type (pointer-to-int); it has type `ident` (see [page 9](#)).

An assertion in Verifiable C’s *separation logic* can be written at either of two levels: The *lifted level*, implicitly quantifying over all local-variable states; or the *base level*, at a particular local-variable state. Program assertions are written at the lifted level, for which the notation is `PROP(...) LOCAL(...) SEP(...)`.

In an assertion `PROP(\vec{P}) LOCAL(\vec{Q}) SEP(\vec{R})`, the propositions in the sequence \vec{P} are all of Coq type `Prop`. They describe things that are forever true, independent of program state. Of course, in the function precondition above, the statement $0 \leq \text{size} \leq \text{Int.max_signed}$ is “forever” true *just within the scope of the quantification of the variable size*; it is bound by **WITH** and spans the **PRE** and **POST** assertions.

The **LOCAL** propositions \vec{Q} are *variable bindings* of type `localdef`. Here, the function-parameters a and n are treated as nonaddressable local variables, or “temp” variables. The `localdef (temp _a a)` says that (in this program state) the contents of C local variable `_a` is the Coq value a . In general, the contents of a C scalar variable is always a `val`; this type is defined by CompCert as,

Inductive `val`: `Type := Vundef: val | Vint: int → val | Vlong: int64 → val`
`| Vfloat: float → val | Vsingle: float32 → val | Vptr: block → int → val.`

The **SEP** conjuncts \vec{R} are *spatial assertions* in separation logic. In this

case, there's just one, a `data_at` assertion saying that at address `a` in memory, there is a data structure of type *array[size] of integers*, with access-permission `sh`, and the contents of that array is the sequence map `Vint` contents.

THE POSTCONDITION is introduced by `POST [tint]`, indicating that this function returns a value of type `int`. There are no `PROP` statements in the postcondition, because no forever-true facts exist in the world that weren't already true on entry to the function. (This is typical!) The `LOCAL` *must not mention* the function parameters, because they are destroyed on function exit; it will only mention the return-temporary `ret_temp`. The `SEP` clause mentions all the spatial resources from the precondition, minus ones that have been freed (deallocated), plus ones that have been malloc'd (allocated).

So, overall, the specification for `sumarray` is this: “At any call to `sumarray`, there exist values *a, sh, contents, size* such that *sh* gives at least read-permission; *size* is representable as a nonnegative 32-bit signed integer; function-parameter `_a` contains value *a* and `_n` contains the 32-bit representation of *size*; and there's an array in memory at address *a* with permission *sh* containing *contents*. The function returns a value equal to `sum_int(contents)`, and leaves the array unaltered.”

INTEGER OVERFLOW. The C language specification says that a C compiler *may* treat signed integer overflow by wrapping around mod 2^n , where n is the word size (e.g., 32). In practice, almost all C compilers (including CompCert) do this wraparound, and it is part of the CompCert C light operational semantics. See [Chapter 20](#). The function `Int.repr`: $\mathbb{Z} \rightarrow \text{int}$ truncates mathematical integers into 32-bit integers by taking the (sign-extended) low-order 32 bits. `Int.signed`: $\text{int} \rightarrow \mathbb{Z}$ injects back into the signed integers.

The postcondition guarantees that the value return is `Int.repr (sum_Z contents)`. But what if $\sum s \geq 2^{31}$, so the sum doesn't fit in a 32-bit signed integer? Then `Int.signed(Int.repr (sum_Z contents)) \neq (sum_Z contents)`. In gen-

eral, for a claim about $\text{Int.repr}(x)$ to be *useful*, one also needs a claim that $0 \leq x \leq \text{Int.max_unsigned}$ or $\text{Int.min_signed} \leq x \leq \text{Int.max_signed}$. The caller of this function will probably need to prove $\text{Int.min_signed} \leq \text{sum_Z contents} \leq \text{Int.max_signed}$ in order to make much use of the post-condition.

What if s is the sequence $[\text{Int.max_signed}; 5; 1 - \text{Int.max_signed}]$? Then $\sum s = 6$. Does the program really work? Answer: Yes, by the miracle of modular arithmetic.

7 *Proof of the sumarray program*

To prove correctness of a whole program,

1. Collect the function-API specs together into Gprog: list funspec.
2. Prove that each function satisfies its own API spec (with a `semax_body` proof).
3. Tie everything together with a `semax_func` proof.

In `progs/verif_sumarray.v`, the first step is easy:

Definition `Gprog := ltac:(with_library prog [sumarray_spec; main_spec]).`

The function specs, built using `DECLARE`, are listed in the same order the functions appear in the program (in particular, the same order they appear in `prog.(prog_defs)`, in `sumarray.v`). [Chapter 57](#) describes `with_library`.

In addition to `Gprog`, the API spec contains `Vprog`, the list of global-variable type-specs. This is computed automatically by the `mk_varspecs` tactic, as shown at the beginning of `verif_sumarray.v`.

Each C function can call any of the other C functions in the API, so each `semax_body` proof is a client of the entire API spec, that is, `Vprog` and `Gprog`. You can see that in the statement of the `semax_body` lemma for the `_sumarray` function:

Lemma `body_sumarray: semax_body Vprog Gprog f_sumarray sumarray_spec.`

Here, `f_sumarray` is the actual function body (AST of the C code) as parsed by `clightgen`; you can read it in `sumarray.v`. You can read `body_sumarray` as saying, *In the context of `Vprog` and `Gprog`, the function body `f_sumarray` satisfies its specification `sumarray_spec`.* We need the context in case the `sumarray` function refers to a global variable (`Vprog` provides the variable's type) or calls a global function (`Gprog` provides the function's API spec).

8 start_function

The predicate `semax_body` states the Hoare triple of the function body, $\Delta \vdash \{Pre\}c\{Post\}$. *Pre* and *Post* are taken from the funspec for *f*, *c* is the body of *F*, and the type-context Δ is calculated from the global type-context overlaid with the parameter- and local-types of the function.

To prove this, we begin with the tactic `start_function`, which takes care of some simple bookkeeping and expresses the Hoare triple to be proved.

Lemma `body_sumarray`: `semax_body Vprog Gprog f_sumarray sumarray_spec`.

Proof.

`start_function`.

The proof goal now looks like this:

```

Espec : OracleKind
a : val
sh : share
contents : list Z
size : Z
Delta_specs := abbreviate : PTree.t funspec
Delta := abbreviate : tycontext
SH : readable_share sh
H : 0 ≤ size ≤ Int.max_signed
H0 : Forall (fun x : Z ⇒ Int.min_signed ≤ x ≤ Int.max_signed) contents
POSTCONDITION := abbreviate : ret_assert
MORE_COMMANDS := abbreviate : statement
----- (1/1)
semax Delta
  (PROP ()
    LOCAL(temp _a a; temp _n (Vint (Int.repr size)))
    SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a))
  (Ssequence (Sset _i (Econst.int (Int.repr 0) tint)) MORE_COMMANDS)
  POSTCONDITION

```

First we have *Espec*, which you can ignore for now (it characterizes the outside world, but `sumarray.c` does not do any I/O). Then `a,sh,contents,size` are exactly the variables of the `WITH` clause of `sumarray_spec`.

The two abbreviations `Delta_spec`, `Delta` are the type-context in which Floyd's proof tactics will look up information about the types of the program's variables and functions. The hypotheses `SH,H,H0` are exactly the `PROP` clause of `sumarray_spec`'s precondition. The `POSTCONDITION` is exactly the `POST` part of `sumarray_spec`.

To see the contents of an abbreviation, either (1) set your IDE to show implicit arguments, or (2) (e.g.,) unfold abbreviate in `POSTCONDITION`.

Below the line we have one proof goal: the Hoare triple of the function body. In this judgment $\Delta \vdash \{P\} c \{R\}$, written in Coq as `semax (Δ: tycontext) (P: environ → mpred) (c: statement) (R: ret_assert)`

Δ is a *type context*, giving types of function parameters, local variables, and global variables; and *specifications* (`funspec`) of global functions.
 P is the precondition;
 c is a command in the C language; and
 R is the postcondition. Because a c statement can exit in different ways (fall-through, continue, break, return), a `ret_assert` has predicates for all of these cases.

Because we do *forward* Hoare-logic proof, we won't care about the postcondition until we get to the end of c , so here we hide it away in an abbreviation. Here, the command c is a long sequence starting with `i=0;...more`, and we hide the *more* in an abbreviation `MORE_COMMANDS`.

The precondition of this `semax` has `LOCAL` and `SEP` parts taken directly from the `funspec` (the `PROP` clauses have been moved above the line). The statement `(Sset _i (Econst_int (Int.repr 0) tint))` is the AST generated by `clightgen` from the C statement `i=0;.`

9 forward

We do Hoare logic proof by forward symbolic execution. On [page 18](#) we show the proof goal at the beginning of the `sumarray` function body. In a forward Hoare logic proof of $\{P\} i = 0; \text{more} \{R\}$ we might first apply the sequence rule,

$$\frac{\{P\} i = 0 \{Q\} \quad \{Q\} \text{more} \{R\}}{\{P\} i = 0; \text{more} \{R\}}$$

assuming we could derive some appropriate assertion Q .

For many kinds of statements (assignments, return, break, continue) this is done automatically by the forward tactic. When we execute forward here, the resulting proof goal is,

Espec, a, sh, contents, size, Delta_spec, SH, H, H0 *as before*

Delta := abbreviate : tycontext

POSTCONDITION := abbreviate : ret_assert

MORE_COMMANDS := abbreviate : statement

----- (1/1)

semax Delta

(PROP ())

LOCAL(temp _i (Vint (Int.repr 0)); temp _a a;

temp _n (Vint (Int.repr size)))

SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a))

(Ssequence (Sset _s (Econst_int (Int.repr 0) tint)) MORE_COMMANDS)

POSTCONDITION

Notice that the precondition of this `semax` is really the *postcondition* of the `i=0;` statement; it is the precondition of the *next* statement, `s=0;`. It's much like the precondition of `i=0;` what has changed?

- The `LOCAL` part contains `temp _i (Vint (Int.repr 0))` in addition to what it had before; this says that the local variable `i` contains integer value zero.

- the command is now $s=0;more$, where `MORE_COMMANDS` no longer contains $s=0$;
- Delta has changed; it now records the information that i is initialized.

Another forward goes through $s=0$; to yield a proof goal with a `LOCAL` binding for the `_s` variable.

FORWARD WORKS ON SEVERAL KINDS OF C COMMANDS. In each of the following cases, the expression E must not contain side effects or function calls. The variable x must be a nonaddressable local variable.

$c_1; c_2$ Sequencing of two commands. The forward tactic will work on c_1 first.

$(c_1; c_2) c_3$ In this case, forward will re-associate the commands using the `seq_assoc` axiom, and work on $c_1; (c_2; c_3)$.

$x=E$; Assignment statement. Expression E must not contain memory dereferences (loads or stores using `*prefix`, `suffix[]`, or `->` operators). No restrictions on the form of the precondition (except that it must be in canonical form). The expression `&p->next` does not actually load or store (it just computes an address) and is permitted.

$x= *E$; Memory load.

$x= a[E]$; Array load.

$x= E \rightarrow fld$; Field load.

$x= E \rightarrow f_1.f_2$; Nested field load.

$x= E \rightarrow f_1[i].f_2$; Fields and subscripts ... When the right-hand side is equivalent to a single memory-load via some access *path* (struct-fields and array-subscripts) from pointer value p , the SEP component of the precondition must contain an appropriately typed item of the form `data_at π t v p` such that the *path* from p in an object of type t leads to a field (or array slot) that can be loaded into `_x`. Or, `field_at π t path' v p'`, such that where *path'* is a suffix of *path*, and p' is the address reached by starting at p and following the prefix. Share π must be a readable_share.

$E_1 = E_2$; Memory store. Expression E_2 must not dereference memory. Expression E_1 must be equivalent to a single memory store via some access *path* (as described above for loads), and there must be an appropriate storable `data.at` or `field.at`. Or E_1 may be an addressable local variable. Share π must be a `writable_share`.

if (E) C_1 else C_2 For an if-statement, use `forward_if` and provide a postcondition.

while (E) C For a while-loop, use the `forward_while` tactic ([page 23](#)) and provide a loop invariant.

break; The forward tactic works.

continue; The forward tactic works.

return E ; Expression E must not dereference memory, and the presence/absence of E must match the nonvoid/void return type of the function. The proof goal left by forward is to show that the precondition (with appropriate substitution for the abstract variable `ret_var`) entails the function's postcondition.

$x = f(a_1, \dots, a_n)$; For a function call, use `forward_call(W)`, where W is a witness, a tuple corresponding (componentwise) to the `WITH` clause of the function specification. (If you do just forward, you'll get a message with advice about the *type* of W .)

This results a proof goal to show that the precondition implies the function precondition and includes an uninstantiated variable: The Frame represents the part of the spacial precondition that is unchanged by the function call. It will generally be instantiated by a call to `cancel`.

10 While loops

To prove a *while* loop by forward symbolic execution, you use the tactic `forward_while`, and you must supply a loop invariant. Take the example of the `forward_while` in `progs/verif_sumarray.v`. The proof goal is,

```

Espec, Delta_specs, Delta
a : val, sh : share, contents : list Z, size : Z
SH : readable_share sh
H : 0 ≤ size ≤ Int.max_signed
H0 : Forall (fun x : Z ⇒ Int.min_signed ≤ x ≤ Int.max_signed) contents
POSTCONDITION := abbreviate : ret_assert
MORE_COMMANDS, LOOP_BODY := abbreviate : statement
-----(1/1)

```

```

semax Delta
(PROP ()
  LOCAL(temp _s (Vint (Int.repr 0)); temp _i (Vint (Int.repr 0));
    temp _a a; temp _n (Vint (Int.repr size)))
  SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a))
(Ssequence
  (Swhile (Ebinop Olt (Etempvar _i tint) (Etempvar _n tint) tint)
    LOOP_BODY)
  MORE_COMMANDS)
POSTCONDITION

```

A loop invariant is an assertion, almost always in the form of an existential `EX...PROP()LOCAL()SEP()`. Each iteration of the loop has a state characterized by a different value of some iteration variable(s), the `EX` binds that value. For example, the invariant for this loop is,

Definition `sumarray_Inv a0 sh contents size :=`

```

EX i: Z,
  PROP(0 ≤ i ≤ size)
  LOCAL(temp _a a0; temp _i (Vint (Int.repr i)); temp _n (Vint (Int.repr size));
    temp _s (Vint (Int.repr (sum_Z (sublist 0 i contents)))))
  SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a0).

```

The existential binds i , the iteration-dependent value of the local variable named $_i$. In general, there may be any number of EX quantifiers.

The `forward_while` tactic will generate four subgoals to be proven:

1. the precondition (of the whole loop) implies the loop invariant;
2. the loop-condition expression type-checks (i.e., guarantees to evaluate successfully);
3. the postcondition of the loop body implies the loop invariant;
4. the loop invariant (and *not* loop condition) is a good precondition for the proof of the `MORE_COMMANDS` after the loop.

Let's take a look at that first subgoal:

(above-the-line hypotheses elided) —1/4

```

ENTAIL Delta,
  PROP()
  LOCAL(temp _s (Vint (Int.repr 0)); temp _i (Vint (Int.repr 0));
        temp _a a; temp _n (Vint (Int.repr size)))
  SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a)
 $\vdash$  EX  $i : \mathbb{Z}$ ,
  PROP( $0 \leq i \leq \text{size}$ )
  LOCAL(temp _a a; temp _i (Vint (Int.repr i));
        temp _n (Vint (Int.repr size));
        temp _s (Vint (Int.repr (sum_Z (sublist 0 i contents)))))
  SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a)
  
```

This is an *entailment* goal; [Chapter 11](#) shows how to prove such goals.

11 Entailments

An *entailment* in separation logic, $P \vdash Q$, says that any state satisfying P must also satisfy Q . What’s in a state? Local-variable environment, heap (addressable memory), even the state of the outside world. VST’s type `mpred`, *memory predicate*, can be thought of as $\text{mem} \rightarrow \text{Prop}$ (but is not quite the same, for quite technical semantic reasons). That is, an `mpred` is a test on the heap only, and cannot “see” the local variables (tempvars) of the C program.

Type `environ` is a local/global variable environment, mapping identifiers (`ident`) to the values of globals, addressable locals, and tempvars (nonaddressable locals). A *lifted predicate* of type $\text{environ} \rightarrow \text{mpred}$ can “see” both the heap and the local/global variables. The Pre/Post arguments of Hoare triples (`semax` Δ Pre `c` Post) are lifted predicates.

At present, Verifiable C has a notion of external-world state, in the `Espec`: `OracleKind`, but it is not well developed; enhancements will be needed for reasoning about input/output.

Our language for lifted predicates uses $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})$, where \vec{R} is a list of `mpreds`. Our language for `mpreds` uses primitives such as `data_at` and `emp`, along with connectives such as the $*$ and $\neg*$ of separation logic. In both languages there is an `EX` operator for existential quantification.

Separation logic’s rule of consequence is shown here

$$\frac{P \vdash P' \quad \{P'\} c \{Q'\} \quad Q' \vdash Q}{\{P\} c \{Q\}} \quad \frac{\Delta, P \vdash P' \quad \text{semax } \Delta P' c Q' \quad \Delta, Q' \vdash Q}{\text{semax } \Delta P c Q}$$

at left in traditional notation, and at right as in Verifiable C. The type-context Δ constrains values of locals and globals. Using this axiom, called `semax_pre_post` on a proof goal `semax` $\Delta P c Q$ yields three subgoals: another `semax` and two (lifted) entailments, $\Delta, P \vdash P'$ and $\Delta, Q' \vdash Q'$.

The standard form of a lifted entailment is $\text{ENTAIL } \Delta, \text{PQR} \vdash \text{PQR}'$, where PQR and PQR' are typically in the form $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})$, perhaps with some EX quantifiers in the front. The turnstile \vdash is written in Coq as $|--$.

Let's consider the entailment arising from `forward_while` in the `progs/verif_sumarray` example:

$$\frac{\begin{array}{l} H : 0 \leq \text{size} \leq \text{Int.max_signed} \\ \text{(other above-the-line hypotheses elided)} \end{array}}{\text{ENTAIL } \Delta, \text{PROP}() \text{LOCAL}(\text{temp_s } (\text{Vint } (\text{Int.repr } 0)); \text{temp_i } (\text{Vint } (\text{Int.repr } 0)); \text{temp_a } a; \text{temp_n } (\text{Vint } (\text{Int.repr } \text{size}))) \text{SEP}(\text{data_at sh } (\text{tarray tint size}) (\text{map Vint } (\text{map Int.repr contents})) a) \vdash \text{EX } i : \mathbb{Z}, \text{PROP}(0 \leq i \leq \text{size}) \text{LOCAL}(\text{temp_a } a; \text{temp_i } (\text{Vint } (\text{Int.repr } i)); \text{temp_n } (\text{Vint } (\text{Int.repr } \text{size})); \text{temp_s } (\text{Vint } (\text{Int.repr } (\text{sum_Z } (\text{sublist } 0 \ i \ \text{contents})))) \text{SEP}(\text{data_at sh } (\text{tarray tint size}) (\text{map Vint } (\text{map Int.repr contents})) a)} \quad 1/4$$

We instantiate the existential with the only value that works here, zero: **Exists 0**. [Chapter 19](#) explains how to handle existentials with **Intros** and **Exists**.

Now we use the `entailer!` tactic to solve as much of this goal as possible (see [Chapter 35](#)). In this case, the goal solves entirely automatically. In particular, $0 \leq i \leq \text{size}$ solves by `omega`; `sublist 0 0 contents` rewrites to `nil`; and `sum_Z nil` simplifies to 0.

THE SECOND SUBGOAL of `forward_while` in `progs/verif_sumarray.v` is a *type-checking entailment*, of the form $\text{ENTAIL } \Delta, \text{PQR} \vdash \text{tc_expr } \Delta \ e$ where e is (the abstract syntax of) a C expression; in the particular case of a *while* loop, e is the negation of the loop-test expression. The

entailment guarantees that e executes without crashing: all the variables it references exist, and are initialized; and it doesn't divide by zero, et cetera.

In this case, the entailment concerns the expression $\neg(i < n)$,

ENTAIL Delta, PROP(...) LOCAL(...) SEP(...)

⊢ tc_expr Delta

(Eunop Onotbool (Ebinop Olt (Etempvar _i tint) (Etempvar _n tint) tint)
tint)

This solves completely via the `entailer!` tactic. To see why that is, instead of doing `entailer!`, do `unfold tc_expr; simpl`. You'll see that the right-hand side of the entailment simplifies down to `!!True`. That's because the typechecker is *calculational*, as Chapter 25 of *Program Logics for Certified Compilers* explains.

12 Array subscripts

THE THIRD SUBGOAL of `forward_while` in `progs/verif_sumarray.v` is the *body* of the while loop: `{x=a[i]; s+=x; i++;}`.

This can be handled by three forward commands, but the first one of these leaves a subgoal—proving that the subscript i is in range. Let's examine the proof goal:

SH : readable_share sh

H : $0 \leq \text{size} \leq \text{Int.max_signed}$

H0 : Forall (fun $x : Z \Rightarrow \text{Int.min_signed} \leq x \leq \text{Int.max_signed}$) contents

$i : Z$

HRE : $i < \text{size}$

H1 : $0 \leq i \leq \text{size}$

----- (1/1)

semex Delta

(PROP ())

LOCAL(temp _a a ; temp _i (Vint (Int.repr i));

temp _n (Vint (Int.repr size));

temp _s (Vint (Int.repr (sum_Z (sublist 0 i contents))))))

SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a))

(Ssequence

(Sset _x

(Ederef

(Ebinop Oadd (Etempvar _a (tptr tint)) (Etempvar _i tint)

(tptr tint)) tint)) MORE_COMMANDS) POSTCONDITION

The Coq variable i was introduced automatically by `forward_while` from the existential variable, the EX $i:Z$ of the loop invariant.

The command `x=a[i];` is a *load* from data-structure a . For this to succeed, there must be a `data_at` (or `field_at`) assertion about a in the SEP clauses of the precondition; the permission share in that `data_at` must grant read access; and the subscript must be in range. Indeed, the `data_at` is there,

and the share is taken care of automatically by the hypothesis SH above the line.

So, forward succeeds; but it leaves an array-bounds subgoal:

```
ENTAIL Delta, PROP(...) LOCAL(...) SEP(...)
├ tc_expr Delta (Etempvar _a (tptr tint)) &&
  local `(tc_val tint (Znth i (map Vint (map Int.repr contents)) Vundef)) &&
    (tc_expr Delta (Etempvar _i tint) && TT)
```

The two `tc_expr` conjuncts are trivial (they are $\beta\eta$ -equal to `TT`) but the middle conjunct is nontrivial. To clean things up, we run `entailer!`, which leaves this subgoal:

```
HRE : i < Zlength (map Vint (map Int.repr contents))
H1 : 0 ≤ i ≤ Zlength (map Vint (map Int.repr contents))
  (other above-the-line hypotheses elided)
────────────────────────────────────────────────────────────────────────────────
is_int I32 Signed (Znth i (map Vint (map Int.repr contents)) Vundef)
```

For the load to succeed, the i element of `(map Vint (map Int.repr contents))` must actually be an integer, not an undefined value. To prove this, we use the `Znth_map` lemma to move the `Znth` inside the `Vint`, leaving the goal,

```
is_int I32 Signed (Vint (Znth i (map Int.repr contents) Int.zero))
```

This is an instance of `is_int I32 Signed (Vint ...)` which is $\beta\eta$ -equal to `True`. However, when we rewrote by `Znth_map`, that left a subgoal,

```
HRE : i < Zlength (map Vint (map Int.repr contents))
H1 : 0 ≤ i ≤ Zlength (map Vint (map Int.repr contents))
  (other above-the-line hypotheses elided)
────────────────────────────────────────────────────────────────────────────────
0 ≤ i < Zlength (map Int.repr contents)
```

This solves straightforwardly as shown in the proof script.

13 Splitting sublists

In `progs/verif_sumarray.v`, at the comment “Now we have reached the end of the loop body,” it is time to prove that the *current* precondition (which is the postcondition of the loop body) entails the loop invariant. This is the proof goal:

$H : 0 \leq \text{size} \leq \text{Int.max_signed}$

$H0 : \text{Forall } (\text{fun } x : Z \Rightarrow \text{Int.min_signed} \leq x \leq \text{Int.max_signed}) \text{ contents}$

$HRE : i < \text{size}$

$H1 : 0 \leq i \leq \text{size}$

(other above-the-line hypotheses elided)

ENTAIL Delta,

PROP()

LOCAL(temp _i (Vint (Int.add (Int.repr i) (Int.repr 1))));

temp _s

(force_val

(sem_add_default tint tint

(Vint (Int.repr (sum_Z (sublist 0 i contents)))))

(Znth i (map Vint (map Int.repr contents)) Vundef));

temp _x (Znth i (map Vint (map Int.repr contents)) Vundef); temp _a a;

temp _n (Vint (Int.repr size)))

SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a)

⊢ EX $a_0 : Z$,

PROP($0 \leq a_0 \leq \text{size}$)

LOCAL(temp _a a; temp _i (Vint (Int.repr a_0)));

temp _n (Vint (Int.repr size));

temp _s (Vint (Int.repr (sum_Z (sublist 0 a_0 contents)))))

SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a)

The right-hand side of this entailment is just the loop invariant. As usual at the end of a loop body, there is an existentially quantified variable that must be instantiated with an iteration-dependent value. In this case it's obvious: the quantified variable represents the contents of C local variable `_i`, so we do, **Exists** (`i+1`).

The resulting entailment has many trivial parts and a nontrivial residue. The usual way to get to the hard part is to run `entailer!`, which we do now. After clearing away the irrelevant hypotheses, we have:

$$\begin{array}{l}
 H : 0 \leq \text{Zlength } (\text{map } \text{Vint } (\text{map } \text{Int.repr } \text{contents})) \leq \text{Int.max_signed} \\
 HRE : i < \text{Zlength } (\text{map } \text{Vint } (\text{map } \text{Int.repr } \text{contents})) \\
 H1 : 0 \leq i \leq \text{Zlength } (\text{map } \text{Vint } (\text{map } \text{Int.repr } \text{contents})) \\
 \text{-----}(1/1) \\
 \text{Vint } (\text{Int.repr } (\text{sum_Z } (\text{sublist } 0 \ (i + 1) \ \text{contents}))) = \\
 \text{force_val} \\
 (\text{sem_add_default tint tint } (\text{Vint } (\text{Int.repr } (\text{sum_Z } (\text{sublist } 0 \ i \ \text{contents})))) \\
 (\text{Znth } i \ (\text{map } \text{Vint } (\text{map } \text{Int.repr } \text{contents})) \ \text{Vundef}))
 \end{array}$$

The `sem_add_default` comes from the semantics of C expression evaluation: adding integers means one thing, but adding an integer to a `Vundef` is undefined, and so on. To clear that sludge out of the way, we move the `Znth` inside the `Vint` just as on [page 29](#), then `simpl`, yielding this goal:

$$\begin{array}{l}
 H : 0 \leq \text{Zlength } \text{contents} \leq \text{Int.max_signed} \\
 HRE : i < \text{Zlength } \text{contents} \\
 H1 : 0 \leq i \leq \text{Zlength } \text{contents} \\
 \text{-----}(1/1) \\
 \text{Vint } (\text{Int.repr } (\text{sum_Z } (\text{sublist } 0 \ (i + 1) \ \text{contents}))) = \\
 \text{Vint } (\text{Int.add } (\text{Int.repr } (\text{sum_Z } (\text{sublist } 0 \ i \ \text{contents}))) \\
 (\text{Int.repr } (\text{Znth } i \ \text{contents } 0)))
 \end{array}$$

The lemma `add_repr: $\forall i \ j, \text{Int.add } (\text{Int.repr } i) (\text{Int.repr } j) = \text{Int.repr } (i + j)$` is useful here; followed by `f_equal`, leaves:

$$\begin{array}{l}
 \text{sum_Z } (\text{sublist } 0 \ (i + 1) \ \text{contents}) = \\
 \text{sum_Z } (\text{sublist } 0 \ i \ \text{contents}) + \text{Znth } i \ \text{contents } 0
 \end{array}$$

Now the lemma `sublist_split: $\forall l \ m \ h \ a \ l, \ 0 \leq l \leq m \leq h \leq |a| \rightarrow \text{sublist } l \ h \ a = \text{sublist } l \ m \ a ++ \text{sublist } m \ h \ a$` is helpful here: rewrite `(sublist_split 0 i (i+1))` by `omega`. A bit more rewriting with the theory of `sum_Z` and `sublist` finishes the proof.

14 Returning from a function

In `progs/verif_sumarray.v`, at the comment “After the loop,” we have reached the return statement. The forward tactic works here, leaving a proof goal that the precondition of the return entails the postcondition of the function-spec. (When this automatically, it leaves no proof goal at all.) The goal is a *lowered* entailment (on `mpred` assertions).

After doing `simpl` to clear away some C-expression-evaluation sludge, we have

```
H4 : Forall (value_fits tint) (map Vint (map Int.repr contents))
H2 : field_compatible (Tarray tint (Zlength ...) noattr) [] a
    (other above-the-line hypotheses elided)
-----
data_at sh (tarray tint (Zlength ...)) (map Vint (map Int.repr contents)) a
⊢ !!(Vint (Int.repr (sum_Z contents)) =
    Vint (Int.repr (sum_Z (sublist 0 i contents))))
```

The left-hand side of this entailment is a spatial predicate (`data_at`). Purely nonspatial facts (`H4` and `H2`) derivable from it have already been inferred and moved above the line by `saturate_local` (see [Chapter 31](#)).

This entailment’s right-hand side has no spatial predicates. That’s because the SEP clause of the funspec’s postcondition had exactly the same `data_at` clause as we see here in the entailment precondition, and the entailment-solver called by `forward` has already cleared it away.

In a situation like this—where `saturate_local` has already been done *and* the r.h.s. of the entailment is purely nonspatial—*almost always* there’s no more useful information in the left hand side that hasn’t already been extracted by `saturate_local`. We can throw away the l.h.s. with `apply prop_right` (or by `entailer!` but that’s a bit slower).

The remaining subgoal solves easily in the theory of sublists. The proof of the function `sumarray` is now complete.

15 *Global variables and* `main()`

C programs may have “extern” global variables, either with explicit initializers or initialized by default. Any function that accesses a global variable must have the appropriate spatial assertions in its funspec’s precondition (and postcondition). But the main function is special: it has spatial assertions for *all* the global variables. Then it may pass these on, piecemeal, to the functions it calls on an as-needed basis.

The function-spec for main always looks the same:

Definition `main_spec` :=

```
DECLARE _main WITH u : unit
  PRE [] main_pre prog u
  POST [ tint ] main_post prog u.
```

`main_pre` calculates the precondition automatically from (the list of extern global variables and initializers of) the program. Then, when we prove that main satisfies its funspec,

Lemma `body_main`: `semax_body Vprog Gprog f_main main_spec`.

Proof.

```
name four _four.
start_function.
```

the `start_function` tactic “unpacks” `main_pre` into an assertion:

```
four : val
----- (1/1)
semax Delta
  (PROP () LOCAL(gvar _four four)
    SEP(data_at Ews (tarray tint 4)
      (map Vint [Int.repr 1; Int.repr 2; Int.repr 3; Int.repr 4]) four))
  (... function body ...)
POSTCONDITION
```

The `LOCAL` clause means that the C global variable `_four` is at memory address *four*. (If we had omitted the name tactic in the proof script above, then `start.function` would have chosen some other name for this value.) See [Chapter 29](#).

The `SEP` clause means that there's data of type “array of 4 integers” at address *four*, with access permission `Ews` and contents `[1;2;3;4]`. `Ews` stands for “external write share,” the standard access permission of extern global writable variables. See [Chapter 38](#).

Now it's time to prove the function-call statement, `s = sumarray(four,4)`. When proving a function call, one must supply a *witness* for the `WITH` clause of the function-spec. The `_sumarray` function's `WITH` clause binds variables `a:val`, `sh:share`, `contents:list Z`, `size: Z`, so the type of the witness will be `(val*(share*(list Z * list Z)))`. To choose the witness, examine your actual parameter values (along with the precondition of the funspec) to see what witness would be consistent; here, we use `(four,Ews,four_contents,4)`.
`forward_call (four,Ews,four_contents,4).`

The `forward_call` tactic (usually) leaves subgoals: you must prove that your current precondition implies the funspec's precondition. Here, these solve easily, as shown in the proof script.

The postcondition of the call statement (which is the precondition of the next return statement) has an existential, `EX vret:val`. This comes directly from the existential in the funspec's postcondition. To move `vret` above the line, simply `Intros vret`.

Finally, we are at the return statement. The `forward` tactic is easily able to prove that the current assertion implies the postcondition of `_main`, because `main_post` is basically an abbreviation for `True`.

16 Tying all the functions together

We build a whole-program proof by composing together the proofs of all the function bodies. Consider `Gprog`, the list of all the function-specifications:

Definition `Gprog : funspecs := sumarray.spec :: main.spec :: nil.`

Each `semax.body` proof says, assuming that *all the functions I might call* behave as specified, then *my own function-body* indeed behaves as specified:

Lemma `body_sumarray: semax.body Vprog Gprog f_sumarray sumarray.spec.`

Note that *all the functions I might call* might even include “myself,” in the case of a recursive or mutually recursive function.

This might seem like circular reasoning, but it is actually sound—by the miracle of step-indexed semantic models, as explained in Chapters 18 and 39 of *Program Logics for Certified Compilers*.

The rule for tying the functions together is called `semax.func`, and its use is illustrated in this theorem, the main proof-of-correctness theorem for the program `sumarray.c`:

Lemma `all_funcs_correct: semax.func Vprog Gprog (prog_func prog) Gprog.`

Proof.

`unfold Gprog, prog, prog_func; simpl.`

`semax.func_skipn.`

`semax.func_cons body_sumarray.`

`semax.func_cons body_main.`

`apply semax.func_nil.`

Qed.

The calls to `semax.func_cons` must appear in the same order as the functions are listed in `Gprog` and the same order as they appear in `prog.(prog_defs)`.

17 Separation logic: EX, *, emp, !!

The *base level* separation logic is built, like any separation logic, from predicates on “heaplets”. The grammar of base-level separation-logic expressions is,

$R ::=$	emp	empty
	\top	True
	FF	False
	$R_1 * R_2$	separating conjunction
	$R_1 \ \&\& \ R_2$	ordinary conjunction
	$\text{field_at } \pi \ \tau \ \vec{fld} \ v \ p$	“field maps-to”
	$\text{data_at } \pi \ \tau \ v \ p$	“maps-to”
	$\text{array_at } \tau \ \pi \ v \ lo \ hi$	array slice
	$!!P$	pure proposition
	$\text{EX } x : T, R$	existential quantification
	$\text{ALL } x : T, R$	universal quantification (rare)
	$R_1 \parallel R_2$	disjunction
	$\text{wand } R \ R'$	magic wand $R \multimap R'$ (rare)
	...	other operators, including user definitions

18 PROP() LOCAL() SEP()

The *lifted* separation logic can “see” local and global variables of the C program, in addition to the contents of the heap (pointer dereferences) that the base level separation logic can see. The *canonical form* of a lifted assertion is $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})$, where \vec{P} is a list of propositions (Prop), where \vec{Q} is a list of local-variable definitions (localdef), and \vec{R} is a list of base-level assertions (mpred). Each list is semicolon-separated.

Lifted assertions can occur in other forms than canonical form; in fact, anything of type $\text{environ} \rightarrow \text{mpred}$ is a lifted assertion. But canonical form is most convenient for forward symbolic execution (Hoare-logic rules).

The existential quantifier EX can also be used on canonical forms, e.g., $\text{EX } x:T, \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})$.

Entailments in canonical form are normally of the form, $\text{ENTAIL } \Delta, PQR \vdash PQR'$, where PQR is a lifted assertion in canonical form, PQR' is a lifted assertion not necessarily in canonical form, and Δ is a type context. The \vdash operator is written $|-$ in Coq.

This notation is equivalent to $(\text{tc_environ } \Delta \ \&\& \ PQR) \vdash PQR'$. That is, Δ just provides extra assertions on the left-hand side of the entailment.

19 EX, Intros, Exists

In a canonical-form lifted assertion, existentials can occur at the outside, or in one of the base-level conjuncts within the SEP clause. This assertion has both:

```

ENTAIL  $\Delta$ ,
  EX  $x:Z$ ,
    PROP( $0 \leq x$ ) LOCAL(temp _i (Vint (Int.repr x)))
    SEP(EX  $y:Z$ ,  $!!(x < y) \ \&\& \text{data\_at } \pi \text{ tint (Vint (Int.repr } y)) \ p$ )
 $\vdash$  EX  $u: Z$ ,
  PROP( $0 < u$ ) LOCAL()
  SEP(data_at  $\pi$  tint (Vint (Int.repr  $u$ ))  $p$ )

```

To prove this entailment, one can first move x and y “above the line” by the tactic **Intros** a b:

```

a: Z
b: Z
H:  $0 \leq a$ 
H0:  $a < b$ 

```

```

ENTAIL  $\Delta$ ,
  PROP() LOCAL(temp _i (Vint (Int.repr  $a$ )))
  SEP(data_at  $\pi$  tint (Vint (Int.repr  $b$ ))  $p$ )
 $\vdash$  EX  $u: Z$ ,
  PROP( $0 < u$ ) LOCAL()
  SEP(data_at  $\pi$  tint (Vint (Int.repr  $u$ ))  $p$ )

```

One might just as well say **Intros** $x \ y$ to use those names instead of a b. Note that the propositions (previously hidden inside existential quantifiers) have been moved above the line by **Intros**. Also, if there had been any separating-conjunction operators $*$ within the SEP clause, those will be “flattened” into semicolon-separated conjuncts within SEP.

Sometimes, even when there are no existentials to introduce, one wants

to move PROP propositions above the line and flatten the $*$ operators into semicolons. One can just say **Intros** with no arguments to do that.

If you want to Intro an existential *without* gratuitous PROP-introduction and $*$ -flattening, you can just use **Intro** a , instead of **Intros** a .

Then, instantiate u by **Exists** b .

$a: Z$

$b: Z$

$H: 0 \leq a$

$H0: a < b$

ENTAIL Δ ,
 PROP() LOCAL(temp _i (Vint (Int.repr a)))
 SEP(data.at π tint (Vint (Int.repr b)) p)
 \vdash PROP($0 < b$) LOCAL()
 SEP(data.at π tint (Vint (Int.repr b)) p)

This entailment proves straightforwardly by entailer!.

20 Integers: nat, Z, int 40 (compcert/lib/Integers.v)

Coq's standard library has the natural numbers `nat` and the integers `Z`.

C-language integer values are represented by the type `Int.int` (or just `int` for short), which are 32-bit two's complement signed or unsigned integers with mod-2³² arithmetic. [Chapter 48](#) describes the operations on the `int` type.

For most purposes, specifications and proofs of C programs should use `Z` instead of `int` or `nat`. Subtraction doesn't work well on naturals, and that screws up many other kinds of arithmetic reasoning. *Only when you are doing direct natural-number induction* is it natural to use `nat`, and so you might then convert using `Z.to_nat` to do that induction.

Conversions between `Z` and `int` are done as follows:

```
Int.repr: Z → int.  
Int.unsigned: int → Z.  
Int.signed: int → Z.
```

with the following lemmas:

$$\begin{array}{l} \text{Int.repr_unsigned} \frac{}{\text{Int.repr}(\text{Int.unsigned } z) = z} \\ \text{Int.unsigned_repr} \frac{0 \leq z \leq \text{Int.max_unsigned}}{\text{Int.unsigned}(\text{Int.repr } z) = z} \\ \text{Int.repr_signed} \frac{}{\text{Int.repr}(\text{Int.signed } z) = z} \\ \text{Int.signed_repr} \frac{\text{Int.min_signed} \leq z \leq \text{Int.max_signed}}{\text{Int.signed}(\text{Int.repr } z) = z} \end{array}$$

`Int.repr` truncates to a 32-bit twos-complement representation (losing information if the input is out of range). `Int.signed` and `Int.unsigned` are different injections back to `Z` that never lose information.

When doing proofs about integers, the recommended proof technique is to make sure your integers never overflow. That is, if the C variable `_x` contains the value `Vint (Int.repr x)`, then make sure `x` is in the appropriate range. Let's assume that `_x` is a signed integer, i.e. declared in C as `int x`; then the hypothesis is,

H: `Int.min_signed ≤ x ≤ Int.max_signed`

If you maintain this hypothesis “above the line”, then Floyd’s tactical proof automation can solve goals such as `Int.signed (Int.repr x) = x`. Also, to solve goals such as,

...

H2 : `0 ≤ n ≤ Int.max_signed`

...

`Int.min_signed ≤ 0 ≤ n`

you can use the `reable_signed` tactic, which is basically just `omega` with knowledge of the values of `Int.min_signed`, `Int.max_signed`, and `Int.max_unsigned`.

To take advantage of this, put conjuncts into the `PROP` part of your function precondition such as `0 ≤ i < n; n ≤ Int.max_signed`. Then the `start_function` tactic will move them above the line, and the other tactics mentioned above will make use of them.

To see an example in action, look at `progs/verif_sumarray.v`. The array size and index (variables `size` and `i`) are kept within bounds; but the *contents* of the array might overflow when added up, which is why `add_elem` uses `Int.add` instead of `Z.add`.

21 Values: Vint, Vptr 42 (compcert/common/Values.v)

Definition block : Type := positive.

Inductive val: Type :=

- | Vundef: val
- | Vint: int → val
- | Vlong: int64 → val
- | Vfloat: float → val
- | Vsingle: float32 → val
- | Vptr: block → int → val.

Vundef is the *undefined* value—found, for example, in an uninitialized local variable.

Vint(i) is an integer value, where i is a CompCert 32-bit integer. These 32-bit integers can also represent short (16-bit) and char (8-bit) values.

Vfloat(f) is a 64-bit floating-point value.

Vsingle(f) is a 32-bit floating-point value.

Vptr $b\ z$ is a pointer value, where b is an abstract block number and z is an offset within that block. Different *malloc* operations, or different extern global variables, or stack-memory-resident local variables, will have different abstract block numbers. Pointer arithmetic must be done within the same abstract block, with $(\text{Vptr } b\ z) + (\text{Vint } i) = \text{Vptr } b\ (z + i)$. Of course, the C-language $+$ operator first multiplies i by the size of the array-element that $\text{Vptr } b\ z$ points to.

Vundef is not always treated as distinct from a defined value. For example, $p \mapsto \text{Vint } 5 \vdash p \mapsto \text{Vundef}$, where \mapsto is the `data_at` operator (Chapter 26). That is, $p \mapsto \text{Vundef}$ really means $\exists v, p \mapsto v$. Vundef could mean “truly uninitialized” or it could mean “initialized but arbitrary.”

CompCert C describes C's type system with inductive data types.

Inductive signedness := Signed | Unsigned.

Inductive intsize := I8 | I16 | I32 | IBool.

Inductive floatsize := F32 | F64.

Record attr : Type := mk_attr {
 attr_volatile: bool; attr_alignas: option N
}.

Definition noattr := { | attr_volatile := false; attr_alignas := None | }.

Inductive type : Type :=

| Tvoid: type
| Tint: intsize → signedness → attr → type
| Tlong: signedness → attr → type
| Tfloat: floatsize → attr → type
| Tpointer: type → attr → type
| Tarray: type → Z → attr → type
| Tfunction: typelist → type → calling_convention → type
| Tstruct: ident → attr → type
| Tunion: ident → attr → type

with typelist : Type :=

| Tnil: typelist
| Tcons: type → typelist → typelist.

We have abbreviations for commonly used types:

Definition tint = Tint I32 Signed noattr.

Definition tuint = Tint I32 Unsigned noattr.

Definition tschar = Tint I8 Signed noattr.

Definition tuchar = Tint I8 Unsigned noattr.

Definition tarray (t: type) (n: Z) = Tarray t n noattr.

Definition tptr (t: type) := Tpointer t noattr.

23 CompSpecs

The C language has a namespace for struct- and union-identifiers, that is, *composite types*. In this example, struct foo {int value; struct foo *tail} a,b; the “global variables” namespace contains a,b, and the “struct and union” namespace contains foo.

When you use CompCert clightgen to parse myprogram.c into myprogram.v, the main definition it produces is prog, the AST of the entire C program:

Definition prog : Clight.program := {| prog_types := composites; ... |}.

To interpret the meaning of a type expression, we need to look up the names of its struct identifiers in a *composite* environment. This environment, along with various well-formedness theorems about it, is built from prog as follows:

Require Import floyd.proofauto. (** Import Verifiable C library **)

Require Import myprogram. (** AST of my program **)

Instance CompSpecs : compspecs. **Proof.** make_compspecs prog. **Defined.**

The make_compspecs tactic automatically constructs the *composite specifications* from the program. As a typeclass Instance, CompSpecs is supplied automatically as an implicit argument to the functions and predicates that interpret the meaning of types:

Definition sizeof {env: composite.env} (t: type) : Z := ...

Definition data_at_ {cs: compspecs} (sh: share) (t: type) (v: val) := ...

@sizeof (@cenv.cs CompSpecs) (Tint l32 Signed noattr) = 4.

sizeof (Tint l32 Signed noattr) = 4.

sizeof (Tstruct _foo noattr) = 8.

@data_at_ CompSpecs sh t v ⊢ data_at_ sh t v

When you have two separately compiled .c files, each will have its own prog and its own compspecs. See [Chapter 64](#).

24 retype

For each C-language data type, we define a *representation type*, the Type of Coq values that represent the contents of a C variable of that type.

Definition `retype {cs: compspecs} (t: type) : Type := ...`

Lemma `retype_ind: $\forall (t: \text{type}),$`

`retype t =`

`match t with`

`| Tvoid \Rightarrow unit`

`| Tint _ _ \Rightarrow val`

`| Tlong _ _ \Rightarrow val`

`| Tfloat _ _ \Rightarrow val`

`| Tpointer _ _ \Rightarrow val`

`| Tarray t0 _ _ \Rightarrow list (retype t0)`

`| Tfunction _ _ _ \Rightarrow unit`

`| Tstruct id _ \Rightarrow retype_structlist (co_members (get_co id))`

`| Tunion id _ \Rightarrow retype_unionlist (co_members (get_co id))`

`end`

`retype_structlist` is the right-associative cartesian product of all the (retypes of) the fields of the struct. For example,

`struct list {int hd; struct list *tl};`

`struct one {struct list *p};`

`struct three {int a; struct list *p; double x};`

`retype (Tstruct _list noattr) = (val*val).`

`retype (Tstruct _one noattr) = val.`

`retype (Tstruct _three noattr) = (val*(val*val)).`

We use `val` instead of `int` for the retype of an integer variable, because the variable might be uninitialized, in which case its value will be `Vundef`.

25 *Uninitialized data*, default_val

CompCert represents uninitialized atomic (integer, pointer, float) values as `Vundef : val`.

The dependently typed function `default_val` calculates the undefined value for any C type:

`default_val`: $\forall \{cs: \text{compspecs}\} (t: \text{type}), \text{reptype } t.$

For any C type t , the default value for variables of type t will have Coq type $(\text{reptype } t)$.

For example:

```
struct list {int hd; struct list *tl;};
```

```
default_val tint = Vundef
```

```
default_val (tptr tint) = Vundef
```

```
default_val (tarray tint 4) = [Vundef; Vundef; Vundef; Vundef]
```

```
default_val (tarray t n) = list_repeat (Z.to_nat n) (default_val t)
```

```
default_val (Tstruct _list noattr) = (Vundef, Vundef)
```

26 data_at

Consider a C program with these declarations:

```
struct list {int hd; struct list *tl;} L;
int f(struct list a[5], struct list *p) { ... }
```

Assume these definitions in Coq:

Definition t_list := Tstruct _list noattr.

Definition t_arr := Tarray t_list 5 noattr.

Somewhere inside `f`, we might have the assertion,

```
PROP() LOCAL(temp _a  $\alpha$ , temp _p  $p$ , gvar _L  $L$ )
SEP(data_at Ews t_list (Vint (Int.repr 0), nullval)  $L$ ;
    data_at  $\pi$  t_arr (list_repeat (Z.to_nat 5) (Vint (Int.repr 1),  $p$ ))  $\alpha$ ;
    data_at  $\pi$  t_list (default_val t_list)  $p$ )
```

This assertion says, “Local variable `_a` contains address α , `_p` contains address p , global variable `_L` is at address L . There is a struct list at L with permission-share `Ews` (“extern writable share”), whose `hd` field contains 0 and whose `tl` contains a null pointer. At address α there is an array of 5 list structs, each with `hd`=1 and `tl`= p , with permission π ; and at address p there is a single list cell that is uninitialized¹, with permission π .”

In pencil-and-paper separation logic, we write $q \mapsto i$ to mean `data_at Tsh tint (Vint (Int.repr i)) q` . We write $L \mapsto (0, \text{NULL})$ to mean `data_at Tsh t_list (Vint (Int.repr 0), nullval) L` . We write $p \mapsto (_, _)$ to mean `data_at π t_list (default_val t_list) p` .

In fact, the definition `data_at_` is useful for the situation $p \mapsto _$:

Definition data_at_ {cs: compspecs} sh t p := data_at sh t (default_val t) p.

¹Uninitialized, or initialized but we don’t know or don’t care what its value is

27 retype', repinj

```

struct a {double x1; int x2;};
struct b {int y1; struct a y2;} p;
repinj:  $\forall t$ : type, retype' t  $\rightarrow$  retype t
retype t_struct_b = (val*(val*val))
retype' t_struct_b = (int*(float*int))
repinj t_struct_b (i,(x,j)) = (Vint i, (Vfloat x, Vint j))

```

TL;DR

The retype function maps C types to the the corresponding Coq types of (possibly uninitialized) values. When we know a variable is definitely initialized, it may be more natural to use int instead of val for integer variables, and float instead of val for double variables. The retype' function maps C types to the Coq types of (definitely initialized) values.

Definition retype' {cs: compspecs} (t: type) : Type := ...

Lemma retype'_ind: $\forall (t$: type),
retype t =

```

match t with
| Tvoid  $\Rightarrow$  unit
| Tint _ _  $\Rightarrow$  int
| Tlong _ _  $\Rightarrow$  Int64.int
| Tfloat _ _  $\Rightarrow$  float
| Tpointer _ _  $\Rightarrow$  pointer_val
| Tarray t0 _ _  $\Rightarrow$  list (retype' t0)
| Tfunction _ _ _  $\Rightarrow$  unit
| Tstruct id _  $\Rightarrow$  retype'_structlist (co_members (get_co id))
| Tunion id _  $\Rightarrow$  retype'_unionlist (co_members (get_co id))
end

```

The function repinj maps an initialized value to the type of possibly uninitialized values:

Definition repinj {cs: compspecs} (t: type) : retype' t \rightarrow retype t := ...

The program `progs/nest2.c` (verified in `progs/verif_nest2.v`) illustrates the use of `retype'` and `repinj`.

```
struct a {double x1; int x2;};
struct b {int y1; struct a y2;} p;

int get(void) { int i; i = p.y2.x2; return i; }
void set(int i) { p.y2.x2 = i; }
```

Our API spec for `get` reads as,

Definition `get_spec` :=

```
DECLARE _get
  WITH v : retype' t_struct_b, p : val
  PRE []
    PROP() LOCAL(gvar _p p)
    SEP(data_at Ews t_struct_b (repinj _ v) p)
  POST [ tint ]
    PROP() LOCAL(temp ret_temp (Vint (snd (snd v))))
    SEP(data_at Ews t_struct_b (repinj _ v) p).
```

In this program, `retype' t_struct_b = (int*(float*int))`, and `repinj t_struct_b (i,(x,j)) = (Vint i, (Vfloat x, Vint j))`.

One could also have specified `get` without `retype'` at all:

Definition `get_spec` :=

```
DECLARE _get
  WITH i: Z, x: float, j: int, p : val
  PRE []
    PROP() LOCAL(gvar _p p)
    SEP(data_at Ews t_struct_b (Vint (Int.repr i), (Vfloat x, Vint j)) p)
  POST [ tint ]
    PROP() LOCAL(temp ret_temp (Vint j))
    SEP(data_at Ews t_struct_b (Vint (Int.repr i), (Vfloat x, Vint j)) p).
```

28 field_at

Consider again the example in `progs/nest2.c`

```
struct a {double x1; int x2;};
struct b {int y1; struct a y2;};
```

The command `i = p.y2.x2;` does a nested field load. We call `y2.x2` the *field path*. The precondition for this command might include the assertion,

```
LOCAL(gvar _pb pb)
SEP( data_at sh t_struct_b (y1,(x1,x2)) pb)
```

The postcondition (after the load) would include the new `LOCALfact`,
`temp _i x2.`

The tactic (`unfold_data_at 1%nat`) changes the `SEP` part of the assertion as follows:

```
SEP(field_at Ews t_struct_b (DOT _y1) (Vint y1) pb;
   field_at Ews t_struct_b (DOT _y2) (Vfloat x1, Vint x2) pb)
```

and then doing (`unfold_field_at 2%nat`) unfolds the second `field_at`,

```
SEP(field_at Ews t_struct_b (DOT _y1) (Vint y1) pb;
   field_at Ews t_struct_b (DOT _y2 DOT _x1) (Vfloat x1) pb;
   field_at Ews t_struct_b (DOT _y2 DOT _x2) (Vint x2) pb)
```

The third argument of `field_at` represents the *path* of structure-fields that leads to a given substructure. The empty path (`nil`) works too; it “leads” to the entire structure. In fact, `data_at $\pi \tau v p$` is just short for `field_at $\pi \tau nil v p$` .

Arrays and structs may be nested together, in which case the field path may also contain array subscripts at the appropriate places, using the notation `SUB i` along with `DOT field`.

29 *Localdefs*: temp, lvar, gvar

The LOCAL part of a PROP()LOCAL()SEP() assertion is a list of localdefs that bind variables to their values or addresses.

Inductive localdef : Type :=
 | temp: ident → val → localdef
 | lvar: ident → type → val → localdef
 | gvar: ident → val → localdef
 | sgvar: ident → val → localdef
 | localprop: Prop → localdef.

temp *i v* binds a nonaddressable local variable *i* to its value *v*.

lvar *i t v* binds an *addressable* local variable *i* (of type *t*) to its *address v*.

gvar *i v* binds a *visible global* variable *i* to its *address v*.

sgvar *i v* binds a *possibly shadowed global* variable *i* to its *address v*.

The *contents* of an addressable (local or global) variable is on the heap, and can be described in the SEP clause.

```
int g=2;
int f(void) { int g; int *p = &g; g=6; return g; }
```

In this program, the global variable *g* is shadowed by the local variable *g*. In an assertion inside the function body, one could write

```
PROP() LOCAL(temp _p q; lvar _g tint q; sgvar _g p}
SEP(data_at Ews tint (Vint (Int.repr 2)) p; data_at Tsh tint (Vint (Int.repr 6)) q)
```

to describe a shadowed global variable *_g* that is still there in memory but (temporarily) cannot be referred to by its name in the C program.

Normally one does not use this tactic directly, it is invoked as the first step of entailer or entailer!

Given a lifted entailment $\text{ENTAIL } \Delta, \text{PROP}(\vec{P}) \text{ LOCAL}(\vec{Q}) \text{ SEP}(\vec{R}) \vdash S$, one often wants to prove it at the base level: that is, with all of \vec{P} moved above the line, with all of \vec{Q} out of the way, just considering the base-level separation-logic conjuncts \vec{R} .

When $\Delta, \vec{P}, \vec{Q}, \vec{R}$ are *concrete*, the `go_lower` tactic does this. Concrete means that the \vec{P}, \vec{Q} are nil-terminated lists (not Coq variables) that every element of \vec{Q} is manifestly a localdef (not hidden in Coq abstractions), the identifiers in \vec{Q} be (computable to) ground terms, and the analogous (tree) property for Δ . It is not necessary that $\Delta, \vec{P}, \vec{Q}, \vec{R}$ be fully *ground terms*: Coq variables (and other Coq abstractions) can appear anywhere in \vec{P} and \vec{R} and in the *value* parts of Δ and \vec{Q} . When the entailment is not fully concrete, or when there existential quantifiers outside PROP , the tactic `old.go_lower` can still be useful.

`go_lower` moves the propositions \vec{P} above the line; when a proposition is an equality on a Coq variable, substitute the variable.

For each localdef in \vec{Q} (such as `temp i v`), `go_lower` looks up i in Δ to derive a type-checking fact (such as `tc_val t v`), then introduces it above the line and simplifies it. For example, if t is `tptr tint`, then the typechecking fact simplifies to `is_pointer_or_null v`.

Then it proves the localdefs in S , if possible. If there are still some local-environment dependencies remaining in S , it introduces a variable `rho` to stand for the run-time environment.

The remaining goal will be of the form $\vec{R} \vdash S'$, with the semicolons in \vec{R} replaced by the separating conjunction `*`. S' is the residue of S after lowering to the base separation logic and deleting its (provable) localdefs.

31 *saturate_local*

Normally one does not use this tactic directly, it is invoked by *entailer* or *entailer!*

To prove an entailment $R_1 * R_2 * \dots * R_n \vdash!! (P'_1 \wedge \dots \wedge P'_n) \&\& R'_1 * \dots * R'_m$, first extract all the *local (nonspatial)* facts from $R_1 * R_2 * \dots * R_n$, use them (along with other propositions above the line) to prove $P'_1 \wedge \dots \wedge P'_n$, and then work on the separation-logic (spatial) conjuncts $R_1 * \dots * R_n \vdash R'_1 * \dots * R'_m$.

An example local fact: $\text{data_at } Ews \text{ (tarray tint } n) \ v \ p \vdash!! (\text{Zlength } v = n)$. That is, the value v in an array “fits” the length of the array.

The Hint database *saturate_local* contains all the local facts that can be extracted from *individual* spatial conjuncts:

field_at_local_facts:

$$\begin{aligned} \text{field_at } \pi \ t \ path \ v \ p \vdash!! & (\text{field_compatible } t \ path \ p \\ & \wedge \text{value_fits (nested_field_type } t \ path) \ v) \\ \text{data_at } \pi \ t \ v \ p \vdash!! & (\text{field_compatible } t \ nil \ p \wedge \text{value_fits } t \ v) \end{aligned}$$

memory_block_local_facts:

$$\text{memory_block } \pi \ n \ p \vdash!! \text{isptr } p$$

The assertion $(\text{Zlength } v = n)$ is actually a consequence of *value_fits* when t is an array type. See [Chapter 33](#).

If you create user-defined spatial terms (perhaps using EX, *data_at*, etc.), you can add hints to the *saturate_local* database as well.

The tactic *saturate_local* takes a proof goal of the form $R_1 * R_2 * \dots * R_n \vdash S$ and adds *saturate-local* facts for *each* of the R_i , though it avoids adding duplicate hypotheses above the line.

32 *field_compatible, field_address*

CompCert C light comes with an “address calculus.” Consider this example:

```
struct a {double x1; int x2;};
struct b {int y1; struct a y2;};
struct a *pa; int *q = &(pa→y2.x2);
```

Suppose the value of `_pa` is p . Then the value of `_q` is $p + \delta$; how can we reason about δ ?

Given type t such as `Tstruct _b noattr`, and $path$ such as `(DOT _y2 DOT _x2)`, then `(nested_field_type t path)` is the type of the field accessed by that path, in this case `tint`; `(nested_field_offset t path)` is the distance (in bytes) from the base of t to the address of the field, in this case (on a 32-bit machine) 12 or 16, depending on the field-alignment conventions of the target-machine.

On the Intel x86 architecture, where doubles need not be 8-byte-aligned, we have,

$$\text{data_at } \pi \text{ t_struct_b } (i, (f, j)) \ p \vdash \\ \text{data_at } \pi \text{ tint } i \ p * \text{data_at } \pi \text{ t_struct_a } (f, j) \ (\text{offset_val } p \ 12)$$

but don't write it that way! For one thing, the converse is not valid:

$$\text{data_at } \pi \text{ tint } i \ p * \text{data_at } \pi \text{ t_struct_a } (f, j) \ (\text{offset_val } p \ 12) \\ \not\vdash \text{data_at } \pi \text{ t_struct_b } (i, (f, j)) \ p$$

The reasons: we don't know that $p + 12$ satisfies the alignment requirements for struct b; we don't know whether $p + 12$ crosses the end-of-memory boundary. That entailment *would* be valid in the presence of this hypothesis: `field_compatible t_struct_b nil p : Prop`.

which says that an entire struct b value *can* fit at address p . Note that

this is a *nonspatial* assertion, a pure proposition, independent of the *contents* of memory.

In order to assist with reasoning about reassembly of data structures, `saturate_local` (and therefore `entailer`) puts `field_compatible` assertions above the line; see [Chapter 31](#).

Sometimes one needs to name the address of an internal field—for example, to pass just that field to a function. In that case, one *could* use `field_offset`, but it better to use `field_address`:

Definition `field_address` (t : type) ($path$: list gfield) (p : val) : val :=
 if `field_compatible_dec t path p`
 then `offset_val (Int.repr (nested_field_offset t path)) p`
 else `Vundef`

That is, `field_address` has “baked in” the fact that the offset is “compatible” with the base address (is properly aligned, has not crossed the end-of-memory boundary). And therefore:

```
data_at  $\pi$  tint  $i$   $p$ 
  * data_at  $\pi$  t_struct.a ( $f, j$ ) (field_address t_struct.b (DOT _y2 DOT _x2)  $p$ )
 $\vdash$  data_at  $\pi$  t_struct.b ( $i, (f, j)$ )  $p$ 
```

33 *value_fits*

The spatial maps-to assertion, $\text{data_at } \pi \ t \ v \ p$, says that there's a value v in memory at address p , filling the data structure whose C type is t (with permission π). A corollary is $\text{value_fits } t \ v$: v is a value that actually *can* reside in such a C data structure.

Value_fits is a recursive, dependently typed relation that is easier described by its induction relation; here, we present a simplified version that assumes that all types t are not volatile:

```

value_fits  $t \ v = \text{tc\_val}' \ t \ v$    (when  $t$  is an integer, float, or pointer type)
value_fits (tarray  $t' \ n$ )  $v = (\text{Zlength } v = \text{Z.max } 0 \ n) \wedge \text{Forall } (\text{value\_fits } t') \ v$ 
value_fits (Tstruct  $i \ \text{noattr}$ )  $(v_1, (v_2, (\dots, v_n))) =$ 
    value_fits (field_type  $f_1 \ v_1$ )  $\wedge \dots \wedge \text{value\_fits } (\text{field\_type } f_n \ v_n)$ 
    (when the fields of struct  $i$  are  $f_1, \dots, f_n$ )

```

The predicate $\text{tc_val}'$ says,

Definition $\text{tc_val}' (t: \text{type}) (v: \text{val}) := v \neq \text{Vundef} \rightarrow \text{tc_val } t \ v$.

Definition $\text{tc_val } (t: \text{type}) (v: \text{val}) :=$

```

    match  $t$  with
    | Tvoid  $\Rightarrow$  False
    | Tint  $\text{sz sg } _ \Rightarrow \text{is\_int } \text{sz sg}$ 
    | Tlong  $_ _ \Rightarrow \text{is\_long}$ 
    | Tfloat F32  $_ \Rightarrow \text{is\_single}$ 
    | Tfloat F64  $_ \Rightarrow \text{is\_float}$ 
    | Tpointer  $_ _$  | Tarray  $_ _ _$  | Tfunction  $_ _ _ \Rightarrow \text{is\_pointer\_or\_null}$ 
    | Tstruct  $_ _$  | Tunion  $_ _ \Rightarrow \text{isptr}$ 
end

```

So, an atomic value (int, float, pointer) fits *either* when it is Vundef or when it type-checks. We permit Vundef to “fit,” in order to accommodate partially initialized data structures in C.

Since τ is usually concrete, `tc_val τ v` immediately unfolds to something like,

```
TC0: is_int l32 Signed (Vint i)
TC1: is_int l8 Unsigned (Vint c)
TC2: is_int l8 Signed (Vint d)
TC3: is_pointer_or_null p
TC4: is_ptr q
```

TC0 says that i is a 32-bit signed integer; this is a tautology, so it will be automatically deleted by `go_lower`.

TC1 says that c is a 32-bit signed integer whose value is in the range of unsigned 8-bit integers (unsigned char). TC2 says that d is a 32-bit signed integer whose value is in the range of signed 8-bit integers (signed char). These hypotheses simplify to,

```
TC1: 0 ≤ Int.unsigned c ≤ Byte.max_unsigned
TC2: Byte.min_signed ≤ Int.signed c ≤ Byte.max_signed
```

The cancel tactic proves associative-commutative rearrangement goals such as $(A_1 * A_2) * ((A_3 * A_4) * A_5) \vdash A_4 * (A_5 * A_1) * (A_3 * A_2)$.

If the goal has the form $(A_1 * A_2) * ((A_3 * A_4) * A_5) \vdash (A_4 * B_1 * A_1) * B_2$ where there is only a partial match, then cancel will remove the matching conjuncts and leave a subgoal such as $A_2 * A_3 * A_5 \vdash B_1 * B_2$.

cancel solves $(A_1 * A_2) * ((A_3 * A_4) * A_5) \vdash A_4 * \text{TT} * A_1$ by absorbing $A_2 * A_3 * A_5$ into TT. If the goal has the form

$$\frac{F := ?224 : \text{list}(\text{environ} \rightarrow \text{mpred})}{(A_1 * A_2) * ((A_3 * A_4) * A_5) \vdash A_4 * (\text{fold_right sepcon emp } F) * A_1}$$

where F is a *frame* that is an abbreviation for an uninstantiated logical variable of type $\text{list}(\text{environ} \rightarrow \text{mpred})$, then the cancel tactic will perform *frame inference*: it will unfold the definition F , instantiate the variable (in this case, to $A_2 :: A_3 :: A_5 :: \text{nil}$), and solve the goal. The frame may have been created by `evvar(F: list(envIRON → mpred))`, as part of forward symbolic execution through a function call.

WARNING: cancel can turn a provable entailment into an unprovable entailment. Consider this:

$$\frac{A * C \vdash B * C}{A * D * C \vdash C * B * D}$$

This goal is provable by first rearranging to $(A * C) * D \vdash (B * C) * D$. But cancel may aggressively cancel C and D, leaving $A \vdash B$, which is not provable. You might wonder, what kind of crazy hypothesis is $A * C \vdash B * C$; but indeed such “context-dependent” cancellations do occur in the theory of linked lists; see ?? and PLCC Chapter 19.

CANCEL DOES *not* USE $\beta\eta$ equality, as this can sometimes be very slow. That means sometimes cancel leaves a residual subgoal $A \vdash A'$ where $A =_\beta A'$, sometimes the only differences are in (invisible) implicit arguments. In any case, apply `derives_refl` to solve such residual goals.

The entailer and entailer! tactics simplify (or solve entirely) entailments in either the lifted or base-level separation logic. The entailer never turns a provable entailment into an unprovable one; entailer! is more aggressive and somewhat more efficient, but sometimes turns a provable entailment into an unprovable one, especially in cases related to the WARNING on [page 58](#); see also [??](#). We recommend trying entailer! first, especially where list segments are not involved.

When `go_lower` is applicable, the entailers start by applying it (see [Chapter 30](#)).

Then: `saturate_local` (see [Chapter 31](#)).

NEXT: on each side of the entailment, gather the propositions to the left: $R_1 * (!!P_1 \&\& (!!P_2 \&\& R_2))$ becomes $!!(P_1 \wedge P_2) \&\& (R_1 * R_2)$.

Move all left-hand-side propositions above the line; substitute variables. Autorewrite with `entailer_rewrite`, a *modest* hint database. If the r.h.s. or its first conjunct is a “`valid_pointer`” goal (or one of its variants), try to solve it.

At this point, entailer tries `normalize` and (if progress) back to NEXT; entailer! applies `cancel` to the spatial terms and `prove_it_now` to each propositional conjunct.

The result is that either the goal is entirely solved, or a residual entailment or proposition is left for the user to prove.

36 *normalize*

The `normalize` tactic performs autorewrite **with** norm and several other transformations. **Normalize can be slow:** Many of these simplifications can be done more efficiently and systematically by `entailer` or `Intros`.

The norm rewrite-hint database uses several sets of rules.

Generic separation-logic simplifications.

$$\begin{array}{llll}
 P * \text{emp} = P & \text{emp} * P = P & P \&\& \text{TT} = P & \text{TT} \&\& P = P \\
 P \&\& \text{FF} = \text{FF} & \text{FF} \&\& P = \text{FF} & P * \text{FF} = \text{FF} & \text{FF} * P = \text{FF} \\
 P \&\& P = P & (\text{EX } _ : _, P) = P & \text{local 'True} = \text{TT}
 \end{array}$$

Pull EX and !! out of *-conjunctions.

$$\begin{array}{ll}
 (\text{EX } x : A, P) * Q = \text{EX } x : A, P * Q & (\text{EX } x : A, P) \&\& Q = \text{EX } x : A, P \&\& Q \\
 P * (\text{EX } x : A, Q) = \text{EX } x : A, P * Q & P \&\& (\text{EX } x : A, Q) = \text{EX } x : A, P \&\& Q \\
 P * (!!Q \&\& R) = !!Q \&\& (P * R) & (!!Q \&\& P) * R = !!Q \&\& (P * R)
 \end{array}$$

Delete auto-provable propositions.

$$P \rightarrow (!!P \&\& Q = Q) \qquad P \rightarrow (!!P = \text{TT})$$

Integer arithmetic.

$$\begin{array}{llllll}
 n + 0 = n & 0 + n = n & n * 1 = n & 1 * n = n & \text{sizeof tuchar} = 1 \\
 \text{align } n \ 1 = n & (z > 0) \rightarrow (\text{align } 0 \ z = 0) & (z \geq 0) \rightarrow (\text{Z.max } 0 \ z = z)
 \end{array}$$

Int32 arithmetic.

$$\text{Int.sub } x \ x = \text{Int.zero}$$

$$\text{Int.sub } x \ \text{Int.zero} = x$$

$$\text{Int.add } x \ (\text{Int.neg } x) = \text{Int.zero}$$

$$\text{Int.add } x \ \text{Int.zero} = x$$

$$\text{Int.add } \text{Int.zero } x = x$$

$$x \neq y \rightarrow \text{offset_val}(\text{offset_val } v \ i) \ j = \text{offset_val } v \ (\text{Int.add } i \ j)$$

$$\text{Int.add}(\text{Int.repr } i)(\text{Int.repr } j) = \text{Int.repr}(i + j)$$

$$\text{Int.add}(\text{Int.add } z \ (\text{Int.repr } i)) \ (\text{Int.repr } j) = \text{Int.add } z \ (\text{Int.repr}(i + j))$$

$$z > 0 \rightarrow (\text{align } 0 \ z = 0)$$

$$\text{force_int}(\text{Vint } i) = i$$

$$(\text{min_signed} \leq z \leq \text{max_signed}) \rightarrow \text{Int.signed}(\text{Int.repr } z) = z$$

$$(0 \leq z \leq \text{max_unsigned}) \rightarrow \text{Int.unsigned}(\text{Int.repr } z) = z$$

$$(\text{Int.unsigned } i < 2^n) \rightarrow \text{Int.zero_ext } n \ i = i$$

$$(-2^{n-1} \leq \text{Int.signed } i < 2^{n-1}) \rightarrow \text{Int.sign_ext } n \ i = i$$

map, fst, snd, ...

$$\text{map } f \ (x :: y) = f \ x :: \text{map } f \ y$$

$$\text{map } \text{nil} = \text{nil}$$

$$\text{fst}(x, y) = x$$

$$\text{snd}(x, y) = y$$

$$(\text{isptr } v) \rightarrow \text{force_ptr } v = v$$

$$\text{isptr } (\text{force_ptr } v) = \text{isptr } v$$

$$(\text{is_pointer_or_null } v) \rightarrow \text{ptr_eq } v \ v = \text{True}$$

Unlifting.

$$'f \ \rho = f \ \text{[when } f \text{ has arity 0]}$$

$$'f \ a_1 \ \rho = f \ (a_1 \ \rho) \ \text{[when } f \text{ has arity 1]}$$

$$'f \ a_1 \ a_2 \ \rho = f \ (a_1 \ \rho) \ (a_2 \ \rho) \ \text{[when } f \text{ has arity 2, etc.]}$$

$$(P * Q)\rho = P\rho * Q\rho$$

$$(P \ \&\& \ Q)\rho = P\rho \ \&\& \ Q\rho$$

$$(!P)\rho = !!P$$

$$!!(P \wedge Q) = !!P \ \&\& \ !!Q$$

$$(\text{EX } x : A, P x)\rho = \text{EX } x : A, P x \rho$$

$$(\text{EX } x : B, P x) = \text{EX } x : B, (P x)$$

$$'(P * Q) = 'P * 'Q$$

$$'(P \ \&\& \ Q) = 'P \ \&\& \ 'Q$$

Type checking and miscellaneous.

$$\text{tc_andp tc_TT } e = e \qquad \text{tc_andp } e \text{ tc_TT} = e$$

$$\text{eval_id } x \text{ (env_set } \rho \text{ } x \text{ } v) = v$$

$$x \neq y \rightarrow (\text{eval_id } x \text{ (env_set } \rho \text{ } y \text{ } v) = \text{eval_id } x \text{ } v)$$

$$\text{isptr } v \rightarrow (\text{eval_cast_neutral } v = v)$$

$$(\exists t. \text{tc_val } t \text{ } v \wedge \text{is_pointer_type } t) \rightarrow (\text{eval_cast_neutral } v = v)$$

Expression evaluation. (autorewrite with eval, but in fact these are usually handled just by simpl or unfold.)

$$\text{deref_noload(tarray } t \text{ } n) = (\text{fun } v \Rightarrow v) \qquad \text{eval_expr(Etempvar } i \text{ } t) = \text{eval_id } i$$

$$\text{eval_expr(Econst_int } i \text{ } t) = \text{'(Vint } i \text{'}$$

$$\text{eval_expr(Ebinop } op \text{ } a \text{ } b \text{ } t) =$$

$$\text{'(eval_binop } op \text{ (typeof } a \text{) (typeof } b \text{)) (eval_expr } a \text{) (eval_expr } b \text{'}$$

$$\text{eval_expr(Eunop } op \text{ } a \text{ } t) = \text{'(eval_unop } op \text{ (typeof } a \text{)) (eval_expr } a \text{'}$$

$$\text{eval_expr(Ecast } e \text{ } t) = \text{'(eval_cast(typeof } e \text{) } t \text{) (eval_expr } e \text{'}$$

$$\text{eval_lvalue(Ederef } e \text{ } t) = \text{'force_ptr (eval_expr } e \text{'}$$

Function return values.

$$\text{get_result(Some } x) = \text{get_result1}(x) \qquad \text{retval(get_result1 } i \text{ } \rho) = \text{eval_id } i \text{ } \rho$$

$$\text{retval(env_set } \rho \text{ ret_temp } v) = v$$

$$\text{retval(make_args(ret_temp :: nil) (v :: nil) } \rho) = v$$

$$\text{ret_type(initialized } i \text{ } \Delta) = \text{ret_type}(\Delta)$$

Postconditions. (autorewrite with ret_assert.)

$$\text{normal_ret_assert FF ek vl} = \text{FF}$$

$$\text{frame_ret_assert}(\text{normal_ret_assert } P) Q = \text{normal_ret_assert } (P * Q)$$

$$\text{frame_ret_assert } P \text{ emp} = P$$

$$\text{frame_ret_assert } P Q \text{ EK_return vl} = P \text{ EK_return vl} * Q$$

$$\text{frame_ret_assert}(\text{loop1_ret_assert } P Q) R =$$

$$\text{loop1_ret_assert } (P * R)(\text{frame_ret_assert } Q R)$$

$$\text{frame_ret_assert}(\text{loop2_ret_assert } P Q) R =$$

$$\text{loop2_ret_assert } (P * R)(\text{frame_ret_assert } Q R)$$

$$\text{overridePost } P (\text{normal_ret_assert } Q) = \text{normal_ret_assert } P$$

$$\text{normal_ret_assert } P \text{ ek vl} = (!!(\text{ek} = \text{EK_normal}) \&\& (!!(\text{vl} = \text{None}) \&\& P))$$

$$\text{loop1_ret_assert } P Q \text{ EK_normal None} = P$$

$$\text{overridePost } P R \text{ EK_normal None} = P$$

$$\text{overridePost } P R \text{ EK_return} = R \text{ EK_return}$$

IN ADDITION TO REWRITING, normalize applies the following lemmas:

$$P \vdash \text{TT} \quad \text{FF} \vdash P \quad P \vdash P * \text{TT} \quad (\forall x. (P \vdash Q)) \rightarrow (\text{EX } x : A, P \vdash Q)$$

$$(P \rightarrow (\text{TT} \vdash Q)) \rightarrow (!P \vdash Q) \quad (P \rightarrow (Q \vdash R)) \rightarrow (!P \&\& Q \vdash R)$$

and does some rewriting and substitution when P is an equality in the goal, $(P \rightarrow (Q \vdash R))$.

Given the goal $x \rightarrow P$, where x is not a Prop, normalize avoids doing an intro. This allows the user to choose an appropriate name for x .

37 Welltypedness of variables

The typechecker ensures this about C-program variables: if a variable is initialized, then it contains a value of its declared type.

Function parameters (accessed by Etempvar expressions) are always initialized. Nonaddressable local variables (accessed by Etempvar expressions) and address-taken local variables (accessed by Evar) may be uninitialized or initialized. Global variables (accessed by Evar) are always initialized.

The typechecker keeps track of the initialization status of local nonaddressable variables, *conservatively*: if on all paths from function entry to the current point—assuming that the conditions on if-expressions and while-expressions are uninterpreted/nondeterministic—there is an assignment to variable x , then x is known to be initialized.

Addressable local variables do not have initialization status tracked by the typechecker; instead, this is tracked in the separation logic, by `data_at` assertions such as $v \mapsto _$ (uninitialized) or $v \mapsto i$ (initialized).

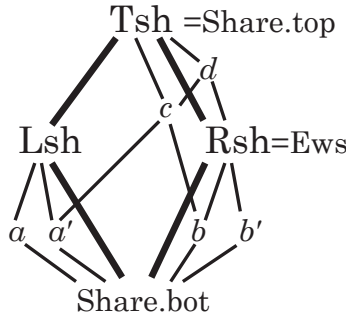
Proofs using the forward tactic will typically generate proof obligations (for the user to solve) of the form,

$$\text{ENTAIL } \Delta, \text{PROP}(\vec{P}) \text{ LOCAL}(\vec{Q}) \text{ SEP}(\vec{R}) \vdash \text{PROP}(\vec{P}') \text{ LOCAL}(\vec{Q}') \text{ SEP}(\vec{R}')$$

Δ keeps track of which nonaddressable local variables are initialized; says that all references to local variables contain values of the right type; and says that all addressable locals and globals point to an appropriate block of memory.

Using `go_lower` or `entailer` on an `ENTAIL` goal causes a `tc_val` assertion to be placed above the line for each initialized tempvar. As explained at [page 56](#), this `tc_val` may be simplified into an `is_int` hypothesis, or even removed if vacuous.

The mapsto operator (and related operators) take a *permission share*, expressing whether the mapsto grants read permission, write permission, or some other fractional permission.



The *top* share, written Tsh or Share.top, gives total permission: to deallocate any cells within the footprint of this mapsto, to read, to write.

Share.split Tsh = (Lsh, Rsh)	
Share.split Lsh = (a, a')	Share.split Rsh = (b, b')
$a' \oplus b = c$	$\text{lub}(c, \text{Rsh}) = a' \oplus \text{Rsh} = d$
$\forall sh. \text{writable_share } sh \rightarrow \text{readable_share } sh$	
writable_share Ews	readable_share b
writable_share d	readable_share c
writable_share Tsh	$\neg \text{readable_share Lsh}$

Any share may be split into a *left half* and a *right half*. The left and right of the top share are given distinguished names Lsh, Rsh.

The right-half share of the top share (or any share containing it such as *d*) is sufficient to grant *write permission* to the data: “the right share is the write share.” A thread of execution holding only Lsh—or subshares of it such as *a, a'*—can neither read or write the object, but such shares are not completely useless: holding any nonempty share prevents other threads from deallocating the object.

Any subshare of Rsh, in fact any share that overlaps Rsh, grants *read* permission to the object. Overlap can be tested using the glb (greatest

lower bound) operator.

Whenever $(\text{mapsto } sh \ t \ v \ w)$ holds, then the share sh must include at least a read share, thus this give permission to load memory at address v to get a value w of type t .

To make sure sh has enough permission to write (i.e., $Rsh \subset sh$, we can say `writable_share sh : Prop`.

Memory obtained from `malloc` comes with the top share Tsh . Writable extern global variables and stack-allocated addressable locals (which of course must not be deallocated) come with the “extern writable share” Ews which is equal to Rsh . Read-only globals come with a half-share of Rsh .

Sequential programs usually have little need of any shares except the Tsh and Ews . However, many function specifications can be parameterized over any share (example: [page ??](#)), and this sort of generalized specification makes the functions usable in more contexts.

In C it is undefined to test deallocated pointers for equality or inequalities, so the Hoare-logic rule for pointer comparison also requires some permission-share; see [page 67](#).

39 *Pointer comparisons*

In C, if p and q are expressions of type pointer-to-something, testing $p=q$ or $p!=q$ is defined only if: p is NULL, or points within a currently allocated object, or points at the end of a currently allocated object; and similarly for q . Testing $p<q$ (etc.) has even stricter requirements: p and q must be pointers into the *same* allocated object.

Verifiable C's enforces this by creating “type-checking” conditions for the evaluation of such pointer-comparison expressions. Before reasoning about the result of evaluating expression $p=q$, you must first prove $\text{tc_expr} \Delta (\text{Ebinop Oeq} (\text{Etempvar } _p (\text{tptr tint})) (\text{Etempvar } _q (\text{tptr tint})))$, where tc_expr is the type-checking condition for that expression. This simplifies into an entailment with the current precondition on the left, and $\text{denote_tc_comparable } p \ q$ on the right.

The entailer(!) has a solver for such proof goals. It relies on spatial terms on the l.h.s. of the entailment, such as $\text{data_at } \pi \ t \ v \ p$ which guarantees that p points to something.

The file `progs/verif_ptr_compare.v` illustrates pointer comparisons.

40 *Proof of the reverse program*

Program Logics for Certified Compilers, Chapter 3 describes the notion of *list segments* and their application to a proof of the list-reverse function. (Chapters 2 and 3 available free [here](#); the whole e-book available cheap [here](#) or [here](#); or buy the [hardcover](#).)

In this chapter we will demonstrate this proof in Verifiable C, on the C program in `progs/reverse.c`. Please open your CoqIDE or Proof General to `progs/verif_reverse.v`.

```
/* reverse.c */
#include <stddef.h>

struct list {int head; struct list *tail;};

struct list three[] = { {1, three+1}, {2, three+2}, {3, NULL} };

struct list *reverse (struct list *p) {
    struct list *w, *t, *v;
    w = NULL;
    v = p;
    while (v) {
        t = v->tail;  v->tail = w;  w = v;  v = t;
    }
    return w;
}

int main (void) {
    struct list *r; int s;
    r = reverse(three);  s = sumlist(r);  return s;
}
```

As usual, in `progs/verif_reverse.v` we import the clightgen-produced file `reverse.v` and build `CompSpecs` and `Vprog` (see [page 13](#), [Chapter 23](#), [Chapter 42](#)).

For the struct list used in *this* program, struct list {int head; struct list *tail;}; we can define the notion of *list segment* $x \overset{\sigma}{\rightsquigarrow} z$ with a recursive definition:

```

Fixpoint lseg (sh: share)
  (contents: list val) (x z: val) : mpred :=
  match contents with
  | h::hs  $\Rightarrow$  !! (x<>z) &&
    EX y:val, data_at sh (Tstruct _list noattr) (h,y) x
    * lseg sh hs y z
  | nil  $\Rightarrow$  !! (ptr_eq x z) && emp
  end.

```

But instead, we make a general theory of list segments (over any C struct type, no matter how many fields). Here, we import the LsegSpecial module of that theory, covering the “ordinary” case appropriate for the reverse.c program.

```
Require Import progs.list_dt. Import LsegSpecial.
```

Then we *instantiate* that theory for our particular struct list by providing the listspec operator with the *names* of the struct (_list) and the link field (_tail).

```
Instance LS: listspec _list _tail.
```

```
Proof. apply mk_listspec; reflexivity. Defined.
```

All other fields (in this case, just _head) are treated as “data” fields.

Now, lseg LS $\pi \sigma p q$ is a list segment starting at pointer p , ending at q , with permission-share π and contents σ .

In general, with multiple data fields, the type of σ is constructed via retype (see [Chapter 24](#)). In this example, with one data field, the type of σ computes to list val.

We’ll skip over the sumlist function and its verification.

The API spec (see also [Chapter 6](#)) for reverse is,

Definition `reverse_spec` :=

```
DECLARE _reverse
  WITH sh: share, contents: list val, p: val
  PRE [  $\neg p$  OF (tptr t_struct_list) ]
    PROP(writable_share sh)
    LOCAL(temp  $\neg p$  p)
    SEP(lseg LS sh contents p nullval)
  POST [ (tptr t_struct_list) ]
    EX p:val,
      PROP() LOCAL(temp ret.temp p)
      SEP(lseg LS sh (rev contents) p nullval).
```

The precondition says (for *p* the function parameter) $p \xrightarrow{\sigma} \text{nil}$, and the postcondition says that (for *p* the return value) $p \xrightarrow{\text{rev } \sigma} \text{nil}$. This is basically the specification given in PLCC Chapter 3, page 20.

Also, the list must have write permission (`writable_share sh`), because the list-reverse is an in-place destructive update.

In your IDE, enter the Lemma `body_reverse` and move after the `start_function` tactic. As expected, the precondition for the function-body is

```
PROP() LOCAL(temp  $\neg p$  p) SEP(lseg LS sh contents p nullval).
```

After forward through two assignment statements (`w=NULL; v=p;`) the LOCAL part also contains `temp $\neg v$ p; temp $\neg w$ (Vint (Int.repr 0))`.

The loop invariant for the while loop is quite similar to the one given in PLCC Chapter 3 page 20:

$$\exists \sigma_1, \sigma_2. \sigma = \text{rev}(\sigma_1) \cdot \sigma_2 \wedge v \xrightarrow{\sigma_2} 0 * w \xrightarrow{\sigma_1} 0$$

It's quite typical for loop invariants to existentially quantify over the values that are different iteration-to-iteration.

Definition $\text{reverse_Inv} (sh: \text{share}) (contents: \text{list val}) : \text{environ} \rightarrow \text{mpred} :=$
 $\text{EX } cts_1: \text{list val}, \text{EX } cts_2: \text{list val}, \text{EX } w: \text{val}, \text{EX } v: \text{val},$
 $\text{PROP}(contents = \text{rev } cts_1 ++ cts_2)$
 $\text{LOCAL}(\text{temp } _w w; \text{temp } _v v)$
 $\text{SEP}(\text{lseg LS } sh \ cts_1 \ w \ \text{nullval}; \text{lseg LS } sh \ cts_2 \ v \ \text{nullval}).$

We apply `forward_while` with this invariant, and (as usual) we have four subgoals: (1) precondition implies loop invariant, (2) loop invariant implies typechecking of loop-termination test, (3) loop body preserves invariant, and (4) after the loop.

(1) To prove the precondition implies the loop invariant, we instantiate cts_1 with `nil` and cts_2 with $contents$; we instantiate w with `NULL` and v with p . But this leaves the goal,

```
ENTAIL  $\Delta$ , PROP() LOCAL(temp  $\_v p$ ; temp  $\_w \text{nullval}$ ; temp  $\_p p$ )
  SEP(lseg LS  $sh \ contents \ p \ \text{nullval}$ )
 $\vdash \text{PROP}(contents = \text{rev } [] ++ contents)$  LOCAL(temp  $\_w \text{nullval}$ ; temp  $\_v p$ )
  SEP(lseg LS  $sh \ [] \ \text{nullval} \ \text{nullval}$ ;
    lseg LS  $sh \ contents \ p \ \text{nullval}$ )
```

The PROP and LOCAL parts are trivially solvable by the entailer. We can remove the SEP conjunct $(\text{lseg LS } sh \ [] \ \text{nullval} \ \text{nullval})$ by rewriting in the theory of list segments:

Lemma $\text{lseg_eq}: \forall (\text{LS} : \text{listspec } _list _tail) (\pi : \text{share}) (l : \text{list } _) (v : \text{val}),$
 $\text{is_pointer_or_null } v \rightarrow$
 $\text{lseg LS } \pi \ l \ v \ v = !!(l = []) \ \&\& \ \text{emp}.$

(2) The type-checking condition is not trivial, as it is a pointer comparison (see [Chapter 39](#)), but the entailer! solves it anyway.

(3) The loop body starts by assuming the *loop invariant* and the truth of the *loop test*. Their propositional parts have already been moved above the line at the comment *(* loop body preserves invariant *)*. That is, HRE: `isptr v` says that the loop test is true, and H: $contents = \text{rev } cts_1 ++ cts_2$ is from the invariant.

The first statement in the loop body, $t = v \rightarrow \text{tail}$; loads from the list cell at v . But our SEP assertion for v is, $\text{lseg LS } sh \text{ } cts_2 \text{ } v \text{ nullval}$. A list-segment isn't necessarily loadable, i.e., we cannot necessarily fetch $v \rightarrow \text{tail}$; what we need to unfold the lseg , using this lemma:

Lemma lseg_nonnull : $\forall (\text{LS} : \text{listspec_list_tail}) (\pi : \text{share}) (l : \text{list_}) v,$
 $\text{typed_true} (\text{tptr } t_struct_list) v \rightarrow$
 $\text{lseg LS } \pi \text{ } l \text{ } v \text{ nullval} =$
 $\text{EX } h::, \text{EX } r::, \text{EX } y::\text{val},$
 $!!(l = h::r \wedge \text{is_pointer_or_null } y) \ \&\&$
 $\text{list_cell LS } \pi \text{ } h \text{ } x *$
 $\text{field_at } \pi \text{ } t_struct_list \text{ (SUB_tail) } y \text{ } x *$
 $\text{lseg LS } \pi \text{ } r \text{ } y \text{ } z.$

That is, if $v \neq \text{nullval}$, then the list-segment $v \xrightarrow{\sigma} \text{nullval}$ is not empty: there exists a record $x \mapsto (h, y)$ and a residual list $y \xrightarrow{\sigma'} \text{nullval}$. Actually, here it is more convenient to use a corollary of this lemma, $\text{semax_lseg_nonnull}$, that is adapted to unfolding *the first lseg in the SEP clause of a semax precondition*. The typed_true premise solves easily by entailer! .

NOW THAT THE FIRST LIST-CELL IS UNFOLDED, it's easy to go forward through the four commands of the loop body. Now we are (* at end of loop body, re-establish invariant *).

We choose appropriate values to instantiate the existentials: **Exists** ($h::cts_1, r, v, y$). Note that for some reason the four separate EX quantifiers have been uncurried into a single 4-tuple EX; this may be adjusted in a future version of Verifiable C. Then entailer! leaves two subgoals:

----- (1/2)
 $\text{rev } cts_1 \text{ } ++ \text{ } h :: r = (\text{rev } cts_1 \text{ } ++ [h]) \text{ } ++ r$
 ----- (2/2)

$\text{list_cell LS } sh \text{ } h \text{ } v * \text{field_at } sh \text{ } t_struct_list \text{ (DOT_tail) } w \text{ } v$
 $* \text{lseg LS } sh \text{ } cts_1 \text{ } w \text{ nullval}$
 $\vdash \text{lseg LS } sh \text{ } (h :: cts_1) \text{ } v \text{ nullval}$

Indeed, *entailer!* always leaves at most two subgoals: at most one propositional goal, and at most one cancellation (spatial) goal. Here, the propositional goal is easily dispatched in the theory of (Coq) lists.

The second subgoal requires unrolling the r.h.s. list segment, which we do with `lseg_unroll`. Then we appropriately instantiate some existentials, call on the *entailer!* again, and the goal is solved.

(4) After the loop, we must prove that the loop invariant *and not the loop-test condition* is a sufficient precondition for the next statement(s). In this case, the next statement is a return; one can *always* go forward through a return, but now we have to prove that our current assertion implies the function postcondition. This is fairly straightforward.

41 *list_cell, assert_PROP*

In `progs/verif_reverse.v`, in the **Lemma** `body_sumlist`, move to the comment `(* Prove that loop body preserves invariant *)`, and then three or four lines to just before `assert_PROP`.

This proof state is very similar to the one in the loop body of the `body_reverse` lemma (page 72):

```

contents, cts1, cts2 : list int;   p, t, y : val;   i : int
SH : readable_share sh
HRE : isptr t
H : contents = cts1 ++ i :: cts2
H1 : is_pointer_or_null y
────────────────────────────────────────
semax Delta
  (PROP () LOCAL(temp _t t; temp _s (Vint (sum_int cts1)))
    SEP(list_cell LS sh (Vint i) t;
      field_at sh list_struct [StructField _tail] y t;
      lseg LS sh (map Vint cts2) y nullval; lseg LS sh (map Vint cts1) p t))
  h = t → head; ...
  POSTCONDITION

```

Here, the operator `list_cell` (from the general theory of list segments) describes “all the fields but the link.” In our particular `LS` there is exactly one data field, which fact we state as a lemma:

Lemma `list_cell_eq`: $\forall sh\ i\ p$,
`sepalg.nonidentity sh` \rightarrow
`field.compatible t_struct_list [] p` \rightarrow
`list_cell LS sh (Vint i) p` =
`field.at sh t_struct_list (DOT _head) (Vint i) p`.

To rewrite by `list_cell_eq`, we need to get a `field.compatible` fact above the line. Such facts are promiscuously introduced by `saturate_local` as part of calling `entailer!`, but we are not currently proving an entailment. No matter; we can prove one artificially:

`assert_PROP (field_compatible t_struct_list nil t) as FC by entailer!.`

The `assert_prop` tactic creates an ENTAIL proof goal with *the current semax precondition* on the left, and the named proposition on the right. That proposition is then put *above the line*; really this is a use of the rule of consequence. It's an easy way to get this `field_compatible` fact above the line.

42 Global variables

In the C language, “extern” global variables live in the same namespace as local variables, but they are shadowed by any same-name local definition. In the C light operational semantics, global variables live in the same namespace as *addressable* local variables (both referenced by the expression-abstract-syntax constructor `Evar`), but in a different namespace from *nonaddressable* locals (expression-abstract-syntax constructor `Etempvar`).¹

In the program-AST produced by `clightgen`, globals (and their initializers) are listed as `Gvars` in the `prog.defs`. These are accessed (automatically) in two ways by the Verifiable C program logic. First, their names and types are gathered into `Vprog` as shown on [page 13](#) (try the Coq command `Print Vprog` to see this list). Second, their initializers are translated into `data_at` conjuncts of separation logic as part of the `main_pre` definition (see [page 33](#)).

When proving `semax_body` for the main function, the `start_function` tactic takes these definitions from `main_pre` and puts them in the precondition of the function body. In VST version 1.6, in some cases this is done using the more-primitive `mapsto` operator², in other cases it uses the higher-level (and more standard) `data_at`³.

¹This difference in namespace treatment cannot matter in a program translated by CompCert `clightgen` from C, because no as-translated expression will exercise the difference.

²For example, examine the proof state in `progs/verif_reverse.v` immediately after `start_function` in Lemma `body_main`; and see the conversion to `data_at` done by the `setup_globals` lemma in that file.

³For example, examine the proof state in `progs/verif_sumarray.v` immediately after `start_function` in Lemma `body_main`.

43 For loops (special case)

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MANY FOR-LOOPS HAVE THIS SPECIAL FORM,
`for (init; i < hi; i++) body` such that the expression `hi` will evaluate to the same value every time around the loop. This upper-bound expression need not be a literal constant, it just needs to be invariant.

For these loops you can use the tactic,

`forward_for_simple_bound n (EX i:Z, PROP(\vec{P}) LOCAL(\vec{Q}) SEP(\vec{R})).`

where n is the upper bound: a Coq value of type Z such that `hi` will evaluate to n .

The loop invariant is given by the expression $(\text{EX } i:Z, \text{PROP}(\vec{P}) \text{LOCAL}(\vec{Q}) \text{SEP}(\vec{R}))$ where i is the value (in each iteration) of the loop iteration variable `id`. This tactic generates simpler subgoals than the general `forward_for` tactic.

You must omit from Q any mention of the loop iteration variable `_i`. The tactic will insert the binding `temp _i i`. Similarly, you need not write `i < hi` in P , the tactic will insert it.

AN EXAMPLE of a for-loop proof is in `progs/verif_sumarray2.v`. This is an alternate implementation of `progs/sumarray.c` (see [Chapter 10](#)) that uses a for loop instead of a while loop:

```
int sumarray(int a[], int n) { /* sumarray2.c */
  int i,s,x;
  s=0;
  for (i=0; i<n; i++) {
    x = a[i];
    s += x;
  }
  return s;
}
```

44 For loops (general case)

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The C-language for loop has the general form, for (*init*; *test*; *incr*) *body* in which *init* and *incr* can be any statements that don't do control flow, *test* can be any expression, and the *body* can contain break or continue statements.

To handle the general case, you cannot use forward_for_simple_bound. Instead, you should unfold Sfor into Ssequence - (Sloop - -), and use Verifiable C's semax_loop rule.

This is demonstrated in the lemma body_sumarray_alt in the file progs/verif_sumarray2.v. The procedure there is straightforward but cumbersome, because you need to define assertions at each of these points:

<pre>int sumarray(int a[], int n) { int i,s,x; s=0; for (i=0; i<n; i++) { x = a[i]; s += x; } return s; }</pre>	<pre>int sumarray(int a[], int n) { int i,s,x; s=0; i=0; <i>Pre</i> for (; ; i++) { <i>Inv</i> if (i<n) ; else break; <i>PreBody</i> x = a[i]; s += x; <i>PostBody</i> } <i>Post</i> return s; }</pre>
--	--

Pre and *Post* are the precondition/postcondition for the loop as a whole. *PreBody* and *PostBody* are the precondition/postcondition for the loop body (not including the increment); and *Inv* is the loop invariant.

In the general case, why is the increment (i++) not attached directly to the end of the loop body? Answer: because the continue statement goes to right *before* the increment.

45 *Manipulating preconditions*

In some cases you cannot go forward until the precondition has a certain form. For example, to go forward through $t=v \rightarrow \text{tail}$; there must be a `data.at` or `field.at` in the SEP clause of the precondition that gives a value for `_tail` field of `t`. [page 72](#) describes a situation where a list segment had to be unfolded to expose such a SEP conjunct.

Faced with the proof goal, $\text{semax } \Delta \text{ (PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})) \text{ } c \text{ } \textit{Post}$ where $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})$ does not match the requirements for forward symbolic execution, you have several choices:

- Use the rule of consequence explicitly:
apply `semax_pre` **with** $\text{PROP}(\vec{P}')\text{LOCAL}(\vec{Q}')\text{SEP}(\vec{R}')$,
then prove $\text{ENTAIL } \Delta, \vec{P};\vec{Q};\vec{R} \vdash \vec{P}';\vec{Q}';\vec{R}'$.
- Use the rule of consequence implicitly, by using tactics ([page 80](#)) that modify the precondition.
- Do rewriting in the precondition, either directly by the standard `rewrite` and `change` tactics, or by `normalize` ([page 60](#)).
- Extract propositions and existentials from the precondition, by using `Intros` ([page 38](#)) or `normalize`.
- Flatten stars into semicolons, in the SEP clause, by `Intros`.
- Use the freezer ([page 107](#)) to temporarily “frame away” spatial conjuncts.

TACTICS FOR MANIPULATING PRECONDITIONS. In many of these tactics we select specific conjuncts from the SEP items, that is, the semicolon-separated list of separating conjuncts. These tactic refer to the list by zero-based position number, 0,1,2,...

For example, suppose the goal is a semax or entailment containing $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(a;b;c;d;e;f;g;h;i;j)$. Then:

focus_SEP $i\ j\ k$. Bring items $\#i,j,k$ to the front of the SEP list.

focus_SEP 5. *results in* $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(f;a;b;c;d;e;g;h;i;j)$.

focus_SEP 0. *results in* $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(a;b;c;d;e;f;g;h;i;j)$.

focus_SEP 1 3. *results in* $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(b;d;a;c;e;f;g;h;i;j)$

focus_SEP 3 1. *results in* $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(d;b;a;c;e;f;g;h;i;j)$

gather_SEP $i\ j\ k$. Bring items $\#i,j,k$ to the front of the SEP list and conjoin them into a single element.

gather_SEP 5. *results in* $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(f;a;b;c;d;e;g;h;i;j)$.

gather_SEP 1 3. *results in* $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(b*d;a;c;e;f;g;h;i;j)$

gather_SEP 3 1. *results in* $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(d*b;a;c;e;f;g;h;i;j)$

replace_SEP $i\ R$. Replace the i th element the SEP list with the assertion R , and leave a subgoal to prove.

replace_SEP 3 R . *results in* $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(a;b;c;R;e;f;g;h;i;j)$.

with subgoal $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(d) \vdash R$.

replace_in_pre $S\ S'$. Replace S with S' anywhere it occurs in the precondition then leave $(\vec{P};\vec{Q};\vec{R}) \vdash (\vec{P};\vec{Q};\vec{R})[S'/S]$ as a subgoal.

frame_SEP $i\ j\ k$. Apply the frame rule, keeping only elements i,j,k of the SEP list. See [Chapter 46](#).

46 The Frame rule

Separation Logic supports the Frame rule,

$$\text{Frame} \frac{\{P\} c \{Q\}}{\{P * F\} c \{Q * F\}}$$

To use this in a forward proof, suppose you have the proof goal,

$$\text{semax} \Delta \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(R_0; R_1; R_2) \ c_1; c_2; c_3 \ \text{Post}$$

and suppose you want to “frame out” R_2 for the duration of $c_1; c_2$, and have it back again for c_3 . First you rewrite by `seq_assoc` to yield the goal

$$\text{semax} \Delta \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(R_0; R_1; R_2) \ (c_1; c_2); c_3 \ \text{Post}$$

Then eapply `semax_seq'` to peel off the first command $(c_1; c_2)$ in the new sequence:

$$\text{semax} \Delta \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(R_0; R_1; R_2) \ c_1; c_2 \ ?88$$

$$\text{semax} \Delta' \ ?88 \ c_3 \ \text{Post}$$

Then `frame_SEP 0 2` to retain only $R_0; R_2$.

$$\text{semax} \Delta \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(R_0; R_2) \ c_1; c_2 \ \dots$$

Now you'll see that (in the precondition of the second subgoal) the unification variable `?88` has been instantiated in such a way that R_2 is added back in.

47 *malloc/free*

If your program uses `malloc` or `free`, you must declare and specify these as external functions. But even then, `free` is difficult to specify: “[How do it know?](#)” the size of the object being freed?

The answer is that the `malloc/free` system maintains an implicit extra field (before the official “beginning” of the object) with the length. One could indeed reason about this in separation logic, but for some applications it is overkill.

For simpler-to-specify memory allocation, you may want to change the interface of the `free` function. We do this in our example definitions of `malloc` and `free` in `progs/queue.c` and their specifications in `progs/verif_queue.v`.

The VST program logic uses CompCert's 32-bit integer type.

Inductive comparison := Ceq | Cne | Clt | Cle | Cgt | Cge.

Int.wordsize: nat = 32.

Int.modulus : $\mathbb{Z} = 2^{32}$.

Int.max_unsigned : $\mathbb{Z} = 2^{32} - 1$.

Int.max_signed : $\mathbb{Z} = 2^{31} - 1$.

Int.min_signed : $\mathbb{Z} = -2^{31}$.

Int.int : Type.

Int.unsigned : int $\rightarrow \mathbb{Z}$.

Int.signed : int $\rightarrow \mathbb{Z}$.

Int.repr : $\mathbb{Z} \rightarrow \text{int}$.

Int.zero := Int.repr 0.

(* Operators of type int->int->bool *)

Int.eq Int.lt Int.ltu Int.cmp(c:comparison) Int.cmpu(c:comparison)

(* Operators of type int->int *)

Int.neg Int.not

(* Operators of type int->int->int *)

Int.add Int.sub Int.mul Int.divs Int.mods Int.divu Int.modu

Int.and Int.or Int.xor Int.shl Int.shru Int.shr Int.rol Int.ror Int.rolm

Lemma eq-dec: $\forall (x\ y: \text{int}), \{x = y\} + \{x <> y\}$.

Theorem unsigned_range: $\forall i, 0 \leq \text{unsigned } i < \text{modulus}$.

Theorem unsigned_range_2: $\forall i, 0 \leq \text{unsigned } i \leq \text{max_unsigned}$.

Theorem signed_range: $\forall i, \text{min_signed} \leq \text{signed } i \leq \text{max_signed}$.

Theorem repr_unsigned: $\forall i, \text{repr } (\text{unsigned } i) = i$.

Lemma repr_signed: $\forall i, \text{repr } (\text{signed } i) = i$.

Theorem unsigned_repr:

$\forall z, 0 \leq z \leq \text{max_unsigned} \rightarrow \text{unsigned } (\text{repr } z) = z$.

Theorem signed_repr:

$$\forall z, \text{min_signed} \leq z \leq \text{max_signed} \rightarrow \text{signed} (\text{repr } z) = z.$$

Theorem signed_eq_unsigned:

$$\forall x, \text{unsigned } x \leq \text{max_signed} \rightarrow \text{signed } x = \text{unsigned } x.$$

Theorem unsigned_zero: unsigned zero = 0.

Theorem unsigned_one: unsigned one = 1.

Theorem signed_zero: signed zero = 0.

Theorem eq_sym: $\forall x \ y, \text{eq } x \ y = \text{eq } y \ x.$

Theorem eq_spec: $\forall (x \ y : \text{int}), \text{if } \text{eq } x \ y \text{ then } x = y \text{ else } x <> y.$

Theorem eq_true: $\forall x, \text{eq } x \ x = \text{true}.$

Theorem eq_false: $\forall x \ y, x <> y \rightarrow \text{eq } x \ y = \text{false}.$

Theorem add_unsigned: $\forall x \ y, \text{add } x \ y = \text{repr} (\text{unsigned } x + \text{unsigned } y).$

Theorem add_signed: $\forall x \ y, \text{add } x \ y = \text{repr} (\text{signed } x + \text{signed } y).$

Theorem add_commut: $\forall x \ y, \text{add } x \ y = \text{add } y \ x.$

Theorem add_zero: $\forall x, \text{add } x \ \text{zero} = x.$

Theorem add_zero_l: $\forall x, \text{add } \text{zero } x = x.$

Theorem add_assoc: $\forall x \ y \ z, \text{add} (\text{add } x \ y) \ z = \text{add } x (\text{add } y \ z).$

Theorem neg_repr: $\forall z, \text{neg} (\text{repr } z) = \text{repr } (-z).$

Theorem neg_zero: $\text{neg } \text{zero} = \text{zero}.$

Theorem neg_involutive: $\forall x, \text{neg} (\text{neg } x) = x.$

Theorem neg_add_distr: $\forall x \ y, \text{neg} (\text{add } x \ y) = \text{add} (\text{neg } x) (\text{neg } y).$

Theorem sub_zero_l: $\forall x, \text{sub } x \ \text{zero} = x.$

Theorem sub_zero_r: $\forall x, \text{sub } \text{zero } x = \text{neg } x.$

Theorem sub_add_opp: $\forall x \ y, \text{sub } x \ y = \text{add } x (\text{neg } y).$

Theorem sub_idem: $\forall x, \text{sub } x \ x = \text{zero}.$

Theorem sub_add_l: $\forall x \ y \ z, \text{sub} (\text{add } x \ y) \ z = \text{add} (\text{sub } x \ z) \ y.$

Theorem sub_add_r: $\forall x \ y \ z, \text{sub } x (\text{add } y \ z) = \text{add} (\text{sub } x \ z) (\text{neg } y).$

Theorem sub_shifted: $\forall x \ y \ z, \text{sub} (\text{add } x \ z) (\text{add } y \ z) = \text{sub } x \ y.$

Theorem sub_signed: $\forall x \ y, \text{sub } x \ y = \text{repr} (\text{signed } x - \text{signed } y).$

Theorem `mul_commut`: $\forall x\ y, \text{mul } x\ y = \text{mul } y\ x$.

Theorem `mul_zero`: $\forall x, \text{mul } x\ \text{zero} = \text{zero}$.

Theorem `mul_one`: $\forall x, \text{mul } x\ \text{one} = x$.

Theorem `mul_assoc`: $\forall x\ y\ z, \text{mul } (\text{mul } x\ y)\ z = \text{mul } x\ (\text{mul } y\ z)$.

Theorem `mul_add_distr_l`: $\forall x\ y\ z, \text{mul } (\text{add } x\ y)\ z = \text{add } (\text{mul } x\ z)\ (\text{mul } y\ z)$.

Theorem `mul_signed`: $\forall x\ y, \text{mul } x\ y = \text{repr } (\text{signed } x * \text{signed } y)$.

and many more axioms for the bitwise operators, shift operators, signed/unsigned division and mod operators.

49 CompCert C abstract syntax

The CompCert verified C compiler translates standard C source programs into an abstract syntax for *CompCert C*, and then translates that into abstract syntax for *C light*. Then VST Separation Logic is applied to the C light abstract syntax. C light programs proved correct using the VST separation logic can then be compiled (by CompCert) to assembly language.

C light syntax is defined by these Coq files from CompCert:

Integers. 32-bit (and 8-bit, 16-bit, 64-bit) signed/unsigned integers.

Floats. IEEE floating point numbers.

Values. The val type: integer + float + pointer + undefined.

AST. Generic support for abstract syntax.

Ctypes. C-language types and structure-field-offset computations.

Clight. C-light expressions, statements, and functions.

You will see C light abstract syntax constructors in the Hoare triples (semax) that you are verifying. We summarize the constructors here.

Inductive `expr : Type :=`

<code>(* 1 *)</code>	<code>Econst_int: int → type → expr</code>
<code>(* 1.0 *)</code>	<code>Econst_float: float → type → expr (* double precision *)</code>
<code>(* 1.0f0 *)</code>	<code>Econst_single: float → type → expr (* single precision *)</code>
<code>(* 1L *)</code>	<code>Econst_long: int64 → type → expr</code>
<code>(* x *)</code>	<code>Evar: ident → type → expr</code>
<code>(* x *)</code>	<code>Etempvar: ident → type → expr</code>
<code>(* *e *)</code>	<code>Ederef: expr → type → expr</code>
<code>(* &e *)</code>	<code>Eaddrrof: expr → type → expr</code>
<code>(* ~e *)</code>	<code>Eunop: unary_operation → expr → type → expr</code>
<code>(* e + e *)</code>	<code>Ebinop: binary_operation → expr → expr → type → expr</code>
<code>(* (int)e *)</code>	<code>Ecast: expr → type → expr</code>
<code>(* e.f *)</code>	<code>Efield: expr → ident → type → expr.</code>

Inductive unary_operation := Onotbool | Onotint | Oneg | Oabsfloat.

Inductive binary_operation := Oadd | Osub | Omul | Odiv | Omod
| Oand | Oor | Oxor | Oshl | Oeq | One | Olt | Ogt | Ole | Oge.

Inductive statement : Type :=

$(* \text{ /**/}; *)$	Sskip : statement
$(* E_1 = E_2; *)$	Sassign : $\text{expr} \rightarrow \text{expr} \rightarrow \text{statement}$ (<i>* memory store *</i>)
$(* x = E; *)$	Sset : $\text{ident} \rightarrow \text{expr} \rightarrow \text{statement}$ (<i>* tempvar assign *</i>)
$(* x = f(...); *)$	Scall: $\text{option ident} \rightarrow \text{expr} \rightarrow \text{list expr} \rightarrow \text{statement}$
$(* x = b(...); *)$	Sbuiltin: $\text{option ident} \rightarrow \text{external_function} \rightarrow \text{typelist} \rightarrow$ $\text{list expr} \rightarrow \text{statement}$
$(* s_1; s_2 *)$	Ssequence : $\text{statement} \rightarrow \text{statement} \rightarrow \text{statement}$
$(* \text{ if() else } \{ \} *)$	Sifthenelse : $\text{expr} \rightarrow \text{statement} \rightarrow \text{statement} \rightarrow \text{statement}$
$(* \text{ for } (;;s_2) s_1 *)$	Sloop: $\text{statement} \rightarrow \text{statement} \rightarrow \text{statement}$
$(* \text{ break}; *)$	Sbreak : statement
$(* \text{ continue}; *)$	Scontinue : statement
$(* \text{ return } E; *)$	Sreturn : $\text{option expr} \rightarrow \text{statement}$
	Sswitch : $\text{expr} \rightarrow \text{labeled_statements} \rightarrow \text{statement}$
	Slabel : $\text{label} \rightarrow \text{statement} \rightarrow \text{statement}$
	Sgoto : $\text{label} \rightarrow \text{statement}$.

50 *C light semantics*

The operational semantics of C light statements and expressions is given in `compcert/cfrontend/Clight.v`. We do not expose these semantics *directly* to the user of Verifiable C. Instead, the *statement* semantics is reformulated as `semax`, an axiomatic (Hoare-logic style) semantics. The *expression* semantics is reformulated in `veric/expr.v` and `veric/Cop2.v` as a *computational big-step evaluation semantics*. In each case, a soundness proof relates the Verifiable C semantics to the CompCert Clight semantics.

Rules for `semax` are given in `veric/SeparationLogic.v`—but the user rarely uses these rules directly. Instead, derived lemmas regarding `semax` are proved in `floyd/*.v` and Floyd’s forward tactic applies them (semi)automatically.

The following functions (from `veric/expr.v`) define expression evaluation:

```
eval_id {CS: compspecs} (id: ident) : environ → val.
    (* evaluate a tempvar *)
eval_var {CS: compspecs} (id: ident) (ty: type) : environ → val.
    (* evaluate an lvar or gvar, addressable local or global variable *)
eval_cast (t t': type) (v: val) : val.
    (* cast value v from type t to type t', but beware! There are
       three types involved, including native type of v. *)
eval_unop (op: unary_operation) (t1 : type) (v1 : val) : val.
eval_binop {CS: compspecs} (op: binary_operation) (t1 t2: type) (v1 v2: val): val.
eval_lvalue {CS: compspecs} (e: expr) : environ → val.
    (* evaluate an l-expression, one that denotes a loadable/storable place*)
eval_expr {CS: compspecs} (e: expr) : environ → val.
    (* evaluate an r-expression, one that is not storable *)
```

The *environ* argument is for looking up the values of local and global variables. However, in most cases where Verifiable C users see `eval_lvalue` or `eval_expr`—in subgoals generated by the forward tactic—all the variables have already been substituted by values. Thus the environment is not

needed.

The expression-evaluation functions call upon several helper functions from `veric/Cop2.v`:

```

sem_cast: type → type → val → option val.
sem_cast.* (* several helper functions for sem_cast *)
bool_val: type → val → option bool.
bool_val.*: (* helper functions *)
sem_notbool: type → val → option val.
sem_neg: type → val → option val.
sem_sub {CS: compspecs}: type → type → val → val → option val.
sem_sub.*: (* helper functions *)
sem_add {CS: compspecs}: type → type → val → val → option val.
sem_add.*: (* helper functions *)
sem_mul: type → type → val → val → option val.
sem_div: type → type → val → val → option val.
sem_mod: type → type → val → val → option val.
sem_and: type → type → val → val → option val.
sem_or: type → type → val → val → option val.
sem_xor: type → type → val → val → option val.
sem_shl: type → type → val → val → option val.
sem_shr: type → type → val → val → option val.
sem_cmp: comparison → type → type → (...) → val → val → option val.
sem_unary_operation: unary_operation → type → val → option val.
sem_binary_operation {CS: compspecs}:
    binary_operation → type → type → mem → val → val → option val.

```

The details are not so important to remember. The main point is that Coq expressions of the form `sem_...` *should* simplify away, provided that their arguments are instantiated with concrete operators, concrete constructors `Vint/Vptr/Vfloat`, and concrete C types. The *int* values (etc.) carried inside `Vint/Vptr/Vfloat` *do not* need to be concrete: they can be Coq variables. This is the essence of proof by symbolic execution.

51 Splitting arrays

Consider this example drawn from the main function of `progs/verif_sumarray2.v`:

`data_at sh (tarray tint k) al p : mpred`

The `data_at` predicate here says that in memory starting at address p there is an array of k slots containing, respectively, the elements of the sequence al .

Suppose we have a function `sumarray(int a[], int n)` that takes an array of length n , and we apply it to a “slice” of p : `sumarray(p+i,k-i)`; where $0 \leq i \leq k$. The precondition of the `sumarray` funspec has `data_at sh (tarray tint n) bl a`. In this case, we would like $a = \&(p[i])$, $n = k - i$, and $bl =$ the sublist of al from i to $k - 1$.

To prove this function-call by `forward_call`, we must split up `(data_at sh (tarray tint k) al p)` into two conjuncts:

`(data_at sh (tarray tint i) (sublist 0 i al) p *
 data_at sh (tarray tint ($k - i$)) (sublist i k al) q),`

where q is the pointer to the array slice beginning at address $p + i$. We write this as, $q = \text{field_address0 (tarray tint k) [ArraySubsc i] } p$. That is, given a pointer p to a data structure described by `(tarray tint k)`, calculate the *address* for subscripting the i th element. (See [Chapter 32](#))

As shown in the `body_main` proof in `progs/verif_sumarray2.v`, the lemma `split_array` proves the equivalence of these two predicates. Then the `data_at ... q)` predicate can satisfy the precondition of `sumarray`, while the p slice will be part of the “frame” for the function call.

52 sublist

Chapter 51 explained that we often need to reason about slices of arrays whose contents are sublists of lists. For that we have a function `sublist i j l` which makes a new list out of the elements $i \dots j-1$ of list l .

These rules comprise the *sublist rewrite database*:

`sublist_nil'`: $i = j \rightarrow \text{sublist } i \ j \ l = []$.

`app_nil_l`: $[] ++ l = l$.

`app_nil_r`: $l ++ [] = l$.

`Zlength_rev`: $\text{Zlength } (\text{rev } l) = \text{Zlength } l$.

`sublist_rejoin'`: $0 \leq i \leq j = j' \leq k \leq \text{Zlength } l \rightarrow$

$\text{sublist } i \ j \ l ++ \text{sublist } j' \ k \ l = \text{sublist } i \ k \ l$.

`subsub1`: $a - (a - b) = b$.

`Znth_list_repeat_inrange`: $0 \leq i \leq n \rightarrow \text{Znth } i \ (\text{list_repeat } (\text{Z.to_nat } n) \ a) \ d = a$.

`Zlength_cons`: $\text{Zlength } (a :: l) = \text{Z.succ } (\text{Zlength } l)$.

`Zlength_nil`: $\text{Zlength } [] = 0$.

`Zlength_app`: $\text{Zlength } (l ++ l') = \text{Zlength } l ++ \text{Zlength } l'$.

`Zlength_map`: $\text{Zlength } (\text{map } f \ l) = \text{Zlength } l$.

`list_repeat_0`: $\text{list_repeat } (\text{Z.to_nat } 0) = []$.

`Zlength_list_repeat`: $0 \leq n \rightarrow \text{Zlength } (\text{list_repeat } (\text{Z.to_nat } n)) = n$.

`Zlength_sublist`: $0 \leq i \leq j \leq \text{Zlength } l \rightarrow \text{Zlength } (\text{sublist } i \ j \ l) = j - i$.

`sublist_sublist`: $0 \leq m \rightarrow 0 \leq k \leq i \leq j - m \rightarrow$

$\text{sublist } k \ i \ (\text{sublist } m \ j \ l) = \text{sublist } (k + m) \ (i + m) \ l$.

`sublist_app1`: $0 \leq i \leq j \leq \text{Zlength } l \rightarrow \text{sublist } i \ j \ (l ++ l') = \text{sublist } i \ j \ l$.

`sublist_app2`: $0 \leq \text{Zlength } l \leq i \rightarrow$

$\text{sublist } i \ j \ (l ++ l') = \text{sublist } (i - \text{Zlength } l) \ (j - \text{Zlength } l) \ l'$.

`sublist_list_repeat`: $0 \leq i \leq j \leq k \rightarrow$

$\text{sublist } i \ j \ (\text{list_repeat } (\text{Z.to_nat } k) \ v) = \text{list_repeat } (\text{Z.to_nat } (j - i)) \ v$.

`sublist_same`: $i = 0 \rightarrow j = \text{Zlength } l \rightarrow \text{sublist } i \ j \ l = l$.

`app_Znth1`: $i < \text{Zlength } l \rightarrow \text{Znth } i \ (l ++ l') \ d = \text{Znth } i \ l \ d$.

`app_Znth2`: $i \geq \text{Zlength } l \rightarrow \text{Znth } i \ (l ++ l') \ d = \text{Znth } i - \text{Zlength } l \ l' \ d$.

`Znth_sublist`: $0 \leq i \rightarrow 0 \leq j < k - i \rightarrow \text{Znth } j \ (\text{sublist } i \ k \ l) \ d = \text{Znth } (j + i) \ l \ d$.

along with miscellaneous \mathbb{Z} arithmetic:

$$\begin{aligned} n - 0 = n \quad 0 + n = n \quad n + 0 = n \quad n \leq m \rightarrow \max(n, m) = m \\ n + m - n = m \quad n + m - m = n \quad m - n + n = m \quad n - n = 0 \\ n + m - (n + p) = m - p \quad \text{etcetera.} \end{aligned}$$

Therefore, autorewrite **with** sublist is a good way to simplify expressions involving sublist, ++, map, Zlength, Znth, and list_repeat.

Many of the Hoare rules, such as the one on page ??,

$$\text{semax_set_forward} \frac{}{\Delta \vdash \{\triangleright P\} \ x := e \ \{\exists v. x = (e[v/x]) \wedge P[v/x]\}}$$

have the operator \triangleright (pronounced “later”) in their precondition.

The modal assertion $\triangleright P$ is a slightly weaker version of the assertion P . It is used for reasoning by induction over how many steps left we intend to run the program. The most important thing to know about \triangleright later is that P is stronger than $\triangleright P$, that is, $P \vdash \triangleright P$; and that operators such as $*$, $\&\&$, ALL (and so on) commute with later: $\triangleright(P * Q) = (\triangleright P) * (\triangleright Q)$.

This means that if we are trying to apply a rule such as `semax_set_forward`; and if we have a precondition such as

`local (tc_expr Δ e) $\&\&$ \triangleright local (tc_temp_id id t Δ e) $\&\&$ ($P_1 * \triangleright P_2$)`

then we can use the rule of consequence to *weaken* this precondition to

`\triangleright (local (tc_expr Δ e) $\&\&$ local (tc_temp_id id t Δ e) $\&\&$ ($P_1 * P_2$))`

and then apply `semax_set_forward`. We do the same for many other kinds of command rules.

This weakening of the precondition is done automatically by the forward tactic, as long as there is only one \triangleright later in a row at any point among the various conjuncts of the precondition.

A more sophisticated understanding of \triangleright is needed to build proof rules for recursive data types and for some kinds of object-oriented programming; see PLCC Chapter 19.

54 Nested Loads

This experimental appeared in VST release 1.5, but is broken in VST 1.6.

To handle assignment statements with nested loads, such as $x[i]=y[i]+z[i]$; the recommended method is to break it down into smaller statements compatible with separation logic: $t=y[i]$; $u=z[i]$; $x[i]=t+u$;. However, sometimes you may be proving correctness of preexisting or machine-generated C programs. Verifiable C has an **experimental** nested-load mechanism to support this.

We use an expression-evaluation relation $e \Downarrow v$ which comes in two flavors:

$\text{rel_expr} : \text{expr} \rightarrow \text{val} \rightarrow \text{rho} \rightarrow \text{mpred}.$

$\text{rel_lvalue} : \text{expr} \rightarrow \text{val} \rightarrow \text{rho} \rightarrow \text{mpred}.$

The assertion $\text{rel_expr } e \ v \ \rho$ says, “expression e evaluates to value v in environment ρ and in the current memory.” The rel_lvalue evaluates the expression as an l -value, to a pointer to the data.

Evaluation rules for rel_expr are listed here:

$\text{rel_expr_const_int} : \quad \forall (i : \text{int}) \ \tau \ (P : \text{mpred}) \ (\rho : \text{enviro}),$

$P \vdash \text{rel_expr} \ (\text{Econst_int } i \ \tau) \ (\text{Vint } i) \ \rho.$

$\text{rel_expr_const_float} : \quad \forall (f : \text{float}) \ \tau \ P \ (\rho : \text{enviro}),$

$P \vdash \text{rel_expr} \ (\text{Econst_float } f \ \tau) \ (\text{Vfloat } f) \ \rho.$

$\text{rel_expr_const_long} : \quad \forall (i : \text{int64}) \ \tau \ P \ \rho,$

$P \vdash \text{rel_expr} \ (\text{Econst_long } i \ \tau) \ (\text{Vlong } i) \ \rho.$

$\text{rel_expr_tempvar} : \quad \forall (\text{id} : \text{ident}) \ \tau \ (v : \text{val}) \ P \ \rho,$

$\text{Map.get} \ (\text{te_of } \rho) \ \text{id} = \text{Some } v \rightarrow$

$P \vdash \text{rel_expr} \ (\text{Etempvar } \text{id } \tau) \ v \ \rho.$

$\text{rel_expr_addrof} : \quad \forall (e : \text{expr}) \ \tau \ (v : \text{val}) \ P \ \rho,$

$P \vdash \text{rel_lvalue } e \ v \ \rho \rightarrow$

$P \vdash \text{rel_expr} \ (\text{Eaddrof } e \ \tau) \ v \ \rho.$

$\text{rel_expr_unop} : \quad \forall P \ (e_1 : \text{expr}) \ (v_1 \ v : \text{val}) \ \tau \ op \ \rho,$

$P \vdash \text{rel_expr } e_1 \ v_1 \ \rho \rightarrow$

$\text{Cop.sem_unary_operation } op \ v_1 \ (\text{typeof } e_1) = \text{Some } v \rightarrow$
 $P \vdash \text{rel_expr } (\text{Eunop } op \ e_1 \ \tau) \ v \ \rho.$
 $\text{rel_expr_binop: } \forall (e_1 \ e_2 : \text{expr}) \ (v_1 \ v_2 \ v : \text{val}) \ \tau \ op \ P \ \rho,$
 $P \vdash \text{rel_expr } e_1 \ v_1 \ \rho \rightarrow$
 $P \vdash \text{rel_expr } e_2 \ v_2 \ \rho \rightarrow$
 $(\forall \ m : \text{Memory.Mem.mem},$
 $\text{Cop.sem_binary_operation } op \ v_1 \ e \ (\text{typeof } e_1) \ v_2 \ (\text{typeof } e_2) \ m = \text{Some } v) \rightarrow$
 $P \vdash \text{rel_expr } (\text{Ebinop } op \ e_1 \ e_2 \ \tau) \ v \ \rho.$
 $\text{rel_expr_cast: } \forall (e_1 : \text{expr}) \ (v_1 \ v : \text{val}) \ \tau \ P \ \rho,$
 $P \vdash \text{rel_expr } e_1 \ v_1 \ \rho \rightarrow$
 $\text{Cop.sem_cast } v_1 \ (\text{typeof } e_1) \ \tau = \text{Some } v \rightarrow$
 $P \vdash \text{rel_expr } (\text{Ecast } e_1 \ \tau) \ v \ \rho.$
 $\text{rel_expr_lvalue: } \forall (a : \text{expr}) \ (\text{sh} : \text{Share.t}) \ (v_1 \ v_2 : \text{val}) \ P \ \rho,$
 $P \vdash \text{rel_lvalue } a \ v_1 \ \rho \rightarrow$
 $P \vdash \text{mapsto } sh \ (\text{typeof } a) \ v_1 \ v_2 * \text{TT} \rightarrow$
 $v_2 <> \text{Vundef} \rightarrow$
 $P \vdash \text{rel_expr } a \ v_2 \ \rho.$
 $\text{rel_lvalue_local: } \forall (\text{id} : \text{ident}) \ \tau \ (b : \text{block}) \ P \ \rho,$
 $P \vdash \text{!!}(\text{Map.get } (\text{ve_of } \rho) \ \text{id} = \text{Some } (b, \tau)) \rightarrow$
 $P \vdash \text{rel_lvalue } (\text{Evar } \text{id} \ \tau) \ (\text{Vptr } b \ \text{Int.zero}) \ \rho.$
 $\text{rel_lvalue_global: } \forall (\text{id} : \text{ident}) \ \tau \ (v : \text{val}) \ P \ \rho,$
 P
 $\vdash \text{!!}(\text{Map.get } (\text{ve_of } \rho) \ \text{id} = \text{None} \wedge$
 $\text{Map.get } (\text{ge_of } \rho) \ \text{id} = \text{Some } (v, \tau)) \rightarrow$
 $P \vdash \text{rel_lvalue } (\text{Evar } \text{id} \ \tau) \ v \ \rho.$
 $\text{rel_lvalue_deref: } \forall (a : \text{expr}) \ (b : \text{block}) \ (z : \text{int}) \ \tau \ P \ \rho,$
 $P \vdash \text{rel_expr } a \ (\text{Vptr } b \ z) \ \rho \rightarrow$
 $P \vdash \text{rel_lvalue } (\text{Ederef } a \ \tau) \ (\text{Vptr } b \ z) \ \rho.$
 $\text{rel_lvalue_field_struct: } \forall (i \ \text{id} : \text{ident}) \ \tau \ e \ (b : \text{block}) \ (z : \text{int}) \ (\text{fList} : \text{fieldlist}) \ \text{att} \ ($
 $\text{typeof } e = \text{Tstruct } \text{id} \ \text{fList} \ \text{att} \rightarrow$
 $\text{field_offset } i \ \text{fList} = \text{Errors.OK } \delta \rightarrow$
 $P \vdash \text{rel_expr } e \ (\text{Vptr } b \ z) \ \rho \rightarrow$
 $P \vdash \text{rel_lvalue } (\text{Efield } e \ i \ \tau) \ (\text{Vptr } b \ (\text{Int.add } z \ (\text{Int.repr } \delta))) \ \rho.$

The primitive nested-load assignment rule is,

Axiom `semax_loadstore`:

$$\begin{aligned} &\forall v0\ v1\ v2\ \Delta\ e1\ e2\ sh\ P\ P', \\ &\quad \text{writable_share}\ sh \rightarrow \\ &\quad P \vdash !!\ (\text{tc_val}\ (\text{typeof}\ e1)\ v2) \\ &\quad \quad \&\&\ \text{rel_lvalue}\ e1\ v1 \\ &\quad \quad \&\&\ \text{rel_expr}\ (\text{Ecast}\ e2\ (\text{typeof}\ e1))\ v2 \\ &\quad \quad \&\&\ (\text{mapsto}\ sh\ (\text{typeof}\ e1)\ v1\ v0) * P') \rightarrow \\ &\text{semax}\ \Delta\ (\triangleright P)\ (\text{Sassign}\ e1\ e2) \\ &\quad (\text{normal_ret_assert}\ (\text{mapsto}\ sh\ (\text{typeof}\ e1)\ v1\ v2) * P')). \end{aligned}$$

but do not use this rule! It is best to use a derived rule, such as,

Lemma `semax_loadstore_array`:

$$\begin{aligned} &\forall n\ vi\ lo\ hi\ t1\ (\text{contents: } Z \rightarrow \text{retype}\ t1)\ v1\ v2\ \Delta\ e1\ ei\ e2\ sh\ P\ Q\ R, \\ &\quad \text{retype}\ t1 = \text{val} \rightarrow \\ &\quad \text{type_is_by_value}\ t1 \rightarrow \\ &\quad \text{legal_alignas_type}\ t1 = \text{true} \rightarrow \\ &\quad \text{typeof}\ e1 = \text{tptr}\ t1 \rightarrow \\ &\quad \text{typeof}\ ei = \text{tint} \rightarrow \\ &\quad \text{PROP}_x\ P\ (\text{LOCAL}_x\ Q\ (\text{SEP}_x\ R)) \\ &\quad \quad \vdash \text{rel_expr}\ e1\ v1 \\ &\quad \quad \&\&\ \text{rel_expr}\ ei\ (\text{Vint}\ (\text{Int.repr}\ vi)) \\ &\quad \quad \&\&\ \text{rel_expr}\ (\text{Ecast}\ e2\ t1)\ v2 \rightarrow \\ &\quad \text{nth_error}\ R\ n = \text{Some}\ (\text{array_at}\ t1\ sh\ \text{contents}\ lo\ hi\ v1)) \rightarrow \\ &\quad \text{writable_share}\ sh \rightarrow \\ &\quad \text{tc_val}\ t1\ v2 \rightarrow \\ &\quad \text{in_range}\ lo\ hi\ vi \rightarrow \\ &\quad \text{semax}\ \Delta\ (\triangleright \text{PROP}_x\ P\ (\text{LOCAL}_x\ Q\ (\text{SEP}_x\ R))) \\ &\quad (\text{Sassign}\ (\text{Ederef}\ (\text{Ebinop}\ \text{Oadd}\ e1\ ei\ (\text{tptr}\ t1))\ t1)\ e2) \\ &\quad (\text{normal_ret_assert} \\ &\quad (\text{PROP}_x\ P\ (\text{LOCAL}_x\ Q\ (\text{SEP}_x \\ &\quad (\text{replace_nth}\ n\ R \\ &\quad \quad \text{array_at}\ t1\ sh\ (\text{upd}\ \text{contents}\ vi\ (\text{valinject}\ _ v2))\ lo\ hi\ v1)))))). \end{aligned}$$

Proof-automation support is available for `semax_loadstore_array` and `rel_expr`, in the form of the `forward_n1` (for “forward nested loads”) tactic. For example, with this proof goal,

`semax Delta`

```
(PROP ())
  LOCAL(`(eq (Vint (Int.repr i))) (eval_id _i); `(eq x) (eval_id _x);
    `(eq y) (eval_id _y); `(eq z) (eval_id _z))
  SEP(`(array_at tdouble Tsh (Vfloat oo fx) 0 n x);
    `(array_at tdouble Tsh (Vfloat oo fy) 0 n y);
    `(array_at tdouble Tsh (Vfloat oo fz) 0 n z)))
(Ssequence
  (Sassign (* x[i] = y[i] + z[i]; *)
    (Ederef (Ebinop Oadd (Etempvar _x (tptr tdouble)) (Etempvar _i tint)
      (tptr tdouble)) tdouble)
    (Ebinop Oadd
      (Ederef (Ebinop Oadd (Etempvar _y (tptr tdouble)) (Etempvar _i tint)
        (tptr tdouble)) tdouble)
      (Ederef (Ebinop Oadd (Etempvar _z (tptr tdouble)) (Etempvar _i tint)
        (tptr tdouble)) tdouble) tdouble))
    MORE_COMMANDS)
  POSTCONDITION
```

the tactic-application `forward_n1` yields the new proof goal,

`semax Delta`

```
(PROP ())
  LOCAL(`(eq (Vint (Int.repr i))) (eval_id _i); `(eq x) (eval_id _x);
    `(eq y) (eval_id _y); `(eq z) (eval_id _z))
  SEP
    (`(array_at tdouble Tsh
      (upd (Vfloat oo fx) i (Vfloat (Float.add (fy i) (fz i)))) 0 n x);
      `(array_at tdouble Tsh (Vfloat oo fy) 0 n y);
      `(array_at tdouble Tsh (Vfloat oo fz) 0 n z)))
  MORE_COMMANDS
  POSTCONDITION
```

This chapter is needed only by “power users.”

Assertions in our Hoare triple of separation are presented as $\text{env} \rightarrow \text{mpred}$, that is, functions from environment to memory-predicate, using our natural deduction system $\text{NatDed}(\text{mpred})$ and separation logic $\text{SepLog}(\text{mpred})$.

Given a separation logic over a type B of formulas, and an arbitrary type A , we can define a *lifted* separation logic over functions $A \rightarrow B$. The operations are simply lifted pointwise over the elements of A . Let $P, Q : A \rightarrow B$, let $R : T \rightarrow A \rightarrow B$ then define,

$$\begin{aligned}
 (P \&\& Q) : A \rightarrow B &:= \text{fun } a \Rightarrow Pa \&\& Qa \\
 (P \parallel Q) : A \rightarrow B &:= \text{fun } a \Rightarrow Pa \parallel Qa \\
 (\exists x. R(x)) : A \rightarrow B &:= \text{fun } a \Rightarrow \exists x. Rxa \\
 (\forall x. R(x)) : A \rightarrow B &:= \text{fun } a \Rightarrow \forall x. Rxa \\
 (P \longrightarrow Q) : A \rightarrow B &:= \text{fun } a \Rightarrow Pa \longrightarrow Qa \\
 (P \vdash Q) : A \rightarrow B &:= \forall a. Pa \vdash Qa \\
 (P * Q) : A \rightarrow B &:= \text{fun } a \Rightarrow Pa * Qa \\
 (P \multimap Q) : A \rightarrow B &:= \text{fun } a \Rightarrow Pa \multimap Qa
 \end{aligned}$$

In Coq we formalize the typeclass instances LiftNatDed , LiftSepLog , etc., as shown below. For a type B , whenever $\text{NatDed } B$ and $\text{SepLog } B$ (and so on) have been defined, the lifted instances $\text{NatDed } (A \rightarrow B)$ and $\text{SepLog } (A \rightarrow B)$ (and so on) are automatically provided by the typeclass system.

Instance $\text{LiftNatDed}(A \ B : \text{Type})\{\text{ND} : \text{NatDed } B\} : \text{NatDed } (A \rightarrow B) := \text{mkNatDed } (A \rightarrow B)$

```

(*andp*) (fun P Q x => andp (P x) (Q x))
(*orp*) (fun P Q x => orp (P x) (Q x))
(*exp*) (fun {T} (F: T -> A -> B) (a: A) => exp (fun x => F x a))
(*allp*) (fun {T} (F: T -> A -> B) (a: A) => allp (fun x => F x a))
(*imp*) (fun P Q x => imp (P x) (Q x))
(*prop*) (fun P x => prop P)
(*derives*) (fun P Q => forall x, derives (P x) (Q x))

```

```

Instance LiftSepLog (A B: Type) {NB: NatDed B}{SB: SepLog B}
  : SepLog (A → B).
  apply (mkSepLog (A → B) _ (fun ρ ⇒ emp)
    (fun P Q ρ ⇒ P ρ * Q ρ) (fun P Q ρ ⇒ P ρ -* Q ρ)).
  (* fill in proofs here *)

```

In particular, if P and Q are functions of type $\text{environ} \rightarrow \text{mpred}$ then we can write $P * Q$, $P \&\& Q$, and so on.

Consider this assertion:

```

fun ρ ⇒ mapsto sh tint (eval_id _x ρ) (eval_id _y ρ)
  * mapsto sh tint (eval_id _u ρ) (Vint Int.zero)

```

which might appear as the precondition of a Hoare triple. It represents $(x \mapsto y) * (u \mapsto 0)$ written in informal separation logic, where x, y, u are C-language variables of integer type. Because it can be inconvenient to manipulate explicit lambda expressions and explicit environment variables ρ , we may write it in lifted form,

```

` (mapsto sh tint) (eval_id _x) (eval_id _y)
* ` (mapsto sh tint) (eval_id _u) `(Vint Int.zero)

```

Each of the first two backquotes lifts a function from type $\text{val} \rightarrow \text{val} \rightarrow \text{mpred}$ to type $(\text{environ} \rightarrow \text{val}) \rightarrow (\text{environ} \rightarrow \text{val}) \rightarrow (\text{environ} \rightarrow \text{mpred})$, and the third one lifts from val to $\text{environ} \rightarrow \text{val}$.

56 *Mapsto and func_ptr* (see PLCC section 24)

Aside from the standard operators and axioms of separation logic, the core separation logic has just two primitive spatial (memory) predicates:

Parameter `address_mapsto`:

`memory_chunk` \rightarrow `val` \rightarrow `share` \rightarrow `share` \rightarrow `address` \rightarrow `mpred`.

Parameter `func_ptr` : `funspec` \rightarrow `val` \rightarrow `mpred`.

`func_ptr` ϕ `v` means that value v is a pointer to a function with specification ϕ ; see ??.

`address_mapsto` expresses what is typically written $x \mapsto y$ in separation logic, that is, a singleton heap containing just value y at address x .

From this, we construct two low-level derived forms:

`mapsto (sh:share) (t:type) (v w: val) : mpred` describes a singleton heap with just one value w of (C-language) type t at address v , with permission-share sh .

`mapsto_ (sh:share) (t:type) (v:val) : mpred` describes an *uninitialized* singleton heap with space to hold a value of type t at address v , with permission-share sh .

From these primitives, `field_at` and `data_at` are constructed.

57 *with_library*: Library functions

A CompCert C program is implicitly linked with dozens of “built-in” and library functions. In the .v file produced by clightgen, the prog-defs component of your prog lists these as External definitions, along with the Internal definitions of your own functions. *Every one of these needs a funspec*, of the form DECLARE...WITH..., and this funspec must be *proved* with a semax_ext proof.

Fortunately, if your program does not use a given library function f , then the funspec DECLARE _f WITH...PRE[...] False POST... with a **False** precondition is easy to prove! The tactic with_library prog [$s_1; s_2; \dots; s_n$] augments your explicit funspec-list [$s_1; s_2; \dots; s_n$] with such trivial funspecs for the other functions in the program prog.

YOU MAY WISH to use standard library functions such as malloc, free, exit. These are axiomatized (with external funspecs) in floyd.library. To use them, **Require Import** floyd.library *after* you import floyd.proofauto. This imports (floyd.library.)with_library tactic hiding the standard (floyd.forward.)with_library tactic; the new one includes *axiomatized* specifications for malloc, free, exit, etc. We haven’t proved the implementations against the axioms, so if you don’t trust them, then don’t import floyd.library.

The next chapters explain the specifications of certain standard-library functions.

58 *Malloc, free*

59 *exit*

60 *Function pointers*

Parameter $\text{func_ptr} : \text{funspec} \rightarrow \text{val} \rightarrow \text{mpred}$.

Definition $\text{func_ptr}' f v := \text{func_ptr } f v \ \&\& \ \text{emp}$.

$\text{func_ptr } \phi v$ means that value v is a pointer to a function with specification ϕ .

$\text{func_ptr}' \phi v$ is a form more suitable to be a conjunct of a SEP clause.

Verifiable C's program logic is powerful enough to reason expressively about function pointers (see PLCC Chapters 24 and 29). However, the Floyd proof-automation system does not have much support for proving such programs at present.

61 Axioms of separation logic (see PLCC 105 Chapter 12)

These axioms of separation logic are often useful, although generally it is the automation tactics (entailer, cancel) that apply them.

pred_ext: $P \vdash Q \rightarrow Q \vdash P \rightarrow P = Q$.
derives_refl: $P \vdash P$.
derives_trans: $P \vdash Q \rightarrow Q \vdash R \rightarrow P \vdash R$.
andp_right: $X \vdash P \rightarrow X \vdash Q \rightarrow X \vdash (P \&\& Q)$.
andp_left1: $P \vdash R \rightarrow P \&\& Q \vdash R$.
andp_left2: $Q \vdash R \rightarrow P \&\& Q \vdash R$.
orp_left: $P \vdash R \rightarrow Q \vdash R \rightarrow P || Q \vdash R$.
orp_right1: $P \vdash Q \rightarrow P \vdash Q || R$.
orp_right2: $P \vdash R \rightarrow P \vdash Q || R$.
exp_right: $\forall \{B: \text{Type}\} (x: B) (P: \text{mpred}) (Q: B \rightarrow \text{mpred}),$
 $P \vdash Q \ x \rightarrow P \vdash \text{EX } x: B, Q$.
exp_left: $\forall \{B: \text{Type}\} (P: B \rightarrow \text{mpred}) (Q: \text{mpred}),$
 $(\forall x, P \ x \vdash Q) \rightarrow \text{EX } x: B, P \vdash Q$.
allp_left: $\forall \{B\} (P: B \rightarrow \text{mpred}) \ x \ Q, P \ x \vdash Q \rightarrow \text{ALL } x: B, P \vdash Q$.
allp_right: $\forall \{B\} (P: \text{mpred}) (Q: B \rightarrow \text{mpred}),$
 $(\forall v, P \vdash Q \ v) \rightarrow P \vdash \text{ALL } x: B, Q$.
prop_left: $\forall (P: \text{Prop}) \ Q, (P \rightarrow (\text{TT} \vdash Q)) \rightarrow !!P \vdash Q$.
prop_right: $\forall (P: \text{Prop}) \ Q, P \rightarrow (Q \vdash !!P)$.
not_prop_right: $\forall (P: \text{mpred}) (Q: \text{Prop}), (Q \rightarrow (P \vdash \text{FF})) \rightarrow P \vdash !(\sim Q)$.

sepcon_assoc: $(P * Q) * R = P * (Q * R)$.
sepcon_comm: $P \ Q, P * Q = Q * P$.
sepcon_andp_prop: $P * (!Q \ \&\& \ R) = !Q \ \&\& \ (P * R)$.
derives_extract_prop: $(P \rightarrow Q \vdash R) \rightarrow !!P \ \&\& \ Q \vdash R$.
sepcon_derives: $P \vdash P' \rightarrow Q \vdash Q' \rightarrow P * Q \vdash P' * Q'$.

62 *Obscure higher-order axioms*

imp_andp_adjoint: $P \&\&Q \vdash R \leftrightarrow P \vdash (Q \longrightarrow R)$.

wand_sepcon_adjoint: $P * Q \vdash R \leftrightarrow P \vdash Q * R$.

ewand_sepcon: $(P * Q) \multimap R = P \multimap (Q \multimap R)$.

ewand_TT_sepcon: $\forall (P \ Q \ R: A),$

$(P * Q) \&\&(R \multimap TT) \vdash (P \&\&(R \multimap TT)) * (Q \&\&(R \multimap TT))$.

exclude_elsewhere: $P * Q \vdash (P \&\&(Q \multimap TT)) * Q$.

ewand_conflict: $P * Q \vdash FF \rightarrow P \&\&(Q \multimap R) \vdash FF$

now_later: $P \vdash \triangleright P$.

later_K: $\triangleright (P \longrightarrow Q) \vdash (\triangleright P \longrightarrow \triangleright Q)$.

later_allp: $\forall T (F: T \rightarrow \text{mpred}), \triangleright (\text{ALL } x:T, F \ x) = \text{ALL } x:T, \triangleright (F \ x)$.

later_exp: $\forall T (F: T \rightarrow \text{mpred}), \text{EX } x:T, \triangleright (F \ x) \vdash \triangleright (\text{EX } x: F \ x)$.

later_exp': $\forall T (\text{any}:T) F, \triangleright (\text{EX } x: F \ x) = \text{EX } x:T, \triangleright (F \ x)$.

later_imp: $\triangleright (P \longrightarrow Q) = (\triangleright P \longrightarrow \triangleright Q)$.

loeb: $\triangleright P \vdash P \rightarrow TT \vdash P$.

later_sepcon: $\triangleright (P * Q) = \triangleright P * \triangleright Q$.

later_wand: $\triangleright (P \multimap Q) = \triangleright P \multimap \triangleright Q$.

later_ewand: $\triangleright (P \multimap Q) = (\triangleright P) \multimap (\triangleright Q)$.

63 Proving larg(ish) programs

When your program is not all in one .c file, see also [Chapter 64](#). Whether or not your program is all in one .c file, you can prove the individual function bodies in separate .v files. This uses less memory, and (on a multicore computer with parallel make) saves time. To do this, put your API spec (up to the construction of Gprog in one file; then each `semax_body` proof in a separate file that imports the API spec.

EXTRACTION OF SUBORDINATE SEMAX-GOALS. To ease memory pressure and recompilation time, it is often advisable to partition the proof of a function into several lemmas. Any proof state whose goal is a `semax`-term can be extracted as a stand-alone statement by invoking tactic `semax_subcommand V G F`. The three arguments are as in the statement of surrounding `semax-body` lemma, i.e. are of type *varspecs*, *funspecs*, and *function*.

The subordinate tactic `mkConciseDelta V G F Δ` can also be invoked individually, to concisely display the type context Δ as the application of a sequence of initializations to the host function's `func_tycontext`.

THE FREEZER. A distinguishing feature of separation logic is the frame rule, i.e. the ability to modularly verify a statement w.r.t. its minimal resource footprint. Unfortunately, being phrased in terms of the syntactic program structure, the standard frame rule does not easily interact with forward symbolic execution as implemented by the Floyd tactics (and many other systems), as these continuously rearrange the associativity of statement sequencing to peel off the redex of the next *forward*, and (purposely) hide the program continuation as the abbreviation `MORE_COMMANDS`.

Resolving this conflict, Floyd's *freezer* abstraction provides a means for flexible framing, by implementing a veil that opaquely hides selected items of a SEP clause from non-symbolic treatment by non-freezer tactics.

The freezer abstraction consists of two main tactics, *freeze* $N\ F$ and *thaw* F , where $N : \text{list nat}$ and F is a user-supplied (fresh) Coq name. The result of applying *freeze* $[i_1; \dots; i_n]\ F$ to a semax goal is to remove items i_1, \dots, i_n from the precondition's SEP clause, inserting the item $FRZL\ F$ at the head of the SEP list, and adding a hypothesis $F := \text{abbreviate}$ to Coq's proof context.

The term $FRZL\ F$ participates symbolically in all non-freezer tactics just like any other SEP item, so can in particular be canceled, and included in a function call's frame. Unfolding a freezer is not tied to the associativity structure of program statements but can be achieved by invoking *thaw* F , which simply replaces $FRZL\ F$ by the the list of F 's constituents. As multiple freezers can coexists and freezers can be arbitrarily nested, SEP-clauses R effectively contain forests of freezers, each constituent being thawable independently and freezer-level by freezer-level.

Wrapping single *forward* or *forward_call* commands in a freezer often speeds up the processing time noticably, as invocations of subordinate tactics *entailer*, *cancel*, etc. are supplied with smaller and more symbolic proof goals. In our experience, applying the freezer throughout the proof of an entire function body typically yields a speedup of about 30% on average with improvements of up to 55% in some cases, while also easing the memory pressure and freeing up valuable real estate on the user's screen.

A more invasive implementation of a freezer-like abstraction would refine the $\text{PROP}(P)\ \text{LOCAL}(Q)\ \text{SEP}(R)$ structure to terms of the form $\text{PROP}(P)\ \text{LOCAL}(Q)\ \text{SEP}(R)\ \text{FR}(H)$ where $H : \text{list mpred}$. Again, terms in H would be treated opaquely by all tactics, and freezing/thawing would correspond to transfer rules between R and H . In either case, forward symbolic execution is reconciled with the frame rule, and the use of the mechanism is sound engineering practice as documentation of programmer's insight is combined with performance improvements.

64 *Separate compilation*, semax_ext

What to do when your program is spread over multiple .c files.

CODE PREPARATION. In order to separate the namespaces of multiple files compiled by CompCert's clightgen tool, it is necessary to apply

```
python fix_clightgen.py file1.v ...fileN.v
```

The script reads in the named files, concisely renames variables etc by making up new positives, and writes the modified files back to the given names.

65 *Catalog of tactics / lemmas*

Below is an alphabetic catalog of the major floyd tactics. In addition to short descriptions, the entries indicate whether a tactic (or tactic notation) is typically user-applied [u], primarily of internal use [i] or is expected to be used at development-time but unlikely to appear in a finished proof script [d]. We also mention major interdependencies between tactics, and their points of definition.

- cancel** (tactic; [page 58](#)) Deletes identical spatial conjuncts from both sides of a base-level entailment.
- derives_refl** (lemma) $A \vdash A$. Useful after **cancel** to handle $\beta\eta$ -equality; see [page 58](#).
- derives_refl'** (lemma) $A = B \rightarrow A \vdash B$.
- entailer** (tactic; [page 59](#), [page 25](#)) Proves (lifted or base-level) entailments, possibly leaving a residue for the user to prove. The more aggressive **entailer!** should usually be used, but it sometimes turns a provable goal into an unprovable goal.
- drop_LOCAL** n (tactic, where $n : nat$). Removes the n th entry of a the LOCAL block of a **semax** or **ENTAIL** precondition.
- forward** (tactic; [page 20](#)) Do forward Hoare-logic proof through one C statement (assignment, break, continue, return).
- forward_call** **ARGS** (tactic; [page 22](#), [page ??](#)) Forward Hoare-logic proof through one C function-call, where **ARGS** is a witness for the **WITH** clause of the **funspec**.
- forward_for** (tactic) This tactic does not work well in VST 1.6. Use **forward_for_simple_bound** when applicable, or see the method described in [Chapter 44](#).
- forward_for_simple_bound** n **Inv** (tactic, [page 77](#)) When a for-loop has the form for (*init*; $i < hi$; $i++$), where n is the *value* of hi , and **Inv** is the loop invariant.
- forward_seq** (tactic)
- mkConciseDelta** V G F Δ (tactic) Applicable to a proof state with a **semax** goal. Simplifies the Δ component to the application of a sequence of initializations to the host function's **func_tycontext**.

Used to prepare the current proof goal for abstracting/factoring out as a separate lemma.

semax_subcommand $V\ G\ F$ (tactic) Applicable to a proof state with a semax goal. Extracts the current proof state as a stand-alone statement that can be copy-and pasted to a separate file. The three arguments should be copied from the statement of surrounding semax-body lemma: V : varspecs, G : funspecs, F : function.

unfold_data_at (tactic; [page 50](#)) When t is a struct (or array) type, break apart `data_at sh t v p` into a separating conjunction of its individual fields (or array elements).

unfold_field_at (tactic; [page 50](#)) Like `unfold_data_at`, but starts with `field_at sh t path v p`.