Mereology and Postmodernism

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Abstract

Almost all work on logic, symbolic reasoning, indeed, most of mathematics, is built on the intuitive foundation of "variables" and "functions" that map between them. There is an alternative viewpoint: that of "connectors", which is developed extensively in the accompanying texts.

This chapter focuses on how connectors have an impact on a number of philosophical arguments, ranging from mereology to postmodernism(!) This is a surprise (to this author, a novice at philosophy.)

Philosophy

Here's a hint of how pervasively important the concept of variables and values can be. Some philosophers use these notions to anchor the idea of "objects" or "things" that "exist":

"An object is anything that can be the value of a variable, that is, anything we can talk about using pronouns, that is, anything."[2, p. 180]

This conception of an object then leads to confused discussions about the identity of indiscernibles, and the importance of location (space-time) in mereology. The notion of connectors can be used as an alternate foundation for the conception of "objects", "identity" and "location". Specifically, one can instead define objects as "things that can participate in relationships".

Relational algebras

In mathematics, a "relational algebra" is a system in which one can define collections of things, and make statements about their relationships to one-another. Examples of relations are "is-a", "has-a", "part-of".

Relations have several notable properties: they provide an ordering (something may have a part which may have a part...) and thus a hierarchy. Formally, such hierarchies are referred to as "directed acyclic graphs" or DAG's. In general, any partial ordering can be represented with arrows – the directed edges of the DAG. That is, relations are usually asymmetric, but not always. If B is a part of A, then A is not a part of B, at least, not in any conventional sense. But if A is-a B, then by convention, B is-a A. One may write an entire book on the algebraic properties of relations; and indeed, more than a few hundred books have been written on the topic, to which the reader is invited to consult, for more information. I recommend [1].

By definition (i.e. so as not to confuse vocabulary), relations manifestly do *not* define functions or mappings or "terms". There is a broader algebraic system, called "term algebra", which does supply these combinatoric elements; the relational algebras are a fragment of term algebras.

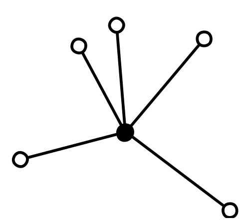
Relational algebras find common deployment in computer science, where they provide an explicit theory for databases (*e.g.* both SQL and no-SQL, and so on.) They are also central to dictionaries of natural language, providing relational explanations aside from part-of-speech tagging (*e.g.* "pitch•fork pĭch´fôrk˝ n. A large, long-handled fork with ...") In between these two extremes is the more general problem of knowledge representation, *i.e.* of capturing knowledge in such a way that a computer can manipulate it, that is, artificial intelligence, which is the ostensible topic of this collection of essays.

Limiting oneself to a single part-whole relationship, one arrives at mereology as a possible alternative to set theory as a foundation for mathematics. But one also arrives the the notion of "gunk" in philosophy (that, roughly speaking, infinite recursion is possible, when considering the structure of physical reality), as well as a number of puzzles dating to Ancient Greece (the Ship of Theseus: what happens when parts are replaced? The Statue and the Lump of Clay: what happens when parts are rearranged?). That there are such puzzles is perhaps endemic to the desire to apply part-whole relationships, and the adduced DAG partial orders as a fundamental ground on which to build philosophical discussions. it also perhaps suggests that a search be undertaken for some other foundational principle. This text offers up "connectors" as the alternative.

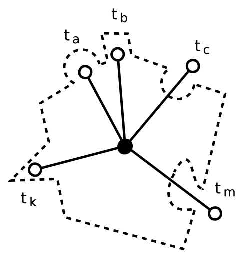
Mereology

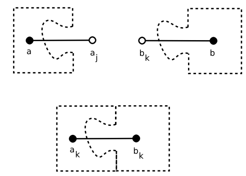
The paradoxes of mereology mentioned just above: the difficulty of defining part-whole relationships in a consistent manner, the problem of the existence or non-existence of "gunk", and even the puzzles from Ancient Greece (the Ship of Theseus, *etc.*) One proposed solution to these riddles is the notion of "restricted composition" (see Wikipedia [3]). The idea is that "things" are still composed of "other things", but the manner in which these can be composed is restricted. The body of ideas is very nearly identical to the concept of seeds and sheaves introduced in the earlier chapter.

To recap, here is a seed:



Think of it as a burr, the black dot in the center being the seed; the white circles being unattached connectors. The connectors may be typed, so that they attach much as jigsaw-puzzle pieces. To make it painfully obvious, some figures:





The explicit labels in the figures above have no particular meaning for this text; rather, keep in mind the general notion of jigsaw-puzzle pieces for the remainder of this text.

Then, quoting directly from the Wikipedia article on mereology, we can can learn that the notions of restricted compositionality include:

Contact — that a complex object Y is composed of X's if and only if the X's are in contact. *viz.* that seeds may attach only if they are sufficiently close to one-another to become connectable.

Fastenation — the X's compose a complex Y if and only if the X's are fastened. *viz.* seeds attach only if the connectors allow mating. Once mated, they are fastened together.

Cohesion — the X's compose a complex Y if and only if the X's cohere (cannot be pulled apart or moved in relation to each other without breaking). The point of connectors is that they are not meant to be pulled apart, once mated. Insofar as the abstract definition of a sheaf given in earlier sections failed to appeal to and force any order of the connectors, or to the length of the edges, the concept of "movement" remains incomplete.[1]

Fusion — the X's compose a complex Y if and only if the X's are fused (fusion is when the X's are joined together such that there is no boundary). Connectors, once connected, serve no further purpose, and are discarded. What mattered was the bond that was made. In this sense, the bonds are the fusion of connectors.[2]

Brutal Composition — "It's just the way things are." There is no true, nontrivial, and finitely long answer.

The last point is perhaps the most interesting notion coming out of restricted composition. It is not an ingredient, it is rather a consequence. If one abandons the explicit DAGs of partial orders, one then also abandons the notion of logical predicates and

[1]This can be repaired in a variety of ways. In practical applications, it is convenient to order the edges, and to give them weights. These weights can sometimes act as distances, e.g. in N-dimensional space. Furthermore, the branch of mathematics known as "algebraic topology" is very highly developed, and can provide very explicit and concrete statements about the structure of space. For example, the dimensionality of the embedding of the network is given by the degree of the largest simplicial complex in the network.

[2]This is reminiscent of Derrida's statement that "deconstruction is not analysis". One cannot break up meaning into atomic parts; there are no self-sufficient atomic parts of meaning. Meaning derives only from the fusion of words into text and into language.

truth assignments. One no longer arranges logical connectives into a tree structure, or even into Horn clauses. The network, rather, represents things "as they are". This is not to deny predicate logic, or Boolean satisfiability: these are tremendously useful concepts. Rather, this is to avoid the symbol grounding problem (ref Wikipedia, again). By discarding the arrow from "signifier" to "significand", one can no longer peer into the gaping chasm of the question "what does this symbol mean?"

Postmodernism

These last observations are very much in line with theories of postmodernism and deconstruction evident in the works of Derrida,[3] Deleuze, Lyotard. There is no difference between appearance and true form; the difference is undecidable. The constructed network of connections and relations is all there is.[4]

In the remaining chapters, the sheaf construction will be used as a tool to create A(G)I representations of reality. Whether the constructed network is an accurate representation of reality is undecidable, and this is true even in a narrow formal sense. Famously, it is known that the question of whether two different presentations of a (group-theoretical) group refer to the same group is undecidable, in the computational sense: there is no algorithm, guaranteed to terminate in finite time, that is capable of making this decision. This is the undecidability problem in group theory. That this result carries over into the fullness of a network built from a sheaf should come as no surprise: the presentation of a group is little more than a certain collection of labeled vertexes and edges (the group elements and the group operations) and decidability requires the graph-rewriting of these into a different form. There are no graph rewrite rules that can always bring two different presentations into normal form. Oddly enough, such "received wisdom" from mathematics can be used as a foundational cornerstone for post-modernism. This is unexpected.[5]

Much of postmodernism is concerned with linguistics. This is more directly attacked in the subsequent chapters.

[3] The différance of Derrida can be seen as a rejection of the linguistic equivalent of mereological nihilism: there is no infinite regression of looking up the meanings of words in a dictionary.

[4] In this sense, it is also a rejection of Searle's "Chinese Rom": there is no additional meaning beyond the vacant activity of the mindless drones performing algorithmic actions. That is all that there is, nothing more. For Searle to pretend that there is something more to meaning, language and translation is effectively an appeal to the existence of some sort of magical metaphysical quintessence of language that simply is not there. Words have meaning insofar as they related to each other; a bad translation rendered by a mindless Chinese Room is nothing more than a bad translation. The Chinese Room neithr demonstrates the existence of intelligence, nor the absence of it.

[5] Despite this, sheaves are still somehow fundamentally "structuralist", not post-structuralist. The bonds are relational, in the end. There is always still a predicate: either an edge exists, or it doesn't. To every graph there is a corresponding adjacency matrix, populated with zeroes and ones. Fortunately, one can wriggle ones way out if this, by assigning numerical weights to edges, making some probable and some improbable. Whether true and false can be replaced by a subobject classifier in this context is perhaps a step too far, but perhaps also an interesting step.

References

[1] Wilifred Hodges. *A Shorter Model Theory*. Cambridge University Press, 1997.

[2] P. Van Inwagen. The number of things. *Philosophical Issues*, 12:176–196, 200s23.

[3] Wikipedia. Mereology, 2020.