

Think of $f \in \mathcal{O}(X)$ or $L^2(X) \approx H(X)$
 where $X = \mathbb{P}^1 - \{0, 1, \infty\}$ holomorphic

so $\mathcal{O}(X)_x = k[x]$ $\mathcal{O}(X)_y = k\left[\frac{1}{x}\right]$
 \parallel
 y

$$\begin{array}{ccc} k[x, y]/(xy) & \longrightarrow & k[x] \\ \downarrow & & \downarrow \\ k[y] & \longrightarrow & k \xrightarrow{\lambda} k \end{array} \quad \begin{array}{l} \lambda \in k^* = GL_1(k) \\ = T_k^1 \\ = SL_1(k) \times T_k^1 \end{array}$$

Since f holomorphic on X , on any open disk \mathbb{D} with $\mathbb{D} \cap \{0, 1, \infty\} = \emptyset$ can uniformly approximate f with polynomials.

On any \mathbb{D}^* punctured disk can uniformly approximate by ~~holomorphic~~ rational functions.

If $f = L(X, s) = \sum_{n \in \mathbb{Z}} \frac{X(n)}{n^s}$

or $= \prod_{p \text{ prime}} (1 - \frac{X(p)}{p^s})^{-1}$

$\prod_{p \text{ prime}} \det \left(1 - \frac{X(p)}{p^s} \right)^{-1}$