The $g^{(2)}$ of noisy squeezed light

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We are interested in calculating the second order coherence function at zero delay

$$g^{(2)} = \frac{\langle N^2 - N\rangle}{\langle N\rangle^2}, \tag{1}$$

for a beam that contains a number of single mode squeezed vacuum states and a "dark mode" with Poisson number statistics. We use the index 0 for the dark mode and Latin indices for the squeezed modes starting at 1. Finally, we use Greek indices for the dark mode together with the squeezed modes thus,

$$N = n_0 + \sum_{i=1} n_i = \sum_{\mu=0} n_\mu. \tag{2}$$

Since the dark mode has Poisson statistics then

$$\langle n_0\rangle = \bar{n}_0, \quad \langle n_0^2\rangle = \bar{n}_0^2 + \bar{n}_0. \tag{3}$$

We can then write the denominator in Eq. (1) as

$$\langle N^2 - N\rangle = \langle \sum_\mu n_\mu \sum_\nu n_\nu\rangle - \langle \sum_\mu n_\mu\rangle \tag{4}$$

$$= \langle n_0^2 + 2n_0 \sum_i n_i + \sum_i \sum_j n_i n_j\rangle - \bar{n}_0 - \bar{M}, \tag{5}$$

where we introduced $\bar{M} = \sum_i \bar{n}_i = \sum_i \langle n\rangle_i$ as the total mean photon number of the squeezed modes, in terms of which we can write

$$\langle N\rangle = \bar{N} = \bar{n}_0 + \bar{M} \tag{6}$$

We also assume that the dark mode is uncorrelated with the squeezed modes allowing us to write

$$\langle N^2 - N\rangle = \bar{n}_0^2 + 2\bar{n}_0\bar{M} + \left\langle \sum_i \sum_j n_i n_j \right\rangle - \bar{M}, \tag{7}$$

where in the last line we used the fact that $\langle n_0^2\rangle - \bar{n}_0 = \bar{n}_0^2$.

We can finally write the last equation as

$$\langle N^2 - N\rangle = \bar{M}^2 \left(\underbrace{\frac{\langle \sum_i \sum_j n_i n_j\rangle - \bar{M}}{\bar{M}^2}}_{\equiv g^{(2)}_{\text{noiseless}}} + 2\frac{\bar{n}_0}{\bar{M}} + \left(\frac{\bar{n}_0}{\bar{M}}\right)^2 \right). \tag{8}$$

In the last equation we introduced $g^{(2)}_{\text{noiseless}}$ as the second order coherence function if there were no noise photons.

One can also easily write the denominator of Eq. (1) as

$$\langle N\rangle^2 = \bar{M}^2 \left(1 + 2\frac{\bar{n}_0}{\bar{M}} + \left(\frac{\bar{n}_0}{\bar{M}}\right)^2\right). \tag{9}$$

With this last piece we finally write

$$g^{(2)} = \frac{g^{(2)}_{\text{noiseless}} + 2\frac{\bar{n}_0}{\bar{M}} + \left(\frac{\bar{n}_0}{\bar{M}}\right)^2}{1 + 2\frac{\bar{n}_0}{\bar{M}} + \left(\frac{\bar{n}_0}{\bar{M}}\right)^2}. \tag{10}$$

The last equation can expanded in the limit where $n_0/M \ll 1$ to obtain

$$g^{(2)} = g^{(2)}_{\text{noiseless}} - 2\left(g^{(2)}_{\text{noiseless}} - 1\right)\frac{n_0}{M} \tag{11}$$

One can now specialize the last equation to two cases. The first one is where on the modes of interest come from a degenerate squeezer

$$g^{(2)}_{\text{noiseless}} = 1 + \frac{1}{\bar{M}} + \frac{2}{K}. \tag{12}$$

where

$$K = \frac{\left(\sum_i \bar{n}_i\right)^2}{\sum_i \bar{n}_i^2} \geq 1. \tag{13}$$

where K is the Schmidt number. The second case is when one has one half of a twin beam for which one has (cf. Christ et al.)

$$g^{(2)}_{\text{noiseless}} = 1 + \frac{1}{K}. \tag{14}$$

Note that in general the amount of noise in the "signal" and "idler" arm of a twin beam can be different leading to different values for the $g^{(2)}$. In the limit where the noise photon number is much smaller than the twin beam photon number one can write the following equality

$$g_s^{(2)} - g_i^{(2)} = -2(g^{(2)}_{\text{noiseless}} - 1)\frac{n_s - n_i}{M} \tag{15}$$