

Ekvivalentumai:

$$\begin{aligned} \text{sin_pi_k: } \sin(\pi \cdot k) &= 0; \\ \sin(\pi k) &= 0 \end{aligned}$$

$$\begin{aligned} \text{sin_pi_m: } \sin(\pi \cdot m) &= 0; \\ \sin(\pi m) &= 0 \end{aligned}$$

$$\begin{aligned} \text{sin_3pi_k: } \sin(3 \cdot \pi \cdot k) &= 0; \\ \sin(3 \pi k) &= 0 \end{aligned}$$

$$\begin{aligned} \text{cos_pi_k: } \cos(\pi \cdot k) &= (-1)^k; \\ \cos(\pi k) &= (-1)^k \end{aligned}$$

$$\begin{aligned} \text{cos_pi_m: } \cos(\pi \cdot m) &= (-1)^m; \\ \cos(\pi m) &= (-1)^m \end{aligned}$$

Integravimas [1]

$$\begin{aligned} \text{fn1: } (1 - \cos(2 \cdot \pi \cdot k \cdot x/2))/2; \\ \frac{1 - \cos(\pi k x)}{2} \end{aligned}$$

$$\begin{aligned} \text{int1: } \text{integrate}(\text{fn1}, x); \\ x - \frac{\sin(\pi k x)}{\pi k} \end{aligned}$$

$$\begin{aligned} \text{ratsimp}(\%); \\ - \frac{\sin(\pi k x) - \pi k x}{2 \pi k} \end{aligned}$$

$$\begin{aligned} \text{bound1_right: } \text{subst}(x=3, \text{int1}); \\ 3 - \frac{\sin(3 \pi k)}{\pi k} \end{aligned}$$

$$\begin{aligned} \text{bound1_left: } \text{subst}(x=0, \text{int1}); \\ 0 \end{aligned}$$

$$\begin{aligned} \text{int1: } \text{bound1_right} - \text{bound1_left}; \\ 3 - \frac{\sin(3 \pi k)}{\pi k} \end{aligned}$$

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int1:subst(sin_3pi_k, int1);
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$$\frac{3}{2}$$

Integravimas [2]

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fn2: (2/3)·sin(%pi·k·x/3)·sin(%pi·m·x/3);
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$$\frac{2 \sin\left(\frac{\pi k x}{3}\right) \sin\left(\frac{\pi m x}{3}\right)}{3}$$

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int2:integrate(fn2,x);
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$$\frac{2 \left[\frac{\sin\left(\left|\frac{\pi k}{3} - \frac{\pi m}{3}\right| x\right)}{2 \left|\frac{\pi k}{3} - \frac{\pi m}{3}\right|} - \frac{\sin\left(\left|\frac{\pi m}{3} + \frac{\pi k}{3}\right| x\right)}{2 \left|\frac{\pi m}{3} + \frac{\pi k}{3}\right|} \right]}{3}$$

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%,trigexpand=true,trigexpandplus=true,expand;
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$$\begin{aligned} & - \frac{2 \cos\left(\frac{\pi k x}{3}\right) \sin\left(\frac{\pi m x}{3}\right)}{2 \pi m + 2 \pi k} - \frac{2 \cos\left(\frac{\pi k x}{3}\right) \sin\left(\frac{\pi m x}{3}\right)}{2 \pi k - 2 \pi m} - \\ & \frac{2 \sin\left(\frac{\pi k x}{3}\right) \cos\left(\frac{\pi m x}{3}\right)}{2 \pi m + 2 \pi k} + \frac{2 \sin\left(\frac{\pi k x}{3}\right) \cos\left(\frac{\pi m x}{3}\right)}{2 \pi k - 2 \pi m} \end{aligned}$$

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int2:ratsimp(%);
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$$\frac{2 k \cos\left(\frac{\pi k x}{3}\right) \sin\left(\frac{\pi m x}{3}\right) - 2 m \sin\left(\frac{\pi k x}{3}\right) \cos\left(\frac{\pi m x}{3}\right)}{\pi m^2 - \pi k^2}$$

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bound2_right:subst(x=3, int2);
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$$\frac{2 k \cos(\pi k) \sin(\pi m) - 2 \sin(\pi k) m \cos(\pi m)}{\pi m^2 - \pi k^2}$$

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bound2_left:subst(x=0, int2);
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$$0$$

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int2:bound2_right - bound2_left;
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$$\frac{2 k \cos(\pi k) \sin(\pi m) - 2 \sin(\pi k) m \cos(\pi m)}{\pi m^2 - \pi k^2}$$

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int2:subst([sin_pi_k, sin_pi_m], %);
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$$0$$

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is(int2=0);
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true

Integravimas [3]

fn3: ((x^2)-2·x)·sin(%pi·m·x/2);

$$(x^2 - 2x) \sin\left(\frac{\pi m x}{2}\right)$$

fn3: ((2/9)·x^2-(2/3)·x) · sqrt(2/3)·sin(%pi·m·x/3);

$$\frac{\sqrt{2} \left| \frac{2x^2}{9} - \frac{2x}{3} \right| \sin\left(\frac{\pi m x}{3}\right)}{\sqrt{3}}$$

int3:integrate(fn3,x);

$$\begin{aligned} & (\sqrt{2} \sqrt{3}) \left(\frac{2 \left| \frac{2 \pi m x \sin\left(\frac{\pi m x}{3}\right)}{3} + \left| 2 - \frac{\pi^2 m^2 x^2}{9} \right| \cos\left(\frac{\pi m x}{3}\right) \right|}{\pi^2 m^2} - \right. \\ & \left. \frac{2 \left| \sin\left(\frac{\pi m x}{3}\right) - \frac{\pi m x \cos\left(\frac{\pi m x}{3}\right)}{3} \right|}{\left((2^{5/2} \pi^{3/2} m^{3/2} \pi m x - 2^{3/2} \pi^{3/2} m^{3/2} \pi m) \sin\left(\frac{\pi m x}{3}\right) + \right. \right.} \right. \\ & \left. \left. (-2^{3/2} \sqrt{3} \pi^2 m^2 x^2 + 2^{3/2} \pi^2 m^2 x + 2^{5/2} \pi^{5/2} \cos\left(\frac{\pi m x}{3}\right)) / (9 \pi^3 m^3) \right) \right) \end{aligned}$$

bound3_right:subst(x=3, int3);

$$\frac{(2^{5/2} \pi^{5/2} m^{3/2} \pi m - 2^{3/2} \pi^{3/2} \pi m) \sin(\pi m) + 2^{5/2} \pi^{5/2} \cos(\pi m)}{9 \pi^3 m^3}$$

bound3_left:subst(x=0, int3);

$$\frac{2^{5/2} \sqrt{3}}{\pi^3 m^3}$$

int3:bound3_right – bound3_left;

$$\frac{(2^{5/2} \pi^{5/2} m^{3/2} \pi m - 2^{3/2} \pi^{3/2} \pi m) \sin(\pi m) + 2^{5/2} \pi^{5/2} \cos(\pi m)}{9 \pi^3 m^3} - \frac{2^{5/2} \sqrt{3}}{\pi^3 m^3}$$

ratsimp(%);

$$\frac{2^{3/2} \sqrt{3} \pi m \sin(\pi m) + 2^{5/2} \sqrt{3} \cos(\pi m) - 2^{5/2} \sqrt{3}}{\pi^3 m^3}$$

subst(sin_pi_m, %);

$$\frac{2^{5/2} \sqrt{3} \cos(\pi m) - 2^{5/2} \sqrt{3}}{\pi^3 m^3}$$

int3:subst(cos_pi_m, %);

$$\frac{2^{5/2} \sqrt{3} (-1)^m - 2^{5/2} \sqrt{3}}{\pi^3 m^3}$$

Integravimas [4]

fn4: (-8*h+(2/3)*x)*sqrt(2/3)*sin(%pi*m*x/3);

$$\frac{\sqrt{2} \left(\frac{2x}{3} - 8h \right) \sin\left(\frac{\pi m x}{3}\right)}{\sqrt{3}}$$

h=-((2/3)*t+1)*x+3*t+3;

$$h = \left(-\frac{2t}{3} - 1 \right) x + 3t + 3$$

subst(%, fn4);

$$\frac{\sqrt{2} \left[\frac{2x}{3} - 8 \left(-\frac{2t}{3} - 1 \right) x + 3t + 3 \right] \sin\left(\frac{\pi m x}{3}\right)}{\sqrt{3}}$$

fn4:ratsimp(%);

$$\frac{\left(2^{9/2} t + 13 \cdot 2^{3/2} \right) x - 9 \cdot 2^{7/2} t - 9 \cdot 2^{7/2} \sin\left(\frac{\pi m x}{3}\right)}{3^{3/2}}$$

int4:integrate(fn4,x);

$$\left(\frac{3 \cdot 2^{9/2} t \left[\sin\left(\frac{\pi m x}{3}\right) - \frac{\pi m x \cos\left(\frac{\pi m x}{3}\right)}{3} \right]}{\pi m} + \right.$$

$$\left. \frac{39 \cdot 2^{3/2} \left[\sin\left(\frac{\pi m x}{3}\right) - \frac{\pi m x \cos\left(\frac{\pi m x}{3}\right)}{3} \right]}{3} + 9 \cdot 2^{7/2} t \cos\left(\frac{\pi m x}{3}\right) + 9 \cdot 2^{7/2} \cos\left(\frac{\pi m x}{3}\right) \right) / (\sqrt{3} \pi m) + \left(-2 \cdot 2^{9/2} \pi m t - 13 \cdot 2^{3/2} \pi m \right) x + 9 \cdot 2^{7/2} \pi m t + 9 \cdot 2^{7/2} \pi m \cos\left(\frac{\pi m x}{3}\right) / (\sqrt{3} \pi^2 m^2)$$

bound4_right:subst(x=3, int4);

$$\left(\cos(\pi m) \left(3 \left(-2^{\frac{9}{2}} \pi m t - 13 \cdot 2^{\frac{3}{2}} \pi m \right) + 9 \cdot 2^{\frac{7}{2}} \pi m t + 9 \cdot 2^{\frac{7}{2}} \pi m \right) + \sin(\pi m) \left(3 \cdot 2^{\frac{9}{2}} t + 39 \cdot 2^{\frac{3}{2}} \right) \right) / (\sqrt{3} \pi^2 m^2)$$

ratsimp(%);

$$\left(\left(3 \cdot 2^{\frac{9}{2}} \sin(\pi m) + 3 \cdot 2^{\frac{7}{2}} \pi m \cos(\pi m) \right) t + 39 \cdot 2^{\frac{3}{2}} \sin(\pi m) - 3 \cdot 2^{\frac{3}{2}} \pi m \cos(\pi m) \right) / (\sqrt{3} \pi^2 m^2)$$

bound4_left:subst(x=0, int4);

$$\frac{9 \cdot 2^{\frac{7}{2}} \pi m t + 9 \cdot 2^{\frac{7}{2}} \pi m}{\sqrt{3} \pi^2 m^2}$$

int4:bound4_right - bound4_left;

$$\left(\cos(\pi m) \left(3 \left(-2^{\frac{9}{2}} \pi m t - 13 \cdot 2^{\frac{3}{2}} \pi m \right) + 9 \cdot 2^{\frac{7}{2}} \pi m t + 9 \cdot 2^{\frac{7}{2}} \pi m \right) + \sin(\pi m) \left(3 \cdot 2^{\frac{9}{2}} t + 39 \cdot 2^{\frac{3}{2}} \right) \right) / (\sqrt{3} \pi^2 m^2) - \frac{9 \cdot 2^{\frac{7}{2}} \pi m t + 9 \cdot 2^{\frac{7}{2}} \pi m}{\sqrt{3} \pi^2 m^2}$$

expand(%);

$$\begin{aligned} & \frac{2^{\frac{9}{2}} \sqrt{3} \sin(\pi m) t}{\pi^2 m^2} + \frac{2^{\frac{7}{2}} \cdot 3^{\frac{3}{2}} \cos(\pi m) t}{\pi m} - \frac{2^{\frac{9}{2}} \sqrt{3} \cos(\pi m) t}{\pi m} - \\ & \frac{2^{\frac{7}{2}} \cdot 3^{\frac{3}{2}} t}{\pi m} + \frac{13 \cdot 2^{\frac{3}{2}} \sqrt{3} \sin(\pi m)}{\pi^2 m^2} + \frac{2^{\frac{7}{2}} \cdot 3^{\frac{3}{2}} \cos(\pi m)}{\pi m} - \frac{13 \cdot 2^{\frac{3}{2}} \sqrt{3} \cos(\pi m)}{\pi m} - \\ & \frac{2^{\frac{7}{2}} \cdot 3^{\frac{3}{2}}}{\pi m} \end{aligned}$$

ratsimp(%);

$$\left(\left(2^{\frac{9}{2}} \sqrt{3} \sin(\pi m) + 2^{\frac{7}{2}} \sqrt{3} \pi m \cos(\pi m) - 2^{\frac{7}{2}} \cdot 3^{\frac{3}{2}} \pi m \right) t + 13 \cdot 2^{\frac{3}{2}} \sqrt{3} \sin(\pi m) - 2^{\frac{3}{2}} \sqrt{3} \pi m \cos(\pi m) - 2^{\frac{7}{2}} \cdot 3^{\frac{3}{2}} \pi m \right) / (\pi^2 m^2)$$

int4:subst(sin_pi_m, %);

$$\left(\left(2^{\frac{7}{2}} \sqrt{3} \pi m \cos(\pi m) - 2^{\frac{7}{2}} \cdot 3^{\frac{3}{2}} \pi m \right) t - 2^{\frac{3}{2}} \sqrt{3} \pi m \cos(\pi m) - 2^{\frac{3}{2}} \cdot 3^{\frac{3}{2}} \pi m \right) / (\pi^2 m^2)$$

ratsimp(%);

$$\frac{(2^{7/2} \sqrt{3} \cos(\pi m) - 2^{7/2} 3^{3/2}) t - 2^{3/2} \sqrt{3} \cos(\pi m) - 2^{7/2} 3^{3/2}}{\pi m}$$

int4:subst(cos_pi_m, %);

$$\frac{(2^{7/2} \sqrt{3} (-1)^m - 2^{7/2} 3^{3/2}) t - 2^{3/2} \sqrt{3} (-1)^m - 2^{7/2} 3^{3/2}}{\pi m}$$

Integravimas [5]

A:3*2^(7/2)*(-1)^k-9*2^(7/2);

$$3 \cdot 2^{7/2} (-1)^k - 9 \cdot 2^{7/2}$$

B:3*2^(3/2)*(-1)^k+9*2^(7/2);

$$3 \cdot 2^{3/2} (-1)^k + 9 \cdot 2^{7/2}$$

C:(9*(%pi*k/3)^2+8);

$$\pi^2 k^2 + 8$$

fn5: (A*s-B)*%e^(C*s)/(sqrt(3)*%pi*k);

$$\frac{\left((3 \cdot 2^{7/2} (-1)^k - 9 \cdot 2^{7/2}) s - 3 \cdot 2^{3/2} (-1)^k - 9 \cdot 2^{7/2} \right) e^{(\pi^2 k^2 + 8)s}}{\sqrt{3} \pi k}$$

int5:integrate(fn5,s);

$$\begin{aligned} & \left(\frac{3 \cdot 2^{7/2} (-1)^k ((\pi^2 k^2 + 8)s - 1) e^{\pi^2 k^2 s + 8s}}{\pi^4 k^4 + 16 \pi^2 k^2 + 64} - \frac{9 \cdot 2^{7/2} ((\pi^2 k^2 + 8)s - 1) e^{\pi^2 k^2 s + 8s}}{\pi^4 k^4 + 16 \pi^2 k^2 + 64} - \frac{3 \cdot 2^{3/2} (-1)^k e^{\pi^2 k^2 s + 8s}}{\pi^2 k^2 + 8} \right) - \\ & \text{int5:ratsimp(\%)} \\ & \frac{9 \cdot 2^{7/2} \pi^2 k^2 s^2 e^{\pi^2 k^2 s + 8s}}{2} \left(\frac{\pi^2 k^2 s^2 e^{\pi^2 k^2 s + 8s}}{(\sqrt{3} \pi k)^2} \right) (-1)^k - 9 \cdot 2^{7/2} \pi^2 k^2 - 9 \cdot 2^{13/2} \left(s + \frac{(-3 \cdot 2^{3/2} \pi^2 k^2 - 9 \cdot 2^{7/2}) (-1)^k - 9 \cdot 2^{7/2} \pi^2 k^2 - 63 \cdot 2^{7/2}}{e^{\pi^2 k^2 s + 8s}} \right) / \\ & (\sqrt{3} \pi^5 k^5 + 16 \sqrt{3} \pi^3 k^3 + 64 \sqrt{3} \pi k) \\ & \text{bound5_right:subst(s=t, int5);} \\ & \left(\left(\frac{3 \cdot 2^{7/2} \pi^2 k^2 + 3 \cdot 2^{13/2}}{(-1)^k - 9 \cdot 2^{7/2} \pi^2 k^2 - 9 \cdot 2^{13/2}} \right) t + \frac{(-3 \cdot 2^{3/2} \pi^2 k^2 - 9 \cdot 2^{7/2}) (-1)^k - 9 \cdot 2^{7/2} \pi^2 k^2 - 63 \cdot 2^{7/2}}{e^{\pi^2 k^2 t + 8t}} \right) / \\ & (\sqrt{3} \pi^5 k^5 + 16 \sqrt{3} \pi^3 k^3 + 64 \sqrt{3} \pi k) \end{aligned}$$

bound5_left:subst(s=0, int5);

$$\frac{(-3 \cdot 2^{3/2} \pi^2 k^2 - 9 \cdot 2^{7/2}) (-1)^k (-9 \cdot 2^{7/2} \pi^2 k^2 - 63 \cdot 2^{7/2})}{\sqrt{3} \pi^5 k^5 + 16 \sqrt{3} \pi^3 k^3 + 64 \sqrt{3} \pi k}$$

int5:bound5_right - bound5_left;

$$\frac{\left(\left((3 \cdot 2^{7/2} \pi^2 k^2 + 3 \cdot 2^{13/2}) (-1)^k (-9 \cdot 2^{7/2} \pi^2 k^2 - 9 \cdot 2^{13/2}) \right) t + (-3 \cdot 2^{3/2} \pi^2 k^2 - 9 \cdot 2^{7/2}) (-1)^k (-9 \cdot 2^{7/2} \pi^2 k^2 - 63 \cdot 2^{7/2}) \% e^{\pi^2 k^2 t + 8 t} \right) / (\sqrt{3} \pi^5 k^5 + 16 \sqrt{3} \pi^3 k^3 + 64 \sqrt{3} \pi k) - (-3 \cdot 2^{3/2} \pi^2 k^2 - 9 \cdot 2^{7/2}) (-1)^k (-9 \cdot 2^{7/2} \pi^2 k^2 - 63 \cdot 2^{7/2})}{\sqrt{3} \pi^5 k^5 + 16 \sqrt{3} \pi^3 k^3 + 64 \sqrt{3} \pi k}$$