[3] LYGTIS, SUSIVEDANTI Į TIESINĘ HOMOGENINĘ DIF. LYGTĮ

```
kill(all)$
```

```
reset(integration_constant_counter)$
```

```
eq:'diff(y,x)=(x+4·y−5)/(x−y−5);
```

$$\frac{d}{d\,x}\,y=\frac{4\,y+x-5}{-y+x-5}$$

```
ode2(eq,y,x);
```

false

Nepavyko išspręsti su ODE2, todėl spręsime "žingsnis po žingsnio" metodu nesinaudodami ODE2

```
L1:num(rhs(eq))=0;
```

$$4\,y+x-5=0$$

```
L2:denom(rhs(eq))=0;
```

$$-y+x-5=0$$

```
solve([L1,L2]);
```

$$[\,[\,y=0\,,x=5\,]\,]$$

```
[x0,y0]:subst(%[1],[x,y]);
```

$$[\,5\,,0\,]$$

```
keit:w(x)=(y−y0)/(x−x0);
```

$$\mathrm{w}(x)=\frac{y}{x-5}$$

```
akeit:solve(%,y)[1];
```

$$y=(x-5)\;\mathrm{w}(x)$$

```
subst(akeit,eq),factor;
```

$$\frac{d}{d\,x}\,((x-5)\;\mathrm{w}(x))=-\frac{4\;\mathrm{w}(x)+1}{\mathrm{w}(x)-1}$$

```
ev(%,diff);
```

$$(x-5)\;\left(\frac{d}{d\,x}\,\mathrm{w}(x)\right)+\mathrm{w}(x)=-\frac{4\;\mathrm{w}(x)+1}{\mathrm{w}(x)-1}$$

```
eq1:%−w(x),factor;
```

$$(x-5)\;\left(\frac{d}{d\,x}\,\mathrm{w}(x)\right)=-\frac{\mathrm{w}(x)^{2}+3\;\mathrm{w}(x)+1}{\mathrm{w}(x)-1}$$

```
eq2:eq1/rhs(eq1)/(x-5);
```

$$-\frac{(\mathrm{w}(x)-1)\left(\frac{d}{d\,x}\,\mathrm{w}(x)\right)}{\mathrm{w}(x)^2+3\,\mathrm{w}(x)+1}=\frac{1}{x-5}$$

```
solve(denom(lhs(%))=0);
```

$$\left[\mathrm{w}(x)=-\frac{\sqrt{5}+3}{2},\mathrm{w}(x)=\frac{\sqrt{5}-3}{2}\right]$$

```
[w1,w2]:[rhs(%[1]),rhs(%[2])];
```

$$\left[-\frac{\sqrt{5}+3}{2},\frac{\sqrt{5}-3}{2}\right]$$

```
eq2:num(lhs(eq2))/((w(x)-w1)·(w(x)-w2))=rhs(eq2);
```

$$-\frac{(\mathrm{w}(x)-1)\left(\frac{d}{d\,x}\,\mathrm{w}(x)\right)}{\left(\mathrm{w}(x)-\frac{\sqrt{5}-3}{2}\right)\left(\mathrm{w}(x)+\frac{\sqrt{5}+3}{2}\right)}=\frac{1}{x-5}$$

```
partfrac(lhs(%),w(x))=rhs(%);
```

$$-\frac{(\sqrt{5}+5)\left(\frac{d}{d\,x}\,\mathrm{w}(x)\right)}{\sqrt{5}\,(2\,\mathrm{w}(x)+\sqrt{5}+3)}-\frac{(\sqrt{5}-5)\left(\frac{d}{d\,x}\,\mathrm{w}(x)\right)}{\sqrt{5}\,(2\,\mathrm{w}(x)-\sqrt{5}+3)}=\frac{1}{x-5}$$

```
integrate(%,x);
```

$$-\frac{(\sqrt{5}+5)\log(2\,\mathrm{w}(x)+\sqrt{5}+3)}{2\sqrt{5}}-\frac{(\sqrt{5}-5)\log(2\,\mathrm{w}(x)-\sqrt{5}+3)}{2\sqrt{5}}=\log(x-5)+\%c1$$

```
%-log(x-5);
```

$$-\frac{(\sqrt{5}+5)\log(2\,\mathrm{w}(x)+\sqrt{5}+3)}{2\sqrt{5}}-\frac{(\sqrt{5}-5)\log(2\,\mathrm{w}(x)-\sqrt{5}+3)}{2\sqrt{5}}-\log(x-5)=\%c1$$

```
subst(keit,%);
```

$$-\frac{(\sqrt{5}+5)\log\left(\frac{2\,y}{x-5}+\sqrt{5}+3\right)}{2\sqrt{5}}-\frac{(\sqrt{5}-5)\log\left(\frac{2\,y}{x-5}-\sqrt{5}+3\right)}{2\sqrt{5}}-\log(x-5)=\%c1$$

Atsakymas:

```
ats:lhs(%)=C;
```

$$-\frac{(\sqrt{5}+5)\log\left(\frac{2\,y}{x-5}+\sqrt{5}+3\right)}{2\sqrt{5}}-\frac{(\sqrt{5}-5)\log\left(\frac{2\,y}{x-5}-\sqrt{5}+3\right)}{2\sqrt{5}}-\log(x-5)=C$$

Patikrinimas:

```
depends(y,x);
```

$$[\,y(x)\,]$$

```
diff(ats,x);
```

$$-\frac{(\sqrt{5}+5)\left(\frac{2\,(\frac{d}{d\,x}\,y)}{x-5}-\frac{2\,y}{(x-5)^{2}}\right)}{2\sqrt{5}\,(\frac{2\,y}{x-5}+\sqrt{5}+3)}-\frac{(\sqrt{5}-5)\left(\frac{2\,(\frac{d}{d\,x}\,y)}{x-5}-\frac{2\,y}{(x-5)^{2}}\right)}{2\sqrt{5}\,(\frac{2\,y}{x-5}-\sqrt{5}+3)}-\frac{1}{x-5}=0$$

```
solve(%,'diff(y,x,1));
```

$$\left[\frac{d}{d\,x}\,y=-\frac{4\,y+x-5}{y-x+5}\right]$$

```
subst(%,eq);
```

$$-\frac{4\,y+x-5}{y-x+5}=\frac{4\,y+x-5}{-y+x-5}$$

```
ratsimp(%);
```

$$-\frac{4\,y+x-5}{y-x+5}=-\frac{4\,y+x-5}{y-x+5}$$

```
is(%);
```

true

Bendrasis integralas, gautas sprendžiant ant popieriaus:

```
(−1/2)·log(w(x)^2+3·w(x)+1)−(sqrt(5)/2)·log(((2·w(x)+3)/sqrt(5))+1)+(sqrt(5)/2)·log(((2·w(x)+3)/sqrt(5))−1)=log(x−5)+%c1;
```

$$-\frac{\log(\mathrm{w}(x)^{2}+3\,\mathrm{w}(x)+1)}{2}-\frac{\sqrt{5}\,\log\left(\frac{2\,\mathrm{w}(x)+3}{\sqrt{5}}+1\right)}{2}+\frac{\sqrt{5}\,\log\left(\frac{2\,\mathrm{w}(x)+3}{\sqrt{5}}-1\right)}{2}=\log(x-5)+\%c1$$

```
%−log(x−5);
```

$$-\frac{\log(\mathrm{w}(x)^{2}+3\,\mathrm{w}(x)+1)}{2}-\frac{\sqrt{5}\,\log\left(\frac{2\,\mathrm{w}(x)+3}{\sqrt{5}}+1\right)}{2}+\frac{\sqrt{5}\,\log\left(\frac{2\,\mathrm{w}(x)+3}{\sqrt{5}}-1\right)}{2}-\log(x-5)=\%c1$$

```
subst(keit,%);
```

$$-\frac{\log\left|\frac{y^{2}}{(x-5)^{2}}+\frac{3\,y}{x-5}+1\right|}{2}-\frac{\sqrt{5}\,\log\left|\frac{\frac{2\,y}{x-5}+3}{\sqrt{5}}+1\right|}{2}+\frac{\sqrt{5}\,\log\left|\frac{\frac{2\,y}{x-5}+3}{\sqrt{5}}-1\right|}{2}-\log(x-5)=\%c1$$

Atsakymas:

```
ats:lhs(%)=C;
```

$$-\frac{\log\left|\frac{y^{2}}{(x-5)^{2}}+\frac{3\,y}{x-5}+1\right|}{2}-\frac{\sqrt{5}\,\log\left|\frac{\frac{2\,y}{x-5}+3}{\sqrt{5}}+1\right|}{2}+\frac{\sqrt{5}\,\log\left|\frac{\frac{2\,y}{x-5}+3}{\sqrt{5}}-1\right|}{2}-\log(x-5)=C$$

Patikrinimas:

```
depends(y,x);
```

$$[\,y(x)\,]$$

```
diff(ats,x);
```

$$-\frac{\frac{2\,y\left(\frac{d}{d\,x}\,y\right)}{(x-5)^{2}}+\frac{3\left(\frac{d}{d\,x}\,y\right)}{x-5}-\frac{2\,y^{2}}{(x-5)^{3}}-\frac{3\,y}{(x-5)^{2}}}{2\left|\frac{y^{2}}{(x-5)^{2}}+\frac{3\,y}{x-5}+1\right|}-\frac{\frac{2\left(\frac{d}{d\,x}\,y\right)}{x-5}-\frac{2\,y}{(x-5)^{2}}}{2\left|\frac{\frac{2\,y}{x-5}+3}{\sqrt{5}}+1\right|}+\frac{\frac{2\left(\frac{d}{d\,x}\,y\right)}{x-5}-\frac{2\,y}{(x-5)^{2}}}{2\left|\frac{\frac{2\,y}{x-5}+3}{\sqrt{5}}-1\right|}-\frac{1}{x-5}=0$$

```
solve(%,'diff(y,x,1));
```

$$\left[\frac{d}{d\,x}\,y=-\frac{4\,y+x-5}{y-x+5}\right]$$

```
subst(%,eq);
```

$$-\frac{4\,y+x-5}{y-x+5}=\frac{4\,y+x-5}{-y+x-5}$$

```
ratsimp(%);
```

$$-\frac{4\,y+x-5}{y-x+5}=-\frac{4\,y+x-5}{y-x+5}$$

```
is(%);
```

true

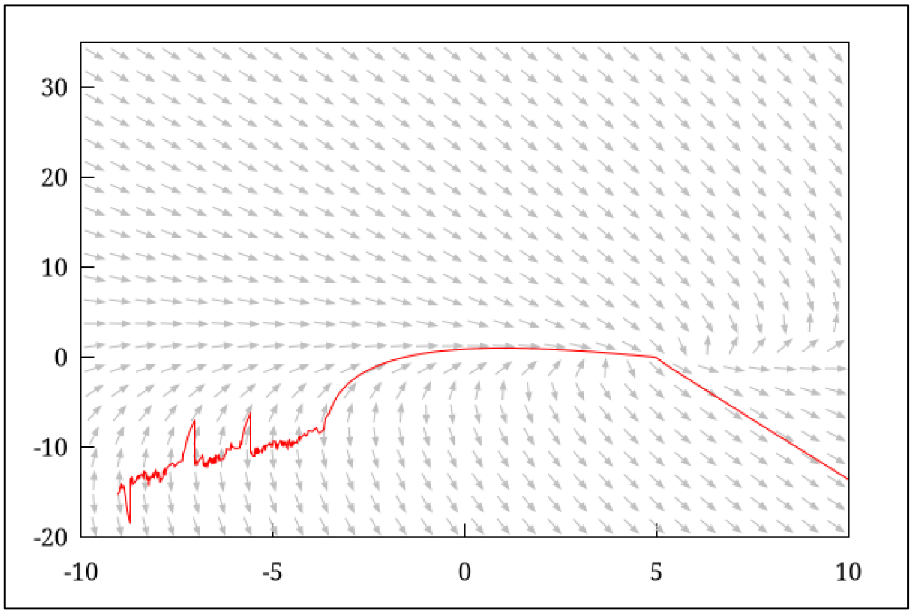
Krypčių laukas (lyg ir ne toks brėžinys turėtų gautis pgl. dėst., pradžioje "tikslumo" (?) trūksta Maximai, todėl tokie "dantukai" kairėj)

```
f: rhs(eq);
```

$$\frac{4\,y+x-5}{-y+x-5}$$

```
load(drawdf)$
```

```
wxdrawdf(f, [x,-10,10], [y,-20,35],
   [trajectory_at,1,1], field_color=gray, key="isocline", color=green, line_width=2,
nticks=100);
```



0

Integralinės kreivės (lyg ir ne tokios turi gautis pgl. dėst.)

```
load(draw)$
```

```
load(implicit_plot)$
```

```
ats:    -((sqrt(5)+5)·log((2·y)/(x-5)+sqrt(5)+3))/(2·sqrt(5))-((sqrt(5)-5)·log((2·y)/(x-5)-sqrt(5)+3))/(2·sqrt(5))-log(x-5)=C;
```

$$-\frac{(\sqrt{5}+5)\,\log\left(\frac{2\,y}{x-5}+\sqrt{5}+3\right)}{2\,\sqrt{5}}-\frac{(\sqrt{5}-5)\,\log\left(\frac{2\,y}{x-5}-\sqrt{5}+3\right)}{2\,\sqrt{5}}-\log(x-5)=C$$

```
f1: makelist(subst(C = k, ats), k, [1/10, 1/2, 1, 2, 5]);
```

$$\left[-\frac{\left(\sqrt{5}+5\right)\log\left(\frac{2\,y}{x-5}+\sqrt{5}+3\right)}{2\,\sqrt{5}}-\frac{\left(\sqrt{5}-5\right)\log\left(\frac{2\,y}{x-5}-\sqrt{5}+3\right)}{2\,\sqrt{5}}-\log\left(x-5\right)=\frac{1}{10},-\frac{\left(\sqrt{5}+5\right)\log\left(\frac{2\,y}{x-5}+\sqrt{5}+3\right)}{2\,\sqrt{5}}-\frac{\left(\sqrt{5}-5\right)\log\left(\frac{2\,y}{x-5}-\sqrt{5}+3\right)}{2\,\sqrt{5}}-\log\left(x-5\right)=\frac{1}{2},-\frac{\left(\sqrt{5}+5\right)\log\left(\frac{2\,y}{x-5}+\sqrt{5}+3\right)}{2\,\sqrt{5}}-\frac{\left(\sqrt{5}-5\right)\log\left(\frac{2\,y}{x-5}-\sqrt{5}+3\right)}{2\,\sqrt{5}}-\log\left(x-5\right)=1,-\frac{\left(\sqrt{5}+5\right)\log\left(\frac{2\,y}{x-5}+\sqrt{5}+3\right)}{2\,\sqrt{5}}-\frac{\left(\sqrt{5}-5\right)\log\left(\frac{2\,y}{x-5}-\sqrt{5}+3\right)}{2\,\sqrt{5}}-\log\left(x-5\right)=2,-\frac{\left(\sqrt{5}+5\right)\log\left(\frac{2\,y}{x-5}+\sqrt{5}+3\right)}{2\,\sqrt{5}}-\frac{\left(\sqrt{5}-5\right)\log\left(\frac{2\,y}{x-5}-\sqrt{5}+3\right)}{2\,\sqrt{5}}-\log\left(x-5\right)=5\right]$$

```
wxdraw2d(grid = true,
color = magenta, key = "c=1/10",
implicit(f1[1], x, 0, 10, y, −10, 10),
color = cyan, key = "c=1/2",
implicit(f1[2], x, 0, 10, y, −10, 10),
color = red, key = "c=1",
implicit(f1[3], x, 0, 10, y, −10, 10),
color = blue, key = "c=2",
implicit(f1[4], x, 0, 10, y,  −10, 10),
color = green, key = "c=5",
implicit(f1[5], x, 0, 10, y,  −10, 10)),
wxplot_size = [500,500]$
```

```
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
```