

Ekvivalentumai:

$$\begin{aligned} \text{sin_pi_k: } \sin(\pi \cdot k) &= 0; \\ \sin(\pi k) &= 0 \end{aligned}$$

$$\begin{aligned} \text{sin_pi_m: } \sin(\pi \cdot m) &= 0; \\ \sin(\pi m) &= 0 \end{aligned}$$

$$\begin{aligned} \text{sin_2pi_k: } \sin(2 \cdot \pi \cdot k) &= 0; \\ \sin(2 \pi k) &= 0 \end{aligned}$$

$$\begin{aligned} \text{cos_pi_k: } \cos(\pi \cdot k) &= (-1)^k; \\ \cos(\pi k) &= (-1)^k \end{aligned}$$

$$\begin{aligned} \text{cos_pi_m: } \cos(\pi \cdot m) &= (-1)^m; \\ \cos(\pi m) &= (-1)^m \end{aligned}$$

Integravimas [1]

$$\begin{aligned} \text{fn1: } (1 - \cos(2 \cdot \pi \cdot k \cdot x/2))/2; \\ \frac{1 - \cos(\pi k x)}{2} \end{aligned}$$

$$\begin{aligned} \text{int1: } \text{integrate}(\text{fn1}, x); \\ x - \frac{\sin(\pi k x)}{\pi k} \end{aligned}$$

$$\begin{aligned} \text{ratsimp}(\%); \\ - \frac{\sin(\pi k x) - \pi k x}{2 \pi k} \end{aligned}$$

$$\begin{aligned} \text{bound1_right: } \text{subst}(x=2, \text{int1}); \\ 2 - \frac{\sin(2 \pi k)}{\pi k} \end{aligned}$$

$$\begin{aligned} \text{bound1_left: } \text{subst}(x=0, \text{int1}); \\ 0 \end{aligned}$$

$$\begin{aligned} \text{int1: } \text{bound1_right} - \text{bound1_left}; \\ 2 - \frac{\sin(2 \pi k)}{\pi k} \end{aligned}$$

$$\begin{aligned} \text{int1: } \text{subst}(\text{sin_2pi_k}, \text{int1}); \\ 1 \end{aligned}$$

Integravimas [2]

fn2: sin(%pi·k·x/2)·sin(%pi·m·x/2);

$$\sin\left(\frac{\pi k x}{2}\right) \sin\left(\frac{\pi m x}{2}\right)$$

int2:integrate(fn2,x);

$$\frac{\sin\left(\left(\frac{\pi k}{2} - \frac{\pi m}{2}\right) x\right)}{2\left(\frac{\pi k}{2} - \frac{\pi m}{2}\right)} - \frac{\sin\left(\left(\frac{\pi m}{2} + \frac{\pi k}{2}\right) x\right)}{2\left(\frac{\pi m}{2} + \frac{\pi k}{2}\right)}$$

%,trigexpand=true,trigexpandplus=true,expand;

$$\begin{aligned} & - \frac{\cos\left(\frac{\pi k x}{2}\right) \sin\left(\frac{\pi m x}{2}\right)}{\pi m + \pi k} - \frac{\cos\left(\frac{\pi k x}{2}\right) \sin\left(\frac{\pi m x}{2}\right)}{\pi k - \pi m} - \\ & \frac{\sin\left(\frac{\pi k x}{2}\right) \cos\left(\frac{\pi m x}{2}\right)}{\pi m + \pi k} + \frac{\sin\left(\frac{\pi k x}{2}\right) \cos\left(\frac{\pi m x}{2}\right)}{\pi k - \pi m} \end{aligned}$$

int2:ratsimp(%);

$$\frac{2 k \cos\left(\frac{\pi k x}{2}\right) \sin\left(\frac{\pi m x}{2}\right) - 2 m \sin\left(\frac{\pi k x}{2}\right) \cos\left(\frac{\pi m x}{2}\right)}{\pi m^2 - \pi k^2}$$

bound2_right:subst(x=2, int2);

$$\frac{2 k \cos(\pi k) \sin(\pi m) - 2 \sin(\pi k) m \cos(\pi m)}{\pi m^2 - \pi k^2}$$

bound2_left:subst(x=0, int2);

0

int2:bound2_right - bound2_left;

$$\frac{2 k \cos(\pi k) \sin(\pi m) - 2 \sin(\pi k) m \cos(\pi m)}{\pi m^2 - \pi k^2}$$

int2:subst([sin_pi_k, sin_pi_m], %);

0

is(int2=0);

true

Integravimas [3]

fn3: ((x^2)-2·x)·sin(%pi·m·x/2);

$$(x^2 - 2x) \sin\left(\frac{\pi m x}{2}\right)$$

int3:integrate(fn3,x);

$$\frac{4 \left(\pi m x \sin\left(\frac{\pi m x}{2}\right) + \left(2 - \frac{\pi^2 m^2 x^2}{4}\right) \cos\left(\frac{\pi m x}{2}\right) \right)}{\pi^2 m^2} - \frac{4 \left(\sin\left(\frac{\pi m x}{2}\right) - \frac{\pi m x \cos\left(\frac{\pi m x}{2}\right)}{2} \right)}{\pi m} \right) / (\pi m)$$

int3:ratsimp(%);

$$\frac{(8 \pi m x - 8 \pi m) \sin\left(\frac{\pi m x}{2}\right) + (-2 \pi^2 m^2 x^2 + 4 \pi^2 m^2 x + 16) \cos\left(\frac{\pi m x}{2}\right)}{\pi^3 m^3}$$

bound3_right:subst(x=2, int3);

$$\frac{8 \pi m \sin(\pi m) + 16 \cos(\pi m)}{\pi^3 m^3}$$

bound3_left:subst(x=0, int3);

$$\frac{16}{\pi^3 m^3}$$

int3:bound3_right - bound3_left;

$$\frac{8 \pi m \sin(\pi m) + 16 \cos(\pi m)}{\pi^3 m^3} - \frac{16}{\pi^3 m^3}$$

ratsimp(%);

$$\frac{8 \pi m \sin(\pi m) + 16 \cos(\pi m) - 16}{\pi^3 m^3}$$

subst(sin_pi_m, %);

$$\frac{16 \cos(\pi m) - 16}{\pi^3 m^3}$$

int3:subst(cos_pi_m, %);

$$\frac{16 (-1)^m - 16}{\pi^3 m^3}$$

Integravimas [4]

fn4: (-6·h+x+1)·sin(%pi·m·x/2);

$$(x - 6h + 1) \sin\left(\frac{\pi m x}{2}\right)$$

h=(-t)·x+(2·t+2);

$$h = -t x + 2t + 2$$

subst(%, fn4);

$$(-6(-tx+2t+2)+x+1)\sin\left(\frac{\pi mx}{2}\right)$$

fn4:ratsimp(%);

$$((6t+1)x-12t-11)\sin\left(\frac{\pi mx}{2}\right)$$

int4:integrate(fn4,x);

$$\frac{12t \left[\sin\left(\frac{\pi mx}{2}\right) - \frac{\pi mx \cos\left(\frac{\pi mx}{2}\right)}{2} \right]}{\pi m} + \frac{2 \left[\sin\left(\frac{\pi mx}{2}\right) - \frac{\pi mx \cos\left(\frac{\pi mx}{2}\right)}{2} \right]}{\pi m} + 12t \cos\left(\frac{\pi mx}{2}\right) + 11 \cos\left(\frac{\pi mx}{2}\right) \Big/ (\pi m)$$

$$\frac{(24t+4)\sin\left(\frac{\pi mx}{2}\right) + ((-12\pi mt-2\pi m)x+24\pi mt+22\pi m)\cos\left(\frac{\pi mx}{2}\right)}{\pi^2 m^2}$$

bound4_right:subst(x=2, int4);

$$\frac{\cos(\pi m)(2(-12\pi mt-2\pi m)+24\pi mt+22\pi m) + \sin(\pi m)(24t+4)}{\pi^2 m^2}$$

ratsimp(%);

$$\frac{24\sin(\pi m)t+4\sin(\pi m)+18\pi m\cos(\pi m)}{\pi^2 m^2}$$

bound4_left:subst(x=0, int4);

$$\frac{24\pi mt+22\pi m}{\pi^2 m^2}$$

int4:bound4_right - bound4_left;

$$\frac{\cos(\pi m)(2(-12\pi mt-2\pi m)+24\pi mt+22\pi m) + \sin(\pi m)(24t+4)}{\pi^2 m^2} - \frac{24\pi mt+22\pi m}{\pi^2 m^2}$$

expand(%);

$$\frac{24\sin(\pi m)t}{\pi^2 m^2} - \frac{24t}{\pi m} + \frac{4\sin(\pi m)}{\pi^2 m^2} + \frac{18\cos(\pi m)}{\pi m} - \frac{22}{\pi m}$$

ratsimp(%);

$$\frac{(24 \sin(\pi m) - 24 \pi m) t + 4 \sin(\pi m) + 18 \pi m \cos(\pi m) - 22 \pi m}{\pi^2 m^2}$$

int4:subst(sin_pi_m, %);

$$\frac{-24 \pi m t + 18 \pi m \cos(\pi m) - 22 \pi m}{\pi^2 m^2}$$

ratsimp(%);

$$- \frac{24 t - 18 \cos(\pi m) + 22}{\pi m}$$

int4:subst(cos_pi_m, %);

$$- \frac{24 t - 18 (-1)^m + 22}{\pi m}$$

Integravimas [5]

fn5:(-24·s+18·(-1)^k-22)/(%pi·k) · %e^((((%pi·k/2)^2)+6)·s);

$$\frac{(-24 s + 18 (-1)^k - 22) \%e^{\left[\frac{\pi^2 k^2}{4} + 6 \right] s}}{\pi k}$$

int5:integrate(fn5,s);

$$\left(- \frac{24 \left((4 \pi^2 k^2 + 96) s - 16 \right) \%e^{\frac{\pi^2 k^2 s}{4} + 6 s}}{\pi^4 k^4 + 48 \pi^2 k^2 + 576} + \frac{18 (-1)^k \%e^{\frac{\pi^2 k^2 s}{4} + 6 s}}{\frac{\pi^2 k^2}{4} + 6} - \right.$$

$$\left. \frac{22 \%e^{\frac{\pi^2 k^2 s}{4} + 6 s}}{\frac{\pi^2 k^2}{4} + 6} \right) / (\pi k)$$

int5:ratsimp(%);

$$\left[\frac{((96 \pi^2 k^2 + 2304) s + (-72 \pi^2 k^2 - 1728) (-1)^k + 88 \pi^2 k^2 + 1728) \%e^{\frac{\pi^2 k^2 s}{4} + 6 s}}{(\pi^5 k^5 + 48 \pi^3 k^3 + 576 \pi k)} \right]$$

bound5_right:subst(s=t, int5);

$$- \frac{((96 \pi^2 k^2 + 2304) t + (-72 \pi^2 k^2 - 1728) (-1)^k + 88 \pi^2 k^2 + 1728) \%e^{\frac{\pi^2 k^2 t}{4} + 6 t}}{\pi^5 k^5 + 48 \pi^3 k^3 + 576 \pi k}$$

bound5_left:subst(s=0, int5);

$$- \frac{(-72 \pi^2 k^2 - 1728) (-1)^k + 88 \pi^2 k^2 + 1728}{\pi^5 k^5 + 48 \pi^3 k^3 + 576 \pi k}$$

int5:bound5_right - bound5_left;

$$\frac{(-72 \pi^2 k^2 - 1728) (-1)^k + 88 \pi^2 k^2 + 1728}{\pi^5 k^5 + 48 \pi^3 k^3 + 576 \pi k} -$$

$$\frac{((96 \pi^2 k^2 + 2304) t + (-72 \pi^2 k^2 - 1728) (-1)^k + 88 \pi^2 k^2 + 1728) \% e^{\frac{\pi^2 k^2 t}{4}} + 6 t}{\pi^5 k^5 + 48 \pi^3 k^3 + 576 \pi k}$$