[1] PIRMOSIOS EILĖS DIFERENCIALINĖ LYGTIS
SU ATSKIRIAMAIS KINTAMAISIAIS

Nenaudojant ODE2 funkcijos

eq:2·x+x·y^2+(4–x^2)^(1/2)·'diff(y,x)=0;

$$\sqrt{4-x^2}\,\left(\frac{\mathrm{d}}{\mathrm{d}\,x}\,y\right)+x\,y^2+2\,x=0$$

eq:eq–(x·y^2+2·x);

$$\sqrt{4-x^2}\,\left(\frac{\mathrm{d}}{\mathrm{d}\,x}\,y\right)=-x\,y^2-2\,x$$

eq:factor(eq);

$$\sqrt{4-x^2}\,\left(\frac{\mathrm{d}}{\mathrm{d}\,x}\,y\right)=-x\,\left(y^2+2\right)$$

eq:eq/(y^2+2)/((4–x^2)^(1/2));

$$\frac{\frac{\mathrm{d}}{\mathrm{d}\,x}\,y}{y^2+2}=-\frac{x}{\sqrt{4-x^2}}$$

subst([y=y(x),dx=1,dy=diff(y(x),x)],eq);

$$\frac{\frac{\mathrm{d}}{\mathrm{d}\,x}\,\mathrm{y}\left(x\right)}{\mathrm{y}\left(x\right)^2+2}=-\frac{x}{\sqrt{4-x^2}}$$

integrate(%,x);

$$\int\frac{\frac{\mathrm{d}}{\mathrm{d}\,x}\,\mathrm{y}\left(x\right)}{\mathrm{y}\left(x\right)^2+2}\,x=\sqrt{4-x^2}+\%c1$$

/* integravimas ranka */

res: (1/sqrt(2))·arctan(y(x)/sqrt(2))=sqrt(4–x^2)+%c1;

$$\frac{\arctan\left(\frac{\mathrm{y}\left(x\right)}{\sqrt{2}}\right)}{\sqrt{2}}=\sqrt{4-x^2}+\%c1$$

res:res–sqrt(4–x^2);

$$\frac{\arctan\left(\frac{\mathrm{y}\left(x\right)}{\sqrt{2}}\right)}{\sqrt{2}}-\sqrt{4-x^2}=\%c1$$

lhs(%)=C;

$$\frac{\arctan\left(\frac{y(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\sqrt{4-x^2}=C$$

ats:subst(y(x)=y,%);

$$\frac{\arctan\left(\frac{y}{\sqrt{2}}\right)}{\sqrt{2}}-\sqrt{4-x^2}=C$$

Naudojant ODE2 funkciją

eq:2·x+x·y^2+(4–x^2)^(1/2)·'diff(y,x)=0;

$$\sqrt{4-x^2}\left(\frac{\mathrm{d}}{\mathrm{d}\,x}y\right)+x\,y^2+2\,x=0$$

eq1:solve(%,'diff(y,x))[1];

$$\frac{\mathrm{d}}{\mathrm{d}\,x}y=-\frac{x\,y^2+2\,x}{\sqrt{4-x^2}}$$

ode2(eq1,y,x);

$$-\frac{\operatorname{atan}\left(\frac{y}{\sqrt{2}}\right)}{\sqrt{2}}=\%c-\sqrt{4-x^2}$$

Pertvarkome šį sprendinį

%+(4–x^2)^(1/2);

$$\sqrt{4-x^2}-\frac{\operatorname{atan}\left(\frac{y}{\sqrt{2}}\right)}{\sqrt{2}}=\%c$$

%·–1;

$$\frac{\operatorname{atan}\left(\frac{y}{\sqrt{2}}\right)}{\sqrt{2}}-\sqrt{4-x^2}=-\%c$$

Atsakymas:

ats:lhs(%)=C;

$$\frac{\operatorname{atan}\left(\frac{y}{\sqrt{2}}\right)}{\sqrt{2}}-\sqrt{4-x^2}=C$$

Patikrinimas:

```
depends(y,x);
```

$$[y(x)]$$

```
diff(ats,x);
```

$$\frac{\frac{d}{dx}y}{2\left(\frac{y^2}{2}+1\right)}+\frac{x}{\sqrt{4-x^2}}=0$$

```
solve(%,'diff(y,x));
```

$$\left[\frac{d}{dx}y=-\frac{x\,y^2+2\,x}{\sqrt{4-x^2}}\right]$$

```
subst(%,eq1);
```

$$-\frac{x\,y^2+2\,x}{\sqrt{4-x^2}}=-\frac{x\,y^2+2\,x}{\sqrt{4-x^2}}$$

```
is(%);
```

true

Nubrėšime keletą integralinių kreivių, kai C=1/10, C=1/2, C=1.

```
kr:makelist(ev(ats),C,[1/10,1/2,1]);
```

$$\left[\frac{\operatorname{atan}\left(\frac{y}{\sqrt{2}}\right)}{\sqrt{2}}-\sqrt{4-x^2}=\frac{1}{10},\frac{\operatorname{atan}\left(\frac{y}{\sqrt{2}}\right)}{\sqrt{2}}-\sqrt{4-x^2}=\frac{1}{2},\frac{\operatorname{atan}\left(\frac{y}{\sqrt{2}}\right)}{\sqrt{2}}-\sqrt{4-x^2}=1\right]$$

```
load(implicit_plot)$
```

```
wximplicit_plot(kr,[x,–20,20],[y,–1,20],same_xy)$
```

rat: replaced 1.414213562373095 by 22619537/15994428 = 1.414213562373096
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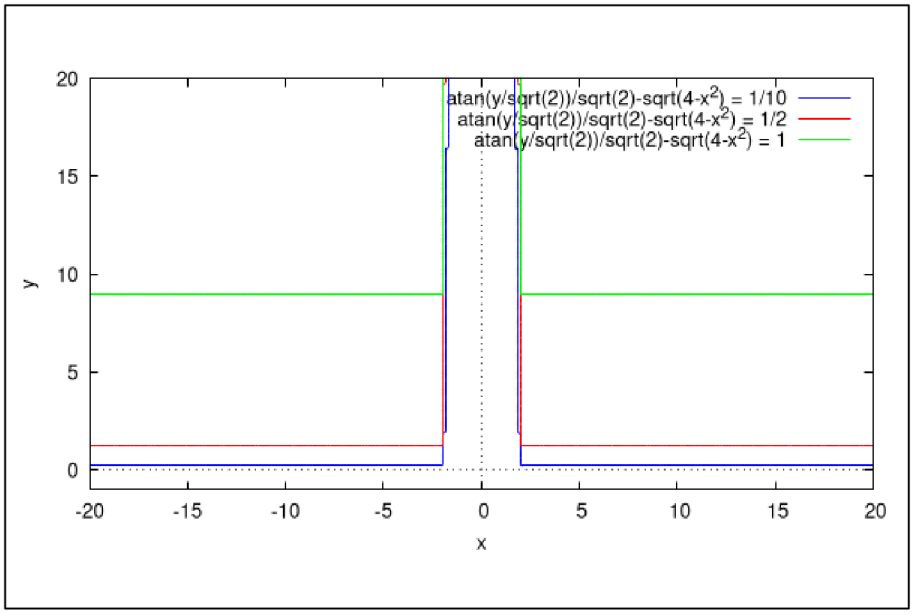

Nubrėšime izoklines ir krypčių lauką.

```
load(drawdf)$
```

```
f:rhs(eq1);
```

$$-\frac{x\,y^2+2\,x}{\sqrt{4-x^2}}$$

```
wxdrawdf(f,[x,–5,5],[y,–5,5],
color=red,
line_width=2,
implicit(kr[1],x,–5,5,y,–5,5),
implicit(kr[2],x,–5,5,y,–5,5),
implicit(kr[3],x,–5,5,y,–5,5),
proportional_axes = 'xy
)$
```

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"/tmp/maxout_278845_1.gnuplot" line 338: warning: Skipping data file with no valid points