[2] HOMOGENINĖ PIRMOSIOS EILĖS DIFERENCIALINĖ LYGTIS

```
kill(all)$
```

```
reset(integration_constant_counter)$
```

```
eq:x·'diff(y,x)=((x^2+2·y^2)^(1/2)+y);
```

$$x\left(\frac{d}{d\,x}\,y\right)=\sqrt{2\,y^2+x^2}+y$$

```
eq1:solve(eq,'diff(y,x))[1];
```

$$\frac{d}{d\,x}\,y=\frac{\sqrt{2\,y^2+x^2}+y}{x}$$

Nenaudojant ODE2 funkcijos

```
lambda: 1/x$
```

```
eq2:lhs(eq1)=subst([x=lambda·x,y=lambda·y], rhs(eq1));
```

$$\frac{d}{d\,x}\,y=\sqrt{\frac{2\,y^2}{x^2}+1}+\frac{y}{x}$$

```
subst(y=u(x)·x,eq2);
```

$$\frac{d}{d\,x}\,(x\,\mathrm{u}(x))=\sqrt{2\,\mathrm{u}(x)^2+1}+\mathrm{u}(x)$$

```
ev(%,diff);
```

$$x\left(\frac{d}{d\,x}\,\mathrm{u}(x)\right)+\mathrm{u}(x)=\sqrt{2\,\mathrm{u}(x)^2+1}+\mathrm{u}(x)$$

```
ratsimp(%-u(x));
```

$$x\left(\frac{d}{d\,x}\,\mathrm{u}(x)\right)=\sqrt{2\,\mathrm{u}(x)^2+1}$$

```
%/rhs(%)/x;
```

$$\frac{\frac{d}{d\,x}\,\mathrm{u}(x)}{\sqrt{2\,\mathrm{u}(x)^2+1}}=\frac{1}{x}$$

```
integrate(%,x);
```

$$\int\frac{\frac{d}{d\,x}\,\mathrm{u}(x)}{\sqrt{2\,\mathrm{u}(x)^2+1}}\,dx=\log(x)+\%c1$$

/* integravimas ranka */;

```
res: (1/sqrt(2))·log(abs(sqrt(2)·u(x) +sqrt(2·u(x)^2+1))) = log(abs(x)) + %c1;
```

$$\frac{\log\left(\left|\sqrt{2\,u(x)^2+1}+\sqrt{2}\,u(x)\right|\right)}{\sqrt{2}}=\log\left(\left|x\right|\right)+\%c1$$

```
%–log(abs(x));
```

$$\frac{\log\left(\left|\sqrt{2\,u(x)^2+1}+\sqrt{2}\,u(x)\right|\right)}{\sqrt{2}}-\log\left(\left|x\right|\right)=\%c1$$

```
logcontract(%);
```

$$-\frac{\sqrt{2}\,\log\left(\left|x\right|\right)-\log\left(\left|\sqrt{2\,u(x)^2+1}+\sqrt{2}\,u(x)\right|\right)}{\sqrt{2}}=\%c1$$

```
subst([u(x)=y/x,%c1=log(C)],%);
```

$$-\frac{\sqrt{2}\,\log\left(\left|x\right|\right)-\log\left(\left|\sqrt{\frac{2\,y^2}{x^2}+1}+\frac{\sqrt{2}\,y}{x}\right|\right)}{\sqrt{2}}=\log\left(C\right)$$

Atsakymas:

```
ats:map(exp,%), ratsimp;
```

$$\frac{\%e^{\frac{\log\left(\frac{\left|x\sqrt{2\,y^2+x^2}+\sqrt{2}\,\left|x\right|\,y\right|}{x^2}\right)}{\sqrt{2}}}}{\left|x\right|}=C$$

Naudojant ODE2 funkciją

```
ats:ode2(eq,y,x);
```

$$x=\%c\,\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,\left|x\right|}}$$

```
%/x/%c;
```

$$\frac{1}{\%c}=\frac{\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,\left|x\right|}}}{x}$$

Atsakymas:

```
ats:rhs(%)=%c;
```

$$\frac{\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,|x|}}}{x}=\%c$$

```
logarc(%), ratsimp;
```

$$\frac{\%e^{\frac{x\log\left(\frac{x\sqrt{2\,y^2+x^2}+\sqrt{2}\,|x|\,y}{x\,|x|}\right)}{\sqrt{2}\,|x|}}}{x}=\%c$$

Patikrinimas:

```
depends(y,x);
```

$$[\,\mathrm{y}(x)\,]$$

```
diff(ats,x);
```

$$\frac{\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,|x|}}\left(\frac{\sqrt{2}\,\left(\frac{d}{d\,x}\,y\right)}{x}-\frac{\sqrt{2}\,y}{x^2}\right)}{\sqrt{2}\,|x|\sqrt{\frac{2\,y^2}{x^2}+1}}-\frac{\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,|x|}}}{x^2}=0$$

```
eq3:solve(%,diff(y,x));
```

$$\left[\frac{d}{d\,x}\,y=\frac{|x|\sqrt{\frac{2\,y^2}{x^2}+1}+y}{x}\right]$$

```
subst(%,eq), ratsimp;
```

$$\sqrt{2\,y^2+x^2}+y=\sqrt{2\,y^2+x^2}+y$$

```
is(%);
```

true

```
f1:rhs(eq3[1]);
```

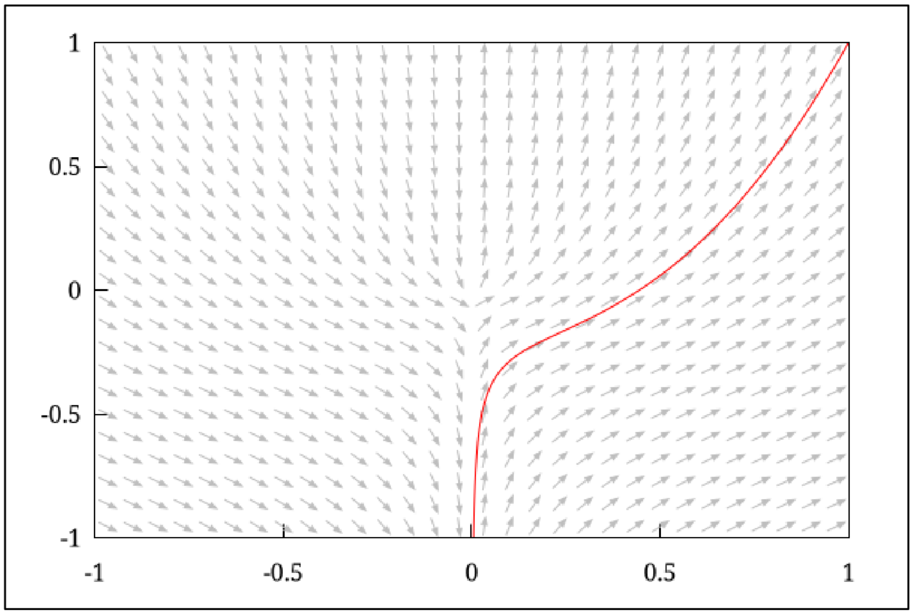
$$\frac{|x|\sqrt{\frac{2\,y^2}{x^2}+1}+y}{x}$$

Krypčių laukas

```
load(drawdf);
```

/usr/share/maxima/5.44.0/share/diffequations/drawdf.mac

```
wxdrawdf(f1, [x, −1,1], [y,−1,1],
[trajectory_at,1,1], field_color=gray, key="isocline", color=green, line_width=2,
nticks=200);
```



0

```
f:makelist(subst(%c=k,ats),k,[−2,−1,−1/2,1/2,1,2]);
```

$$\left[\frac{\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,|x|}}}{x}=-2,\frac{\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,|x|}}}{x}=-1,\frac{\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,|x|}}}{x}=-\frac{1}{2},\frac{\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,|x|}}}{x}=\frac{1}{2},\frac{\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,|x|}}}{x}=1,\frac{\%e^{\frac{x\,\mathrm{asinh}\left(\frac{\sqrt{2}\,y}{x}\right)}{\sqrt{2}\,|x|}}}{x}=2\right]$$

Integralinės kreivės

```
load(draw)$
```

```
wxdraw2d(grid = true,
color = red, key = "c=-2",
implicit(f[1], x, −1, 1, y, −2, 2),
color = blue, key = "c=-1",
implicit(f[2], x, −1, 1, y, −2, 2),
color = green, key = "c=-1/2",
implicit(f[3], x, −1, 1, y, −2, 2),
color = violet, key = "c=1/2",
implicit(f[4], x, −1, 1, y, −2, 2),
color = brown, key = "c=1",
implicit(f[5], x, −1, 1, y, −2, 2),
color = orange, key = "c=2",
implicit(f[6], x, −1, 1, y, −2, 2)),
wxplot_size = [500,500]$
```

rat: replaced 1.414213562373095 by 22619537/15994428 = 1.414213562373096
rat: replaced 1.414213562373095 by 22619537/15994428 = 1.414213562373096

Refusing to factor polynomial $\left| \%e^{\frac{x \operatorname{asinh}\left(\frac{22619537\, y}{15994428\, x}\right)}{22619537\, |x|}} \right|^{15994428} + 2\, x$

because its degree exceeds factor_max_degree (1000)
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Refusing to factor polynomial $2 \left| \%e^{\frac{x \operatorname{asinh}\left(\frac{22619537\, y}{15994428\, x}\right)}{22619537\, |x|}} \right|^{15994428} + x$

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