[3] KRAŠTINIO UŽDAVINIO ŠILUMOS LAIDUMO LYGČIAI SPRENDIMAS FURJĖ, ARBA KINTAMŲJŲ ATSKYRIMO, METODU (6 VARIANTAS).

```
load(draw);
```

/usr/share/maxima/5.44.0/share/draw/draw.lisp

1 Uždavinio formulavimas

```
[a,alpha,f,L,u0,u1,u2] : [3,8,3,3,(2/9)·x^2−(5/3)·x+3,3·t+3,t];
```

$$\left[3,8,3,3,\frac{2x^2}{9}-\frac{5x}{3}+3,3t+3,t\right]$$

```
eq:diff(u(x,t),t)=a^2·diff(u(x,t),x,2)−alpha·u(x,t)+f;
ic:u(x,0)=u0;    /* "ic" - initial condition           */
bc1:u(0,t)=u1;  /* bc1 - boundary condition 1 */
bc2:u(L,t)=u2;  /* bc2 - boundary condition 2 */
```

$$\frac{d}{dt}u(x,t)=9\left(\frac{d^2}{dx^2}u(x,t)\right)-8u(x,t)+3$$

$$u(x,0)=\frac{2x^2}{9}-\frac{5x}{3}+3$$

$$u(0,t)=3t+3$$

$$u(3,t)=t$$

2 Tikrinės reikšmės ir tikrinės funkcijos

Tikrinių reikšmių uždavinio
v''(x) + λv(x) = 0,
v(0)=0, v(L)=0
tikrinės funkcijos(tf) ir tikrinės reikšmės(tr) yra:

```
define(tf(k),sqrt(2/L)·sin(%pi·k·x/L));
```

$$\mathrm{tf}(k):=\frac{\sqrt{2}\sin\left(\frac{\pi k x}{3}\right)}{\sqrt{3}}$$

Dėstytojas tf(k) pažymėjo v_k

```
v_k:rhs(%);
```

$$\frac{\sqrt{2}\sin\left(\frac{\pi k x}{3}\right)}{\sqrt{3}}$$

```
define(tr(k),((%pi·k)/L)^2);
```

$$\mathrm{tr}(k):=\frac{\pi^2 k^2}{9}$$

Dėstytojas tr(k) pažymėjo λ_k

```
λ_k:rhs(%);
```

$$\frac{\pi^2 k^2}{9}$$

Šių formulių išvedimą kompiuteriu žr. A. Domarko lab. darbe (5-7 psl.)

Nuoroda:
https://klevas.mif.vu.lt/aleksas/MatematinisModeliavimas/3%20LaboratorinisDarbas/silumos-new.pdf

Detalus tikrinių funkcijų ir tikrinių reikšmių išvedimas pateiktas raštu.

Tikrinės funkcijos sudaro ortonormuotą sistemą Hilberto erdvėje L[2](0,L).
Šioje erdvėje norma apibrėžiama lygybe:

```
norma(u):=sqrt(integrate(u^2,x,0,L));
```

$$\mathrm{norma}(u):=\sqrt{\int_0^L u^2\, x}$$

Skaliarinė sandauga:

```
ss(u,v):=integrate(u·v,x,0,L);
```

$$\mathrm{ss}(u,v):=\int_0^L u\, v\, x$$

```
declare([k,m],integer)$
```

```
norma(tf(k));
```

$$1$$

```
ss(tf(k),tf(m));
```

$$0$$

```
ss(tf(k),tf(k));
```

$$1$$

3 Suvedimas į uždavinį su nulinėmis kraštinėmis sąlygomis

Atliksime keitinį u(x,t)=w(x,t) + h(x,t), kad gautume uždavinį su nulinėmis kraštinėmis sąlygomis.
Čia h(x,t) yra tiesė pagal x, einanti per taškus (0,u1) ir (L,u2), nes h(0)=u1 ir h(L)=u2:

```
define(h(x,t),((u2−u1)·x)/L+u1);
```

$$\mathrm{h}(x,t):=\frac{(-2\,t-3)\,x}{3}+3\,t+3$$

```
h(0,t);
```

$$3\,t+3$$

```
h(L,t);
```

$$t$$

Suvedimui į uždavinį su homogeninėmis kraštinėmis sąlygomis atliksime keitinį

```
keit:u(x,t)=w(x,t)+h(x,t);
```

$$u(x,t)=w(x,t)+\frac{(-2\,t-3)\,x}{3}+3\,t+3$$

Apskaičiuojame naują lygties laisvąjį narį g (vietoje f):

```
subst(keit,eq);
```

$$\frac{d}{d\,t}\left(w(x,t)+\frac{(-2\,t-3)\,x}{3}+3\,t+3\right)=9\left(\frac{d^2}{d\,x^2}\left(w(x,t)+\frac{(-2\,t-3)\,x}{3}+3\,t+3\right)\right)-8\left(w(x,t)+\frac{(-2\,t-3)\,x}{3}+3\,t+3\right)+3$$

```
ev(%, nouns);
```

$$\frac{d}{d\,t}\,w(x,t)-\frac{2\,x}{3}+3=9\left(\frac{d^2}{d\,x^2}\,w(x,t)\right)-8\left(w(x,t)+\frac{(-2\,t-3)\,x}{3}+3\,t+3\right)+3$$

```
expand(rhs(%)-lhs(%));
```

$$9\left(\frac{d^2}{d\,x^2}\,w(x,t)\right)-\frac{d}{d\,t}\,w(x,t)-8\,w(x,t)+\frac{16\,t\,x}{3}+\frac{26\,x}{3}-24\,t-24$$

```
g:subst(w(x,t)=0,%);
```

$$\frac{16\,t\,x}{3}+\frac{26\,x}{3}-24\,t-24$$

Apskaičiuojame naują pradinę funkciją (vietoje u0), kurią žymėsime w0:

```
subst([t=0,ic],keit);
```

$$\frac{2\,x^2}{9}-\frac{5\,x}{3}+3=w(x,0)-x+3$$

```
solve(%,w(x,0) )[1];
```

$$w(x,0)=\frac{2\,x^2-6\,x}{9}$$

```
w0:rhs(%);
```

$$\frac{2\,x^2-6\,x}{9}$$

Gavome uždavinį su nulinėmis kraštinėmis sąlygomis funkcijai w(x,t):

```
eqn:diff(w(x,t),t)=a^2·diff(w(x,t),x,2)−alpha·w(x,t)+g;
icn:w(x,0)=w0; /* "ic new" */
bc1n:w(0,t)=0; /* "bc1 new" */
bc2n:w(L,t)=0; /* "bc2 new" */
```

$$\frac{d}{d\,t}\,w(x,t)=9\left(\frac{d^2}{d\,x^2}\,w(x,t)\right)-8\,w(x,t)+\frac{16\,t\,x}{3}+\frac{26\,x}{3}-24\,t-24$$

$$w(x,0)=\frac{2\,x^2-6\,x}{9}$$

$$w(0,t)=0$$

$$w(3,t)=0$$

4 Sprendinio radimas

Uždavinio su nulinėmis kraštinėmis sąlygomis sprendinys randamas pavidalu

```
w(x,t)=sum(T[k](t)·'tf(k),k,1,inf);
```

$$w(x,t)=\sum_{k=1}^{\infty}\left(\mathrm{tf}(k)\,T_k(t)\right)$$

Koeficientai T[k](t) yra uždavinių
y'(t) + a^2·tr(k)·y(t) + alpha·y(t)= d(k), y(0)=c(k), k = 1, 2, ...
sprendiniai. Čia
d(k) = integrate(g·tf(k),x,0,L),
c(k) = integrate(w0·tf(k),x,0,L),
g = d(1)·tf(1) + d(2)·tf(2) + ...   (1)
w0= c(1)·tf(1) + c(2)·tf(2) +...   (2)

skaičiuojame d(k). Pirmas daugiklis visada bus = 1.

```
d(k):=1/integrate(tf(k)^2,x,0,L)·integrate(g·tf(k),x,0,L);
```

$$d(k):=\frac{1}{\int_0^L \mathrm{tf}(k)^2\,dx}\int_0^L g\,\mathrm{tf}(k)\,dx$$

skaičiuojame c(k). Pirmas daugiklis visada bus = 1.

```
c(k):=1/integrate(tf(k)^2,x,0,L)·integrate(w0·tf(k),x,0,L);
```

$$c(k):=\frac{1}{\int_0^L \mathrm{tf}(k)^2\,dx}\int_0^L w0\,\mathrm{tf}(k)\,dx$$

Įrodymui pakanka lygybes (1) ir (2) skaliariškai padauginti iš tf(k) ir atsižvelgti, kad tikrinių funkcijų sistema yra ortogonali.

```
ratsimp(c(k));
```

$$\frac{3\,2^{5/2}\,(-1)^k-3\,2^{5/2}}{\sqrt{3}\,\pi^3\,k^3}$$

```
makelist(c(k),k,1,10);
```

$$[-\frac{2^{7/2}\sqrt{3}}{\pi^3},0,-\frac{2^{7/2}}{3^{5/2}\,\pi^3},0,-\frac{2^{7/2}\sqrt{3}}{125\,\pi^3},0,-\frac{2^{7/2}\sqrt{3}}{343\,\pi^3},0,-\frac{2^{7/2}}{3^{11/2}\,\pi^3},0]$$

```
ratsimp(d(k));
```

$$\frac{(3\,2^{7/2}\,(-1)^k-9\,2^{7/2})\,t-3\,2^{3/2}\,(-1)^k-9\,2^{7/2}}{\sqrt{3}\,\pi\,k}$$

```
makelist(d(k),k,1,10);
```

$$[\frac{\sqrt{2}\,(-\frac{72\,t+72}{\pi}-\frac{24\,t-6}{\pi})}{\sqrt{3}},\frac{\sqrt{2}\,(\frac{12\,t-3}{\pi}-\frac{36\,t+36}{\pi})}{\sqrt{3}},\frac{\sqrt{2}\,(-\frac{24\,t+24}{\pi}-\frac{8\,t-2}{\pi})}{\sqrt{3}},\frac{\sqrt{2}\,(\frac{12\,t-3}{2\,\pi}-\frac{18\,t+18}{\pi})}{\sqrt{3}},\frac{\sqrt{2}\,(-\frac{72\,t+72}{5\,\pi}-\frac{24\,t-6}{5\,\pi})}{\sqrt{3}},\frac{\sqrt{2}\,(\frac{4\,t-1}{\pi}-\frac{12\,t+12}{\pi})}{\sqrt{3}},\frac{\sqrt{2}\,(-\frac{72\,t+72}{7\,\pi}-\frac{24\,t-6}{7\,\pi})}{\sqrt{3}},\frac{\sqrt{2}\,(\frac{12\,t-3}{4\,\pi}-\frac{9\,t+9}{\pi})}{\sqrt{3}},\frac{\sqrt{2}\,(-\frac{8\,t+8}{\pi}-\frac{8\,t-2}{3\,\pi})}{\sqrt{3}},\frac{\sqrt{2}\,(\frac{12\,t-3}{5\,\pi}-\frac{36\,t+36}{5\,\pi})}{\sqrt{3}}]$$

Duotojo uždavinio sprendinys randamas pavidalu (žr. [1], p. 196)

```
u(x,t)=sum(T[k](t)·'tf(k),k,1,inf)+h(x,t);
```

$$u(x,t)=\frac{(-2\,t-3)\,x}{3}+\left|\sum_{k=1}^{\infty}(\mathrm{tf}(k)\,T_k(t))\right|+3\,t+3$$

Koeficientai T[k](t) yra uždavinių
diff(y,t)' + a^2*tr(k)*y + alpha*y= d(k), y(0)=c(k), k = 1, 2, ...
sprendiniai.

Skaičiuojame w(x,t). Vietoj begalinės sumos imame sumą iki 5. Tai mums duoda pakankamą tikslumą.

```
S:0$
for k thru 5 do
(
ode2('diff(y,t)+a^2·tr(k)·y + alpha·y=d(k),y,t),
ic1(%%, t=0, y=c(k)),
ratsimp(%%),
S:S+rhs(%%)·tf(k)
)$
```

S: expand(S);

$$\frac{17500\sqrt{3}\,\pi^{4}\,\%e^{-25\pi^{2}t-8t}\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^{7}+50000\sqrt{3}\,\pi^{5}+8000\sqrt{3}\,\pi^{3}}+$$

$$\frac{800\sqrt{3}\,\pi^{2}\,\%e^{-25\pi^{2}t-8t}\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^{7}+50000\sqrt{3}\,\pi^{5}+8000\sqrt{3}\,\pi^{3}}-\frac{1024\sqrt{3}\,\%e^{-25\pi^{2}t-8t}\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^{7}+50000\sqrt{3}\,\pi^{5}+8000\sqrt{3}\,\pi^{3}}$$

$$-\frac{40000\sqrt{3}\,\pi^{4}\,t\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^{7}+50000\sqrt{3}\,\pi^{5}+8000\sqrt{3}\,\pi^{3}}-$$

$$\frac{12800\sqrt{3}\,\pi^{2}\,t\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^{7}+50000\sqrt{3}\,\pi^{5}+8000\sqrt{3}\,\pi^{3}}-\frac{27500\sqrt{3}\,\pi^{4}\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^{7}+50000\sqrt{3}\,\pi^{5}+8000\sqrt{3}\,\pi^{3}}-$$

$$\frac{800\,3^{5/2}\,\pi^{2}\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^{7}+50000\sqrt{3}\,\pi^{5}+8000\sqrt{3}\,\pi^{3}}+\frac{26\sqrt{3}\,\pi^{2}\,\%e^{-16\pi^{2}t-8t}\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^{5}+32\sqrt{3}\,\pi^{3}+8\sqrt{3}\,\pi}+$$

$$\frac{4\,3^{3/2}\,\%e^{-16\pi^{2}t-8t}\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^{5}+32\sqrt{3}\,\pi^{3}+8\sqrt{3}\,\pi}-\frac{16\sqrt{3}\,\pi^{2}\,t\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^{5}+32\sqrt{3}\,\pi^{3}+8\sqrt{3}\,\pi}-$$

$$\frac{8\sqrt{3}\,t\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^{5}+32\sqrt{3}\,\pi^{3}+8\sqrt{3}\,\pi}-\frac{26\sqrt{3}\,\pi^{2}\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^{5}+32\sqrt{3}\,\pi^{3}+8\sqrt{3}\,\pi}-$$

$$\frac{4\,3^{3/2}\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^{5}+32\sqrt{3}\,\pi^{3}+8\sqrt{3}\,\pi}+\frac{28\,3^{9/2}\,\pi^{4}\,\%e^{-9\pi^{2}t-8t}\sin(\pi x)}{3^{15/2}\,\pi^{7}+16\,3^{11/2}\,\pi^{5}+64\,3^{7/2}\,\pi^{3}}+$$

$$\frac{32\,3^{5/2}\,\pi^{2}\,\%e^{-9\pi^{2}t-8t}\sin(\pi x)}{3^{15/2}\,\pi^{7}+16\,3^{11/2}\,\pi^{5}+64\,3^{7/2}\,\pi^{3}}-\frac{1024\sqrt{3}\,\%e^{-9\pi^{2}t-8t}\sin(\pi x)}{3^{15/2}\,\pi^{7}+16\,3^{11/2}\,\pi^{5}+64\,3^{7/2}\,\pi^{3}}-$$

$$\frac{64\,3^{9/2}\,\pi^{4}\,t\sin(\pi x)}{3^{15/2}\,\pi^{7}+16\,3^{11/2}\,\pi^{5}+64\,3^{7/2}\,\pi^{3}}-\frac{512\,3^{5/2}\,\pi^{2}\,t\sin(\pi x)}{3^{15/2}\,\pi^{7}+16\,3^{11/2}\,\pi^{5}+64\,3^{7/2}\,\pi^{3}}-$$

$$\frac{44\,3^{9/2}\,\pi^{4}\sin(\pi x)}{3^{15/2}\,\pi^{7}+16\,3^{11/2}\,\pi^{5}+64\,3^{7/2}\,\pi^{3}}-\frac{32\,3^{9/2}\,\pi^{2}\sin(\pi x)}{3^{15/2}\,\pi^{7}+16\,3^{11/2}\,\pi^{5}+64\,3^{7/2}\,\pi^{3}}+$$

$$\frac{26\sqrt{3}\,\pi^{2}\,\%e^{-4\pi^{2}t-8t}\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^{5}+16\sqrt{3}\,\pi^{3}+16\sqrt{3}\,\pi}+\frac{16\,3^{3/2}\,\%e^{-4\pi^{2}t-8t}\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^{5}+16\sqrt{3}\,\pi^{3}+16\sqrt{3}\,\pi}-$$

$$\frac{16\sqrt{3}\,\pi^{2}\,t\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^{5}+16\sqrt{3}\,\pi^{3}+16\sqrt{3}\,\pi}-\frac{32\sqrt{3}\,t\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^{5}+16\sqrt{3}\,\pi^{3}+16\sqrt{3}\,\pi}-$$

$$\frac{26\sqrt{3}\,\pi^{2}\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^{5}+16\sqrt{3}\,\pi^{3}+16\sqrt{3}\,\pi}-\frac{16\,3^{3/2}\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^{5}+16\sqrt{3}\,\pi^{3}+16\sqrt{3}\,\pi}+$$

$$28\sqrt{3}\,\pi^{4}\,\%e^{-\pi^{2}t-8t}\sin\left(\frac{\pi x}{}\right)\qquad 32\sqrt{3}\,\pi^{2}\,\%e^{-\pi^{2}t-8t}\sin\left(\frac{\pi x}{}\right)$$

Užrašome galutinį sprendinį u(x,t)

spr:u(x,t)=S+h(x,t);

$$u(x,t)=\frac{17500\sqrt{3}\,\pi^4\,\%e^{-25\pi^2 t-8t}\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^7+50000\sqrt{3}\,\pi^5+8000\sqrt{3}\,\pi^3}+$$

$$\frac{800\sqrt{3}\,\pi^2\,\%e^{-25\pi^2 t-8t}\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^7+50000\sqrt{3}\,\pi^5+8000\sqrt{3}\,\pi^3}-\frac{1024\sqrt{3}\,\%e^{-25\pi^2 t-8t}\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^7+50000\sqrt{3}\,\pi^5+8000\sqrt{3}\,\pi^3}$$

$$-\frac{40000\sqrt{3}\,\pi^4\,t\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^7+50000\sqrt{3}\,\pi^5+8000\sqrt{3}\,\pi^3}-$$

$$\frac{12800\sqrt{3}\,\pi^2\,t\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^7+50000\sqrt{3}\,\pi^5+8000\sqrt{3}\,\pi^3}-\frac{27500\sqrt{3}\,\pi^4\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^7+50000\sqrt{3}\,\pi^5+8000\sqrt{3}\,\pi^3}-$$

$$\frac{800\,3^{5/2}\,\pi^2\sin\left(\frac{5\pi x}{3}\right)}{78125\sqrt{3}\,\pi^7+50000\sqrt{3}\,\pi^5+8000\sqrt{3}\,\pi^3}+\frac{26\sqrt{3}\,\pi^2\,\%e^{-16\pi^2 t-8t}\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^5+32\sqrt{3}\,\pi^3+8\sqrt{3}\,\pi}+$$

$$\frac{4\,3^{3/2}\,\%e^{-16\pi^2 t-8t}\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^5+32\sqrt{3}\,\pi^3+8\sqrt{3}\,\pi}-\frac{16\sqrt{3}\,\pi^2\,t\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^5+32\sqrt{3}\,\pi^3+8\sqrt{3}\,\pi}-$$

$$\frac{8\sqrt{3}\,t\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^5+32\sqrt{3}\,\pi^3+8\sqrt{3}\,\pi}-\frac{26\sqrt{3}\,\pi^2\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^5+32\sqrt{3}\,\pi^3+8\sqrt{3}\,\pi}-$$

$$\frac{4\,3^{3/2}\sin\left(\frac{4\pi x}{3}\right)}{32\sqrt{3}\,\pi^5+32\sqrt{3}\,\pi^3+8\sqrt{3}\,\pi}+\frac{28\,3^{9/2}\,\pi^4\,\%e^{-9\pi^2 t-8t}\sin(\pi x)}{3^{15/2}\,\pi^7+16\,3^{11/2}\,\pi^5+64\,3^{7/2}\,\pi^3}+$$

$$\frac{32\,3^{5/2}\,\pi^2\,\%e^{-9\pi^2 t-8t}\sin(\pi x)}{3^{15/2}\,\pi^7+16\,3^{11/2}\,\pi^5+64\,3^{7/2}\,\pi^3}-\frac{1024\sqrt{3}\,\%e^{-9\pi^2 t-8t}\sin(\pi x)}{3^{15/2}\,\pi^7+16\,3^{11/2}\,\pi^5+64\,3^{7/2}\,\pi^3}-$$

$$\frac{64\,3^{9/2}\,\pi^4\,t\sin(\pi x)}{3^{15/2}\,\pi^7+16\,3^{11/2}\,\pi^5+64\,3^{7/2}\,\pi^3}-\frac{512\,3^{5/2}\,\pi^2\,t\sin(\pi x)}{3^{15/2}\,\pi^7+16\,3^{11/2}\,\pi^5+64\,3^{7/2}\,\pi^3}-$$

$$\frac{44\,3^{9/2}\,\pi^4\sin(\pi x)}{3^{15/2}\,\pi^7+16\,3^{11/2}\,\pi^5+64\,3^{7/2}\,\pi^3}-\frac{32\,3^{9/2}\,\pi^2\sin(\pi x)}{3^{15/2}\,\pi^7+16\,3^{11/2}\,\pi^5+64\,3^{7/2}\,\pi^3}+$$

$$\frac{26\sqrt{3}\,\pi^2\,\%e^{-4\pi^2 t-8t}\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^5+16\sqrt{3}\,\pi^3+16\sqrt{3}\,\pi}+\frac{16\,3^{3/2}\,\%e^{-4\pi^2 t-8t}\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^5+16\sqrt{3}\,\pi^3+16\sqrt{3}\,\pi}-$$

$$\frac{16\sqrt{3}\,\pi^2\,t\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^5+16\sqrt{3}\,\pi^3+16\sqrt{3}\,\pi}-\frac{32\sqrt{3}\,t\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^5+16\sqrt{3}\,\pi^3+16\sqrt{3}\,\pi}-$$

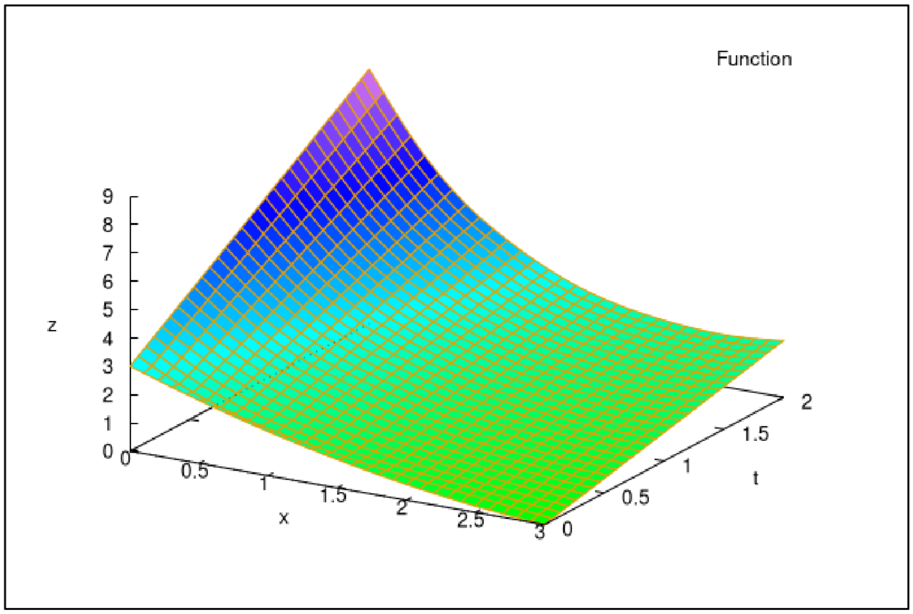
$$\frac{26\sqrt{3}\,\pi^2\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^5+16\sqrt{3}\,\pi^3+16\sqrt{3}\,\pi}-\frac{16\,3^{3/2}\sin\left(\frac{2\pi x}{3}\right)}{4\sqrt{3}\,\pi^5+16\sqrt{3}\,\pi^3+16\sqrt{3}\,\pi}+$$

5 Grafinis sprendinio vaizdavimas

```
define(ats(x,t),rhs(spr))$
```

Trimatis brėžinys:

```
wxplot3d(ats(x,t), [x,0,L], [t,0,2])$
```



Iš grafiko matosi, kad esant pradinei sąlygai ("ic") t = 0, kai x kinta nuo 0 iki 3,
z-x plokštumoje matome atvaizduotą parabolę z = u(x,0)=(2·x^2)/9-(5·x)/3+3, ir matome,
kad teisinga, kad kai x = 0, u(0,0) = 3, o kai x = 3, u(3,0) = 0.

Taip pat matome, kad esant pradinėms kraštinėms sąlygoms:
x = 0, u(0,t) = 3·t+3 ("bc1")
x = 3, u(3,t) = t ("bc2")
grafike tai irgi atsispindi: kai x = 0, plokštumoje z-t matome tiesę z = 3·t+3,
o kai x = 3, plokštumoje z-t matome tiesę z = t.

Apskaičiuojame sprendinio ir h(x,t) reikšmes taške x=0.5, t=2:

```
ev(spr,[x=0.5,t=2]),numer;
```

$$u(0.5,2)=5.741204396547597$$

```
ev(h(x,t),[x=0.5,t=2]);
```

$$7.833333333333333$$

```
subst(t=0,spr),ratsimp;
```

$$u(x,0)=-\frac{432\sin\left(\frac{5\pi x}{3}\right)+2000\sin(\pi x)+54000\sin\left(\frac{\pi x}{3}\right)+3375\pi^3 x-10125\pi^3}{3375\pi^3}$$

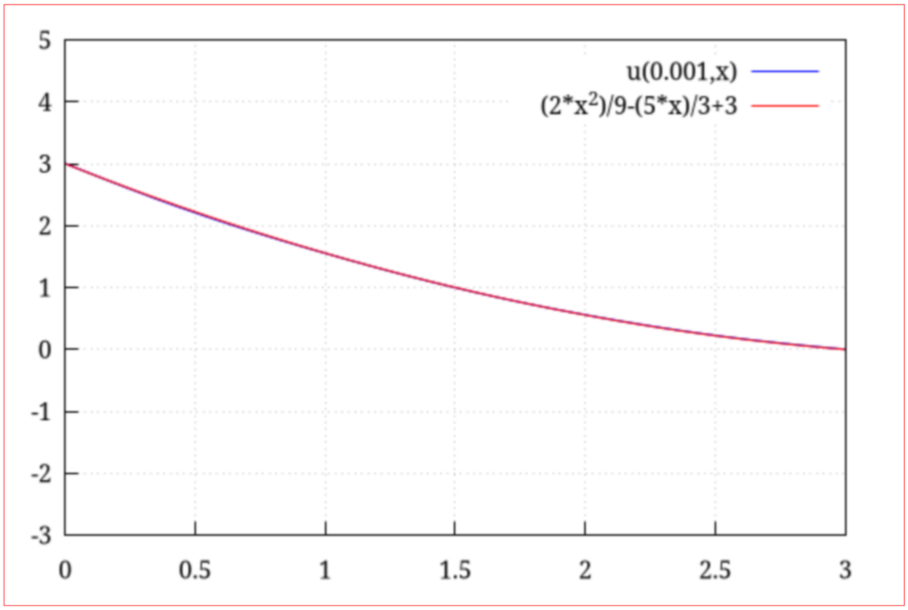
```
fpprintprec:4$
```

Pasirenkame animacijos parametro t reikšmių sąrašą. Arčiau nulio reikšmes imame tankiau.

```
t_list:append(makelist(t·0.0001,t,0,100,10),makelist(t·0.1,t,1,20,2));
```

[0 , 0.001 , 0.002 , 0.003 , 0.004 , 0.005 , 0.006 , 0.007 , 0.008 , 0.009 , 0.01 , 0.1 , 0.3 , 0.5 , 0.7 , 0.9 , 1.1 , 1.3 , 1.5 , 1.7 , 1.9]

```
with_slider_draw(
 t,t_list,
 yrange=[−3,5],
 key=string(u(t,x)),
 explicit(ats(x,t),x,0,L),
 key=string(u0),
 color=red,
 explicit(u0,x,0,L),grid=true);
```

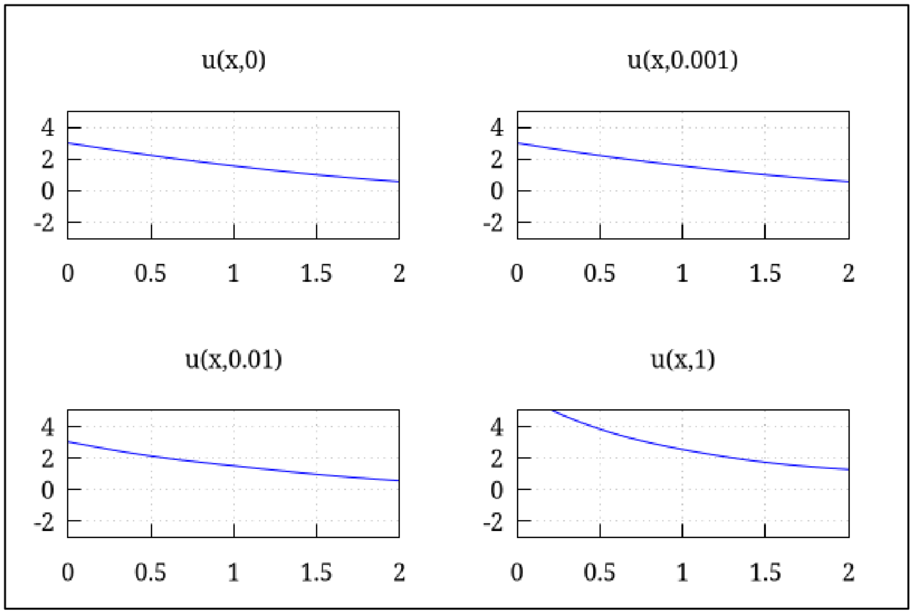


Animacijos mometus dar galima pavaizduoti taip:

```
set_draw_defaults( xrange = [0, 2],
 yrange = [−3,5],
 xtics=1/2,
 ytics = 2,
 color = blue,
 grid = true)$
```

```
sc1: gr2d(title="u(x,0)", explicit(ats(x,0),x,0,2))$
sc2: gr2d(title="u(x,0.001)", explicit(ats(x,0.001),x,0,2))$
sc3: gr2d(title="u(x,0.01)", explicit(ats(x,0.01),x,0,2))$
sc4: gr2d(title="u(x,1)", explicit(ats(x,1),x,0,2))$
```

```
wxdraw(sc1, sc2, sc3, sc4, columns = 2)$
```



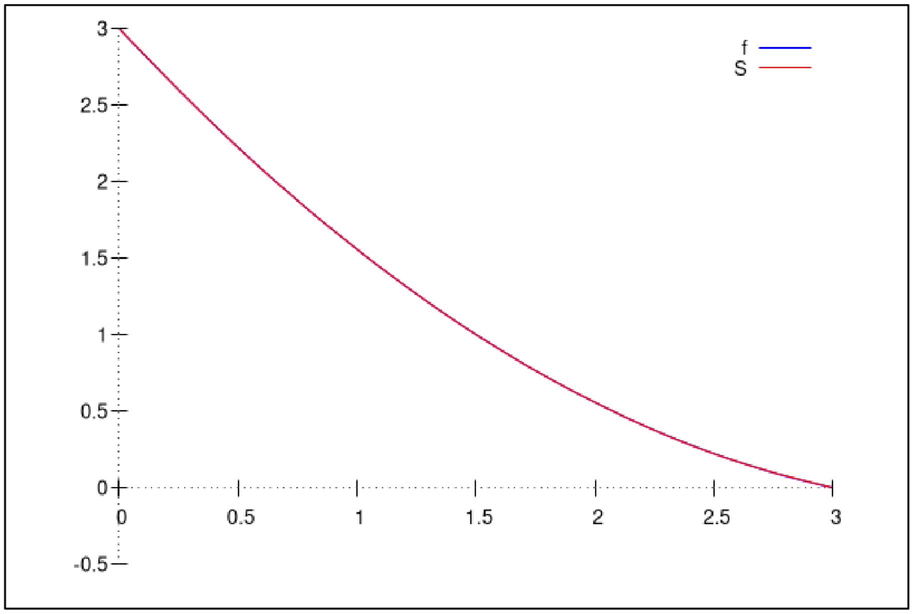
Patikrinimas

Patikriname pradinę sąlygą:

```
subst(t=0,spr),ratsimp;
```

$$u(x,0)=-\frac{432\sin\left(\frac{5\pi x}{3}\right)+2000\sin(\pi x)+54000\sin\left(\frac{\pi x}{3}\right)+3375\pi^3 x-10125\pi^3}{3375\pi^3}$$

```
wxplot2d([u0,rhs(spr)], [x,0,L],[box, false],[legend, "f", "S"]),t=0$
```



Matome, kad dalinė suma S, kai t=0, gerai aproksimuoja pradinę funkciją f.

Patikriname kraštines sąlygas:

```
ev(S,x=0);
ev(S,x=L);
```

0

0

```
subst(spr,eq)$
ev(%,nouns)$
expand(lhs(%)-rhs(%))$
ratsimp(%);
```

$$-((192\ t+132)\sin\left(\frac{5\pi x}{3}\right)+(120\ t+195)\sin\left(\frac{4\pi x}{3}\right)+(320\ t+220)\sin(\pi x)+(240\ t+390)\sin\left(\frac{2\pi x}{3}\right)+(960\ t+660)\sin\left(\frac{\pi x}{3}\right)+(80\pi t+130\pi)x-360\pi t-360\pi)/(15\pi)$$

```
eq;
```

$$\frac{d}{d\,t}\,u(x,t)=9\left(\frac{d^2}{d\,x^2}\,u(x,t)\right)-8\,u(x,t)+3$$