

Hadamard Transform

A Hadamard transform is then used as a precoding to build packets of the same importance and the same energy.

From: [Academic Press Library in Mobile and Wireless Communications](#), 2014

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Learn more about Hadamard Transform

Raman and Infrared Microspectroscopy

Pina Colarusso, ... E. Neil Lewis, in [Encyclopedia of Spectroscopy and Spectrometry](#), 1999

Hadamard transform imaging

Spatial [encoding methods](#) such as [Hadamard transform](#) imaging also can be used for the spectroscopic visualization of samples. More recently, developments in digital [microarray](#) technology will likely provide a convenient new approach for spatial encoding from the mid-infrared to the ultraviolet. In one common arrangement, the entire sample area is irradiated with wide-field, epi-illumination (Figure 4D). Part of the Raman signal emanating from the sample is blocked with a mask containing a series of [apertures](#). The spatially [filtered signal](#) is focused on to an entrance slit of a monochromator, which disperses the signal across a two-dimensional [array detector](#). The slit preserves one image axis while the Hadamard mask is used to encode the other axis. Subsequent measurements are carried out with the mask in different positions. Each measurement corresponds to the Raman signal from the unmasked points on the sample along one spatial axis over the entire [spectral range](#) of interest. The experiment is designed such that the number of [independent measurements](#)

equals the number of points on the sample. The spatially dependent images are then converted to spectroscopic images through a Hadamard transform. Unlike the other methods mentioned in this article, Hadamard [spectroscopic imaging](#) systems are limited to research activities and are not yet commercially available.

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Transforming Images for Video Coding

David R. Bull, in [Coding Pictures](#), 2014

5.3.2 The Walsh-Hadamard transform

The Walsh and Walsh-Hadamard transforms are simple but effective ways of compressing data. They have the significant advantage that the basic transform requires no multiplications, only additions and subtractions. While the coding gain is lower than transforms such as the DCT, which we will discuss shortly, they do find application in modern video compression, such as H.264/AVC, where they are used in intra-frame coding.

The discrete Walsh-Hadamard transform (DWT) is obtained from a simple rearrangement of the discrete Hadamard matrix. The Hadamard matrix is an $N \times N$ matrix with the property that $H^T H = N I$, where I is the identity matrix. This property can be used to find the inverse transform by iteratively applying the following operation:

$$(5.19) \quad Y = \frac{1}{N} H^T X$$

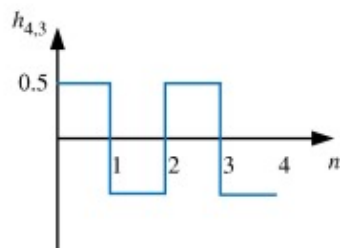
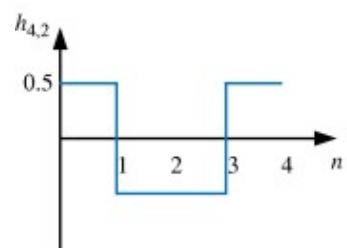
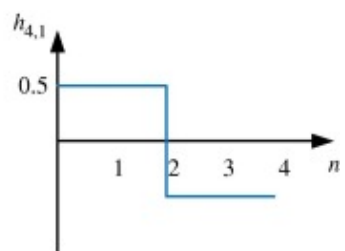
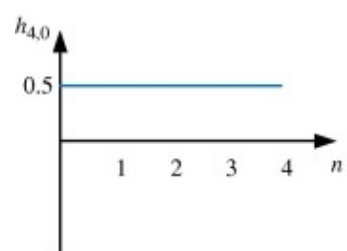
For example:

$$(5.20) \quad Y = \frac{1}{N} H^T X$$

The DWT is simply obtained by finding the corresponding Hadamard matrix by normalizing and rearranging the rows and columns (in sequence of the number of sign changes). Therefore the DWT is given by:

$$(5.21) \quad Y = \frac{1}{N} H^T X$$

The basis functions for the 1-D DWT are depicted in Figure 5.4.



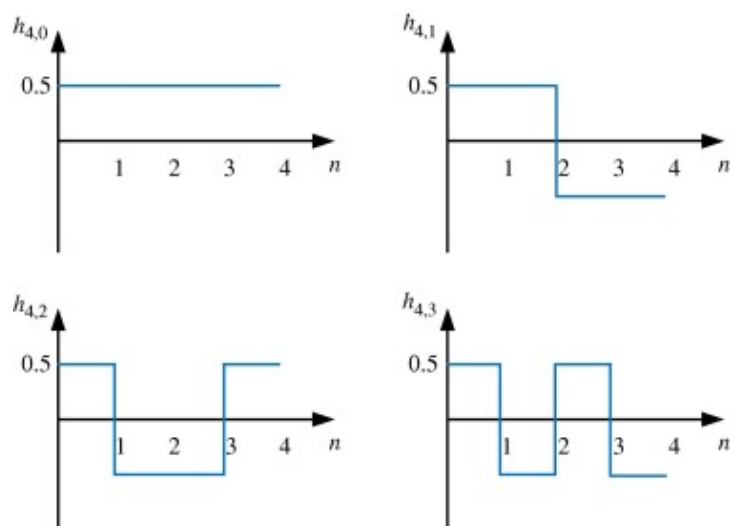


Figure 5.4. DWHT basis functions for $N=4$.

In the case of the 2-D transform, the basis functions are formed from the outer product of the 1-D basis functions. For basis functions for the 2-D DWHT are shown in Figure 5.5. **Example 5.3** Compute the 1-D DWHT for the data vector $x = [1, 2, 3, 4]^T$. **Solution** Using the basis functions from Figure 5.4, we can use equation (5.14) to compute the 1-D DWHT. The result is $X = [1, 1, 1, 1]^T$. **Example 5.4** Compute the 2-D DWHT of the input image block shown below. **Solution** Since the 2-D DWHT is separable, we can use equation (5.14) to compute the 2-D DWHT. The result is $X = [1, 1, 1, 1]^T$. **Example 5.5** Taking the result of Example 5.4, set all small-valued coefficients (coefficients with magnitude less than 0.5) to zero and perform the inverse transformation. What is the PSNR of the reconstructed signal? **Solution** The original signal was represented with a 6 bit word length. The reconstructed signal is represented with a 4 bit word length. The MSE = 3 and the PSNR of the reconstruction is:

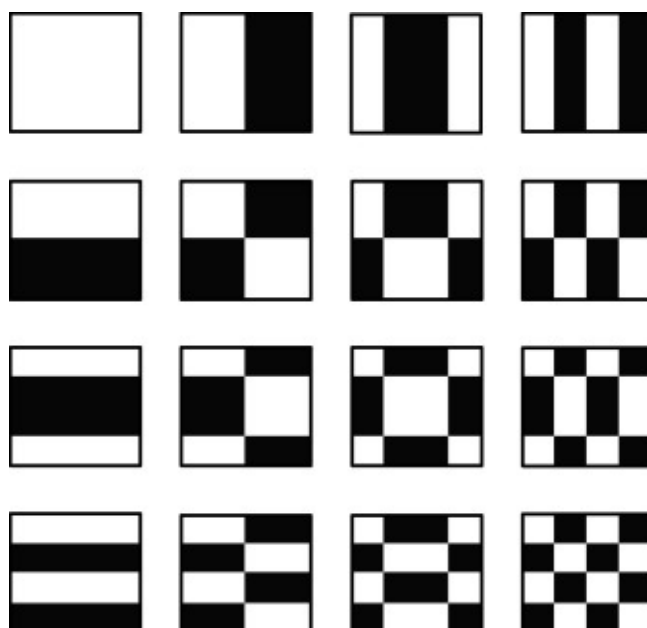


Figure 5.5. Basis functions for the 2-D DWHT.

Figure 5.5. Basis functions for the 2-D DWHT.

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Application of Data Hiding in Massive Computing

Anuj R. Shah, ... N. Anuj Z. Shihjin. [Advances in Computers](#), 2010

3.1.3.2 Real-Time Processing with High-Performance Computing Hardware Architectures

We have implemented the design for the inverse Hadamard computation on a single node of the full Cray XE6 node. The design uses 22% of the slices (5201 out of 23,616) of the XE6 node. It uses 36% of the blocks (36% of the blocks) and 1% of the 18 x 18 multipliers (4 out of 232). The design also uses 4-MB QDR SRAM memory banks available on the node. The design achieved a clock frequency of 142 MHz out of the 199 MHz available. The maximum data transfer bandwidth on the XE6 node is 4.2 GBps. However, this bandwidth can only be achieved only by a design that operates at the maximum frequency of 199 MHz. Designers operating at lower clock frequencies achieve a proportional bandwidth. In our case, we achieved a bandwidth of 1.01 GBps.

Our data set consists of 100 frames of 640 x 480 pixels, of 16-bit elements per scan. The total size of the data set is 400 MB. The streaming time at approximately 900 MBps is 0.44 s. The processing time for a single frame takes 0.11 s and is twice as fast as the processing time for the algorithm on the host 2.4-GHz AMD Opteron CPU. The data write-back time for the inverse transform is 4 ms.

Our research has determined that it is feasible to use FPGAs for processing mass spectrometry data at high speeds (GBps range). We have shown that it is possible to implement a high-speed algorithm for the inverse Hadamard transform to operate with high-speed data capture.

> [Read full chapter](#)

Type I (Linear) Data Hiding

Husrev T. Sencar, ... Ali N. Akansu, in [Data Hiding Fundamentals and Applications](#),
2004

to processing noise. Similarly, low-quality JPEG affects the high-frequency bands of subband decomposition (using an 8-tap binomial QMF [Daubechies] filter) to a much larger extent than the high-frequency Hadamard bands. We already know that **low-frequency bands** are not efficient channels due to the presence of high image noise. If the high-frequency bands are also affected by processing, it leaves us a small number of useful midfrequency bands. Transforms with lower GTC have many more of these useful midfrequency bands than the high-GTC transforms due to their **spectral properties** at higher processing noise scenarios. Therefore, *decompositions unsuitable for compression would, in general, be more immune to processing noise than decompositions with high GTC*. Also, recall that in Section 4.3, embedding in the image domain (or using identity transform for the transform blocks in Fig. 4-2) was found to be very robust to processing noise. The identity transform, which has the lowest GTC, has the highest robustness to processing noise. It is relevant to point out here that the term *robustness* is a measure of the change in *overall capacity* with a change in the processing noise (or processing scenario). The more robust the decomposition, the less is the reduction in capacity for a scenario of increased processing noise (or lower-quality compression). One should note that the robustness of the *low-frequency bands* of, say, the DCT decomposition will be much higher than the robustness of the single band coefficients (pixels) in the image domain. However, the low-frequency bands of the DCT have very little capacity due to high image noise. The reduced robustness of DCT is because of the drastic reduction in the *overall capacity* due to the significant increase of processing noise in the high-frequency bands.

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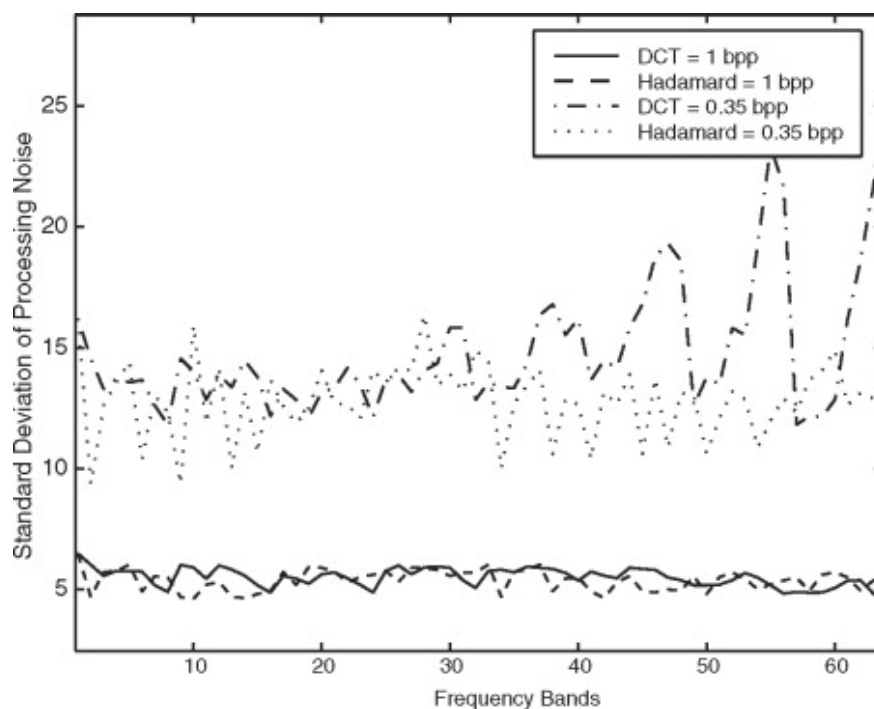


Figure 4-6. Comparison of standard deviation of processing noise for DCT and Hadamard decompositions. The source of processing noise is SPIHT compression at 1 bpp and 0.35 bpp.

The next question is whether the absence of the first band of the fourth order of bands for the decomposition or size of positions for size of the Eq. (4.15), we see from Eq. (4.15) is possible that a decomposition will not hurt. At worst, it may cause a worst case scenario. The information from the decomposition of each subchannel of, for instance, a 16-band decomposition can be further decomposed into four subchannels can only improve the capacity for data handling, city of data handling, processing noise is low.

[> Read full chapter](#)

Calibration, Transfer, Path, and The Instrumentation Aspects

Howard Mark, Jerry Workman, Jerry Workman, in [Spectrometry \(Second Edition\)](#), 2018

Instrument Types

Spectrometers, or spectrophotometers, come in many design types. There are instruments based on the grating with mechanical drive, grating monochromator with motor drive, Michelson interferometer in various forms, dispersive gratings with interferometer, interference and linear variable filter instruments with single detectors, single variable filters with array detection, MEMS-based Fabry-Pérot-based filters, digital transform actuator technologies, acousto-optic filters (AOFs), fiber-optic filters, and fiber-optic multivariate optical computing, and others. Calibration design type spectrometer to the same type and the same type is challenging enough, but getting enough sophisticated multifactor PLS models of different design types can be truly daunting. The requirement of precise spectral shape acquisition requires more sophisticated transformations (wavelength and frequency) and Y-axes (photometric value, potential value) correlation. Multivariate calibration models have been developed for many data dimensions and require delicate adjustments across this multidimensional space to fit measurements precisely. The data processing, data spacing, photometric response, photometric resolution, resolution, and symmetry, and other parameters and other parameters must be carefully calibrated to yield equivalent prediction results from different design types of spectrometers.

[> Read full chapter](#)

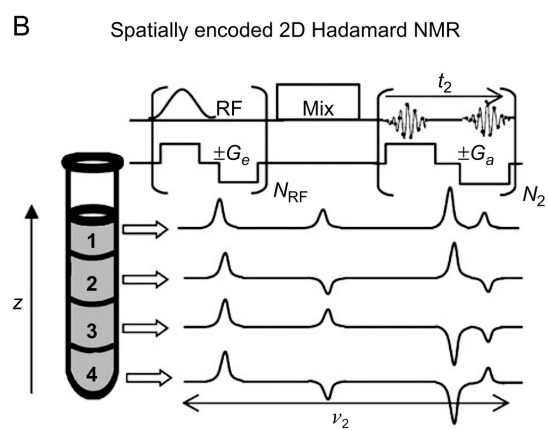
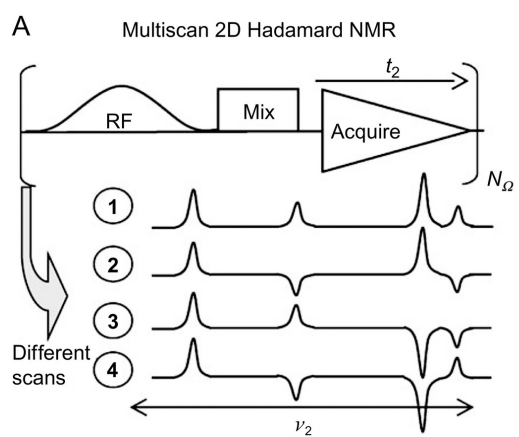
Ultrafast 2D NMR: Methods and Applications

Ultrafast 2D NMR: Methods and Applications

Boris Gouilleux, ... Patrice Guillemaud, Patrick Garavito, in *NMR Spectroscopy*, 2018

2.3.1.2 Hadamard 2D Hadamard

As for the spatial/spatial/spectral, only regions of interest are encoded in multiscan 2D Hadamard NMR. This had performed by modifying the phases of the different regions of the different scans [23]. Thus, one encoded spectral region of the Hadamard matrix. In multiscan 2D Hadamard NMR, traces are obtained by FT in the direct dimension (Fig. 5A), which is followed by the application of the Hadamard Transform (HT) in the indirect dimension, leading to the 2D spectrum.



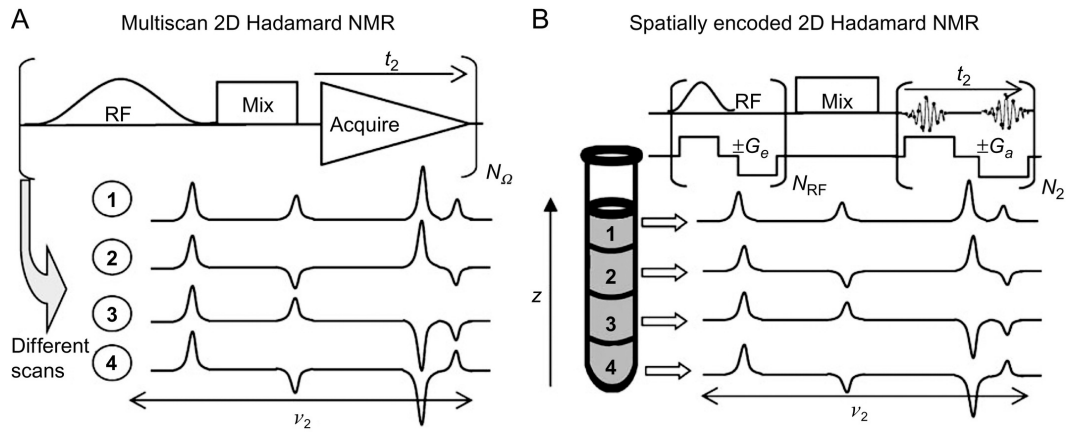


Fig. 5. (A) Original Fig. 5. (A) Original scheme proposed for obtaining 2D NMR spectra. (B) UF 2D Hadamard approach for obtaining 2D NMR spectra. (B) UF 2D Hadamard approach for obtaining 2D NMR spectra. Both information processes are based on the basis of a Hadamard transform. The spectral weights are illustrated for the illustrated example—to afford the desired information.

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Reproduced from A. Tal, B. Shapira, L. Frydman, Single-scan 2D Hadamard NMR spectroscopy, *Angew. Chem. Int. Ed.* 48 (2009) 2732–2736 with permission from John Wiley and Sons.

In ultrafast 2D Hadamard NMR, the Hadamard matrix is spatially encoded, thanks to the use of a special pulse sequence. During the EPR detection block, spatially resolved traces are obtained, without FT (Fig. 5B). As in its multiscan counterpart, it is applied to the 1D dimension to retrieve the 2D spectrum.

The use of Hadamard encoding is particularly interesting when the ultrafast domain is sparse, as the gain is inversely proportional to the size of the Hadamard matrix (related to the number of regions to be encoded). This technique is thus suitable for crowded spectra and for the spatial/spectral encoded experiments, requiring no knowledge of the 1D spectrum.

[Read full chapter](#)

Advanced implementations

Husrev T. Sencar, H. Akar, Ali N. Akar, in *Data Mining Applications and Applications*, 2004

6.2.7 Implementation Simulation Results

Optimum codebook selection depends on designing the set of transformation bases properly (i.e., they should be able to generate maximally separated transformation signals, see Eq. (6.13)). One intuitive way of picking such a set of transformation bases is by choosing the **rotation matrices** so that each transformation is a rotated version of the others. The multiple codebook data hiding is implemented by designing the transformation bases using **orthogonal transformations** in that can be employed to rotate a given vector with a chosen angle.

A particular transformation is obtained by multiplication of number of **orthogonal matrices**, all with the determinant equal to 1. Each orthogonal matrix is derived from the identity matrix by introducing $\cos \theta_k$ terms at (i, i) and (j, j) locations and $\sin \theta_k$ at (i, j) and (j, i) locations in order to rotate the (i, j) coordinate plane with the designated angle θ_k . The **rotation angles** θ_k , $k = 1, \dots, n$, are chosen by uniformly sampling.

By setting the signal size to N and the number of messages to M , the size of the **codebooks** utilized by the embedder is fixed to $M \times N$. The watermark signals that are embedded into the host signal are generated using a **Hadamard transform** matrix due to its simplicity. The Hadamard transform matrix of size $N \times N$ and its negated version are combined into a $2N \times N$ binary valued matrix. Every row of the combined matrix is indexed from 1 to $M = 2N$, scaled by $\frac{1}{\sqrt{M}}$ for maximum separation, and assigned to the **watermark signal** \mathbf{W}_j , $1 \leq j \leq M$, such that $i \neq j + N$. The host signal and channel noise are *iid* zero-mean **Gaussian** vectors with σ^2 . Prior to embedding, the permitted **embedding distortion** P_E is fixed, and the optimal values for the embedding parameter Δ are derived for the considered WNRs. The Δ values are also revealed to the detector. The parameters α and β , however, are properly adjusted for each embedding in order to ensure an embedding distortion of P_E and are not known to the detector. The simulations are done for a different number of transformations L and signal sizes N by embedding and detecting randomly chosen message indices.

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Multiple codebook data hiding is implemented in the type-I method, while the type-II method is based on thresholding and distortion-compensation types of processing. Figures 6-23 and 6-24 display the performance of the multiple codebook data hiding method using both maximum correlation and minimum distance criteria. Message embedding and detection with up to 25 codebooks is performed for codebook sizes of 64×32 , 128×64 , and 256×128 bits. The WNR and the WNR figures of 0.1 and 0.6-24 display the probability of success results obtained, respectively, for $L = 1, 3$ and $L = 1, 4$ with varying N values. The postprocessing is the postprocessing. This thresholding method, where the embedding signal size N , of a codebook, improves the [detection statistics](#) because normalized correlation and normalized distance give more reliable results with the large signal sizes. On the other hand, Figs. 6-25 and 6-26 display the performance of the thresholding type of postprocessing when $N = 128$ and $L = 1, 3, 5, 9, 14, 25$ using the 9, 14, 25 using the corresponding results for the distortion-compensation type of processing in Figs. 6-27 and 6-28 for both criteria. It is observed from these plots that the multiple codebook data hiding method has superior performance compared to the corresponding single codebook method at the same N .

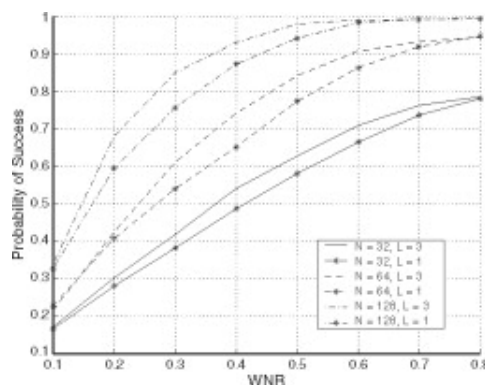


Figure 6-23. Probability of success performance for three-codebook hiding based on thresholding processing and maximum correlation criterion for various watermark signal sizes of $N = 32$, $N = 64$, and $N = 128$.

Figure 6-23. Probability of success performance for three-codebook hiding based on thresholding processing and maximum correlation criterion for various watermark signal sizes of $N = 32$, $N = 64$, and $N = 128$.

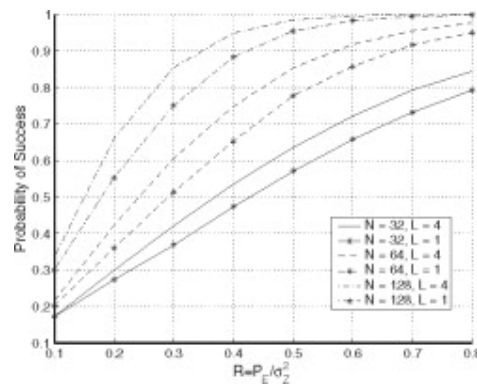


Figure 6-24. Probability of success performance for three-codebook hiding based on thresholding processing and maximum correlation criterion for various watermark signal sizes of $N = 32$, $N = 64$, and $N = 128$.

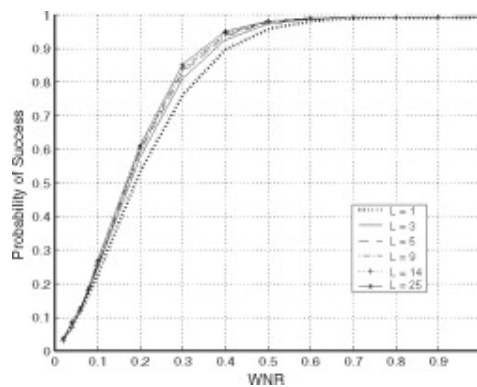


Figure 6-25. Probability of success performance for three-codebook hiding based on thresholding type of processing and maximum correlation criterion for $L = 1, 3, 5, 9, 14, 25$ and $N = 32, 64$, and $N = 128$.

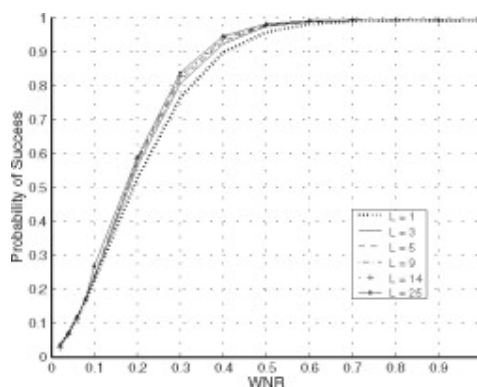
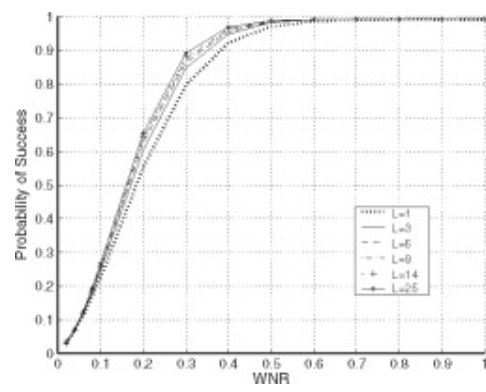


Figure 6-26. Probability of success performance for three-codebook hiding based on thresholding type of processing and maximum correlation criterion for $L = 1, 3, 5, 9, 14, 25$ and $N = 32, 64$, and $N = 128$.



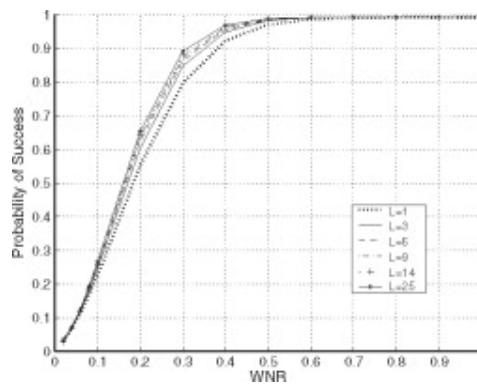


Figure 6-27. Probability of success for competitive codebook hiding based on distortion-compensation type of processing for $L = 1, 3, 5, 9, 14, 25$ and $N = 128$.

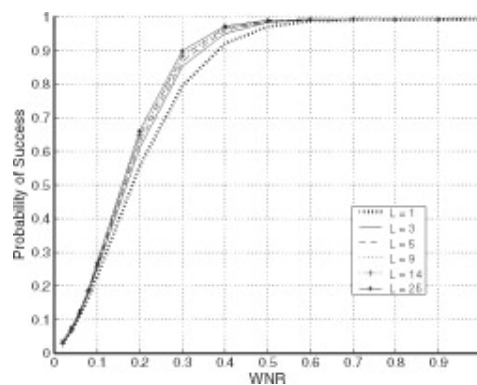


Figure 6-28. Probability of success for competitive codebook hiding based on distortion-compensation type of processing for $L = 1, 3, 5, 9, 14, 25$ and $N = 128$.

The computational complexity of the proposed method depends on the number of codebooks employed. Multiple codebook embedding, when compared with single codebook embedding, requires the embedding of the watermarking signal into transformations of the host signals of the comparison basis. At the detector, extraction should be repeated for each basis. Therefore, the computational complexity increases linearly with the number of codebooks (see Fig. 6-10).

[Read full chapter](#)

Video Coding Standards

David R. Bull, in [Coding Pictures](#), 2014

12.7.5 The video coding layer

Intra-coding

standard also includes an transform as this is often better suited to content with higher [spatial resolutions](#).

standard also includes an transform as this is often better suited to content with higher spatial resolutions.

Multiple reference frames Multiple reference frames (32 fields (depending on profile) can be buffered in the H.264 reference buffer. The reference buffer does not have to contain the 16 most recent frames, but is updated as and according to their utility, provided that the decoder remains debuffered. For further details see H.264 Annex E, the back to Chapter 9.

Prediction structure The H.264 coding structure is much more flexible approach to temporal prediction. In the case of low delay, low memory constraints, almost all frames are predicted by the baseline profile. If some delay is allowed, the ability to predict B-frames, and multiple reference frames, then superior compression performance can be achieved. An example of this is shown in Figure 12.16, where a hierarchical prediction structure is proposed, which has been demonstrated [12] that, if the quantisation is controlled in a controlled fashion, with layer, then more efficient compression can be achieved. Figure 12.16. Hierarchical prediction structure for H.264.

Weighted prediction This is used in H.264, that enables the weighting/offsetting of prediction samples for B-frames. This provides coding gains in the case of accelerating motion during scene cuts.

Quarter-pixel prediction H.264 enables the availability of subpixel motion compensation to quarter-pixel accuracy. This has been estimated to provide up to 20% saving over the pixel accuracy. A comprehensive description of the H.264/AVC subpixel motion estimation and compensation process is provided in Chapter 9. H.264/AVC uses a five-tap filter for half-pixel interpolation and a two-tap filter for local refinement to quarter-pixel accuracy.

Deblocking operations

The H.264 deblocking filter has been assessed to be rate for the same subjective quality by 10% as the coded bit rate for the same subjective quality by 10%. This is a highly content adaptive non-linear filter removes blocking artifacts and does not remove content. Its structure and operation are described in Chapter 9.

Variable length coding

- Exp-Golomb coding:** This is a highly variable length coding method, used in H.264 for all symbols apart from transform coefficients.

CAVLC: Context-Adaptive Variable Length Coding is a more complex but effective method, used as the Baseline and Extended Baseline and Extended Profiles for entropy coding of transform coefficients. Local context is used at the encoder to select between different VLC tables according to the table statistics in the local neighborhood. The number of non-zero coefficients in neighboring blocks.

- **CABAC:** CABAC is a more complex, arithmetic coding-based method that is only supported in the High Profile Main and High Profile. It uses local contexts to adjust its conditioning probabilities. CABAC has been found to reduce bit rate by between 10 and 20% compared to CAVLC, depending on content type and quantization level.

Further details on all the details on top of the above entropy coding method can be found in Chapter 7.

Coder control

Coder control is a key element in achieving the optimum performance from the standard, sometimes without good RDO, many of the advanced features (e.g., block size selection) will not deliver major savings. The goal of coder control is to select what parts of the video signal should be encoded using which coding tools and with what settings. Often a Lagrangian optimization approach is adopted where the distortion measure, is based on sum of squares and the difference between the bits associated with the decision including header information, and transform data. Further details on the RDO used in H.264/AVC can be found in Chapter 10.

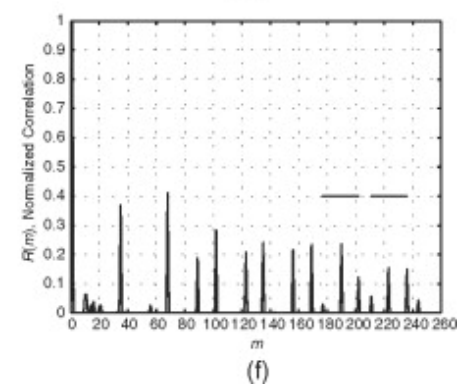
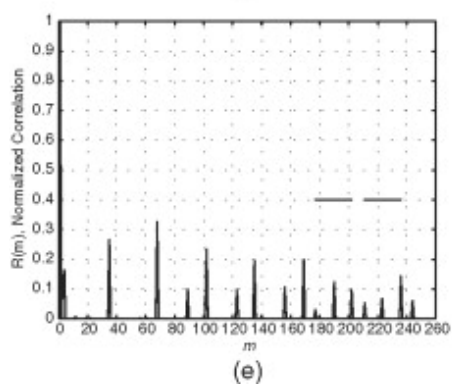
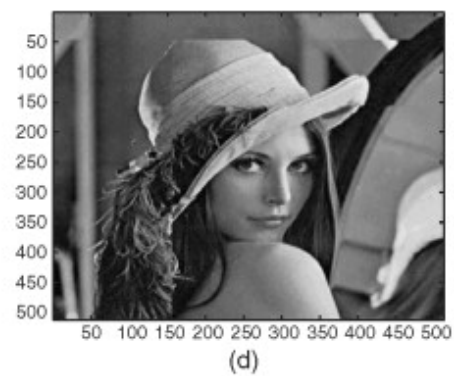
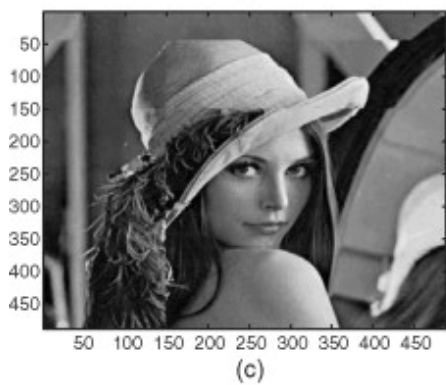
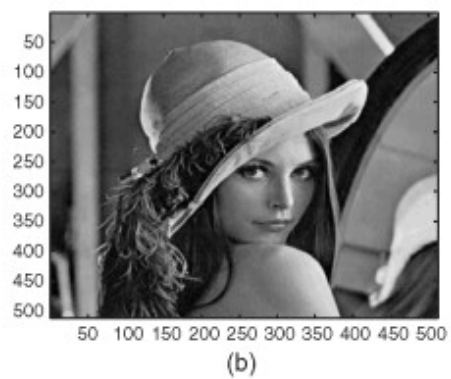
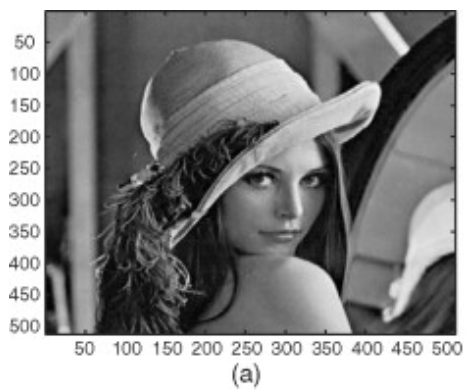
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Major Design Issues

Husrev T. Sencar, H. Akar, Ali N. Akar, in *Data Hiding Applications*, 2004

7.2.4 Results

We implemented the 512x512 grayscale image, Fig. 7-3-a. Message m is assumed to be 32 bits. The signal W takes the form of the watermark signal corresponding to the correlation properties. Hadamard transform is designed as the orthogonal rows are mapped to codebook.



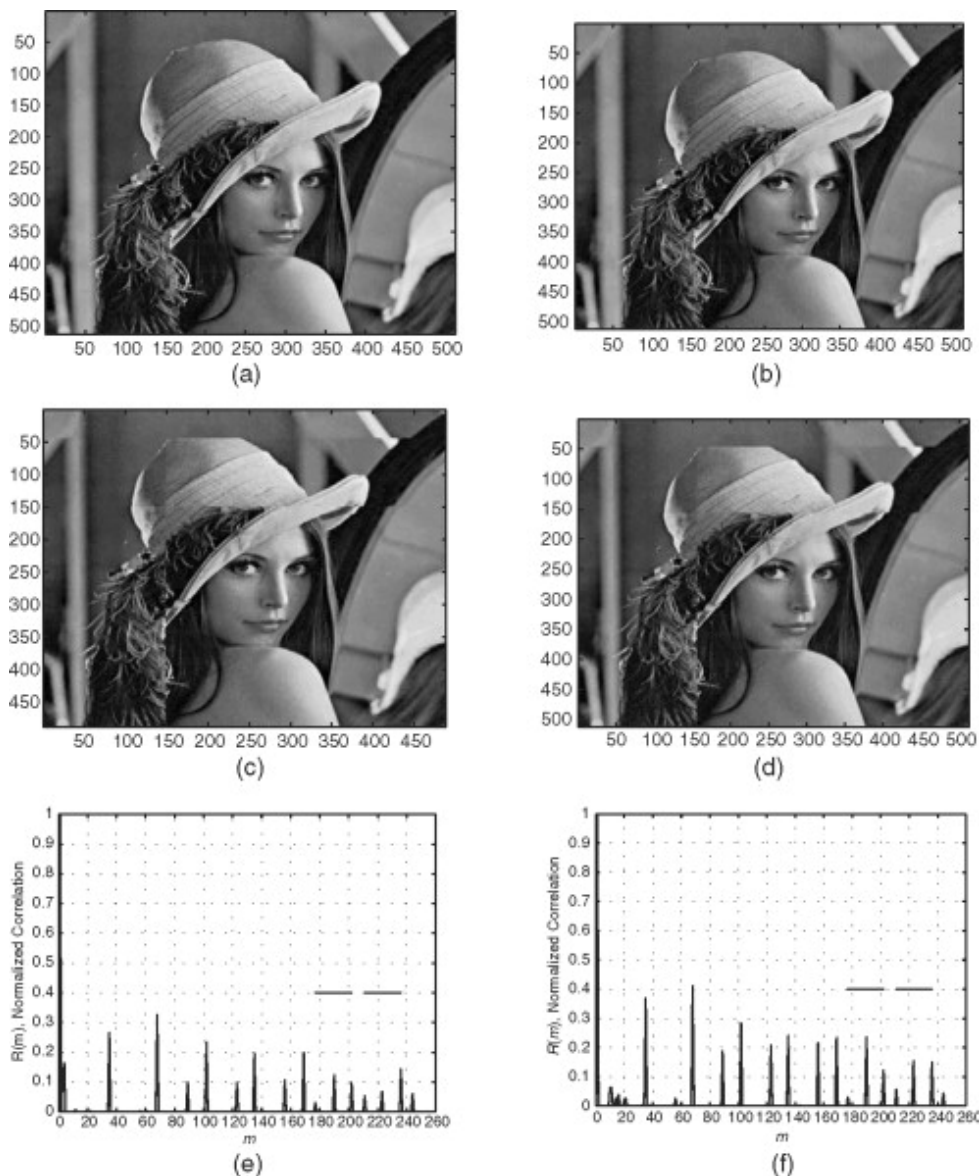


Figure 7-3. (a) Lena image, (b) Watermarked image, (c) Cropped image, (d) Resampled image, (e) Resampled and cropped image, (f) Autocorrelation function. The figure shows the process of watermarking, cropping, and resampling an image, along with the resulting autocorrelation function.

The message bit sequence is divided into message words. The message words are redundancy coded using RS codes. Using the codes, the encoded message words are mapped to phase samples. The phase samples are modulated and ordered in a way that fulfills the frequency domain requirements for the phase of the all-pass filter in order to generate the watermark signal. The watermark signal is chosen to be a 32 x 32 all-pass filter, which provides a simple way to modulate by the coded message. The message of the watermark signal are embedded through the whole image.

The watermarked image is cropped and is then resampled to its original size. At the original size, a copy of the watermarked, cropped image is divided into partitions of size W . Watermark detection is followed by the two-dimensional cyclic

autocorrelation of the detected set of signals. Using correlation peak pattern, resampling factor α is estimated. The extractor, knowing an estimate of the total cropped amount but not their locations, resamples the image back to its size after [cropping](#). Hence, the disturbing effects of the resampling can be reversed or at least minimized. This image is then repartitioned and the watermark extracted. Since extracted watermark signals may have been cropped and translated, an immediate detection of message m is not possible. RS codes are used to detect the message m from the extracted watermark signal, since they are capable of correcting burst error. The two-dimensional signal is shifted in rows and columns until an errorless decoding is possible. High redundancy coding helps in detecting message m even under severe signal loss.

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Figure 7-3a–d displays the results of the proposed scheme for the depicted watermark applied on the Lena image, Fig. 7-3a. The watermark is displayed in Fig. 7-3b, where the MSE per coefficient MSE per coefficient is 6.19 (40 dB) and 5.19 (40 dB) PSNR. Figure 7-3c is the watermarked image with both horizontal and vertical dimensions of 488 × 488. Each cropping is the erasing of 12 lines of pixels of 12 lines of pixels in vertical dimension. The cropped image is resampled back to its original size of 512 × 512 in 7-3d. Figure 7-3e–f shows the projections of the cyclic autocorrelation onto horizontal and vertical dimensions. The distance between the first and last peaks in each period, corresponding to the size of the watermark, is the resampling factor, of the cyclic autocorrelation is 25, which has an estimation error of 1 line of pixels in both dimensions. Both dimensions used in the Fig. 7-3e–f as 33 at some shifts and 34 at shifts of the 34th row of the image in Fig. 7-3d is resampled to a size of 24 lines of pixels in each dimension, partitioned in 32 × 32 blocks and 32 × 32 blocks extracted. Extracted signals from each block are averaged. Then the averaged signal block is shifted by α in α rows and columns until an errorless decoding is possible. For the proposed implementation, the redundancy rate is 25, RS codes are used to detect the 32-bit message m with no errors.

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Fast DCT/IDST Algorithms

Vladimir Britanak, Vladimir Britanak, K. R. Rao, and S. C. Goswami, 2007 [Discrete Transforms](#), 2007

4.4.3.2 DCT-II via W-algorithm

Since any EOT (see Section 4.3) can be expressed in EOT through a conversion matrix [34], the DCT-II with [34], the DCT-II with even symmetry/structure

of [basis vectors](#) can be realized via other simpler EOT [4, 28, 34, 37] such as sequency (Walsh) ordered [Walsh–Hadamard transform](#) (WHT) [1] whose basis vectors consist of ± 1 elements only.

of **basis vectors** can be realized via other simpler EOT [4, 28, 34, 37] such as sequency (Walsh) ordered **Walsh–Hadamard transform** (WHT) [1] whose basis vectors consist of ± 1 elements only.

Denote the DCT-II and WHT respectively by \mathbf{D}_N and \mathbf{W}_N respectively in notation (the **normalization factors** are omitted) as

$$(4.58) \quad \mathbf{D}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} \cos \frac{\pi}{2N} (n-1/2) (m-1/2) \\ \vdots \\ \cos \frac{\pi}{2N} (n-1/2) (N-1/2) \end{bmatrix}$$

$$(4.58) \quad \mathbf{W}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 \\ \vdots \\ (-1)^{m-1} \end{bmatrix}$$

Examination of \mathbf{D}_N and \mathbf{W}_N matrices off and given at shows that there is a **one-to-one** correspondence between the sequences (the sequences) of crossings in sign) of the rows of these **the rows of these transform matrices**. This implies that, even symmetry/even antisymmetry structure of the WHT matrix of the WHT matrix is determined by the DCT-II matrix. If we **rearrange** the rows of \mathbf{D}_N and \mathbf{W}_N in order, then (4.58) can be rewritten as

$$(4.59) \quad \mathbf{D}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} \cos \frac{\pi}{2N} (n-1/2) (m-1/2) \\ \vdots \\ \cos \frac{\pi}{2N} (n-1/2) (N-1/2) \end{bmatrix}$$

$$(4.59) \quad \mathbf{W}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 \\ \vdots \\ (-1)^{m-1} \end{bmatrix}$$

Since \mathbf{D}_N is an orthonormal matrix, substituting it into (4.59) we get [34]

$$(4.60) \quad \mathbf{D}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} \cos \frac{\pi}{2N} (n-1/2) (m-1/2) \\ \vdots \\ \cos \frac{\pi}{2N} (n-1/2) (N-1/2) \end{bmatrix}$$

$$(4.60) \quad \mathbf{W}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 \\ \vdots \\ (-1)^{m-1} \end{bmatrix}$$

where T_N is the conversion matrix which takes the Walsh domain vector and converts it to the DCT-II domain. The conversion matrix T_N has two important properties:

1. It is orthonormal, being the product of two orthonormal matrices and
2. It has a sparse **block diagonal** structure, where the rows of \mathbf{D}_N and \mathbf{W}_N are in bit-reversed order.

Equation (4.60) defines the DCT algorithm for DCT-II computation via WHT involving two steps: the efficient steps: **coefficients** and **computation** and the **conversion** of the conversion matrix T_N . In order to illustrate the DCT-II computation, let us consider $N = 8$. The \mathbf{W}_8 matrix is given by [1]

and the conversion matrix T_N in **binary form** is given by

$$\text{where } T_N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{where } T_N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The $M \times M$ **block matrix** T_N in the **binary form** is given by [38]

$$(4.61) \quad T_N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(4.61) \quad T_N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

can be further factorized into a product of sparse matrices. The higher-order conversion matrix T_{2N} can be generated recursively [37], where having T_N and the block derived the block

matrices for T_N we need to derive for the T_2N only the [block matrix](#) U_N taking into account its above general structure.

matrices for T_N we need to derive for the T_{2N} only the **block matrix** U_N taking into account its above general structure.

Among the existing sparse factorization [35, 37], the factor [25, 37], the **factorization** presented in Ref. [37] is preferred. It is preferred for its simplicity and regularity and it is defined as it is defined as

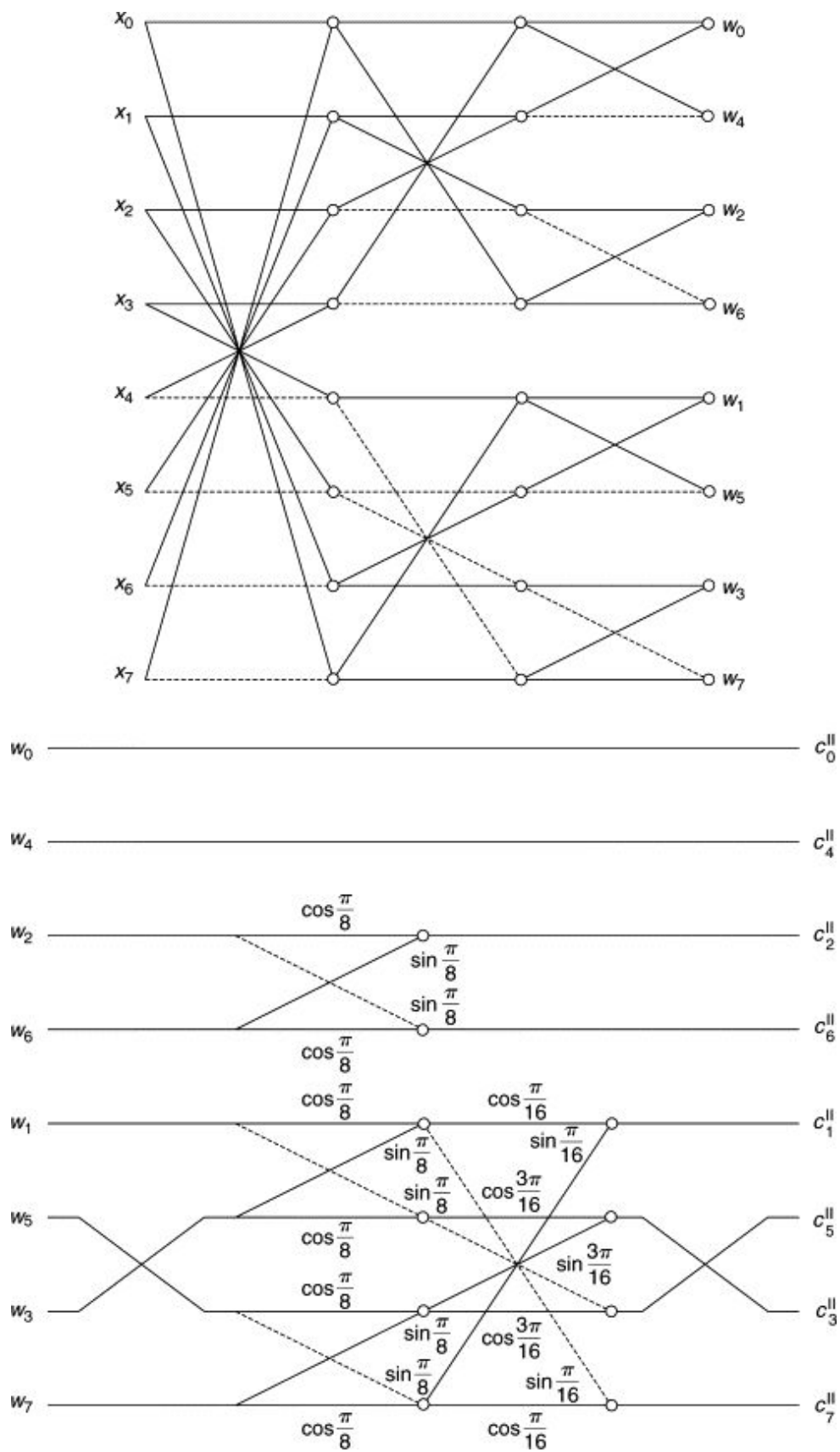
where where

For the fast WHT computation, the algorithm, is presented in Ref. [1] requiring the input data sequence to be in reversed order. Since WHT is an EOT according to (4.17) according to (4.17) we can factorize it into factorization of matrix as

where where

and and

defining the new algorithm for fast WHT. The corresponding **signal flow graph** for the **DCT graph** for the DCT computation via WHT for $N=4$ is shown in Fig. 4.12. in Fig. 4.12.



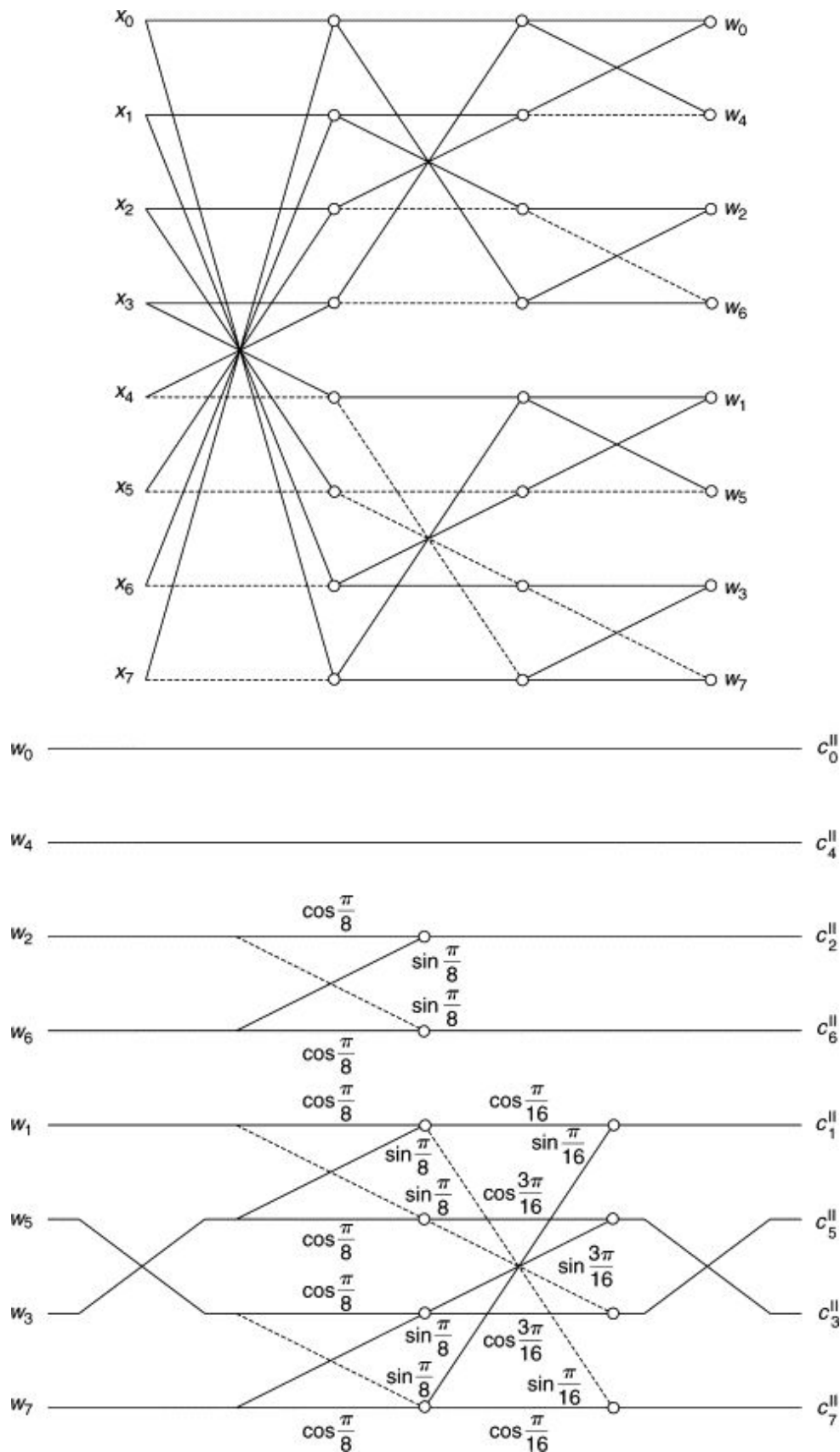


Fig. 4.12. The signal flow graph for the DCT-II and DCT-III for $N=8$ (the fast WHT and conventional WHT conversion matrix).

If we compare the signal flow graph for $N=8$ in Fig. 4.12 with the DCT-II implementation via WHT presented in Ref. [35] (see Fig. 4.11), the implementation in Fig. 4.12 has the following advantages:

Saves two bit-reversal permutations. Saves two bit-reversal permutations.

- For 5 plane rotations the number of rotation angles is reduced from four to three.
- The implementation is more regular.

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