Basic concept of problem solving

A FIRST PROBLEM

1. **Determine how many zeros end the number 100! .**

⇨ We know that,

100!= 100*99*98----*3*2*1

Adding zero the end of a product occurs precisely when we multiply when we multiply by 10. In fact the prime factorization of 10 = 5*2.

In the num **1-10**, only the number 5 & 10 have factors of 5. The number 5 may be paired with 2 to yield 10 & the number 10 doesn’t need to be paired. So, two zeros to full product that forms the factorial.

In the num **11-20**, only the number 15 and 20 have factors of 5. Reasoning as in last paragraph, we count two additional zeros.

The numbers between **21- 30** are a bit different. As before 25 and 30 are only number having a factor of 5, but 25 has two factors of 5. Thus this will contribute two zeros. Thus the range 21- 30 contributes a total of three zeros.

The range **31-40** is simple one, like the first two ranges we considered, it contribute two zeros.

The range **41-50** is special because 45 contributes one factor of 5 but 50 contributes two factors. Thus this range of number contributes three zeros.

The range **51-60** & range **61-70** are like the first two. There are no multiple of factors of 5 and each range contributes two zeros.

The range **71-80** is special because 75 contributes two factors of 5 and 80 contributes one factor of 5. The total contribution is three zeros.

The range **91-100** contains 95 and 100(factors of 5). 95 contributes one factor of 5 and 100 contributes two zeros. Thus three zeros are added.

Taking an of our analysis, we have six ranges of numbers that contributes two zeros and four ranges that contributes three zeros. The given of total of 24 zeros that will appear at the end of 100!

2. Determine how many zeros end the number $14^{56} * 25^{64} * 15^{34}$.

Given, $14^{56} * 25^{64} * 15^{34}$

= $(7*2)^{56} * (5*5)^{64} * (5*3)^{34}$

= $(7)^{56} * (2)^{56} * (5)^{64} * (5)^{64} * (5)^{34} * (3)^{34}$

= $(5*2)^{56} * (7)^{56} * (5)^{106} * (3)^{34}$

= $(10)^{56} * (7)^{56} * (5)^{106} * (3)^{34}$

Therefore, total no. of zeros at the end of $14^{56} * 25^{64} * 15^{34}$ is 56.

3. Determine how many zeros end the number 100! + 55!.

From 100!

Multiple of 5 = $\frac{100}{5}$ = 20

Multiple of 25 = $\frac{100}{25}$ = 4

Total no. of 5's in 100! = 20+4 = 24

:. From 100! we get 24 zeros at the end.

From 55!

Multiple of 5 = $\frac{55}{5}$ = 11

Multiple of 25 = $\frac{55}{25}$ = 2.2 ≈ 2

Total no. of 5's in 55! = 11+2 = 13

:. From 55! we get 13 zeros at the end.

Hence, total no. of zeros at the end of 100! + 55! = 13 zeros.

*Note: (**In case of addition and subtraction, the total zeros must be count as common. Eg. Look above**)*

4. Determine how many zeros end the number 122! * 76!

From 122!

Multiple of 5 = $\frac{122}{5}$ ≈ 24

Multiple of 25 = $\frac{122}{25}$ ≈ 4

Total no. of 5's in 100! = 24+4 = 28

:. From 122! we get 28 zeros at the end.

From 76!

Multiple of 5 = $\frac{76}{5} \approx 15$

Multiple of 25 = $\frac{76}{25} \approx 3$

Total no. of 5’s in 76! = 15+3 = 18

:. From 76! we get 18 zeros at the end.

Hence, total no. of zeros at the end of 122! * 76! = 28+18=46 zeros.

5. **A class has 12 students. At the beginning of each class hour, each student shakes hand with each of the other students. How many handshakes take palce?**

Suppose that there are just 2 students. Then only one handshake is possible.

Now suppose a new students walks in the door. He/she must shake hands with each of student that already in the room. The total number of handshake is 1+2=3.

If a fourth students walks in the door, then he/she must shake hands with each of students already in the room. Then total number of handshakes is 1+2+3=6.

When we get up to twelve students, we will have required

1+2+3+ - - - - - +9+10+11= 66 handshakes.

6. **At the beginning of each class hour, each students shakes hands with each of the other students and number of handshakes is 820. Find the number of handshakes.**

Let total number of students be n.

Only one handshake is possible if there are just two students.

Only three handshakes is possible if new students walk in the door and handshakes with two student that already in room and so on.

820 handshakes is possible if n number of student are there in the room and must handshakes with each other.

Then,

¿, $\frac{n(n-1)}{2}=820$

¿, $n^2-n=1640$

¿, $n^2-41n+40n-1640=0$

¿, n¿

Either, $n-41=0$, $n+40=0$

$n=41$ $n=-40$(neglect -ve value)

Hence, the total number of students is 41.

7. Assume that k is a positive integer. What is the sum of the integers s = 1+2+3-------+ (k-1)+k ?

For instance if g(k)= k^2

g(k+1)-g(k)=$(k+1)^2-k^2$ = 2k +1

Here,

$2^2-1^2=2.1+13^2-2^2=2.2+1$

$k^2-(k-1)^2=2(k-1)+1$

$(k+1)^2-k^2=2K+1$

adding the columns,

¿, $(k+1)^2-1=2[1+2+..+k]+[1+1+..+1]$

¿, $k^2+2k=2s+k$

Where s is that sum we wish to calculate,

$$s=\frac{k^2+k}{2}$$

8. Assume that k is a positive integer. What is the sum of the integers s = 1^2+2^2+3^2-------+ $(k-1)^2$+k^2 ?

For instance if g(k)= k^3

g(k+1)-g(k)=$(k+1)^3-k^3$ =$3k^2+3k+1$

Here,

$2^3-1^3=3.1^2+3.1+13^3-2^3=3.2^2+3.2+1$

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| | | |

$|\qquad\quad|\qquad\quad|\qquad\quad|$

$k^3-(k-1)^3=3.(k-1)^2+3(k-1)+1$

$(k+1)^3-k^3=3k^2+3k+1$

adding the columns,

¿, $(k+1)^3-1^3=3.[1^2+2^2+..+k^2]+3.[1+2+..+k]+[1+1+..+1]$

¿, $k^3+3k^2+3k+1-1=3.s+3.\left(\frac{k^2+k}{2}\right)+k$

Where s is that sum we wish to calculate,

¿, $2k^3+6k^2+6k=6s+3k^2+3k+2k$

¿, $6s=2k^3+3k^2+k$

¿, $6s=k$¿]

¿, $6s=k$¿]

$\therefore s=\frac{k(k+1)(2k+1)}{6}$

9. Assume that k is a positive integer. What is the sum of the integers s = 1^3**+**2^3**+**3^3**-------+** $(k-1)^3$**+**k^3 **?**

⇨ For instance if g(k)= k^4

g(k+1)-g(k)=$(k+1)^4-k^4$ =$4k^3+6k^2+4k+1$

Here,

$2^4-1^4=4.1^3+6.1^2+4.1+1$

$3^4-2^4=4.2^4+6.2^2+4.2+1$

$|\qquad\quad|\qquad\quad|\qquad\quad|$

$|\qquad\quad|\qquad\quad|\qquad\quad|$

$|\qquad\quad|\qquad\quad|\qquad\quad|$

$k^4-(k-1)^4=4.(k-1)^3+6(k-1)^2+4(k-1)+1$

$(k+1)^4-k^4=4k^3+6k^2+4k+1$

adding the columns,

¿, $(k+1)^4-1^4=4s+6.[1^2+2^2+..+k^2]+4.[1+2+..k]+[1+1+..1]$

¿, $k^4+4k^3+6k^2+4k+1-1=4.s+6.\frac{k(k+1)(2k+1)}{6}+4.\left(\frac{k^2+k}{2}\right)+k$

Where s is that sum we wish to calculate,

¿, $k^4+4k^3+6k^2+4k+1-1=4.s+2k^3+3k^2+k+2k^2+2k+k$

¿, $k^4+2k^3+k^2=4.S$

$¿, S=\frac{n^2(n^2+2n+1)}{4}$

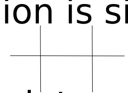
$:. S=¿$

10. What is the greatest number of region into which three straight lines can divide the plane?

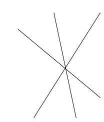
If all three straight lines are coincide, then plane divided into two regions.

- If two lines out of three coincide, then total number of region is three.

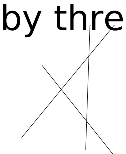
- If two parallel lines intersected by a transversal line then total no. of region is six.



- If all three lines intersect at a single point, then total no. region is six.



- If three lines intersect at two points, the total number of region is 7, which is maximum no. of region made by three straight lines.



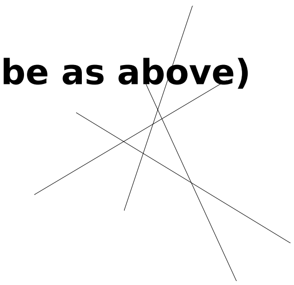
11. What is the greatest number of region into which four straight lines can divide the plane?

If all four straight lines are coincide, then plane divided into two regions.

- If three lines out of four coincide, then total number of region is three.
- If three parallel line intersected by a transversal line, the total no. of region is 8.
- If all four lines intersect at a single point, then total no. of region is eight.
- If two line are parallel & rest of two line are intersected at single point, the total no. of region is 10

- If four line intersect at three points, the total number of region is 11,which is maximum no. region made by four straight lines.

  Note:(Make figure and describe as above)



12. Suppose now that there are k students in the class. If k is even then will the number of handshakes that takes place be even or odd? If k is odd then will the number of handshakes that take place be even or odd?

⇨ If number of students k=0 → number of handshake = 0 “even”

If number of students k=1 → number of handshake = 0 “even”

If number of students k=2 → number of handshake = 1 “odd”

If number of students k=3 → number of handshake = 3 “odd”

If number of students k=4 → number of handshake = 6 “even”

If number of students k=5 → number of handshake = 10 “even”

If number of students k=6 → number of handshake = 15 “odd”

If number of students k=7 → number of handshake = 21 “odd”

If number of students k=8 → number of handshake = 28 “even”

If number of students k=9 number of handshake = 36 "even"

We see that the first two number for handshakes are even, then there are two odd, then there are two even and so forth.

13. Calculate the sum is closed from $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{n}{(n+1)!}$

$$¿\frac{(2-1)}{2!}+\frac{(3-1)}{3!}+\frac{(4-1)}{4!}+\ldots+\frac{(n+1)-1}{(n+1)!}$$

$$¿\frac{2}{2!}-\frac{1}{2!}+\frac{3}{3!}-\frac{1}{3!}+\frac{4}{4!}-\frac{1}{4!}+\ldots+\frac{(n+1)}{(n+1)!}-\frac{1}{(n+1)!}$$

Same terms are cancels, remaining term

$$¿\frac{1}{1!}-\frac{1}{(n+1)!}$$

$$¿1-\frac{1}{(n+1)!}$$

How to count

14. We are given k objects{$a_1,a_2,\ldots..a_k$}. How many different ordered pairs may be made up from those k objects?

There are k possible choices for the first element of ordered pairs. So, if we chose $a_1$for the first element then we may choose any of $a_2,a_3,\ldots..a_k$ for the second element-that's (n-1) choices.

If we chose a_2 for the first element that we may chose any of $a_1,a_3,\ldots..a_k$ for the second element – that's (k-1) choices and so forth.

In summary, there are k choices for the first element and (k-1) choices for second element of the ordered pair. Thus, the total number of possible ordered pairs, chosen from among **{** $a_1,a_2,\ldots..a_k$**}** is k.(k-1).

15. We are given k object {$a_1,a_2,\ldots..a_k$}. How many different ordered may be made up from those k objects?

Suppose that we have k position into which to put the objects. There are k different object ($a_1, a_2, \ldots.. a_k$) that we may put in the first position. Having placed k object into the first position, there remains (k-1) different objects to put into second position. Thus, there are k.(k-1) choices of pairs of objects to put into the first two position.

Having chosen two objects to put into the first two positions, we see that there remain (k-2) objects to put into the third position. Thus there are k.(k-1).(k-2) choices of objects to put into the first three positions and so forth.

In end there are

k.(k-1).(k-2).-----3.2.1=k! possible different ordering of k objects $\{a_1, a_2, \ldots.. a_k\}$.

16. How many different 5 card poker hands may be had from a deck of 52 cards?

52_{C_5} different way for 5 card poker hand be had from a deck of 52 cards.

$$52_{C_5} = \frac{52!}{(52-5)!5!} = ¿$$ 2,598,960 different ways.

17. How many pairs of bridge hands may be dealt from a deck of 52 cards?

The bridge is played by two teams of two people. Each person is dealt 13 cards. Consider one of the team.

The first team member is dealt 13 cards from among the total of 52 cards. The number of possible hands that this person could dealt is

$$C_1 = 52_{C_{13}} = \frac{52!}{(52-13)!13!} = \frac{52!}{39!13!}$$

The second team member is also dealt 13 cards, chosen from among the remaining 39 cards. Thus the total number of possibilities for second team member could be dealt is

$$C_2 = 39_{C_{13}} = \frac{39!}{(39-13)!13!} = \frac{52!}{26!13!}$$

Hence, the two number of pairs of hand for two member is

$$C_1 * C_2 = 52_{C_{13}} * 39_{C_{13}} = \frac{\frac{52!}{39!13!} * 52!}{26!13!} \approx 5.1578 * 10^{21}$$ different ordered pair.

18. From 4 engineers, 2 physicists and 3 economists. Find the number of committees that can be formed consisting of 2 engineers, 1 physicist and 1 economist.

From four engineers 2engineers can be selected. The number of possible ways to be selected is

$$C_1 = 4_{C_3} = \frac{4!}{2!2!} = 6 \, different \, ways$$

Again, from 2 physicists 1 physicist can be selected. The number of possible ways that can be formed is

$$C_2 = 2_{C_1} = \frac{2!}{1!1!} = 2 \, different \, ways$$

Again, from 3 economists 1 economist can be selected. The number of possible ways that can be formed is

$$C_3 = 3_{C_1} = \frac{3!}{2!1!} = 3 \, different \, ways$$

:. The number of committees that can be formed consisting of 2 engineers,1 physicist and 1 economist is
$C_1 * C_2 * C_3$= 6*2*3= 36 different committees formed.

19. There are six doors in a hostel. In how many ways can a student enter the hostel and leave by a different door.

There are six different doors(D_1,D_2, D_3, D_4, D_5, D_6) in the hostel. By the question, the student from which door he enter must not leave the hostel through same door. He must choose different door.
Suppose, for student no. 1 there are e6 different ways to enter the hostel. He may choose any of the doors. If he choose D_1 door to enter the hostel then he must leave the hostel through D_2, D_3, D_4, D_5, D_6not from D_1. So there are five different ways to leave the hostel.

Again, if he chose D_2door to enter then he must leave the hostel from other door not from D_2. So there are five different ways and so on.
Hence, the students can enter the hostel and leave the hostel by different door in

6*5=30 different ways.

Contradiction problem

20. Prove that square root of 2 is a irrational number.

Proof:- let $\sqrt{2}$is a rational number.

So, $\sqrt{2}=\frac{p}{q}$,where p & q are integers, q≠0 & HCF(p,q)=1 that means p and q have no common factor other than 1.

¿ $, p=q\sqrt{2}$

Squaring on both side

$p^2=2q^2$------(i)

$p^2$is divisible by 2.

:.p is divisible by 2.

$so, 2 is a factor of \ p$

We shall put p=2m in eqn(i)

$2q^2=4m^2$

$q^2=2m^2$

$q^2$is divisible by 2.

:.q is divisible by 2.

$so, 2 is a factor of \ q$

:. 2 is common factor of p & q.

But this contradicts that fact that p & q have no common factor other than 1.

:. Thus contradiction occurs because of our assumption that $\sqrt{2}$ is rational number, which was wrong assumption. Hence, square root of 2 is a irrational number.

21. Prove that square root of 3 is a irrational number.

Proof:- let $\sqrt{3}$is a rational number.

So, $\sqrt{3}=\frac{p}{q}$,where p & q are integers, q≠0 & HCF(p,q)=1

that means p and q have no common factor other than 1.

$¿, p=q\sqrt{3}$

Squaring on both side

$p^2=3q^2$-----(i)

$p^2$is divisible by 3.

:.p is divisible by 3.

$so, 3 is\ a\ factor\ of\ p$

We shall put p=3m in eqn(i)

$3q^2=9m^2$

$q^2=3m^2$

$q^2$is divisible by 3.

:.q is divisible by 3.

$so, 3 is\ a\ factor\ of\ q$

:. 3 is common factor of p & q.

But this contradicts that fact that p & q have no common factor other than 1.

:. Thus contradiction occurs because of our assumption that $\sqrt{3}$ is rational number, which was wrong assumption. Hence, square root of 3 is a irrational number.

22. Explain why there are infinitely many prime numbers.

Suppose there are only finite many primes, let's say n of them. We denote them by $p_1, p_2, \ldots . p_n$. Now construct a new number

$$p=p_1 * p_2 * \ldots . * p_n + 1$$

Clearly, p is larger than any of the primes, so it doesn't equal one of them. Since, $p_1, p_2, \ldots . p_n$constitute all primes p can't be prime. Thus it must be divisible by at least one of our finitely many primes, say p_n. But when we divide p by p_n we get remainder 1. That's contradiction, so or original assumption that there are finitely many primes must be false. Thus there are infinitely many primes number.

23. Explain why, if two positive real numbers sum to 100, then their product cannot be 3000?

Let x &y be two positive real number then,

$x+y=100---(i)$

$if\ the\ product\ of\ these\ two\ number\ is\ 3000$

i.e $x*y=3000$

$¿, x(100-x)=3000$

$¿, x^2-100x+3000=0$

∴ $x=\frac{-(-100)\pm\sqrt{(-100)^2-4*1*300}}{2*1}=\frac{100\pm\sqrt{-200}}{2}$

which gives imaginary number but by question x & y are positive real number. Hence by contradiction, product of these two number never 3000.

Induction principle

Step:- to prove for p(j)=1
Suppose it is satisfy for n=j
We have to show that, it also satisfy for n=j+1

24. Verify the formula $S=\frac{k(k+1)}{2}$ by induction method where s=1+2+3…+k

If k=1, then s=1

Suppose that it is true for k=j

$p(j)=\frac{j(j+1)}{2}$

We have to show that it is true for k=j+1

$p(j+1)=\frac{j+1(j+1+1)}{2}$

$p(j+1)=\frac{j+1(j+2)}{2}$

By induction, it is true for k=j+1, which verify the formula.

25. **Verify the formula** $s=\frac{k(k+1)(2k+1)}{6}$ **by induction method where s =** $1^2+2^2+3^2$**-------+** $(k-1)^2+k^2$

For k =1

$1^2=\frac{1(1+1)(2.1+1)}{6}$

:.1=1

Which is true for k=1

Suppose that it is true for k=j

$1^2+2^2+3^2$-------$+j^2=\frac{j(j+1)(2j+1)}{6}$ ------------------(i)

we have ¿ show it is true for $k=j+1$

Adding $(j+1)^2$ on both side of equation (i) we get,

$1^2+2^2+3^2$-------$+j^2+(j+1)^2=\frac{j(j+1)(2j+1)}{6}+(j+1)^2$

¿, $1^2+2^2+3^2$-------$+j^2+(j+1)^2=(j+1)$¿]

¿, $1^2+2^2+3^2$-------$+j^2+(j+1)^2=(j+1)$¿]

¿, $1^2+2^2+3^2$-------$+j^2+(j+1)^2=(j+1)$¿]

¿, $1^2+2^2+3^2$-------$+j^2+(j+1)^2=\frac{(j+1)(2j+3)(j+2)}{6}$

By induction, it is true for k=j+1 which verify the formula.

26. **Verify the formula** $s=\frac{n^2(n+1)^2}{4}$ **by induction method where s =** $1^3+2^3+3^3$**-------+** $(k-1)^3+k^3$

For k =1

$1^3=\frac{1^2(1+1)^2}{4}$

:.1=1

Which is true for k=1

Suppose that it is true for k=j

$1^3+2^3+3^3$-------$+j^3=\frac{j^2(j+1)^2}{4}$ ------------------(i)

we have ¿ show it is true for $k=j+1$

Adding $(j+1)^3$ on both side of equation (i) we get,

$$1^3+2^3+3^3\text{-------}+j^3+(j+1)^3=\frac{j^2(j+1)^2}{4}+(j+1)^3$$

$$¿,1^3+2^3+3^3\text{-------}+j^3+(j+1)^3=(j+1)^2¿]$$

$$¿,1^3+2^3+3^3\text{-------}+j^3+(j+1)^3=(j+1)^2¿]$$

$$¿,1^3+2^3+3^3\text{-------}+j^3+(j+1)^3=\frac{(j+1)^2(j+2)^2}{4}$$

By induction, it is true for k=j+1 which verify the formula.

27. Prove by mathematical induction that $\sum 3^i=\frac{1}{2}\left(3^{n+1}-1\right)$

For n=0

$$3^0=\frac{1}{2}\left(3^{0+1}-1\right)$$

or, 1= $\frac{3-1}{2}$

:. 1=1

which is true for n=0.

Suppose that it is true for n=k

$3^0+3^1----\mp 3^k=\frac{1}{2}\left(3^{k+1}-1\right)$------------(i)

We have to show that it is true for n=k+1

Adding 3^{k+1}on the both side of equation i.

$$3^0+3^1----\mp 3^k+3^{k+1}=\frac{1}{2}\left(3^{k+1}-1\right)+\ 3^{k+1}$$

or, $3^0+3^1----\mp 3^k+3^{k+1}=\frac{1}{2}\left(3^{k+1}\right)-\frac{1}{2}+¿\ 3^{k+1}$

or, $3^0+3^1----\mp 3^k+3^{k+1}=\frac{3}{2}\left(3^{k+1}\right)-\frac{1}{2}$

which is true for n=k+1, This complete proof.

28. Show that $n^3 - n \, is \, divisible \, by \, 3 \, for \, every \, positive \, integer.$

For n=1

$1^3 - 1 = 0 \, is \, divi \, sible \, by \, 3$

Which is true for n=1.

Suppose that it is for n=k

p(k)=$k^3 - k \, is \, divisible \, by \, 3.$

We have to show that it is true for n=k+1

p(k+1)=$(k+1)^3 - (k+1) \, is \, divisible \, by \, 3.$

= ($k^3 + 3k^2 + 3k + 1$¿−¿(k+1)

= ($k^3 - k$) + 3($k^2 + k$)

By inductive hypothesis ($k^3 - k$) is divisible by 3 and 3($k^2 + k$) is divisible by 3 because it is 3 times on integer, So p(k+1) is divisible by 3.

which is true for n=k+1. This completes the proof.

29. Show that $n^5 - n \, is \, divisible \, by \, 5 \, for \, every \, positive \, integer.$

For n=1

$1^5 - 1 = 0 \, is \, divisible \, by \, 5$

Which is true for n=1.

Suppose that it is for n=k

p(k)=$k^5 - k \, is \, divisible \, by \, 5.$

We have to show that it is true for n=k+1

p(k+1)=$(k+1)^5 - (k+1) \, is \, divisible \, by \, 5.$

= ($k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1$¿−¿(k+1)

= ($k^5 - k$) + 5($k^4 + k$) +10($k^3 + k^2$)

= (k^5-k) + $5(k^4+2k^3+2k^2+k)$

By inductive hypothesis (k^5-k) is divisible by 5 and $5(k^4+2k^3+2k^2+k)$ is divisible by 5 because it is 5 times on integer, So p(k+1) is divisible by 5.

which is true for n=k+1. This completes the proof.

EXTRA

30. Sum of 1^{st} k odd positive integer.

S=1+2+3+-----------------+(k-1)+k
S=0+1+2+3+-----------------+(k-2)+(k-1)+k
= =
2s= 1+3+5+-------------+(2k-3)+ (2k-1)+k
where s= sum of k positive integer.

or, $2\frac{k(k+1)}{2}=S_k+k$

where, S_k= Sum on 1^{st} k odd integer.

$k^2+k=S_k+k$

:. $S_k=k^2$

Therefore, Sum of 1^{st} k odd positive integer is k^2

31. Find all pairs of integer m,n such that m.n=m+n.

Given, m.n=m+n
or, m.n-m=n
or, m(n-1)=n
or, m= $\frac{n}{n-1}$

Here only possible of n is 0 & 2
n=0 , m=0
n=2 , m=2
:. Possible pairs are (0,0) & (2,2)

32. Determine how many zeros end the number 800!.

From 800!

Multiple of 5 = $\frac{800}{5}$ = 160

Multiple of 25 = $\frac{800}{25}$ = 32

Multiple of 125 = $\frac{800}{125}$ = 6

Multiple of 625 = $\frac{800}{625}$ = 1

Total no. of 5's in 800! = 160+32+6+1 = 199
We have sufficient number of 2's in 800!.
:. From 800! we get 199 zeros at the end.

33. Calculate the sum $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\frac{-----+n}{(n+1)!}$

Given,

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\frac{-----+n}{(n+1)!}$$

$$= \frac{2-1}{2!}+\frac{3-1}{3!}+\frac{4-1}{4!}+\frac{-----+(n+1)-1}{(n+1)!}$$

$$= 1-\frac{1}{2!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{3!}-\frac{----+1}{n!}-\frac{1}{(n+1)!}$$

$$=1-\frac{1}{(n+1)!}$$

34. How many zeros at the end of

$$\frac{16^{300}*15^{500}*250!}{20^{150}*3^{50}}$$

¿ $\frac{(2^4)^{300}*(3*5)^{500}*250!}{(2*2*5)^{150}*3^{50}}$

¿ $\frac{2^{1200}*3^{500}*5^{500}*250!}{2^{300}*5^{150}*3^{50}}$

¿ $2^{900}*5^{350}*3^{450}*250!$

¿ $(10)^{350}*2^{550}*3^{450}*250!$

From 250!

Multiple of 5 = $\frac{250}{5}$ = 50

Multiple of 25 = $\frac{250}{25}$ = 10

Multiple of 125 = $\frac{250}{125}$ = 2

Total no. of 5's in 250! = 50+10+2=62

:. From 250! We get 62 zeros at the end.

Now, total no. of zeros= 350+62= 412

35. What is the last digit of 3^{4798}?

We have,

$3^1=3$

$3^2=9$

$3^3=27$

$3^4=81$

$3^5=243$ & so on.

The Possible number at the last digit of any power of 3 are 3,9,7,1 and the repeats. Here 1 is special case i.e 1.1=1 which gets from 4th power of 3.

Given,

$3^{4798}=(3^4)^{4798}*3^2$

$¿(81)^{4798}*3^2$

The First part $(81)^{4798}$ gives 1 at the last digit, which is multiplied by 3^2 i.e 9 gives the last digit is 9.

```
4)4798(1199
       -4
   -------------
        798
       -4
   -------------
        398
       -36
   -------------
        38
       -36
   -------------
          2
```

36. What is the last digit of 7^{65432}?

We have,

$7^1=7$

$7^2=49$

$7^3=343$

$7^4=2401$

$7^5=16807$ & so on.

The possible number at the last digit of any power of 7 are 7,9,7,1 and the repeats. Here 1 is special case i.e 1.1=1 which gets from 4th power of 7.

Given,

$3^{65432}=(7^4)^{16358}*7^0$

¿$(2401)^{16358}*7^0$

4)65432(16358
-65432

0

The First part $(2401)^{16358}$ gives 1 at the last digit, which is multiplied by 7^0 i.e 1 gives the last digit is 1.

37. Suppose you are told in advance that 10 of the cattle present are lame, & only have three feet. But the count yields 120 heads & 300 feet. How many cattle and how many people are there?

Total no. of head =120

Total no. of feet =300

Let x denote number of cattle and y denote the number of people.

Then, x+y=120 -----(i)

& 2y+10*3+(x-10)*4=300 ------(ii)

Solving (i) & (ii)

2(120-x)+30+4x-40=300

or, 240+2x-10=30

x=35

And, y=120-35

:. y= 85

Hence, The total no. cattle is 35 and the total no. people is 85.

38. How many digits are used to number the pages of a book having 750 pages – numbered from 1- 100?

Page from 1 to 9 = total no. of digit =9

Page from 10 to 99 = total no. of digit =90*2= 180

Page from 100 to 750 = total no. of digit =1953

Total no. of digit = 9+180+1953= 2142

Hence, the total no. pages of book have page 1 to 750 is 2142.

39. A heard of cows invades a dance causing damage in the field. The boys were chasing them. There are 130 heads & 300 feet but 5 cows have 3 legs & 2 men have 1 leg. Find the no. cows and boys.

Total no. of head =130
Total no. of feet =300
Let x denote number of cows and y denote the number of boys.
Then, x+y=130 -----(i)
& 5*3 + (x-5)*4 + 2*1 + (y-2)*2=300 -----(ii)
Solving eqn (i) & (ii)
15 + 4x – 20 + 2 + 2*(130-x) -4 =300
or, 15 + 4x – 20 + 2 + 260 - 2x – 4= 300
or, 2x=47

:. x= $\frac{47}{2}$

& y=$\frac{213}{2}$

Here, no. of cows are $\frac{47}{2}$ & no. boys are $\frac{213}{2}$

40. Suppose S is set with k elements. Show that S has property 2^k subsets.

Let p(k) is" if a set contain k element then S has 2^k subsets"

If k= 0, i.e s={}, So the S has only one subset i,e $2^0 = 1$

Assume p(j) is valid i.e it is true for k= j

We have to show that is true for k= j+1

Now, let S ={$S_1, S_2, S_3 ---- S_{j+1}$} be a set with j+1 elements.

Let S' = {$S_1, S_2, S_3 ---- S_j$}, so it has 2^j subsets which is also the subset of S.

Let A ⊂ S'

A U {S_{j+1}}⊂S'⊂S

that add another 2^j subset of S'. We thus have a total $2^j + 2^j = 2^j(1+1) = 2^{j+1}$ subset of the set.

41. Assume that K is positive integer. If (k+1) letter are delivered to K mailboxes, then show that one mailbox must contain at least two letter.

Let p(k) is "if (k+1) letters delivered to k mailbox then some mailbox contain at least two letter"

For k=1, then number of mailbox is 1 and no. of letter is 1+1=2. Hence, the mailbox contain 2 letter.

Suppose, it is true for k=j i.e no. of letter is j+1 & mailbox is j.

We have to show that is true for p(j+1)

- If last mailbox is empty, then (j+2) letter delivered to J mailbox. At least (j+1) (indeed j+2) letters have been delivered to these first j mailbox. So by assumption one of them will contain at least two letters.

- If the last mailbox contain only one letter, then remaining (j+1) letters are delivered to j mailbox, which is true by assumption.
- If last mailbox contain two letters, then obviously we say some mailbox contain at least two letters.

42. There are more adults than boys, more boys than girls, more girl than families. If no family member has fewer than three children, then what is the least number of families that there could be?

Adults = A

Boys = B

Girls = G

Families = F "A>B>G>F"

- If there were just one family then there would be at least two girls, at least 3 boys at least 4 adults, which make two families which is a contradiction.
- If there were just 2 families then there would be at least 3 girls, at least 4 boys at least 5 adults. But 5 adults cannot be just two families which is a contradiction.
- If there were just 3 families then there would be at least 4 girls, at least 5 boys at least 6 adults. That is not a contradiction. Thus three families might satisfy the conditions.

The answer is that three families is the smallest number that there could be.

43. Case-By-Case(Referred to book **)**

44. We wish to bring from the river 1 quart of water, but we are only have 8 quart container

and 5 quart container and no other container, How do we do it?

Step 1: Fill 8 quart container then pour it into 5 quart container. Then 3 quart remains in 8 quart container.

Step 2: Empty 5 quart and pour 3 quart from 8 quart container to it then fill up 8 quart again and top-off the 5 quart container using the contest of 8 quart container. 6 quart still remains in 8 quart container.

Step 3: Then pour 6 quart of 8 quart container in 5 quart container the 1 quart remaining in the quart container.