IOE, TU, Questions and Solutions
2067 Ashadh to 2071 Magh

Engineering Physics
(for BE first year)

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PREFACE

The pattern of the questions and the depth of knowledge required for answering to them are fully revealed to the students by this time. Because in this country a student pursuing the engineering stream is seldom in position to purchase so many books on a particular subject. And even if the text books are collected, due to several problems he/she can hardly afford ample time to go through them for finding appropriate answers to the different questions asked in the examination. To get rid of this lacuna a complete IOE solution especially in Engineering Physics was already in demand for the part of the bachelor level students. But unfortunately no writer showed any interest so long as this aspect. Though every effort is taken to make the volume most suitable, owing to extreme hurriedness there may be some shortfall in it in some cases. Even then, having the book before the forthcoming examination the student can gain confidence on the subject matter indeed. Suggestion from the students and teachers for betterment of this volume is thankfully solicited. Finally I will think my labour to be fruitful if this IOE Engineering Physics caters to the need of the students, as tried for.

31st January, 2015 Maha Datta Paudel

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2067 Ashadh Regular (BEX, BCT, BEL, BIE, B. Agri)

1. *Show that there are four collinear points within compound pendulum having same time period. Give their physical significance.*

Sol^n: The time period of oscillation for compound pendulum is

$$T = 2\pi\sqrt{\frac{l^2+k^2}{lg}}$$

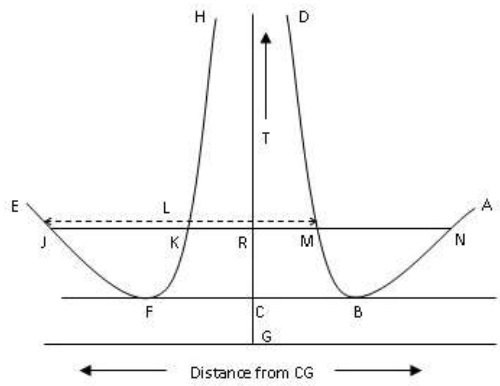
Now squaring this equation on both sides, we get

$\Rightarrow T^2 g l = 4\pi^2(l^2 + k^2)$ $\qquad\qquad$ $\Rightarrow 4\pi^2 l^2 + 4\pi^2 k^2 = gT^2 l$

$\Rightarrow 4\pi^2 l^2 - gT^2 l + 4\pi^2 k^2 = 0$

Which is quadratic in l. So, l has two values with same time period T in one side of the compound pendulum. The values are

$l = \frac{gT^2 \pm \sqrt{(gT^2)^2 - 64\pi^4 k^2}}{8\pi^2}$. These two values are different points. Similarly, there exist another two points in another side with same time period T. So, there are four collinear points within compound pendulum. The graph between length of the pendulum and the time period of oscillation is as shown in side figure. According to graph at points J, K, M and N the time period of oscillation is same. The point J is the point of suspension and M is the corresponding point of oscillation. Similarly, if point M is point of suspension then J is point of oscillation. Also, same for point K and N. The point of suspension is that point about which the pendulum is suspended and point of oscillation is that point about which the oscillation is taken.



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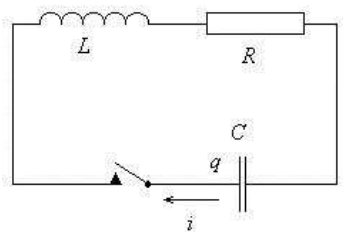
OR

Derive the differential equation of damped harmonic oscillation in LCR circuit. Solving the equation find the damped frequency of the oscillation and explain its significance.



Sol^n: If we consider the effect of resistor in LC circuit then the oscillation is called damped LCR oscillation. In the circuit there is energy lose due to the present of the resistor and the rate of energy lose by the circuit is – i^2R.

i.e. $\frac{dU}{dt} = -i^2R$

$\Rightarrow \frac{d}{dt}\left(\frac{q^2}{2C} + \frac{1}{2}Li^2\right) = i^2R \qquad \Rightarrow \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = i^2R$

$\Rightarrow \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0$ ---------------------------- (1)

This is the differential equation of damped LCR oscillation. The solution of this equation is

$q = q_m\, e^{-\frac{Rt}{2L}} \cos(\beta t \pm \theta)$, where β is the angular frequency of the system.

Now, differentiating this equation with respect to t, we get

$$\frac{dq}{dt} = q_m e^{-\frac{Rt}{2L}}\left[-\frac{R}{2L}\cos(\beta t \pm \theta) - \beta\sin(\beta t \pm \theta)\right]$$

Again, differentiating this with respect to t, we have

$$\frac{d^2q}{dt^2} = q_m e^{-\frac{Rt}{2L}}\left[\left(\frac{R}{2L}\right)^2\cos(\beta t \pm \theta) + \frac{R\beta}{2L}\sin(\beta t \pm \theta) + \frac{R\beta}{2L}\sin(\beta t \pm \theta) - \beta^2\cos(\beta t \pm \theta)\right]$$

$$= q_m e^{-\frac{Rt}{2L}}\left[\left(\frac{R^2}{4L^2} - \beta^2\right)\cos(\beta t \pm \theta) + \frac{R\beta}{L}\sin(\beta t \pm \theta)\right]$$

With the values of q, $\frac{dq}{dt}$, and $\frac{d^2q}{dt^2}$, Eq. (1) becomes

$$q_m e^{-\frac{Rt}{2L}}\left[\left(\frac{R^2}{4L^2}-\beta^2\right)\cos(\beta t \pm \theta)+\frac{R\beta}{L}\sin(\beta t \pm \theta)-\frac{R^2}{2L^2}\cos(\beta t \pm \theta)-\frac{R\beta}{L}\sin(\beta t \pm \theta)+\frac{1}{LC}\cos(\beta t \pm \theta)\right]=0$$

$$\Rightarrow \left[\frac{R^2}{4L^2}-\beta^2-\frac{R^2}{2L^2}+\omega^2\right] q_m e^{-\frac{Rt}{2L}}\cos(\beta t \pm \theta)=0 \quad \text{where } \omega^2=\frac{1}{LC}$$

$$\Rightarrow \left[\frac{R^2}{4L^2}-\beta^2-\frac{R^2}{2L^2}+\omega^2\right]=0 \qquad \Rightarrow \beta^2=\omega^2-\frac{R^2}{4L^2}$$

$$\therefore \beta=\sqrt{\omega^2-\frac{R^2}{4L^2}} \qquad \text{and} \qquad f=\frac{1}{2\pi}\sqrt{\omega^2-\frac{R^2}{4L^2}}$$

This is the frequency of damped LCR oscillation. This expression shows that the frequency of damped LCR circuit is slightly smaller than that of the LC oscillation. There may be arises three cases as

(i) Overdamped ($\omega^2<\frac{R^2}{4L^2}$): The system returns (exponentially decays) to steady state without oscillating. Larger values of the damping factor return to equilibrium slower.

(ii) Critically damped ($\omega^2=\frac{R^2}{4L^2}$): The system returns to steady state as quickly as possible without oscillating. This is often desired for the damping of systems such as doors.

(iii) Underdamped ($\omega^2>\frac{R^2}{4L^2}$): The system oscillates (with a slightly different frequency than the undamped case) with the amplitude gradually decreasing to zero.

2. *A uniform circular disc whose radius R is 12.6 cm is suspended as a physical pendulum from a point on its rim. (a) What is its period? (b) At what radial distance r < R is there a pivot point that gives the same period?*

Sol^n: Here the length of the pendulum is equal to the radius of the disc (l) = R = 12.6 cm

For the disc $mk^2 = \frac{mR^2}{2} \Rightarrow k = \sqrt{\frac{12.6^2}{2}} = 8.91$ cm

(a) Time period of oscillation is given by $T = 2\pi\sqrt{\frac{l^2+k^2}{lg}}$

So, $T = 2\pi\sqrt{\frac{12.6^2+8.91^2}{12.6\times 981}} = 0.87$ sec

(b) The distance which lies at $\frac{k^2}{l}$, the time period is same as that of the pivot point. The radial distance $r = \frac{8.91^2}{12.6} = 6.3$ cm (less than R) the period is same as that of pivot point.

3. *Define absorption coefficient of sound. Derive a relation between reverberation time and absorption coefficient for acoustically good hall.*

Sol^n: The ratio of sound energy absorbed by a surface to the total amount of sound energy incident on that surface is absorption coefficient. It is dimensionless parameter. Also, the ratio of perfectly absorbing area to total available area is absorption coefficient.

$\alpha = \frac{\text{perfectly absorbing area (A)}}{\text{total available area (S)}}$

Let I be the instantaneous intensity of sound in the hall, ΔI be the change in intensity in small time Δt and α be the average absorption coefficient of the hall then according to Sabine's assumption

$\Delta I \propto \alpha\, n\, I\, \Delta t \qquad \Rightarrow \Delta I = -\,\alpha\, n\, I\, \Delta t$

Where n is the number of reflections per second and negative sign indicates that intensity of sound decreases with increase in time.

If V be the volume and S be the surface area of the hall, then the average distance travelled by the sound in two successive reflections is $\frac{4V}{S}$.

If v be the velocity of sound then time for two successive reflections is $\frac{4V}{Sv}$. So, number of reflections per second is $n = \frac{Sv}{4V}$.

Thus, $\Delta I = -\alpha \frac{Sv}{4V} I \Delta t$ $\qquad\Rightarrow \frac{\Delta I}{\Delta t} = -\alpha \frac{Sv}{4V} I$

$$\lim_{\Delta t \to 0} \frac{\Delta I}{\Delta t} = \frac{dI}{dt} \qquad \therefore \frac{dI}{I} = -\alpha \frac{Sv}{4V} dt$$

Integrating from 0 to t, we have

$$\int_{I_0}^{I} \frac{dI}{I} = -\int_0^t \alpha \frac{Sv}{4V} dt \qquad \Rightarrow \text{Log}_e\left(\frac{I}{I_0}\right) = -\alpha \frac{Sv}{4V} t$$

$$\therefore I = I_0 e^{-\frac{\alpha Svt}{4V}}$$ For t = T (reverberation time), $I = 10^{-6} I_0$

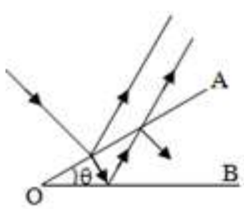
$$\Rightarrow 10^{-6} = e^{-\frac{\alpha SvT}{4V}}$$ again, v = 350 m/s

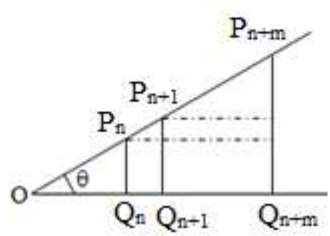
$$T = \frac{6 \times \ln 10 \times 4V}{350 \times S\alpha} = \frac{0.158V}{\alpha S}.$$

This is the relation between reverberation time and average absorption coefficient for a good hall.

4. *Explain how interference fringes are formed by a thin wedge shaped film, when examined by normally reflected light. How will you establish the difference of film thickness between two points?*

Sol^n^: Consider two plane surfaces OA and OB inclined at an angle θ encloses a wedge shaped air film as shown in figure. Thickness of the air film increases from O to A. The interference effect is best observed when the angle of incidence is small i.e., cos r = 1. Suppose the n^{th} bright fringe occurs at P_n where the thickness of air film is P_nQ_n. Applying the relation for





bright fringe reflected light, $2\mu t \cos r = (2n + 1)\frac{\lambda}{2}$, Here, $\mu = 1$, $\cos r = 1$ and $t = P_nQ_n$.

$\therefore\ 2P_nQ_n = (2n + 1)\frac{\lambda}{2}$ -------- (1)

And at P_{n+1}, next bright fringe occurs so that

$\therefore\ 2P_{n+1}Q_{n+1} = (2n + 3)\frac{\lambda}{2}$ -------- (2)

Subtracting Eq. (1) form Eq. (2), we get

$P_{n+1}Q_{n+1} - P_nQ_n = \frac{\lambda}{2}$

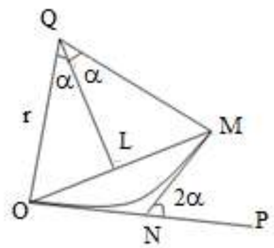
Hence, the thickness of the film increases in each bright fringes by $\frac{\lambda}{2}$.

If the wedge shaped film has refractive index μ then the thickness of the film increases by $\frac{\lambda}{2\mu}$ in consecutive bright or dark fringe.

OR

Show that the intensity of the second order maxima of Fraunhoffer's single slit diffraction is $\left(\frac{2}{5\pi}\right)^2$ times the intensity of central maxima.

Sol^n: Bending of light from the edge of the obstacle and spreading around the geometrical shadow is known as diffraction of light. The total phase difference between the wavelets from the top and bottom edge of the slit of width a is α. As the wave front is divided into a large number of strips, the resultant amplitude due to all the individual small strips can be obtained by the vector polygon method. Here, the amplitudes are small and the phase difference increases by infinitesimally small amounts from strip to strip. Thus the vibration polygon coincides with the circular arc OM. OP gives the direction of the



initial vector and NM the direction of the final vector due to the secondary waves from A. Q is the center of the circular arc.

$\angle MNP = 2\alpha$ and $\angle OQM = 2\alpha$

In the ΔOQL, $\sin\alpha = \frac{OL}{r}$; $OL = r\sin\alpha$

Where r is the radius of the circular arc.

$\therefore$ Chord $OM = 2OL = 2r\sin\alpha$ -------- (1)

The length of the arc OM is proportional to the width of the slit.

$\therefore$ Length of the arc OM = Ka, where K is a constant and a is the width of the slit. Also, $2\alpha = \frac{\text{arcOM}}{\text{radius}} = \frac{Ka}{r}$ $\Rightarrow 2r = \frac{Ka}{\alpha}$

Substituting this value of 2r in Eq. (1)

Chord $OM = \frac{Ka}{\alpha}\sin\alpha$

But, Chord OM = A is the amplitude of resultant.

$A = (Ka)\frac{\sin\alpha}{\alpha}$ $\Rightarrow A = A_0\frac{\sin\alpha}{\alpha}$

The intensity I at the point is given by $I = A^2 = A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$

A phase difference of 2π corresponds to a path difference of λ. Therefore a phase difference of 2α is given by

$2\alpha = \frac{2\pi}{\lambda}a\sin\theta$, where $a\sin\theta$ is the path difference between the secondary waves from A and B.

For second primary maxima $\alpha = \frac{\pi}{\lambda}a\sin\theta = \frac{\pi}{\lambda}(2n+1)\frac{\lambda}{2}$ for n = 2

$\alpha = \frac{5\pi}{2}$ $\therefore I = I_0\left(\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2 = \left(\frac{2}{5\pi}\right)^2 I_0 = 1.62\%$ of I_0. Proved

5. *In a Newton's ring arrangement s source emitting two wavelengths 6×10^{-7} m and 5.9×10^{-7} m is used. It is found that n^{th} dark ring due to* *one wavelength coincides with (n + 1)th dark ring due to other. Find the diameter of the nth dark ring if radius of curvature of lens is 0.9m.*

Sol^n: Two wavelengths are $\lambda_1 = 6 \times 10^{-7}$ m and $\lambda_2 = 5.9 \times 10^{-7}$ m

Diameter of the nth dark ring due to λ_1 is $D_n = 2\sqrt{n\lambda_1 R}$ and diameter of the n+1th dark ring due to λ_2 is $D_{n+1}' = 2\sqrt{(n+1)\lambda_2 R}$

According to question, $2\sqrt{n\lambda_1 R} = 2\sqrt{(n+1)\lambda_2 R}$

$\Rightarrow n\lambda_1 = (n+1)\lambda_2$ $\quad\Rightarrow n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{5.9}{6-5.9} = \frac{5.9}{0.1} = 59$

Radius of curvature of lens (R) = 0.9m

Diameter of the nth dark ring is $D_n = 2\sqrt{n\lambda_1 R} = 2\sqrt{59 \times 6 \times 10^{-7} \times 0.9}$

$\therefore D_n = 0.0112$ m = 1.12 cm

6. *Calculate the thickness of quarter wave plate for light of wavelength 5893Å. Given refractive indices of ordinary and extraordinary ray are 1.544 and 1.553 respectively.*

Sol^n: Wavelength of light (λ) = 5.893×10^{-5} cm

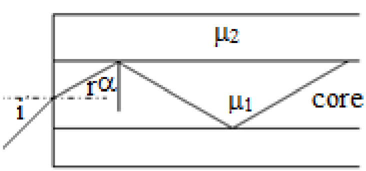
$\mu_0 = 1.544$ and $\mu_e = 1.553$

Thickness of quarter wave plate (t) =?

$t = \frac{\lambda}{4(\mu_e - \mu_0)} = \frac{5.893 \times 10^{-5}}{4(1.553 - 1.544)} = 1.64 \times 10^{-3}$ cm

7. *Define acceptance angle of an optical fiber. Derive the relation for Numerical Aperture (NA) of the optical fiber. Also write down its significance.*

Sol^n: Consider an optical fiber and light is send from one end which is known as lunching end. Let the refractive index of core and cladding to be μ_1 and μ_2 respectively.



When the light beam enters at an angle i to the axis of the fiber, the ray refracted at an angle r. For angle α more than the critical angle C, the light will suffer total internal reflection. By applying snell's law at lunching end, μ_0 sini = μ_1 sinr

Where μ_0 is the refractive index of the medium through which light is incident on the fiber.

μ_0 sini = μ_1 cosα ------- (1)

Let us consider a particular angle α = C then at core-cladding interface

μ_1 sinα = μ_2 sin90° = μ_2

sinα = $\frac{\mu_2}{\mu_1}$ = sinC

From Eq. (1) μ_0 sini = $\mu_1\sqrt{1-\sin^2\alpha}$ = $\mu_1\sqrt{1-\frac{\mu_2^2}{\mu_1^2}} = \sqrt{\mu_1^2-\mu_2^2}$

Sini = $\frac{\sqrt{\mu_1^2-\mu_2^2}}{\mu_0}$ i = $\sin^{-1}\left(\frac{\sqrt{\mu_1^2-\mu_2^2}}{\mu_0}\right)$

This is called acceptance angle of the fiber. Acceptance angle may be defined as the maximum angle that a light ray can have relative to the axis of fiber and propagate down the fiber. The fractional refractive index is $\Delta = \frac{\mu_1-\mu_2}{\mu_1}$

The numerical aperature (NA) is defined as the sine of the acceptance angle. So, NA = sini$_{max}$ = $\sqrt{\mu_1^2-\mu_2^2}$

Again, $\mu_1^2 - \mu_2^2 = (\mu_1 + \mu_2)(\mu_1 - \mu_2) = \frac{(\mu_1+\mu_2)}{2}\frac{(\mu_1-\mu_2)}{\mu_1}2\mu_1 \cong \mu_1\Delta 2\mu_1$

So, NA = $\sqrt{\mu_1\Delta 2\mu_1}$ = $\mu_1\sqrt{2\Delta}$.

The **numerical aperture** (**NA**) of an optical system is a dimensionless number that characterizes the range of angles over which the system can accept or emit light.

8. *Two thin converging lenses of focal lengths 0.2m and 0.3m are placed coaxially 0.10m apart in air. An object is located 0.6m in front of* *the lens of smaller focal length. Find the position of the two principal points and that of image.*

Sol^n: Focal length of first lens (f_1) = 20 cm, focal length of the second lens (f_2) = 30cm, separation between two lenses (d) = 10cm, object distance from first lens (u) = 60cm

$f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{20\times30}{20+30-10} = 15$ cm

first principal point (α) = $\frac{fd}{f_2} = \frac{15\times10}{30} = 5$cm

second principal point (β) = $-\frac{fd}{f_1} = -\frac{15\times10}{20} = -7.5$ cm

distance of the object from first lens (u) = 60 cm

object position from first principal point (U) = - 65cm

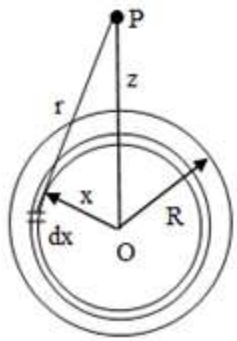
From lens formula, $\frac{1}{f} = \frac{1}{V} - \frac{1}{U}$ $\quad\Rightarrow \frac{1}{15} = \frac{1}{V} + \frac{1}{65}$

$\Rightarrow \frac{1}{V} = \frac{1}{15} - \frac{1}{65}$ $\quad\therefore V = 19.5$ cm

So, the image distance from the second lens is 12 cm towards right.

9. *Derive an expression for electric potential at any point on the axis of the uniformly charged disk. Extend your result to calculate electric filed.*

Sol^n: Consider a disc of radius R has uniform surface charge density σ. To find the electric potential at a point P along its central axis at z distance from the center, we consider the disc as a set of concentric rings. We calculate the electric potential at P due to one ring.



The ring of radius x and width dx has surface area $2\pi x dx$. So, the charge on the ring of width dx is $dq = 2\pi\sigma x dx$.

Now, the electric potential at P due to this ring is

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{2\pi\sigma x dx}{4\pi\varepsilon_0 (z^2+x^2)^{1/2}}$$

$$= \frac{\sigma}{2\varepsilon_0} \frac{x dx}{(z^2+x^2)^{1/2}}$$

Total potential at P is given by

$$V = \int_0^R dV = \frac{\sigma}{2\varepsilon_0} \int_0^R (z^2 + x^2)^{-1/2} x dx$$

$$V = \frac{\sigma}{2\varepsilon_0} \left[(z^2 + x^2)^{\frac{1}{2}}\right]_0^R = \frac{\sigma}{2\varepsilon_0} \left(\sqrt{z^2 + R^2} - z\right)$$

Now, the electric field E at point P is given by $E = -\frac{dV}{dz}$

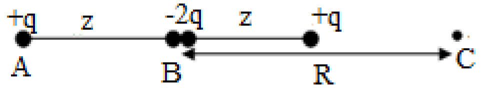
$$E = -\frac{dV}{dz} = \frac{d}{dz}\left[\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)\right] = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}\left(\sqrt{z^2 + R^2} - z\right)$$

$$E = -\frac{\sigma}{2\varepsilon_0}\left(\frac{2z}{2\sqrt{z^2+R^2}} - 1\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

This gives the electric field at point P due to charged disc.

OR

Derive an expression for the electric field at any point on the axis of the short linear quadrupole.



Sol^n: The arrangement of four equal and opposite charges as shown in the figure forms the electric quadrupole.

Take a linear quadrupole of separation 2z and consider a point P at a distance R from the center of the quadrupole on the axial line. The electric field at P due to charges at points A, B and C are

$E_A = \frac{q}{4\pi\varepsilon_0 (R+z)^2}$, $E_B = \frac{-2q}{4\pi\varepsilon_0 R^2}$ and $E_C = \frac{q}{4\pi\varepsilon_0 (R-z)^2}$ respectively.

So, net electric field at point P is given by

$E = E_A + E_B + E_C$

$$E = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(R+z)^2} - \frac{2}{R^2} + \frac{1}{(R-z)^2}\right) = \frac{2qz^2}{4\pi\varepsilon_0}\left[\frac{3R^2-z^2}{R^2(R^2-z^2)^2}\right]$$

$E = \frac{Q}{4\pi\varepsilon_0}\left[\frac{3R^2-z^2}{R^2(R^2-z^2)^2}\right]$, Where $Q = 2qz^2$ is electric dipole moment

This gives the magnitude of electric field intensity of a quadrupole along its axis. For a short quadrupole R >> z, then

$E = \frac{3Q}{4\pi\varepsilon_0 R^4}$.

10. *A capacitor slab of thickness b is inserted into a parallel plate capacitor exactly half way between the plates. If the separation of the plate is d and the area of each plate is A, show that the change in capacitance is equal to* $\frac{\varepsilon_0 Ab}{(d-b)d}$.

Sol^n: In this case the parallel plate capacitor may be thought of as an arrangement of two capacitors with the plates separation of $\frac{d-b}{2}$ after the introduction of a slab of thickness b exactly halfway between the plates, and the capacitance of each capacitor is $C_1 = C_2 = \frac{2\varepsilon_0 A}{(d-b)}$

So, the equivalent capacitance is given by $C' = \frac{C_1C_2}{C_1+C_2} = \frac{C_1}{2} = \frac{\varepsilon_0 A}{(d-b)}$

The original capacitance of the capacitor is $C = \frac{\varepsilon_0 A}{d}$

Now, the change in capacitance is $\Delta C = C' - C = \frac{\varepsilon_0 A}{(d-b)} - \frac{\varepsilon_0 A}{d}$

$= \frac{\varepsilon_0 Ab}{(d-b)d}$

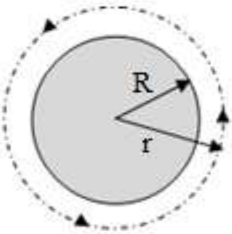
11. *What is the drift speed of the conduction electrons in a copper wire (molecular mass = 63.54 gm/mol, density = 8.96 gm/cc) with radius 900μm when it has a uniform current 17mA flowing in the wire?*

Sol^n: There is one conducting electron per atom and molar mass is 63.54gm. We have the number of electrons per unit volume of the copper is $n = \frac{N_A \rho}{M} = \frac{6.02\times10^{23}\times8.96\times10^{3}}{63.54\times10^{-3}} = 8.48 \times 10^{28}\ m^{-3}$

Now the drift speed is given by

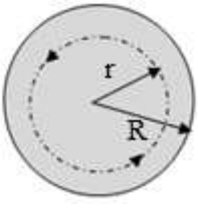
$$v_d = \frac{I}{neA} = \frac{17\times10^{-3}}{8.48\times10^{28}\times1.6\times10^{-19}\times\pi\times(9\times10^{-4})^2} = 4.9 \times 10^{-7}\ m/s$$

12. *A long straight wire of radius R carries uniformly distributed current I. Calculate magnetic fields at any points inside and outside the wire.*



cSol^n: Consider a straight wire of radius R carries a steady current I. The current is uniformly distributed over the cross sectional area of the wire. Let us consider a point at a distance r from the center of the wire in which we have to calculate the magnetic field.

(i) Outside (r > R): Figure shows cross section of the straight wire that carries uniformly distributed current I, directed out of the page. To find the magnetic field for the region r > R, draw a circular Amperian loop of radius r that encloses the wire as shown in figure. From symmetry, B must be constant in magnitude and parallel to elemental length at every point on this circle. From Ampere's law,



$$\oint \vec{B} \cdot \vec{dl} = B \oint dl = B\,(2\pi r) = \mu_0 I$$

$$\therefore\ B = \frac{\mu_0 I}{2\pi r} \qquad \text{for, } r > R$$

(ii) Inside (r < R): To find he magnetic field at a point inside the wire we can again use the Amperian loop of radius r as shown in figure. Let I' be the current enclosed by the loop, then from Ampere's law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I'$$

We know that current density J is constant because the current is uniformly distributed over the conductor.

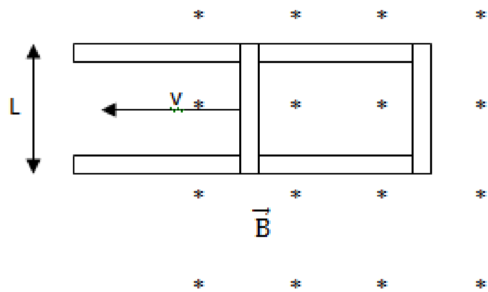
$J = \frac{I}{\pi R^2} = \frac{I'}{\pi r^2}$ $\Rightarrow I' = \left(\frac{r}{R}\right)^2 I$

$\Rightarrow B\,(2\pi r) = \mu_0 \left(\frac{r}{R}\right)^2 I$

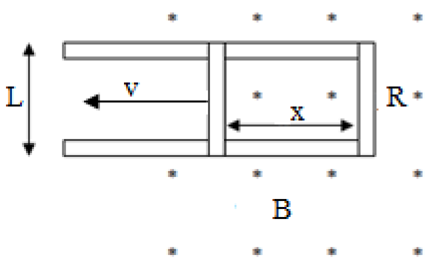
$\therefore\ B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$

B is zero at the center and maximum at the surface r = R.

13. *The conducting rod shown in figure has length L and is being pulled along horizontal, frictionless conducting rails at a constant velocity $\vec{v}$. The rails are connected at one end with a metal strip. A uniform magnetic field $\vec{B}$, directed out of the page, fills the region in which the rod moves. Derive an expression for the rate of thermal energy being generated in the rod.*



Sol^n: Consider a straight rod of length L is moving through a uniform magnetic field directed perpendicular into the plane with constant velocity v under some external force. The force experience by the electron is



$\vec{F} = q(\vec{v} \times \vec{B})$

Due to this force electrons move to the lower end and accumulate there, leaving a net positive charge at the upper end of the conductor.

Thus at equilibrium, qE = Bqv $\therefore$ E = vB

The potential difference produced in the conductor across the ends is

V = EL = vBL

Let us assume the bar of length L has no resistance and the rest part of the closed circuit has resistance R, then the force produced on the free charge set up an induced emf due to which the magnetic flux is

$\phi_B = BA = BLx$,   where Lx is the area enclosed by the circuit.

Using Faraday's law, the induced emf is $\varepsilon = -\frac{d\phi_B}{dt} = -vBL$

Now, the induced current in the circuit is given from the conservation of energy as, $i = \frac{\varepsilon}{R} = \frac{vBL}{R}$

The rate at which the thermal energy appeared in the loop when the conductor is pulled or pushed is $P = i^2R = \frac{B^2L^2v^2}{R}$

14. *A coil has an inductance of 53 mH and a resistance of 0.35Ω. If a 12V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value? After how many time constants will half this equilibrium be stored in the magnetic field?*

Sol^n: Here, inductance of the coil (L) = 53 mH = 0.053 H

Resistance (R) = 0.35Ω, applied emf (ε_0) = 12V

At equilibrium, the current through the circuit (i_0) = $\frac{\varepsilon_0}{R}$ = 34.3 Amp

Energy stored in the inductor (U_B) = $\frac{1}{2} L\, i^2{}_0 = \frac{1}{2} \times 0.053 \times 34.3^2 = 31.15$ J

Also, $i = i_0\, e^{-t/\tau}$ where $\tau = \frac{L}{R} = 0.053/0.35 = 0.15$ sec

From question, $\frac{1}{2} L\, i^2 = \frac{1}{4} L\, i^2{}_0$ $\Rightarrow i_0{}^2\ (1 - e^{-t/\tau})^2 = \frac{1}{2} i^2{}_0$

$\Rightarrow \frac{t}{\tau} = \ln(\frac{1}{0.29})$ $\Rightarrow t = \tau \ln(\frac{1}{0.29}) = 1.2\ \tau$

So, after 1.2 times time constant half of equilibrium value of energy will be stored.

OR

In a certain cyclotron a proton moves in a circle of radius 0.5m. The magnitude of the magnetic field is 1.20T. What is the oscillator frequency? What is the kinetic energy of the proton in eV?

Sol^n^: Radius of the path (R) = 0.5m, magnetic field (B) = 1.2T

Oscillator frequency (f) =?

We have, $f = \frac{qB}{2\pi m} = \frac{1.6\times10^{-19}\times1.2}{2\pi\times1.67\times10^{-27}} = 0.18\times10^{8}$ Hz = 18 MHz

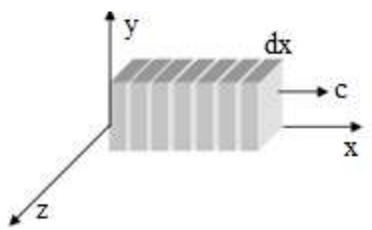
Kinetic energy of the electron (KE) =?

$\text{K.E.} = \frac{q^2B^2R^2}{2m} = \frac{(1.6\times10^{-19}\times1.2\times0.5)^2}{2\times1.67\times10^{-27}} = 1.7\times10^{7}$ eV = 17 MeV

15. *Define Poynting vector. Prove that $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$, where the symbols have their usual meanings.*

Sol^n^: In case of the electromagnetic wave the energy transmitted through unit area in unit time is measured in terms a vector which is known as Poynting vector. It is defined as $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$.

Where E and B are the magnitude of electric and magnetic fields at a point due to the propagation of electromagnetic wane and S is poynting vector. The magnitude of Poynting vector gives the amount of energy transmitted through unit area in unit time. The direction of Poynting vector is perpendicular to the plane containing the vectors E and B.



Let us consider an electromagnetic wave is propagating through a medium with velocity c as shown in figure. Let us divide the medium into infinite number of small layer of thickness dx as shown in figure.

Now the total energy developed in small layer due to propagation of electromagnetic wave is

$dU = dU_E + dU_B$

$$= (u_E + u_B)\,(A\,dx) = \left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\varepsilon_0 E^2\right) A\,dx \quad \text{since } u_e = u_b$$

$$dU = \varepsilon_0 E^2 A\,dx = \frac{EB}{c\mu_0} A dx \qquad \text{since} \quad \frac{E}{B} = c$$

The time taken by the electromagnetic wave to travel the distance dx is dt which is given by $dt = \frac{dx}{c}$.

Hence the magnitude of S, in terms of energy flow per unit time per unit area is

$$S = \frac{dU}{dt\ A} = \frac{EBA\ dx}{(\mu_0 c)(dx/c)A} = \frac{1}{\mu_0} EB$$

In vector notation, $\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$

16. *An electron is trapped in an one dimensional infinite potential well of width 'a' such that*

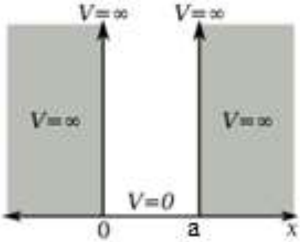
$$V = \begin{cases} \infty \ for\ 0 \le x \ and\ x \ge a \\ 0\ for\ 0 < x < a \end{cases}$$

Using boundary condition, prove that the total energy of the system is $E = \frac{n^2\pi^2}{2ma^2}\hbar^2$. *Where symbols carry their usual meanings.*

Sol^n^: Let us consider one dimensional potential well of infinite height and width a as shown in figure with potential as

$V(x) = 0,\ 0 < x < a$ and

$V(x) = \infty,\ x \le 0,\ x \ge a.$



In the outside of the well the wave function is zero and inside the well the wave function is finite and non-zero. The time independent Schrodinger wave equation is

$$\frac{d^2\varphi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\,\varphi = 0$$

Inside the well, V = 0,

$\Rightarrow \frac{d^2\varphi}{dx^2} + \frac{2m}{\hbar^2} E\, \varphi = 0$

$\Rightarrow \frac{d^2\varphi}{dx^2} + \alpha^2\, \varphi = 0$ ----- (1) where $\alpha^2 = \frac{2m}{\hbar^2} E$ ------- (2)

The solution of Eq. (1) is $\varphi(x) = A \sin\alpha x + B \cos\alpha x$

At $x = 0$, $\varphi(x = 0) = 0$ $\Rightarrow \varphi(0) = B = 0$

Again, at $x = a$, $\varphi(a) = 0$ $\Rightarrow \varphi(a) = A \sin\alpha a = 0$

Since, $A \neq 0 \Rightarrow \sin\alpha a = 0 = \sin(n\pi)$

$\Rightarrow \alpha a = n\pi,$ $n = 0, 2, 3, \ldots\ldots\ldots\ldots\ldots$

$\alpha = \frac{n\pi}{a},$ $n = 0, 2, 3, \ldots\ldots\ldots\ldots\ldots$

Thus $\frac{n^2\pi^2}{a^2} = \frac{2mE_n}{\hbar^2}$ $\Rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$

This gives the energy of the particle confined in infinite well potential.

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2067 Ashwin Back (BEX, BCT, BEL, BIE, B. Agri)

1. *Define forced oscillation. Show that the total energy of the damped oscillation decreases with increasing time.*

Sol^n: If the oscillation in a system is due to continuous application of periodic external force then the vibration is known as forced oscillation. In this oscillation the frequency of the system is the frequency of the applied force. The amplitude of oscillation depends upon the frequency of the applied force. When the frequency of the applied force is equal to the natural frequency of the system then the amplitude of oscillation is maximum. This condition is known as resonance.

In case of simple harmonic motion the total energy is directly proportional to the square of the amplitude. In this oscillation the amplitude of oscillation is constant, due to which total energy of the system is also constant. By using same analogy the total energy of the damped oscillator is given by E $\propto$ (amplitude)^2

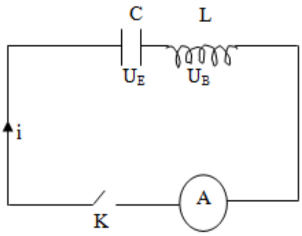
In case of damped oscillator, the amplitude A(t) = R $e^{-bt/2m}$

In case of the damped harmonic motion amplitude of oscillation depends upon the time. So, total energy of damped harmonic motion is given by $E = \frac{1}{2}KR^2 e^{-bt/m}$. This expression shows that total energy of the damped harmonic motion decreasing with increasing in time. $E(t) = \frac{1}{2}KR^2 e^{-bt/m}$

OR

Derive a differential equation for LC oscillation. Solve the equation and show that the maximum values of electric and magnetic energies stored in LC circuit are equal.

Sol^n: Let a fully charged capacitor (C) is connected with an inductor (L) as shown in figure. Consider q be the instantaneous charge in the capacitor and i be the current through the circuit then the energy stored in the capacitor and inductor are $U_E = \frac{q^2}{2C}$ and $U_B = \frac{1}{2} L i^2$ respectively. So, total energy in the circuit is



$U = U_E + U_B = \frac{q^2}{2C} + \frac{1}{2} L i^2$

If there is no any resistive element in the circuit then rate of energy lost in the circuit is zero.

i.e. $\frac{dU}{dt} = 0 \Rightarrow \frac{d}{dt}\left(\frac{q^2}{2C} + \frac{1}{2}Li^2\right) = 0 \qquad \Rightarrow \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = 0$

$\Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$ ---------------------------- (1)

This is the differential equation of LC oscillation. The solution of this equation is

$q = q_m \cos(\omega t \pm \theta)$, where $\omega = \frac{1}{\sqrt{LC}}$ the angular frequency of the LC oscillation. So, the current through the circuit is

$i = \frac{dq}{dt} = - q_m\omega \sin(\omega t \pm \theta)$.

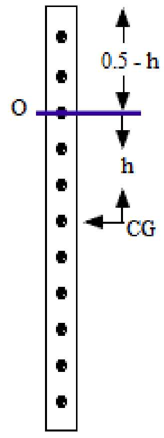
$U_E = \frac{q_m^2}{2C} \cos^2(\omega t \pm \theta) \qquad \therefore (U_E)_{max} = \frac{q_m^2}{2C}$

Similarly, $U_B = \frac{1}{2} L q_m^2 \omega^2 \sin^2(\omega t \pm \theta)$

$(U_B)_{max} = \frac{1}{2} L q_m^2 \omega^2 = \frac{1}{2} L q_m^2 \frac{1}{LC} = \frac{q_m^2}{2C} = (U_E)_{max}$ Proved.

2. *A meter stick swings about pivot at one end, at distance 'h' from the stick's center of mass. Calculate the period of oscillation using parallel axis theorem.*

Sol^n: Consider a meter scale of mass 'm' is suspended at point O. The center of gravity of the body is CG at a distance h from point O. If the meter scale is displaced by small angle θ (less than 4°) and released the restoring moment of force is



$\tau = -\,mgh\theta$ --------------- (1)

From Newton's second law of motion,

$\tau = I\,\alpha$ --------------- (2)

where I is the moment of inertia of the scale about the axis passing through point O and α is angular acceleration. From Eq. (1) and Eq. (2)

$-\,mgh = I\frac{d^2\theta}{dt^2} \quad \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mgh}{I}\theta \quad \Rightarrow \frac{d^2\theta}{dt^2} \propto -\,\theta$

This shows the motion of the rigid body is simple harmonic. So, $\omega^2 = \frac{mgh}{I} \quad \Rightarrow \omega = \sqrt{\frac{mgh}{I}} \quad \Rightarrow T = 2\pi\sqrt{\frac{I}{mgh}}$

Using parallel axes theorem, $I = I_{CG} + mh^2 = mk^2 + mh^2$.

$\therefore\ T = 2\pi\sqrt{\frac{h^2+k^2}{hg}}$ --------------- (3)

Also, $I_{CG} = mk^2 = \frac{mL^2}{12}$, here L = 1 m $\qquad \therefore\ k = \frac{1}{\sqrt{12}}$

So, $T = 2\pi\sqrt{\frac{12h^2+1}{12hg}}$

This is the required expression.

3. *Give an account of bad acoustic properties of a hall. Derive the expression for reverberation time in a good acoustics of a hall.*

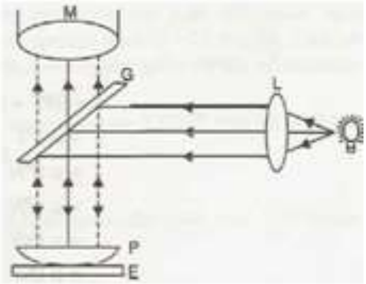
Sol^n: The acoustics of building is a branch of sound engineering. It has an important role in civil and architectural engineering. Acoustics of building is connected with the hearing to speakers and musicians in halls and auditoriums. It is found that some auditoriums are acoustically good and some bad. In bad, sound lack in distinctness. It is sometimes observed that a speech made in a certain hall or sound produced in a theatre is not audible at certain places. In some areas there is so much interference that it is sometimes difficult to understand what is being said. Therefore it is in fact necessary to bear certain points in mind while a place of public speaking is designed so that one can follow the utterances from every point clearly without interference.

(For Second part see in 2067 Ashadh Regular Q. No. 3)

4. *What are coherent sources? Describe a method for determining the refractive index of transparent liquid film using the interference phenomenon.*

Sol^n: Two sources are said to be coherence if they emit the light waves of having same amplitude, same frequency, moving in same direction with either same phase or constant phase difference. Newton's rings experiment is used to determine the refractive of transparent liquid film.

When a Plano – convex lens is placed in contact with a flat glass surface, a thin air film is formed. When such film is exposed by monochromatic light, a series of concentric fringes are formed which are called Newton's rings.



The experimental arrangement for the determination of wavelength of light using Newton's rings is shown in figure. A parallel beam of light from lens system is reflected by a glass plate G which is inclined at an angle 45° with the horizontal. A plano convex lens of large radius of curvature is exposed to the reflected light from the glass plate E which encloses an air film as shown. The interference of the reflected light is observed using an eye piece of a travelling microscope. Using the same microscope we measure the diameter of dark rings.

Let the diameter of n^{th} dark ring is D_n which is $D_n^2 = 4n\lambda R$

Similarly, the diameter of $(n + m)^{th}$ dark ring is $D^2_{n+m} = 4(n + m) \lambda R$

$D^2_{n+m} - D_n^2 = 4m\lambda R$ --------- (1)

Gently put the liquid into the air film space without disturbing the entire arrangement. Measure the diameters of n^{th} and $(n+m)^{th}$ dark rings. Let D'_n and D'_{n+m} be the diameters of n^{th} and $(n+m)^{th}$ rings. For normal incidence and with film of liquid of refractive index μ,

$D'^2_n = \frac{4n\lambda R}{\mu}$ and $D'^2_{n+m} = \frac{4(n+m)\lambda R}{\mu}$

$\therefore D'^2_{n+m} - D'^2_n = \frac{4m\lambda R}{\mu}$ ---------- (2)

From Eq. (1) and Eq. (2), we get

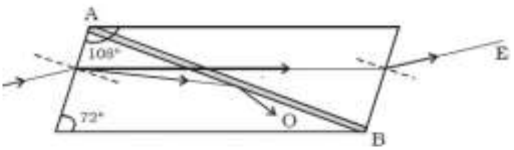
$\mu = \frac{D^2_{n+m} - D^2_n}{D'^2_{n+m} - D'^2_n}$

This gives the refractive index of the transparent liquid film.

OR

Describe the construction of Nicol prism. Explain it can be used as polarizer and analyzer.

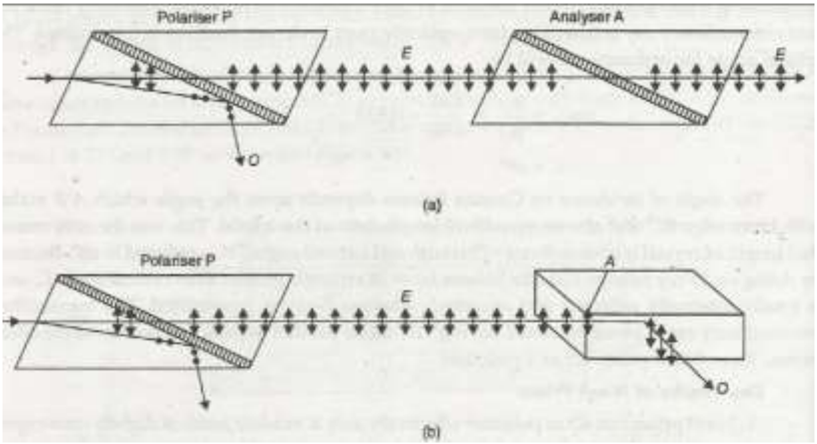
Sol^n: Nicol Prism: It is an optical device made from calcite crystal and used in many instruments for producing and analyzing plane polarized light.



A calcite crystal about 3 times as long its diagonal as that of width is taken. The diagonal surfaces are grounded, polished optically flat and cemented with Canada balsam which is clear transparent cement whose refractive index lies mid-way between the refractive indices of calcite for the ordinary and the extra-ordinary rays. The sides of prism are blackened to absorb the totally reflected light.

When a ray of light is incident on Nicol prism, it splits the ray into extra-ordinary ray and ordinary ray due to double refraction of the crystal. For ordinary ray, Canada balsam works as rarer medium, so it suffers total internal reflection. For extra-ordinary ray, Canada balsam works as denser medium; so it does not escape out from the prism and only e-ray is observed after refraction.

Consider two Nicol prisms arranged coaxially one after another. When a beam of unpolarized light is incident on the first prism P, the emergent beam is plane polarized with its vibrations in principal section of first prism. This prism is called polarizer. When principal section of both prisms are parallel then intensity of emergent light is maximum. But when the principal sections are at right angles to each other the, intensity of



emergent light is minimum i.e., there is no light it transmitted through the second prism. Here first prism produced plane polarized light and second prism detects and analyses it.

5. *A diffraction grating is used at normal incidence. In such arrangement a green line (λ = 5400Å) of certain order is superimposed on the violet line (λ = 4050Å) of the next order. If the angle of diffraction is 30°, how many lines are there in 1 centimeter?*

Sol^n: Wavelength of green line (λ_g) = 5400Å = 5.4×10^{-5} cm

Wavelength of violet line (λ_v) = 4050 Å = 4.05×10^{-5} cm

From question, $(a + b) \sin\theta_n = n\lambda_g = (n + 1) \lambda_v$

$\therefore\ n = \frac{\lambda_v}{\lambda_g - \lambda_v} = \frac{4.05}{5.4 - 4.05} = 3$

$(a + b) \sin 30° = 3 \times 5.4 \times 10^{-5}$ cm

$\therefore$ $a + b = 0.324 \times 10^{-3}$ cm

$N = \frac{1}{a+b} = \frac{1000}{0.324} = 3086$ lines per cm

6. *A light source emits light of two wavelengths 4300Å and 5100Å. The source is used in a double slit experiment. The distance between the sources and screen is 1.5m and the distance between the slits is 0.025mm. Calculate the separation between the third order bright fringes due to these two wavelengths.*

Sol^n: Here, $\lambda_1 = 4.3 \times 10^{-7}$ m, $\lambda_2 = 5.1 \times 10^{-7}$ m,

Distance between source and screen (D) = 1.5 m

Distance between the slits (d) = 0.025mm = 2.5×10^{-5} m

Position of the n^{th} order bright fringe from center maxima $y_n = \frac{n\lambda D}{d}$

For λ_1, $y_n = \frac{n\lambda_1 D}{d} = \frac{3\times4.3\times10^{-7}\times1.5}{2.5\times10^{-5}} = 7.74 \times 10^{-2}$ m = 7.74 cm

For λ_2, $y_n' = \frac{n\lambda_2 D}{d} = \frac{3\times5.1\times10^{-7}\times1.5}{2.5\times10^{-5}} = 9.18 \times 10^{-2}$ m = 9.18 cm

Hence, the separation between the third order bright fringes is given by

$\Delta y = y_n' - y_n = 9.18 - 7.74 = 1.44$ cm

7. *A thin convex and thin concave lens, each of focal length 50cm, are coaxially situated and separated by 10cm. find the position and nature of the final image formed of an object placed 20cm from the convex lens.*

Sol^n: Focal length of convex lens (f_1) = 50 cm, focal length of the concave lens (f_2) = – 50cm, separation between two lenses (d) = 10cm, object distance from convex lens (u) = 20cm

$f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{50\times(-50)}{50-50-10} = 250$ cm

first principal point (α) = $\frac{fd}{f_2} = \frac{250\times10}{-50} = -50$cm second principal point (β) = $-\frac{\mathrm{fd}}{\mathrm{f}_1} = -\frac{250\times10}{50}$ = - 50 cm

object position from first principal point (U) = 30cm right from P_1

From lens formula, $\frac{1}{f} = \frac{1}{V} - \frac{1}{U}$ $\Rightarrow \frac{1}{250} = \frac{1}{V} - \frac{1}{30}$

$\Rightarrow \frac{1}{V} = \frac{1}{250} + \frac{1}{30}$ $\therefore$ V = 26.79 cm right from P_2

$M = \frac{V}{U} = \frac{26.79}{30} = 0.893$

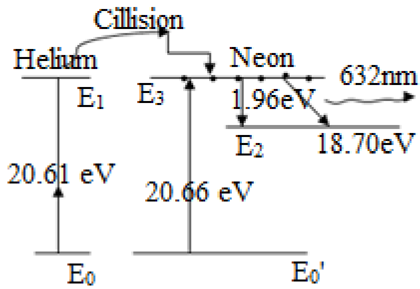
So, the final image distance from the concave lens = 23.21cm towards left and the final image is diminished and virtual.

8. *What is population inversion? Explain the lasing action of a gas laser with necessary energy level diagram.*

Sol^n: The principle of laser is the light amplification by stimulated emission of radiation. The stimulated emission is used for amplifying light waves.

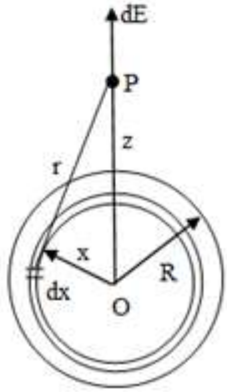
The number of atoms in ground state is maximum and decreases exponentially as we go to higher states. For lasing action, it is necessary that stimulated emission predominate over spontaneous emission. This happens only if $N_2 > N_1$. The situation in which the upper level are more populated than the lower levels is called population inversion. Generally, population inversion is achieved by exciting the medium with suitable form of energy. Such a phenomenon of population inversion occurs with a process called pumping. A light source is used to supply luminous energy in optical pumping. Usually this energy occurs in the form of light flashes. In some lasers, the excitation by electrical discharge provides the initial excitation, raising one type of atoms to their excited states. Then it inelastically collides with another type of atoms.

The gas laser to be operated successfully was the He – Ne laser. The He-Ne laser consists of a long and narrow discharge tube of diameter about 2-8mm and length 10-100cm. The lasing material is the mixture of the gases with a concentration of about 15% Helium and 85% Neon. The mixture works as lasing material because of same compatible properties of the two gases. The electrodes in the discharge tube are connected to a high voltage source. So an electric discharge takes place within the gas. With this high voltage some of the He atoms are excited to a metastable state at E_1 = 20.61eV above the ground state as shown in figure. It so happens that Ne has a metastable state at nearly the same energy, E_3= 20.66eV. The He atoms do not quickly return to the ground state by spontaneous emission. Rather it transfers the energy to Ne atoms during collision. With such collision the energy of excited He atoms will be transferred and it drops to ground state. However, getting the excess energy Ne atom is excited to the state E_3. The small difference of 0.05eV is supplied the kinetic energy of the atoms. In this way the higher state E_3 of Neon, becomes the metastable state, than E_2. Therefore the population inversion is achieved Ne atoms. Hence the lasing action takes place by stimulated emission between E_3 and E_2 states of Neon. The laser light emitted is of about 632.8nm.



9. *Consider a circular plastic disk of radius R that has positive surface charge of uniform density on its upper surface. Find the electric field at any point at a distance z from the center of the disk along its central axis.*

Sol^n: Consider a disc of radius R has uniform surface charge density σ. To find the electric field at a point P along its central axis at z distance from the center, we consider the disc as a set of concentric rings. We calculate the electric field at P due to one such ring. Let the radius of ring be x and thickness of the ring be dx. By symmetry, the field at an axial point must be along the central axis.



The ring of radius x and width dx has surface area 2πxdx. So, the charge on the ring of width dx is dq = 2πσxdx.

Now, the electric field at P due to this ring is

$$dE = \frac{dqz}{4\pi\varepsilon_0(z^2+x^2)^{3/2}} = \frac{2\pi\sigma zxdx}{4\pi\varepsilon_0(z^2+x^2)^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\frac{xdx}{(z^2+x^2)^{3/2}}$$

Total field at P is given by $E = \int_0^R dE = \frac{\sigma z}{2\varepsilon_0}\int_0^R \frac{xdx}{(z^2+x^2)^{3/2}}$

Put x = z tanθ $\Rightarrow$ dx = z sec^{2}θ dθ, when x = 0, θ = 0°, and when x = R, $\theta = \tan^{-1}\left(\frac{R}{z}\right)$

$$E = \frac{\sigma z}{2\varepsilon_0}\int_0^\theta \frac{z\tan\theta.z.\sec^2\theta\, d\theta}{(z^2+z^2\tan^2\theta)^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\int_0^\theta \frac{z^2\tan\theta.\sec^2\theta\, d\theta}{z^3(1+\tan^2\theta)^{3/2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\int_0^\theta \frac{\tan\theta.\sec^2\theta\, d\theta}{(\sec\theta)^3} = \frac{\sigma}{2\varepsilon_0}\int_0^\theta \tan\theta\cos\theta d\theta$$

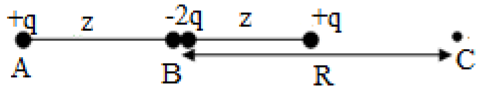
$$= \frac{\sigma}{2\varepsilon_0}\int_0^\theta \sin\theta d\theta = \frac{\sigma}{2\varepsilon_0}[-\cos\theta]_0^\theta = \frac{\sigma}{2\varepsilon_0}[1-\cos\theta]$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{(\sqrt{z^2+R^2})}\right)$$

OR

Define electric quadrupole. Calculate the electric potential of linear quadrupole of separation 2z at an axial distance R from its center.

Sol^n: The arrangement of four equal and opposite charges as shown in the figure forms the electric quadrupole.



Take a linear quadrupole of separation 2z and consider a point P at a distance R from the center of the quadrupole on the axial line. The electric potential at P due to charges at points A, B and C are

$V_A = \frac{q}{4\pi\varepsilon_0(R+z)}$, $V_B = \frac{-2q}{4\pi\varepsilon_0 R}$ and $V_C = \frac{q}{4\pi\varepsilon_0(R-z)}$ respectively.

So, net potential at point P is given by

$V = V_A + V_B + V_C$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R+z} - \frac{2}{R} + \frac{1}{R-z}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{R+z+R-z}{R^2-z^2} - \frac{2}{R}\right)$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{2R}{R^2-z^2} - \frac{2}{R}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{2R^2-2R^2+2z^2}{R(R^2-z^2)}\right)$$

$$V = = \frac{2qz^2}{4\pi\varepsilon_0 R(R^2-z^2)} = \frac{Q}{4\pi\varepsilon_0 R(R^2-z^2)}$$

Where $Q = 2qz^2$ is electric dipole moment.

10. *As a parallel plate capacitor with circular plates 20cm in diameter is being charged, the current density of the displacement current in the region between the plates is uniform and has a magnitude of 20A/m^2. Calculate the magnitude of magnetic field (B) at a distance r = 50mm from the axis of symmetry of this region. Also, calculate $\frac{dE}{dt}$ in this region.*

Sol^n: (a) Using Ampere's law in the region between the plates we have

$\oint \vec{B}.\vec{dl} = \mu_0 I_{enclosed}$ $\Rightarrow B \,.\, 2\pi r = \mu_0 I_{enclosed}$

$$\Rightarrow B = \frac{\mu_0 I_{enclosed}}{2\pi r} = \frac{\mu_0 J_d \pi r^2}{2\pi r} = \frac{1}{2}\mu_0 J_d r = \frac{1}{2} \times 4\pi \times 10^{-7} \times 20 \times 0.05$$

$\therefore B = 6.3\times 10^{-7}T = 0.63\mu T$

(b) The displacement current is $i_d = \varepsilon_0 \pi r^2 \frac{dE}{dt}$

$$\Rightarrow \frac{dE}{dt} = \frac{J_d}{\varepsilon_0} = \frac{20}{8.85\times10^{-12}} = 2.26 \times 10^{12}\ V\,m^{-1}s^{-1}$$

11. *Assuming that each atom of copper contributes one free electrons, calculate the drift velocity of free electrons in copper conductor of cross sectional area 10^{-4} m^2 carrying a current of 200A. Given: Atomic weight of copper = 63.5 g/mole, Density of copper = 8.94×10^3 kg/m^3, Charge of an electron = 1.6×10^{-19} C.*

Sol^n: We have the number of electron per unit volume of the conductor is given by $n = \frac{N_A \rho}{M} = \frac{6.02\times10^{23}\times8.94\times10^3}{63.5\times10^{-3}} = 8.5 \times 10^{28}\ m^{-3}$

Now, the drift velocity is given by $v_d = \frac{I}{neA} = \frac{200}{8.5\times10^{28}\times1.6\times10^{-19}\times10^{-4}}$

$V_d = 1.47\times 10^{-4}$ m/s

12. *State Ampere's law. Use this law to find magnetic field that a current produces inside and outside a long straight wire of circular cross section.*

Sol^n: Ampere's law states that the line integral of magnetic induction B around any closed loop in a vacuum is equal to μ_0 times total current enclosed by the loop $\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{enclosed}$.

(For second pert see in 2067 Ashadh Regular Q. No. 12)

OR

Derive an expression for energy stored in an inductor. Show that the magnetic energy density is proportional to the square of the magnetic flux density. How can you compare electric energy density with this result?

Sol^n: ***Energy Density in Magnetic Field***

We now derive an expression for the energy density u_b in a magnetic field. Consider a very long solenoid of cross-sectional area A whose interior contains no material. A portion of length *l* far from either end encloses a volume A*l*. The magnetic energy stored in this portion of the solenoid must lie entirely within this volume because the magnetic field outside the solenoid is zero. Thus we can write the energy density as

$$u_b = \frac{U_B}{Al} \qquad \text{Since } U_B = \frac{1}{2}Li^2 \qquad \therefore u_b = \frac{Li^2}{2Al}$$

Also, B = μ_0 i n and L = μ_0 n^2lA, where n is the number of turns per unit length. So, $u_b = \frac{1}{2}\frac{\mu_0 n^2 l A i^2}{Al} = = \frac{1}{2}\frac{(\mu_0 n i)^2}{\mu_0}$

$$\therefore u_b = \frac{B^2}{2\mu_0} \qquad \text{---------} \ (1)$$

Energy Density in Electric Field

We know the energy stored in capacitor is $U_E = \frac{1}{2}CV^2$, where C is the capacitance of the capacitor and V is the potential across the capacitor. It is assumed that in parallel plate capacitor the electric field has the same value for all points between the plates. The flow energy density u_e, which is the stored energy per unit volume and is given by

$$u_e = \frac{CV^2}{2Ad} \quad \text{Also, } C = \varepsilon_0 \frac{A}{d} \quad \text{So, } u_e = \varepsilon_0 \frac{AV^2}{2Ad^2} = \frac{\varepsilon_0}{2}\left(\frac{V}{d}\right)^2$$

$$\therefore u_e = \frac{\varepsilon_0}{2}E^2 \qquad \text{--------}(2)$$

Eq. (1) and Eq. (2) shows that the energy per unit volume in electric field and magnetic field are proportional to the square of their fields.

13. *A cyclotron which has the dees of radius 42cm and magnetic field of flux density 0.5 Weber/m^2 is employed to accelerate protons. If the final velocity of the proton is 2.02×10^7m/s, calculate the charge to mass ratio for the proton and the frequency of the alternating potential between the dees.*

Sol^n: Radius of Dees (R) = 42 cm = 0.42 m

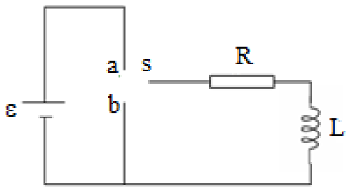
Magnetic field (B) = 0.5 w/m^2, velocity of the proton = 2.02×10^7m/s

Charge to mass ratio $\left(\frac{e}{m}\right)$ =?, frequency of alternating potential (f) =?

We have, $\frac{e}{m} = \frac{v}{RB} = \frac{2.02\times10^7}{0.42\times0.5} = 9.62 \times 10^7$ C/kg

And, $f = \frac{v}{2\pi R} = \frac{2.02\times10^7}{2\pi\times0.42} = 7.65\times10^6$ Hz = 7.65 MHz

14. *In the given figure, when switch S is closed on a, the current rises and approaches a limiting value $\frac{\varepsilon}{R}$. (a) find the current through the inductor as a function of time. (b) When the switch is closed on b, the current reduces to zero. Find the rate of decay of current through the inductor.*



Sol^n: Let a resistor R and inductor L are connected in series with a battery of emf ε and switch (a, b) as in figure.

When the switch is on at a, the current in the resistor starts to rise. If the inductor is not present, the current would quickly rise to a steady value $\frac{\varepsilon}{R}$. The inductance of the inductor results in a back emf, and inductor in the circuit opposes change in the current in the circuit. When switch is at b, current flows in anti-clockwise direction. For the switch is at a,

$\varepsilon = V_L + V_R \quad \Rightarrow \quad \varepsilon = L\frac{di}{dt} + iR$

$L\frac{di}{dt} + R\left(i + \frac{\varepsilon}{R}\right) = 0 \quad \Rightarrow \quad \frac{di}{dt} = -\frac{R}{L}(i - i_0);$

$\frac{\varepsilon}{R} = i_0$ = maximum current, again $\frac{L}{R} = \tau_L$ = inductive time constant.

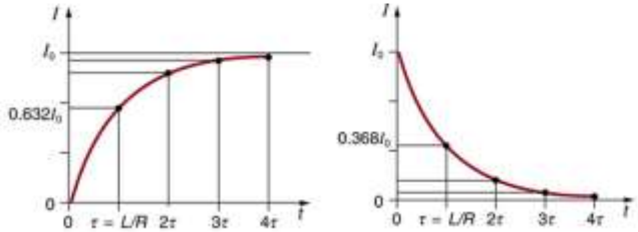
$\frac{di}{(i-i_0)} = -\frac{dt}{\tau_L}$ Integrating both sides

$\int_0^i \frac{di}{(i-i_0)} = -\frac{1}{\tau_L}\int_0^t dt \qquad \therefore\ i = i_0\left(1 - e^{-t/\tau_L}\right)$

This gives the growth of current in LR circuit.

If we put $t = \tau_L$ then $i = 0.63\, i_0$

Thus the time constant is in the circuit to reach about 63% of its final equilibrium value i_0. From figure, it $t \to \infty$, $e^{-t/\tau_L} = 0$.



Thus initially, an inductor acts to oppose changes in the current through it. After long time, it acts like ordinary connecting wire.

Now, if the switch is thrown from a to b as the current reaches i_0 then decaying of current starts due to absence of source in the circuit.

$L\frac{di}{dt} + iR = 0 \qquad \Rightarrow i = i_0\, e^{-t/\tau_L}$

This gives decay of current. The decrease in current is slow, if τ_L is large.

If $t = \tau_L$ then $i = 0.37\, i_0$.

15. *State Maxwell equations in integral form. Convert them into differential form. Explain each of these equations.*

Sol^n: Maxwell's equations are

1. $\oint_S \vec{E}.\overrightarrow{dS} = q/\varepsilon_0$: This is Gauss law in electrostatics, which states that the outward flux of $\vec{E}$ throughout any surface is $1/\varepsilon_0$ times the net charge (q) enclosed by the Gaussian the surface.

We know, flux through the small area dS is $d\Phi = \vec{E}\cdot d\vec{S}$

$\therefore \Phi = \oint_S \vec{E}.d\vec{S} = q/\varepsilon_0$

Now to convert in differential form, let us change the surface integral to volume integral as $\quad \oint_S \vec{E}.\overrightarrow{dS} = \frac{q}{\varepsilon_0}$

$\Rightarrow \varepsilon_0 \oint_S \vec{E}.d\vec{S} = q \qquad [\, dq = \rho dV \qquad \Rightarrow q = \oint_V \rho dV]$

$\Rightarrow \varepsilon_0 \oint_V (\nabla.\vec{E})\, dV = \oint_V \rho dV$ [$\because$ Using Gauss divergence theorem]

$\Rightarrow \oint_V (\nabla.\vec{E} - \frac{\rho}{\varepsilon_0}) dV = 0$ $\quad\quad\quad$ $\therefore \nabla.\vec{E} = \rho/\varepsilon_0$

2. $\oint_S \vec{B}.\vec{dS} = 0$: This is Gauss law in magnetism. Let us consider a surface of surface area S and volume V in a magnetic field B. The flux through the surface = $\oint_S \vec{B}.d\vec{S} = \oint_V (\nabla.\vec{B})dV$

As there is no real magnetic poles, total magnetic flux through a closed surface = 0.

$\therefore \oint_V (\nabla.\vec{B})dV = 0$ $\quad\quad$ $\Rightarrow \nabla.\vec{B} = 0$

3.$\oint \vec{E}.d\vec{l} = -\frac{\partial\vec{B}}{\partial t}$: This is the Faraday's law of electromagnetic induction. Let us consider a circuit of resistance R carrying current i. The magnetic flux linking to this circuit is denoted by $\Phi_B = \oint_S \vec{B}.d\vec{S}$ and induced emf is $e = -\frac{d\Phi_B}{dt}$ The induced emf (e) around the closed path is given by the line integral of the induced electric field E along the wire.

$\therefore \oint \vec{E}.d\vec{l} = e = -\frac{\partial\Phi_B}{\partial t}$ $\quad\quad\quad$ $\Rightarrow \oint \vec{E}.d\vec{l} = -\frac{\partial}{\partial t}\oint_S \vec{B}.d\vec{S}$

By Stoke's theorem, $\oint_S (\nabla \times \vec{E}).d\vec{S} = -\frac{\partial}{\partial t}\oint_S \vec{B}.d\vec{S}$

$\therefore \nabla \times \vec{E} = -\frac{\partial\vec{B}}{\partial t}$

4. $\oint \vec{B}.d\vec{l} = \mu_0(i + i_d) = \mu_0\left(i + \varepsilon_0 \frac{\partial\Phi_B}{\partial t}\right)$: This equation is extended form of Ampere's law by Maxwell. Maxwell extends Ampere's law by placing a coil of wire in a changing electric field. He found that if a coil of wire is placed in the changing electric field then $i_d \propto \frac{\partial\Phi_E}{\partial t}$

$\Rightarrow i_d = \varepsilon_0 \frac{\partial\Phi_E}{\partial t}$. So, the Ampere's law extended by Maxwell is

$$\oint \vec{B}.d\vec{l} = \mu_0(i + i_d) = \mu_0\left(i + \varepsilon_0 \frac{\partial\Phi_B}{\partial t}\right)$$

The surface integral of current density gives the current through the given circuit and the surface integral of electric field gives the electric flux, i.e; i = $\oint_S \vec{J}.d\vec{S}$ and $\Phi_E = \oint_S \vec{E}.d\vec{S}$

Using Stoke's theorem, $\oint_S (\nabla \times \vec{B}).d\vec{S} = \mu_0 \left(\oint_S \vec{J}.d\vec{S} + \varepsilon_0 \frac{\partial}{\partial t}\oint_S \vec{E}.d\vec{S}\right)$

$\therefore \nabla \times \vec{B} = \mu_0 \left[\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right]$

16. *Discuss the significance of the wave function and deduce the time independent Schrodinger equation.*

Sol^n: In classical mechanics, the wave is represented by

y = R $e^{-\frac{i}{\hbar}(Et-px)}$

This function in general is a complex quantity and is dependent upon space and time is given by

$\psi(x,t) = Re^{-\frac{i}{\hbar}(Et-px)}$ --------(1)

This function has no physical significance itself. The quantity having physical meaning is the square of its magnitude. i.e., P = $\psi\psi^* = |\psi|^2$.

This is called probability density, where ψ^* is complex conjugate of ψ. The probability of finding a particle in the given volume element dV can be expressed as $|\psi|^2$ dV and the probability of finding particle in entire space is $\int_{-\infty}^{+\infty} |\psi|^2 dV = 1$.

Differentiating Eq. (1) with respect to x, twice times, we get

$\frac{d\psi}{dx} = R\left(\frac{i}{\hbar}\right) pe^{-\frac{i}{\hbar}(Et-px)}$ and $\frac{d^2\psi}{dx^2} = R\left(\frac{i}{\hbar}\right)^2 p^2 e^{-\frac{i}{\hbar}(Et-px)} = -\frac{p^2}{\hbar^2}\psi$

$P^2\psi = -\hbar^2 \frac{d^2\psi}{dx^2}$ --------- (2)

Total energy associated with the particle of mass "m" and moving with velocity v is E = K. E. + P. E.= $\frac{P^2}{2m} + V$

$P^2 = 2m(E - V)$

Multiplying this equation both sides by ψ, we get

$P^2\psi = 2m(E - V)\psi$

$\Rightarrow -\hbar^2 \frac{d^2\psi}{dx^2} = 2m(E - V)\psi$

This is time independent Schrodinger equation in one dimension.

In three dimension,

$\therefore -\frac{\hbar^2}{2m}\nabla^2\psi = 2m(E - V)\psi$

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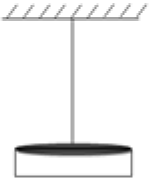
2067 Mangsir Regular (BCE, BME)

1. *What is torsional pendulum? Obtain an expression for its time period and explain why, unlike a simple or a compound pendulum the time period in this case remains unaffected even if the amplitude be large?*

Sol^n: If a rigid body of any shape suspended from a rigid support with the help of metallic wire constitutes torsion pendulum. If a circular disc is suspended by a rigid support with the help of metallic wire of length *l* as shown in figure and the disc is displaced by an angle θ and released then the disc oscillates in turning motion due to restoring couple



Torsion pendulum

$\tau = - c\,\theta$ ----------- (1)

where c is torsional constant of the wire.

If I be the moment of inertia of the disc about ax axis passing through wire as an axis then according to Newton's second law

$\tau = I\frac{d^2\theta}{dt^2}$ ------------- (2)

From Eq. (1) and Eq. (2), we have

$I\frac{d^2\theta}{dt^2} = - c\theta \quad \Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{c}{I}\theta \quad \Rightarrow \frac{d^2\theta}{dt^2} \propto -\theta$

This shows motion of torsion pendulum is angular harmonic.

Also, $\omega^2 = \frac{C}{I} \quad \Rightarrow \omega = \sqrt{\frac{C}{I}}$

$\therefore\ T = 2\pi\sqrt{\frac{I}{C}}$

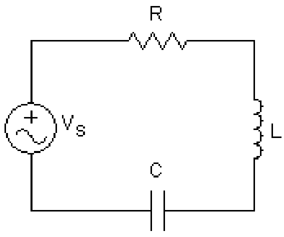
This gives time period of torsion pendulum. Notice that in the derivation of time period of torsion pendulum, no approximation is required as in simple pendulum or a compound pendulum. Hence, the time of torsional oscillation remains the same for large amplitude oscillation provided that the elastic limit of the suspension wire has not been exceeded.

OR

Derive the differential equation of the forced oscillation of LCR circuit with ac source and find the expression for the current amplitude.

Sol^n: Let an inductor (L), capacitor (C) and resistor (R) are connected in series with an ac source of emf $E = E_0 \sin\omega t$ as shown in figure. Consider i be the current through the circuit and q be the charge in the capacitor then potential drop across capacitor, resistor and inductor are

$V_C = \frac{q}{C}$, $V_R = iR$ and $V_L = L\frac{di}{dt}$ respectively.



According to Kirchhoff's voltage equation,

$V_C + V_R + V_L = E$

$\Rightarrow L\frac{di}{dt} + iR + \frac{q}{C} = E_0 \sin\omega t$

$$\Rightarrow \frac{d^2i}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = \frac{E_0}{L}\sin\omega t \qquad \text{----------- (1)}$$

This is the differential equation of forced LCR circuit. Now the solution of Eq. (1) is

$q = A\sin\omega t + B\cos\omega t$, where A and B are constants whose values are

$A = -\frac{E_0}{\omega Z^2}\left(L\omega - \frac{1}{C\omega}\right)$ and $B = -\frac{E_0}{\omega Z^2}R$.

$$\therefore q = -\frac{E_0}{\omega Z^2}\left(L\omega - \frac{1}{C\omega}\right)\sin\omega t - \frac{E_0}{\omega Z^2}R\cos\omega t$$

Now current through the circuit is

$$i = \frac{dq}{dt} = \frac{E_0}{Z^2}R\sin\omega t - \frac{E_0}{Z^2}\left(L\omega - \frac{1}{C\omega}\right)\cos\omega t$$

$$= \frac{E_0}{Z}\left[\frac{R}{Z}\sin\omega t - \frac{\left(L\omega - \frac{1}{C\omega}\right)}{Z}\cos\omega t\right]$$

If $\frac{R}{Z} = \cos\phi$, then $\frac{\left(L\omega - \frac{1}{C\omega}\right)}{Z} = \sin\phi$

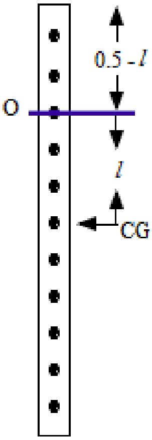
$i = i_0 \sin(\omega t - \phi)$

where $i_0 = \dfrac{E_0}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}$ is the current amplitude.

2. *A meter stick suspended from one end swings as a physical pendulum (a) what is the period of oscillation (b) what would be the length of the simple pendulum that would have the same period.*

Sol^n: Consider a meter scale of mass 'm' is suspended at point O. The center of gravity of the body is CG at a distance *l* from point O. If the meter scale is displaced by small angle θ (less than 4°) and released the restoring moment of force is

$\tau = - mgl\theta$ --------------- (1)



From Newton's second law of motion,

$\tau = I\,\alpha$ --------------- (2)

where I is the moment of inertia of the scale about the axis passing through point O and α is angular acceleration. From Eq. (1) and Eq. (2)

$- mgl = I\dfrac{d^2\theta}{dt^2} \quad \Rightarrow \dfrac{d^2\theta}{dt^2} = -\dfrac{mgl}{I}\theta \quad \Rightarrow \dfrac{d^2\theta}{dt^2} \propto -\theta$

This shows the motion of the rigid body is simple harmonic. So,

$\omega^2 = \dfrac{mgl}{I} \quad \Rightarrow \omega = \sqrt{\dfrac{mgl}{I}} \quad \Rightarrow T = 2\pi\sqrt{\dfrac{I}{mgl}}$

Using parallel axes theorem, $I = I_{CG} + m\,l^2 = mk^2 + m\,l^2$.

$\therefore\ T = 2\pi\sqrt{\dfrac{l^2+k^2}{lg}}$ --------------- (3)

Also, $I_{CG} = mk^2 = \dfrac{mL^2}{12}$, here L = 1 m

$\therefore\ k = \dfrac{1}{\sqrt{12}}$

So, T = 2π $\sqrt{\frac{12l^2+1}{12lg}}$

This is the required expression.

Comparing this expression with the time period of simple pendulum, as

T = 2π $\sqrt{\frac{L}{g}}$, we get

L = $l + \frac{1}{12l}$, this length is known as equivalent length of the simple pendulum.

For the bar suspended from the end point l = 0.50m = 50cm

∴ T = 2π $\sqrt{\frac{50^2+(28.86)^2}{50\times981}}$ = 1.64 sec

Equivalent length of simple pendulum is L = 50 + $\frac{(28.86)^2}{50}$ = 66.67 cm

3. *Calculate the minimum intensity of audiability in watts per square cm from a note of 1000 Hz if the amplitude of vibration is 10^{-9} cm. Given density of air is 0.0013 gm/cc and velocity of sound in air is 340 m/s.*

Sol^n: Frequency of wave (f) = 1000 Hz, amplitude (R) = 10^{-9} m

Density of air (ρ) = 0.0013 gm/cc, velocity of sound (v) = 34000cm/s

Intensity of sound (I) =?

We have, $I = 2\pi^2\rho v f^2 R^2$

$\Rightarrow I = 2\pi^2\times0.0013\times34000\times10^6\times10^{-18} = 8.73\times10^{-10}$ W/cm^2

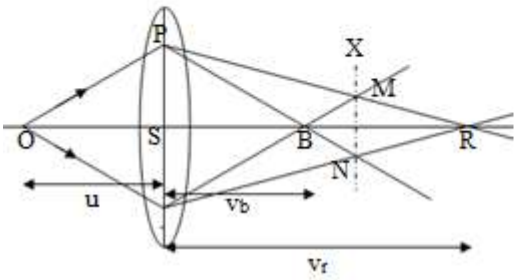
4. *What is diffraction of light? Discuss the intensity distribution with special reference to diffraction of light in a single slit.*

Sol^n: The bending of light from the edge of the obstacle and spreading around the geometrical shadow is known as diffraction of light. Also, bending of light when passes thorough the small opening is diffraction.

(For Remaining Part see in 2067 Ashadh Regular Q. No. 4 OR)

OR

Define circle of least confusion and show that d = $\frac{1}{2}\omega D$, where d = diameter of circle of least confusion, ω = dispersive power and D = diameter of the lens aperture.



Sol^n: Consider white light illuminated point object O on the principal axis. Rays from this point object enters a lens of focal length f. After refraction through the lens, blue and red image will form at B and R respectively. If the screen XY is placed as shown, the image of least chromatic aberration is observed in the screen. On the screen there form a circular form of image. That is known as circle of least confusion. Let the object distance be u, the blue and red image distances be v_b and v_r respectively. So,

$$\frac{1}{v_b}-\frac{1}{u}=\frac{1}{f_b} \quad \text{and} \quad \frac{1}{v_r}-\frac{1}{u}=\frac{1}{f_r}$$

$$\frac{1}{v_b}-\frac{1}{v_r}=\frac{1}{f_b}-\frac{1}{f_r} \qquad \Rightarrow \frac{v_r-v_b}{v_r v_b}=\frac{f_r-f_b}{f_r f_b}$$

But, $v_b\, v_r \approx v^2$ and $f_b\, f_r \approx f^2$

$$\frac{v_r-v_b}{v^2}=\frac{f_r-f_b}{f^2} \qquad \Rightarrow v_r - v_b = \frac{\omega v^2}{f} \qquad \text{---------- (1)}$$

From geometry, $\Delta PQR \cong \Delta MNR$

$$\frac{SR}{ZR}=\frac{PQ}{MN} \qquad \Rightarrow \frac{SR}{PQ}=\frac{ZR}{MN} \qquad \text{-------- (2)}$$

Also, $\Delta PBQ \cong \Delta MBN$

$$\frac{SB}{BZ}=\frac{PQ}{MN} \qquad \Rightarrow \frac{SB}{PQ}=\frac{ZB}{MN} \qquad \text{--------(3)}$$

Adding Eq. (2) and Eq. (3), we have

$$\frac{SR+SB}{PQ} = \frac{ZR+ZB}{MN} \qquad \Rightarrow \frac{v_r+v_b}{D} = \frac{v_r-v_b}{d}$$

Where MN = d is diameter of circle of least confusion, PQ = D is the diameter of the lens. Also, $v_r + v_b = 2v$

So, $2vd = (v_r - v_b)\, D$

$$\Rightarrow d = \frac{v_r-v_b}{2v} D = \frac{\omega v^2}{2vf} D \qquad \therefore\ d = \frac{1}{2} D\omega$$

Hence, the diameter of a circle of least confusion is independent of the focal length of a lens.

5. *A Plano-convex lens of radius 300cm is placed on an optically flat glass plate and is illuminated by monochromatic light. The diameter of the 8th dark in the transmitted system is 0.72cm. Calculate the wavelength of the light used.*

Solⁿ: Radius of curvature of the plano-convex lens (R) = 300 cm

Diameter of the 8^{th} dark ring (D_8) = 0.72cm

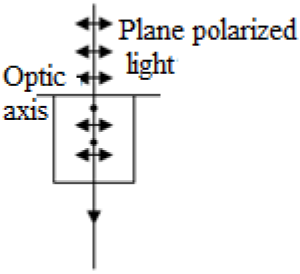
Wavelength of the light (λ) =?

The diameter of the dark ring for transmitted light (D_n) = $\sqrt{2\lambda R(2n-1)}$

$$\Rightarrow D_8 = \sqrt{30\lambda R} \qquad \Rightarrow D_8^2 = 30\lambda R \qquad \Rightarrow \lambda = \frac{D_8^2}{30R}$$

$$\therefore\ \lambda = \frac{(0.72)^2}{30\times 300} = 5760 \times 10^{-8}\,\text{cm} = 5760\ \text{Å}$$

6. *What is double refraction? Using the concept of double refraction show that the plane polarized and circularly polarized light are the special cases of elliptically polarized light*



Solⁿ: In certain crystal, if a light is passed then it splits into two components which are known as double refraction.

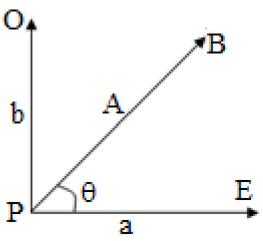
Consider monochromatic light incidents on a Nicol prism. The emergent light is plane polarized. Let this emergent light falls normally on a doubly refracting crystal whose faces are cut parallel to the optic axis. The plane polarized light is splitted into O and E components. Both of these O and E ray travel in the same direction however, they have different velocities. Let δ be the phase difference between the two rays after the crystal be represented by two simple harmonic motions perpendicular to each other as shown in figure. Let the amplitude of the incident plane polarized light be A. It makes an angle θ with the optical axis. The amplitude of the O ray which is vibrating along a direction PO is given by Asinθ while that of E ray along PE by Acosθ. Also, these rays are represented by

$y = A\sin\theta \sin\omega t$ ----- for O ray

$x = A\cos\theta \sin(\omega t + \delta)$ ----- for E ray

Let $A\cos\theta = a$, $A\sin\theta = b$ then $x = a \sin(\omega t + \delta)$ and $y = b \sin\omega t$

$\Rightarrow \sin\omega t = \frac{y}{b}$ and $\cos\omega t = \sqrt{1 - \left(\frac{y}{b}\right)^2}$



$x = a [\sin\omega t \cos\delta + \cos\omega t \sin\delta]$

$\Rightarrow \frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1 - \left(\frac{y}{b}\right)^2}\sin\delta$

$\Rightarrow \left(\frac{x}{a} - \frac{y}{b}\cos\delta\right)^2 = \left[1 - \left(\frac{y}{b}\right)^2\right]\sin^2\delta$

$\Rightarrow \frac{x^2}{a^2} - \frac{2xy}{ab}\cos\delta + \frac{y^2}{b^2} = \sin^2\delta$

Which is the general equation of ellipse.

Case (i): If $\delta = 0°$ then $\frac{x^2}{a^2} - \frac{2xy}{ab} + \frac{y^2}{b^2} = 0$

$\Rightarrow \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$ $\Rightarrow y = \frac{b}{a}x$. This is the equation of straight line.

So, the light is linearly polarized.

Case (ii): If $\delta = 90°$, and a = b then $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \quad \Rightarrow x^2 + y^2 = a^2$. This is the equation of circle. So, the light is circularly polarized.

7. *Two similar thin convex lenses of focal length 10cm each are placed co-axially 5cm apart. Find the equivalent focal length and the position of principal points. Also, find the position of the object for which the image is formed at infinity.*

Sol^n: Focal length of both lenses (f_1) = (f_2) = 10 cm, separation between lenses (d) = 5cm

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{10 \times 10}{10 + 10 - 5} = 6.67\text{cm}$$

first principal point (α) = $\frac{fd}{f_2} = \frac{20 \times 5}{3 \times 10} = 3.33$ cm

second principal point (β) = $-\frac{fd}{f_1} = -\frac{20 \times 5}{3 \times 10} = -3.33$ cm

distance of the image (V) = ∞

From lens formula, $\frac{1}{f} = \frac{1}{V} - \frac{1}{U} \qquad \Rightarrow f = -U$

Object distance from first principal point (U) = 6.67 cm left

So, the object distance from the first lens is 3.34 cm towards left.

8. *What is an optical fiber? Discuss its types. Derive the relation for Numerical Aperture (NA) in an optical fiber.*

Sol^n: Optical fibers are made up of glass or plastic which are as thin as human's hair of diameter of about 150μm which is designed to guide light waves along the length of the fiber with the help of total internal reflection. There are two types of fibers.

Monomode fiber: The monomode fiber has a very narrow core of diameter 5μm or less than this. So the cladding is relatively thick than the core.

Multimode fiber: The diameter of the multimode fiber is relatively thick is about diameter of 50μm. These fibers are further divided into two types.

Step index optical fiber: The multimode fiber in which the refractive index μ_1 of core is constant and the refractive index of cladding is also constant value is called step index optical fiber. Thus at boundary of core-cladding interfere, the refractive index jumps from μ_1 to μ_2.

Graded index optical fiber: The multimode optical fiber in which the refractive index decreases continuously from core to the outer surface of the fiber and there is no noticeable boundary between the core and cladding is called the graded index optical fiber.

(For second part see in 2067 Ashadh Regular Q. No. 7)

9. *Find the electric field at a distance x above the center of the flat circular disc of radius R which carries a uniform surface charge density σ. Extend your result in the limit $R \to \infty$.*

Sol^n: See in 2067 Ashwin Back Q. No. 9 up to

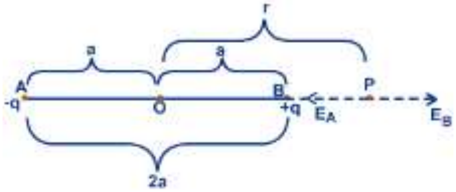
$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{(\sqrt{x^2+R^2})}\right)$$

If $R \to \infty$, then $E = \frac{\sigma}{2\varepsilon_0}$.

OR

Show that the electric field due to a short dipole at a point on the axial line is twice as that of a point on the equatorial line.

Sol^n: ***Field along axial line of the dipole:*** Consider a dipole AB separated by distance 2a, electric field at P at distance r from center of dipole due to +q and – q charge is to be



measured.

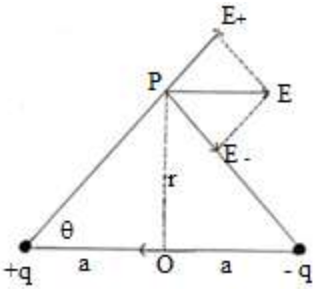
Here, the resultant field is $E = E_A + E_B$

$E = \frac{q}{4\pi\varepsilon_0(r-a)^2} - \frac{q}{4\pi\varepsilon_0(r+a)^2} = \frac{q}{4\pi\varepsilon_0}\frac{4ra}{(r^2-a^2)^2}$; $P = 2aq$ is dipole moment.

$E = \frac{2Pr}{4\pi\varepsilon_0(r^2-a^2)^2}$ For short dipole, r >> a

$$\therefore \vec{E}_{axial} = \frac{\vec{P}}{2\pi\varepsilon_0 r^3} \quad \text{----------(1)}$$

Field along equatorial line:



+ q and – q charges set up electric field E_+ and E_- respectively. So, $|\vec{E}_+| = |\vec{E}_-|$

Total electric field at P is vector sum and is expressed as

$|\vec{E}| = |\vec{E}_+| \cos\theta + |\vec{E}_-| \cos\theta$

$$\Rightarrow E = \frac{q\cos\theta}{4\pi\varepsilon_0(r^2+a^2)} + \frac{q\cos\theta}{4\pi\varepsilon_0(r^2+a^2)} = \frac{2qa}{4\pi\varepsilon_0(r^2+a^2)^{3/2}} = \frac{P}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$$

For short dipole, r >> a, $\vec{E}_{eq} = \frac{\vec{P}}{4\pi\varepsilon_0 r^3}$ ---------- (2)

From Eq. (1) and Eq. (2), we get

$\therefore \vec{E}_{axial} = 2\vec{E}_{eq}$

10. *Differentiate between polar and non-polar dielectrics. Using Gauss's laws in dielectrics establish relation of electric field with displacement vector and polarization vector. Hence obtain the relation for free and induced charge in the dielectric.*

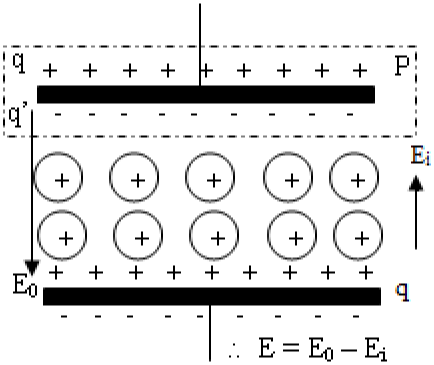
Sol^n: Polar means having electrical poles (i.e. electrical polarity). The molecules in which the arrangement or geometry of the atoms is such that one end of the molecule has a positive electrical charge and the other side has a negative charge are called as polar molecules. Examples of polar molecules are Water (H_2O), Ammonia (NH_3), Hydrochloric acid (HCl), Sulfur Dioxide (SO_2), Hydrogen Sulfide (H_2S), Carbon Monoxide (CO) etc.

A non-polar molecule is that in which the electrons are distributed more symmetrically and thus does not have an excess /abundance of charges at the opposite sides. The charges all cancel out each other. e.g. $CO_2, H_2, N_2, O_2, CH_4, CCl_4$ etc.

According to Gauss law in electrostatics, the total flux is given by

$\phi = \oint_S \overrightarrow{E_0}.\overrightarrow{da} = \frac{q}{\varepsilon_0}$ $\Rightarrow E_0 = \frac{q}{A\varepsilon_0}$

Here E_0 is the electric field between the plates of capacitor without dielectrics. Now consider the case of dielectric slab inserted in the space between the plates, we assume that the net free charge q on the plates is same in both situation, but electric field is decreased from E_0 to E as shown in figure. Inside the dielectric dipole is produced and on the surface of it induced charges q' is produced. In the figure the dotted rectangular represents the Gaussian surface within



which, the Gauss law is $\oint_S \overrightarrow{E_0}.\overrightarrow{da} = E\,A = \frac{q-q'}{\varepsilon_0}$

So, $E = \frac{q-q'}{\varepsilon_0 A}$ --------- (1)

The effect of the dielectric is to weaken the original field E_0 by a factor of dielectric constant k, so we have $E = \frac{E_0}{k} = \frac{q}{kA\varepsilon_0}$

From above two equations, $q \neq -q'$ and $q' = q\left(1 - \frac{1}{k}\right)$

This shows that q' is always less than q and is equal to zero if no dielectric is present i.e. k = 1.

Hence Gauss law in dielectric is given by $\oint_S k\,\overrightarrow{E_0}\,.\,\overrightarrow{da} = \frac{q}{\varepsilon_0}$

Again, from Eq. (1), $\varepsilon_0 E = \frac{q}{A} - \frac{q'}{A}$

Where q/A is called electric displacement vector $\vec{D}$ which is the product of dielectric constant and electric field i.e. $\vec{D} = k\,\vec{E}$.

It is equivalent to free charge density. The term q'/A is called polarization (P). It represents the capacity of dipole formation due to applied field and is equivalent to induced surface charge density. Hence,

$\varepsilon_0\vec{E} = \vec{D} - \vec{P}$ i.e. $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

This is the required relation.

11. *What is the average time between collisions of free electrons in a copper wire? (Atomic weight = 63 gm/mole, density = 9 gm/cc and resistivity = 1.7 × 10^{-8} Ωm, N_A = 6.02 × 10^{23}ml^{-1}).*

Sol^n: The number of electrons per unit volume of the copper is given by

$n = \frac{N_A\rho}{M} = \frac{6.02\times10^{23}\times9\times10^{3}}{63\times10^{-3}} = 8.6\times 10^{28}\ m^{-3}$

The average time between collisions of free electrons (τ) = $\frac{m\sigma}{ne^2}$

$\Rightarrow \tau = \frac{9.1\times10^{-31}}{8.6\times10^{28}\times(1.6\times10^{-19})^2\times1.7\times10^{-8}} = 2.4\times10^{-14}$ sec

12. *A spherical drop of water carrying a charge of 3 μC has a potential of 500V at its surface. What is the radius of drop? If two such drops of same charge and radius are combined, what is potential of the single new drop formed?*

Sol^n: Charge on the drop (q) = 3 × 10^{-6} C, potential (V) = 500V

Potential is $V = \frac{q}{4\pi\varepsilon_0 r}$ $\Rightarrow r = \frac{q}{4\pi\varepsilon_0 V} = \frac{3\times10^{-6}}{4\pi\times8.85\times10^{-12}\times500} = 53.95$ m

If two such drops are combined then the radius of the new drop will be given by 2(volume of single drop) = volume of new drop

$R = 2^{1/3}\, r$ and total charge will be q’ = 2q

So, potential of the single drop is $V = \frac{2q}{4\pi\varepsilon_0 r}\, 2^{-1/3} = \frac{q}{4\pi\varepsilon_0 r}\, 2^{2/3}$

$\Rightarrow V = 500\times 2^{2/3}$ $\therefore V = 793.7$ V

13. *A variable field of 10^{12}V/m s is applied to a parallel plate capacitor with circular plates of diameter 10cm. Calculate (a) induced magnetic field and (b) displacement current.*

Sol^n: Plate diameter (D) = 10cm, radius of plate (r) = 5 cm

$\frac{dE}{dt} = 10^{12}$ V/ms

(a) The induced magnetic field is $(B) = \frac{1}{2}\mu_0\, r\, J_d$

(b) Displacement current $(i_d) = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0\, \pi r^2 \frac{dE}{dt}$

$\Rightarrow i_d = \pi \times 8.85\times 10^{-12} \times (0.05)^2 \times 10^{12} = 0.07$ Amp

Now, $J_d = \frac{i_d}{\pi r^2} = \frac{0.07}{\pi\times 0.05^2} = 8.91$ A/m^2

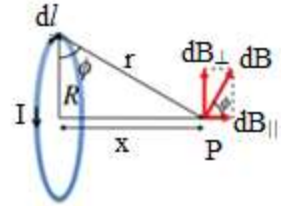
So, $B = 0.5 \times 4\pi \times 10^{-7} \times 8.91 = 5.6 \times 10^{-6}$T $= 5.6\ \mu$T

14. *A circular coil having radius R carries a current i. Calculate the magnetic flux density at an axial distance x from the center of the coil. Explain how the coil behaves for a large distance point and at what condition field will be maximum?*

Sol^n: Figure shows the circular conductor with radius R that carries a current i. To find the magnetic field at P on the axis of the loop, at a distance x from the center, we can use Biot-Savart’s law. Consider an element d*l* of the coil at right angles to the plane of paper. This set up of field dB at P, in the plane of the paper and right angles to the radius vector $\vec{r}$.

Let us resolve dB into two components: $dB_{||}$ along the axis of the loop and $dB_{\perp}$ at right angles to the axis. Only $dB_{||}$ contributes to total induction B at P.

$\therefore B = \int dB_{||} = \int dB \sin\theta$



Using Biot – Savart's law

$dB = \frac{\mu_0 I dl \sin(\pi/2)}{4\pi r^2} = \frac{\mu_0 I dl}{4\pi(x^2+R^2)}$

So, $B = \int \frac{\mu_0 I dl}{4\pi(x^2+R^2)} \sin\theta$ Also, $\sin\theta = \frac{R}{\sqrt{x^2+R^2}}$

$$B = \int \frac{\mu_0 I dl R}{4\pi(x^2+R^2)^{3/2}} = \frac{\mu_0 I R}{4\pi(x^2+R^2)^{3/2}} \int_0^{2\pi R} dl$$

$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

For N number of turns $B = \frac{\mu_0 NI R^2}{2(x^2+R^2)^{3/2}}$

The dipole moment is $\mu = \pi I R^2$

So, $B = \frac{\mu_0 N \pi I R^2}{2\pi(x^2+R^2)^{3/2}} = \frac{\mu_0 \mu N}{2\pi(x^2+R^2)^{3/2}}$

If x >> R then $B = \frac{\mu_0 \mu N}{2\pi x^3}$

This shows that circular current carrying coil behaves as magnetic dipole for a large distance. The field is maximum at x = 0 (at center of the coil) and is given by $B_{max} = \frac{\mu_0 NI R^2}{2(R)^3} = \frac{\mu_0 NI}{2R}$

OR

Show that the energy per unit volume in electric field and magnetic field are proportional to the square of their fields.

Sol^n: (See in 2067 Ashwin Back Q. No. 12 OR)

15. *Derive the Schrodinger time dependent wave equation. What is the physical significance of wave functions?*

Sol^n: In classical mechanics, the wave is represented by

$y = R\, e^{-\frac{i}{\hbar}(Et-px)}$ This function in general is a complex quantity and is dependent upon space and time is given by $\psi(x,t) = Re^{-\frac{i}{\hbar}(Et-px)}$ ----(1)

This function has no physical significance itself. The quantity having physical meaning is the square of its magnitude. i.e., $P = \psi\psi^* = |\psi|^2$.

This is called probability density, where ψ^* is complex conjugate of ψ. The probability of finding a particle in the given volume element dV can be expressed as $|\psi|^2$ dV and the probability of finding particle in entire space is $\int_{-\infty}^{+\infty} |\psi|^2 dV = 1$.

Differentiating Eq. (1) with respect to t, we have

$$\frac{d\psi}{dt} = R\left(-\frac{i}{\hbar}\right)Ee^{-\frac{i}{\hbar}(Et-px)} = \left(-\frac{i}{\hbar}\right)E\psi \quad \Rightarrow i\hbar\frac{d\psi}{dt} = E\psi \quad \text{-------(2)}$$

Differentiating Eq. (1) with respect to x twice times, we get

$$\frac{d\psi}{dx} = R\left(\frac{i}{\hbar}\right)pe^{-\frac{i}{\hbar}(Et-px)} \text{ and } \frac{d^2\psi}{dx^2} = R\left(\frac{i}{\hbar}\right)^2 p^2e^{-\frac{i}{\hbar}(Et-px)} = -\frac{p^2}{\hbar^2}\psi$$

$$P^2\psi = -\hbar^2\frac{d^2\psi}{dx^2} \quad \text{--------- (3)}$$

Total energy associated with the particle of mass “m” and moving with velocity v is $E = K.E. + P.E. = \frac{P^2}{2m} + V \quad \Rightarrow P^2 = 2m(E - V)$

Multiplying this equation both sides by ψ, we get

$$P^2\psi = 2m(E - V)\psi \quad \Rightarrow -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = i\hbar\frac{d\psi}{dt}$$

This is time dependent Schrodinger equation in one dimension.

In three dimension, $i\hbar\frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$

16. *Using Maxwell’s equations in free space, derive electromagnetic wave equations for $\vec{E}$ and $\vec{B}$. Write its plane wave solution.*

Sol^n: Maxwell equations in free are

$\nabla \cdot \vec{E} = 0$ --------- (1) $\nabla \cdot \vec{B} = 0$ --------- (2)

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ------ (3) $\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$----- (4)

Now, taking curl on both sides of Eq. (3)

$$\Rightarrow \nabla \times \nabla \times \vec{E} = -\frac{\partial(\nabla \times \vec{B})}{\partial t}$$

$$\Rightarrow \nabla\left(\nabla \cdot \vec{E}\right) - (\nabla \cdot \nabla)\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\Rightarrow -\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \therefore\ \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \text{---------- (5)}$$

Again, taking the curl on both sides of Eq. (4), we have

$$\nabla \times \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial(\nabla \times \vec{E})}{\partial t}$$

$$\Rightarrow \nabla\left(\nabla \cdot \vec{B}\right) - (\nabla \cdot \nabla)\vec{B} = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{B}}{\partial t}\right)$$

$$\Rightarrow -\nabla^2 \vec{B} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \qquad \therefore\ \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \qquad \text{-------- (6)}$$

Eq. (5) and Eq. (6) are electromagnetic wave equations for E and B respectively.

Now, the plane solution of these equations are

$E = E_0 \sin(\omega t - kr)$ and $B = B_0 = \sin(\omega t - kr)$ respectively.

Where E_0 and B_0 are maximum values of E and B. Also, $k = \frac{2\pi}{\lambda}$ and

$\omega = 2\pi f$ is angular frequency.

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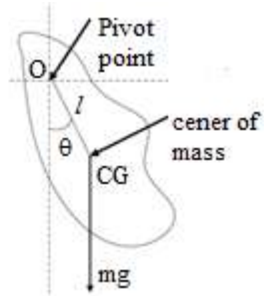
2067 Chaitra Back (BCE, BME)

***1.** Derive a relation for the time period of a compound pendulum and compare it with that of simple pendulum to locate the center of oscillation.*



Sol^n: Consider a rigid body of mass 'm' is suspended at point O. The center of gravity of the body is CG at a distance l from point O. If the meter scale is displaced by small angle θ (less than 4°) and released the restoring moment of force is $\tau = -\, mgl\theta$ --------------- (1)

From Newton's second law of motion, $\tau = I\,\alpha$ --------------- (2)

Where I is the moment of inertia of the rigid body about the axis passing through point O and α is angular acceleration. From Eq. (1) and Eq. (2)

$$-\, mgl = I\frac{d^2\theta}{dt^2} \qquad \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta \qquad \Rightarrow \frac{d^2\theta}{dt^2} \propto -\,\theta$$

This shows the motion of the rigid body is simple harmonic. So,

$$\omega^2 = \frac{mgl}{I} \qquad \Rightarrow \omega = \sqrt{\frac{mgl}{I}} \qquad \Rightarrow T = 2\pi\sqrt{\frac{I}{mgl}}$$

Using parallel axes theorem, $I = I_{CG} + m\,l^2 = mk^2 + m\,l^2$.

$$\therefore\ T = 2\pi\sqrt{\frac{l^2+k^2}{lg}} \qquad \text{--------------- (3)}$$

Comparing this equation with the time period of simple pendulum, we get, $L = l + \frac{k^2}{l}$, this length is known as equivalent length of simple pendulum. Any point which lies at a distance of $\frac{k^2}{l}$ from the center of gravity opposite to point of suspension is known as center of oscillation or point of oscillation.

OR

Obtain differential equation for forced oscillation. Write its solution. Explain the statement "quality factor (Q) is a measure of the sharpness of resonance in the case of a driven oscillator".

Sol^n: If the vibration in a system is due to continuous application of periodic external force then the vibration is known as forced vibration. In this oscillation there exist three forces which are used to explain the motion of the system: Restoring force (F_S) = – Kx, where K is force constant, Damping force (F_d) = – bv, where b is damping constant and Applied periodic force (F_{ext}) = F_m sinαt, where F_m is the maximum value of applied force and α is angular frequency. So, total force in this system is $F = F_S + F_d + F_{ext} = -Kx - bv + F_m \sin\alpha t$

If m be the mass of the body which is in vibration then according to Newton's second law of motion F = ma.

From above two equations,

$ma = -Kx - bv + F_m \sin\alpha t \qquad \Rightarrow m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_m \sin\alpha t$

$\Rightarrow \frac{d^2x}{dt^2} = -\omega^2 x - \frac{b}{m}\frac{dx}{dt} + F_0 \sin\alpha t$ ----------- (1) where $F_0 = \frac{F_m}{m}$

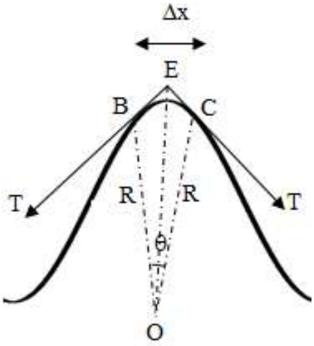
This is the differential equation of forced vibration. The solution of this equation is

$x = A\cos(\alpha t + \phi)$ where A is the amplitude of this system and ϕ is phase difference between displacement and applied periodic force. Also, the value of A is $A = \pm\frac{F_m}{\sqrt{m^2(\alpha^2-\omega^2)^2+b^2\alpha^2}}$. When the frequency of the periodic force is equal to the natural frequency of the body then the amplitude of oscillation is maximum and this condition is known as resonance. At resonance, α = ω and $A_{max} = \pm\frac{F_m}{b\omega}$. This equation shows that the amplitude of oscillation is inversely proportional to damping constant (b) of the medium.

Quality factor of a system measures the quality of a system and its value is $Q = \omega \frac{m}{b}$. For small value of b, Q is large for such system sharpness of resonance curve is large. So, it is a measure of the sharpness of resonance in the case of driven oscillator.

***2.** Derive a relation for speed of transverse wave in a stretched string and show that the average rate of energy transfer is $\frac{1}{2} \mu v \omega^2 A^2$, where the symbols are having usual meanings.*

Sol^n: Consider a string of linear density μ through which a transverse wave is propagating with speed v along positive x-direction. Take a part and consider an elemental arc BC of length Δx as shown in figure. So, the mass of this arc BC is μ Δx.



The tangents drawn at the extremities of this arc represent the tension produced on the end of the arc BC. O is the center of curvature of the arc BC and OB = OC = R is the center of curvature of arc BC. Let ∠BOC = θ and ∠BEC = α then ∠BOE = ∠EOC = θ/2 and ∠BEO = ∠OEC = α/2.

Now, the resultant tension (F) is given by

$$F = \sqrt{T^2 + T^2 + 2TT\cos\alpha} = \sqrt{2T^2(1 + \cos\alpha)}$$

$$F = \sqrt{2T^2 2\cos^2\frac{\alpha}{2}} = 2T\cos\frac{\alpha}{2} = 2T\sin\frac{\theta}{2}$$

For small arc $F = T\theta$ ------- (1)

This resultant tension is given from centripetal force

$$F_{cen} = \frac{mv^2}{R} = \frac{\mu\Delta x v^2}{R} = \mu\frac{\Delta x}{R}v^2 = \mu\theta\, v^2 \quad \text{------- (2)}$$

From Eq. (1) and Eq. (2), we get

$\Rightarrow \mu\theta\, v^2 = T\theta$ $\therefore\ v = \sqrt{\frac{T}{\mu}}$

This gives the speed of transverse wave in a stretched string. Also, the intensity of wave in a medium is $I = 2\pi^2\rho v f^2 A^2 = \frac{\text{Power (P)}}{\text{area (S)}}$

So, the average rate of energy transfer i.e. power developed in the string due to the propagation of wave is

$P = 2\pi^2\rho v f^2 A^2\, S = 2\pi^2 v f^2 A^2 \frac{m}{SL} S = \frac{1}{2}\mu\, v\, \omega^2 A^2$, where μ is mass per unit length of the string.

3. *Write down the requirements for a good acoustic hall and derive a relation for reverberation time.*

Sol^n: Acoustics of buildings is concerned with the hearing to speakers and musicians in hall and auditorium. It is found that some auditoriums are acoustically good and some bad. In a good auditorium the following conditions should be satisfied,

(a) The sound must sufficiently be loud everywhere.

(b) Successive syllables must be clear and distinct i.e; each syllable should die away sufficiently quickly to give place to the next syllable.

(c) There should be no echoes or distortion of the original sound.

(d) There should neither be any focusing of sound nor any zone of silence in any part of the hall.

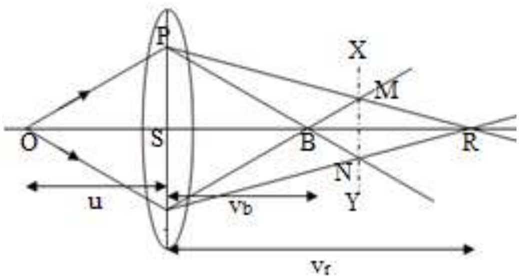
(e) The quality of sound must not change. The relative intensities of the several components of a complex sound must be maintained.

(For second part see 2067 Ashadh Regular Q. No. 3)

4. *Explain chromatic aberration. Show that the longitudinal chromatic aberration is equal to the product of dispersive power and mean focal length of the lens.*

Sol^n: A lens is composed of large number of small angle prisms of varying refracting angles placed one after other. The refracting angle of the prism goes on decreasing at a uniform rate from its center to outwards in convex lens. If a ray, consisting of various colors is allowed to fall on a lens, it is dispersed into its constituent colors. The deviation for different colors is different. Therefore the dispersed rays focus at different points in the axis and there will be colored images. This phenomenon is chromatic aberration.

Consider white light illuminated point object O on the principal axis. Rays from this point object enters a lens of focal length f. After refraction through the lens, blue and red image will form at B and R respectively. If the screen XY is placed as shown, the image of least chromatic aberration is observed in the screen. On the screen there form a circular form of image. That is known as circle of least confusion.



Let the object distance be u, the blue and red image distances be v_b and v_r respectively. The focal length of a lens is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \Rightarrow \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f(\mu - 1)}$$

The focal lengths for blue and red colors are

$$\frac{1}{f_b} = \frac{\mu_b - 1}{f(\mu - 1)} \text{ and } \frac{1}{f_r} = \frac{\mu_r - 1}{f(\mu - 1)}$$

Where μ_b and μ_r are refractive indices for blue and red colors respectively.

$\frac{1}{f_b} - \frac{1}{f_r} = \frac{\mu_b - \mu_r}{f(\mu - 1)}$ $\Rightarrow \frac{f_r - f_b}{f_r f_b} = \frac{\omega}{f}$

Where ω is the dispersive power of the lens.

Also, $f_r f_b \approx f^2$ and $f_r - f_b$ is longitudinal chromatic aberration.

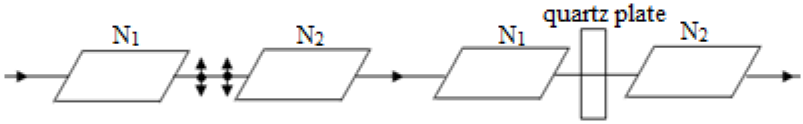
$\therefore\ f_r - f_b = \omega f$

So, the longitudinal chromatic aberration is equal to the product of dispersive power and mean focal length of the lens.

OR

Define the term "optical activity". Derive a relation for the specific rotation of any optically active substance. Also write down its applications.

Sol^n^: Consider a set of Nicol prisms placed in such a way that there is no emergent light coming out from the analyzer, as shown in figure below.



But, if a quartz plate which is cut with its face parallel to the optic axis is kept between the prism in such a way that the emergent ray from N_1 falls at angle 90° with the plate there is light emerging out from N_2 as well. The plane of vibration of plane polarized light from N_1 entering the quartz is gradually rotated. The amount of rotation of the plane of vibration depends mainly on the thickness of the quartz plate and wavelength of light. This property of rotating the plane of vibration by a crystal is known as Optical Activity. Optical activity is performed in materials which are optically active.

Solution of optically active substances rotates the plane of linearly polarized light. The angle through which the plane polarized light is rotated depends on the thickness of the medium, concentration of the solute in the solution, the wavelength of the light used and temperature of the substance.

The specific rotation is the rotation produced by a decimeter long solution containing 1gm of optically active substance in one cubic centimeter of the solution. i.e; $S = \frac{10\theta}{lc}$, where θ is the angle of rotation, *l* is the length of the solution in cm, c is the concentration in gm/cm^3 and S is the specific rotation at a given temperature for a given wavelength of light.

Uses of specific rotation; to measure the concentration of a solution and to test whether the medium is optically active or not.

5. *A glass clad fiber is made with the core glass of refractive index 1.5 and the cladding is doped to give a fractional index change of 0.0005. Determine (a) The cladding index, (b) The acceptance angle and (c) The numerical aperture.*

Sol^n: Refractive index of core (μ_1) = 1.5, $\Delta = 0.0005$

We have, $\Delta = \frac{\mu_1 - \mu_2}{\mu_1} = 0.0005$

(a) $\mu_2 = ?$ $\Rightarrow 0.0005\mu_1 = \mu_1 - \mu_2$ $\Rightarrow \mu_2 = 1.5 - 0.0005 \times 1.5$

$\mu_2 = 1.49925$

(b) $\text{Sin}i = \sqrt{\mu_1^2 - \mu_2^2} = \sqrt{1.5^2 - 1.49925^2} = 0.047$

Acceptance angle (i) = 2.72°

(c) Numerical Aperture (NA) = $\mu_1\sqrt{2\Delta} = 1.5\sqrt{2 \times 0.0005} = 0.047$

6. *Newton's rings are observed in reflected light of wavelength 5900Å. The diameter of 10th dark ring is 50mm. Find the radius of curvature of lens and thickness of air film.*

Sol^n: Here, $\lambda = 5900Å = 5.9 \times 10^{-5}$ cm

Diameter of 10th dark ring (D_{10}) = 50mm = 5cm

Radius of curvature of plano-convex lens (R) = ?

Thickness of air film for 10th dark ring (t) = ?

Diameter of the n^{th} order dark ring for reflected light is given by

$D_n = \sqrt{n\lambda R}$ $\Rightarrow D_{10} = \sqrt{10\lambda R}$

$\Rightarrow R = \frac{{D_{10}}^2}{10\lambda} = \frac{25}{10\times 5.9\times 10^{-5}} = 423.73$ m

Also, thickness of air film is (t) = $\frac{n\lambda}{2} = \frac{10\times 5.9\times 10^{-7}}{2} = 2.95 \times 10^{-6}$ m

7. *In a grating the sodium doublet (5890Å, 5896Å) is viewed in third order at 30° to the normal and is resolved. Determine the grating element and the total width of the rulings.*

Sol^n: Here, order of maxima (n) = 3, angle of diffraction (θ) = 30°,

$\lambda_1 = 5.89 \times 10^{-5}$ cm, $\lambda_2 = 5.896 \times 10^{-5}$ cm, $\Delta\lambda = 0.006 \times 10^{-5}$ cm,

a + b =? and width of the grating =?

Let the total number of lines required on the grating be N, then the resolving power is $\frac{\lambda}{\Delta\lambda} = nN$ $\Rightarrow N = \frac{\lambda}{n\Delta\lambda} = \frac{5.89\times 10^{-5}}{3\times 0.006\times 10^{-5}} = 327$

Again, $(a + b) \sin\theta_n = n\lambda$

$\Rightarrow (a + b) \sin 30° = 3 \times 5.89 \times 10^{-5}$ $\Rightarrow a + b = 3.53 \times 10^{-4}$cm

So, $N' = (a + b)^{-1} = 2830$ lines per cm

Width of the ruling = $\frac{327}{2830} = 0.12$ cm.

8. *Calculate the thickness of (i) a quarter wave plate and (ii) a half wave plate given that $\mu_E = 1.553$, $\mu_0 = 1.544$ and $\lambda = 5\times 10^{-5}$ cm.*

Sol^n: Here, $\mu_E = 1.553$, $\mu_0 = 1.544$ and $\lambda = 5\times 10^{-5}$ cm

(i) For quarter wave plate, $t = \frac{\lambda}{4(\mu_E - \mu_0)} = \frac{5\times 10^{-5}}{4(1.553-1.544)} = 1.39 \times 10^{-3}$ cm

(ii) For half wave plate, $t = \frac{\lambda}{2(\mu_E - \mu_0)} = \frac{5\times 10^{-5}}{2(1.553-1.544)} = 2.78 \times 10^{-3}$ cm

9. *What is electric Quadrupole? Finding an expression for electric potential at any point on an axial line at a distance 'r' from center of short Quadrupole, show that electric field at that point is inversely proportional to r^4.*

Sol^n: (For first part see in 2067 Ashwin Back Q. No. 9 OR)

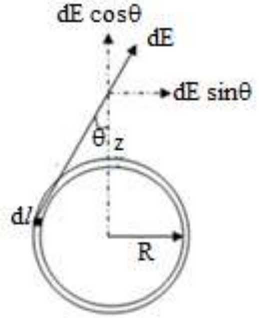
For second part, we have $\therefore$ $V = \frac{Q}{4\pi\varepsilon_0 r^3}$

Also, the electric field is $E = -\frac{dV}{dr} = -\frac{Q}{4\pi\varepsilon_0 r^3}\frac{d}{dr}\left(r^{-3}\right) = \frac{3Q}{4\pi\varepsilon_0 r^4}$

This shows for short quadrupole, the electric field at a point on the axial line is inversely proportional to forth power of the distance.

OR

A ring of radius 'R' is carrying a uniformly distributed charge 'q'. Find an expression for electric field at any point on the axial line. Locate the point at which electric field is maximum.



Sol^n: Consider a ring of radius R carrying uniformly distributed positive charge q with linear charge density λ. The ring is divided into elementary segments each of length dl. Let the electric field intensity dE due to this segment makes an angle θ with vertical. So, it can be resolved into two components dE sinθ and dE cosθ. If we consider the effect of whole ring, dE sinθ components gets cancelled out and resultant field is

$E = \int dE\cos\theta$, where $dE = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{\lambda dl}{4\pi\varepsilon_0 (R^2+z^2)}$, $\cos\theta = \frac{z}{\sqrt{R^2+z^2}}$

$\Rightarrow E = \frac{\lambda z}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}\int_0^{2\pi R} dl = \frac{qz}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}$ $\qquad (q = 2\pi\lambda R)$

For E maximum $\frac{dE}{dz} = 0$

$$\frac{dE}{dz} = \frac{q}{4\pi\varepsilon_0}\frac{d}{dz}\left(\frac{z}{(R^2+z^2)^{3/2}}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{(R^2+z^2)^{3/2} - z\times\frac{3}{2}\times(R^2+z^2)^{1/2}\times 2z}{(R^2+z^2)^3}\right)$$

$$\Rightarrow \frac{dE}{dz} = \frac{q}{4\pi\varepsilon_0}\left(\frac{R^2+z^2-3z^2}{(R^2+z^2)^{5/2}}\right)$$

Now, $\frac{dE}{dz} = 0$ $\Rightarrow R^2 + z^2 - 3z^2 = 0$ $\therefore z = \pm\frac{R}{\sqrt{2}}$

At, $z = \pm\frac{R}{\sqrt{2}}$, $\frac{d^2E}{dz^2} < 0$. Hence, the electric field is maximum at $\pm\frac{R}{\sqrt{2}}$.

10. *A parallel plate capacitor each of area 100 cm^2 has a p.d. of 50V and capacitance of 100×10^{-6} μF. If a mica of dielectric constant 5.4 is inserted between plates find the magnitude of (a) Electric field in mica, (b) Displacement vector and (c) Polarization vector.*

Sol^n: Area of plates (A) = 100 cm^2 = 10^{-2} m^2, potential difference between the plates (V) = 50V, capacitance (C) = 100×10^{-6} μF = 10^{-10}F

Dielectric constant of mica sheet (k) = 5.4

(a) Electric field in mica (E) =?

We have, $q = CV = 5\times10^{-9}$ C

$$E = \frac{E_0}{k} = \frac{q}{k\varepsilon_0 A} = \frac{5\times10^{-9}}{5.4\times8.85\times10^{-12}\times10^{-2}} = 1.04\times10^4 \text{ V/m}$$

(b) Displacement vector (D) = $\frac{q}{A} = \frac{5\times10^{-9}}{10^{-2}} = 5\times10^{-7}$ C/m^2

(c) Polarization vector (P) = $D - \varepsilon_0 E = 5\times10^{-7} - 8.85\times10^{-12}\times1.04\times10^4$

$\therefore$ P = = $(5-0.9)\times10^{-7} = 4.1\times10^{-7}$ C/m^2

11. *Compare the methods of Biot Savart law and Ampere's law to calculate the magnetic fields due to a current carrying conductor. Calculate magnetic field at an axial distance 'r' from the center of the circular coil carrying current.*

Sol^n: Biot-Savart's law states that the magnitude of magnetic field dB due to the current element d*l* at any point P at a distance r from of conductor carrying current I is $dB \propto \frac{Idl\sin\theta}{r^2}$ $\Rightarrow$ $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$ and magnetic field at a point due to current carrying conductor is $B = \frac{\mu_0 I}{2\pi r}$.

Ampere's law states that the line integral of magnetic induction B around any closed loop in a vacuum is equal to μ_0 times total current enclosed by the loop $\oint \vec{B}\cdot\vec{dl} = \mu_0$ I, and magnetic induction at a distance r from the center of the conductor is $B = \frac{\mu_0 I}{2\pi r}$

But, inside $B = \frac{\mu_0 I'}{2\pi r}$ where $I' = \frac{Ir^2}{R^2}$, R is the radius of the conductor.

(For second part see in 2067 Mansir Regular Q. No. 14)

12. *A current of 1.2×10^{-10} A exists in a copper wire (At. Wt. = 63 and density = 9gm/cc) whose diameter is 2.5 mm. Assuming current to be uniform, calculate: (a) Current density, (b) Electrical conductivity and, (c) Mobility of electrons.*

Sol^n: Current through wire (I) = 1.2×10^{-10} A, At. Wt. of copper = 63, Density of copper (ρ) = 9gm/cm^3, diameter of copper wire (d) = 2.5mm

$\therefore$ radius of wire (r) = 0.125cm

(a) Current density (J) = $\frac{I}{A} = \frac{I}{\pi r^2} = \frac{1.2\times10^{-10}}{\pi\times0.125^2} = 2.45 \times 10^{-9}$ A/m^2

(b) Electrical conductivity (σ) = $\frac{1}{\rho_{resistivity}}$

ρ_{resis} of the copper = 1.7×10^{-8} Ohm-m $\therefore \sigma = 5.9 \times 10^7$ mho/m

(c) Mobility of electrons (μ_e) =?

We have, $\sigma = ne\mu_e$ $\Rightarrow \mu_e = \frac{\sigma}{ne}$

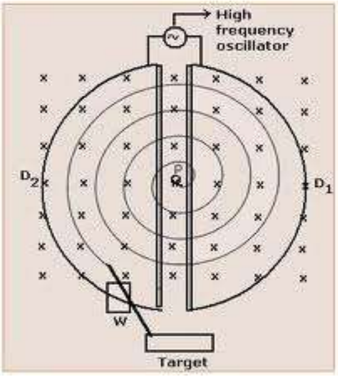
$n = \frac{\rho N_A}{M_{at}} = \frac{9\times6.02\times10^{23}}{63} = 8.6 \times 10^{22}$ /cm^3

$\therefore \mu_e = \frac{\sigma}{ne}$ $\Rightarrow \mu_e = \frac{5.9\times10^7}{8.6\times10^{28}\times1.6\times10^{-19}} = 0.42 \times 10^{-2}\,m^2V^{-1}s^{-1}$.

13. *What is Cyclotron? Find an expression to show that maximum kinetic energy of charge particles coming out of dees of Cyclotron is directly proportional to square of frequency of oscillator.*

Sol^n: The cyclotron is a particle accelerator which is consists of two large dipole magnets designed to produce a semicircular region of uniform magnetic field, directed uniformly downward. Two Dee's are placed back-to-back with their straight sides parallel but slightly separated.

Now, in order to produce an electric field across this gap there apply an oscillating voltage. Particles, which are injected into the magnetic field region of a D, trace out a semicircular path until they reach the gap. However, as the particles pass across the gap they are accelerated by the applied electric field. After gaining energy, these particles follow a semicircular path in the next D with larger radius.



The cyclotron uses electric and magnetic fields and the whole accelerator remains in a uniform magnetic field. In this accelerator, charged particles move in the field feel force acting at 90° to their direction of motion. Hence they move in circle. Here the Lorentz force due to the magnetic field provides the centripetal force for the circular motion with radius R. It means for a charged particle of charge q and mass m circulating with velocity v is Bqv = $\frac{mv^2}{R}$

$\therefore$ R = $\left(\frac{m}{qB}\right)$ v $\qquad\qquad \Rightarrow$ v = $\frac{qBR}{m}$

The time period of this motion is T = $\frac{2\pi R}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$

Now, the frequency of the cyclotron is given by $f = \frac{qB}{2\pi m}$

The kinetic energy of the charge particle is K.E. = $\frac{1}{2}$ m v^2, it will be maximum when the velocity of the particle is maximum.

i.e; $(K.E.)_{max} = \frac{1}{2} m v^2_{max} = \frac{q^2B^2R^2_{max}}{2m} = \left(\frac{qB}{2\pi m}\right)^2 2\pi^2 mR^2_{max}$

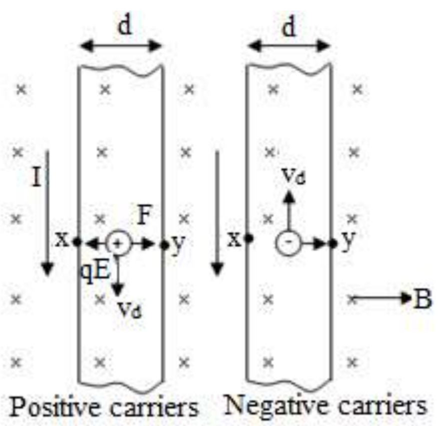
$\Rightarrow (K.E.)_{max} = 2\pi^2 m\, R_{max}^2\, f^2$

$\Rightarrow (K.E.)_{max} \propto f^2$ Proved.

OR

What is Hall-effect? Derive an expression for Hall coefficient and establish the relation with mobility of charge carrier and conductivity of material of wire.

Sol^n: When a conductor carrying current is placed in a uniform magnetic field perpendicular to the direction of the current, a transverse field is set up across the conductor. That field is called Hall field and corresponding potential difference is called Hall voltage and its value depend on the magnetic field strength, nature of the material and applied current. This effect is known as Hall Effect.



Let us consider, a strip of material of width d carrying current I as shown in figure magnetic field B is applied in the direction perpendicular to the direction of current. Let us assume that the current is due to the flow of charges of particular sign (-ve or +ve). The magnetic field exerts magnetic force on the charges towards right. As the charges drift towards right, an electric field is developed inside the conductor, so as to oppose the further sideway motion of the charges (the electric field is developed towards left). The sideway force is $\vec{F} = q(\vec{v_d} \times \vec{B})$

And the Hall field is $E_H = \frac{V_H}{d}$

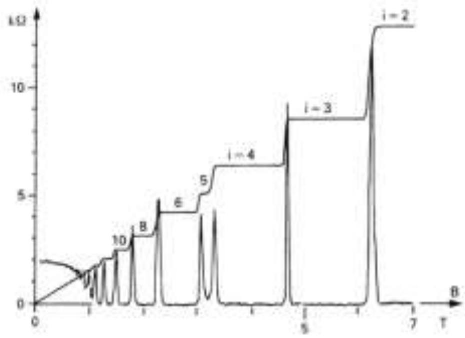
Eventually, an equilibrium is reached in which the sideways magnetic deflecting force on the charge carriers is just cancelled by oppositely directed electric force,

$q\vec{E}_H = -\, q(\vec{v_d} \times \vec{B})$ $\Rightarrow \vec{E}_H = -\,(\vec{v_d} \times \vec{B})$

Since v_d and B are right angles so, $E_H = -\, v_dB$

$\Rightarrow E_H = -\frac{J}{ne}B$ where $v_d = \frac{J}{ne}$; J is current density.

Also, $\frac{E_H}{JB} = -\frac{1}{ne} = R_H$ is hall coefficient and n is number of charge carriers per unit volume.



So, $n = \frac{JB}{eE_H} = \frac{\left(I/_{dt}\right)B}{e\left(\frac{V_H}{d}\right)} = \frac{BI}{etV_H}$

$\therefore V_H = \frac{BI}{net}$, This expression gives the Hall voltage.

Now, the Hall resistance is given by $R = \frac{V_H}{I} = \frac{B}{net}$.

From graph, the Hall resistance did not increase linearly with the field, instead, the plot shows a series of stair steps as shown in figure. Such effect has become is known as quantized Hall Effect.

The drift velocity acquired in unit applied electric field is known as the mobility of the carrier and is denoted by μ_H and is also called Hall mobility, $\mu_H = \frac{v_d}{E} = \frac{J}{neE} = \sigma\, R_H = \frac{R_H}{\rho}$.

14. *A proton with speed of 3 × 10^5m/s orbits just outside a charged sphere of radius 1cm. What is the charge on the sphere?*

Sol^n: Let V the potential at the surface of the sphere of radius r having charge q. So, K.E. of proton = energy experienced by proton from the potential of the charged sphere

i.e; $\frac{1}{2}mv^2 = eV$ $\Rightarrow V = \frac{m \times v^2}{2e} = \frac{1.67\times10^{-27}\times(3\times10^5)^2}{2\times1.6\times10^{-19}} = 470$ V

Also, potential at the surface of sphere of radius R is

$V = \frac{q}{4\pi\varepsilon_0 R}$ $\Rightarrow q = 4\pi\varepsilon_0 RV = 4\pi\times8.85\times10^{-12}\times10^{-2}\times470$

$\therefore q = 5.22 \times 10^{-10}$ C

So, charge on the surface of the sphere is 5.22×10^{-10} C

15. *Write Maxwell equations in integral form. Convert them in differential form. Explain each equation.*

Sol^n: (See in 2067 Ashwin Back Q. No. 15)

16. *A free particle is confined in a box with width L. Find an expression for energy Eigen value to show that the particle can have only discrete energy and momentum.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 16)

The momentum of the particle is $P_n = \sqrt{2mE_n}$, since E_n is discrete energy values, so the momentum is also discrete.

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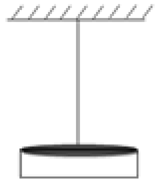
2068 Baishakh Regular (BEX, BCT, BEL, BIE, B. Agri)

1. *What is torsion pendulum? Describe how you will determine modulus of rigidity of a thin metallic wire which supports the disc.*

Sol^n: If a rigid body of any shape is suspended from a rigid support with the help of a metallic wire forms torsion pendulum. This is so called because, if the rigid body is turned by an angle about the wire as an axis and released then it oscillates in twisting or turning or torsionally. If a disc is suspended by a rigid support with the help of metallic wire of length l as shown in figure and the disc is displaced by an angle θ and released then the disc oscillates in turning motion due to restoring couple

$\tau = - c\,\theta$ ----------- (1)



Torsion pendulum

where c is torsional constant of the wire and its value is $c = \frac{\pi\eta r^4}{2l}$, where η is the modulus of rigidity and r is radius of the wire respectively.

If I be the moment of inertia of the disc about ax axis passing through wire as an axis then according to Newton's second law

$\tau = I\frac{d^2\theta}{dt^2}$ ------------- (2)

From Eq. (1) and Eq. (2), we have

$I\frac{d^2\theta}{dt^2} = - c\theta$ $\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{c}{I}\theta$

$\Rightarrow \frac{d^2\theta}{dt^2} \propto -\theta$

This shows motion of torsion pendulum is angular harmonic.

Also, $\omega^2 = \frac{c}{I}$ $\Rightarrow \omega = \sqrt{\frac{c}{I}}$

$\therefore T = 2\pi\sqrt{\frac{I}{c}}$

Now, squaring on both sides of this equation, we have

$\Rightarrow T^2 = 4\pi^2 \frac{I}{c}$

$\Rightarrow T^2 = 4\pi^2 \frac{2Il}{\pi\eta r^4}$

$\therefore \eta = \frac{8\pi I l}{T^2 r^4}$ This gives the modulus of rigidity of the wire.

OR

LC oscillations are called em oscillations, why? Derive the differential equation for damped electromagnetic oscillations and find the amplitude and frequency of that oscillation.

Sol^n: In case of LC oscillations, there is variation of charge stored in the capacitor and current through the circuit. Due to change in charge in the capacitor and current through the inductor, there is change in electric field between the plates of the capacitor and magnetic field surrounding the inductor. So, LC oscillations are called em oscillations.

(For second part see in 2067 Ashadh Regular Q. No. 1 OR up to f only)

2. *Prove that if a transverse wave is travelling along a stretched string, the slope at any point of the string is numerically equal to the ratio of the particle speed to the wave speed at that point.*

Sol^n: The equation of the transverse wave moving along the positive x-direction is $y = R \sin(\omega t - kx)$

Now, the slope at any point is

$m = \frac{dy}{dx} = -kR\cos(\omega t - kx)$

Also, speed of a particle at point x is

$u = \frac{dy}{dt} = R\omega\cos(\omega t - kx)$

$\frac{m}{u} = -\frac{k}{\omega} = -\frac{2\pi}{2\pi\lambda f} = -\frac{1}{v}$

$\therefore\ m = -\frac{u}{v}$

Hence, the slope at any point of the string is numerically equal to the ratio of the particle speed to the wave speed at that point.

3. *The time of reverberation of any empty hall and with 500 audiences in the hall is 1.5 sec and 1.4 sec respectively. Find the reverberation time with 800 audiences in the hall.*

Sol^n: The reverberation time for empty hall (T_0) = 1.5 sec

The reverberation time for entry of 500 audiences (T_{500}) = 1.4 sec

The reverberation time for entry of 800 audiences (T_{800}) = ?

Decrease in reverberation time due to 500 audiences = 1.5 – 1.4 = 0.1 sec

Decrease in reverberation time due to 800 audiences = $0.1\times\frac{800}{500}$ = 0.16sec

The reverberation time for 800 audiences (T_{800}) = T_0 – 0.16 = 1.34 sec

4. *What are Haidengers fringes? Describe the interference phenomenon in wedge shape thin film and determine the relation of path difference.*

Sol^n: In thin films interference, fringes are produced due to path difference 2μt cosr between the overlapping rays. For a given film the path difference may arises due to (i) the angle of refraction r inside the film and (ii) the change in thickness. We can express the change in path difference δ(D) by differentiating 2μt cosr i.e.

$\delta(D) = 2\mu t\,.\delta(\cos r) + 2\mu\cos r\,(\delta t)$, if t is constant then $\delta(D) = 2\mu t\,.\delta(\cos r)$ is very small. And if t is very large then δ(D) > high for small r.

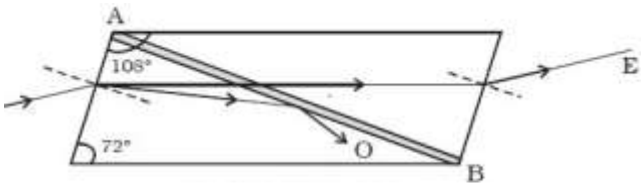
If the thickness of the film is large in thin film experiment, a very small change in r will change in path difference appreciably. Here the ray through the plates appears as parallel beam and generally it is viewed by the eye. Fringes are produced in this case is due to superposition of rays, which are equally inclined to the normal. These fringes are called Haidingers fringes.

(For remaining part see in 2067 Ashadh Regular Q. no. 4)

OR

What is double refraction? Discuss, how we can recognize that the given light is plane polarized, circularly polarized, elliptically polarized or unpolarized?

Sol^n: Light passing through a calcite crystal is split into two rays. This process, first reported by Erasmus Bartholinus in

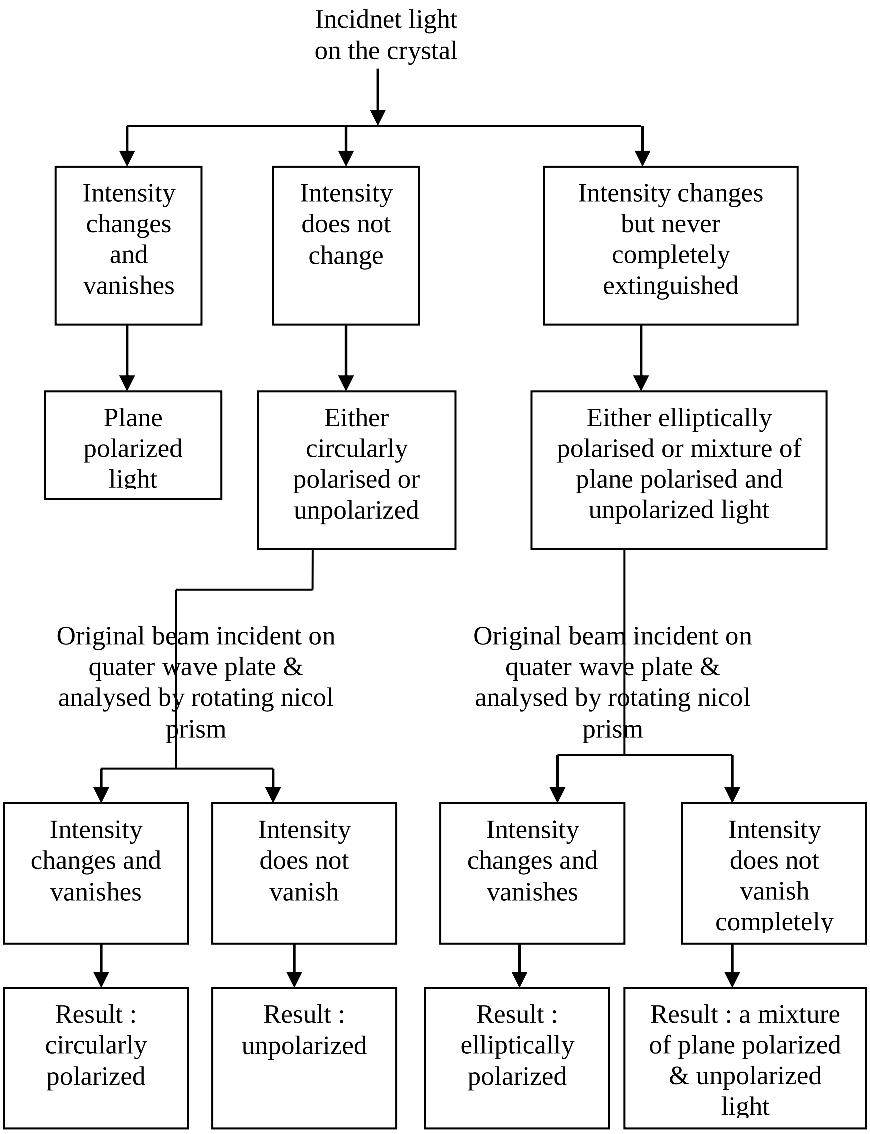


1669, is called double refraction. The two rays of light are each plane polarized by the calcite such that the planes of polarization are mutually perpendicular. For normal incidence (a Snell's law angle of 0°), the two planes of polarization are also perpendicular to the plane of incidence.

For normal incidence (a 0° angle of incidence), Snell's law predicts that the angle of refraction will be 0°. In the case of double refraction of a normally incident ray of light, at least one of the two rays must violate Snell's Law as we know it. For calcite, one of the two rays does indeed obey Snell's Law; this ray is called the ordinary ray (or O-ray). The other ray (and any ray that does not obey Snell's Law) is an extraordinary ray (or E-ray).

We can test the light in the following way to recognize that the given light is plane polarized, circularly polarized, elliptically polarized or unpolarized.

Intensity explanation



5. *Light is incident normally on a grating of total ruled width 5×10^{-3}m with 2500 lines in all. Find the angular separation of the sodium lines in the first order spectrum. Wavelengths of lines are 589 $\times 10^{-9}$ m and 589.6 $\times 10^{-9}$m. Can they be seen distinctly?*

Sol^n: Width of the grating (w) = 5×10^{-3} m = 0.5cm

Number of lines (N) = 2500

$\lambda_1 = 589 \times 10^{-9}$ m $\lambda_2 = 589.6 \times 10^{-9}$m

Number of lines per cm (N') = $\frac{2500}{0.5}$ = 5000 lines/cm

For $\lambda_1 = 5.89 \times 10^{-5}$ cm, n = 1

$(a + b) \sin\theta_1 = \lambda_1 \quad \Rightarrow \frac{1}{5000} \sin\theta_1 = 5.89 \times 10^{-5}$

$\Rightarrow \sin\theta_1 = 0.2945 \quad \therefore \theta_1 = 17.1°$

For $\lambda_2 = 5.896 \times 10^{-5}$ cm, n = 1

$(a + b) \sin\theta_1' = \lambda_2 \quad \Rightarrow \frac{1}{5000} \sin\theta_1' = 5.896 \times 10^{-5}$

$\Rightarrow \sin\theta_1' = 0.2948 \quad \therefore \theta_1' = 17.2°$

$\therefore \Delta\theta = 0.1°$ The condition for just resolution is $\frac{\lambda}{d\lambda} = nN$, $\lambda = 5.89 \times 10^{-5}$ cm, $d\lambda = 0.006 \times 10^{-5}$ cm, n = 1,

N = 982

As the total number of lines on the grating is 2500, the two lines will appear well resolved.

6. *Plane polarized light is incident on a piece of quartz cut parallel to the axis. Find the least thickness for which the ordinary and extra-ordinary rays combine to form plane polarized light. Given, $\mu_0 = 1.5442$, $\mu_E = 1.5533$, $\lambda = 5 \times 10^{-5}$ cm.*

Sol^n: The least thickness for the ordinary and extra-ordinary rays when they combine to form plane polarized light is $t = \frac{\lambda}{2(\mu_E - \mu_O)}$

$t = \frac{5 \times 10^{-5}}{2(1.5533 - 1.5442)} = 2.75 \times 10^{-3}$ cm = 0.0275 mm

7. *Show that the diameter of circle of least confusion depends on the diameter of lens aperture and the dispersive power of the material of the lens but is independent of the focal length of the lens.*

Sol^n: (See in 2067 Mangsir Regular Q. No. 4 OR)

8. *Calculate the refractive indices of the core and cladding materials of a fiber from following data. Numerical aperture (NA) = 0.22 and fractional refractive index change, Δ = 0.012.*

Sol^n: Here, Numerical aperture (NA) = 0.22 and fractional refractive index change, Δ = 0.012

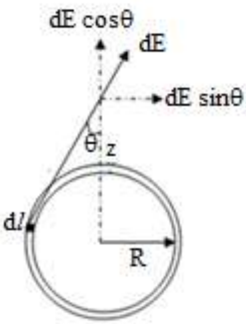
We have, NA = $\mu_1\sqrt{2\Delta}$ $\Rightarrow \mu_1 = \frac{NA}{\sqrt{2\Delta}} = \frac{0.22}{\sqrt{2\times0.012}} = 1.42$

Also, $\Delta = \frac{\mu_1-\mu_2}{\mu_1}$ $\Rightarrow 0.012 = \frac{1.42-\mu_2}{1.42}$

$\Rightarrow 0.017 = 1.42 - \mu_2$ $\therefore \mu_2 = 1.403$

9. *A thin ring made of plastic of radius R is uniformly charged with linear charge density λ. Calculate the electric field at any point P at a distance x from the center. Show that the motion of an electron is simple harmonic if electron is constrained to be in axial line of the same ring provided that x << R.*

Sol^n: Consider a ring of radius R carrying uniformly distributed positive charge q with linear charge density λ. The ring is divided into elementary segments each of length d*l*. Let the electric field intensity dE due to this segment makes an angle θ with vertical. So, it can be resolved into two components dE sinθ and dE cosθ. If we consider the effect of whole ring, dE sinθ components gets cancelled out and resultant field is



E = $\int$ dE cosθ, where $dE = \frac{dq}{4\pi\varepsilon_0 r^2}$

$\Rightarrow E = \frac{\lambda dl}{4\pi\varepsilon_0(R^2+x^2)}$, $\cos\theta = \frac{x}{\sqrt{R^2+x^2}}$

$\Rightarrow$ E = $\frac{\lambda x}{4\pi\varepsilon_0(R^2+x^2)^{3/2}} \int_0^{2\pi R} dl$

$\therefore$ E = $\frac{qx}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}$ where $q = 2\pi R\lambda$

If x << R, then E = $\frac{qx}{4\pi\varepsilon_0 R^3}$

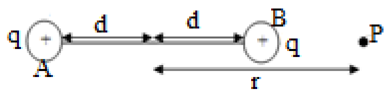
If a electron (- e) is placed near the center of this charge ring, we get a force, F = - eE = $-\left(\frac{eq}{4\pi\varepsilon_0 R^3}\right)x$

$\Rightarrow$ F $\propto$ - x. So, the motion of the electron will be simple harmonic.

OR

If both charges of dipole of charge q and separation 2d are positive, show that the electric intensity at any point P at a distance r from the center of such dipole for r >> d is E = $\frac{2q}{4\pi\varepsilon_0 r^2}$.

Sol^n: Consider both charges of the dipole are +q at a distance 2d as shown in figure. Consider a point P at a distance r from the center of the dipole. Now, the electric field at P due to charges at A and B are $E_A = \frac{q}{4\pi\varepsilon_0(r+d)^2}$ and $E_B = \frac{q}{4\pi\varepsilon_0(r-d)^2}$ along the same direction. So, net field at P is $E = E_A + E_B$

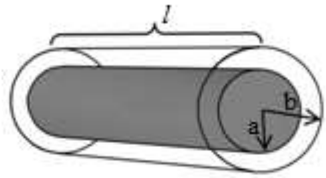


$$E = \frac{q}{4\pi\varepsilon_0(r+d)^2} + \frac{q}{4\pi\varepsilon_0(r-d)^2} = \frac{q}{4\pi\varepsilon_0}\frac{r^2-2rd+d^2+r^2+2rd+d^2}{(r^2-d^2)^2}$$

$= \frac{2q}{4\pi\varepsilon_0}\frac{r^2+d^2}{(r^2-d^2)^2}$ If r >> d then $E = \frac{2q}{4\pi\varepsilon_0}\frac{r^2}{r^4} = \frac{2q}{4\pi\varepsilon_0 r^2}$

10. *Prove that the capacitance per unit length of a cylindrical capacitor varies inversely with logarithm of ratio of external and internal radii. Obtain an expression for energy stored per unit volume in a parallel plate capacitor.*

Sol^n: Figure shows a section of a cylindrical capacitor of length l, inner radius a and outer radius b. The inner cylinder is a solid rod carrying a charge – q uniformly distributed over its surface and outer conductor is a co-axial cylindrical shell carrying charge + q which is also uniformly distributed. As a Gaussian surface, choose a cylinder of length l and radius r, closed by the end caps, and placed as shown in figure.



From Gauss's law $\oint_S \vec{E}\,.\,d\vec{a} = \frac{q}{\varepsilon_0} \Rightarrow E\,.\,2\pi r l = \frac{q}{\varepsilon_0}$

$\therefore\ E = \frac{q}{2\pi\varepsilon_0 l r}$

Now, the potential difference between a and b is $V = V_{ab} = +\int_a^b \vec{E}\,.\,\overrightarrow{dr}$

$\Rightarrow V = \frac{q}{2\pi\varepsilon_0 l r}\int_a^b \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 l r}\ln\left(\frac{b}{a}\right)$ $\qquad \therefore\ C = \frac{q}{V} = \frac{2\mu\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$

Energy stored per unit volume in a parallel plate capacitor

We know the energy stored in capacitor is $U_E = \frac{1}{2}CV^2$, where C is the capacitance of the capacitor and V is the potential across the capacitor.

It is assumed that in parallel plate capacitor the electric field has the same value for all points between the plates. The flow energy density u_e, which is the stored energy per unit volume and is given by

$u_e = \frac{CV^2}{2AD}$ also, $C = \varepsilon_0\frac{A}{d}$

So, $u_e = \frac{\varepsilon_0 AV^2}{2Ad^2} = \frac{\varepsilon_0}{2}\left(\frac{V}{d}\right)^2 = \frac{\varepsilon_0}{2}(E)^2$

This expression gives the energy stored per unit volume of the parallel plate capacitor.

11. *A copper wire of cross section area $5 \times 10^{-6}\ m^2$ carries a steady current of 50A. Assuming one free electrons per atom, calculate:*

(a) Free electron density and (b) Average drift velocity.Given: Density of copper = 8.9 × 10^3 kg/m^{-3}, Avogadro's No = 6.02 × 10^{23} mol^{-1}, Molar mass of copper = 64.

Sol^n^: Cross section area of wire (A) = 5 × 10^{-6} m^2, current (I) = 50 Amp

(a) $n = \frac{\rho N_A}{M_{at}} = \frac{8.9\times10^3\times6.02\times10^{23}}{64\times10^{-3}} = 8.4 \times 10^{28}$ electrons/m^3

(b) $I = nev_dA \qquad \Rightarrow v_d = \frac{I}{neA} = \frac{50}{8.4\times10^{28}\times1.6\times10^{-19}\times5\times10^{-6}}$

$\therefore v_d = 0.74 \times 10^{-3}$ m/s

12. *What is Ampere's law? Derive an expressions for magnetic flux density outside & inside a long straight conductor carrying a current i.*

Sol^n^: Ampere's law states that the line integral of magnetic induction B around any closed loop in a vacuum is equal to μ_0 times total current enclosed by the loop is $\oint \vec{B}\cdot\vec{dl} = \mu_0 I$

Ampere's law is more useful and simplified method to calculate the magnetic field for a current carrying conductor.

(For second part see in 2067 Ashadh Regular Q. No. 12)

OR

Describe cyclotron with necessary theory. Find the expression for maximum energy of a rotating particle in a cyclotron. Give limitations of cyclotron and how is it modified?

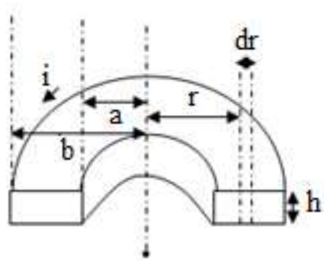
Sol^n^: (For first part see in 2067 Chaitra Back Q. No. 13)

Last part:

The cyclotron fails to operate at high energies because one of its assumptions that the frequency of rotation of an ion circulating in a magnetic field is independent of its speed is true only for speeds much less than that of light. As particle speed increases, we must use the relativistic mass in above equations.

Another difficulty associated with the acceleration of particles to high energies is that the size of the magnet that would be required to guide such particle in a circular path is very large. For a 30BeV proton, for example, in a field of 15000 gauss the radius of curvature is 65 meters. The magnet of cyclotron type of this size would be prohibitively expensive.

13. *Find an expression of the self inductance of a toroid having N number of turns, radius r and carrying a current i.*



Sol^n: Let a toroid of rectangular cross-section area A carrying current i be considered. The external and internal radius of toroid are b and a. The elementary part of the cross section be dr at a distance r from the center and h be width of the strip. The magnetic flux through the cross-section of the toroid is $\Phi_B = \oint_S \vec{B}.d\vec{a} = \int_a^b B(hdr)$ Also, $B = \frac{\mu_0 N i}{2\pi r}$

So, $\Phi_B = \frac{\mu_0 iNh}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 iNh}{2\pi}\ln\left(\frac{b}{a}\right)$ $\therefore$ $L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)$

This shows that the inductance of toroid depends on permeability, shape and size of the coil.

14. *Calculate the displacement current between the capacitor plates of area 1.5×10^{-2} m^2 and rate of electric field change is 1.5×10^{12} V/m.s. Also, calculate displacement current density.*

Sol^n: Here, area of plate (A) = 1.5×10^{-2} m^2, rate of electric field change $(\frac{dE}{dt}) = 1.5 \times 10^{12}$ V/m.s

(i) Displacement current $(i_d) = \varepsilon_0 A \frac{dE}{dt}$

$\Rightarrow i_d = 8.85\times10^{-12}\times1.5\times10^{-2}\times1.5\times10^{12} = 0.199$ Amp

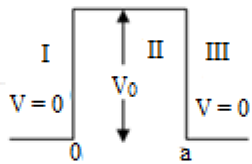
(ii) Displacement current density (J_d) = $\varepsilon_0 \frac{dE}{dt}$ = $8.85\times10^{-12} \times 1.5 \times 10^{12}$

(J_d) = 13.275 A/m^2

15. *Write Maxwell's equations in integral form and explain the laws on which these equations are based. Convert them into differential form.*

Sol^n: (See in 2067 Ashwin Back Q. No. 15)

16. *What is barrier tunneling? Discuss and write the Schrodinger wave equation in each region. Also write the formula of transmission coefficient, T in this case.*



Sol^n: The phenomenon of tunneling, which has no counterpart in classical physics, is an important consequence of quantum mechanics. Consider a particle with energy E in the inner region of a one-dimensional potential well V(x). (A potential well is a potential that has a lower value in a certain region of space than in the neighboring regions.) In classical mechanics, if E < V (the maximum height of the potential barrier), the particle remains in the well forever; if E > V, the particle escapes. In quantum mechanics, the situation is not so simple. The particle can escape even if its energy E is below the height of the barrier V, although the probability of escape is small unless E is close to V. In that case, the particle may tunnel through the potential barrier and emerge with the same energy E. The probability of finding the particle in third region by penetrating the barrier is known as barrier tunneling.

The potential is defined as V(x) = 0, for x < 0 and x > a

= V_0, for 0 < x < a

The Schrodinger wave equations in I, II and III regions are

$$\frac{d^2\varphi_I}{dx^2} + \frac{2mE}{\hbar^2}\varphi_I = 0 \quad \Rightarrow \frac{d^2\varphi_I}{dx^2} + \beta^2\varphi_I = 0 \quad \text{where } \beta^2 = \frac{2mE}{\hbar^2}$$

$$\frac{d^2\varphi_{II}}{dx^2} + \frac{2m(E-V_0)}{\hbar^2}\varphi_{II} = 0 \quad \Rightarrow \frac{d^2\varphi_{II}}{dx^2} + \alpha^2\varphi_{II} = 0$$

where $\alpha^2 = \frac{2m(E-V_0)}{\hbar^2}$

$\frac{d^2\varphi_{III}}{dx^2} + \frac{2mE}{\hbar^2}\varphi_{III} = 0 \Rightarrow \frac{d^2\varphi_{III}}{dx^2} + \beta^2\varphi_{III} = 0$ where $\beta^2 = \frac{2mE}{\hbar^2}$

The value of wave functions in these regions are

$\varphi_I = Ae^{i\beta x} + Be^{-i\beta x}$, where A is the amplitude of the incident wave and B is the amplitude of the reflected wave at x = 0.

$\varphi_{II} = Ce^{-\alpha x} + De^{\alpha x}$, where C is the amplitude of the incident wave and B is the amplitude of the reflected wave at x = a.

and $\varphi_{III} = Fe^{i\beta x}$, where F is the amplitude of the transmitted wave at x = a. The transmission coefficient represents the probability that the particle penetrates to the other side of the barrier. It is

$T = \frac{|F|^2}{|A|^2} = \frac{16E}{V_0}\left(1 - \frac{E}{V_0}\right)e^{-2\alpha a} = T_0\, e^{-2\alpha a}$

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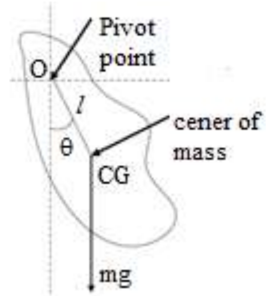
2068 Shrawan Back (BEX, BCT, BEL, BIE, B. Agri)

1. *List the common pendulums in practice. Which of them is a physical pendulum and why? Show that point of suspension and point of oscillation are interchangeable.*

Sol^n: Common pendulums are: simple pendulum, compound pendulum and torsion pendulum. Among these compound pendulum is the physical pendulum because this is formed by a physical body of any shape and size. Consider a rigid body of mass 'm' is suspended at point O. The center of gravity of the body is G at a distance l from point O. If the rigid body is displaced by small angle θ (less than 4°) and released the time period of oscillation for this body is



$$\Rightarrow T = 2\pi\sqrt{\frac{I}{mgl}}$$

Using parallel axes theorem, $I = I_{CG} + ml^2 = mk^2 + ml^2$.

$$\therefore\ T = 2\pi\sqrt{\frac{l^2+k^2}{lg}} \qquad \text{---------------}\ (1)$$

Any point which lies at a distance of $\frac{k^2}{l}$ from the center of gravity is known as point of oscillation. In figure point O' is the point of oscillation. For the point O the time period of oscillation is given from Eq. (1). If we suppose the point O' as the point of suspension then the length of the pendulum becomes $\frac{k^2}{l}$. For this case l should be replaced by $\frac{k^2}{l}$. So, the time period of oscillation for the point O' as point of suspension is given by

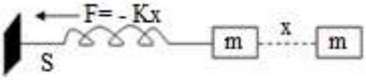
$$T' = 2\pi\sqrt{\frac{\left(\frac{k^2}{l}\right)^2+k^2}{\frac{k^2}{l}g}} = 2\pi\sqrt{\frac{k^2+l^2}{lg}} \qquad \text{------------------}\ (2)$$

From Eq. (1) and Eq. (2) T = T'. So, point of suspension and point of oscillation are interchangeable

OR

Explain the theory of a simple spring mass system. Develop the relation for time period and frequency of two springs constants K_1 and K_2 supporting a mass 'm' between them on a frictionless horizontal table.

Sol^n: Consider a mass m attached to the free end of a mass-less spiral spring, with its other end fixed to a rigid support, like a wall etc. If the mass be displaced through a distance x, as shown, a linear restoring force F = - Kx at once starts acting on the spring, tending to bring it back into its original condition, where K is the force constant of the spring. The negative sign of the force simply indicates that it is directed oppositely to the displacement of the mass.



Imagining the system to lie on a smooth horizontal surface, if the mass be released, it starts oscillating back and forth due to the spring getting alternately compressed and extended under the action of this force. If d^2x/dt^2 be the acceleration set up in the spring, the force acting on the mass is also equal to m. d^2x/dt^2. We, therefore, have

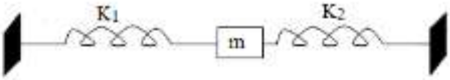
$$m\frac{d^2x}{dt^2} = -Kx \qquad \text{or,} \qquad \frac{d^2x}{dt^2} = -\frac{K}{m}x$$

Thus, $\frac{d^2x}{dt^2} \propto$ - x and is directed oppositely to it. Hence, the horizontal spring-mass executes a simple harmonic motion and its time period is given by comparing this equation with the differential equation of simple harmonic motion, $T = 2\pi\sqrt{\frac{m}{K}}$

For second part, let x be the displacement on the mass m then $F_1 = -K_1x$ and $F_2 = -K_2x$, both have same direction. So, total restoring force,

$F = F_1 + F_2 = -(K_1 + K_2)\,x = ma$

i.e; $\frac{d^2x}{dt^2} = -\frac{K_1+K_2}{m}x$



$\Rightarrow \frac{d^2x}{dt^2} \propto -x \qquad \therefore\ \omega^2 = \frac{K_1+K_2}{m}$

$\Rightarrow T = 2\pi\sqrt{\frac{m}{K_1+K_2}}$ and frequency $f = \frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$.

2. *A rod vibrating at 12Hz generates harmonic waves with amplitude of 1.5mm in a string of linear density 2 gm/m. If the tension in the string is 15N, what is the average power supplied by the source.*

Sol^n: The average power supplied by the source is $P_{ave} = 2\pi^2\mu v f^2R^2$

$P_{ave} = 2\pi^2\mu\sqrt{\frac{T}{\mu}}f^2R^2 = 2\pi^2f^2R^2\sqrt{T\mu}$

$\therefore\ P_{ave} = 2\pi^2\times12^2\times(1.5\times10^{-3})^2\times\sqrt{15\times0.002} = 1.11\times10^{-3}$ Watt

3. *Derive a necessary equation for reverberation time and mention the factors affecting the acousting of building.*

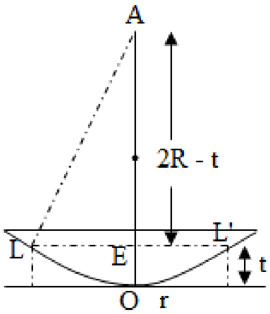
Sol^n: (See in 2067 Ashadh Regular Q. No. 3)

4. *Why Newton's are rings circular? Discuss and derive the necessary theory of Newton's ring experiment for transmitted light.*

Sol^n: Due to the symmetrical distribution of air film in all direction the Newton's rings are circular. The path difference between the rays transmitted on the upper and lower surface of the film is $2\mu t\cos r$. For normal incidence of light in air film, $r \approx 0$ and $\mu = 1$, so path difference is 2t. At the point of contact t = 0, so the path difference is zero. Hence the central spot is bright. The condition for the bright rings is $2t = n\lambda$; $n = 0, 1, 2, \ldots$

And the condition for dark rings is

$2t = (2n - 1)\frac{\lambda}{2}$; n = 1, 2, ……….



From figure, $\frac{LE}{OE} = \frac{AE}{EL'}$

$\Rightarrow$ LE × EL' = OE × AE

$\Rightarrow r \times r = (2R - t) \times t$

$\Rightarrow r^2 = 2Rt$, since t^2 is very small.

$\therefore\ t = \frac{r^2}{2R}$

Thus for bright rings, $2t = n\lambda$ $\Rightarrow \frac{r^2}{R} = n\lambda$

$\Rightarrow r^2 = n\lambda R$

So, the diameter of the nth order bright fringe is $D_n = 2\sqrt{n\lambda R}$ ------ (1)

Similarly, for dark rings, $2t = (2n - 1)\frac{\lambda}{2}$

$\frac{r^2}{R} = (2n - 1)\frac{\lambda}{2}$ $\Rightarrow r^2 = (2n - 1)\frac{\lambda R}{2}$

$\therefore\ D_n = \sqrt{(2n - 1)2\lambda R}$ ---------- (2)

Thus the diameter of bright rings are proportional to the square roots of natural numbers and center of the rings is bright.

$D_n^2 = 4n\lambda R$ and $D^2_m = 4m\lambda R$

From these, $\lambda = \frac{D_m^2 - D_n^2}{4(m-n)R}$

OR

Show that the intensity distribution pattern of Franhoffer's single slit diffraction is $I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$, where symbols carry usual meanings.

Sol^n: (See in 2067 Ashadh Regular Q. No. 4 OR)

5. *A 20cm long tube having sugar solution rotates the plane of polarization by 20°. If the specific rotation of sugar is 66°, calculate the strength of the solution.*

Sol^n: S = $\frac{10\theta}{LC}$ $\Rightarrow$ C = $\frac{10\theta}{LS} = \frac{10\times20°}{20\times66°}$ = 0.152 gm/cc

6. *A screen containing two slits 0.1mm apart is 1m from the viewing screen. Light of wavelength λ = 500nm falls on the slits from a distant source. Approximately how far apart will the bright interference fringes be seen on the screen?*

Sol^n: Fringe width (β) = $\frac{\lambda D}{d} = \frac{5\times10^{-7}\times1}{1\times10^{-4}}$ = 5×10^{-3} m = 5 mm

7. *Write down the principle of optical fiber and show that numerical aperture (NA) = $\mu_{core}\sqrt{2\Delta}$, where symbols have their own meaning.*

Sol^n: Optical fibers are made up of glass or plastic which are as thin as human's hair of diameter of about 150μm which is designed to guide light waves along the length of the fiber with the help of total internal reflection.

(For remaining part see in 2067 Ashadh Regular Q. No. 7)

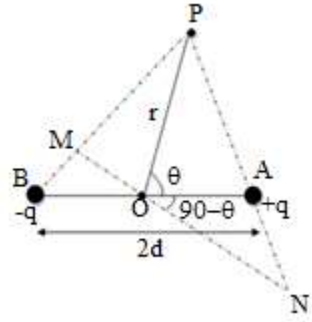
8. *Two thin convex lenses having focal lengths 10cm and 4cm are coaxially separated by a distance of 5cm, find the equivalent focal length of the combination. Determine also the positions of the principal points.*

Sol^n: Here, f_1 = 10cm, f_2 = 4cm, d = 5cm

f = $\frac{f_1 f_2}{f_1+f_2-d}$ = $\frac{10\times4}{10+4-5}$ = 4.44cm, first principal point (α) = $\frac{fd}{f_2}$ = $\frac{40\times5}{9\times4}$ = 5.55 cm, second principal point (β) = $-\frac{fd}{f_1}$ = $-\frac{40\times5}{9\times10}$ = -2.22cm

9. *For a given short electric dipole, show that the electric potential at any point at a distance r is V = $\frac{P\cos\theta}{r^2}$, where θ is the angle made by r to the dipole and P is its dipole moment. Using above relation find an expression for resultant electric intensity at that point.*

Sol^n: Let AB be an electric dipole with charge – q and + q with separation 2d and O is middle point of it. Electric potential at point P is to be determined at a distance r from the center of dipole O and ∠POA = θ and an arc with center P and radius OP is drawn to meet PA and PB produced at N and M. But if the dipole is very short 2d<<r, then MN is nearly a straight line perpendicular to both PB and PN. So, MP = OP = NP = r



MB = NA and from ΔAON we have NA = OA sin(90° - θ) = d cosθ

The electric potential at P due to dipole is

$$V = \left(\frac{q}{PA} - \frac{q}{PB}\right) = q\left(\frac{1}{PN-AN} - \frac{1}{PM+MB}\right)$$

$$= q\left(\frac{1}{r-d\cos\theta} - \frac{1}{r+d\cos\theta}\right) = q\left(\frac{2d\cos\theta}{r^2-d^2\cos^2\theta}\right)$$

For $r^2 >> d^2$, $V = \frac{P\cos\theta}{r^2}$, where P = 2qd is electric dipole moment.

To find the electric field at P, we choose coordinate system with origin at P, x-axis along r and y-axis perpendicular to r and in the plane containing P = 2qd and z-axis perpendicular to xy plane, such that dx = dr, dy = rdθ.

So, $E_r = E_x = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{P\cos\theta}{r^2}\right) = \frac{2P\cos\theta}{r^3}$

$E_\theta = E_y = -\frac{1}{r}\frac{d}{d\theta}\left(\frac{P\cos\theta}{r^2}\right) = \frac{P\sin\theta}{r^3}$ and $E_Z = -\frac{dV}{dz} = 0$

Now, resultant field is $E = \sqrt{E_r^2 + E_\theta^2} = \frac{P}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$

$E = \frac{P}{r^3}\sqrt{3\cos^2\theta + 1}$

This field will make an angle ϕ with r called resultant direction which is

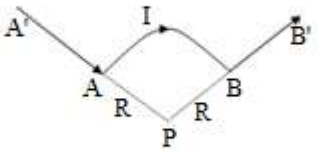
$\phi = \tan^{-1}\left(\frac{E_\theta}{E_r}\right) = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$

OR

State and explain Gauss law in electrostatics. Explain the meaning of three electric vectors $\vec{P}$, $\vec{E}$ and $\vec{D}$. And show that $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$.

Sol^n: (See in 2067 Mangsir Regular Q. No. 10 from statement of Gauss law)

10. *Show that the magnetic field due to a curve wire segment carrying current I and circular arc of radius R is $\frac{\mu_0 I}{8R}$.*



Sol^n: The wire shown in side figure carries a current I and consists of three segments A'A, BB' and AB where A'A and BB' are straight sections and AB is circular arc of radius R and constant angle $\pi/2$ radian. Let B_1, B_2 and B_3 are magnetic fields at P due to section A'A, BB' and AB respectively. The angle between $\vec{dl}$ and $\vec{r}$ is 0° for straight sections and hence $\vec{dl} \times \vec{r} = 0$. Thus the magnetic fields due to these sections are zero. i.e., $B_1 = B_2 = 0$.

However, at every point on AB, $\vec{dl}$ is perpendicular to $\vec{r}$.

i.e. $\vec{dl} \times \vec{r} = dl$

$dB_3 = \frac{\mu_0 I}{4\pi}\frac{dl \sin 90°}{R^2} = \frac{\mu_0 I}{4\pi}\frac{dl}{R^2}$ Also $dl = R\,d\theta$

$\Rightarrow B_3 = \frac{\mu_0 I}{4\pi R}\int_0^{\pi/2} d\theta = \frac{\mu_0 I}{4\pi R}\;\frac{\pi}{2}$

$\therefore B_3 = \frac{\mu_0 I}{8R}$

OR

State and explain Faraday law of induction. Show in induction, the mechanical energy is converted into electrical and finally into heat energy.

Sol^n: Faraday's law of induction states that the induced emf in the circuit equals to the negative of the time rate of change of magnetic flux through the circuit. This is the principle of electric generator, the direction of mechanical force, electric current and magnetic field is determined by Fleming's left hand rule.

(Remaining part see in 2067 Ashadh Regular Q. No. 13)

This is equivalent work done that he conductor is pulled through a magnetic field at constant velocity, which manifests itself as a small increase in the temperature of the loop, because the rate of work done on the rod through a magnetic field is $P_{applied} = Fv = BiLv = \frac{B^2L^2v^2}{R}$

Therefore, the work done that we do in pulling the conductor through the magnetic field appears as thermal energy. In short form

Mechanical energy $\rightarrow$ Electrical energy $\rightarrow$ Thermal energy.

11. *Two copper wires of same length l and cross sectional area A and 2A are connected to a battery. What will be the ratio of drift velocities when the wires are in (a) series and (b) parallel?*

Sol^n: The general expression for the drift velocity is $v_d = \frac{I}{neA}$

Here, both wires are of same material. So n is same for both wires.

(a) When wires are in series then current through both is same.

$$\therefore \frac{v_d}{v_d'} = \frac{I/neA}{I/ne2A} = 2:1$$

(b) When wires are in parallel then, $R_1 = \frac{\rho l}{A}$ and $R_2 = \frac{\rho l}{2A}$

So, equivalent resistance is $R = \frac{R_1R_2}{R_1+R_2} = \frac{\rho l/A \, \rho l/2A}{\rho l/A(1+1/2)} = \frac{\rho l}{3A}$

The current through the combination is $I = \frac{V}{R} = \frac{3AV}{\rho l}$

Now, the current through wire of area A is $I_a = I\frac{R_{2a}}{R} = \frac{3AV}{\rho l}\frac{\frac{\rho l}{2A}}{\frac{\rho l}{3A}} = \frac{9VA}{2\rho l}$

Similarly, through the wire of area 2A is $I_{2a} = I\frac{R_a}{R} = \frac{3AV}{\rho l}\frac{\frac{\rho l}{A}}{\frac{\rho l}{3A}} = \frac{9VA}{\rho l}$

$\therefore v_d = \frac{9VA}{2\rho lneA} = \frac{9V}{2\rho nel}$ and $v_d' = \frac{9VA}{\rho ne2Al} = \frac{9V}{2\rho nel}$ $\therefore \frac{v_d}{v_d'} = 1 : 1$

12. *A copper strip 2cm wide and 10mm thick is placed in a magnetic field of 1.5T. If a current of 200A is set up in the strip, calculate (i) Hall voltage (ii) Hall mobility if the no. of electrons per unit volume is 8.4 × 10^{28} m^{-3} and resistivity is 1.72 × 10^{-8} Ohm-m.*

Sol^n: (i) Hall voltage $(V_H) = \frac{Bi}{net} = \frac{1.5\times200}{8.4\times10^{28}\times1.6\times10^{-19}\times10^{-2}}$

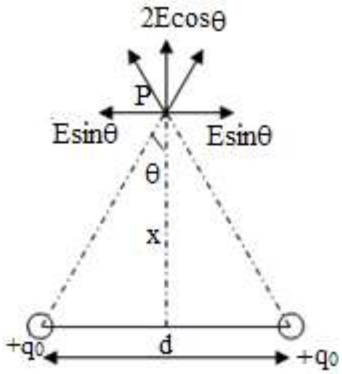
$V_H = 2.23 \times 10^{-6}$ T = 2.23 μT

(ii) Hall mobility $(\mu_H) = \frac{R_H}{\rho} = \frac{1}{ne\rho} = \frac{1}{8.4\times10^{28}\times1.6\times10^{-19}\times1.72\times10^{-8}}$

$\mu_H = 4.3 \times 10^{-3}\, m^2\, V^{-1}\, s^{-1}$.

13. *A particle of charge '-q' and mass 'm' is placed midway between two equal positive charges 'q_0' of separation 'd'. If the negative charge (- q) is placed in perpendicular direction to the line joining them and released, show that the particle describe a SHM with a period given by*

$T = \left[\frac{\varepsilon_0 m\pi^3 d^3}{qq_0}\right]^{1/2}$.



Sol^n: Let the particle of charge – q and mass m be at perpendicular distance x from the line joining two charges $+q_0$. The electric field produced by q_0 at point P is $E = \frac{q_0}{4\pi\varepsilon_0\left(\frac{d^2}{4}+x^2\right)}$ ------(1)

This field can resolved into two components E cosθ along vertical direction and E sinθ along horizontal direction, horizontal components cancel each other being equal and opposite.

So, resultant electric field along vertical direction due to both charges is

$$E' = 2E\cos\theta = \frac{2q_0\cos\theta}{4\pi\varepsilon_0\left(\frac{d^2}{4}+x^2\right)} = \frac{2q_0}{4\pi\varepsilon_0\left(\frac{d^2}{4}+x^2\right)}\frac{x}{\sqrt{\left(\frac{d^2}{4}+x^2\right)}} = \frac{2q_0x}{4\pi\varepsilon_0\left(\frac{d^2}{4}+x^2\right)^{3/2}}$$

Force experienced by charge – q at point P is

$$F_x = \frac{-2qq_0x}{4\pi\varepsilon_0\left(\frac{d^2}{4}+x^2\right)^{3/2}} \qquad \Rightarrow m\frac{d^2x}{dt^2} = \frac{-2qq_0x}{4\pi\varepsilon_0\left(\frac{d^2}{4}+x^2\right)^{3/2}}$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{2qq_0x}{4\pi\varepsilon_0 m\left(\frac{d^2}{4}+x^2\right)^{3/2}} = 0$$

This is the differential equation of simple harmonic motion with angular frequency $\omega^2 = \frac{2qq_0}{4\pi\varepsilon_0 m\left(\frac{d^2}{4}+x^2\right)^{3/2}}$ and at center x = 0,

$$\omega^2 = \frac{2qq_0}{4\pi\varepsilon_0 m\frac{d^3}{8}} = \frac{4qq_0}{\pi\varepsilon_0 md^3} \qquad \Rightarrow \omega = 2\sqrt{\frac{qq_0}{\pi\varepsilon_0 md^3}}$$

$$\text{Time period (T)} = \frac{2\pi}{\omega} = \left(\frac{\varepsilon_0 m\pi^3 d^3}{qq_0}\right)^{1/2}$$

14. *An inductor L is connected to a battery of emf ε through a resistor R. Show that the p.d. across the inductance after time t is $V_L = \varepsilon e^{-\left(\frac{R}{L}\right)t}$. At what time is the p.d. across the inductance equals to that across the resistance such that $i = \frac{i_0}{2}$.*

Sol^n: The growth of current through LR circuit is

$$i = i_0\left(1 - e^{-\frac{t}{\tau}}\right) = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

Potential difference across the inductor is

$V_L = L\frac{di}{dt} = -Li_0\left(-\frac{R}{L}\right)e^{-\frac{Rt}{L}} = i_0R\,e^{-\frac{Rt}{L}} = \varepsilon\,e^{-\frac{Rt}{L}}$

When the potential difference across inductor (V_L) = potential difference across resistor (V_R) $\Rightarrow \varepsilon\,e^{-\frac{Rt}{L}} = iR$

$e^{\frac{Rt}{L}} = \frac{\varepsilon}{iR} = \frac{i_0R}{iR} = \frac{i_0}{i}$

According to question, $\frac{i_0}{i} = 2$ $\Rightarrow e^{\frac{Rt}{L}} = 2$

$t = 0.693\frac{L}{R}$ sec

15. *Starting from Maxwell's equations in free space, obtain differential equations for electromagnetic waves. Find the plane wave solutions.*

Sol^n: (See in 2067 Mangsir Regular Q. No. 16)

16. *Derive an expression for the energy of a particle in an one dimensional infinite deep potential well.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 16)

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2068 Bhadra Regular (BCE, BME)

***1.** What is forced oscillation? Derive differential equation for forced oscillation and show that amplitude at resonance is inversely proportional to damping constant of medium.*

Sol^n: If the vibration in a system is due to continuous application of periodic external force then the vibration is known as forced vibration. In this oscillation there exist three forces which are used to explain the motion of the system:

Restoring force (F_S) = – Kx, where K is force constant,

Damping force (F_d) = – bv, where b is damping constant and

Applied periodic force (F_{ext}) = $F_m \sin\alpha t$,

Where F_m is the maximum value of applied force and α is angular frequency.

So, total force in this system is

$F = F_S + F_d + F_{ext} = -Kx - bv + F_m \sin\alpha t$

If m be the mass of the body which is in vibration then according to Newton's second law of motion F = ma.

From above two equations,

$ma = -Kx - bv + F_m \sin\alpha t \qquad \Rightarrow m\frac{d^2x}{dt^2} = -k\,x - b\frac{dx}{dt} + F_m \sin\alpha t$

$\Rightarrow \frac{d^2x}{dt^2} = -\omega^2 x - \frac{b}{m}\frac{dx}{dt} + F_0 \sin\alpha t$ ----------- (1) where $F_0 = \frac{F_m}{m}$

This is the differential equation of forced vibration. The solution of this equation is

$x = A\cos(\alpha t + \phi)$, where A is the amplitude of this system and ϕ is phase difference between displacement and applied periodic force.

$\frac{dx}{dt} = -A\alpha \sin(\alpha t + \phi)$ and $\frac{d^2x}{dt^2} = -A\alpha^2 \cos(\alpha t + \phi)$

With these values Eq. (1) becomes

$\Rightarrow -A\alpha^2\cos(\alpha t+\phi) - \frac{b}{m}A\alpha\sin(\alpha t+\phi) + \omega^2 A\cos(\alpha t+\phi) = F_0\sin\alpha t$

$\Rightarrow -A(\alpha^2-\omega^2)\cos(\alpha t+\phi) - \frac{b}{m}A\alpha\sin(\alpha t+\phi) = F_0\sin(\alpha t+\phi-\phi)$

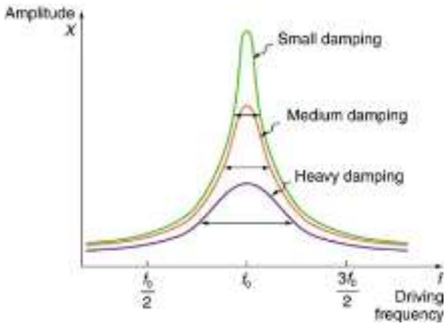
$= F_0\cos\phi\sin(\alpha t+\phi) - F_0\cos\phi\sin(\alpha t+\phi)$

Now comparing coefficients of $\cos(\alpha t+\phi)$ and $\sin(\alpha t+\phi)$ on both sides, we have

$-A(\alpha^2-\omega^2) = F_0\sin\phi$ --------- (2) and $\frac{b}{m}A\alpha = F_0\cos\phi$ ------- (3)

Solving Eq. (2) and Eq. (3), we get

$A = \pm\frac{F_m}{\sqrt{m^2(\alpha^2-\omega^2)^2+b^2\alpha^2}}$ is the amplitude of vibration. When the frequency of the periodic force is equal to the natural frequency of the body then the amplitude of oscillation is maximum and this condition is known as resonance.



At resonance, $\alpha = \omega$ and $A_{max} = \pm\frac{F_m}{b\omega}$.

This equation shows that the amplitude of oscillation is inversely proportional to damping constant (b) of the medium. From above curve it is clear that for small value of b the resonance curve is very sharp. So, for sharpness the value of damping coefficient should be very small.

OR

Derive the differential equation for damped LCR oscillation. Obtain an expression for current and frequency of oscillation.

Sol^n: (See in 2067 Ashadh Regular Q. No. 1 OR)

Now, the current flowing through the circuit is

$i = \frac{dq}{dt} = q_m e^{-\frac{Rt}{2L}}\left[-\frac{R}{2L}\cos(\beta t\pm\theta) - \beta\sin(\beta t\pm\theta)\right].$

2. *Prove that if a transverse wave is travelling along a stretched string, the slope at any point of the string is numerically equal to the ratio of the particle speed to the wave speed at that point.*

Sol^n: (See in 2068 Baishakh Regular Q. No. 2)

3. *The volume of a room is 600 m^3, wall area of room is 220 m^2, the floor and ceiling area each is 120m^2. If average absorption coefficient for walls is 0.03, for ceiling is 0.80 and for floor is 0.06, calculate average absorption coefficient and reverberation time.*

Sol^n: Volume of room (V) = 600m^3

Area of the wall (S_w) = 220m^2, absorption coefficient of wall (α_w) = 0.03

Area of the ceiling (S_c) = 120m^2,

absorption coefficient of ceiling (α_c) = 0.08

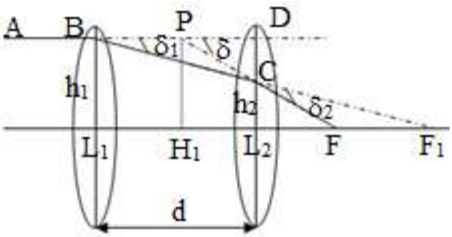
Area of the floor (S_f) = 120m^2, absorption coefficient of floor (α_f) = 0.06

Now, the average absorption coefficient of the room is given by

$$\alpha = \frac{S_w\alpha_w+S_c\alpha_c+S_f\alpha_f}{S_w+S_c+S_f} = \frac{220\times0.03+120\times0.08+120\times0.06}{220+120+120} = 0.051$$

Reverberation time is $T = \frac{0.158V}{\alpha S} = \frac{0.158\times600}{0.051\times460} = 4.05$ sec.

4. *Two thin lenses of power P_1 and P_2 are separated by a distance d. Find an expression to show that equivalent power of the combination is given as $P = P_1 + P_2 - dP_1P_2$.*



Sol^n: Consider two lenses L_1 and L_2 having focal lengths f_1 and f_2 with power P_1 and P_2 are placed co-axially as shown in figure and are separated by a distance d. Consider a monochromatic ray AB, parallel to principal axis is refracted towards second principal focus of the first lens. Therefore the deviation produced by the first lens is $\delta_1 = \frac{h_1}{f_1}$. Similarly the deviation for the second lens is $\delta_2 = \frac{h_2}{f_2}$.

If we produce the first incident ray AB and final emergent ray, they will meet at a point P. Therefore, if a thin lens is placed at PH_2, it will produce the same deviation as the lenses L_1 and L_2 together. This imaginary lens of focal length H_2F_2 (f) is called an equivalent lens. For this case deviation produced is $\delta = \frac{h_1}{f}$.

From geometry, $\delta = \delta_1 + \delta_2$

$\Rightarrow \frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2}$

Also, ΔBL_1F_1 and ΔCL_2F_1 are similar

So, $\frac{BL_1}{L_1F_1} = \frac{CL_2}{L_2F_1}$ $\quad\Rightarrow \frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$ $\quad\Rightarrow h_2 = h_1\left(\frac{f_1 - d}{f_1}\right)$

So, $\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1}{f_2}\left(\frac{f_1 - d}{f_1}\right)$ $\quad\Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

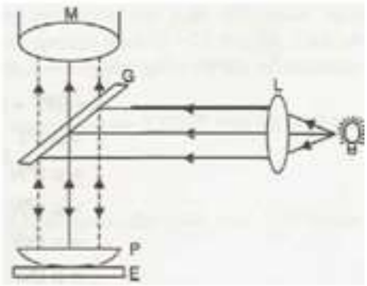
Also, power of lens is $P = \frac{1}{f}$, $P_1 = \frac{1}{f_1}$, $P_2 = \frac{1}{f_2}$

$\therefore$ $P = P_1 + P_2 - d\,P_1 P_2$. This is the equivalent power of the combination.

5. *Explain the formation of Newton's ring in reflected light. Prove that, in reflected light the diameter of the dark rings are proportional to the square root of natural numbers and diameter of bright rings are proportional to the square root of odd numbers.*

Sol^n: When monochromatic light is incident in the experimental set up as shown in figure below, Newton's rings are observed. The path difference between the rays reflected on the upper and lower surface of the thin film is $2\mu t \cos r + \frac{\lambda}{2}$.

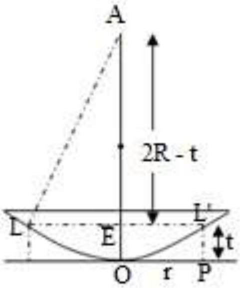
For almost normal incident in air film, $r \approx 0$ and $\mu = 1$. So, the path difference is $2t + \frac{\lambda}{2}$. At the point O, $t = 0$, so the path difference is $\frac{\lambda}{2}$. Hence the center spot is dark.



The condition for the bright rings is $2t + \frac{\lambda}{2} = n\lambda$

$\Rightarrow 2t = (2n - 1)\frac{\lambda}{2}$ --------(1), where n = 0, 1, 2…….and the condition for the dark rings is $2t + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$ ------- (2) where n = 0, 1, 2…….

Let a plan-convex lens of radius of curvature R is placed on the plane glass plate AOB, the curved surface LOL' is a part of spherical surface having radius of curvature R. At this particular case r is the radius of Newton's ring corresponding to film thickness t. From figure



$\frac{LE}{OE} = \frac{AE}{EL'}$ $\Rightarrow LE \times EL' = OE \times AE$

$\Rightarrow r \times r = t(2R - t)$ $\Rightarrow r^2 \cong 2Rt$ since t is very small, so t^2 can be neglected.

For bright fringe, $2t = (2n - 1)\frac{\lambda}{2}$.

$\Rightarrow \frac{r^2}{R} = (2n - 1)\frac{\lambda}{2}$ $\Rightarrow r_n^2 = (2n - 1)R\frac{\lambda}{2}$ and

diameter is $D_n = \sqrt{2(2n-1)\lambda R} = \sqrt{2\lambda R}\sqrt{(2n-1)}$

Hence, the diameter of bright rings are proportional to the square root of odd numbers. Similarly, for dark rings $D_n = 2\sqrt{n\lambda R}$.

The diameter of dark rings are proportional to the square root of natural numbers.

OR

Write down the physical meanings of dispersive power and resolving power of plane transmission grating. Show that both resolving and dispersive power have proportional relation with the order of spectrum.

Sol^n: Spreading the diffraction lines associated with various wavelengths by the grating is called dispersion. The capacity of spreading of the diffraction lines of the grating is called its dispersive power. It is defined as the ratio of the difference in the angle of diffraction of any two neighboring spectral lines to the difference in wavelength between two spectral lines. The relation for the path difference for maxima in diffraction grating is $(a + b)\sin\theta_n = n\lambda$

Differentiating on both sides, we get $\Rightarrow (a + b)\cos\theta_n\, d\theta = n\, d\lambda$

$$\Rightarrow \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} \qquad \Rightarrow \frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta} \quad \text{-------------------- (1)}$$

Where n is the number of order in the spectrum.

The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to the difference in wavelength between this line and a neighboring line such that the two lines appears to be just resolved. The relation for the path difference for maxima in diffraction grating is

$(a + b)\sin\theta_n = n\lambda$

From Rayleigh's criterion, if the principal maximum wavelength $\lambda + d\lambda$ falls on the first minimum of wavelength λ, then the wavelengths are said to be resolved. Let this common diffraction angle be represented by $(\theta_n + d\theta)$, so for n^{th} order spectrum, the two wavelengths λ and $\lambda + d\lambda$ will be just resolved if the following conditions are satisfied

$(a + b)\sin(\theta_n + d\theta) = n(\lambda + d\lambda)$ and $\qquad (a + b)\sin(\theta_n + d\theta) = n\lambda + \frac{\lambda}{N}$.

So, $n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N} \qquad \therefore \qquad \frac{\lambda}{d\lambda} = nN$ --------------- (2)

Hence, from Eq. (1) and Eq. (2) both dispersive power and resolving power have proportional relation with the order of maxima of the spectrum.

6. *A 200 mm long tube containing 48 cm^3 of sugar solution produces an optical rotation of 11° when placed in a polarimeter. If the specific rotation of sugar solution is 66°, calculate quantity of sugar contained in the form of solution.*

Sol^n: Here, Volume of the solution (V) = 48 cm^3,

length of the tube (l) = 200mm = 20 cm, optical rotation (θ) = 11°, specific rotation (S) = 66°, concentration of the solution (C) = ?

We have, $S = \frac{10\theta}{LC}$ $\qquad \Rightarrow C = \frac{10\theta}{SL} = \frac{10\times11}{66\times20} = \frac{1}{12}$ gm/cc

Hence, the amount of sugar in the solution (m) = V× C = 4 gm.

7. *Light is incident normally on a grating 0.5cm wide with 2500 lines. Find the angles of diffraction for the principal maxima of the two sodium lines in the first order spectrum, λ_1 = 5890Å and λ_2 = 5896Å. Are the two lines resolved?*

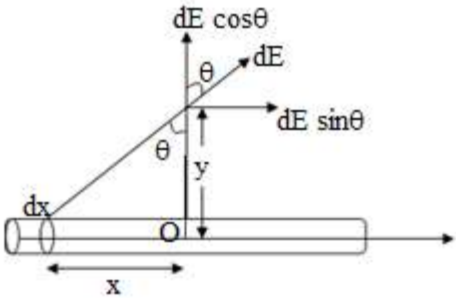
Sol^n: (See in 2068 Baishakh Regular Q. No. 5)

8. *What is principle of laser? Discuss how population inversion is carried out? With the help of energy level diagram, explain how He-Ne laser works.*

Sol^n: (See in 2067 Ashwin Back Q. No. 8)

9. *A thin non conducting rod of finite length l carries a total charge q spread uniformly along it. Show that the electric field at any point at a distance y above from the center of rod is $E = \frac{q}{2\pi\varepsilon_0 y}\sqrt{\frac{1}{l^2+4y^2}}$. Extend this result for infinite length.*

Sol^n: Let us consider a long thin rod having uniformly distributed charges q with linear charge density λ lies along x-axis. To find the electric field by the rod at a point located at the perpendicular bisector of the rod at a distance y, the rod is divided into small elements each of length dx.



Consider one of the element at a distance x from center of the rod. It contains an element of charge given by dq = λdx. This produces an electric field at P at a distance of r is dE = $\frac{dq}{4\pi\varepsilon_0 r^2} = \frac{\lambda dx}{4\pi\varepsilon_0(x^2+y^2)}$

This electric field dE makes an angle θ with vertical. The resultant electric field due to the charged rod is

$$E = \int dE\cos\theta = \frac{\lambda}{4\pi\varepsilon_0}\int\frac{\cos\theta dx}{(x^2+y^2)} = \frac{\lambda y}{4\pi\varepsilon_0}\int\frac{dx}{(x^2+y^2)^{3/2}}$$

Case (i) If the rod is finite length (l) then E = $\frac{\lambda y}{2\pi\varepsilon_0}\int_0^{l/2}\frac{dx}{(x^2+y^2)^{3/2}}$

Put x = y tanθ ⇒ dx = y sec^{2}θ dθ

$$E = \frac{\lambda y}{2\pi\varepsilon_0}\int\frac{y\sec^2\theta\, d\theta}{y^3\sec^3\theta\, d\theta} = \frac{\lambda}{2\pi\varepsilon_0 y}\int\cos\theta\, d\theta = \frac{\lambda}{2\pi\varepsilon_0 y}[\sin\theta]$$

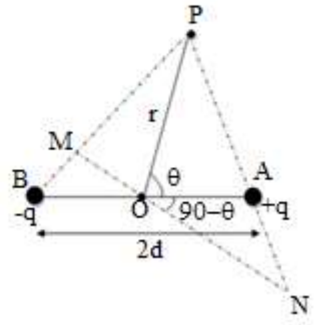
$$= \frac{\lambda}{2\pi\varepsilon_0 y}\frac{l/2}{\sqrt{(l/2)^2+y^2}} = \frac{\lambda l/2}{2\pi\varepsilon_0 y\times\frac{1}{2}\sqrt{l^2+4y^2}} = \frac{q}{2\pi\varepsilon_0 y\sqrt{l^2+4y^2}}$$

Case (ii) For infinite rod, E = $\frac{\lambda}{2\pi\varepsilon_0 y}$.

OR

Find the potential at any point at an angle θ at a distance r from the center of the short dipole. What result do you obtain if the point is along axial line?

Son^n: Let AB be an electric dipole with charge – q and + q with separation 2d and O is middle point of it. Electric potential at point P is to be determined at a distance r from the center of dipole O and ∠POA = θ and an arc with center P and radius OP is drawn to meet PA and PB produced at N and M. But if the dipole is very short 2d<<r, then MN is nearly a straight line perpendicular to both PB and PN. So,



MP = OP = NP = r

MB = NA and from ΔAON we have NA = OA sin(90° - θ) = d cosθ

The electric potential at P due to dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{PA} - \frac{q}{PB}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{PN-AN} - \frac{1}{PM+MB}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r-d\cos\theta} - \frac{1}{r+d\cos\theta}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{2d\cos\theta}{r^2-d^2\cos^2\theta}\right)$$

For $r^2 >> d^2$, we get

$V = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2}$, where P = 2qd is electric dipole moment.

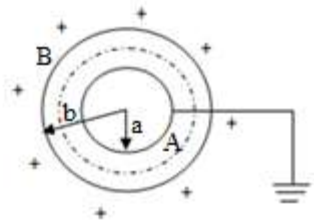
The dipole moment is a vector whose direction is along BA from the negative to positive side.

$\therefore V = \frac{\vec{P}.\vec{r}}{r^3}$ Along the axial line θ = 0°

So, $V = \frac{P}{4\pi\varepsilon_0 r^2}$

10. *A capacitor is made of two concentric spherical plates of radii a and b of inner and outer spheres respectively. If outer plate is positively charged and inner sphere is earthed, prove that the capacitance of such capacitance is given as* $C = 4\pi\varepsilon_0\left[\frac{b^2}{b-a}\right]$.

Sol^n: Let +q charge be given to the outer sphere B. Some of these charges will remain on the outer surface and the remaining charge will be distributed over the inner surface. Thus the total charge $q = q_1 + q_2$.



Due to charge q_2 on the inner surface of A induces q charge $-q_2$ on the outer surface of A and q_2 on the inner surface of the inner sphere which goes to earth being free. So two capacitors C_1 and C_2 are formed connected in parallel. C_1 = isolated capacitor of radius b formed by the earth and outer surface of sphere B = $4\pi\varepsilon_0 b$ and C_2 = capacitance of capacitor consisting of inner surface of B and outer surface of A

$$= 4\pi\varepsilon_0 \frac{ab}{b-a}$$

$$\therefore\ C = C_1 + C_2 = 4\pi\varepsilon_0 b + 4\pi\varepsilon_0 \frac{ab}{b-a} = 4\pi\varepsilon_0 b \left(1 + \frac{a}{b-a}\right) = 4\pi\varepsilon_0 b \left(\frac{b-a+a}{b-a}\right)$$

$$= 4\pi\varepsilon_0 \left(\frac{b^2}{b-a}\right)$$

11. *Calculate the relaxation time for the electrons of sodium atom. The number of atoms per cm^3 in sodium is 2.5×10^{22}, and the electrical conductivity is 1.9×10^7 s/m.*

Sol^n: Number of atoms (n) = $2.5 \times 10^{22}\ cm^{-3} = 2.5 \times 10^{28}\ cm^{-3}$

Electrical conductivity (σ) = 1.9×10^7 s/m

Relaxation time (τ) = ?

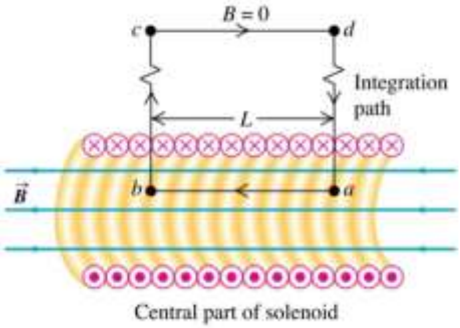
We have, $\tau = \frac{\sigma m}{ne^2} = \frac{1.9\times10^7\times9.1\times10^{-31}}{2.5\times10^{28}(1.6\times10^{-19})^2} = 2.7 \times 10^{-14}$ sec.

12. *List and explain methods to calculate magnetic field due to a current carrying conductor. Derive an expression for the magnetic field on the axial line of a long solenoid carrying current.*

Sol^n: Following methods can be used to calculate the magnetic field due to a current carrying conductor.

(a) Ampere's law – inside, outside and on the surface

(b) Biot – Savart law



A solenoid consists of a long wire winding closely around an insulating conductor in helical form.

Since the solenoid field is the vector sum of the fields set up by all turns make up the solenoid. The mark ⊙represents that the current is out of the page and the mark ⊗ represents that the current is into the page. The magnetic field is set up by the upper part of the solenoid turns which points to the left and tends to cancel the field set up by the lower part of the solenoid; the magnetic field outside the solenoid is zero. Taking the external field to be zero is an excellent assumption for a real solenoid if its length is much greater than its diameter. The direction of the magnetic field along the solenoid axis is given by right hand thumb rule.

Let us apply Ampere's law to the rectangular Amperian loop abcd in the ideal solenoid of above figure, where B is uniform within the solenoid and zero outside of it.

$\therefore \oint_C \vec{B}.\vec{dl} = \mu_0 I'$ ----------(1)

Where I' is the current enclosed by Amperian loop.

But, $\oint_C \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl}$

Here, $\int_a^b \vec{B}.\vec{dl} = \int_a^b B dl \cos 0° = \int_a^b B dl = B\int_a^b dl = BL$

Where L is the arbitrary length of the path from a to b.

$\int_b^c \vec{B}.\vec{dl} = \int_d^a \vec{B}.\vec{dl} = 0$ [since θ is 90°]

and $\int_c^d \vec{B}.\vec{dl} = 0$ [since B = 0, outside the solenoid]

$\therefore \oint_C \vec{B}.\vec{dl} = BL$ -------------- (2)

From Eq. (1) and Eq. (2)

$BL = \mu_0 I'$ $\therefore B = \frac{\mu_0 I'}{L}$ Also, $I' = I n L$

So, $B = \frac{\mu_0 nIL}{L}$ $\therefore B = \mu_0 n I$

OR

What is self inductance? Calculate the inductance of a circular Toroid. From your result, show that inductance is a property of a coil and depends on permeability and shape and size of the coil.

Slo^n: The phenomenon in which the induced emf is produced as a result of change in the current passing through the coil is known as self induction. **(Second part see in 2068 Baishakh Regular Q. No. 13)**

13. *Suppose a cyclotron is operated at an oscillator frequency of 12MHz and has a dee of radius 53cm. (a) What is the magnitude of the magnetic field needed for deuteron to be accelerated in the cyclotron? (b) What is the resulting kinetic energy of the deuteron? Give: mass of deuteron = 3.34×10^{-34}kg.*

Sol^n: Here, frequency of oscillator (f) = 12 MHz = 1.2×10^7 Hz

Dee radius (R) = 53 cm

Mass of deuteron (m) = 3.34×10^{-27} kg

Charge of an electron (q) = 1.6×10^{-19} C

(a) The frequency of the oscillator is given by $f = \frac{qB}{2\pi m}$

$$B = \frac{2\pi mf}{q} = \frac{2\pi\times3.34\times10^{-27}\times1.2\times10^{7}}{1.6\times10^{-19}} = 1.57 \text{ T}$$

(b) The kinetic energy of the deuteron is given by K.E. = $\frac{q^2B^2R^2}{2m}$

= $\frac{(1.6\times10^{-19}\times1.57\times0.53)^2}{2\times3.34\times10^{-27}}$ = 17× 10^6 eV = 17 MeV.

14. *What must be the magnitude of a uniform electric field if it is to have the same energy density that passed by 0.50T magnetic field?*

Sol^n: The energy density due to electric field is (u_e) = $\frac{\varepsilon_0}{2}$ E^2 and the energy density due to magnetic field is (u_b) = $\frac{B^2}{2\mu_0}$

Here, the energy density due to electric field = energy density due to magnetic field. If u_e = u_b then $\frac{E}{B} = c$.

So, E = cB = 3 × 10^8 × 0.5 V/m = 1.5 × 10^8 V/m

15. *What is Poynting vector? Show that the intensity of an electromagnetic wave equals the average magnetic energy density times the speed of light.*

Sol^n: The energy transferred through unit area in unit time in electromagnetic waves is measured in terms of a vector which is known as Poynting vector. It is given by $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$.

Where E and B are electric and magnetic fields at a point due to the propagation of electromagnetic wave and S is the Poynting vector. Its magnitude gives the intensity of electromagnetic wave. For plane electromagnetic wave E and B are perpendicular to each other.

$S = \frac{EB}{\mu_0}$

Again, E = E_m sin(ωt – kx) and B = B_m sin(ωt – kx)

So, $S = \frac{E_mB_m}{\mu_0}\sin^2(\omega t - kx)$

$S_{ave} = \frac{E_mB_m}{2\mu_0}$

This gives the intensity of electromagnetic wave. i.e. $I = \frac{E_m B_m}{2\mu_0}$

Also, $\frac{E_m}{B_m} = c \quad \Rightarrow E_m = c\, B_m$

$\therefore\ I = \frac{cB_m^2}{2\mu_0} = c\, u_b$

This shows the intensity of an electromagnetic wave equals the average magnetic energy density times the speed of light.

16. *A particle is moving in one dimensional potential well of infinite height and width a. Find the expression for energy of the particle.*

Sol^n: (See in 2067 Ashadh Regular Q. no. 16)

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2068 Magh Back (BCE, BME)

1. *Define free oscillation and write two differences between compound and torsion pendulum. Show that time period of compound pendulum is minimum when k = l.*

Sol^n: If the oscillation in a system is due to only the restoring force produce in that system then the oscillation is called free oscillation. In this oscillation we neglect the effect of damping force.

Compound Pendulum	**Torsion pendulum**
1. Rigid body of any shape suspended from a rigid support about its one point.	1. Rigid body of any shape suspended from a rigid support with the help of a metallic wire.
2. Oscillation due to restoring moment of force produce in the rigid body which is in to and fro motion.	2. Oscillation is due to restoring couple produced is the wire which is in turning oscillation.

The time period of oscillation for compound pendulum is

$$\therefore\ T = 2\pi\sqrt{\frac{l^2+k^2}{lg}}$$

Squaring on both sides, we have

$$T^2 = \frac{4\pi^2}{g}\left(l + \frac{k^2}{l}\right)$$

Differentiating on both sides with respect to l, we get

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left(1 - \frac{k^2}{l^2}\right) \qquad \text{---------} \ (1)$$

For T to be minimum or maximum $\frac{dT}{dl} = 0$

$$\Rightarrow \frac{l^2-k^2}{l^2} = 0 \qquad \therefore\ l = k$$

Again, differentiating Eq. (1) with respect to *l*, we get

$$\left(\frac{dT}{dl}\right)^2 + T\,\frac{d^2T}{dl^2} = \frac{2\pi^2}{g} \times \frac{2k^2}{l^3}$$

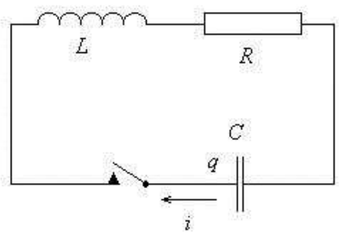
At $l = k$

$$T\,\frac{d^2T}{dl^2} = \frac{4\pi^2}{gk} \qquad \Rightarrow \frac{d^2T}{dl^2} = \frac{4\pi^2}{gTk} > 0$$

So, time period of compound pendulum is minimum when $l = k$.

OR

Why the electromagnetic oscillations are always damping? Explain how can we keep the oscillations continue? Derive the differential equation of the damped em oscillation and write its solution.



Sol^n: In LC oscillation, we neglect the the internal resistance of the inductor and capacitor. But, in actual practice, which is impossible, we cannot neglect the internal resistance of the inductor and capacitor due to which electromagnetic oscillations are always damping. We can keep the oscillations continue by applying the external source of certain frequency.

If we consider the effect of resistor in LC circuit then the oscillation is called damped LCR oscillation. In the circuit there is energy lose due to the present of the resistor and the rate of energy lose by the circuit is $-i^2R$. **i.**e. $\frac{dU}{dt} = -i^2R$

$$\Rightarrow \frac{d}{dt}\left(\frac{q^2}{2C} + \frac{1}{2}Li^2\right) = -i^2R \qquad \Rightarrow \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = -i^2R$$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0 \text{ ---------------------------- (1)}$$

This is the differential equation of damped LCR oscillation. The solution of this equation is

$q = q_m e^{-\frac{Rt}{2L}} \cos(\beta t \pm \theta)$, where β is the angular frequency of the system.

and $\beta = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$.

2. *A wave of frequency 500 Hz has a phase velocity of 200 m/s. (i) How far apart two points 30° out of phase? (ii) What is the phase difference between two displacement at a point at time 10^{-3} s apart?*

Solⁿ: Frequency of wave (f) = 500 Hz

Wave velocity (v) = 200 m/s

(i) Path difference (x) = ?, when phase difference (ϕ) = 30° = $\frac{\pi}{6}$.

$x = \frac{\lambda}{2\pi}\phi = \frac{v}{2\pi f} = \frac{200}{2\pi\times 500}\times\frac{\pi}{6} = 3.33$ cm

(ii) Phase difference (ϕ) = ?, when time (t) = 10^{-3} sec

$\phi = \frac{2\pi}{T}\times t = 2\pi f \times t = 2\pi\times 500\times 10^{-3} = \pi$ radian = 180°

3. *Sound waves are emitted uniformly in all direction from the speaker in a large hall. Prove that the amplitude of the sound waves change with the distance r at any point from the speaker is $a = \frac{1}{r}\sqrt{\frac{P}{2\pi\rho v\omega^2}}$. Where P is power of the sound wave moving with velocity v in the medium of density ρ and ω is the angular frequency.*

Sol^n: The displacement in a medium due to propagation of wave along positive x – direction is $y = a\sin(kx - \omega t)$

The velocity of particle is $u = -a\omega\cos(kx - \omega t)$

And acceleration is $a = -\omega^2 y$

Now, the force produced on the accelerated particle is $F = m\omega^2 y$

If dy be the small displacement then small amount of work done is

$dU = Fdy = m\omega^2 y\, dy$ and $U = \frac{1}{2}m\omega^2 y^2$

$\therefore$ P. E. = $\frac{1}{2}$ mω^2a^2 sin^2(kx - ωt)

Also, kinetic energy is (K.E.) = $\frac{1}{2}$ mu^2 = $\frac{1}{2}$ mω^2a^2 cos^2(kx - ωt)

$\therefore$ Total energy (E) = K.E. + P.E. $\Rightarrow$ E = $\frac{1}{2}$ mω^2a^2

If n be the total number of particles then total energy is E = $\frac{1}{2}$ nmω^2a^2

E = $\frac{1}{2}$ Mω^2a^2 where M is the total mass of the medium.

Now, energy density i.e; energy associated with unit volume is

$\varepsilon = \frac{1}{2}\ \rho\omega^2 a^2$, ρ is the density of the medium.

If l be the length and A is the cross-section of the medium then the total energy is E = $\frac{1}{2}\ \rho\omega^2 a^2\, l$A

The power delivered in forward direction is P = $\frac{E}{t} = \frac{1}{2}\ \rho\omega^2 a^2\, \frac{l}{t}$ A

P = $\frac{1}{2}\ \rho\omega^2 a^2$ vA

Now, the intensity of the wave is given by

I = $\frac{P}{A} = \frac{1}{2}\ \rho\omega^2 a^2$ v $\Rightarrow$ a^2 = $\frac{2P}{\rho\omega^2 vA} = \frac{2P}{\rho\omega^2 v4\pi r^2} = \frac{P}{2\rho\omega^2 v\pi r^2}$

$\therefore$ a = $\frac{1}{r}\sqrt{\frac{P}{2\pi\rho v\omega^2}}$ Proved.

4. *Define the diffraction of light. Show that the intensity of second primary maxima is 1.62% of central maxima in Fraunhoffer's single slit diffraction.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 4 OR)

5. *What are Newton's rings? How are they formed? Show that the diameter of dark Newton's rings by reflected system of light is proportional to the square root of natural number.*

Sol^n: When a Plano-convex lens is placed in contact with a flat glass surface, as in figure, a thin air film is formed. When such film is exposed by monochromatic light, a series of concentric fringes are formed which are called Newton's rings.

(Remaining part see in 2068 Bhadra Regular Q. No. 5)

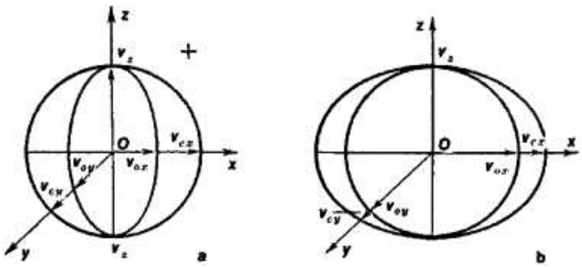
OR

Differentiate between quarter wave plate and half wave plate. Use the reference of double refraction to describe how you distinguish positive and negative crystal. [Describe with a neat diagram]

Sol^n: The difference between quarter and half wave plate are

Quarter wave plate	**Half wave plate**
1. It produces a path difference of $\frac{\lambda}{4}$ and phase difference of 90° between o-ray and e-ray.	1. It produces path difference of $\frac{\lambda}{2}$ and phase difference of 180° between o-ray and e-ray.
2. Thickness of quarter wave plate is given by t = $\frac{\lambda}{4(\mu_0-\mu_e)}$	2. thickness of half wave plate is given by t = $\frac{\lambda}{2(\mu_0-\mu_e)}$

Light passing through a calcite crystal is split into two rays. This process, first reported by Erasmus Bartholinus in 1669, is called double refraction. The two rays of light are each plane polarized by the calcite such that the planes of polarization are mutually perpendicular. For normal incidence (a Snell's law angle of 0°), the two planes of polarization are also perpendicular to the plane of incidence.



For normal incidence (a 0° angle of incidence), Snell's law predicts that the angle of refraction will be 0°. In the case of double refraction of a normally incident ray of light, at least one of the two rays must violate Snell's Law as we know it. For calcite, one of the two rays does indeed obey Snell's Law; this ray is called the ordinary ray (or O-ray). The other ray (and any ray that does not obey Snell's Law) is an extraordinary ray (or E-ray).

Consider a point source of light O in a calcite crystal. The sphere is the wave surface for the ordinary ray and the ellipsoid is the wave surface for the extraordinary ray. The ordinary wave surface lies within the extraordinary wave surface. Such crystals are known as negative crystals. For crystals like quartz, which are known as positive crystals, the extraordinary wave surface lies within the ordinary wave surface.

For the negative uni-axial crystals $\mu_0 > \mu_e$. The velocity of the extraordinary ray varies as the radius vector of the ellipsoid. It is least and equal to the velocity of the ordinary ray along the optic axis but it is maximum at right angles to the direction of the optic axis.

For the positive uniaxial crystals $\mu_e > \mu_0$. The velocity of the extraordinary ray is least in a direction at right angles to the optic axis. It is maximum and is equal to the velocity of the ordinary ray along the optic axis.

6. *Define acceptance angle in optical fiber. Show that, Numerical Aperture (NA) =* $\mu_1\sqrt{2\Delta}$*; where* μ_1*: refractive index of core of optical fiber,* Δ *: fractional refractive index change.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 7)

7. *Two thin converging lenses of focal lengths 20cm and 40cm are placed co-axially 20cm apart. An object is located at a distance of 48 cm from first lens. Find the positions of principal points and image.*

Sol^n: Here, f_1 = 20cm, f_2 = 40cm and d = 20cm

We have the equivalent focal length of the combination is

$$f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{20\times40}{20+40-20} = 20\text{cm}$$

first principal point (α) = $\frac{fd}{f_2} = \frac{20\times20}{40}$ = 10cm

second principal point (β) = $-\frac{fd}{f_1} = -\frac{20\times20}{20}$ = -20cm

distance of the object from first lens (u) = 48 cm

object position from first principal point (U) = - 58cm

From lens formula

$$\frac{1}{f} = \frac{1}{V} - \frac{1}{U} \qquad \Rightarrow \frac{1}{20} = \frac{1}{V} + \frac{1}{58}$$

$$\Rightarrow \frac{1}{V} = \frac{1}{20} - \frac{1}{58} \qquad \therefore V = 30.53 \text{ cm}$$

So, the image distance from the second lens = 10.53cm towards right.

8. *A 200 mm long tube containing 48cm^3 of sugar solution produces an optical rotation of 11° when placed on a Sacchari meter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of solution.*

Sol^n: (See in 2068 Bhadra Regular Q. No. 6)

9. *Derive an expression for the electric field at a point P at a distance 'Z' from a circular plastic disc of radius R along its central axis. Explain what will happen to the electric field if (i) R →∞, with keeping Z finite and (ii) Z→∞; while keeping R finite.*

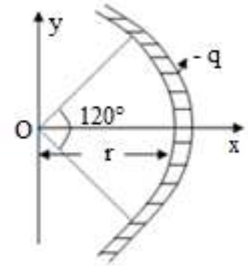
Sol^n: (See in 2067 Ashwin Back Q. No. 9)

(i) If R → ∞, with keeping z finite E = $\frac{\sigma}{2\varepsilon_0}$

(ii) If z → 0, and keeping R finite. E = $\frac{\sigma}{2\varepsilon_0}$.

OR

A plastic rod contains uniformly distributed charge – q. The rod has been bent in 120° circular arc of radius r as shown in figure below. Prove that the electric intensity at the center of the bent rod is E = $\frac{0.83q}{4\pi\varepsilon_0 r^2}$.



Sol^n: Let the coordinate system be located at the center of the circle. Let us choose x-axis so that it divides the bent rod into two equal parts. Now the charge on the elemental length dλ is dq = $\frac{q}{\lambda}$ dλ, where λ is half length of the rod. dq = $q\frac{d\theta}{\theta_0/2} = 2q\frac{d\theta}{\theta_0}$

The field at O, the center of the circle is dE = $\frac{dq}{4\pi\varepsilon_0 r^2} = \frac{2qd\theta}{4\pi\varepsilon_0\theta_0 r^2}$

Since x-component of the field due to dq is

$dE_x = dE\cos\theta = \frac{2qd\theta}{4\pi\varepsilon_0\theta_0 r^2}\cos\theta = \frac{2q\cos\theta}{4\pi\varepsilon_0\theta_0 r^2}d\theta$

As y-component of the field would be vanish upon integration.

$\therefore\ E = \int dE_x = \int_0^{\theta_0/2}\frac{2q\cos\theta}{4\pi\varepsilon_0\theta_0 r^2}\,d\theta = \frac{2q}{4\pi\varepsilon_0\theta_0 r^2}\int_0^{\theta_0/2}\cos\theta d\theta$

$= \frac{2q}{4\pi\varepsilon_0\theta_0 r^2}\ [\sin\theta]_0^{60^0} = \frac{q}{2\pi\varepsilon_0\theta_0 r^2}\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{\theta_0}\times\frac{q}{4\pi\varepsilon_0 r^2}$

$\theta_0 = \frac{22}{7\times180}\times 120 = 2.095$

$E = \frac{0.83q}{4\pi\varepsilon_0 r^2}$.

10. *If a parallel plate capacitor is to be designed to operate in an environment of fluctuating temperature. Prove that the rate of change of capacitance with temperature T is given by* $\frac{dC}{dT} = C\left[\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right]$. *Where symbols carry their usual meanings.*

Sol^n: For a parallel plate capacitor of plate separation x, the capacitance is $C = \frac{\varepsilon_0 A}{x}$, where A is the cross-section of the plate.

Now, taking log on both sides

$\ln C = \ln\varepsilon_0 + \ln A - \ln x$

Differentiating with respect to T, we have

$$\frac{d}{dT}(\ln C) = \frac{d}{dT}(\ln\varepsilon_0) + \frac{d}{dT}(\ln A) - \frac{d}{dT}(\ln x)$$

$$\frac{1}{C}\frac{dC}{dT} = \frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}$$

$$\therefore \frac{dC}{dT} = C\left(\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right)$$

11. *Calculate the relaxation time for electron of sodium atom. The number of atoms per cubic cm in sodium is 2.5×10^{22}, and the electrical conductivity is 1.9×10^{7} s/m.*

So^n: (See in 2068 Bhadra Regular Q. No. 11)

12. *Derive the relation for magnetic field on the axis of a circular loop and show that a circular current carrying coil behaves as a magnetic dipole for a large distance.*

Sol^n: (See in 2067 Mangsir Regular Q. No. 14)

13. *What radius is needed in proton synchrotron to attain particle energy of 15 GeV, assuming that a guide field of 2.0 wb/m^3 is available? Rest mass of the proton is 1.007529 amu.*

Sol^n: The rest mass of proton is 1.007529 amu, so its corresponding energy, $M_0c^2 = M_0 \times 931$ MeV $= M_0 \times 0.931$ GeV $= 1.007529 \times 0.931$ GeV $= 0.938$ GeV

Hence, the total energy required = (15 + 0.938) GeV and equivalent mass of proton = 35.41 unit = $35.41 \times 1.67 \times 10^{-27}$ kg

Hence, $Bev = \frac{mv^2}{r}$

$r = \frac{mv}{Be} = \frac{35.41 \times 1.67 \times 10^{-27} \times 3 \times 10^{8}}{2 \times 1.6 \times 10^{-19}} = 55.12$ m

OR

A parallel plate capacitor with circular plates is being charged by time varying electric field of 1.5 $\times 10^{12}$V/ms. Calculate the induced magnetic field if the radius of the plate is 55mm and displacement current of the system.

Sol^n: The rate of change of electric field $\frac{dE}{dt} = 1.5 \times 10^{12}$ V m^{-1} s^{-1}

Radius of the plate (r) = 55 mm = 5.5×10^{-2} m

The displacement current (i_d) = $\varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \times \pi r^2 \times \frac{dE}{dt}$

$i_d = 8.85 \times 10^{-12} \times \pi \times (5.5 \times 10^{-2})^2 \times 1.5 \times 10^{12} = 126$ mA

Displacement current density (J_d) = $\frac{i_d}{\pi r^2} = \frac{0.126}{\pi \times (5.5 \times 10^{-2})^2} = 13.26$ A/m^2

Induced magnetic field (B) = $\frac{1}{2} \mu_0 r J_d$

$B = \frac{1}{2} \times 4\pi \times 10^{-7} \times 5.5 \times 10^{-2} \times 13.26 = 4.58 \times 10^{-7}$ T = 458 nT

14. *Derive the relation for rise and fall of current in LR circuit. Plot a graph between current and time and explain the figure.*

Sol^n: **(See in 2067 Ashwin Back Q. No. 14)**

15. *Write Maxwell's electromagnetic wave equations in dielectric medium. Obtain electromagnetic wave equations for $\vec{E}$ and $\vec{B}$ in both dielectric medium and in free space. Compare velocity of electromagnetic wave in dielectric medium to free space.*

Sol^n: Maxwell equations for any medium are

$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$ $\qquad$ $\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\qquad$ $\nabla \times \vec{B} = \mu\left(\sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$

For dielectric medium charge density is zero and conductivity is also zero. So,

$\nabla \cdot \vec{E} = 0$ ---------- (1) $\quad$ $\nabla \cdot \vec{B} = 0$ ---------- (2)

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ---------- (3) $\quad$ $\nabla \times \vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$ ---------- (4)

Now, taking curl on both sides of Eq. (3)

$\Rightarrow \nabla \times \nabla \times \vec{E} = -\frac{\partial(\nabla \times \vec{B})}{\partial t}$ $\quad$ $\Rightarrow \nabla(\nabla \cdot \vec{E}) - (\nabla \cdot \nabla)\vec{E} = -\frac{\partial}{\partial t}\left(\mu\varepsilon \frac{\partial \vec{E}}{\partial t}\right)$

$\Rightarrow -\nabla^2\vec{E} = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ $\quad$ $\therefore\ \nabla^2\vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ ---------- (5)

Again, taking the curl on both sides of Eq. (4), we have

$\nabla \times \nabla \times \vec{B} = \mu\varepsilon \frac{\partial(\nabla \times \vec{E})}{\partial t}$ $\quad$ $\Rightarrow \nabla(\nabla \cdot \vec{B}) - (\nabla \cdot \nabla)\vec{B} = -\frac{\partial}{\partial t}\left(\mu\varepsilon \frac{\partial \vec{B}}{\partial t}\right)$

$\Rightarrow -\nabla^2\vec{B} = -\mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$ $\quad$ $\therefore\ \nabla^2\vec{B} = \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$ -------- (6)

Comparing Eq. (5) and Eq. (6) with $\nabla^2\Psi = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$, we get $v = \frac{1}{\sqrt{\mu\varepsilon}}$

Also, $\mu = \mu_r\mu_0$ and $\varepsilon = \varepsilon_r\varepsilon_0$ both μ_r and ε_r greater than one.

$\therefore\ v = \frac{1}{\sqrt{\mu_r\varepsilon_r\mu_0\varepsilon_0}} = \frac{1}{\sqrt{\mu_r\varepsilon_r}}\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$

This shows that electromagnetic wave travels with velocity less than velocity of light in such medium.

16. *Calculate the permitted energy levels of an electron in one dimensional potential well of width 0.2nm.*

Sol^{n}: Width of the potential well (a) = 0.2nm = 0.2×10^{-9} m

The permitted energy levels are given by

$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} = \frac{\pi^2 n^2 (1.05\times10^{-34})^2}{2\times9.1\times10^{-31}\times(0.2\times10^{-9})^2} = 9.34\ n^2$ eV

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2068 Chaitra Regular (BEX, BCT, BEL, B. Agri.)

1. Differentiate between linear and angular harmonic motion. Show that the motion of torsion pendulum is angular harmonic motion. Also find its time period.

Sol^n : Difference between linear and angular motion:

Linear harmonic motion	Angular harmonic motion
1. If the displacement of the simple harmonic motion is measured in terms of linear displacement then the motion is linear harmonic.	1. If the displacement is measured in terms of angle then the motion is angular harmonic motion.
2. In linear harmonic motion, acceleration is $a = \omega^2 y$.	2. In the angular harmonic motion $\alpha = \omega^2\theta$.
3. Newton's second law of motion is $F = ma$.	3. Newton's second law of motion is $\tau = I\alpha$.

(Second part see in 2068 Baishakh Regular Q. No. 1)

OR

Derive the differential equation of the forced oscillation of LCR circuit with an AC source and find the expression for the current amplitude. Hence explain the condition of current resonance in such circuit.

Sol^n : (See in 2067 Mangsir Regular Q. No. 1 OR)

When $L\omega = \frac{1}{C\omega}$ then current amplitude in the circuit is maximum. This condition is current resonance. At current resonance current amplitude is $(i_0)_{max} = \frac{E_0}{R}$.

2. *A 750 gm block oscillates on the end of a spring whose force constant, k = 56 N/m. The mass moves in a fluid which offers a resistive* *force F = - bv, where b = 0.162 Ns/m. What is the period of the oscillation?*

Sol^n: Here, mass of block (m) = 750gm = 0.75 kg

Force constant (K) = 56 N/m

Damping constant or coefficient (b) = 0.162 N s/m

Angular frequency for damped oscillation is

$$\beta = \sqrt{\frac{K}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{56}{0.75} - \frac{0.162^2}{4\times0.75^2}} = 8.64$$

$$\therefore\ T = \frac{2\pi}{\beta} = 0.727 \text{ sec}$$

3. *A room has dimensions 6m × 4m × 5m. Find: (i) mean free path of sound wave in the room (ii) the number of reflections made per second by the sound wave with the walls of the room. (Take velocity of sound in air = 350m/s).*

Sol^n: Volume of the room (V) = 6×4×5 = 120 m^3

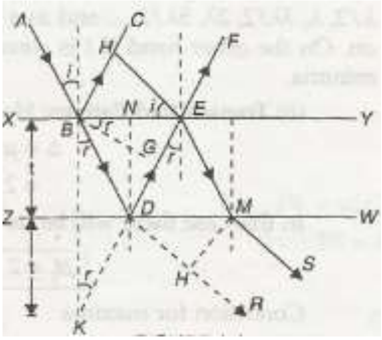
Total area of the room (S) = 2 × (4 × 6 + 4 × 5 + 6 × 5) = 148 m^2

(i) Mean free path (λ) = $4\frac{V}{S} = 4 \times \frac{120}{148} = 3.24$ m

(ii) Number of reflection per second (n) = $\frac{v}{\lambda} = \frac{350}{3.24} = 108$

4. *Define interference. Show that interference in thin film due to reflected and transmitted lights are complementary.*

Sol^n: The phenomenon of non-uniform distribution of intensity in a medium due to the superposition of light waves from two coherent sources is known as interference.



Interference in thin film due to reflected light:

Consider a film of uniform thickness t as shown in figure. Suppose a ray AB be incident on its upper surface at an angle of incidence i. Part of the incident light is reflected towards BC and the other part is refracted into the medium of refractive index μ towards BD. At D, the ray gets partly reflected towards DE and part of that emerges out as ray EF at E. Let r be angle of refraction of the ray at B.

Now the path difference between two reflected rays is

$x = \mu(BD + DE) – BH = \mu(BD + DE) – BE \sin i$

$\Rightarrow x = \mu\left(\frac{DN}{\cos r} + \frac{DN}{\cos r}\right) – BE \sin i = \frac{2\mu t}{\cos r} – 2\ BN \sin i$

$\Rightarrow x = \frac{2\mu t}{\cos r} – 2t \tan r\ \mu \sin i\ = \frac{2\mu t}{\cos r} – 2\mu t\ \frac{\sin^2 r}{\cos r}$

$\Rightarrow x = \frac{2\mu t}{\cos r}(1 – \sin^2 r) = 2\mu t \cos r$

This is the optical path difference. We know, when light encounters a medium of higher refractive index the reflected wave suffers a phase change of π i.e. a path change of $\frac{\lambda}{2}$. So, net path difference is

$\Delta = 2\mu t \cos r + \frac{\lambda}{2}$

For maxima, $2\mu t \cos r + \frac{\lambda}{2} = n\lambda$

$2\mu t \cos r = n\lambda - \frac{\lambda}{2} = (2n – 1)\frac{\lambda}{2}$; n = 1, 2, 3, ………..

For minima, $2\mu t \cos r + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$

$2\mu t \cos r = n\lambda$; n = 0, 1, 2, 3, ……..

In case of transmitted light there is no phase change i.e. actual path difference is $2\mu t \cos r$.

For maxima, $2\mu t \cos r = n\lambda$; n = 0, 1, 2, 3, ……..

For minima, $2\mu t \cos r = (2n – 1)\frac{\lambda}{2}$; n = 1, 2, 3, ………..

From above relations, we get that interference in thin film due to reflected and transmitted lights are complementary.

OR

What are Newton's rings? How can you determine the refractive index of given liquid using Newton's rings experiment?

Sol^n: (See in 2067 Ashwin Back Q. No. 4)

5. *Explain the dispersive and resolving power of a diffraction grating. Derive expressions and develop a relation between them.*

Sol^n: (See in 2068 Bhadra Regular Q. No. 5 OR)

From Eq. (1) and Eq. (2) $\quad \therefore \frac{d\theta}{d\lambda} = \frac{1}{\cos} \frac{\lambda}{d\lambda}$

This gives the relation between dispersive power and resolving power of the grating.

6. *A 200 mm long tube containing 48cm^3 of sugar solution produces an optical rotation of 11° when placed on a saccharimeter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of solution.*

Sol^n: (See in 2068 Bhadra Regular Q. No. 6)

7. *Prove that the condition for achromatism for the combination of two lenses of focal length f_1 and f_2 having dispersive power ω_1 and ω_2 placed at a separate distance x is* $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{x}{f_1 f_2}(\omega_1 + \omega_2)$.

Sol^n: We know the equivalent focal length for the combination of two lenses of focal lengths f_1 and f_2 and dispersive powers ω_1 and ω_2 placed at a distance x is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} \text{ ----------- (1)}$$

Differentiating Eq. (1), we have

$$-\frac{df}{f^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2} - x(-\frac{df_1}{f_2 f_1^2} - \frac{df_2}{f_1 f_2^2})$$

$$\Rightarrow \frac{df}{f^2} = \frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} + \frac{x}{f_2 f_1}\left(-\frac{df_1}{f_1} - \frac{df_2}{f_2}\right)$$

But for achromatism, df = 0.

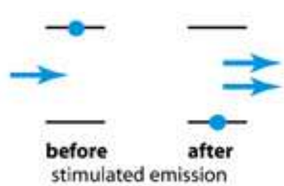
Again, $\frac{df_1}{f_1} = -\omega_1$, $\frac{df_2}{f_2} = -\omega_2$ $\quad \Rightarrow -\frac{\omega_1}{f_1} - \frac{\omega_2}{f_2} + \frac{x}{f_1 f_2}(\omega_1 + \omega_2) = 0$

$\Rightarrow \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{x}{f_1 f_2}(\omega_1 + \omega_2)$ Proved

8. *Differentiate between spontaneous and stimulated emission of radiation. Explain the construction and working of He-Ne laser with a suitable energy level diagram.*

Sol^n: A laser gain medium contains some kind of laser-active atoms or ions, which have different energy levels (states), and a mechanism to put the atoms (or ions) into a certain excited state.

If an atom is in an excited state, it may spontaneously decay into a lower energy level after some time, releasing energy in the form of a photon, which is emitted in a random direction. This process is called spontaneous emission. It is also possible that the emission is stimulated by incoming photons, which is called stimulated emission. The emission then goes into the same direction as the incoming photon. In effect, the incoming radiation is amplified. This is the physical basis of light amplification in amplifiers and lasers.



Of course, stimulated emission can only occur for incoming photons that have photon energy close to the energy of the laser transition. Therefore, the laser gain occurs only for optical frequencies (or wavelengths) within a limited gain bandwidth. A laser normally operates at the optical wavelength where the gain medium provides the highest gain.

(Second part see in 2067 Ashwin Back Q. No. 8)

9. *Derive an expression for the electric field and at a point P at a distance X from a circular plastic disc of radius a along its central axis. Does this expression for E reduces to an expected result for x >>a?*

Sol^n: (See in 2067 Ashwin Back Q. No. 9)

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{(\sqrt{x^2+a^2})}\right)$$

If x >> a, E = 0

10. *A capacitor of capacitance 'C' is discharged through a resistor of resistance 'R'. After how many time constants is the energy stored becomes one fourth of initial value?*

Sol^n: The energy stored in the capacitor is given by $U_E = \frac{q^2}{2C}$ Also, in case of discharging, $q = q_0\, e^{-t/RC}$ $\Rightarrow U_E = \frac{q_0^2}{2C} e^{-\frac{2t}{RC}} = U_0\, e^{-\frac{2t}{RC}}$ where $U_0 = \frac{q_0^2}{2C}$

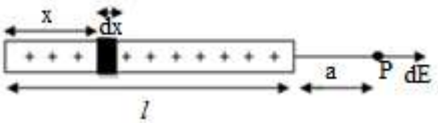
According to question, $U = \frac{U_0}{4}$ $\Rightarrow \frac{U_0}{4} = U_0\, e^{-\frac{2t}{RC}}$ $\Rightarrow \ln(4) = \frac{2t}{RC}$

$$t = \frac{RC\ln(4)}{2} = \frac{\tau_c \ln(4)}{2} = 0.693\tau_c$$

where τ_c = RC is capacitive time constant.

11. *Calculate the electric field due to a uniformly charged rod of length l at a point along its long axis at a distance 'a' from its nearest end.*

Sol^n: Let the rod be lying along x-axis and has uniform positive charge per unit length λ. The rod be divided into elementary segment of length dx and the



electric field intensity due to this segment which is at a distance x from far end of the rod is

$$dE = \frac{\lambda dx}{4\pi\varepsilon_0 (l+a-x)^2}$$

Total field at P due to all segments is

$$E = \int dE = \int_0^l \frac{\lambda dx}{4\pi\varepsilon_0(l+a-x)^2} = \frac{\lambda}{4\pi\varepsilon_0}\int_0^l \frac{dx}{(l+a-x)^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{l+a-x}\right)_0^l = \frac{\lambda}{4\pi\varepsilon_0}\frac{l}{a(l+a)} = \frac{q}{4\pi\varepsilon_0 a(l+a)}$$

12. *Explain the principle and working of cyclotron. Show that the time spent by the particle in a dee is independent of its speed and radius of its circular path.*

Sol^n: (See in 2067 Chaitra Back Q. No. 13)

The time period of the motion is

$$T = \frac{2\pi R}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

This shows the time spent by the particle in a Dee is independent of its speed and radius of the circular path.

OR

Use Biot-Savart law to calculate magnetic field on the axial line of a current carrying circular coil. Explain how the coil behaves for a large distance point.

Sol^n: (See in 2067 Mangsir Regular Q. No. 14)

13. *A copper strip 150μm thick is placed in a magnetic field of strength 0.65T perpendicular to the plane of the strip and current of 23Amp is set up in the strip. Calculate (i) the Hall voltage (ii) Hall coefficient and (iii) Hall mobility, if the number of electrons per unit volume is $8.5\times10^{28}m^{-3}$ and resistivity is 1.72×10^{-8} Ohm-m.*

Sol^n: Here, electron density (n) = $8.5\times10^{28}m^{-3}$

Resistivity of the copper (ρ) = 1.72×10^{-8} Ohm-m

Thickness of the strip (t) = 150μm = 1.5×10^{-4} m

Strength of magnetic field (B) = 0.65 T

Current through the strip (i) = 23 Amp

(i) Hall voltage (V_H) = $\frac{\mathrm{Bi}}{\mathrm{net}} = \frac{0.65\times23}{8.5\times10^{28}\times1.6\times10^{-19}\times1.5\times10^{-4}}$ = 7.33 μV

(ii) Hall coefficient (R_H) = $\frac{1}{\mathrm{ne}} = \frac{1}{8.5\times10^{28}\times1.6\times10^{-19}}$ = 7.35×10^{-11}

(iii) Hall mobility (μ_H) = $\frac{R_H}{\rho} = \frac{7.35\times10^{-11}}{1.72\times10^{-8}}$ = $4.27 \times 10^{-3}\, m^2\, V^{-1} s^{-1}$

14. *A parallel plate capacitor with circular plates of 10 cm radius is charged producing uniform displacement current of magnitude 20A/m^2. Calculate (i) $\frac{dE}{dt}$ in the region (ii) displacement current density and (iii) induced magnetic field.*

Sol^n: Here, radius of the capacitor plate (r) = 10cm = 0.1 m

Displacement current density (J_d) = 20 A/m^{-2}

(i) $i_d = \varepsilon_0 A \frac{dE}{dt} \Rightarrow \frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{J_d}{\varepsilon_0} = \frac{20}{8.85\times10^{-12}} = 2.3 \times 10^{12}\ Vm^{-1} s^{-1}$

(ii) J_d = 20 A/m^{-2}

(iii) Induced magnetic field (B) = $\frac{1}{2} \mu_0 r J_d$

$B = \frac{1}{2} \times 4\pi \times 10^{-7} \times 0.1 \times 20 = 1.26 \times 10^{-6} T = 1.26\ \mu T$

15. *Obtain an expression for energy transfer rate by electromagnetic wave. From your result show that $I \propto E^2_{rms}$. Where I is the intensity em wave and E_{rms} is root mean square value of electric field.*

Soln: (First part see in 2067 Ashadh Regular Q. No. 15)

Again, $S = \frac{E_m B_m}{\mu_0} \cos^2(\omega t - kx)$, So, $I = S_{ave} = \frac{E_m B_m}{2\mu_0}$

$I = S_{ave} = \frac{E_{rms} B_{rms}}{\mu_0} = \frac{E^2{}_{rms}}{c\mu_0} \quad \Rightarrow I \propto E^2{}_{rms}$.

16. *Derive the Schrodinger time independent wave equation. Also, what do you mean by a potential barrier?*

Sol^n: The wave function is $\Psi = A\ e^{-\frac{i}{\hbar}(Et-px)}$

Differentiating this equation with respect to x, twice times, we get

$$\frac{d\Psi}{dx} = \frac{Aip}{\hbar} e^{-\frac{i}{\hbar}(Et-px)} \text{ and } \frac{d^2\Psi}{dx^2} = \frac{Ai^2p^2}{\hbar^2} e^{-\frac{i}{\hbar}(Et-px)} = -\frac{p^2}{\hbar^2}\Psi$$

$$P^2\Psi = -\hbar^2 \frac{d^2\Psi}{dx^2}$$

If a particle of mass m and potential energy V is moving with a velocity v, then total energy of the system is

$$E = \frac{1}{2} mv^2 + V = \frac{p^2}{2m} + V \qquad \Rightarrow E\Psi = \frac{p^2\Psi}{2m} + V\Psi$$

$$\Rightarrow P^2\Psi = 2m(E - V)\Psi \qquad \Rightarrow -\hbar^2 \frac{d^2\Psi}{dx^2} = 2m(E - V)\Psi$$

$$\therefore \frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\Psi = 0$$

This is time independent Schrodinger wave equation in one dimension. For three dimension,

$$\nabla^2\Psi + \frac{2m}{\hbar^2}(E - V)\Psi = 0, \text{ where } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

If the obstacle for the particle is of potential region then that is known as potential barrier.

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2069 Ashad Back (BEL, BEX, BCT, BIE B. Agri)

1. *Obtain an expression for the time period of a compound pendulum and show that its time period is unaffected by the fixing of a small additional mass to it at its center of suspension.*

Sol^n: (First part see in 2067 Chaitra Back Q. No. 1)

Second part:

$\therefore$ T = $2\pi\sqrt{\frac{l^2+k^2}{lg}}$ This expression shows that the time period of oscillation is independent on the mass of the body. So, the period of compound pendulum is unaffected by the fixing of a small additional mass to its center of suspension.

OR

What is electromagnetic oscillation? Derive differential equation of damped LCR oscillation and find its frequency.

Sol^n: If there is variation in electric and magnetic energy continuously in the circuit then the oscillation is known as electromagnetic oscillation.

(Remaining see in 2067 Ashadh Regular Q. No. 1 OR)

2. *A particle is moving with simple harmonic motion in a straight line. If it has a speed v_1 when the displacement is x_1 and speed v_2 when the displacement is x_2 then show that the amplitude of the motion is*

$$a = \left[\frac{v_2^2x_1^2-v_1^2x_2^2}{v_2^2-v_1^2}\right]^{1/2}.$$

Sol^n: Here, the speed of a particle executing simple harmonic motion when the displacement x_1 is v_1, and when the displacement x_2 is v_2. We have, the speed of a particle executing simple harmonic motion is

$v = \omega\sqrt{a^2 - x^2}$

So, $v_1 = \omega\sqrt{a^2 - x_1^2}$ and $v_2 = \omega\sqrt{a^2 - x_2^2}$

Now, squaring and dividing these equations, we have $\frac{v_1^2}{v_2^2} = \frac{a^2 - x_1^2}{a^2 - x_2^2}$

$\Rightarrow a^2 v_1^2 - v_1^2 x_2^2 = a^2 v_2^2 - v_2^2 x_1^2$ $\qquad \Rightarrow a^2 (v_2^2 - v_1^2) = v_2^2 x_1^2 - v_1^2 x_2^2$

$\Rightarrow a^2 = \frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}$ $\qquad \therefore a = \left[\frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}\right]^{1/2}$

3. *In the progressive wave, show that the potential energy and kinetic energy of every particle will change with time but the average kinetic energy per unit volume and potential energy per unit volume remains constant.*

Sol^n: The displacement of a particle in a medium due to the propagation of wave along the positive x-direction is $y = R \sin(\omega t - kx)$ and the velocity of that particle is $u = \omega R \cos(\omega t - kx)$. Both displacement and velocity depends upon the position of the particle. When the particle is in extreme position, the particle velocity is minimum and displacement is maximum and when the particle is in mean position then the particle velocity is maximum and displacement is minimum.

The potential energy associated with the particle is P. E. $= \frac{1}{2} m\omega^2 y^2$

P.E. $= \frac{1}{2} m\omega^2 R^2 \sin^2(\omega t - kx)$.

Similarly, the kinetic energy associated with that particle is K.E. $= \frac{1}{2} mu^2$

K.E. $= \frac{1}{2} m\omega^2 R^2 \cos^2(\omega t - kx)$.

These equations shows that both kinetic and potential energies are depends upon time.

Kinetic energy per unit volume (k.e.) $= \frac{1}{2} \rho\omega^2 R^2 \cos^2(\omega t - kx)$, where ρ is the density of the medium. Now, the average kinetic energy per unit volume $= \frac{\int_0^T \rho\omega^2 R^2 \cos^2(\omega t - kx)}{2T} = \pi^2 \rho f^2 R^2$

Also, potential energy per unit volume (p.e.) = $\frac{1}{2}\rho\omega^2R^2\sin^2(\omega t - kx)$

The average potential energy per unit volume is = $\frac{\int_0^T \rho\omega^2R^2\sin^2(\omega t - kx)}{2T}$

$=\pi^2 \rho f^2 R^2$

From above relations the average kinetic energy per unit volume and average potential energy per unit volume are independent in time.

4. *Two coherent sources having constant phase δ but different amplitudes A_1 and A_2 superimpose, prove that the intensity of superimposed beam is $I = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$.*

Sol^n: Consider two coherent sources producing waves of amplitude A_1 and A_2 with phase difference of δ then two waves are

$y_1 = A_1 \sin\omega t$ and $y_2 = A_2 \sin(\omega t + \delta)$

Resultant wave due to the superposition of these waves is $y = y_1 + y_2$

$y = A_1 \sin\omega t + A_2 \sin(\omega t + \delta) = A_1 \sin\omega t + A_2 \cos\delta \sin\omega t + A_2 \sin\delta \cos\omega t$

$y = (A_1 + A_2 \cos\delta) \sin\omega t + A_2 \sin\delta \cos\omega t$

Let, $A_1 + A_2 \cos\delta = R\cos\phi$ ---(1) and $A_2 \sin\delta = R\sin\phi$ ----- (2)

Then, $y = R \sin\omega t \cos\phi + R \sin\phi \cos\omega t$

$= R \sin(\omega t + \phi)$, R is the amplitude of resultant wave. Squaring and adding Eq. (1) and Eq. (2), we have

$(A_1 + A_2 \cos\delta)^2 + (A_2 \sin\delta)^2 = R^2$

$\Rightarrow R^2 = A_1^2 + A_2^2 \cos^2\delta + 2A_1 A_2 \cos\delta + A_2^2 \sin^2\delta$

$\Rightarrow R^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos\delta$

Since $I \approx R^2$

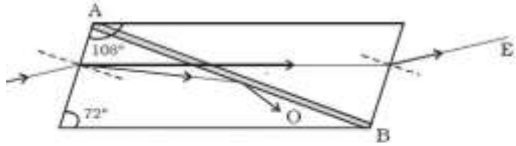
$\therefore I = A_1^2 + A_2^2 + 2A_1 A_2 \cos\delta$ Proved.

OR

Explain the phenomenon of double refraction. Describe the construction and action of Nicol prism.

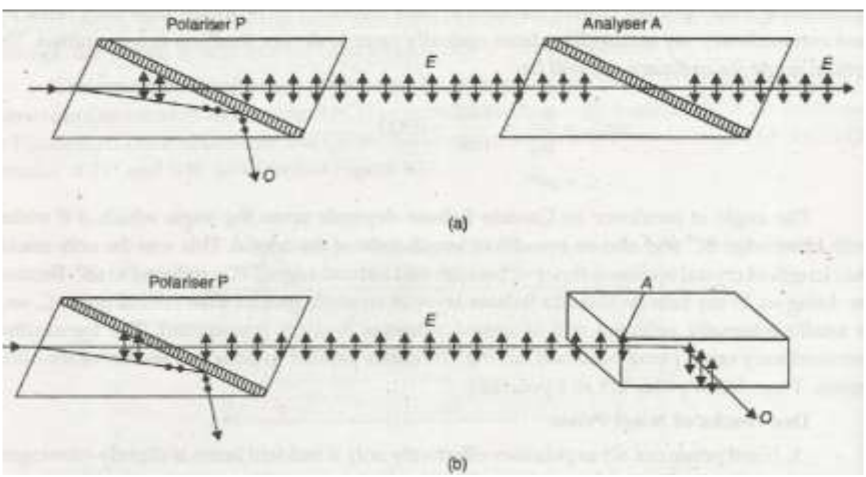
Sol^n: Light passing through a calcite crystal is split into two rays. This process, first reported by Erasmus Bartholinus in 1669, is called double refraction. The two rays of light are each plane polarized by the calcite such that the planes of polarization are mutually perpendicular. For normal incidence (a Snell's law angle of 0°), the two planes of polarization are also perpendicular to the plane of incidence.

For normal incidence (a 0° angle of incidence), Snell's law predicts that the angle of refraction will be 0°. In the case of double refraction of a normally incident ray of light, at least one of the two rays must violate Snell's Law as we know it. For calcite, one of the two rays does indeed obey Snell's Law; this ray is called the ordinary ray (or O-ray). The other ray (and any ray that does not obey Snell's Law) is an extraordinary ray (or E-ray).



Nicol Prism: It is an optical device made from calcite crystal and used in many instruments for producing and analyzing plane polarized light.

A calcite crystal about 3 times as long its diagonal as that of width is



taken. The diagonal surfaces are grounded, polished optically flat and cemented with Canada balsam which is clear transparent cement whose refractive index lies mid-way between the refractive indices of calcite for the ordinary and the extra-ordinary rays. The sides of prism are blackened to absorb the totally reflected light.

When a ray of light is incident on Nicol prism, it splits the ray into extra-ordinary ray and ordinary ray due to double refraction of the crystal. For ordinary ray, Canada balsam works as rarer medium, so it suffers total internal reflection. For extra-ordinary ray, Canada balsam works as denser medium; so it does not escape out from the prism and only e-ray is observed after refraction.

5. *White light is incident on a soap film at an angle* $sin^{-1}\left(\frac{4}{5}\right)$ *and the reflected light on examination by a spectrometer shows dark bands. The consecutive dark bands are overlapping correspond to wavelength* 6.1×10^{-5} *cm and* 6.0×10^{-5} *cm. If* $\mu = 1.33$ *for the film, calculate its thickness.*

Sol^n: Here, $\lambda_1 = 6.1\times10^{-5}$ cm $\lambda_2 = 6.0\times10^{-5}$ cm, $i = \sin^{-1}\left(\frac{4}{5}\right)$, $\mu = 1.33$

According to question nth order dark band of λ_1 overlap with the (n+1)th order dark band of λ_2.

$\therefore n\,\lambda_1 = (n+1)\,\lambda_2$

$\Rightarrow n \times = 6.1\times10^{-5} = (n+1)\times6.0\times10^{-5}$ $\qquad \therefore n = 60$

$\sin i = \frac{4}{5}$, $\qquad$ also, $\mu = \frac{\sin i}{\sin r}$ $\quad \Rightarrow \sin r = \frac{\sin i}{\mu} = 0.6$

$\cos r = \sqrt{1-\sin^2 r} = 0.8$

$2\mu t \cos r = n\lambda_1$

$$t = \frac{n\lambda_1}{2\mu\cos r} = \frac{60\times6.1\times10^{-5}}{2\times1.33\times0.8} = 1.72\times10^{-5}\text{ m} = 1.72\times10^{-2}\text{ mm}$$

6. *Light of wavelength 600nm is incident normally on slit of width 0.1mm. Calculate the intensity at θ = 0.2°.*

Sol^n: Here, the wavelength of light (λ) = 600nm = 6×10^{-7} m,

Slit width (a) = 0.1 mm = 0.1×10^{-3} m, angle of diffraction (θ) = 0.2°

The intensity at a point for an angle of diffraction (θ) is given by

$I_\theta = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2$,

where $\alpha = \frac{\pi}{\lambda} a \sin\theta = \frac{\pi}{6\times10^{-7}} \times 0.1 \times 10^{-3} \times \sin(0.2°) = 1.83$ rad

$\therefore\ I_\theta = I_0 \left(\frac{\sin 1.83}{1.83}\right)^2 = 0.28\ I_0.$

7. *Two lenses of focal lengths 8cm and 4cm are placed at a certain distance apart. Calculate the position of principal points if they form an achromatic combination.*

Sol^n: Here, f_1 = 8cm, f_2 = 4cm

For achromatic combination, $d = \frac{f_1+f_2}{2} = 6$cm

We have, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ $\Rightarrow f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{8\times4}{8+4-6} = \frac{16}{3}$ cm

$\beta = -\frac{df}{f_1} = -\frac{6\times16}{3\times8} = -4$ cm

$\alpha = \frac{df}{f_2} = \frac{6\times16}{3\times4} = 8$ cm

8. *An optical fiber has a NA of 0.2 and a cladding refractive index of 1.59. Determine acceptance angle for the fiber in water which has a refractive index of 1.33.*

Sol^n: Numerical Aperture (NA) = 0.2,

Cladding refractive index (μ_2) = 1.59

Water of refractive index (μ_0) = 1.33

$NA = \sqrt{\mu_1^2 - \mu_2^2}$ $\Rightarrow 0.2 = \sqrt{\mu_1^2 - 1.59^2}$

$\Rightarrow 0.04 = \mu_1^2 - 2.5281$ $\Rightarrow \mu_1^2 = 2.5681$

$\therefore$ $\mu_1 = 1.6025$

When the fiber is in water

$$NA = \frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_0} = \frac{\sqrt{1.6025^2 - 1.59^2}}{1.33} = 0.15$$

Acceptance angle (i_0) = $\sin^{-1}(0.15) = 8.6°$

9. *A ring has a charge q uniformly distributed in it. Find the expression for the electric field at any point on the axial line of the ring. Locate the point at which the field is maximum.*

Sol^n: (See in 2067 Chaitra Back Q. No. 9 OR)

OR

Prove that electric field due to short dipole at axial point is twice that at equatorial point.

Sol^n: (See in 2067 Mangsir Regular Q. No. 9 OR)

10. *A particle of charge '- q' and mass 'm' is placed midway between two equal positive charges 'q_0' of separation 'd'. If the negative charge (- q) is placed in perpendicular direction to the line joining them and released, show that the particle describe a SHM with a period given by*

$$T = \left[\frac{\varepsilon_0 m \pi^3 d^3}{q q_0}\right]^{1/2}.$$

Sol^n: (See in 2068 Shrawan Back Q. No. 13)

11. *A cylindrical capacitor has radii a and b. Show that half the stored electric potential energy lies within a cylinder of radius r =* $\sqrt{ab}$.

Sol^n: Capacitance of a cylindrical capacitor having internal radius a and external radius b is $C = \dfrac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$ ----------- (1)

Capacitance of cylindrical capacitor with inner radius a and outer radius $\sqrt{ab}$ is $C' = \dfrac{2\pi\varepsilon_0 l}{\ln\left(\frac{\sqrt{ab}}{a}\right)} = \dfrac{4\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$ ----------- (2)

Now, energy in first case is E = $\frac{q^2}{2C} = \frac{q^2}{2\times2\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right)$ ------- (3)

Energy in second case, E' = $\frac{q^2}{2\times4\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right)$ -------- (4)

Dividing Eq. (4) by Eq. (3), we get

$\frac{E'}{E} = \frac{1}{2}$ $\Rightarrow$ E' = $\frac{1}{2}$ E

Hence, half the stored electric potential energy lies within a cylinder of radius r = $\sqrt{ab}$.

***12.** A flat silver strip of width 1.5cm and thickness 1.5mm carries a current of 150A. A magnetic field of 2.0 Tesla is applied perpendicular to the flat face of the strip. The emf developed across the width of strip is measured to be 17.9μV. Estimate the number density of free electrons in the metal.*

Sol^n: Here, Hall voltage (V_H) = 17.9 μV = 1.75 × 10^{-5} V

Width of the strip (w) = 1.5 cm = 1.5 × 10^{-2} m

Thickness of the strip (t) = 1.5 mm = 1.5 × 10^{-3} m

Current (i) = 150 Amp, magnetic field (B) = 2.0 T

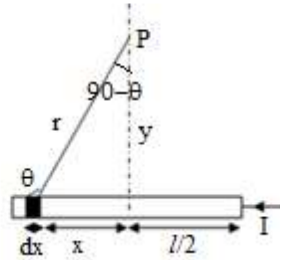
$V_H = \frac{Bi}{net}$ $\Rightarrow n = \frac{Bi}{V_H et}$

$n = \frac{2\times150}{1.79\times10^{-5}\times1.6\times10^{-19}\times1.5\times10^{-3}}$ = 6.98 × 10^{28} m^{-3}

***13.** A straight wire segment of length l carries current I. Show that the magnetic field B produced by that segment at a distance y from it along a perpendicular bisector is B = $\frac{\mu_0 I}{2\pi y}(l^2+4y^2)$.*

Sol^n: Consider an elemental length dx at a distance x from the center of the rod of the length *l*. The magnitude of the magnetic field at the point P due to elemental length dx located at distance r is dB = $\frac{\mu_0 i}{4\pi}\frac{dx\sin\theta}{r^2}$

The direction of dB in the figure is that of the vector $\overrightarrow{dx} \times \hat{r}$ namely, directed into page.



$$B = 2\int_0^{l/2} dB = \frac{\mu_0 i}{2\pi}\int_0^{l/2}\frac{\sin\theta dx}{r^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{y}{\sqrt{x^2+y^2}}$$

$$\text{So, } B = \frac{\mu_0 i}{2\pi}\int_0^{l/2}\frac{y dx}{(x^2+y^2)^{3/2}} = \frac{\mu_0 i}{2\pi y}\left[\frac{x}{(x^2+y^2)^{1/2}}\right]_0^{l/2}$$

$$B = \frac{\mu_0 i}{2\pi y}\frac{l}{(l^2+4y^2)^{1/2}}.$$

14. *Find the inductance of a toroid having N number of turns and radius R.*

Sol^n: (See in 2068 Baishakh Regular Q. No. 13)

OR

Show that the energy per unit volume in electric field and magnetic field are proportional to the square of their fields.

Sol^n: (See in Ashwin Back Q. No. 12 OR)

15. *State and explain Maxwell's equations. Derive the continuity equation:* $\nabla.\vec{J} = -\frac{\partial\rho}{\partial t}$.

Sol^n: Maxwell's equations for any medium in integral form are

$\oiint \vec{E}\cdot\overrightarrow{da} = \frac{q}{\varepsilon}$ Gauss law in electrostatics

$\oiint \vec{B}\cdot\overrightarrow{da} = 0$ Gauss law in magnetism

$\oint \vec{E}\cdot\overrightarrow{dl} = -\frac{\partial\phi_B}{\partial t}$ Faraday's law of electromagnetic induction

$\oint \vec{B}\cdot\overrightarrow{dl} = \mu\left(i + \varepsilon\frac{\partial\phi_E}{\partial t}\right)$ Ampere's modified law

Maxwell's equations in differential form are

$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon}$ $\nabla\cdot\vec{B} = 0$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu\left(\vec{J} + \varepsilon\frac{\partial \vec{E}}{\partial t}\right) \qquad \text{---------} \ (1)$$

Taking the divergence of Eq. (1) on both sides, we get

$$\nabla \cdot \nabla \times \vec{B} = \mu\left(\nabla \cdot \vec{J} + \varepsilon\frac{\partial \nabla \cdot \vec{E}}{\partial t}\right)$$

Since the divergence of curl of any vector is zero.

$$\therefore\ \mu\left(\nabla \cdot \vec{J} + \varepsilon\frac{\partial(\nabla \cdot \vec{E})}{\partial t}\right) = 0 \qquad \Rightarrow \nabla \cdot \vec{J} + \varepsilon\frac{\partial(\rho/\varepsilon)}{\partial t} = 0$$

$$\Rightarrow \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \qquad \therefore\ \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

16. *Determine the total energy of a particle using Schrodinger equation, when the potential energy has value $V = 0$ for $0 < x < a$, and $V = \infty$ for $x \le 0$ and $a \le x$.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 16):

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2069 Bhadra Regular (BCE, BME)

1. *What are drawbacks of simple pendulum? Show that the period of torsion oscillations remain unaffected even if the amplitude be large, provided that the elastic limit of the suspension wire is not exceeded.*

Sol^n: *Drawbacks of a simple pendulum:*

- It is just an ideal conception, non realizable in actual practice, since it is impossible to have both a point mass and a weightless string. So that, the string too has a moment of inertia about the axis of suspension.

- The resistance and buoyancy of the air appreciably affect the motion of the bob.

- The expression of time period is true only for oscillations of infinitely small amplitude.

- The motion of the bob is not strictly linear. It has also a rotator motion about the axis of suspension.

- The bob also has a relative motion with respect to the string at the extremities of its amplitude on either side.

(Remaining part see in 2067 Mangsir Regular Q. No. 1)

OR

In simple harmonic motion, when the displacement is one-half the amplitude, what fraction of the total energy is kinetic energy and what fraction is potential energy? At what displacement is half kinetic energy and half potential energy?

Sol^n: Here, displacement (y) = $\frac{R}{2}$, Kinetic energy (K.E.) = $\frac{1}{2}$ mv^2

The velocity of a particle executing simple harmonic motion at any instant (v) = $\omega\sqrt{R^2 - y^2}$, $\therefore$ K.E. = $\frac{1}{2}m\omega^2(R^2 - y^2)$ and at y = $\frac{R}{2}$

$$= \frac{1}{2}m\omega^2\left(R^2 - \frac{R^2}{4}\right) = \frac{3}{4} \times \frac{1}{2}m\omega^2R^2 = \frac{3}{4}E$$

$\therefore \frac{\text{K.E.}}{\text{E.}} = \frac{3}{4}$ and similarly, $\frac{\text{P.E.}}{\text{E.}} = \frac{1}{4}$

Also, P. E. = K. E. $\Rightarrow \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2(R^2 - y^2)$ $\Rightarrow y^2 = R^2 - y^2$

$\therefore y = \pm \frac{R}{\sqrt{2}}$, half energy is kinetic and half energy is potential.

2. *Derive a differential equation of LC oscillation. With the solution of this equation, show that the maximum value of electric and magnetic energies stored in LC circuits is equal.*

Sol^n: (See in 2067 Ashwin Back Q. No. 1 OR)

3. *How much acoustic power enters the window of area 1.58m^2, through the sound wave (standard intensity level 10^{-16} W/cm^2)? The window opens on a street where the street noise results in an intensity level at the window of 60dB.*

Sol^n: Intensity level at window = 60 dB

Area of the window (A) = 1.58 m^2

Threshold intensity (I_0) = 10^{-16} W/cm^2 = 10^{-12} W/m^2

We have $\Delta L = 10 \log\left(\frac{I}{I_0}\right)$ dB $\therefore I = 10^{-6}$ W/m^2

Acoustic power = intensity × area = 1.58×10^{-6} Watt

4. *Explain circle of least confusion. Show that the diameter of a circle of least confusion is independent of the focal length of a lens.*

Sol^n: **(See in 2067 Mangsir Regular Q. No. 4 OR)**

5. *A glass clad fiber is made with sore glass of refractive index 1.5 and cladding is doped to give a fractional index difference of 0.005. Find (i) the cladding index (ii) the critical internal reflection angle (iii) the external critical acceptance angle (iv) Numerical Aperture (v) acceptance angle.*

Sol^n: Core of refractive index (μ_1) = 1.5

Fractional index (Δ) = 0.005

(i) We have, $\Delta = \frac{\mu_1 - \mu_2}{\mu_1}$ $\Rightarrow 0.005 = \frac{1.5 - \mu_2}{1.5}$

$\Rightarrow 0.0075 = 1.5 - \mu_2$ $\therefore \mu_2 = 1.4925$

(ii) Critical internal reflection angle (C) = ?

$C = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right) = \sin^{-1}\left(\frac{1.4925}{1.5}\right) = 84.3°$

(iii) External critical acceptance angle is given by $\mu_0 \sin C_0 = \mu_1 \sin\theta$

$C_0 = \sin^{-1}[1.5 \sin(90° - 84.3°]$ since $\mu_0 = 1$ for air

$C_0 = 8.56°$

(iv) $NA = \mu_1 \sqrt{2\Delta} = 1.5 \times \sqrt{2 \times 0.005} = 0.15$

(v) Acceptance angle,

$i = \sin^{-1}(\sqrt{\mu_1^2 - \mu_2^2}) = \sin^{-1}(\sqrt{1.5^2 - 1.4925^2} = 8.62°$

6. *A parallel beam of light (λ = 5890Å) is incident on a thin glass plate (μ = 1.5) such that the angle of refraction is 60°. Calculate the smallest thickness of the plate which will appear dark by reflection.*

Sol^n: Wavelength of the light (λ) = 5890Å = 5.89×10^{-5} cm

$\mu = 1.5, r = 60°$ $2\mu t \cos r = n\lambda$ for n = 1

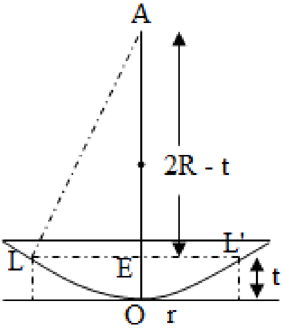
$t = \frac{n\lambda}{2\mu \cos r} = \frac{1 \times 5.89 \times 10^{-5}}{2 \times 1.5 \times \cos 60°} = 3.9 \times 10^{-5}$ cm

7. *How are Newton's rings formed? How is the ring diameter and film thickness related? How can Newton's rings experiment be used to determine refractive index of a liquid?*

Sol^n: When a Plano – convex lens is placed in contact with a flat glass surface, a thin air film is formed. When such film is exposed by monochromatic light, a series of concentric fringes are formed which are called Newton's rings.

The path difference between the rays reflected on the upper and lower surface of the thin film is $2\mu t \cos r + \frac{\lambda}{2}$. For almost normal incident in air film, $r \cong 0$ and $\mu = 1$. So, the path difference is $2t + \frac{\lambda}{2}$. At the point O, $t = 0$, so the path difference is $\frac{\lambda}{2}$. Hence the center spot is dark. The condition for the dark rings is $2t + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$ -------- (2)

Let a plan-convex lens of radius of curvature R is placed on the plane glass plate AOB, the curved surface LOL' is a part of spherical surface having radius of curvature R. At this particular case r is the radius of Newton's ring corresponding to film thickness t. From figure



$\frac{LE}{OE} = \frac{AE}{EL'}$ $\Rightarrow LE \times EL' = OE \times AE$

$\Rightarrow r \times r = t\,(2R - t)$

$\Rightarrow r^2 \cong 2Rt$ since t^2 can be neglected.

$\Rightarrow r = \sqrt{2Rt}$ $\therefore D_n = 2r = 2\sqrt{2Rt}$

For dark fringe, $2t = n\lambda$. $\Rightarrow \frac{r^2}{R} = n\lambda$

$r_n^2 = n\lambda R$ and diameter is $D_n = 2\sqrt{n\lambda R}$. $\Rightarrow D_n^2 = 4n\lambda R$

Similarly, the diameter of $(n + m)^{th}$ dark ring is $D^2_{n+m} = 4(n + m)\lambda R$

$D^2_{n+m} - D_n^2 = 4m\lambda R$ --------- (1)

Gently put the liquid into the air film space without disturbing the entire arrangement. Measure the diameters of n^{th} and $(n+m)^{th}$ dark rings. Let D'_n and D'_{n+m} be the diameters of n^{th} and $(n+m)^{th}$ rings. For normal incidence and with film of liquid of refractive index μ,

$D_n'^2 = \frac{4n\lambda R}{\mu}$ and $D_{n+m}'^2 = \frac{4(n+m)\lambda R}{\mu}$ $\therefore D_{n+m}'^2 - D_n'^2 = \frac{4m\lambda R}{\mu}$ ----- (2)

From Eq. (1) and Eq. (2), we get $\mu = \frac{D_{n+m}^2 - D_n^2}{D_{n+m}'^2 - D_n'^2}$

OR

What is double refraction? How can we experimentally distinguish between plane polarized, circularly polarized and elliptically polarized light?

Sol^n: (See in 2068 Baishakh Regular Q. No. 4 OR)

8. *Assume that the limits of the visible spectrum are arbitrary chosen as 430 nm and 680 nm. Calculate the no. of rulings per millimeter of a grating that will spread the first-order spectrum through an angle of 20°.*

Sol^n: The angular positions of the first order diffraction lines are given by d sinθ = λ. Let λ_1 be the shorter wavelength (430nm) and θ be the angular position of the line associated with it. Let λ_2 be the longer wavelength (680nm), and let θ + Δθ be the angular position of the line associated with it. Here, Δθ = 20°. Then,

$$\lambda_1 = d\sin\theta \quad \Rightarrow \sin\theta = \frac{\lambda_1}{d} \text{ and } \quad \cos\theta = \sqrt{1-\frac{\lambda_1^2}{d^2}} = \sqrt{\frac{d^2-\lambda_1^2}{d^2}}$$

$$\lambda_2 = d\sin(\theta+\Delta\theta) \quad \Rightarrow \sin(\theta+\Delta\theta) = \frac{\lambda_2}{d}$$

$$\text{Sin}(\theta+\Delta\theta) = \sin\theta\cos\Delta\theta + \cos\theta\sin\Delta\theta$$

$$\Rightarrow \frac{\lambda_2}{d} = \frac{\lambda_1}{d}.\cos\Delta\theta + \frac{\sqrt{d^2-\lambda_1^2}}{d}\sin\Delta\theta$$

$$\Rightarrow \lambda_2 = \lambda_1\cos\Delta\theta + \sqrt{d^2-\lambda_1^2}\sin\Delta\theta$$

$$\Rightarrow (d^2 - \lambda_1^2)\sin^2\Delta\theta = (\lambda_2 - \lambda_1\cos\Delta\theta)^2$$

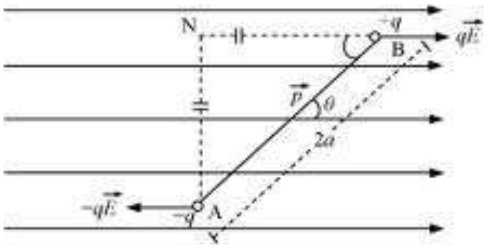
$$\therefore d = \frac{\sqrt{(\lambda_2-\lambda_1\cos\Delta\theta)^2+\lambda_1^2\sin^2\Delta\theta}}{\sin\Delta\theta} = \frac{\sqrt{(680-430\cos20)^2+430^2\sin^2 20}}{\sin 20}$$

$$= 914 \text{ nm} = 9.14\times10^{-4}\text{ mm}$$

$$\therefore N = \frac{1}{d} = 1094 \text{ lines per mm.}$$

9. *Define an electric dipole. How does a dipole behave in electric field? Obtain the conditions for maximum torque and maximum potential energy in an electric field.*

Sol^n: Two equal and opposite charges separated by a certain distance with no net charge is called an electric dipole.



Consider an electric dipole AB of separation 2a and magnitude of charge q be placed in an external electric field of strength $\vec{E}$ at equilibrium condition as shown in figure. At the ends of the dipole, electrostatic forces F_A and F_B act in opposite direction with the same magnitude $q\vec{E}$. Thus the net force exerted on the dipole by the field is zero; however these forces exert a net torque (τ) on dipole about its center of mass, which we can take to be midway along the line connecting the charged ends. The total torque acted on the dipole is τ = F $\times$ perpendicular distance = qE $\times$ 2a sinθ = 2aqE sinθ

In terms of electric dipole moment

$\tau = pE \sin\theta$ $\qquad\qquad$ $\therefore \tau = \vec{p} \times \vec{E}$

Thus torque is maximum when $\vec{p}$ and $\vec{E}$ are perpendicular to each other and is zero when they are parallel or anti-parallel.

When a dipole changes direction in a field, the electric torque does work on it: with a corresponding change in potential energy. The work done or potential energy is minimum when it is in its equilibrium orientation. The work done dw by the torque τ during an infinitesimal displacement dθ is dw = τdθ, Because the torque in the direction of decreasing of θ.

$\tau = - pE \sin\theta$

$\Rightarrow dw = - pE \sin\theta \, d\theta$

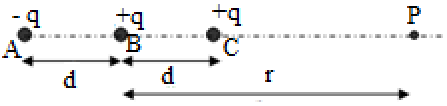
$W = pE\cos\theta_2 - pE\cos\theta_1$

If $\theta_1 = 90°$ and $\theta_2 = \theta$, then potential energy is

$U = -pE\cos\theta$ This will be maximum when $\theta = 180°$ and $U_{max} = pE$.

OR

For the charge configuration of the figure, show that V(r) at a point P on the line assuming r >> a is given by $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{2qa}{r^2}\right)$.



Sol[n]: Potential at P due to charge at A is

$V_A = -\frac{q}{4\pi\varepsilon_0(r+a)}$, due to charge at B is $V_B = \frac{q}{4\pi\varepsilon_0 r}$,

and due to charge at C is $V_C = \frac{q}{4\pi\varepsilon_0(r-a)}$

$\therefore V = V_A + V_B + V_C$

$\Rightarrow V = \frac{q}{4\pi\varepsilon_0(r-a)} + \frac{q}{4\pi\varepsilon_0 r} - \frac{q}{4\pi\varepsilon_0(r+a)}$ $\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r-a} + \frac{1}{r} - \frac{1}{r+a}\right)$

$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left(\frac{r^2-a^2+r^2+ra-r^2+ra}{r(r^2-a^2)}\right)$ $\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left(\frac{r^2-a^2+2ra}{r(r^2-a^2)}\right)$

If r >> a then, $\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left(\frac{r^2+2ra}{r^3}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} + \frac{2a}{r^2}\right)$

$\therefore V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{2aq}{r^2}\right)$

10. *A long cylindrical conductor has length 1m and is surrounded by a co-axial cylindrical conducting shell with inner radius double that of long cylindrical conductor. Calculate the capacitance for this capacitor assuming that there is vacuum in space between cylinders.*

Sol[n]: Consider a be the inner radius and b be the outer radius of cylindrical conducting shell of length 1m. The capacitance is

$C = 2\pi\varepsilon_0 \frac{l}{\ln\left(\frac{b}{a}\right)}$, according to question b = 2a

$C = 2\pi\varepsilon_0 \frac{l}{\ln(2)} = 2 \times \pi \times \frac{8.85\times10^{-12}}{\ln 2} = 8.02 \times 10^{-11}$ F = 80.2 PF

11. *Charges of uniform volume density 3.2μC/m³ fill a non conducting solid sphere of radius 5cm. What is the magnitude of the electric field at (a) 3.5 cm (b) 8 cm from the center of the sphere?*

Sol^{n}: Since, the charge distribution is uniform, we can find the total charge q by multiplying ρ by the spherical volume $(4/3)\pi r^3$ with r = R = 0.05m. This gives $q = 3.2 \times 10^{-6} \times \frac{4}{3} \times \pi \times (0.05)^3 = 1.68 \times 10^{-9}$ C

(a) Electric field inside the sphere is $E = \frac{qr}{4\pi\varepsilon_0 R^3}$, here r = 0.035 m

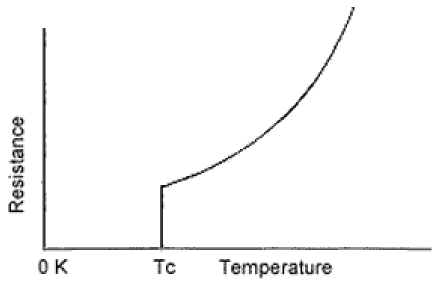
$E = 9 \times 10^9 \times 1.68 \times 10^{-9} \times 0.035 \times \frac{1}{0.05^3} = 4.23 \times 10^3$ N/C

(b) Electric field outside the sphere is $E = \frac{q}{4\pi\varepsilon_0 r^3}$, here r = 0.08 m

$E = 9 \times 10^9 \times 1.68 \times 10^{-9} \times \frac{1}{0.08^2} = 2.4 \times 10^3$ N/C

12. *What are superconductors? How they differ from perfect conductors? Give basic properties and uses of superconductors.*

Sol^{n}: The electrical resistance of metals and alloys decreases as the temperature is lower. If we study the variation of resistance with temperature, it is found that at very low temperature, the resistance becomes immeasurable. This phenomenon in which the electrical resistivity suddenly drops to zero when the material is cooled to a sufficiently low temperature is called superconductivity. This material is known as super conductor.



Difference between superconductor and conductor:

A perfect conductor will have absolutely no losses. A super conductor will be essentially lossless if it can be kept at a specific temperature. As it deviates from this temperature, its' losses will increase. Superconductors, in addition to having no electrical resistance, exhibit quantum effects such as the Meissner effect and quantization of magnetic flux.

In perfect conductors, the interior magnetic field must remain fixed but can have a zero *or* nonzero value. In real superconductors, all magnetic flux is expelled during the phase transition to superconductivity (the Meissner effect), and the magnetic field is *always* zero within the bulk of the superconductor.

Properties of superconductor:

a) If a ring of superconducting material is cooled in magnetic field from a temperature above critical temperature T_C to below T_C, and then magnetic field is switched off, an induced current is set up in the ring. This current continues for a very long time and is known as persistent current.

b) Critical temperature of superconductor varies with isotopic mass $T_C \propto M^{-1/2}$

c) Thermal conductivity of material changes discontinuously during the transfer from normal to super conducting state.

Uses of superconductor

a) It is used in supercomputers.

b) It is used in generation and transmission of electric power.

c) It is used in medical diagnosis.

13. *Derive the relation for rise and fall of current in LR circuit. Plot a graph between current and time and explain the graph.*

Sol^n: (See in 2067 Ashwin Back Q. No. 14)

OR

In a Hall-effect experiment, a current of 3A sent length wise through a conductor 1 cm wide, 4 cm long and 10 μm thick produces a transverse (across the width) Hall potential differences of 10μV, when a magnetic field of 1.5T is passes perpendicularly through the thickness of conductor. From these data, find: (a) the drift velocity of the charge carrier and (b) the number density of charge carrier.

Sol^n: Current through the conductor (i) = 3Amp

Breadth of the conductor (b) = 1 cm = 10^{-2} m

Length of the conductor (l) = 4cm = 4×10^{-2} m

Thickness of the conductor (t) = 10μm = 10^{-5} m

Hall voltage (V_H) = 10μV = 10^{-5} V, Applied magnetic field (B) = 1.5 T

(a) Drift velocity of charge carrier (v_d) = ?

At equilibrium, e E_H = Bev_d

$$v_d = \frac{E_H}{B} = \frac{V_H}{Bb} = \frac{10^{-5}}{1.5\times 10^{-2}} = 6.7 \times 10^{-4} \text{ m/s}$$

(b) Number density of charge carrier (n) = ?

$$V_H = \frac{Bi}{net} \qquad \Rightarrow n = \frac{Bi}{V_H et} = \frac{1.5 \times 3}{10^{-5}\times 1.6\times 10^{-19}\times 10^{-5}} = 2.81 \times 10^{29} \text{ m}^{-3}.$$

14. *A particular cyclotron is designed with dees of radius R = 75 cm and with magnets that can provide a field of 1.5T. (i) To what frequency should be oscillator be set if deuterons are to be accelerated?(ii) What is the maximum energy of deuterons that can be obtained? Given mass of the deuterons is 3.34×10^{-27} kg.*

Sol^n: Radius of dee (R) = 75 cm = 0.75m

Magnetic field (B) = 1.5T

Mass of the deuterons is 3.34×10^{-27} kg.

(i) Frequency of the oscillqtor (f) = ?

f = $\frac{\text{qB}}{2\pi\text{m}} = \frac{1.6\times10^{-19}\times1.5}{2\pi\times3.34\times10^{-27}}$ = 11.43×10^6 Hz = 11.44 MHz

(ii) Maximum energy of the deuteron (E_{max}) = ?

$E_{max} = \frac{(\text{qBR})^2}{2\text{m}} = \frac{\left(1.6\times10^{-19}\times1.5\times0.75\right)^2}{2\times3.34\times10^{-27}}$ = 4.85×10^{-12} J = 30.3 eV

15. *Define Poynting vector. Prove that* $\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$.

Sol^n: (See in 2067 Ashadh Regular Q. No. 15)

16. *Prove that the energy levels are quantized, when the electron is confined in an infinite potential well of width a.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 16)

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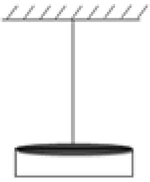
2069 Poush Back (BCE, BME)

1. *Derive the relation for the time period of a torsional pendulum and write the technique to find the moment of inertia a body using torsional pendulum.*



Torsion pendulum

Sol^n: If a disc is suspended by a rigid support with the help of metallic wire of length *l* as shown in figure and the disc is displaced by an angle θ and released then the disc oscillates in turning motion due to restoring couple $\tau = -c\,\theta$ ----------- (1)

where c is torsional constant of the wire. If I be the moment of inertia of the rigid body about ax axis passing through wire as an axis then according to Newton's second law, $\tau = I\frac{d^2\theta}{dt^2}$ ------------- (2)

From Eq. (1) and Eq. (2), we have

$$I\frac{d^2\theta}{dt^2} = -c\theta \quad \Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{c}{I}\theta \quad \Rightarrow \frac{d^2\theta}{dt^2} \propto -\theta$$

This shows motion of torsion pendulum is angular harmonic.

Also, $\omega^2 = \frac{c}{I} \quad \Rightarrow \omega = \sqrt{\frac{c}{I}} \quad \therefore T = 2\pi\sqrt{\frac{I}{c}}$,

When the circular disc is slightly rotated in a horizontal plane and is released, it executes torsional oscillations, which is simple harmonic motion. The period of oscillation is

$$T_1 = 2\pi\sqrt{\frac{I_1}{C}} \qquad \text{---------------------------- (3)}$$

Where I_1 is the moment of inertia of the circular disc along with its combination about the wire as axis and c is the restoring couple per unit twist in the wire, which is given by $c = \frac{\pi\,\eta r^4}{2l}$ where r, *l* and η are radius, length and modulus of elasticity of the wire respectively.

If a circular ring is placed on the circular disc coaxially with the wire, then the period of oscillation for this combination is given by

$T_2 = 2\pi\sqrt{\frac{I_1+I_2}{c}}$ --- (4) Moment of inertia of the ring $I_2 = \frac{M(R_1^2+R_2^2)}{2}$

Where R_1 and R_2 are internal and external radii of the ring and M is mass of the ring. Now squaring and subtracting Eq. (3) and Eq. (4), we get

$T_2^2 = 4\pi^2 \frac{I_1+I_2}{c}$ and $T_1^2 = 4\pi^2 \frac{I_1}{c}$ $\Rightarrow T_2^2 - T_1^2 = 4\pi^2 \frac{I_2}{c}$

So, $\frac{T_2^2-T_1^2}{T_1^2} = \frac{I_2}{I_1}$ $\therefore\ I_1 = \left[\frac{T_1^2}{T_2^2-T_1^2}\right]I_2$

Thus the moment of inertia of the circular disc is determined by using this equation.

OR

What is quality factor? Prove that quality factor of a damped harmonic oscillation depend on the damping constant.

Sol^n: The quality factor is a number which measures the quality of a system. If the quality factor is high then the efficiency of that system is also high and vice versa. Mathematically, quality factor is 2π times the ratio of instantaneous energy to energy loss per period.

i.e. $Q = 2\pi \frac{E(t)}{\Delta E}$,

In case of simple harmonic motion total energy is directly proportional to the square of the amplitude. By using same analogy the total energy of the damped oscillator is given by $E \propto (\text{amplitude})^2$

In case of damped oscillator, the amplitude $A(t) = R\, e^{-bt/2m}$

So, total energy is given by $E(t) = \frac{1}{2}KR^2e^{-bt/m}$

After one cycle energy of the system becomes

$E(t + T) = \frac{1}{2}KR^2e^{-b(t+T)/m}$

The energy loss per cycle is $\Delta E = E(t) - E(t + T)$

$$\Rightarrow \Delta E = = \frac{1}{2}KR^2e^{-bt/m} - \frac{1}{2}KR^2e^{-b(t+T)/m} = E(t)\left(1 - e^{-bT/m}\right)$$

Now, $e^{-bT/m} = 1 - \frac{bT}{m} + \left(\frac{bT}{m}\right)^2 - \left(\frac{bT}{m}\right)^3 + \cdots \ldots \ldots \ldots$

For small value of b, higher powers can be neglected. $\therefore\ e^{-bT/m} = 1 - \frac{bT}{m}$.

$\therefore\ \Delta E = \frac{bT}{m} E(t)$ Thus, $Q = 2\pi \frac{E(t)}{\frac{bT}{m}E(t)} = \frac{2\pi}{T}\frac{m}{b} = \omega\frac{m}{b}$

This shows that the quality factor of a damped harmonic oscillation depend on the damping constant.

2. *A string has linear density 525 gm/m and tension 45N. When sinusoidal wave of frequency 120 Hz and amplitude 8.5 mm is sent along the string, at what average rate does the wave transport energy?*

Sol^n: Linear density of string (μ) = 525 gm/m = 0.525 kg/m

Tension on the string (T) = 45N, frequency of wave (f) = 120Hz,

Amplitude (R) = 8.5 mm = 8.5×10^{-3} m

Average power (P_{ave}) = ? We have, $P_{ave} = \frac{1}{2}\mu v\omega^2R^2 = 2\pi^2\mu vf^2R^2$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{45}{0.525}} = 9.26 \text{ m/s}$$

$\Rightarrow P_{ave} = 2\pi^2 \times 0.525 \times 9.26 \times (120)^2 \times (8.5 \times 10^{-3})^2 = 99.8$ Watt.

3. *You are an engineer and would like to design a sound friendly auditorium hall in your city. What suggestions would you provide the concerned authority in order to incorporate (a) structural design (b) materials to be used (c) reverberation?*

Sol^n: Since, reverberation is due to repeated reflections. It may be reduced by increasing absorption of sound in the room.

Distribution of intensity throughout the hall should be uniform. Usual design of the hall should be parabolic in back side of the listener. At the speaker's end each pitch of sound should be distributed in such a way that each word is heard distinctly and there is no reinforcement causing the change in quality.

In good auditorium, there should be provision to absorb the unnecessary reflected sound. Some absorbing materials like pieces of wood, cloths, cushions, etc should be used and they are placed at various positions of the hall. Open window is considered as perfect absorber of the sound, since there is no reflection and sound simply passes through.

Reverberation is the intermixing between the original and reflected sound wave in a hall due to which sound received by an observer is somewhat sustained or extended. The intensity of sound decreases exponentially with increasing time in a hall. So, it will take longer time to become zero. Due to multiple reflections from wall, ceiling and floor sound reverberates and persist in side room for longer time. For good acoustic, reverberation time should be optimum value. If small, sound vanishes instantaneously and gives the hall dead effect. If very large, there is more reverberation, due to which there by causing confusion. Different frequency of sound may interfere differently at some point. Thus, quality of sound may change. This produces unpleasant effect.

These are some suggestions for sound friendly auditorium hall.

4. *What do you mean by acceptance angle and numerical aperture? Show that numerical aperture (NA) is proportional to square root of fractional refractive index change (Δ).*

Sol^n: (See in 2067 Ashadh Regular Q. No. 7)

5. *Two thin converging lenses of focal lengths 6cm and 8cm are placed co-axially in air and are separated by 4cm. An object is placed 8cm in front of the first lens. Find the position and nature of the final image.*

Sol^n: Here, f_1 = 6cm, f_2 = 8cm, separation between lenses (d) = 4cm

We have, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ $\Rightarrow f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{6\times8}{6+8-4} = 4.8$ cm

$\beta = -\frac{df}{f_1} = -\frac{4\times4.8}{6} = -3.2$ cm and $\alpha = \frac{df}{f_2} = \frac{4\times4.8}{8} = 2.4$ cm

Object distance from first lens (u) = - 8cm

The object distance from first principal point (U) = - (u + α) = - 10.4 cm

$\frac{1}{f} = \frac{1}{V} - \frac{1}{U}$ $\Rightarrow \frac{1}{V} = \frac{1}{U} + \frac{1}{f} = -\frac{1}{10.4} + \frac{1}{4.8}$

$\therefore$ V = 8.91 cm

Distance of the final image from second lens is v = V + β = 8.91 – 3.2

= 5.71cm right from second lens. The final image is real.

6. *Show that the intensity of second primary maxima of Fraunhoffer's single slit diffraction is $\frac{1}{62}$ of its central maxima.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 4 OR)

OR

What is double refraction? Prove that linearly and circularly polarized light are special cases of elliptically polarized light.

Sol^n: (See in 2067 Mangsir Regular Q. No. 6)

7. *A beam of monochromatic light of wavelength 5.82×10^{-7} m falls normally on a glass wedge with the wedge angle of 20 seconds of an arc. If the refractive index of glass is 1.5, find the number of interference fringes per cm of the wedge length.*

Sol^n: Wavelength of light (λ) = 5.82×10^{-7} m,

Refractive index of glass (μ) = 1.5,

angle of wedge (θ) = 20'= $\frac{20\times\pi}{3600\times180}$ rad = 9.7×10^{-5} rad

We have, fringe width (β) = $\frac{\lambda}{2\mu\theta} = \frac{5.82\times10^{-7}}{2\times1.5\times9.7\times10^{-5}}$ = 0.2 cm

Number of fringes per cm = $\frac{1}{0.2}$ = 5

8. *Find the slit separation of the double slit arrangement that will produce interference fringes 0.018 radian apart on a distance screen when the light has wavelength 589nm?*

Sol^n: Wavelength of light (λ) = 589nm $=5.89 \times 10^{-7}$ m

Separation between slits (d) = ?

$\theta = \frac{x}{D} = 0.018$ radian

We have, $\theta = \frac{x}{D} = \frac{\lambda}{d} \Rightarrow d = \frac{\lambda}{\theta} = \frac{5.89\times10^{-7}}{0.018} = 3.27 \times 10^{-5}$ m

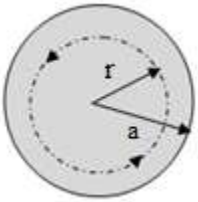
$\therefore d = 3.27 \times 10^{-2}$ mm

9. *Charges are uniformly distributed on a long thin plastic scale. Calculate electric field intensity at an equilateral distance r from the center of the scale.*

Sol^n: (See in 2068 Bhadra Regular Q. No. 9, use r in place of y)

10. *A spherical charge distribution has volume charge density (a) $\rho = Ar^n$ at $r < a$ (b) $\rho = \rho_0$ for $r > a$, where a is the radius of the sphere. Find the electric field in both cases.*

Sol^n: Figure shows the Gaussian surface. From Gauss law we have,

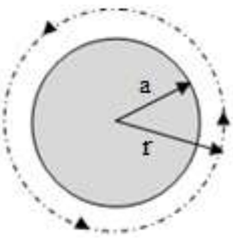


$$\oint_S \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\oint_V \rho dV$$

$$\Rightarrow E . 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r \rho 4\pi r^2 dr$$

$$\Rightarrow E = \frac{1}{\varepsilon_0 r^2}\int_0^r \rho r^2 dr$$

(a) $\rho = Ar^n$ for $r < a$



$$\Rightarrow E = \frac{1}{\varepsilon_0 r^2}\int_0^r Ar^n r^2 dr = \frac{1}{\varepsilon_0 r^2}\int_0^r Ar^{n+2} dr$$

$$\therefore E = \frac{Ar^{n+3}}{\varepsilon_0(n+3)r^2} = \frac{Ar^{n+1}}{\varepsilon_0(n+3)}$$

(b) $\rho = \rho_0$ for $r > a$

$\Rightarrow E = \frac{1}{\varepsilon_0 r^2}\int_0^a \rho_0 r^2 dr = \frac{\rho_0}{\varepsilon_0 r^2}\frac{a^3}{3} = = \frac{\rho_0}{4\pi\varepsilon_0 r^2} 4\pi \frac{a^3}{3} = \frac{\rho_0 V}{4\pi\varepsilon_0 r^2}$

$\therefore E = \frac{q}{4\pi\varepsilon_0 r^2}$

11. *A neutral water molecule in its vapor state has an electric dipole moment of magnitude 7.1 × 10 ⁻ ³⁰ Cm. If the molecule is placed in an electric field of 2.5 × 10⁴ N/C, (i) what maximum torque can the field exert on it? (ii) How much work must an external agent do to turn this molecule end for end in this field?*

Sol^n: Electric dipole moment (P) = 7.1×10^{-30} Cm,

Electric field (E) = 2.5×10^4 N/C

(i) The torque is given by $\tau = PE \sin\theta$, for maximum torque, $\theta = 90°$

$\tau_{max} = PE = 7.1 \times 10^{-30} \times 2.5 \times 10^4 = 1.78 \times 10^{-25}$ Nm

(ii) The work done is given by

$W_a = U_{180°} - U_{0°} = - PE \cos180° + PE \cos0°$

$\Rightarrow W_a = 2\,PE = 2 \times 7.1 \times 10^{-30} \times 2.5 \times 10^4 = 3.56 \times 10^{-25}$J

12. *Calculate the displacement current between the capacitor plates of area 2.3 × 10 ⁻ ² m² and rate of electric field change is 2 × 10 ¹² V/ms. Also, calculate the displacement current density and induced magnetic field for r = R = 70mm.*

Sol^n: Plates of area (A) = 2.3×10^{-2} m^2

The rate of electric field change $\frac{dE}{dt} = 2 \times 10^{12}$ V/ms.

Displacement current (i_d) = $\varepsilon_0 A \frac{dE}{dt} = 8.85 \times 10^{-12} \times 2.3 \times 10^{-2} \times 2 \times 10^{12}$

$i_d = 0.407$ Amp

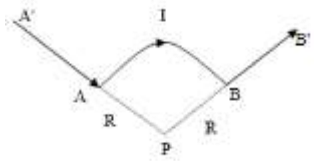
Displacement current density (J_d) = $\varepsilon_0 \frac{dE}{dt} = 8.85 \times 10^{-12} \times 2 \times 10^{12}$

$J_d = 17.70$ A/m^2

Induced magnetic field (B) = $\frac{1}{2}\mu_0 r J_d = 0.5 \times 4\pi \times 10^{-7} \times 17.71 \times 0.086$

$= 9.5 \times 10^{-7}$ T = 950 nT

13. *Derive the magnetic field at a point P due to curve wire segment flowing current I as shown in below figure. [R is the radius of circle of Arc]*



Sol^n: (See in 2068 Shrawan Back Q. No. 10)

14. *Find an expression of the self inductance of a toroid having N number of turns, radius r and carrying current i.*

Sol^n: (See in 2068 Baishakh Regular Q. No. 13)

15. *Write Maxwell's electromagnetic wave equation in dielectric medium. Obtain electromagnetic wave equations for E and B in both dielectric and free space.*

Sol^n: (See in 2068 Magh Back Q. No. 15)

16. *A free particle is confined in a box of width L. Find an expression for energy Eigen value and show that the particle can have only discrete energy.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 16)

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2069 Chaitra Regular (BEL, BEX, BCT, BIE, B. Agri.)

1. *Point out the similarities and dissimilarities between the oscillations of bar pendulum and torsional pendulum. Show that the radius of gyration is equal to distance from center of suspension to center of gravity of compound pendulum, when time period is minimum.*

Sol^n: *Similarities*

a. In both pendulum motion are angular harmonic.

b. Both are formed by rigid body and are used to determine the moment of inertia of the body.

c. Both oscillate about the center of gravity.

Dissimilarities

a. In bar pendulum motion is to and fro whereas in torsion pendulum motion is turning.

b. Bar pendulum is used to measure the acceleration due to gravity while torsion pendulum is used to determine the modulus of rigidity of the metallic wire.

c. In bar pendulum amplitude of oscillation should be very small while amplitude does not affect the motion of the torsion pendulum.

d. There are two positions in bar pendulum with same time period whereas there are not such points with same period in torsion pendulum.

(Second part see in second part of 2068 Magh Back Q. No. 1)

2. *Derive a differential equation for LC oscillation. Show that the maximum value of electric and magnetic energies stored in LC circuit is equal.*

Sol^n: **(See in 2067 Ashwin Bach Q. No. 1 OR)**

OR

Prove that if a transverse wave is travelling along a string, then the slope at any point of the string is numerically equal to the ratio of the particle speed to the wave speed at that point.

Sol^n: (See in 2068 Baishakh Regular Q. No. 2)

3. *The time of reverberation of an empty hall is 1.5 sec with 500 audiences present in the hall; the reverberation time falls to 1.4 sec. Find the number of persons present in the hall if the reverberation time falls down to 1.32 sec.*

Sol^n: The reverberation time for empty hall (T_0) = 1.5 sec

The reverberation time for entry of 500 audiences (T_{500}) = 1.4 sec

The reverberation time for entry of n audiences (T_n) = 1.32 sec

Decrease in reverberation time due to entry of 500 audiences = 1.5 – 1.4 = 0.1 sec

Decrease in reverberation time due to entry of n audiences = 1.5 – 1.32 = 0.18 sec

i.e. 0.1 sec decrease for 500 audiences.

$\Rightarrow$ 0.18 sec will be decrease for n audiences which is (n) = $\frac{500}{0.1} \times 0.18$ = 900 audiences.

So, the reverberation time will be 1.32 sec for 900 audiences.

4. *Show that the intensity of the first subsidiary maxima of Fraunhoffer's diffraction at a single slit is 4.5% of that of principal maxima.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 4 OR)

For first subsidiary maxima (n = 1)

$\alpha = \frac{\pi}{\lambda} a\sin\theta = \frac{\pi}{\lambda}(2n+1)\frac{\lambda}{2}$ for n = 1, $\quad \alpha = \frac{3\pi}{2}$

$$\therefore\ I = I_0\left(\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = \frac{I_0}{21} = 0.045\ I_0. = 4.5\%\text{ of } I_0.$$

OR

What is double refraction? Explain how Nicol prism can be used as polarizer and analyzer?

Sol^n: (See in 2069 Ashadh Back Q. No. 4 OR)

5. *In a Newton's ring experiment, the radius of curvature of the lens is 5cm and the lens diameter is 20mm. (a) How many bright rings are produced? Assume that λ = 589 nm (b) How many bright rings would be produced if the arrangement were immersed in water (μ = 1.33)?*

Sol^n: Radius of curvature of lens (R) = 5 cm

Diameter of nth ring (D_n) = 20 mm, Radius of nth ring (r_n) = 1 cm

Wavelength of light (λ) = 589 nm = 5.89×10^{-5}cm

(a) The radius of the nth order bright fringe is given by $r_n^2 = (2n + 1)\frac{\lambda}{2}R$

$$\Rightarrow n = \frac{r_n^2}{\lambda R} - \frac{1}{2} = \frac{1}{5.89\times10^{-5}\times5} - \frac{1}{2} = 3395$$

Since the first order bright fringe corresponds to n = 0. n = 3395 corresponds to the 3396th bright fringe.

(b) We have to replace λ by $\lambda' = \frac{\lambda}{\mu}$

$$\text{So, } n' = \frac{\mu r_n^2}{\lambda R} - \frac{1}{2} = \frac{1.33\times1}{5.89\times10^{-5}\times5} - \frac{1}{2} = 4515$$

This corresponds to the 4516th bright fringe.

6. *A diffraction grating 3cm wide produces the second order at 33° with light of wavelength 600nm. What is the total number of lines on the grating?*

Sol^n: Width of the grating (w) = 3cm, angle of diffraction for second order (θ_2) = 33°, wavelength of light (λ) = 600nm = 6×10^{-5} cm

Total number of lines (N') = ?

We have, $(a + b)\sin\theta_n = n\lambda$, for n = 2, θ_2 = 33°

$\Rightarrow (a + b)\sin33° = 2 \times 6 \times 10^{-5}$ $\qquad\Rightarrow (a + b) = 2.2 \times 10^{-4}$ cm

So, N = $\frac{1}{a+b} = \frac{1}{2.2} \times 10^4$ = 4538 lines per cm

∴ Total number of lines in the grating = wN = 3 × 4538 = 13615 lines

7. *What is population inversion? Explain why laser action cannot occur without population inversion between atomic levels?*

Sol^n: Population inversion: In normal condition, the number of atoms in lower energy level is greater than that in higher energy level. The establishment of a situation in which the number of atoms in the higher energy level is greater than that in the lower energy level is called population inversion.

To understand the concept of a population inversion, it is necessary to understand some thermodynamics and the way that light interacts with matter. To do so, it is useful to consider a very simple assembly of atoms forming a laser medium. Assume there are a group of *N* atoms, each of which is capable of being in one of two energy states, either

1. The *ground state*, with energy E_1; or

2. The *excited state*, with energy E_2, with $E_2 > E_1$.

The number of these atoms which are in the ground state is given by N_1, and the number in the excited state N_2. Since there are *N* atoms in total, $N_1 + N_2 = N$. The energy difference between two states is given by $\Delta E_{12} = E_2 - E_1$, determines the characteristic frequency ν_{12} of light which will interact with the atoms; this is given by the relation $\Delta E = h\nu_{12}$, where *h* being Planck's constant.

If the group of atoms is in thermal equilibrium, it can be shown from thermodynamics that the ratio of the number of atoms in each state is given by the Boltzmann factor:

$$\frac{N_2}{N_1} = \exp\left[\frac{-(E_2 - E_1)}{k_B T}\right]$$

where T is the thermodynamic temperature of the group of atoms, and k_B is Boltzmann's constant.

We may calculate the ratio of the populations of the two states at room temperature ($T \approx 300$ K) for an energy difference ΔE that corresponds to light of a frequency corresponding to visible light ($\nu \approx 5\times10^{14}$ Hz). In this case $\Delta E = E_2 - E_1 \approx 2.07$ eV, and $k_BT \approx 0.026$ eV. Since $E_2 - E_1 \gg k_BT$, it follows that the argument of the exponential in the equation above is a large negative number, and as such N_2/N_1 is vanishingly small; i.e., there are almost no atoms in the excited state. When in thermal equilibrium, then, it is seen that the lower energy state is more populated than the higher energy state, and this is the normal state of the system. As T increases, the number of electrons in the higher-energy state (N_2) increases, but N_2 never exceeds N_1 for a system at thermal equilibrium; rather, at infinite temperature, the populations N_2 and N_1 become equal. In other words, a population inversion ($N_2/N_1 > 1$) can never exist for a system at thermal equilibrium. To achieve population inversion therefore requires pushing the system into a non-equilibrated state.

If light (photons) of frequency ν_{12} pass through the group of atoms, there is a possibility of the light being absorbed by atoms which are in the ground state, which will cause them to be excited to the higher energy state. The rate of absorption is proportional to the radiation intensity of the light, and also to the number of atoms currently in the ground state, N_1. Laser works on the principle of stimulated emission which is possible only when the atoms jump from higher level to lower level by some trigger. So, continuous transition from higher level to lower level is not possible. Population inversion creates the number of atoms in higher state more so it is possible to jump these excited atoms to ground state by emitting radiation. Hence, laser action cannot occur without population inversion between atomic levels.

8. *What are cardinal points of optical system? Determine the equivalent focal length of a combination of two thin lenses separated by a finite distances.*

Sol^n: There are three pairs of different points on the principal axis which are reference point to measure various distances in the system of coaxial lenses. These three pairs of reference points are called cardinal points. They are: two principal points, two principal focal points and two nodal points. **(Remaining part see in 2068 Bhadra Regular Q. No. 4)**

9. *A ring has a charge q uniform distributed in it. Derive an expression for the electric field at any point on the axial line of the ring. Extend your result to find the potential.*

Sol^n: (First part see in 2067 Chaitra Back Q. No. 9 OR)

For potential, we know that E = - ∇V

$$\Rightarrow V = \int E\,dz = \frac{q}{4\pi\varepsilon_0}\int \frac{z\,dz}{(R^2+z^2)^{3/2}} = \frac{q}{4\pi\varepsilon_0\sqrt{(R^2+z^2)}}$$

OR

Write an expression for electric field at any point in the axial line of a charged ring. Using this equation, calculate the electric field at any point in the axial line of a charged disk.

Sol^n: (See in 2067 Ashwin Back Q. No. 9)

10. *What is the magnitude of the electric field at the point (3, 2) m if the electric potential is given by $V = 2x + 5xy + 3y^2$ volts. What acceleration does an electron experiences in the x-direction.*

Sol^n: Here, $V(x, y) = 2x + 5xy + 3y^2$.

Now, the electric field along different directions are

$$E_x = -\frac{\partial V}{\partial x} = -2 - 5y,\ E_y = -\frac{\partial V}{\partial y} = -5x - 6y\ \&\ E_z = -\frac{\partial V}{\partial z} = 0$$

At (3, 2) ; $E_x = -16$, $E_y = -27$ & $E_z = 0$

The magnitude of the electric field at (3, 2)m is

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = 31.38 \text{ V/m}$$

Force along the x-direction is $F = -eE_x = 1.6 \times 10^{-19} \times 16$

So, acceleration of the electron along x-direction is

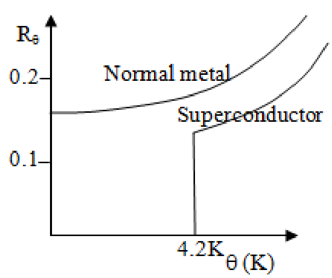
$$a = \frac{eE_x}{m_e} = \frac{1.6 \times 10^{-19} \times 16}{9.1 \times 10^{-31}} = 2.81 \times 10^{12} \text{ m/s}^2$$

11. *Derive an equation $\vec{J} = \sigma\vec{E}$. Explain why resistivity of a conductor increases with increasing temperature. Plot a graph between R_θ (Resistance at any temperature θ) and temperature. Based on the graph, explain what is superconductor? How they differ from perfect conductor? Describe the characteristics of superconductor.*

Sol^n: Let an electron of mass m and charge e be considered in an electric field $\vec{E}$, it experiences a force $\vec{F} = e\,\vec{E}$. The corresponding acceleration given by Newton's law is $a = \frac{\vec{F}}{m} = \frac{e\vec{E}}{m}$

If τ be the average time between the collisions, the drift speed $v_d = a\tau$ then $v_d = \frac{e\vec{E}}{m}\tau$ We have, the current density is $\vec{J} = nev_d = ne\frac{e\vec{E}}{m}\tau$

$$\Rightarrow \vec{E} = \frac{m}{ne^2\tau}\vec{J} = \frac{1}{\sigma}\vec{J} \qquad \therefore \vec{J} = \sigma\vec{E}$$



The resistance for most materials changes with temperature. For many materials including metals, the relation between resistance (R_θ) and temperature (θ) can be given the relation $R_\theta = R_0(1 + \alpha\theta)$.

Where R_0 and R_θ are the values of resistance of a material at 0°C and θ°C respectively such that the temperature difference is not too large and

α is the temperature coefficient of the resistance. In case of resistivity also $\rho_\theta = \rho_0 [1 + \alpha(\theta - \theta_0)]$

(Remaining part see in 2069 Chaitra Regular Q. No. 12)

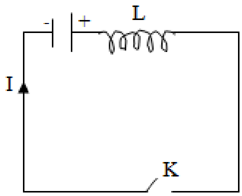
12. *Derive an expression for energy stored in magnetic field. Show that the energy stored per unit volume is directly proportional to the square of the magnetic flux density. Compare this result with electric energy density.*

Sol^n: (See in 2067 Ashwin Back Q. no. 12 OR)

OR

What is self induction? Define inductance of a coil. Show by calculation inductance of a coil depends on the permeability of a medium and the geometry of the coil.

Sol^n: The phenomenon in which the induced emf is produced as a result of change in the current passing through the coil is known as self inductance.



Consider a coil of inductance L in series with a battery and key K. Let I be the current passing through the coil, we have

$\phi \propto I \qquad \Rightarrow \phi = LI,$

Where L is a constant known as coefficient of self induction or self inductance of a coil. Now, $L = \frac{\phi}{I}$

Again, the induced or back emf through the coil is given by

$\varepsilon = -\frac{d\phi}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt} \qquad \therefore L = \frac{\varepsilon}{dI/dt}$

If l be the length of the solenoid having N number of turns, area of cross section be A and I be the current through it then self inductance of the solenoid is given by $L = \frac{\mu_0 N^2 A}{l}$

Similarly for toroid of width h, internal and external radii a and b the self-inductance is given by $L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$

From these relations, it is clear that inductance of coil depends on permeability of the medium and geometry of the coil.

13. *A long circuit coil consisting of 50 turns with diameter 1.2m carries a current of 10Amp. (a) Find the magnetic field at a point along the axis 90cm from the center. (b) At what distance from the center, along the axis, the field is $\frac{1}{8}$ greater as at the center.*

Sol^n: Number of turns in the coil (N) = 50

Diameter of the coil (D) = 1.2m, radius of the coil (R) = 0.6m

Current through the coil (I) = 10 Amp

(a) Magnetic field at 90cm on axial line (B) =?

$$B = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}} = \frac{4\pi\times10^{-7}\times50\times10\times0.6^2}{2(0.9^2+0.6^2)^{3/2}} = 8.9 \times 10^{-5}\,T$$

(b) B at x = $\frac{1}{8}$ B_0, B_0 is the field at the center of foil.

$$\Rightarrow \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}} = \frac{1}{8}\frac{\mu_0 NIR^2}{R^3} \qquad \Rightarrow (2R)^3 = (x^2 + R^2)^{3/2}$$

$$\Rightarrow (2R) = (x^2 + R^2)^{1/2} \qquad \Rightarrow 4R^2 = x^2 + R^2 \qquad \Rightarrow x^2 = 3R^2$$

$$\therefore\ x = \pm R\sqrt{3} = 0.6 \times \sqrt{3} = 1.04\text{ m}$$

14. *Describe the principle and working of Cyclotron. Show that the time taken by the ion in a Dee to travel a semicircle is exactly same whatever be its radius and velocity.*

Sol^n: (See in 2067 Chaitra Back Q. No. 13)

$\therefore\ R = \left(\frac{m}{qB}\right)v$, The time travel a semicircle is $t = \frac{\pi R}{v} = \frac{\pi}{v}\frac{mv}{qB} = \frac{\pi m}{qB}$

This shows the time spent by the particle in a Dee is independent of its speed and radius of the circular path.

15. *Write Maxwell's equations in free space and dielectric medium. With the help of Maxwell's equations, derive the charge conservation theorem.*

Sol^n: Maxwell equations in dielectric medium are

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu\left[\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right]$$

For free space, Maxwell's equations becomes

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0\left[\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right] \quad \text{-------- (1)}$$

Now, taking the divergence on both sides of Eq. (1), we have

$$\nabla . \nabla \times \vec{B} = \mu_0\left[\nabla . \vec{J} + \varepsilon_0 \frac{\partial (\nabla . \vec{E})}{\partial t}\right]$$

Since, the divergence of curl of any vector is zero.

$$\Rightarrow \mu_0\left[\nabla . \vec{J} + \varepsilon_0 \frac{\partial (\nabla . \vec{E})}{\partial t}\right] = 0 \qquad \Rightarrow \nabla . \vec{J} + \varepsilon_0 \frac{\partial (\nabla . \vec{E})}{\partial t} = 0$$

$$\Rightarrow \nabla . \vec{J} = -\varepsilon_0 \frac{\partial (\frac{\rho}{\varepsilon_0})}{\partial t}$$

$$\therefore \nabla . \vec{J} = -\frac{\partial \rho}{\partial t}$$

This is the charge conservation theorem.

16. *A beam of electrons having energy of each 3eV is incident on a potential barrier of height 4eV. If the width of the barrier is 20Å, calculate the transmission coefficient of the beam through the barrier.*

Sol^n: Energy of the electron (E) = 3eV, height of the potential (V_0) = 4eV

Width of the barrier (a) = 20Å = 2×10^{-9} m,

transmission coefficient (T) =?

We have, $T = T_0\, e^{-2\alpha a}$ $\qquad \Rightarrow T = \frac{16E}{V_0}\left(1 - \frac{E}{V_0}\right) e^{-2\alpha a}$

Where $\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar} = \frac{\sqrt{2\times 9.1\times 10^{-31}(4-3)\times 1.6\times 10^{-19}}}{1.05\times 10^{-34}} = 5.14 \times 10^{9}$

So, $T = \frac{16\times 3}{4}\left(1 - \frac{3}{4}\right) e^{-2\times 5.14\times 10^{9}\times 2\times 10^{-9}} = 3.54 \times 10^{-9}$

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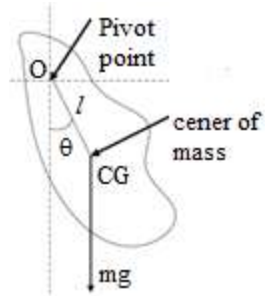
2070 Ashad Back (BEL, BEX, BCT, BIE, B. Agri)

1. *Derive an expression for the time period of a physical pendulum and establish the interchangeability of the center of oscillation and suspension.*



Sol^n^: Consider a rigid body of mass 'm' is suspended at point O. The center of gravity of the body is G at a distance l from point O. If the rigid body is displaced by small angle θ (less than 4°) and released the restoring moment of force is $\tau = -mgl\theta$ --------------- (1)

From Newton's second law of motion, $\tau = I\alpha$ --------------- (2)

where I is the moment of inertia of the body about the axis passing through point O and α is angular acceleration. From Eq. (1) and Eq. (2)

$-mgl = I\frac{d^2\theta}{dt^2}$ $\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta$ $\Rightarrow \frac{d^2\theta}{dt^2} \propto -\theta$

This shows the motion of the rigid body is simple harmonic. So,

$\omega^2 = \frac{mgl}{I}$ $\Rightarrow \omega = \sqrt{\frac{mgl}{I}}$ $\Rightarrow T = 2\pi\sqrt{\frac{I}{mgl}}$

Using parallel axes theorem, $I = I_{CG} + ml^2 = mk^2 + ml^2$.

$\therefore\ T = 2\pi\sqrt{\frac{l^2+k^2}{lg}}$ --------------- (3)

Any point which lies at a distance of $\frac{k^2}{l}$ from the center of gravity is known as point of oscillation. In figure point O' is the point of oscillation. For the point O the time period of oscillation is given from Eq. (3). If we suppose the point O' as the point of suspension then the length of the pendulum becomes $\frac{k^2}{l}$. For this case l should be replaced by

$\frac{k^2}{l}$. So, the time period of oscillation for the point O' as point of suspension is given by

$$T' = 2\pi\sqrt{\frac{\left(\frac{k^2}{l}\right)^2 + k^2}{\frac{k^2}{l}g}} = 2\pi\sqrt{\frac{k^2+l^2}{lg}} \qquad \text{------------------ (4)}$$

From Eq. (3) and Eq. (4) T = T'. So, point of suspension and point of oscillation are interchangeable.

2. *Give the necessary theory of forced vibration and deduce the condition for resonance amplitude.*

Sol^n: (See in 2068 Bhadra Regular Q. No. 1)

OR

Show that the fractional change in frequency of damped oscillation is $\frac{1}{8Q^2}$ where Q is quality factor.

Sol^n: The angular frequency of oscillation for the free oscillation is $\omega = \sqrt{\frac{k}{m}}$ and for the case of damped oscillation is $\beta = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$.

Now,

$$\frac{\omega^2-\beta^2}{\omega^2} = 1 - \left(\frac{\beta}{\omega}\right)^2 = 1 - \frac{\beta^2}{\omega^2}$$

$$\Rightarrow \left(\frac{\omega-\beta}{\omega}\right)\left(\frac{\omega+\beta}{\omega}\right) = 1 - \frac{\frac{k}{m} - \frac{b^2}{4m^2}}{\frac{k}{m}} = 1 - \frac{\omega^2 - \frac{1}{4\tau^2}}{\omega^2} \qquad \text{where } \tau = \frac{m}{b}$$

$$= 1 - \left(1 - \frac{1}{4\omega^2\tau^2}\right) = \frac{1}{4Q^2} \qquad \text{since } Q = \omega\tau$$

Also, $\left(\frac{\omega+\beta}{\omega}\right) \approx 2$ $\qquad \therefore \frac{\omega-\beta}{\omega} = \frac{1}{8Q^2}$ Proved

3. *The reverberation time for empty hall is 1.5 sec. With 500 audiences present in the hall, the reverberation time falls to 1.4 sec. Find the* *number of persons present in the hall if the reverberation time falls down to 1.312 sec.*

Sol^n: The reverberation time for empty hall (T_0) = 1.5 sec

The reverberation time for entry of 500 audiences (T_{500}) = 1.4 sec

The reverberation time for entry of n audiences (T_n) = 1.312 sec

Decrease in reverberation time due to entry of 500 audiences = 1.5 – 1.4 = 0.1 sec

Decrease in reverberation time due to entry of n audiences = 1.5 – 1.312 = 0.188 sec

i.e. 0.1 sec decrease for 500 audiences.

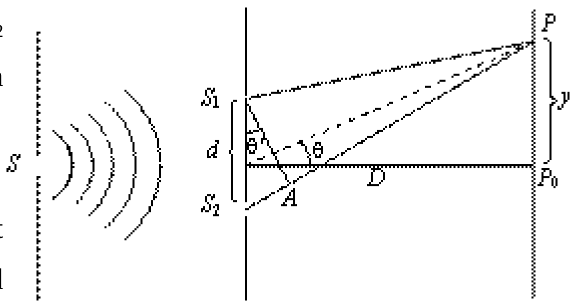
$\Rightarrow$ 0.188 sec will be decrease for n audiences (n) = $\frac{500}{0.1} \times 0.188 = 940$

So, the reverberation time will be 1.312 sec for 940 audiences.

***4.** What is interference? Explain the intensity distribution in interference with mathematical treatment.*

Sol^n: The non uniform distribution of intensity in a medium due to the superposition of two waves from coherent sources is known as interference.

Consider a monochromatic light source S which is emitting waves of wavelength λ. S_1 and S_2 are small pinholes which are lies at same distance from the light source S and acts as two coherent source at a distance of d from each other. Let R be the amplitude of the waves and ϕ is the phase difference between two waves reaching at point P on the screen which lies at D from pinholes. If y_1 and y_2 are displacements then



$y_1 = R \sin\omega t$, $y_2 = R \sin(\omega t+\phi)$

So, the displacement of the resultant wave is

$y = y_1 + y_2 = R\sin\omega t + R\sin(\omega t+\phi)$

$= R\sin\omega t + R\sin\omega t\cos\phi + R\cos\omega t\sin\phi$

$= R\sin\omega t\,(1 + \cos\phi) + R\cos\omega t\sin\phi$

Let $R(1 + \cos\phi) = A\cos\theta$ ----- (1) and $R\sin\phi = A\sin\theta$ -----(2)

$\Rightarrow y = A\sin\omega t\cos\theta + A\cos\omega t\sin\theta = A\sin(\omega t + \theta)$ which represents equation of simple harmonic vibration of amplitude A. Squaring and adding Eq. (1) and Eq. (2), we get

$\Rightarrow A^2\sin^2\theta + A^2\cos^2\theta = R^2\sin^2\phi + R^2\cos^2\phi + 2R^2\cos\phi + R^2$

$\Rightarrow A^2 = 2R^2 + 2R^2\cos\phi = 2R^2(1 + \cos\phi) = 2R^2 \times 2\cos^2(\phi/2)$

$\therefore A^2 = 4R^2\cos^2(\phi/2)$

Intensity is the square of amplitude of resultant wave.

$\therefore \; I = A^2 = 4R^2\cos^2(\phi/2)$

Case (a): When the phase difference $\phi = 0, 2\pi, 4\pi, \ldots\ldots\ldots$ or phase difference $x = 0, \lambda, 2\lambda, \ldots\ldots$, we get

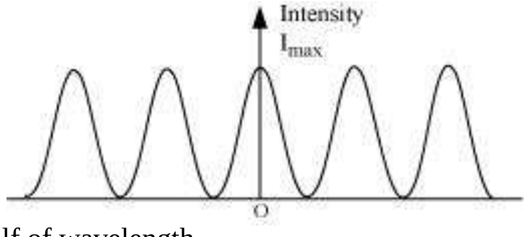
$I = 4R^2$.

The intensity is maximum when the path difference is equal to integral multiple of the wavelength.

Case (b): When the phase difference $\phi = \pi, 3\pi, 5\pi, \ldots\ldots\ldots$ or phase difference $x = (2n + 1)\frac{\lambda}{2}$,

we get, $I = 0$

The intensity is minimum when the path difference is odd number multiple of half of wavelength.



OR

Show that the intensity of second primary maxima is 1.62% of central maxima in Fraunhoffer's single slit diffraction.

Sol^n: (See in 2067 Ashadh Regular Q. No. 4 OR)

5. *A beam of plane polarized light is converted into circularly polarized light by passing it through a crystal slice of thickness 3 × 10^{-5} m. Calculate the difference in the refractive indices of the two rays inside the crystal. Wavelength of light is 600nm.*

Sol^n: Wavelength of light (λ) = 600 nm = 6×10^{-7} m

Thickness of the plate (t) =3×10^{-5} m

We know that when the plane polarized light is passed through the quarter wave plate then change into circularly polarized. Therefore, thickness of the quarter wave plate is given by

$t = \frac{\lambda}{4(\mu_e - \mu_o)} \Rightarrow \mu_e - \mu_o = \frac{\lambda}{4t} = \frac{6 \times 10^{-7}}{4 \times 3 \times 10^{-5}} = 0.5 \times 10^{-2}$ m = 0.005 m

6. *What are active medium, population inversion and optical pumping? Give the importance in the study of LASER. Write a method for getting He-Ne LASER.*

Sol^n: **Active medium**: It is a medium in which light gets amplified and this medium may be solid, liquid or gas. It is important to note that only a fraction of a particular medium is responsible to stimulated emission and remaining part of the medium merely supports the active centers.

Population inversion: In normal condition, the number of atoms in lower energy level is greater than that in higher energy level. The establishment of a situation in which the number of atoms in the higher energy level is greater than that in the lower energy level is called population inversion.

Optical Pumping: The process of achievement of population inversion is known as pumping. In optical pumping, active medium is illuminated by light of suitable frequency $f = \frac{E_2 - E_1}{h}$. As a result, atoms in the lower energy level absorb incident photon of energy hf and excited to higher energy level.

Study of laser is very important in various fields.

(a) Laser is used to measure distance.

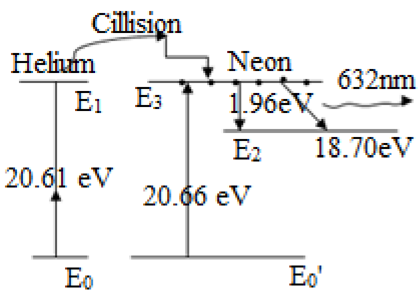
(b) It is used welding, cutting and drilling in industries.

(c) It is used in automatic control of spaceship and rocket.

(d) It is used in eye surgery.

(e) It is used in fighting.

The He-Ne laser consists of a long and narrow discharge tube of diameter about 5mm and length 10-100cm. The lasing material is the mixture of the gases with a concentration of 15% helium and 85% neon. The electrodes in the discharge tube are connected to a high voltage source. So an electric discharge takes place within the gas. With this high voltage some of the He atoms are raised to a metastable state at E_1 = 20.61eV above the ground state as shown in figure. It so happens that Ne has a metastable state at nearly the same energy, E_3 = 20.66 eV. The He atoms do not quickly return to the ground state by spontaneous emission. Rather it transfers the energy to Ne atoms during collision. With such collision the energy of excited He atoms will be transferred and it drops to ground state. However, getting the excess energy Ne atom is excited to the state E_3. The small difference of 0.05eV is supplied the kinetic energy of the atoms. In this way the higher state E_3 of Neon, becomes the metastable state, than E_2. Therefore, the population inversion is achieved for Ne atoms. Hence the lasing action takes place



by stimulated emission between the E_3 and E_2 states of neon. The laser light emitted is of about 632.8nm.

7. *Write the physical meaning of dispersive power and resolving power of plane transmission grating. Show that the product of the total number of ruling and the order of the spectrum gives the resolving power of the plane transmission grating.*

Sol^n: **(See in 2068 Bhadra Regular Q. No. 5 OR)**

8. *Two thin identical convexes lenses of focal lengths 8cm and each are coaxial and 4cm apart. Find the principal points and the position of object for which image is formed at infinity.*

Sol^n: Here, $f_1 = f_2 = 8$cm, separation between lenses (d) = 4cm

We have, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ $\Rightarrow f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{8\times8}{8+8-4} = \frac{16}{3}$ cm

$\beta = -\frac{df}{f_1} = -\frac{4\times16}{3\times8} = -\frac{8}{3}$ cm

$\alpha = \frac{df}{f_1} = \frac{6\times16}{3\times4} = +\frac{8}{3}$ cm

Again, $\frac{1}{f} = \frac{1}{V} - \frac{1}{U}$, for, $V = \infty$,

$\therefore U = -f = -\frac{16}{3}$ cm So, $u = U + \alpha = -\frac{16}{3}$ cm $+ \frac{8}{3}$ cm $= -2.67$ cm

Thus, position of the object is 2.67 cm left from the first lens.

9. *What is electric quadrupole? Calculate the electric potential of a linear quadrupole of separation 2z at an axial distance R from its center.*

Sol^n: (See in 2067 Ashwin Back Q. No. 9 OR)

OR

A ring radius "R" is carrying a uniformly distributed charge "q". Find an expression for electric field at any point on the axial line. Find the point at which electric field is maximum.

Sol^n: (See in 2067 Chaitra Back Q. No. 9 OR)

10. *A cylindrical resistor of radius 6mm and length 2.5cm is made of material that has a resistivity of 4×10^{-5} Ω.m. What are (i) the magnitude of the current density and (ii) the potential difference when the energy dissipation rate in the resistor is 2 watt?*

Sol^n: Radius of cylindrical resistor (R) = 6mm = 6×10^{-3} m

Length (l) = 2.5 cm = 2.5×10^{-2} m, Resistivity (ρ) = 4×10^{-5} Ωm

Dissipation rate of the resistor (P) = 2 Watt

(i) Current density (J) = ?

We have, $P = i^2R = J^2 A^2 R$

$$\Rightarrow J = \frac{1}{A}\sqrt{\frac{P}{R}} = \frac{1}{A}\sqrt{\frac{PA}{\rho l}} \qquad \text{since } R = \frac{\rho l}{A}$$

$$\Rightarrow J = \sqrt{\frac{P}{A\rho l}} = \sqrt{\frac{2}{\pi(6\times10^{-3})^2\times4\times10^{5}\times0.025}} = 1.33 \times 10^5 \text{ A/m}^2$$

(ii) Potential difference (V) = ?

Power (P) = iV = JAV

$$\therefore V = \frac{P}{JA} = \frac{2}{1.33\times10^5\times\pi\times(6\times10^{-3})^2} = 0.133 \text{ V}$$

11. *A solenoid 2.6m long and 1.3cm in diameter carry a current of 9A. The magnetic field inside the solenoid is 20mT. Find the length of the wire forming the solenoid.*

Sol^n: Length of the solenoid (l) = 2.6 m, diameter of solenoid (d) = 1.3cm, radius of the solenoid (r) = 0.65×10^{-2} m

Current through the solenoid (i) = 9A

Magnetic field inside the solenoid (B) = 0.02 T

We have, $B = \mu_0 n i = \frac{\mu_0 i N}{l}$ $\qquad \Rightarrow N = \frac{Bl}{\mu_0 i}$

Thus, the total length of wire used in making the solenoid is $2\pi rN$

$$\Rightarrow 2\pi rN = \frac{2\pi rBl}{\mu_0 i} = \frac{2\pi\times0.65\times10^{-2}\times0.02\times2.6}{4\pi\times10^{-7}\times9} = 188 \text{ m}$$

12. *Compare the methods of Biot Savart law and Ampere's law to calculate magnetic fields due to current carrying conductor. Calculate magnetic field at an axial distance "x" from the center of the circular coil carrying current.*

Sol^n: (See in 2067 Chaitra Back Q. No. 11)

13. *In a Hall experiment, a current of 25A is passed through a long foil of silver, which is 0.1 mm thick and 3 m long. Calculate the Hall voltage produces across the width by a flux of 1.4 Wb/m^2. If the conduction of silver is 6.8 $\times$ 10^7 mho/m, estimate the mobility of the electrons in silver.*

Sol^n: Current through the foil (i) = 25A, thickness of foil (t) = 0.1mm

Length of the foil (l) = 3m, flux density (B) = 1.4 wb/m^2

Conductivity of the silver (σ) = 6.8 $\times$ 10^7 mho/m

Hall voltage (V_H) = ?, and electron mobility in silver (μ_e) = ?

$$V_H = \frac{Bi}{net} \text{ and } n = \frac{N_A \rho}{M} = \frac{6.02\times10^{23}\times9.32\times10^{3}}{107.67\times10^{-3}} = 5.2 \times 10^{28}\ /m^3$$

$$\text{So, } V_H = \frac{1.4\times25}{5.2\times10^{28}\times1.6\times10^{-19}\times10^{-4}} = 4.2 \times 10^{-5}\ V$$

$$\mu_e = R_H\sigma = \frac{\sigma}{ne} = \frac{6.8\times10^{7}}{5.2\times10^{28}\times1.6\times10^{-19}} = 0.81 \times 10^{-2}\ m^2\ V^{-1}\ s^{-1}$$

14. *A parallel plate capacitor with circular plates is charged by current "i" (a) What is the magnitude of $\int B.ds$ in terms of μ_0 and i between the plates if r = (a/5) from the center? What is the magnitude of induced magnetic field for r = (a/5) in terms of displacement current?*

Sol^n: Using Ampere's law

(a) $\oint_S \vec{B}\cdot d\vec{s} = \mu_0\, i_{enclosed}$

$$= \mu_0 \frac{i\, A'}{A} = \mu_0 \frac{i\,.\pi\left(\frac{a}{5}\right)^2}{\pi a^2} = \mu_0 \frac{i}{25}$$

(b) $\oint_S \vec{B}\cdot d\vec{s} = \mu_0\, i_{enclosed}$ $\Rightarrow$ B . 2πr = $\mu_0\, i_{enclosed}$

$\Rightarrow$ B = $\frac{\mu_0}{2\pi r}$ (J_d A) = $\frac{\mu_0}{2\pi r}$ (J_d πr^2) = $\frac{\mu_0}{2}$ (J_d r) = $\frac{\mu_0}{2}\frac{i_d}{A}$ r = $\frac{\mu_0}{2}\frac{i_d}{\pi r^2}$ r = $\frac{\mu_0}{2}\frac{i_d}{\pi r}$

$\Rightarrow$ B = $\frac{\mu_0}{2}\frac{i_d}{\pi\left(a/5\right)}$ $\therefore$ B = $\frac{5\mu_0}{2}\frac{i_d}{\pi a}$

OR

An inductance L is connected to a battery of emf E through a resistance. Show that the potential difference across the inductance after t is $V_L = E\, e^{\left(-\frac{R}{L}\right)t}$*. At what time is the potential difference across the inductance equal to that across the resistance such that* $i = \frac{i_0}{2}$.

Sol^n: (See in 2068 Shrawan Back Q. No. 14)

15. *Write Maxwell equations in integral form. Convert them in differential form. Explain the physical meaning of each equation.*

Sol^n: (See in 2067 Ashwin Back Q. No. 15)

16. *Describe the physical significance of the wave function. Derive an expression of time dependent Schrodinger equation.*

Sol^n: (See in 2067 Mangsir Regular Q. No. 15)

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2070 Bhadra Regular (BCE, BGE, BME)

1. *Derive a relation to determine the radius of gyration of a compound pendulum. Why determination of the acceleration due to gravity is more accurate from a compound pendulum than a simple pendulum.*

Sol^n: The time period of oscillation for the compound pendulum is

$$T = 2\pi\sqrt{\frac{l^2+k^2}{lg}} \qquad \text{------ (1)}$$

Now squaring this equation on both sides, we get

$\Rightarrow T^2gl = 4\pi^2(l^2 + k^2)$ $\qquad \Rightarrow 4\pi^2l^2 + 4\pi^2 k^2 = gT^2l$

$\Rightarrow 4\pi^2l^2 - gT^2l + 4\pi^2 k^2 = 0$

Which is quadratic in l. So, l has two values with same time period T in one side of the compound pendulum. The values are

$l = \frac{gT^2 \pm\sqrt{(gT^2)^2-64\pi^4k^2}}{8\pi^2}$. These two values are different points and let these values are

$l_1 = \frac{gT^2+\sqrt{(gT^2)^2-64\pi^4k^2}}{8\pi^2}$ and $l_2 = \frac{gT^2-\sqrt{(gT^2)^2-64\pi^4k^2}}{8\pi^2}$

Adding these two equations, we get $\Rightarrow l_1 + l_2 = \frac{gT^2}{4\pi^2}$.

$\therefore T = 2\pi\sqrt{\frac{l_1 + l_2}{g}}$ -------- (2) Comparing Eq. (1) and Eq. (2), we have, if $l = l_1$, then $k^2/l = l_2$ $\qquad \Rightarrow k^2 = l_1l_2 = \sqrt{l_1 l_2}$

This gives the radius of gyration of compound pendulum. Simple pendulum is ideal where as compound pendulum is real. So, the determination of acceleration due to gravity by using compound pendulum is more accurate than from simple pendulum.

OR

Define the quality factor (Q). Derive a relation of quality factor (Q) from the damped harmonic motion and show that the quality factor (Q) is inversely proportional to damping constant (b).

Sol^n: (See in 2069 Poush Back Q. No. 1 OR)

2. *An oscillatory motion of a body is represented by $y = a\, e^{i\omega t}$ where y is displacement in time t, a is amplitude and ω is angular frequency. Show that the motion is simple harmonic.*

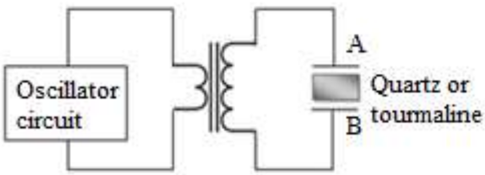
Sol^n: The displacement of a body executing motion at any instant is $y = a\, e^{i\omega t}$ Now, the velocity of that particle is $v = \frac{dy}{dt} = i\omega a\, e^{i\omega t}$ and the acceleration of that particle is $a = \frac{d^2y}{dt^2} = (i\omega)^2\, ae^{i\omega t} = -\,\omega^2\, y$;

Since, the angular frequency is constant. So, $a \propto -\,y$

This shows that the motion is simple harmonic.

3. *What is ultrasound? How these waves are produced? Differentiate such waves from ordinary sound wave.*

Sol^n: If the frequency of a sound is very high and greater than 20kHz than such sound is known as ultrasound. Ultrasound is produced from piezo-electric generator.



If a transverse electric field is applied along the certain crystal like quartz or tourmaline then mechanical stress is produced on the crystal. This effect of electric field on the crystal is known as piezo-electric effect, which is used to produce the ultrasound.

An ac field produced by the oscillator circuit is applied in two metallic plates A and B with the help of setup transformer there is a crystal like quartz or tourmaline as a dielectric between two plates as shown in above figure. On the application high transverse electric field the crystal starts to vibrate and surrounding the crystal there produced a wave whose frequency is very high. That wave is ultrasound.

Difference between ordinary sound and ultrasound:

Ordinary sound	Ultrasound
Its frequency range from 20Hz to 20kHz.	Its frequency is greater than 20kHz.
It can be received by normal person.	It cannot be heard.
	It has many applications in engineering field.

4. Why colors are observed when soap bubble is exposed to sunlight? Show that the consecutive bright or dark fringes are observed when the thickness of the film increases by $\frac{\lambda}{2}$ in an inclined plane.

Sol^n: Sunlight is the combination of seven different colors. When white light is incident on a thin film, the light which comes from any point from it will not include the color whose wavelength satisfies the equation $2\mu t \cos r = n\lambda$, in the reflected system. Therefore, the film will appear colored and the color will depend upon the thickness and the angle of inclination. If r and t are constant, the color will be uniform. In the case of oil on water, different colors are seen because r and t vary.

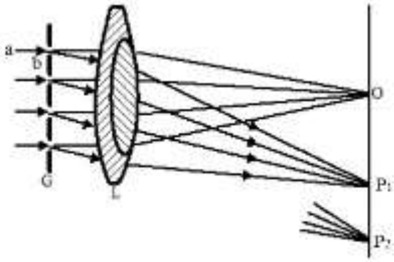
(Second part see in 2067 Ashadh Regular Q. No. 4)

OR

What is plane diffraction grating? How is it used to find the wavelength of a monochromatic light experimentally?

Sol^n: A plane diffraction grating is formed by ruling a number of lines at equidistant in a plane glass plate. The lines divide the glass plate into opaque and transparent portions; the width of transparent is of the order of wavelength of visible light. The region where a line is ruled becomes opaque whereas the space between the two lines is transparent.

Let the width of the transparent is a and that of opaque is b. The distance (a + b) is called grating element. The path difference between the rays from upper point of the slit and lower point of the opaque for a line of the grating is (a + b) $\sin\theta_n$, where θ_n is angle of diffraction for n^{th} order. The point P_1 will be bright or dark according to the rays interfere with one another. They will reinforce and give bright if



$$(a + b) \sin\theta_n = n\lambda \qquad \text{-------} (1)$$

They would interfere and produce dark of minima if

$$(a + b) \sin\theta_n = (2n - 1) \frac{\lambda}{2}$$

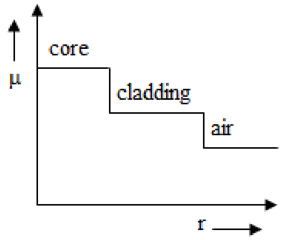
To find the wavelength of monochromatic light by plane diffraction grating, the angle of diffraction is measured for different order, then by using Eq. (1) λ can be determined.

5. *What is an optical fiber? How is it made? Write down the main differences between step index and graded index multimode optical fibers with well diagrams.*

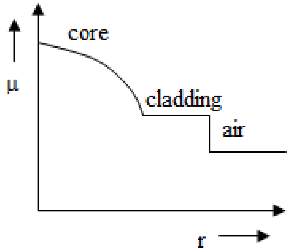
Sol^n: Optical fibers are made up of glass or plastic which are as thin as human's hair of diameter of about 150μm which is designed to guide light waves along the length of the fiber with the help of total internal reflection. It is formed by arranging concentric cylinders of different refractive index. The innermost part is core which is made of material of high refractive index. The part outside the core is known as cladding whose refractive index is less than that of core. The outermost layer is used for safety, which is known as jacket. There are two types of fibers.

Monomode fiber: The monomode fiber has a very narrow core of diameter 5μm or less than this. So the cladding is relatively thick than the core.

Multimode fiber: The diameter of the multimode fiber is relatively thick is about diameter of 50μm. These fibers are further divided into two types.



Step index optical fiber: The multimode fiber in which the refractive index μ_1 of core is constant and the refractive index of cladding is also constant value is called step index optical fiber. Thus at boundary of core-cladding interfere, the refractive index jumps from μ_1 to μ_2.



Graded index optical fiber: The multimode optical fiber in which the refractive index decreases continuously from core to the outer surface of the fiber and there is no noticeable boundary between the core and cladding is called the graded index optical fiber. In this fiber, there is low transmission loss due to self focusing.

6. *A 200mm long glass tube is filled with a solution of sugar, containing 15 gm of sugar in 100 ml of water. The plane of polarized light, passing through this solution, is rotated through 25°17'. Find the specific rotation of sugar.*

Sol^n: Here, length of tube (l) = 200 mm = 20 cm, concentration of solution (c) = 15gm/100ml = 150gm/cc, angle of rotation (θ) = 20°17' = $\left(25+\frac{17}{60}\right)°$ = 25.28°, specific rotation (S) = ?

We have, $S = \frac{10\theta}{lC} = \frac{10\times25.28}{20\times0.150} = 84.27° = 84°16'$

7. *Two thin converging lenses of focal lengths 0.2 m and 0.3 m are placed coaxially 0.1 m apart in air. An object is located 0.6 m in front of the lens of smaller focal length. Find the position of principal points and that of image.*

Sol^{n}: Here, focal length of first lens (f_1) = 0.2m = 20cm

Focal length of second lens (f_2) = 0.3m = 30cm

Separation between two lenses (d) = 0.1m = 10cm

We have the equivalent focal length of the combination is

$$f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{20\times30}{20+30-10} = 15\text{cm}$$

first principal point (α) = $\frac{fd}{f_2} = \frac{15\times10}{30}$ = 5cm

second principal point (β) = $-\frac{fd}{f_1} = -\frac{15\times10}{20}$ = - 7.5 cm

distance of the object from first lens (u) = 0.6 m = 60 cm

object position from first principal point (U) = - 65 cm

From lens formula

$$\frac{1}{f} = \frac{1}{V} - \frac{1}{U} \qquad \Rightarrow \frac{1}{15} = \frac{1}{V} + \frac{1}{65}$$

$$\Rightarrow \frac{1}{V} = \frac{1}{15} - \frac{1}{65} \qquad \therefore V = 19.5 \text{ cm}$$

Position of the image from second lens = 19.5 cm – 7.5 cm = 12 cm

The final image is formed at a distance of 12 cm to right of second lens.

8. *What is double refraction? Show that a beam of plane polarized light is converted into elliptically polarized light when it passes through a quarter wave plate.*

Sol^{n}: (See in 2067 Mangsir Regular Q. No. 6).

9. *Obtain an expression for electric field at an axial distance x from the center of the flat circular disk of radius R that carries a uniform surface* *charge density σ. Extend your result to calculate potential at a distance x.*

Sol^n: (First part see in 2067 Ashwin Back Q. No. 9)

The electric potential is given by V = - ∫ Edx

$$\Rightarrow V = -\frac{\sigma}{2\varepsilon_0}\int_0^R\left(1-\frac{x}{\left(\sqrt{x^2+R^2}\right)}\right)dx = -\frac{\sigma}{2\varepsilon_0}\left\{\int_0^R dx - \int_0^R\frac{xdx}{\left(\sqrt{x^2+R^2}\right)}\right\}$$

$$\Rightarrow V = \frac{\sigma}{2\varepsilon_0}\left[\left(\sqrt{x^2+R^2}\right)-R\right] \quad \text{Since } \int_0^R\frac{xdx}{\left(\sqrt{x^2+R^2}\right)} = \sqrt{x^2+R^2}$$

This is the required expression for potential at distance x from the center of the disc.

OR

A thin ring made of plastic of radius R is uniformly charged with linear charge density λ. Calculate the electric field intensity at any point at an axial distance y from the center. If electron is constrained to be an axial line of the same ring, show that the motion of electron is simple harmonic.

Sol^n: (See in 2068 Baishakh Regular Q. No. 9)

10. *A copper strip 2.5 cm wide and 1.5 mm thick is placed in magnetic field with B = 2.5 T perpendicular to the plane of the strip and away from the reader. If a current of 250 A is set up in the strip, what Hall potential difference appears across the strip? Charge density in copper is $8.5\times10^{28}/m^3$.*

Sol^n: We have, the Hall potential difference in the strip is

$$V_H = \frac{BI}{net} = \frac{2.5\times250}{8.5\times10^{28}\times1.6\times10^{-19}\times1.5\times10^{-3}} = 30.6\times10^{-6}\,V = 30.6\ \mu V$$

11. *Compare Ampere's law with Biot-Savart law. Obtain expressions for magnetic field intensity inside and outside the long straight wire carrying current.*

Sol^n: Biot-Savart's law states that the magnitude of magnetic field dB due to the current element dl at any point P at a distance r from of conductor carrying current I is

dB $\alpha \frac{Idl\sin\theta}{r^2}$ $\Rightarrow$ dB = $\frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$.

Ampere's law states that the line integral of magnetic induction B around any closed loop in a vacuum is equal to μ_0 times total current enclosed by the loop is $\oint \vec{B}\cdot\vec{dl} = \mu_0$ I

Ampere's law is more useful and simplified method to calculate the magnetic field for a current carrying conductor.

(Second part see in 2067 Ashadh Regular Q. No. 12)

12. *A spherical drop of water carrying a charge of 30 pC has a potential of 500 V at its surface (with V = 0 at infinity). (a) What is the radius of the drop? (b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop?*

Sol^n: Charge on the capacitor (q) = 30pC = 30×10^{-12} C

Potential at the surface of drop (V) = 500V

(a) Radius of the drop (r) =?

$V = \frac{q}{4\pi\varepsilon_0 r}$ $\Rightarrow r = \frac{q}{4\pi\varepsilon_0 V} = 9 \times 10^9 \times \frac{30\times10^{-12}}{500} = 5.4 \times 10^{-4}$ m

(b) If two such drops are combine to form a single drop then the charge on the new drop (Q) = 2q and the potential at the surface is given by

$V' = \frac{Q}{4\pi\varepsilon_0 R}$ $\Rightarrow \frac{V}{V'} = \frac{R}{2r}$ $\Rightarrow V' = \frac{2r}{R}V$

Also, the volume of the new drop = volume of 2 small drops

$\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3$ $\Rightarrow R = 2^{1/3}$ r

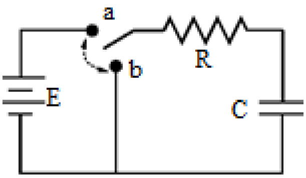
So, $V' = 2^{2/3}$ V $= 2^{2/3} \times 500 = 793.4$ V

13. *Calculate the displacement current between the capacitor plates of area 1.5×10^{-2} m² and rate of electric field charge is 1.5×10^{12} V/ms. Also, find the value of displacement current density.*

Solⁿ: (See in 2068 Baishakh Regular Q. No. 14)

14. *Obtain expressions for growth and decay of charges in the RC circuits. Explain how you will measure experimentally the capacitance of the given capacitor.*

Solⁿ: Consider a source of emf E containing a capacitor (C) and resistor (R) in series as shown in figure. When switch a is on, it starts charging because outer loop is complete. Consider q be the instantaneous charge in the capacitor and I be the current through the circuit then the potential drop across capacitor and resistor are $V_c = \frac{q}{C}$ and $V_R = IR$ respectively.



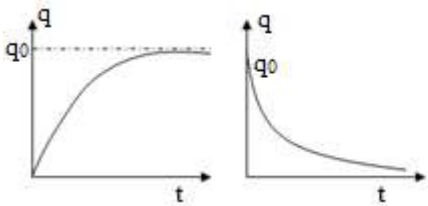
According to KVL,

$E = V_C + V_R$

$\Rightarrow E = \frac{q}{C} + IR = \frac{q}{C} + R\frac{dq}{dt}$

$\Rightarrow R\frac{dq}{dt} = E - \frac{q}{C} = \frac{EC-q}{C}$

$\Rightarrow \frac{dq}{(EC-C)} = \frac{dt}{RC}$



Integrating, we get $q = q_0\left(1 - e^{-\frac{t}{RC}}\right)$ This is the expression of growth of charge. Where $q_0 = EC$ is maximum charge in capacitor.

When the capacitor is fully charged and the key b is on, then discharging takes place through resistor. In this case

$V_C + V_R = 0 \qquad \Rightarrow \frac{q}{C} + IR = 0 \qquad \Rightarrow R\frac{dq}{dt} = -\frac{q}{C} \qquad \Rightarrow \frac{dq}{q} = -\frac{dt}{RC}$

Integrating on both sides, we get

$q = q_0\, e^{-\frac{t}{RC}}$ This is the expression for decay of charge.

Also, the current through the circuit is

$I = \frac{dq}{dt} = I_0\, e^{-\frac{t}{RC}}$ at $I = \frac{I_0}{2}$, $C = \frac{T_{1/2}}{0.6932R}$

Measuring the values of $T_{1/2}$, and R, the capacitance of a capacitor can be determined.

15. *Write down Maxwell equation in integral from with their physical meanings. Convert these equations into differential form.*

Sol^n: (See in 2067 Ashwin Back Q. No. 15)

16. *An electron is confined to an infinite height box of size 0.1 nm. Calculate the ground state energy of the electron. How this electron can be put to the third energy level?*

Sol^n: Width of the infinite potential box (a) = 0.1 nm = 10^{-10} m

Energy states is $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$

For ground state, n = 1,

$\therefore\ E_0 = \frac{\pi^2\hbar^2}{2ma^2} = \frac{\pi^2\times(1.05\times10^{-34})^2}{2\times9.1\times10^{-31}\times(10^{-10})^2} = 5.98\times10^{-18}$ J = 37.4 eV

Similarly, for third state, E_3 = 336.2 eV

Energy difference in these levels = $E_3 - E_1$ = 336.2 – 37.4 = 298.8 eV

Also, $E = \frac{hc}{\lambda}$ $\Rightarrow \lambda = \frac{hc}{E} = \frac{6.62\times10^{-34}\times3\times10^{8}}{298.8\times1.6\times10^{-19}} = 4.15\times10^{-9}$ m

By striking the electron with photon of wavelength 4.15 nm, electron can be put to the third energy level.

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2070 Magh Back (BCE, BGE, BME)

1. *A uniform circular disk of radius R oscillates in a vertical plane about a horizontal axis. Show that disk will oscillate with the minimum time period when the distance of the axis of rotation from the center is* $\frac{R}{\sqrt{2}}$.

Sol^n: Here, the uniform circular disc of radius R forms the compound pendulum. The time period of oscillation for the disc which is suspended at l from center is $T = 2\pi\sqrt{\frac{l^2+k^2}{lg}}$

For disc, $I_{CG} = I_{cener}$ $\Rightarrow Mk^2 = M\frac{R^2}{2} \Rightarrow k^2 = \frac{R^2}{2}$ $\therefore k = \frac{R}{\sqrt{2}}$

When the length of the pendulum is equal to the radius of gyration then the time period is minimum. So, T_{min} when $l = k = \frac{R}{\sqrt{2}}$.

OR

In the progressive wave show that the potential energy and kinetic energy of every particle will change with time but the average energy per unit volume remains constant.

Sol^n: (See in 2069 Ashadh Back Q. No. 3)

2. *A 2μF capacitor is charged up to 50 volt. The battery is disconnected and 50mH is connected across the capacitor so that the LC oscillation occurs. Calculate the maximum value of the current in the circuit.*

Sol^n: Capacitance of capacitor (C) = 2μF = 2×10^{-6} F,

Maximum potential (V) = 50 V,

Inductance of inductor (L) = 50mH = 5×10^{-2} H

Maximum charge in the capacitor (q_m) = VC = 10^{-4} C

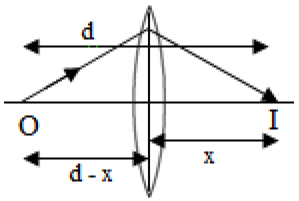
$(U_E)_{max} = \frac{q_m^2}{2C} = \frac{(10^{-4})^2}{2\times2\times10^{-6}} = 2.5 \times 10^{-3}$ Joule

Also, $(U_B)_{max} = \frac{1}{2} L\, i_{max}^2 = (U_E)_{max}$

$\Rightarrow i_{max} = \sqrt{\frac{2(U_E)_{max}}{L}} = \sqrt{\frac{2\times2.5\times10^{-3}}{5\times10^{-2}}} = 0.32$ Amp

3. *Show that the least possible distance between an object and its real image in a convex lens is four times the focal length of the lens.*

Sol^n: Consider a thin convex lens of focal length f. O is the point object and I is the real image. Let the distance between O and I is d and the distance of image from the lens be x.



$\therefore u = -(d - x)$ and $v = x$

From lens formula,

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\quad\Rightarrow \frac{1}{f} = \frac{1}{x} - \frac{1}{d-x}$ $\quad\Rightarrow \frac{1}{f} = \frac{d}{x(d-x)}$

$\Rightarrow xd - x^2 = fd$ $\quad\Rightarrow x^2 - xd + fd = 0$

Which is quadratic in x. So x has two values as

$x = \frac{d\pm\sqrt{d^2-4fd}}{2}$

for real image, the distance x should be real. For this

$\Rightarrow d^2 - 4fd \geq 0$ $\quad\Rightarrow d \geq 4f$ Proved.

4. *What is path difference and phase difference in interference? Explain why we have to make compensation in path difference in interference of light in parallel film in reflected system. Hence find out the condition for obtaining maxima in interference in this film by reflected light.*

Sol^n: Path difference is the difference in path traversed by the two waves, measured in terms of wavelength of the associated wave. It has a direct relation with phase difference. Phase difference decides the nature of interference pattern but phase difference is found out by path difference. Phase difference is related to quantum mechanics. If path difference between two waves is integral multiple of wavelength, which satisfies condition for constructive interference. Whereas, if path difference between two waves is odd multiple of half number wavelength, it satisfies condition for destructive interference.

Whereas phase difference is the difference between some reference point in two waves. It's how much one wave is shifted from the other. For example, at the origin, if one wave's displacement is zero and the other one has some displacement, then there is a shift, which is their phase difference. For eg. sine wave is zero at the origin, but cosine wave is not zero at the origin, its zero at $\pi/2$. So the phase difference is $\pi/2$. In fact, cosine wave is just the sine wave phase-shifted.

When light encounters a medium of higher refractive index, the reflected wave suffers a phase change of π i.e., a path change of $\lambda/2$. So, this has to be compensated in the expression of the path difference.

When light is incident at an angle of incident if i and undergoes refraction about the angle r the optical path difference between two reflected rays is $2\mu t \cos r$.

Therefore the true optical path difference Δ in above case is

$\Delta = 2\mu t \cos r + \lambda/2$

Hence the condition for constructive interference $2\mu t \cos r + \lambda/2 = n\lambda$

$2\mu t \cos r = (2n - 1)\lambda/2$ where n = 1, 2, 3, ...

OR

What is Nicol prism? How it is constructed? Discuss some of its applications.

Sol^n: (See in 2067 Ashwin Back Q. No. OR)

5. *A diffraction grating is used at normal distance gives a green line λ= 5400Å in a certain order superimposed on the violet line λ = 4500Å of the next higher order. If the angle of diffraction is 10°, how many lines are there per centimeter in the grating?*

Sol^n^: Wavelength of green line (λ_g) = 5400Å = 5.4×10^{-5} cm

Wavelength of violet line (λ_v) = 4500 Å = 4.5×10^{-5} cm

From question, $(a + b) \sin\theta_n = n\lambda_g = (n + 1) \lambda_v$

$\therefore\ n = \frac{\lambda_v}{\lambda_g - \lambda_v} = \frac{4.5}{5.4-4.5} = 5$

$(a + b) \sin 10° = 5 \times 5.4 \times 10^{-5}$ cm $\qquad \therefore\ a + b = 1.55 \times 10^{-3}$ cm

$N = \frac{1}{a+b} = \frac{1000}{1.55} = 643$ lines per cm

6. *What is Resolving power and dispersive power of a diffraction grating? Show that the resolving power of a grating depends on the order and number of rulings of grating.*

Sol^n^: (See in 2068 Bhadra Regular Q. No. 5 OR)

7. *Calculate the reverberation time in a hall measuring 40×10×20 ft with the following parameters. (i) 7500 sq. ft of plaster, α_1 = 0.03 (ii) 400 sq. ft of glass, α_2 = 0.025 (iii) 6000 sq. ft of wood and floor etc, α_3 = 0.06 (iv) 600 seats, α_4 = 0.03 and (v) audience of 500 persons, α_5 = 4.0 person.*

Sol^n^: Volume of the hall (V) = 40×10×20 = 8000 ft^3

Total absorption (αS) = $\alpha_1 S_p + \alpha_2 S_g + \alpha_3 S_{wf} + \alpha_4 S_s + \alpha_5 n_a$

$\alpha S = 0.03\times7500 + 0.025\times400 + 0.06\times6000 + 0.03\times600 + 4.0\times500$

$= 2613$

Reverberation time (T) = $\frac{0.05V}{\alpha S} = \frac{0.05\times8000}{2613} = 0.15$ sec

8. *What do you mean by Numerical Aperture and acceptance angle? Show that Numerical Aperture (NA) is proportional to square root of fractional refractive index change.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 7)

9. *Derive an expression for the electric field intensity at any point in the axial line of a ring of charge q. From your result show that electric field is maximum at x =* $\frac{a}{\sqrt{2}}$*, where a is the radius of the ring.*

Sol^n: (See in 2067 Chaitra Back Q. No. OR)

OR

A capacitor of capacitance C is charged through a resistor obtains an expression for charging current. Show the variation of current with time. How will you use this information to calculate capacitance C?

Sol^n: (See in 2070 Bhadra Regular Q. No. 14 for charging only)

10. *What will be the force per unit area with which plates of parallel plate capacitor attract each other if they are separated by 1mm and maintained at 100V potential difference and electric constant of the medium in unity.*

Sol^n: If q be the charge in the plate of the capacitor and A be the area of the plate then the force with which plates of parallel plate capacitor attract each other is $F = \frac{q^2}{2\varepsilon A}$, also, q = VC

So, $F = \frac{V^2C^2}{2\varepsilon A}$, again $C = \frac{\varepsilon A}{d}$

$\Rightarrow F = \frac{V^2\varepsilon^2A^2}{2\varepsilon A d^2} = \frac{V^2\varepsilon A}{2d^2}$ $\Rightarrow \frac{F}{A} = \frac{V^2\varepsilon}{2d^2}$

Here, V = 100 V, d = 10^{-3} m, ε = 1

$\Rightarrow \frac{F}{A} = \frac{100^2}{2\times10^{-6}} = 5 \times 10^9$ N/m^2

11. *Obtain Ohm's law in term of $\vec{J} = \sigma\vec{E}$, explain why and how resistance of a conductor varies with temperature. Based on this information explain superconductor. Give at least two characteristics of superconductors.*

Sol^n: (See in 2069 Chaitra Regular Q. No. 11)

12. *Compare Ampere's law with Biot-Savart's law. Which is more useful for calculating B for a current carrying conductor? Calculate the magnetic field inside and outside a long straight wire carrying current I.*

Sol^n: (See in 2070 Bhadra Regular Q. No. 11)

OR

State Faraday's law of Electromagnetic induction. Show that in electromagnetic induction the mechanical energy is converted into electric and finally in to heat energy.

Sol^n: (See in 2068 Shrawan Back Q. No. 10 OR)

13. *A solenoid having an inductance of 6.3μH is connected in series with 1.2KΩ resistor. (i) If a 14V battery is connected across the pair, how long will it take for the current to reach 80% of its equilibrium value? (ii) What is the current through the resistor at time t = τ_L.*

Sol^n: Inductance of solenoid (L) = 6.3×10^{-6} H, resistor (R) = 1200Ω

Emf of battery (ε) = 14V

(i) $i = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$ where $\tau_L = \frac{L}{R}$

$\Rightarrow 0.8 = 1 - e^{-t/\tau_L}$ $\Rightarrow e^{-t/\tau_L} = 0.2$ $\Rightarrow -t/\tau_L = \ln(0.2) = -1.609$

$\therefore t = 1.609\,\tau_L = 1.609 \times \frac{6.3\times10^{-6}}{1200} = 8.45 \times 10^{-9}$ sec

(ii) At $t = \tau_L$ $\Rightarrow i = \frac{\varepsilon}{R}(1 - e^{-1}) = \frac{14}{1200}(1 - e^{-1}) = 7.37 \times 10^{-3}$ Amp

14. *What is Hall Effect? Obtain an expression for Hall resistance. Show in graph how hall resistance varies with magnetic field.*

Sol^n: (See in 2067 Chaitra Back Q. No. 13 up to R = $\frac{B}{net}$)

15. *Calculate the magnitude of the Poynting vector and the amplitude of the electric and magnetic fields at a distance of 10 cm from a radio station which is radiating power of 10^5 watt uniformly over a hemisphere with radio station as center.*

Sol^n: Power of the source (P) = 10^5 watt,

radius of the hemisphere (r) = 10 cm = 0.1 m

Magnitude of Poynting vector (S) =?, E_m =?, B_m =?

$I = \frac{P}{2\pi r^2} = \frac{10^5}{2\times\pi\times0.1^2} = \frac{10^7}{2\pi}$ W/m^2

We have $I = \frac{E_m^2}{2\mu_0 c}$ $\quad\Rightarrow E_m = \sqrt{2\mu_0 cI}$

$\Rightarrow E_m = \sqrt{2\times4\pi\times10^{-7}\times3\times10^8\times{}^{10^7}/_{2\pi}} = 3.5\times10^4$ V/m

Similarly, $B_m = \frac{E_m}{c} = \frac{3.5\times10^4}{3\times10^8} = 1.2\times10^{-4}\,T = 120\,\mu T$

And, magnitude of Poynting vector (S) = $\frac{E_m B_m}{\mu_0} = \frac{3.5\times10^4\times1.2\times10^{-4}}{4\pi\times10^{-7}}$

$S = 3.34\times10^6$W/m^2

16. *Consider an electron of mass m is confined in an one dimensional infinite potential well of width l such that $V = \infty$ for $0 \le x$ and $x \ge l$, $V = 0$ for $0 < x < l$. Show that inside the well electron can only have discrete energy values.*

Sol^n: (See in 2067 Ashadh Regular Q. No. 16)

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2070 Chaitra Regular (BEX, BCT, BEL, BIE, B. Agri)

1. *Distinguish between free and forced vibrations. Write the differential equation of forced oscillation. Determine the amplitude for forced oscillation and hence explain sharpness of the resonance.*

Sol^n: *Difference between free and forced oscillation:*

Free vibration	Forced vibration
i. If the oscillation in a system is due to restoring force produced in the system then the oscillation is free oscillation.	i. If the oscillation in a system is due to continuous application of external force then the oscillation is forced oscillation.
ii. Frequency of oscillation is constant.	ii. Frequency of oscillation depends upon the frequency of applied force.
iii. Differential equation is $m\frac{d^2x}{dt^2} + kx = 0.$	iii. Differential equation is $m\frac{d^2x}{dt^2} + b\ \frac{dx}{xt} + kx = 0$

(Remaining part see in 2068 Bhadra Regular Q. No. 1)

OR

Define simple harmonic motion. Show that the average kinetic energy is half of the total energy of a particle executing simple harmonic motion.

Sol^n: If the motion of a body is vibrating such that acceleration produced on the body is directed towards the mean position and directly proportional to the displacement then the motion of the body is said to be simple harmonic. The displacement of a body executing simple harmonic motion at any instant is $y = R \sin(\omega t \pm \phi)$, the kinetic energy at any instant is K.E. $= \frac{1}{2} m\omega^2 R^2 \cos^2(\omega t \pm \phi)$ and the potential energy is

P.E. $= \frac{1}{2} m\omega^2 R^2 \sin^2(\omega t \pm \phi)$

So, the total energy of that body is E = K.E + P.E. = $\frac{1}{2}m\omega^2 R^2$

This shows total energy is independent on time whereas K.E. and P.E. both depends upon the time.

Now the average kinetic energy of the particle over a complete cycle is given by $KE_{ave} = \frac{1}{T}\int_0^T \frac{1}{2} m\omega^2 R^2 \cos^2(\omega t + \varphi)dt$

$$= \frac{m\omega^2 R^2}{4T}\int_0^T 2\cos^2(\omega t + \varphi)dt = \frac{m\omega^2 R^2}{4T}\int_0^T \{1 + \cos 2(\omega t + \varphi)\}dt$$

Again, the average value of both sine and cosine function for a complete cycle or whole time T is 0, we have

$$KE_{ave} = \frac{m\omega^2 R^2}{4T} \left| t \right|_0^T = \frac{m\omega^2 R^2}{4T} T = \frac{m\omega^2 R^2}{4} = \frac{E}{2}$$

This shows that the average kinetic energy is half of the total energy of a particle executing simple harmonic motion.

2. *A 2μF capacitor is charged up to 50V. The battery is disconnected and 50mH coil is connected across the capacitor so that LC oscillation to occur. Calculate the maximum value of the current in the circuit.*

Sol^n: Capacitance of capacitor (C) = 2μF = 2×10^{-6} F, maximum potential of capacitor (V) = 50V, inductance of inductor (L) = 50mH = 5×10^{-2} H

The maximum energy stored in the capacitor is equal to maximum energy stored in the inductor i.e; $\frac{q_m^2}{2C} = \frac{1}{2} L\, i^2_{max}$

$$\Rightarrow i_{max} = \frac{q_m}{\sqrt{LC}} = = \frac{10^{-4}}{\sqrt{5\times10^{-2}\times2\times10^{-6}}} = 0.32 \text{ Amp}$$

3. *The elastic limit of steel forming a piece of wire is equal to 2.70 × 10^8 Pa. What is the maximum speed at which transverse wave pulses can* *propagate along this wire without exceeding this stress? (density of steel = 7.89 × 10^3kg/m^3)*

Sol^n: The speed of the wave is given by $v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{2.7\times10^8}{7.89\times10^3}} = 184.99$ m/s

The maximum transverse speed is equal to velocity of wave. So, maximum transverse speed of wave is 184.99 m/s.

4. *What are Newton's rings? How can you use these rings to determine the refractive index of a given liquid?*

Sol^n: (See in 2067 Ashwin Back Q. No. 4)

OR

Discuss the phenomenon of Fraunhoffer's diffraction at single slit. Show that the relative intensities of the successive maxima are 1: $\frac{4}{9\pi^2} : \frac{4}{25\pi^2} :$ ...

Sol^n: (First part see in 2067 Ashadh Regular Q. No. 4 OR)

The intensity I at the point is given by $I = A^2 = A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$

A phase difference of 2π corresponds to a path difference of λ. Therefore a phase difference of 2α is given by

$2\alpha = \frac{2\pi}{\lambda}$ asinθ, where asinθ is the path difference between the secondary waves from A and B. For first primary maxima $\alpha = \frac{\pi}{\lambda}$ asin$\theta = \frac{\pi}{\lambda}(2n + 1)\frac{\lambda}{2}$ for n = 2, $\alpha_1 = \frac{3\pi}{2}$

$$\therefore I_1 = I_0\left(\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = \frac{4I_0}{9\pi^2}$$

For second primary maxima $\alpha = \frac{\pi}{\lambda}$ asin$\theta = \frac{\pi}{\lambda}(2n+1)\frac{\lambda}{2}$ for n = 2, $\alpha_2 = \frac{5\pi}{2}$

$$\therefore I_2 = I_0\left(\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2 = \frac{4I_0}{25\pi^2}$$

So, the relative intensities of the successive maxima are 1: $\frac{4}{9\pi^2}$: $\frac{4}{25\pi^2}$:

… Proved

5. *Light of wavelength 6000Å falls normally on a thin wedge shaped film of refractive index 1.4, forming fringes that are 2mm apart. Find the angle of the wedge.*

Sol^n: Wavelength of light (λ) = 6×10^{-5} cm, refractive index of the film enclosed by thin wedge (μ) = 1.4, fringe width (β) = 2 mm = 0.2 cm, angle of wedge (θ) =?

We have, $\beta = \frac{\lambda}{2\mu\theta}$ $\Rightarrow \theta = \frac{\lambda}{2\mu\beta} = \frac{6\times10^{-5}}{2\times1.4\times0.2} = 1.07 \times 10^{-4}$ radian

6. *If the plane of vibration of the incident beam makes an angle of 30° with the optic axis, compare the intensities of extraordinary and ordinary light.*

Sol^n: Let A be the amplitude of vibrations in the incident light making an angle 30° with the optic axis. Amplitude of the e-ray = A cos30° = $\frac{\sqrt{3}}{2}$ A and amplitude of the o-ray = A sin30° = $\frac{A}{2}$.

The intensity of e-ray is $I_E \propto \left(\frac{\sqrt{3}}{2}A\right)^2$ and intensity of o-ray $I_o \propto \left(\frac{1}{2}A\right)^2$

$\therefore \frac{I_E}{I_0} = 3:1$

7. *Show that the diameter of circle of least confusion depends on the diameter of lens aperture and dispersive power of the material of the lens but is independent of the focal length of the lens.*

Sol^n: (See in 2067 Mangsir Regular Q. No. 4 OR)

8. *An optical fiber has a numerical aperture of 0.22 and core refractive index 1.62. Determine the acceptance angle for the fiber in a liquid which has a refractive index of 1.25. Also, determine the fractional refractive index change.*

Sol^n: Here, NA = 0.22, μ_1 = 1.62, i_{max}= ? for μ_0 = 1.25

Acceptance angle (i_{max}) = $\sin^{-1}\left(\frac{NA}{\mu_0}\right) = \sin^{-1}\left(\frac{0.22}{1.25}\right) = 10.14°$

$NA = \mu_1\sqrt{2\Delta} \quad \Rightarrow \Delta = \frac{NA^2}{2\mu_1^2} = \frac{0.22^2}{2\times1.62^2}$

So, fractional refractive index change (Δ) = 0.0092

9. *Prove that the electric field due to short dipole at axial point is twice that at equatorial point.*

Sol^n: (See in 2067 Mangsir Regular Q. No. 9 OR)

10. *A capacitor of capacitance C is discharging through a resistor of resistance R. After how many time constants is the stored energy $\frac{1}{8}$ of its initial value?*

Sol^n: In discharging, the charge at any instant in the capacitor is

$q = q_0\, e^{-t/RC}$ and the energy stored in the capacitor is $U_E = \frac{q^2}{2C}$

$\Rightarrow U_E = \frac{{q_0}^2}{2C} e^{-2t/RC} \qquad \Rightarrow U_E(t = 0) = \frac{{q_0}^2}{2C}$

According to question, $U_E(t) = \frac{1}{8} U_E(t = 0)$

$\Rightarrow \frac{{q_0}^2}{2C} = \frac{1}{8}\frac{{q_0}^2}{2C} e^{-2t/RC} \qquad \Rightarrow e^{2t/RC} = 8$

$\Rightarrow \frac{2t}{\tau} = \ln(8) \qquad \Rightarrow t = \frac{\ln(8)}{2}\times\tau$

$\therefore$ t = 1.04 τ

11. *Give a general method to calculate electric field and potential due to continuous charge distribution. Using your method, calculate electric field at an equatorial distance y due to a long charged rod having linear charge density λ.*

Sol^n: In the case of continuous charge distribution, the distances between the charges are much smaller than the distance from the point of interest. In such situation, charge distribution is divided into small elements of small charge in terms of linear charge density λ, surface charge density σ and volume charge density ρ according to the dimensions of the charged body. The electric field and potential at any point P due to charge element dq is

$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$ and $dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$. The total electric field and potential are given by $E = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}$ and $V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$

(Remaining part see in 2068 Bhadra Regular Q. No. 9)

12. *Consider a circular coil of radius R carrying current I. Find the magnetic field at any point on the axis of the loop at a distance z from the center of the loop. Show that the circular current carrying coil behaves as a magnetic dipole for large distance.*

Sol^n: (See in 2067 Mangsir Regular Q. No. 14)

13. *In a Hall Effect experiment, a current of 3.2A lengthwise in a conductor 1.2cm wide, 4.0cm long and 9.5μm thick produces a transverse Hall voltage (across the width) of 40μV when a magnetic field of 1.4T is passed perpendicularly through the thin conductor. From this data, find (a) the drift velocity of the charge carriers and (b) the number density of charge carriers.*

Sol^n: Here, current through conductor (I) = 3.2 Amp, width (t) = 9.5 μm $= 9.5 \times 10^{-6}$ m, Hall voltage (V_H) = 40 μV $= 4 \times 10^{-5}$ V

We have, $V_H = \frac{BI}{net}$ $\Rightarrow n = \frac{BI}{etV_H}$

$\Rightarrow n = \frac{1.4\times3.2}{1.6\times10^{-19}\times9.5\times10^{-6}\times4\times10^{-5}} = 7.4 \times 10^{28}\ m^{-3}$

Also, drift velocity $(v_d) = \frac{I}{neA} = \frac{3.2}{7.4\times10^{28}\times1.6\times10^{-19}\times0.012\times0.04}$

$\therefore v_d = 5.63 \times 10^{-7}$ m/s

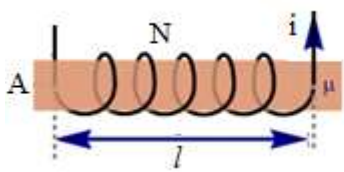
14. *Derive an expression for growth and decay of current in LR circuit. Explain inductive time constant by sketching graph between current and time for both cases.*

Sol^n: (See in 2067 Ashwin Back Q. No. 14)

OR

Derive an expression for inductance of a Solenoid and Toroid. Then show that inductance is the property of the coil.

Sol^n: Consider a solenoid of length l, number of turns N, cross sectional area A, then the magnetic field inside the solenoid carrying current i is



$B = \frac{\mu_0 N i}{l}$

Total flux linkages in the length l and area A is

$N\phi_B = NBA = \frac{\mu_0 N^2 A i}{l}$

The inductance of the solenoid is $L = \frac{N\phi_B}{i} = \frac{\mu_0 N^2 A}{l}$ ------ (1)

(For toroid see in 2068 Baishakh Regular Q. No. 13)

15. *Write and explain Ampere's law in magnetism. How Maxwell modified if. Based on this modified equation, explain the term displacement current. Prove displacement current is equal to conduction current.*

Sol^n: Ampere's law states that the line integral of magnetic field around the closed path is equal to μ_0 times current through the circuit.

i.e; $\oint \vec{B} . \overrightarrow{dl} = \mu_0$ i,

This is valid only if current i is steady. In other words this is true in any electric fields present are constant in time and not used for time-varying electric fields. So, the Ampere's circuital law is incomplete. By using the Faraday's law of electromagnetic induction Maxwell modified the

Ampere's law. The Faraday's law deals how a changing magnetic field induces an electric field as $\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d\phi_B}{dt}$.

Similarly, for reverse phenomenon, $\oint \vec{B} \cdot \overrightarrow{dl} = -\frac{d\phi_E}{dt}$. But, in this there are two things wrong; experiment requires positive sign in place of negative sign which is a kind of symmetry and the equation is dimensionally incorrect. To make it correct, to correct this we must multiply the right side by a factor $\mu_0\varepsilon_0$. So, the correct counterpart is $\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$.

This is called Maxwell's law of induction.

So, Maxwell modified the Ampere's law as in following form,

$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0\, i + \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$.

In this equation $\varepsilon_0 \frac{d\phi_E}{dt}$ is known as displacement current.

Consider conduction current i is passed through a capacitor with parallel plates. The current enters from the positive plate and leaves the negative plate. The conduction current is not possible across the capacitor gap because no charge is actually transported across this gap. So, to explain the continuity of current there set up a new current across the gap called displacement current.

If q be the charge in the plates of the capacitor then the electric field between the plates of the capacitor is $E = \frac{q}{\varepsilon_0 A} \Rightarrow q = \varepsilon_0\, A\, E$

So, $\frac{dq}{dt} = i_d = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d\phi_E}{dt}$

This is the displacement current which is equal to the conduction current in the wire.

16. *Explain Schrodinger wave equation. Derive time independent Schrodinger wave equation. Use this equation to find energy for a particle in a box of Infinite Square well potential.*

Sol^n: An Australian theoretical physicist Schrodinger derived a fundamental equation of quantum mechanics in the sense of that the Newton's law of motion in classical mechanics. The equation is the wave equation which represents the wave particle duality in the variable Ψ(x,t).

This function in general is a complex quantity and is dependent upon space and time is given by

$\psi(x,t) = Re^{-\frac{i}{\hbar}(Et-px)}$ --------(1)

Differentiating Eq. (1) with respect to x, twice times, we get

$\frac{d\psi}{dx} = R\left(\frac{i}{\hbar}\right)pe^{-\frac{i}{\hbar}(Et-px)}$ and $\frac{d^2\psi}{dx^2} = R\left(\frac{i}{\hbar}\right)^2 p^2e^{-\frac{i}{\hbar}(Et-px)} = -\frac{p^2}{\hbar^2}\psi$

$P^2\psi = -\hbar^2\frac{d^2\psi}{dx^2}$ --------- (2)

Total energy associated with the particle of mass "m" and moving with velocity v is E = K. E. + P. E.= $\frac{P^2}{2m} + V$

$P^2 = 2m (E - V)$

Multiplying this equation both sides by ψ, we get

$P^2\psi = 2m (E - V)\psi$

$\Rightarrow -\hbar^2\frac{d^2\psi}{dx^2} = 2m (E - V)\psi$

This is time independent Schrodinger equation in one dimension.

In three dimension,

$\therefore -\frac{\hbar^2}{2m}\nabla^2\psi = 2m (E - V)\psi$

(For second part see in 2067 Ashadh Regular Q. No. 16)

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2071 Shawan Back (BEX, BCT, BEL, BIE, B. Agri)

1. *Derive a relation to find the moment of inertia of a rigid body about an axis passing through its center of gravity using the torsional pendulum.*

Sol^n: (See in 2069 Poush Back Q. No. 1)

OR

What is resonance? Formulate the differential equation of forced electromagnetic oscillation. Then determine the expression for resonant frequency.

Sol^n: When the frequency of the applied force is equal to the natural frequency of the system then, the amplitude of oscillation is maximum in mechanical system. This condition is known as resonance. Whereas in electromagnetic system when the frequency of the applied emf is equal to the natural frequency of LC circuit then the current through the circuit is maximum. This is known as resonance.

(See in 2067 Mangsir Regular Q. No. 1 OR)

When $L\omega = \frac{1}{C\omega}$ then current amplitude in the circuit is maximum. This condition is current resonance. At current resonance current amplitude is $(i_0)_{max} = \frac{E_0}{R}$. The resonant frequency is $f = \frac{1}{2\pi\sqrt{LC}}$.

2. *A string has a linear density of 625 gm/m and is stretched with a tension 50N. A wave, whose frequency and amplitude are 160Hz and 10mm respectively, is travelling along the string. At what average rate is the wave transporting energy along the string?*

Sol^n: The average power supplied by the source is $P_{ave} = 2\pi^2\mu v f^2R^2$

$$P_{ave} = 2\pi^2\mu\sqrt{\frac{T}{\mu}}f^2R^2 = 2\pi^2f^2R^2\sqrt{T\mu}$$

$$\therefore\ P_{ave} = 2\pi^2\times(160)^2\times(10\times10^{-3})^2\times\sqrt{50\times0.625} = 282.48 \text{ Watt}$$

3. *Why is it important to study the reverberation time, before the construction of a Cinema Hall? Derive a relation for reverberation time based on absorption coefficient, volume and surface area of the hall.*

Sol^n: (See in 2067 Chaitra Back Q. No. 3

4. *What happens to the energy when waves perfectly cancel to each other in interference? Derive the relations for thin film interference by reflected light.*

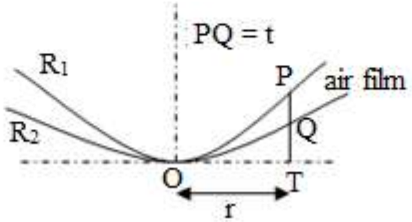
Sol^n: The phenomenon of non-uniform distribution of intensity in a medium due to the superposition of light waves from two coherent sources is known as interference. It appears that the energy "disappear" in the position of the destructive interference, but the law of conservation of energy states that it can't be destructed. The kinetic energy is transformed into potential energy.

(Second part see in 2068 Chaitra Regular Q. No. 4)

OR

Show that the diameters of Newton's rings when two surfaces of radii R_1 and R_2 are placed in contact are related by the relation $\frac{1}{R_1} - \frac{1}{R_2} = \frac{4n\lambda}{d_n^2}$, *where n is the integer number of the fringes.*

Sol^n: Consider two curved surfaces of radii of curvature R_1 and R_2 in contact at point O. A thin air film is enclosed between the two surfaces. The dark and bright rings are formed and can be viewed with a travelling microscope.



Suppose the radius of n^{th} dark ring = r. The thickness of air film at P, is PQ = PT – QT

From geometry, PT = $\frac{r^2}{2R_1}$ and QT = $\frac{r^2}{2R_2}$ $\therefore$ PQ = $\frac{r^2}{2R_1}$ - $\frac{r^2}{2R_2}$, But PQ = t

For reflected light , $2\mu t \cos r = n\lambda$, for dark rings. Here, for air $\mu = 1$, and for normal incident $\cos r = 1$.

$\therefore\ 2t = n\lambda \qquad \Rightarrow 2\left(\frac{r^2}{2R_1} - \frac{r^2}{2R_2}\right) = n\lambda \qquad \Rightarrow r^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = n\lambda$

For diameter D_n, $\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{4n\lambda}{D_n^2}$ Proved

5. *A grating with 250 grooves/mm is used with an incandescent light source. Assume the visible spectrum to range in wavelength from 400 to 700 nm. In how many orders can one see the entire visible spectrum?*

Sol^n: Here, N = 250 grooves/mm = 2500 grooves/cm

$\lambda_1 = 400\text{nm} = 4 \times 10^{-5}$ cm and $\lambda_2 = 700\text{nm} = 7 \times 10^{-5}$ cm

The relation for the maxima is, $(a + b)\sin\theta_n = n\lambda$

Now, $a + b = \frac{1}{N} = \frac{1}{2500}$ cm $= 4 \times 10^{-4}$ cm

For maximum order $\sin\theta_n = 1$

For $\lambda_1 = 400\text{nm} = 4 \times 10^{-5}$ cm $\qquad \Rightarrow (a + b) = n\lambda_1$

$\Rightarrow n = \frac{a+b}{\lambda_1} = \frac{4\times10^{-4}}{4\times10^{-5}} = 10$

Similarly, for $\lambda_2 = 700\text{nm} = 7 \times 10^{-5}$ cm $\Rightarrow n' = \frac{a+b}{\lambda_1} = \frac{4\times10^{-4}}{7\times10^{-5}} = 5$

$\therefore$ Number of visible orders are = n – n' = 10 – 5 = 5

6. *Define the polarization of light. Write its importance in different optical instruments. Derive the relation for the thickness of quarter wave plate and half wave plate.*

Sol^n: Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion i.e; whether the light waves are longitudinal or transverse, or whether the vibrations are linear, circular or torsional. The phenomenon which explains the transverse nature of light is polarization.

If a ray of light is passes through the optical instrument like Nicol prism then the light splits in two components. Among them one satisfies the Snell's law whereas another cannot obey the Snell's law. The light passes through the Nicol prism is polarized. It is used to test whether the medium is optically active or not, to measure the thickness of the retardation plates. Also, is it used to measure the concentration or strength of the solution by using the half shade polarimeter. So, polarization is very useful in optical instruments.

Retardation plates are plates of uniaxial crystals cut with optic axis parallel to the faces of the plate. When a polarized light is incident normally on such a plate, the vibrations in the beam break at the first face into one vibration along the optic axis and other perpendicular to it. These components travel the plate in same direction but with different velocities. One is ordinary and another is extra ordinary. The path difference between these components is $t\,|\mu_0 - \mu_e|$.

If this path difference produced by the plate is equal to half of the wavelength of the incident beam then the plate is known as half wave plate. For this $t\,|\mu_0 - \mu_e| = \lambda/2$. $\therefore\ t = \frac{\lambda}{2|\mu_0 - \mu_e|}$.

And if the path difference is equal to quarter of wavelength then the plate is known as quarter wave plat. For this $t\,|\mu_0 - \mu_e| = \lambda/4$.

$\therefore\ t = \frac{\lambda}{4|\mu_0 - \mu_e|}$.

7. *Two thin converging lenses of focal length 3cm and 4cm respectively are placed coaxially in air and separated by distance of 2cm. An object is placed 4cm in front of the first lens. Find the position of the nature of the image and its lateral magnification.*

Sol^n: Here, focal length of first lens (f_1) = 3cm

Focal length of second lens (f_2) = 4cm

Separation between two lenses (d) = 2cm

We have the equivalent focal length of the combination is

$f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{3\times4}{3+4-2} = 2.4$ cm

first principal point (α) = $\frac{fd}{f_2} = \frac{2.4\times2}{4} = 1.2$ cm

second principal point (β) = $-\frac{fd}{f_1} = -\frac{2.4\times2}{3} = -1.6$ cm

distance of the object from first lens (u) = 4 cm

object position from first principal point (U) = - 5.2 cm

From lens formula

$\frac{1}{f} = \frac{1}{V} - \frac{1}{U}$ $\Rightarrow \frac{1}{2.4} = \frac{1}{V} + \frac{1}{5.2}$

$\Rightarrow \frac{1}{V} = \frac{1}{2.4} - \frac{1}{5.2}$ $\therefore$ v = 4.46 cm

So, the image distance from the second lens = 2.86 cm to right.

$M = \frac{V}{U} = \frac{4.46}{5.2} = 0.86$

So, the final image is real and diminished.

8. *A glass - clad fiber is made with a core glass of refractive index 1.55 and the cladding is doped to give a fractional index difference of 5.5×10^{-4}. Determine (i) cladding index (ii) the critical internal reflection angle (iii) the external critical acceptance angle and (iv) numerical aperture (NA).*

Sol^n: Core of refractive index (μ_1) = 1.55

Fractional index (Δ) = 0.00055

(i) Cladding refractive index (μ_2) = ?

We have, $\Delta = \frac{\mu_1-\mu_2}{\mu_1}$ $\Rightarrow 0.00055 = \frac{1.55-\mu_2}{1.55}$

$\Rightarrow 0.0008525 = 1.55 - \mu_2$

$\therefore\ \mu_2 = 1.54915$

(ii) Critical internal reflection angle (C) = ?

$C = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right) = \sin^{-1}\left(\frac{1.54915}{1.55}\right) = 88.1°$

(iii) External critical acceptance angle is given by

$\mu_0 \sin C_0 = \mu_1 \sin\theta$

$C_0 = \sin^{-1}[1.55 \sin(90° - 88.1°)]$ since $\mu_0 = 1$ for air

$C_0 = 2.95°$

(iv) Numerical Aperture (NA) =?

$NA = \mu_1 \sqrt{2\Delta} = 1.55 \times \sqrt{2 \times 0.00055}$

$\therefore$ NA = 0.0514

9. *A particle of charge – q and mass m is placed midway between two equal positive charges q_0 of separation d. If the negative charge – q is displaced in perpendicular direction to the line joining them and released. Show that the particle describes a SHM with a period T =* $\sqrt{\frac{\epsilon_0 m\pi^3 d^3}{q q_0}}$.

Sol^n: (See in 2068 Shrawan Back Q. No. 13)

OR

Calculate electric field at any point in axial distance due to a dipole and a quadrupole. What conclusion you can draw from your results.

Sol^n: (For dipole see in 2067 Mangsir Regular Q. No. 9 OR first part)

$\therefore\ E = \frac{P}{2\pi\varepsilon_0 r^3}$

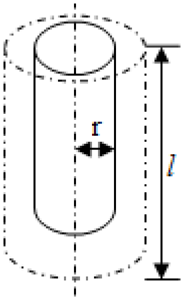
This shows that electric field at a point on the axial line due to short dipole is inversely proportional to the cubic power of the distance.

(For quadrupole see in 2067 Ashadh Regualr Q. No. 9 OR)

$\therefore\ E = \frac{3Q}{4\pi\varepsilon_0 R^4}$

This shows the electric field at a point on the axial line is inversely proportional to forth power of the distance in short quadrupole.

10. *Charges are uniformly distributed throughout the volume of an infinitely large cylinder of radius 'a'. Show that the electric field at a distance 'r' from the cylinder axis r < a is given by* $E = \frac{\rho r}{2\varepsilon_0}$ *where ρ is the volume charge density.*



Sol^n: Let r be the radius of cylindrical Gaussian surface with length *l*. The charge enclosed by the surface is given by Gauss law i.e; $\oint_S \vec{E}.\overrightarrow{da} = \frac{q}{\varepsilon_0}$

$\Rightarrow q = \varepsilon_0 E (2\pi r l)$

If ρ be the charge density of the cylinder then

$\Rightarrow \rho\pi r^2 l = \varepsilon_0 E (2\pi r l)$ $\therefore E = \frac{\rho r}{2\varepsilon_0}$ Proved

11. *A cylindrical capacitor has radii a and b. Show that half the stored electric potential energy lies within a cylinder whose radius is r =* $\sqrt{ab}$.

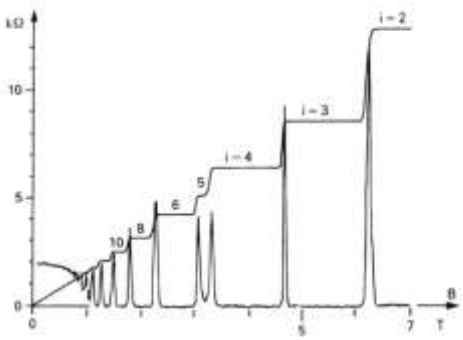
Sol^n: (See in 2069 Ashadh Back Q. No. 11)

12. *Explain Hall Effect. Derive a relation for Hall resistance. From this relation explain the meaning of quantization of hall resistance.*

Sol^n: (See in 2067 Chaitra Back Q. No. 13 OR)

Now, the Hall resistance is given by $R = \frac{V_H}{I} = \frac{B}{net}$.

From graph, the Hall resistance did not increase linearly with the field, instead, the plot shows a series of stair steps as shown in figure. Such effect has become is known as quantized Hall effect.



13. *The current density in a cylindrical wire of radius R = 2mm and uniform cross-sectional area is given by J = 2 × 10⁵A/m². What is the current through the outer portion of the wire between the radial distance* $\frac{R}{2}$ *and R?*

Sol^n: Here, current density (J) = 2×10^5 A/m^2,

Radius of the wire (R) = 2mm = 0.002 m

The wire can be divided into large number of small concentric rings of radius varies from 0 to R. Consider a ring of thickness dx and radius x then the current through that ring will be di = J da = J 2πx dx

So, current through the outer portion of wire between radial distance R/2 and R is given from the integration of above expression.

$i = \int_{R/2}^{R} J \times 2\pi x dx = 2\pi J \int_{R/2}^{R} x dx = \pi J \left|x^2\right|_{R/2}^{R} = \pi J \frac{3}{4} R^2$

$= \frac{3}{4} \times \pi \times 2 \times 10^5 \times (0.002)^2 = 1.88$ Amp

14. *Explain the phenomenon of "self-induction". Find an expression for the self-induction of a toroid having N numbers of turns, radius r and carrying current i.*

Sol^n: An induced emf E is appears in a coil if we change the current in the coil. This phenomenon is called self induction and emf appeared is called a self induced emf. The coefficient of the self induction depends on the geometry of the coil and independent of the magnitude of the current.

(See in 2067 Ashwin Back Q. No. 12)

OR

State Ampere's law. Find the expression for magnetic field outside and inside the long straight wire by using this law.

Sol^n: **(See in 2067 Ashwin Back Q. No. 12)**

15. *Write down the Maxwell's equations for non conducting medium. Find the equation of propagation of plane electromagnetic wave for E-field and B-field for such medium. Show that electromagnetic wave travels with velocity less than velocity of light in such medium.*

Sol^{n}: (See in 2068 Magh Back Q. No. 15)

16. *Derive Schrodinger time independent wave equation. A particle is moving in one dimensional potential well of infinite height and width 'a'. Find the expression for energy of the particle.*

Sol^{n}: (See in 2070 Chaitra Regular Q. No. 16)

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2071 Bhadra Regular (BCE, BME, BGE)

1. *Develop and solve the differential equation of damped harmonic oscillator subjected to a sinusoidal force. Then obtain an expression for its maximum amplitude and quality factor.*

Sol^n: (See in 2068 Bhadra Regular Q. No. 1)

The quality factor is a number which measures the quality of a system. If the quality factor is high then the efficiency of that system is also high and vice versa. Mathematically, quality factor is 2π times the ratio of instantaneous energy to energy loss per period.

i.e. $Q = 2\pi \frac{E(t)}{\Delta E}$

Also, $E(t) = \frac{1}{2} KR^2 e^{-bt/m}$ The energy loss per cycle is $\Delta E = \frac{bT}{m} E(t)$

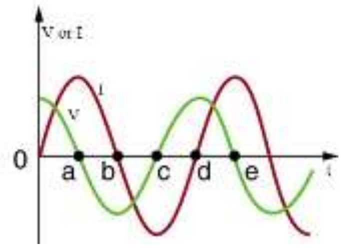
Thus, $Q = 2\pi \frac{E(t)}{\frac{bT}{m} E(t)} = \frac{2\pi}{T}\frac{m}{b} = \omega\tau$

OR

Obtain an expression for current in a driven LCR circuit and discuss how the current leads or lags the applied voltage in phase: (a) When the net reactance in the circuit is inductive and (b) When the net reactance in the circuit is equal to resistance. Illustrate it with the help of a figure.

Sol^n: (See in 2067 Mangsir Regular Q. No. 1 OR)

$\phi = \tan^{-1}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)$



(a) When net reactance in the circuit is inductive then the applied voltage leads the current through the circuit.

(b) Where $i_0 = \frac{E_0}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}$ is the current amplitude. When $L\omega = \frac{1}{C\omega}$

i.e; the reactance in the circuit is equal to resistance then current amplitude in the circuit is maximum. This condition is current resonance. At current resonance current amplitude is $(i_0)_{max} = \frac{E_0}{R}$. In this case applied voltage and current are in same phase.

2. *A circuit has L = 1.2mH, C = 1.6μF and R = 1.5Ω. (a) After what time t will the amplitude of the charge oscillations drop to one-half of its initial value? (b) To how many periods of oscillations does this corresponds?*

Sol^n: Here, L = 1.2mH = 1.2×10^{-3}, C = 1.6μF = 1.6×10^{-6} and

(a) Charge amplitude at time t is $q(t) = q_m\, e^{-\frac{Rt}{2L}}$

At t = 0, initial charge $q(0) = q_m$

According to question, $\frac{q_m}{2} = q_m\, e^{-\frac{Rt}{2L}}$ $\quad \Rightarrow \frac{Rt}{2L} = \ln(2)$

$\therefore\ t = \frac{2L\ln(2)}{R} = \frac{2\times1.2\times10^{-3}\times\ln(2)}{1.5} = 1.11 \times 10^{-3}$ sec

(b) The angular frequency for damped circuit is $\beta = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

$\Rightarrow \beta = \sqrt{\frac{1}{1.2\times10^{-3}\times1.6\times10^{-6}} - \frac{1.5^2}{4\times(1.2\times10^{-3})^2}} = 2.54 \times 10^4$ rad/sec

$\therefore\ T = \frac{2\pi}{\beta} = 2.47 \times 10^{-4}$ sec

So, number of oscillations (n) = $\frac{t}{T} \approx 4$.

3. *Calculate the reverberation time for a hall of volume 1400 m^3, which has seating capacity of 110 persons with full capacity of audience when absence are occupying only cushioned seats. The relevant data are:*

S. No.	*Surface*	*Area (m^2)*	*Coefficient of Absorption*
1	*Plastered wall*	*98*	*0.03*
2	*Plastered ceiling*	*144*	*0.04*
3	*Wooden door*	*15*	*0.06*
4	*Cushioned chair*	*88*	*1.00*
5	*Audience*	*150*	*4.70*

Sol^n: Here, total absorption

$(\alpha s) = 98\times0.03+144\times0.04+15\times0.06+88\times1+110\times4.70 = 614.60$

We have reverberation time is (T) = $\frac{0.158V}{\alpha s} = \frac{0.158\times1400}{614.60}$ = 0.36 sec

4. *Prove that the condition for achromatism for the combination of two lenses of focal lengths f_1 and f_2 of dispersive powers ω_1 and ω_2 separated by a distance x is $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{x}{f_1 f_2}(\omega_1 + \omega_2)$. Also, prove that the distance between two lenses is equal to focal length of lens if $f_1 = f_2$.*

Sol^n: (See in 2068 Chaitra Regular Q.No. 7)

$$\Rightarrow \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{x}{f_1 f_2}(\omega_1 + \omega_2)$$

when $f_1 = f_2$, then $x = f_1$, Proved.

5. *In He-Ne laser, the lasing action is due to neon gas. Then what is the role of the gas in it? Explain how the He-Ne laser works with a suitable energy level diagram on the basis of four level scheme for its action.*

Sol^n: (See in 2067 Ashwin Back Q. No. 9)

6. *Two sources of intensities 4I and I are used in an interference experiment. Obtain the intensities at points where the waves from two sources superimpose with a phase difference of (a) 0 (b) $\frac{\pi}{2}$ (c) π.*

Sol^n: Here, I_1 = 4I and I_2 = I, the resultant intensity due to the superposition of two waves is $I_{res} = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos\theta}$

(a) When $\theta = 0$ then, $I_{res} = \sqrt{16I^2 + I^2 + 8I^2} = 5I$

(b) When $\theta = \frac{\pi}{2}$ then, $I_{res} = \sqrt{16I^2 + I^2} = 4.12\ I$

(c) When $\theta = \pi$ then, $I_{res} = \sqrt{16I^2 + I^2 - 8I^2} = 3I$

OR

Explain the dispersive and resolving power of a diffraction grating. Prove that the ratio of dispersive power to resolving power is equal to the ratio of half width of peak and wavelength of the incident light.

Sol^n: Spreading the diffraction lines associated with various wavelengths by the grating is called dispersion. The capacity of spreading of the diffraction lines of the grating is called its dispersive power. It is defined as the ratio of the difference in the angle of diffraction of any two neighboring spectral lines to the difference in wavelength between two spectral lines. The relation for the path difference for maxima in diffraction grating is $(a + b) \sin\theta_n = n\lambda$

Differentiating on both sides, we get $\Rightarrow (a + b) \cos\theta_n\, d\theta = n\, d\lambda$

$\Rightarrow \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$ $\quad\Rightarrow \frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta}$

$nN = \cos\theta \frac{d\theta}{d\lambda}$-------------------- (1)

According to Rayleigh's dispersion criterion, if two monochromatic waves of wavelength λ_1 and λ_2 are to be distinguishable in an optical instrument, the angle $\Delta\theta$ between them must be at least equal to width of their diffraction peaks. To correlate the criterion with the dispersive power let's take the phase difference δ related with the path difference as, $\delta = \frac{2\pi}{\lambda} (a + b) \sin\theta$

Let $\Delta\theta_0$ the angular position of the minimum next to the peak at $\theta = 0°$. This corresponds to an extra path difference of $(a + b) \sin\Delta\theta_0$.

So, $\frac{\delta}{2\pi} = \frac{(a+b)\sin\Delta\theta_0}{\lambda}$

For N slits, the minima occurs when the phase difference is $\frac{2\pi}{N}$. So, $\delta = \frac{2\pi}{N}$

Therefore, above equation becomes $\frac{2\pi}{N.2\pi} = \frac{(a+b)\sin\Delta\theta_0}{\lambda}$

$\Rightarrow \sin\Delta\theta_0 = \frac{\lambda}{N(a+b)}$

Here, N is very large and $\Delta\theta_0$ is small. Therefore, $\sin\Delta\theta_0 \approx \Delta\theta_0$.

$\Delta\theta_0 = \frac{\lambda}{N(a+b)}$

To determine the half width of higher order peaks, $\Delta\theta_n$ for order n,

$\Delta\theta = \frac{d\delta}{d\theta}\Delta\theta = \frac{2\pi}{\lambda}(a + b)\cos\theta\ \Delta\theta = \Delta\theta_n$

If $\Delta\theta_n$ represents the half width of a peak of order n (n = 1, 2, 3, ……). This is the angle between the peak maximum and minimum to either side. $\Delta\theta_n = \frac{\lambda}{N(a+b)\cos\theta_n} = \frac{\lambda}{nN}\frac{d\theta}{d\lambda}$

$\Rightarrow \frac{d\theta}{d\lambda} = \frac{nN}{\lambda}\Delta\theta_n$ ------------ (2)

Where n is the number of order in the spectrum.

The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to the difference in wavelength between this line and a neighboring line such that the two lines appears to be just resolved. The relation for the path difference for maxima in diffraction grating is

$(a + b)\sin\theta_n = n\lambda$

From Rayleigh's criterion, if the principal maximum wavelength $\lambda + d\lambda$ falls on the first minimum of wavelength λ, then the wavelengths are said to be resolved. Let this common diffraction angle be represented by $(\theta_n + d\theta)$, so for n^{th} order spectrum, the two wavelengths λ and $\lambda + d\lambda$ will be just resolved if the following conditions are satisfied

$(a + b) \sin(\theta_n + d\theta) = n(\lambda + d\lambda)$ and $\quad (a + b) \sin(\theta_n + d\theta) = n\lambda + \frac{\lambda}{N}$.

So, $n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N} \qquad \therefore \qquad \frac{\lambda}{d\lambda} = nN$ ---------- (3)

Dividing Eq. (2) by Eq. (3)

$\frac{\omega}{R} = \frac{\Delta\theta_n}{\lambda}$.

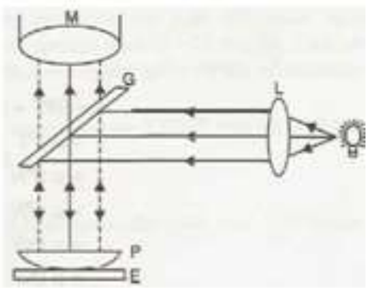
This shows that the ratio of dispersive power to resolving power is equal to the ratio of half width of peak and wavelength of the incident light.

7. *Derive the necessary formula for linearly, circularly and elliptically polarized light when light is emerged out of the doubly refracting crystal.*

Sol^n: (See in 2067 Mangsir Regular Q. No. 6)

8. *What are Newton's rings? Derive the relation for the diameter of the bright rings. What is the difference between the rings observed by reflected light and by transmitted light? Explain how does the pattern appear when white light is used?*

Sol^n: When a Plano – convex lens is placed in contact with a flat glass surface, a thin air film is formed. When such film is exposed by monochromatic light, a series of concentric fringes are formed which are called Newton's rings.



When monochromatic light is incident in the experimental set up as shown in figure above, Newton's rings are observed. The path difference between the rays reflected on the upper and lower surface of the thin film is $2\mu t \cos r + \frac{\lambda}{2}$.

For almost normal incident in air film, $r \approx 0$ and $\mu = 1$. So, the path difference is $2t + \frac{\lambda}{2}$.

At the point O, t = 0, so the path difference is $\frac{\lambda}{2}$. Hence the center spot is dark. The condition for the bright rings is $2t + \frac{\lambda}{2} = n\lambda$

$\Rightarrow 2t = (2n - 1)\frac{\lambda}{2}$ ---------- (1), where n = 0, 1, 2…….

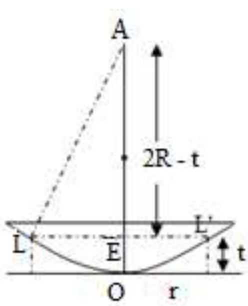
Let a plan-convex lens of radius of curvature R is placed on the plane glass plate AOB, the curved surface LOL' is a part of spherical surface having radius of curvature R. At this particular case r is the radius of Newton's ring corresponding to film thickness t. From figure

$\frac{\text{LE}}{\text{OE}} = \frac{\text{AE}}{\text{EL}'}$ $\Rightarrow$ LE × EL' = OE × AE $\Rightarrow$ r×r = t (2R – t)

$\Rightarrow r^2 \cong 2Rt$ since t is very small, so t^2 can be neglected.

For bright fringe, $2t = (2n - 1)\frac{\lambda}{2}$. $\Rightarrow \frac{r^2}{R} = (2n - 1)\frac{\lambda}{2}$

$r_n^2 = (2n - 1)R\frac{\lambda}{2}$ and diameter is



$D_n = \sqrt{2(2n-1)\lambda R} = \sqrt{2\lambda R}\sqrt{(2n-1)}$

Hence, the diameter of bright rings are proportional to the square root of odd numbers for the reflected light.

Similarly, transmitted light the path difference for the bright fringe is

$2t = n\lambda$

Solving this then the diameter of the nth order bright fringe is

$D_n = 2\sqrt{n\lambda R}$.

The diameter of bright rings are proportional to the square root of natural numbers for the transmitted light.

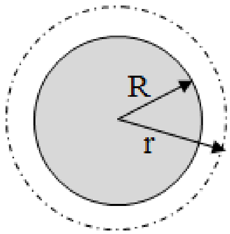
With monochromatic light, Newton's rings are alternately dark and bright. The diameter of the rings depends upon the wavelength of the using light. When the white light is used, the diameter of the rings of the different colors will be different and colored rings are observed. Only the first few rings are clear and after that due to overlapping of the rings of different colors, the rings cannot be viewed.

9. *Define the electric displacement vector. Develop a relation between electric displacement vector, electric field and polarization. Also prove that induced charge in dielectric is always less than free charge.*

Sol^n: (See in 2068 Shrawan Back Q. No. 9 OR)

OR

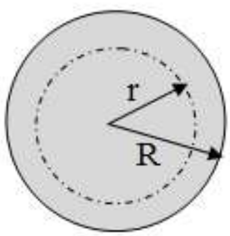
A dielectric sphere of radius R is charged uniformly. Obtain expression for electric field intensity (a) outside (b) at the surface and (c) inside the sphere.



Sol^n: Side figure shows a dielectric sphere of radius R is charged uniformly. The charge density ρ at a point depends only on the distance of the point from the center and not on the direction. To find the expression for E at points outside, and inside, we use Gauss law.

(a) Field outside the sphere: Applying Gauss law to a spherical Gaussian surface of radius r (r > R)

$\oint_s \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$ $\Rightarrow$ $E.4\pi r^2 = \frac{q}{\varepsilon_0}$ $\therefore$ $E = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}$



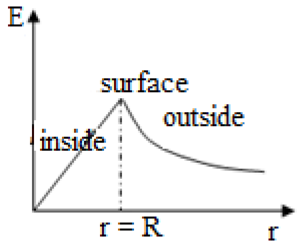
Where q is the total charge. Thus for points outside a spherically uniformly distribution of charge, the electric field has the value that it would have if the charge were concentrated at its center.

(b) At the surface: At surface r = R, then the

electric field is $E = \frac{q}{4\pi\varepsilon_0}\frac{1}{R^2}$

(c) Field inside the sphere: Side figure shows a spherical Gaussian surface of radius r drawn inside the charge distribution. Gauss's law gives $\varepsilon_0 \oint_s \vec{E}.\overrightarrow{ds} = q'$ $\Rightarrow \varepsilon_0 E\, 4\pi r^2 = q'$

$\therefore E = \frac{q'}{4\pi\varepsilon_0}\frac{1}{r^2}$



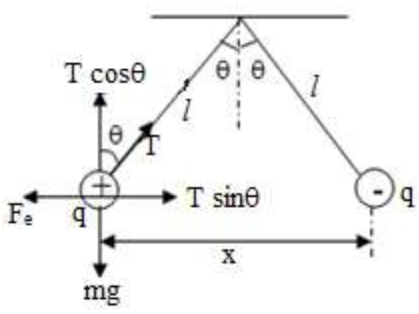
Where q' is that part of q contained within the sphere of radius r. The part of q that lies outside this sphere makes no contribution to E at radius r. Now, $q' = q\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = q\left(\frac{r}{R}\right)^3$

$\therefore E = \frac{q}{4\pi\varepsilon_0}\frac{r}{R^3}$

This will be zero at r = 0.

10. *Two similar balls each of mass m are hung from silk threads of length l and carry similar charges q. Assume that the angle made by each thread with vertical, θ is very small. Show that* $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$, *where x is separation between the balls. Also calculate the charge q on the hung mass if l = 1.2 m, m = 20 gm and x = 3 cm.*

Sol^n: There acting three forces on the ball; weight of the ball (mg), tension on the string (T) and electrical force F_e. Direction of these forces is as shown in the figure. Resolving T into two rectangular components, at equilibrium we have, T sinθn= F_e, T



 By: Maha Datta Paudel

$\cos\theta = mg$,

$\Rightarrow \tan\theta = \frac{F_e}{mg}$, -------- (1)

For small angle $\tan\theta \approx \sin\theta \approx \theta = \frac{x}{2l}$ and $F_e = \frac{q^2}{4\pi\varepsilon_0 x^2}$

From Eq. (1), $mg \times \frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 x^2} \Rightarrow x^3 = \frac{lq^2}{2\pi\varepsilon_0 mg}$

$\therefore x = \left(\frac{lq^2}{2\pi\varepsilon_0 mg}\right)^{1/3}$

At, x = 3cm, l = 1.2 m and m = 20gm

$\Rightarrow q = \left(\frac{2\pi\varepsilon_0 mgx^3}{l}\right)^{1/2} = \left(\frac{2\pi\times8.85\times10^{-12}\times0.02\times9.81\times(0.03)^3}{1.2}\right)^{1/2}$

$\therefore q = 1.57 \times 10^{-8}$ C = 15.7 nC

11. *The parallel plates in a capacitor, with a plate area 8.5 cm^2 and air filled separation of 3 mm are charged by a 6V battery. They are then disconnected from the battery and pulled apart to a separation of 8mm. Neglecting fringing, find (a) the potential difference between the plates (b) the initial energy stored and (c) final energy stored.*

Sol^n: Capacitance of parallel plate capacitor (C) = $\frac{\varepsilon_0 A}{d}$ and q = CV

(a) Initial potential (V_i) = 6V and initial separation (d_i) = 3 mm

Final potential (V_f) = ?, final separation (d_f) = 8 mm

The initial charge in capacitor = final charge in the capacitor

$\Rightarrow q = \frac{\varepsilon_0 A V_i}{d_i} = \frac{\varepsilon_0 A V_f}{d_f}$ $\Rightarrow V_f = \frac{d_f}{d_i} \times V_i$

$\therefore V_f = \frac{8}{3} \times 6 = 16$ volt

(b) The initial energy stored, $\frac{1}{2} CV_i^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d} V_i^2$

$= \frac{8.85\times10^{-12}\times8.5\times10^{-4}\times36}{2\times3\times10^{-3}} = 4.51 \times 10^{-11}$ Joule

(c) The final energy stored, $\frac{1}{2}CV_f^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V_f^2$

$= \frac{8.85\times10^{-12}\times8.5\times10^{-4}\times16^2}{2\times8\times10^{-3}} = 1.2\times10^{-10}$ Joule

OR

A capacitor discharges through a resistor R. (a) After how many times constant (τ_c) does its charge fall to one half of its original value? (b) After how many time constants does the stored energy drop to half of its initial value?

Sol[n]: In discharging, the charge in capacitor $q(t) = q_m e^{-\frac{t}{\tau_c}}$ where $\tau_c = RC$

(a) According to question, $q(t_1) = \frac{1}{2}q_m$

$\Rightarrow q_m e^{-\frac{t_1}{\tau_c}} = \frac{1}{2}q_m \quad \Rightarrow e^{\frac{t_1}{\tau_c}} = 2 \quad \Rightarrow \frac{t_1}{\tau_c} = \ln(2)$

$\therefore t_1 = \tau_c \ln(2) = 0.69\tau_c$

(b) $U_E = \frac{q^2}{2C} = \frac{q_m^2}{2C}e^{-2t/\tau_c}$

According to question, $U_E(t_2) = \frac{q_m^2}{4C}$

$\Rightarrow \frac{q_m^2}{2C}e^{-2t_2/\tau_c} = \frac{q_m^2}{4C} \quad \Rightarrow e^{-2t_2/\tau_c} = \frac{1}{2} \quad \Rightarrow e^{2t_2/\tau_c} = 2$

$\Rightarrow \frac{2t_2}{\tau_c} = \ln(2) \quad \Rightarrow t_2 = \frac{\tau_c}{2}\ln(2) \quad \therefore t_2 = 0.35\,\tau_c$

12. *What is Biot-Savart law? Derive an expression for flux density due to a current carrying circular loop at its axial point.*

Sol[n]: Biot-Savart's law states that the magnitude of magnetic field dB due to the current element d*l* at any point P at a distance r from of conductor carrying current I is

$dB \propto \frac{I\,dl\sin\theta}{r^2} \quad \Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$

(Remaining part see in 2067 Mangsir Regular Q.No. 14)

13. *If a parallel plate capacitor with circular plate be charged, prove that the induced magnetic field at a distance r in the region between the plates be* $B = \frac{1}{2}\mu_0\varepsilon_0 r\frac{dE}{dt}$ *for* $r \le R$ *and* $B = \frac{1}{2}\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}$ *for* $r \ge R$.

Sol^n: For $r \le R$, $\oint \vec{B}.\vec{dl} = \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$

$\Rightarrow B \times 2\pi r = \mu_0\varepsilon_0 A \frac{dE}{dt} = \mu_0\varepsilon_0 \pi r^2 \frac{dE}{dt}$

$\therefore B = \frac{1}{2}\mu_0\varepsilon_0 r \frac{dE}{dt}$

For $r \ge R$, $\oint \vec{B}.\vec{dl} = \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$

$\Rightarrow B \times 2\pi r = \mu_0\varepsilon_0 A \frac{dE}{dt} = \mu_0\varepsilon_0 \pi R^2 \frac{dE}{dt}$

$\therefore B = \frac{\mu_0\varepsilon_0}{2r} R^2 \frac{dE}{dt}$

14. *In a Hall-effect experiment, a current of 3A sent lengthwise through a conductor 1 cm wide, 4cm long and 1 μm thick produces a transverse Hall voltage of 10μV, when a magnetic field of 1.5T is passed perpendicularly through the thickness of the conductor. Calculate (a) drift velocity of the charge carriers and (b) the number density of charge carriers.*

Sol^n: (a) At equilibrium, $e E_H = Bev_d$

$\Rightarrow v_d = \frac{E_H}{B} = \frac{V_H}{dB} = \frac{10\times10^{-6}}{1.5\times10^{-2}} = 6.67 \times 10^{-4}$ m/s

(b) $n = \frac{Bi}{V_H et} = \frac{1.5\times3}{10\times10^{-6}\times1.6\times10^{-19}\times10^{-6}} = 2.81 \times 10^{30}$ /m^3

15. *Define Poynting vector and develop an expression of it in terms of electric and magnetic fields. Using the Poynting vector calculates the maximum electric and magnetic fields for sun-light if the solar constant is 1.4 KW/m^2.*

Sol^n: (First part see in 2067 Ashadh Regular Q. No. 15)

Second part, $I = 1.4\ KW/m^2 = 1400\ W/m^2$

We have, $I = \frac{E_m B_m}{2\mu_0}$ and $\frac{E_m}{B_m} = c$ $\Rightarrow E_m = cB_m$

$\therefore\ I = \frac{cB_m^2}{2\mu_0}$ $\Rightarrow B_m = \sqrt{\frac{2I\mu_0}{c}} = \sqrt{\frac{2\times1400\times4\pi\times10^{-7}}{3\times10^8}}$

$B_m = 3.4 \times 10^{-6}\ T = 3.4\ \mu T$

And $E_m = c\ B_m = 3 \times 10^8 \times 3.4 \times 10^{-6} = 1027\ V/m$

16. *A bean of electrons having energy of each 3 eV is incident on a potential barrier of finite height 4 eV. If the width of the barrier is 20Å, calculate the percentage transmission of the beam through the barrier.*

Sol^n: We have transmission coefficient for $E < V_0$ is

$T = T_0\ e^{-2\alpha a} = \frac{16E}{V_0}\left(1 - \frac{E}{V_0}\right) e^{-2\alpha a}$

Where $\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar} = \frac{\sqrt{2\times9.1\times10^{-31}(4-3)\times1.6\times10^{-19}}}{1.05\times10^{-34}} = 5.14 \times 10^9$

Hence the transmission coefficient is

$T = \frac{16\times3}{4}\left(1 - \frac{3}{4}\right) e^{-2\times5.12\times10^9\times20\times10^{-10}} = 3.54 \times 10^{-9}$

Percentage of transmission = $3.54 \times 10^{-7}\%$

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2071 Magh Back (BCE, BME, BGE)

1. *Define physical pendulum and derive its formula of time period. Also differentiate mechanical oscillations and electromagnetic oscillations.*

Sol^n: A rigid body of any shape which is suspended about the horizontal axis passing through its own point forms the physical pendulum, that point is known as the point of suspension and the distance between point of suspension and center of gravity is the effective length of the pendulum.

(Second part see in 2067 Chaitra Regular Q. No. 1)

Mechanical oscillation	***Electromagnetic oscillation***
• There is a particle is in motion. • There is variation in displacement. • Differential equation is $\frac{d^2y}{dt^2} = -\omega^2 y$, where $\omega^2 = \frac{k}{m}$	• There is charge is in motion. • There is variation of charge in the capacitor. • Differential equation is $\frac{d^2q}{dt^2} = -\omega^2 q$, where $\omega^2 = \frac{1}{LC}$

2. *The speed of transverse wave on a string is 170 m/s when the string tension is 120N. To what value must the tension be changed to raise the wave speed to 180m/s?*

Sol^n: Speed of wave (v_1) = 170 m/s when tension (T_1) = 120N

Tension on the string (T_2) = ? when speed (v_2) = 180 m/s

We have, $v = \sqrt{\frac{T}{\mu}}$ $\Rightarrow v_1 = \sqrt{\frac{T_1}{\mu}}$ $\Rightarrow v_2 = \sqrt{\frac{T_2}{\mu}}$

$\Rightarrow T_2 = \frac{v_2^2}{v_1^2} \times T_1 = \left(\frac{180}{170}\right)^2 \times 120 = 134.54$ m/s

3. *The volume of a room is 980 m^3. The wall area of the room is 150m^2, ceiling area 95 m^2 and floor area is 90 m^2. The average sound* *absorption coefficients for wall, ceiling and floor are respectively 0.03, 0.5 and 0.06. Derive Sabine's relations in acoustics and calculate reverberation time using above data.*

Sol^n: (Sabine's relation see in 2067 Ashadh Regular Q. No. 3)

The total absorption is given by,

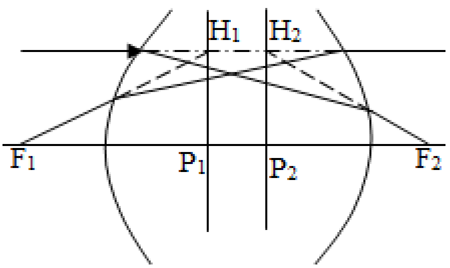
$\alpha s = \alpha_w s_w + \alpha_c s_c + \alpha_f s_f = 0.03\times150+0.8\times95+0.06\times90 = 85.9$

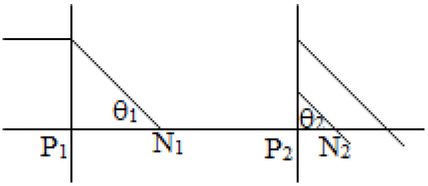
and volume (V) = 980 m^3

$\therefore$ T = $\frac{0.158\times980}{85.9}$ = 1.8 sec

4. *What are coaxial optical system and cardinal points? State their properties and show their positions in diagram. Illustrate the use of these points in the formation of an image by a lens system.*

Sol^n: If two lenses are placed in such a way that their principal axes are common or overlapping to each other than such system are coaxial optical system. In this system, there are three pairs of different points on the principal axis which are reference point or measure various distances in the system of coaxial lenses. These three pair of reference points is called cardinal points. They are: two principal points, two principal focal points and two nodal points.





Consider two coaxial lens systems, the system has two principal foci F_1 and F_2 as shown in figure. Also, at point of intersection H_1 and H_2 the perpendicular drawn on the principal axis meet at P_1 and P_2 are known as principal points. These are the positions of equivalent lens for image at infinity and object at infinity. Nodal points are a pair of conjugate points on the principal axis which have unit angular magnification. If a ray of light is directed towards one of the nodal points after refraction, it appears to proceed from the second conjugate point in the parallel direction. In figure N_1 and N_2 are nodal points.

5. *Explain why population inversion is required for laser operation, and explain how population inversion is achieved in a four-level laser.*

Sol^n: (See in 2067 Ashwin Back Q. No. 8)

6. *Derive an expression for n^{th} dark ring of reflected system in Newton's ring experiment. Also show that central of the Newton's ring is dark.*

Sol^n: (See in 2068 Bhadra Regular Q. No. 5)

OR

Discuss and derive the necessary formula of the intensity distribution in the diffraction pattern due to single slit.

Sol^n: (see in 2067 Ashadh Regular Q. No. 4 OR)

7. *In a double-slit experiment, the distance between slits is 5mm and the slits are 1m from the screen. Two interference patterns can be seen on the screen: one due to light of wavelength 480nm, and the other due to light of wavelength 600nm. What is the separation on the screen between the third-order bright fringes of two interference patterns?*

Sol^n: Slits separation (d) = 5 mm = 5×10^{-3} m, distance between slits and screen (D) = 1 m

λ_1 = 480 nm = 4.8×10^{-7} m, λ_2 = 600 nm = 6.0×10^{-7} m

The position of the third order bright fringe is y = $\frac{3\lambda D}{d}$

For λ_1, $y_3 = \frac{3\lambda_1 D}{d}$ and for λ_2 $y_3' = \frac{3\lambda_2 D}{d}$

So, separation between third order bright fringe (Δy) = $\frac{3D}{d}$ (λ_2 - λ_1)

$\Rightarrow \Delta y = \frac{3 \times 1}{5 \times 10^{-3}} \times (6.0 - 4.8) \times 10^{-7} = 7.2 \times 10^{-5}$ m

8. *A beam of polarized light is sent into a system of two polarizing sheets. Relative to the polarization direction of that incident light, the polarizing directions of the sheets are at angle θ for the first sheet and 90° for the second sheet. If 0.1 of the incident intensity is transmitted by the two sheets, what is θ?*

Sol^n: If the intensity of polarized light is I_0 then its intensity when passes through first crystal becomes $I_1 = I_0 \cos^2\theta$.

After passing through the second crystal which makes 90° with the first crystal the intensity becomes $I_2 = I_1 \cos^2 (90° - \theta) = I_0 \cos^2\theta \sin^2\theta$

According to question, $I_2 = 0.1\ I_0$

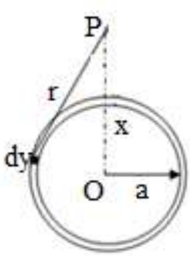
$\Rightarrow I_0 \cos^2\theta \sin^2\theta = 0.1\ I_0$ $\Rightarrow \sin 2\theta = \pm\sqrt{\frac{2}{5}}$

$\Rightarrow 2\theta = \sin^{-1}\left(\pm\sqrt{\frac{2}{5}}\right)$

$\therefore$ θ = 19.6° and 70.4°

9. *Find an expression for the electric potential due to ring of charge of radius "a" at a distance "x" from the center of the ring. Extend to your result to calculate the electric field intensity at that point.*

Sol^n: Consider a charged ring of radius "a" and linear charge density λ. Let O is the center of the ring and take a point P at a distance x from O. Since the ring consists of a continuous distribution of charge, so consider an elementary charge dq is taken in an elementary section dy. So, dq = λ dy.



The potential at P due to dq charge is $dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\lambda dy}{4\pi\varepsilon_0\sqrt{a^2+x^2}}$

So, total potential at P due to charge on the ring is

$$V = \frac{\lambda}{4\pi\varepsilon_0\sqrt{a^2+x^2}}\int_0^{2\pi a} dy = \frac{q}{4\pi\varepsilon_0\sqrt{a^2+x^2}}$$

Also, electric field intensity is $E = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{q}{4\pi\varepsilon_0\sqrt{a^2+x^2}}\right)$

$$E = \frac{qx}{4\pi\varepsilon_0(a^2+x^2)^{3/2}}$$

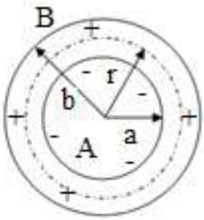
10. *A particle of charge – q and mass m is placed midway between two equal positive charges q_0 of separation d. If the negative charge – q is displaced in perpendicular direction to the line joining them and released. Show that the particle describes a simple harmonic motion and calculate its frequency of oscillation.*

Sol^n: (See in 2068 Shrawan Back Q. No. 13)

11. *Give general method of calculating capacitance of a capacitor. Use the method to calculate capacitance of a spherical and a cylindrical capacitor.*

Sol^n: General method of calculating capacitance of a capacitor of different geometry is as

- Assume a charge q on the plates
- Calculate the electric field E between the plates, using Gauss' law $\oint \vec{E}.\vec{da} = \frac{q}{\epsilon_0}$
- Calculate the potential difference V between the plates, using the relation $V = \int_-^+ Eds$
- Calculate capacitance C from $C = \frac{q}{V}$



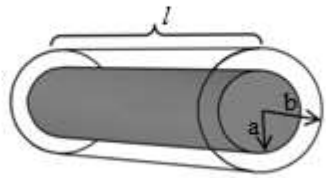
Spherical Capacitor: Let us consider a spherical capacitor consists of two metallic concentric spheres A and B of radii a and b respectively and insulated from each other. The inner sphere is solid sphere and outer is hollow spherical shell of radius b. Draw a Gaussian surface of radius r concentric with two shells. In the region, a < r <b, the electric field is

$E = \frac{q}{4\pi\varepsilon_0 r^2}$

So, $V = \int_{-}^{+} Eds = \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$

$\therefore$ The capacitance of the spherical capacitor is $C = \frac{q}{V} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

Cylindrical Capacitor: Figure shows a section of a cylindrical capacitor of length *l*, inner radius a and outer radius b. The inner cylinder is a solid rod carrying a charge – q uniformly distributed over its surface and outer conductor is a co-axial cylindrical shell carrying charge + q which is also uniformly distributed. As a Gaussian surface, choose a cylinder of length *l* and radius r, closed by the end caps, and placed as shown in figure.



From Gauss's law $\oint_S \vec{E}.d\vec{a} = \frac{q}{\varepsilon_0}$ $\Rightarrow E\,.\,2\pi r l = \frac{q}{\varepsilon_0}$

$\therefore E = \frac{q}{2\pi\varepsilon_0 l r}$

Now, the potential difference between a and b is $V = V_{ab} = +\int_a^b \vec{E}.\overrightarrow{dr}$

$\Rightarrow V = \frac{q}{2\pi\varepsilon_0 l r}\int_a^b \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 l r}\ln\left(\frac{b}{a}\right)$ $\therefore C = \frac{q}{V} = \frac{2\mu\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$

OR

A capacitor discharges through a resistor R. (a) After how many time constants (τ_c) does it charge fall to one half of its initial value? (b) After how many time constants does the stored energy drop to half of its initial value?

Sol^n: (See in 2071 Bhadra Regular Q. No. 11 OR)

12. *What is Hall Effect? Obtain an expression for the Hall resistance "R" in a conductor placed in a magnetic field "B". What conclusions can be drawn by Hall-Effect measurements?*

Sol^n: (See in 2067 Chaitra Back Q. No. 13)

13. *Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1mm. Conductor B is hollow tube of outside diameter 2 mm and inside diameter 1 mm. What is the resistance ratio $\frac{R_A}{R_B}$ measured between their ends?*

Sol^n: The general expression for the resistance of a conductor is $R = \frac{\rho l}{A}$

For conductor A, $R_A = \frac{\rho l}{\pi r_A^2}$ and for conductor B, $R_B = \frac{\rho l}{\pi(r_o^2 - r_i^2)}$

$\therefore \frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = 4 - 1 = 3$

14. *What is inductance? In what factors do it depends? Calculate the inductance of a solenoid and a toroid.*

Sol^n: In electromagnetism, inductance is the property of a conductor by which a change in current flowing through it induces (creates) a voltage (electromotive force) in both the conductor itself (self-inductance) and in any nearby conductors (mutual inductance). It depends upon the geometry of the circuit (or more commonly, by the geometry of individual circuit elements).

(For second part See in 2070 Chaitra Regular Q. No. 14 OR)

OR

A solenoid 1.3 m long and 2.6 cm in diameter carries a current of a current of 18A. The magnetic field inside the solenoid is 23mT. Find the length of the wire forming the solenoid. Also calculate the inductance of the solenoid.

Sol^n: Length of the solenoid (l) = 1.3m, diameter (d) = 2.6cm = 0.026m

Current (I) = 18A and Magnetic field (B) = 2.3×10^{-2} T

Length of the wire forming the solenoid (L') =?

B = $\mu_0 ni = \mu_0 i \frac{N}{l}$ $\therefore$ N = $\frac{Bl}{\mu_0 i}$

Total length of the wire forming solenoid is L' = $2\pi r$ N = πd N

L' = $\pi d \frac{Bl}{\mu_0 i} = \pi \times 0.026 \times \frac{2.3\times10^{-2}\times1.3}{4\pi\times10^{-7}\times18}$ = 108 m

Also, inductance of the solenoid is L = $\frac{B^2 l}{\mu_0 i^2} A = \frac{B^2 l}{\mu_0 i^2} \frac{\pi d^2}{4}$

L = $\frac{(2.3\times10^{-2})^2\times1.3}{4\pi\times10^{-7}\times18\times18} \times \frac{\pi\times0.026^2}{4}$ = 8.97×10^{-4} H = 90 mH

15. *A beam of electrons having energy of each 3eV is incident on a potential barrier of finite height 4eV. If the width of the barrier is 20Å, calculate the percentage transmission of the beam through the barrier.*

Sol^n: (See in 2069 Chaitra Regular Q. No. 16)

16. *Write and explain Maxwell equations in integral from. Convert them into differential form.*

Sol^n: (See in 2067 Ashwin Back Q. No. 15)

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