

# Circle

The equation of a circle is given by

$$x^2 + y^2 = r^2$$

To apply the midpoint method, we define a circle function

as  $F_{\text{circle}}(x,y) = x^2 + y^2 - r^2$

now  $F_{\text{circle}}(x,y) < 0$  if  $(x,y)$  is inside the circle boundary  
 $= 0$  if  $(x,y)$  is on the circle boundary  
 $> 0$  if  $(x,y)$  is outside the circle boundary

This circle function  $F_{\text{circle}}(x,y)$  serves as the decision parameter

Select next pixel along the circle path according to the sign of circle function evaluated at the midpoint between two candidate pixels.

Start at  $(0,y)$  take unit steps in 'x' direction (sample in 'x' direction  $x_{k+1} = x_k + 1$ )

Assuming position  $(x_k, y_k)$  has been selected at previous step we determine next position

$(x_{k+1}, y_{k+1})$  as either  $(x_{k+1}, y_k)$  or  $(x_{k+1}, y_k - 1)$  along circle path by evaluating the decision parameter (circle function). The decision parameter is the circle function evaluated at the midpoint between these two pixels

$$P_k = F_{\text{circle}}(x_k + 1, y_k - \frac{1}{2})$$

$$= (x_k + 1)^2 + (y_k - \frac{1}{2})^2 - r^2 \dots\dots\dots (i)$$

At the next sampling position  $(x_{k+1} + 1 = x_k + 2)$ , the decision parameter is evaluated as

$$P_{k+1} = F_{\text{circle}}(x_{k+1} + 1, y_{k+1} - \frac{1}{2})$$

$$= [(x_k + 1) + 1]^2 + (y_{k+1} - \frac{1}{2})^2 - r^2 \dots\dots\dots (ii)$$

Now subtracting eq (i) and (ii),

$$P_{k+1} = P_k + 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1 \dots\dots\dots (iii)$$

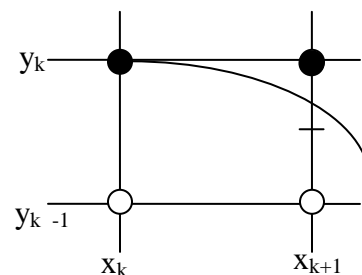
where  $y_{k+1}$  is either  $y_k$  or  $y_k - 1$  depending on the sign of  $P_k$ .

## Case 1:

If  $P_k < 0$  then the mid point is inside the circle, so pixel on scanline ' $y_k$ ' is closer to the circle boundary and  $y_{k+1} = y_k$

From equation (iii)

or  $P_{k+1} = P_k + 2x_{k+1} + 1 \dots\dots\dots (a)$



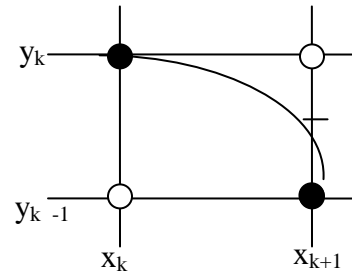
Where  $x_{k+1} = x_k + 1$

or  $2x_{k+1} = 2x_k + 2$

### Case 2:

If  $P_k \geq 0$  then the mid point is outside or on the boundary of the circle, so we select the pixel on scan line ' $y_k - 1$ ' then  $y_{k+1} = y_k - 1$  i.e. from equation (iii)

$$\text{or } P_{k+1} = P_k + 2x_{k+1} - 2y_{k+1} + 1 \quad \text{----- (b)}$$



$$\text{Where } 2y_{k+1} = 2y_k - 2$$

$$\text{or } 2x_{k+1} = 2x_k + 2$$

The initial decision parameter  $P_0$  is obtained by evaluating the circle function at the start position  $(x_0, y_0) = (0, r)$

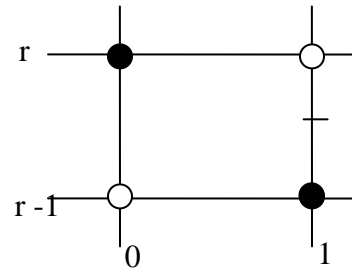
Next pixel to plot is either  $(1, r)$  or  $(1, r - 1)$

So, midpoint coordinate position is  $(1, r - \frac{1}{2})$

$$F_{\text{circle}}(1, r - \frac{1}{2}) = 1 + (r - \frac{1}{2})^2 - r^2$$

Thus,

$$P_0 = 5/4 - r$$



If the radius ' $r$ ' is specified as an integer, we can simply round  $P_0$  to  $P_0 = 1 - r$  (for ' $r$ ' an integer)

### Midpoint Circle Algorithm

1. Input radius  $r$  and circle center  $(x_c, y_c)$ , and obtain the first point on the circumference of a circle centered on the origin as  $(x_0, y_0) = (0, r)$

2. Calculate the initial value of the decision parameter as

$$P_0 = 5/4 - r$$

3. At each  $x_k$  position, starting at  $k = 0$ , perform the following test:

If  $P_k < 0$ , the next point along the circle centered on  $(0,0)$  is  $(x_{k+1}, y_k)$  and  $P_{k+1} = P_k + 2x_{k+1} + 1$

Otherwise, the next point along the circle is  $(x_k + 1, y_k - 1)$  and  $P_{k+1} = P_k + 2x_{k+1} - 2y_{k+1} + 1$

where  $2x_{k+1} = 2x_k + 2$  and  $2y_{k+1} = 2y_k - 2$ .

4. Determine symmetry points in the other seven octants.

5. Move each calculated pixel position  $(x, y)$  onto the circular path centered on  $(x_c, y_c)$  and plot the coordinate values:

$$x = x + x_c, \quad y = y + y_c$$

6. Repeat steps 3 through 5 until  $x \geq y$ .