Midterm

Start Assignment

Due Tuesday by 12pm **Points** 100 **Submitting** a file upload
Available after Oct 25 at 12pm

This is a take-home exam. You must LaTex your answer and submit both the compiled PDF and all the LaTex sources.

For each of the following problems, you must provide an algorithm in pseudo-code, proof of correctness, and analysis of the running time together with any explanation you deem necessary. **Do not provide an implementation in an actual programming language**.

1. [40] Consider the problem of providing change to an arbitrary amount N using US currency denominations, i.e $0.01, $0.05, $0.10, $0.25, $1, $5, $10, $20, $50, $100. Find a polynomial algorithm that, when given N, finds the exact change (or indicates that such change is not possible) using the minimum number of coins/banknotes.
2. [30+10] Given a tree, provide an efficient algorithm that finds the length of and the actual sequence for the longest path starting at the root and terminating at a leaf [30]. If we now assume that tree edges have weights, how does the algorithm need to be modified to accommodate the generalization?
3. [20] Suppose you are given an array A[1..n] of sorted integers that have been circularly shifted k positions to the right (for an unknown k). For example, [35, 42, 5, 15, 27, 29] is a sorted array that has been
   circularly shifted k = 2 positions, while [27, 29, 35, 42, 5, 15] has been shifted k = 4 positions.
   We can obviously find the largest element in A in O(n) time. Describe an O(log n) algorithm.
4. [30 bonus] For problem 1, find the most general set of currency so that the algorithm you found is still correct. Your solution will be judged based on generality. Unless the solution is correct and the generalization is non-trivial, **no points will be awarded**.