COT 5615 Math for Intelligent Systems Fall 2021 Homework #3

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Problem 5.2

A surprising discovery

Solution

According to me, the supervisor is wrong and the intern is probably correct. Here because of the independence-dimension inequality rule, any set collection of n+1 or more n-vectors is linearly dependent; and she is analysing 400 250-vector stocks, and thus this set is linearly dependent. Thus, return of any stock i.e. Google can be expressed as a linear combination of the return of other stocks. Here, this fact is only valid for present return stock value; and in the near future, this fact might change i.e. Google's return stock might not be expressed as a linear combination of the return of other stocks and thus, is not very useful from monetary perspective.

Problem 5.5

Orthogonalizing vectors

Solution

Two vectors are orthogonal, if their inner product is zero i.e. we have to find γ such that $(a - \gamma b)^T b = a^T b - \gamma b^T b = 0$. Now, if $b = 0$, then any value of γ will yield $(a - \gamma b) \perp b$ true as all vectors are orthogonal to 0. If $b \neq 0$, $b^T b = \|b\|^2 \neq 0$; then we can take $\gamma = a^T b / b^T b$, which proves that $(a - \gamma b) \perp b$ is true.

Problem 5.9

Solution

The Gram-Schmidt algorithm requires $n \cdot k^2$ flops, and thus for $n = 10^4$ and $k = 2$, $2 \cdot 10^{10}$ flops are calculated in 2 seconds. Therefore, for $\widetilde{n} = 10^3$ and $\widetilde{k} = 500$, we can get the run-time of the Gram-Schmidt Algorithm as follows: $(2(2 \cdot 1000 \cdot (500)^2))/(2 \cdot 10^{10}) = 0.05$ seconds.

Problem 6.17

Stacked matrix

Solution

a Let's assume $Sx = 0$, thus $Sx = (Ax, x) = 0$, which implies $x = 0$. In conclusion, S always has linearly independent columns.

b S has m+n rows and each row is n-dimension wide. Thus, according to the independence-dimension inequality rule, S can never have linearly independent rows i.e. rows are dependent.

Problem A6.8

Solution

a The columns of matrix A (mXn) may be linearly independent if for $Ax = 0$ implies $x = 0$ when the number of columns is less than or equal to number of rows $[n \leq m]$.

b The rows of matrix A (mXn) may be linearly independent if for $Ax = 0$ implies $x = 0$ when the number of rows is less than or equal to number of columns $[m \leq n]$.

Problem 6.18

Vandermonde matrices

Solution

Here Vc vector represents the values of the polynomial at $t_1, t_2, \ldots, t_m$ as follows:

$$\begin{aligned} Vc &= (c_1 + c_2t_1 + c_3t_1^2 + \ldots + c_nt_1^{n-1}, c_1 + c_2t_2 + c_3t_2^2 + \ldots + c_nt_2^{n-1}, \ldots, c_1 + c_2t_m + c_3t_m^2 + \ldots + c_nt_m^{n-1}) \\ &= (p(t_1), p(t_2), \ldots, p(t_n)) \end{aligned}$$

Now, if $Vc = 0$, then $p(t_i)$ is also 0 for $i = 1, 2, \ldots, m$; thus p(t) has atleast m distinct roots $t_1, t_2, \ldots, t_m$. This is only possible if all the coefficients of p are 0 i.e. c=0. Therefore, $Vc = 0$ implies $c = 0$, which proves that the columns of V [Vandermonde Matrix] are linearly independent.

Problem A6.2

Vandermonde matrices in Julia

Solution

```julia
using LinearAlgebra
function Vandermonde_generator(n,m)
    v = ones(length(m),n)
    for i in 1:n
        mnew = m.^(i-1)
        for j in 1:length(m)
            v[j,i] = mnew[j]
        end
    end
    display(v)
end
Vandermonde_generator(5,[5,6,7])
Vandermonde_generator(5,[7,8,6])
```