# COT 5615 Math for Intelligent Systems, Fall 2021

## Homework 6

**A12.1** (16 pts)

    (a) The commands are

- `x=A\b`
- `x=(A'*A)\(A'*b)`
- `x=pinv(A)*b`

    The solutions are very close to each other.

    (b) The expected inequality holds.

**A12.2** (15 pts)

    It's correct if you run the code a few times and output reasonable time such as 0.4 seconds.

**A12.4** (15 pts)

$$\boldsymbol{R}_{ij} = \begin{cases} 1 & \text{passenger } i \text{ went through link } j \\ 0 & \text{otherwise.} \end{cases}$$

$\boldsymbol{c}$ is a $m$-vector where $c_i = f_i - s_i$.

**A12.7** (15 pts)

$$\boldsymbol{A}_{120\times 20} = \begin{bmatrix} \boldsymbol{c}_1 & \boldsymbol{c}_2 & \dots & \boldsymbol{c}_{20} \end{bmatrix}$$

$$\boldsymbol{b}_{120\times 1} = \boldsymbol{l}$$

**A13.3** (24 pts)

    (a) $\boldsymbol{A}_{(N-M)\times M} = \begin{bmatrix} x_M & x_{M-1} & \cdots & x_1 \\ x_{M+1} & x_M & \cdots & x_2 \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-1} & x_{N-2} & \cdots & x_{N-M} \end{bmatrix}, \boldsymbol{b}_{(N-M)\times 1} = \begin{bmatrix} x_{M+1} \\ x_{M+2} \\ \vdots \\ x_N \end{bmatrix}.$

    (b) The values of the mean-square error on the train and test series is shown in columns 2 and 3 in the figure. $J$ for the two simple predictors on the test series is shown in columns 4 and 5.

    $M = 12$ minimizes `J_train`. $M = 6$ minimizes `J_test`. We choose $M = 6$.

**A13.5** (15 pts)

$$\boldsymbol{A} = \begin{bmatrix} z_2 & z_1 & z_2 z_1 \\ z_3 & z_2 & z_3 z_2 \\ \vdots & \vdots & \vdots \\ z_{T-1} & z_{T-2} & z_{T-1} z_{T-2} \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} z_3 \\ z_4 \\ \vdots \\ z_T \end{bmatrix}.$$

| M | J_train | J_test | J_simple1 | J_simple2 |
|---|---------|--------|-----------|-----------|
| 2 | 0.030494624025126198 | 0.03874425534739428 | 1.7241452300471292 | 3.2572019102483196 |
| 3 | 0.030292696561256324 | 0.03743353941193069 | 1.7419193213195725 | 3.2884608706828913 |
| 4 | 0.02449655127211178 | 0.03385594443279199 | 1.7594331030340384 | 3.3221230354311952 |
| 5 | 0.024394903527141285 | 0.031685969813553636 | 1.777615740057924 | 3.3570454326633645 |
| 6 | 0.02299631429887345 | 0.03040027620370789 | 1.796504988834193 | 3.3922242693425284 |
| 7 | 0.019842912056501266 | 0.03524397169474641 | 1.8128663148111765 | 3.426229645547467 |
| 8 | 0.019137191183089056 | 0.03459784623483322 | 1.828744528501508 | 3.4498932560621385 |
| 9 | 0.019081546282888478 | 0.0370501542405899 | 1.8484085827740073 | 3.4860889695333133 |
| 10 | 0.01899948835215663 | 0.03768621129443151 | 1.8673865520400723 | 3.5211755743296793 |
| 11 | 0.019200812978895986 | 0.03815483757921519 | 1.8876931739069915 | 3.5605507956580524 |
| 12 | 0.01891584139384952 | 0.04030160698971953 | 1.908861817436543 | 3.5991679277128976 |

Figure 1: 13.3(b)