COT 5615 Math for Intelligent Systems, Fall 2021

Homework 4

7.6 (10 pts) The sum of the rows is zero, *i.e.*, $\mathbf{1}^T A = 0$.

7.8 (10 pts) Multiply $0 = Ax + s$ on the left by $\mathbf{1}^T$ to get

$$0 = \mathbf{1}^T(Ax + s) = (\mathbf{1}^T A)x + \mathbf{1}^T s = \mathbf{1}^T s$$

$\mathbf{1}^T A = 0$ because $\mathbf{1}^T A$ is the row vector of columns sums of A, which are all exactly zero since each column has one $+1$ and one -1. So we get $\mathbf{1}^T s = 0$.

7.15 (18 pts)

(a) The equalizer output is $z = h * (c * u) = (h * c) * u$. This is a vector of size $n + m + k - 2$. Since $h * c \approx e_1$, we have

$$z \approx e_1 * u = (u_1, \cdots, u_m, 0, \cdots, 0)$$

(b) Fig.1. Fig.2. Fig.3. show u, y, z for a randomly chosen signal u. These three vectors have size 50, 52, and 59, respectively.

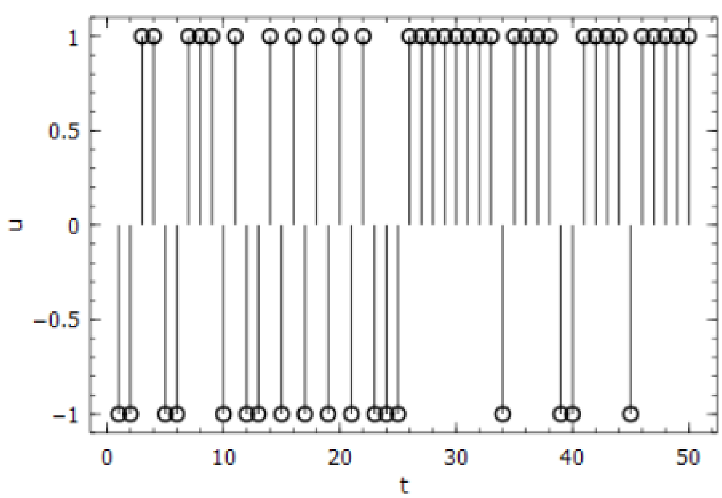

Figure 1: 7.15(b): Plot u.

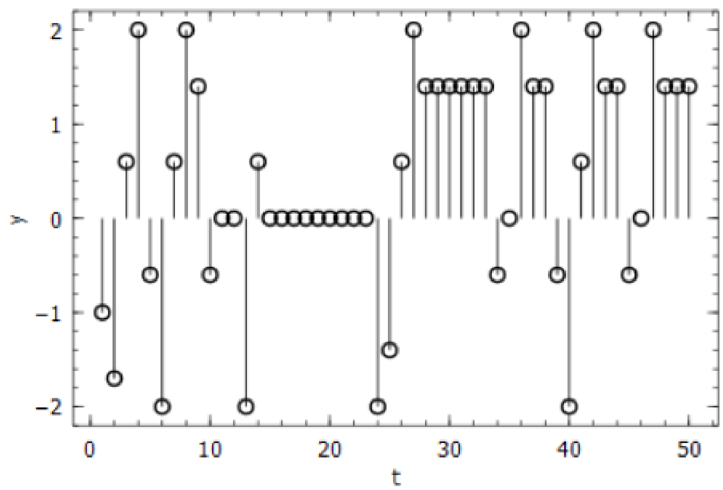

Figure 2: 7.15(b): Plot y.

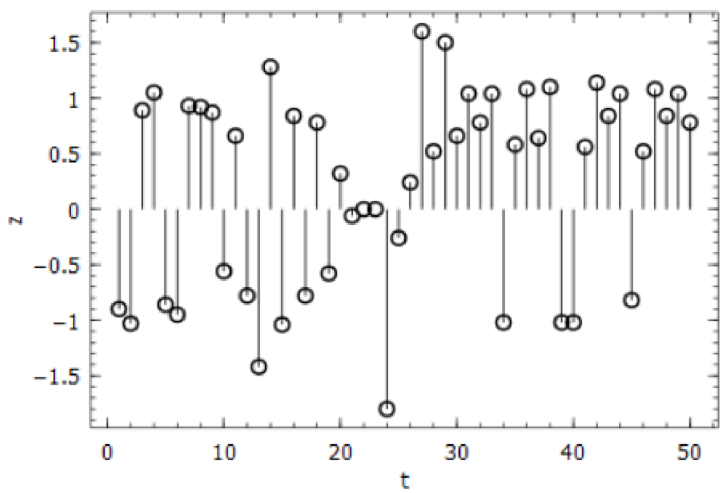

Figure 3: 7.15(b): Plot z.

If we compare u with the first 50th elements of z, we see that they are reasonably close. In fact, $u = \text{sign}(z_{1:50})$. In practice, one wants to transmit the signal u to the receiver through the channel c; the receiver receives y, 'decodes' the signal via the equalizer h, and recovers the original transmitted signal as $\text{sign}(z_{1:50})$.

A7.1 (18 pts)

(a) Plot c, h, and $h * c$ as Fig.4.

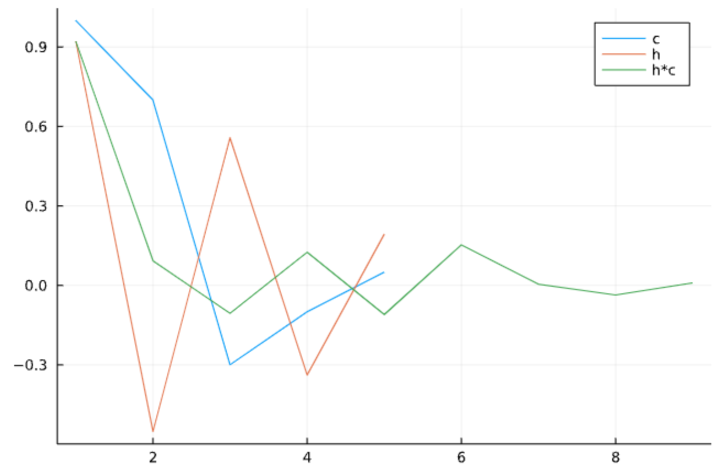

Figure 4: A7.1(a): Plot c, h, and $h * c$

The equalized channel is very close to e_1; the convolution between e_1 and any signal would output a signal that is very close to the input.

(b) Plot s, y, and $\tilde{y}$ as Fig.5.

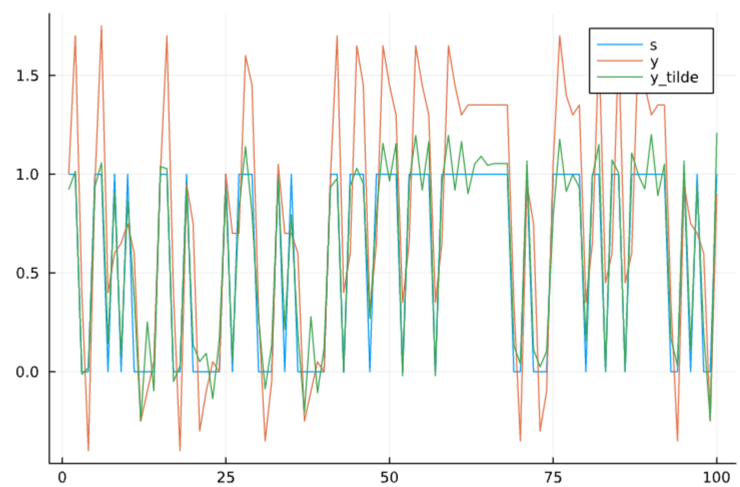

Figure 5: A7.1(b): Plot s, y, and $\tilde{y}$.

It's clear from the plot that estimating s from $\tilde{y}$ is better than from y.

(c) The BER for $\hat{s}$ is randomly about 0.1-0.4. The BER for $\hat{s}^{eq}$ is 0.

A7.3 (12 pts)

(a) The audio is blurred after convolution.

(b) Because the 441000-vector x is a 10-second recording, so $k = 11025$ samples in 0.25

seconds. Then y is a 452025-vector and h^{echo} is a 11026-vector. Due to $y_i = x_i + 0.5x_{i-k}$,

$$h^{\text{echo}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0.5 \end{bmatrix}$$

Similarly, $h^{\text{echo}} * h^{\text{echo}} * x$ adds an echo of $h^{\text{echo}} * x$ 0.25 delayed again. We hear in total two echos, the first echo is as strong as the original one (since it's the superposition of 1/2 from the first filter and 1/2 from the second filter) and the second echo is significantly weaker.

8.7 (10 pts) The four equations are

$$\begin{aligned} c_1 &= 0 \\ c_2 &= 0 \\ c_1 + c_2 + c_3 + c_4 + c_5 &= 1 \\ c_2 + 2c_3 + 3c_4 + 4c_5 &= 0 \end{aligned}$$

They can be written in the form $Ac = b$ as

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

The set of linear equations is underdetermined (4 equations in 5 variables).

8.11 (10 pts) We square and expand the right-hand sides:

$$x^T x - 2a_i^T x + a_i^T a_i = \rho_i^2, \quad i = 1, 2, 3, 4.$$

The quadratic term $x^T x$ can be eliminated by subtracting one equation from the three others. For example,

$$\begin{bmatrix} -2(a_1 - a_4)^T \\ -2(a_2 - a_4)^T \\ -2(a_3 - a_4)^T \end{bmatrix} x = \begin{bmatrix} \rho_1^2 - \rho_4^2 - \|a_1\|^2 + \|a_4\|^2 \\ \rho_2^2 - \rho_4^2 - \|a_2\|^2 + \|a_4\|^2 \\ \rho_3^2 - \rho_4^2 - \|a_3\|^2 + \|a_4\|^2 \end{bmatrix}$$

9.5 (12 pts) The Fibonacci sequence is described the linear dynamical system

$$x_{t+1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} x_t, \quad x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The values of y_t for $t = 0, \cdots, 20$ are

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181.$$

The modified Fibonacci sequence is described the linear dynamical system

$$z_{t+1} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} z_t, \quad z_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The values of z_t for $t = 0, \cdots, 20$ are

$$0, 1, 1, 0, -1, -1, 0, 1, 1, 0, -1, -1, 0, 1, 1, 0, -1, -1, 0, 1.$$