COT 5615 Math for Intelligent Systems Fall 2021 Homework #8

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Problem A16.2

Julia timing test for least-norm

Solution

Following is the code to calculate the time take for the least-norm problem:

```
m = 600
n = 4000
A = rand(m,n);
b = rand(m);
@time A\b;
@time A\b;
@time A\b;
@time A\b;
@time A\b;
#1.609742 seconds (488.74 k allocations: 69.135 MiB, 11.70% compilation time)
#1.308426 seconds (3.64 k allocations: 41.110 MiB)
#1.295313 seconds (3.64 k allocations: 41.110 MiB, 0.97% gc time)
#1.341974 seconds (3.64 k allocations: 41.110 MiB, 3.30% gc time)
#1.316347 seconds (3.64 k allocations: 41.110 MiB)
```

The approximate time taken is 1.312 *seconds*. The approximate flop rate is given as : $\frac{2nm^2}{average\_time} = \frac{2*4000*(600)^2}{1.312} = 2.195\ GFlops/second$.

Problem A16.3

Constrained least squares in Julia

Solution

The following code is used to find constrained least square solution along with the Lagrangian's multiplier:

```
using LinearAlgebra
m = 10000
n = 1000
p = 100

A = rand(m,n);
b = rand(m,1);
C = rand(p,n);
d = rand(p,1);
KKT =  [2*A'A C';C zeros(p,p)]\[2*A'b;d];

print("x hat (solution): \n");
display(KKT[1:n,1]);
```

```
    print("\n Lagrange Multiplier: \n");
    display(KKT[n+1:n+p,1]);
```

Problem A16.4

Varying the right-hand sides in linearly constrained least squares

Solution

The colleague is correct. We can find F and G using the formula for finding $\hat{x}$ from the QR factorization:

$$
\begin{aligned}
R\hat{x} &= Q_1^T b - (1/2)Q_2^T w \\
R\hat{x} &= Q_1^T b - (1/2)Q_2^T (z - 2d) \ [\because w = z - 2d] \\
R\hat{x} &= Q_1^T b - (1/2)Q_2^T z + Q_2^T d \\
R\hat{x} &= Q_1^T b + Q_2^T d \ [\because \textit{The columns of } Q_2^T \textit{ are linearly independent which implies that } z = 0] \\
\hat{x} &= R^{-1}Q_1^T b + R^{-1}Q_2^T d
\end{aligned}
$$

Thus, for the solution eqaution $\hat{x} = Fb + Gd$, we get $F = R^{-1}Q_1^T$ and $G = R^{-1}Q_2^T$.

Problem 4

LDLT Factorization for constrained least squares

Solution

Here, we are trying to minimize $\|y\|^2$ with constraints $Ax+y = b$. This can also be written as $\|A_n X_n - b_n\|^2$ with $CX_n = d$, where $X_n = (x, y)$, $C = [A_n; I]$, $A_n = (0, 0; 0, I)$, $b_n = (0, 0)$, $d = b$.
Now, $(2A_n^T A_n, C; C^T, 0)(X_n, z) = (0, d)$ [$\because$ *Using KKT Matrix*].
This is simplified as follows: $(2A_n, C; C^T, 0)(X_n, z) = (0, d)$.
From solving these tqo equations by expanding all the variables, we get the following two equations:
$A^T y = 0$ and $Ax + Iy = b$, which are the final two equations.
These equations can also be expressed as the following matrix: $(0, A; A^T, I)(x, y) = (0, b)$.