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MIS

Mid-Term-1

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A2.1

$$n = 2m-1$$

$$m = \frac{(n+1)}{2}$$

We have $a^T x =$

$$a_1 x_1 + a_2 x_2 + \cdots + a_m x_m + \cdots$$

$$\cdots + a_n x_n$$

for $f(x)$ we have $\underline{a}$ as the following:

For $a \Rightarrow$ $\boxed{a_i = \begin{cases} -\frac{1}{n} & i \neq m \\ -\left(1-\frac{1}{n}\right) & i = m \end{cases}}$ $\xrightarrow{i=1,\dots,n}$

where $m = \frac{(n+1)}{2}$

$$\therefore \quad f(x) = x_m - \frac{1}{n} \sum_{i=1}^{n} x_i$$

3.25

$$p = \theta r + (1-\theta)\mu^{rf}\mathbf{1}$$

(a) $\text{avg}(p) = \text{avg}\left(\theta r + (1-\theta)\mu^{rf}\mathbf{1}\right)$

$\text{avg}(p) = \theta\,\text{avg}(r) + (1-\theta)\mu^{rf}\,\text{avg}(1)$

$$\boxed{\text{avg}(p) = \theta\,\mu + (1-\theta)\mu^{rf}}$$

[where, $\text{avg}(r) = \mu$ and $\text{avg}(1) = 1$]

$\text{std}(p) = \text{std}\left(\theta r + (1-\theta)\mu^{rf}\mathbf{1}\right)$

$$= \frac{\| p - \text{avg}(p)\mathbf{1} \|}{\sqrt{T}}$$

$$= \frac{\| \theta r + (1-\theta)\mu^{rf}\mathbf{1} - \left(\theta\mu + (1-\theta)\mu^{rf}\right) \|}{\sqrt{T}}$$

$$= \frac{\| \theta (r - \mu\mathbf{1}) \|}{\sqrt{T}}$$

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$$= \frac{|\theta| \, \|r - \mu 1\|}{\sqrt{T}}$$

$$= |\theta| \, std(r)$$

$$\boxed{std(P) = |\theta| \, \sigma} \quad [\because std(r) = \sigma]$$

(b)

To get target risk σ^{tar},

$$|\theta| = \frac{\sigma^{tar}}{\sigma}$$

$$\Rightarrow \theta = \frac{\sigma^{tar}}{\sigma}, \quad \theta = -\frac{\sigma^{tar}}{\sigma}$$

$\Rightarrow$ Now choosing θ with portfolio return, we have the following:

$$\theta \mu + (1-\theta)\mu^{rf} = \mu^{rf} + \theta(\mu - \mu^{rf}) \rightarrow (1)$$

$\Rightarrow$ If $\mu > \mu^{rf}$, choose θ positive

elseif $\mu < \mu^{rf}$, choose θ negative

[$\because$ We short when return is less than risk-free return] to maximize eq $\rightarrow$ (1)

In part a

for $\theta = 0$,

$$\boxed{\begin{gathered} avg(P) = \mu^{rf} \\ \& \\ std(P) = 0 \end{gathered}}$$

for $\theta = 1$,

$$\boxed{\begin{gathered} avg(P) = \mu \\ \& \\ std(P) = \sigma \end{gathered}}$$

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c By taking θ from part b,
we have following 4 cases:

i If $[\mu > \mu^{rf}]$ (asset return is more than the risk-free return)

and $[\sigma > \sigma^{tar}]$ (risk is higher than target risk,

$\boxed{\text{We hedge.}}$

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ii Now if $(\mu > \mu^{ref})$ &

$(\sigma < \sigma^{tar})$ i.e.

asset return is more than risk-free return & the asset risk is less than the target risk,

$\boxed{\text{we leverage.}}$

iii & iv : Now if the return is less than risk-free return i.e. $(\mu < \mu^{rf})$,

for both $(\sigma < \sigma^{tar})$ & $(\sigma > \sigma^{tar})$

$\boxed{\text{we short.}}$

A8.1

$P(0) = C_1 \rightarrow (1)$

$P(1) = C_1 + C_2 + C_3 + C_4 + C_5 \rightarrow (2)$

$P'(0) = C_2 \rightarrow (3)$

$P'(1) = C_2 + 2C_3 + 3C_4 + 4C_5 \rightarrow (4)$

$$\underbrace{\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 3 & 4 \end{bmatrix}}_{A} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{b}$$

$[P(1) = P(0)]$

$0 = C_2 + C_3 + C_4 + C_5$ [(1) & (2)]

$[P'(0) = P'(1)]$

$0 = 2C_3 + 3C_4 + 4C_5$ [(3) & (4)]

$\text{Size}(A) = 2 \times 5$

$\text{Size}(B) = 2 \times 1$

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8.8

$$f(t) = \frac{c_1 + c_2 t + c_3 t^2}{1 + d_1 t + d_2 t^2}$$

$$\frac{c_1 + c_2 t_i + c_3 t_i^2}{1 + d_1 t_i + d_2 t_i^2} = y_i, \quad i = 1, \dots, K$$

⇒ Linear Equation of the above is:

$$c_1 + c_2 t_i + c_3 t_i^2 - y_i d_1 t_i - y_i d_2 t_i^2$$

$$= y_i, \quad i = 1, \dots, K$$

⇒ For $\theta = (c_1, c_2, c_3, d_1, d_2)$ & $A\theta = b$,

$$A = \begin{bmatrix} 1 & t_1 & t_1^2 & -y_1 t_1 & -y_1 t_1^2 \\ 1 & t_2 & t_2^2 & -y_2 t_2 & -y_2 t_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & t_K & t_K^2 & -y_K t_K & -y_K t_K^2 \end{bmatrix}$$

$$b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_K \end{bmatrix}$$

Q.4

$$x_{t+1} = A_1 x_t + A_2 x_{t-1}, \quad t = 2, 3, \ldots$$

$$z_t = (x_t; x_{t-1})$$

Thus, from the above equations; z_{t+1} can be expressed as a dynamical system eq. as fol:-

$$\boxed{z_{t+1} = \begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix} z_t}$$

A10.5

(a) 10×10 for $A^T A + C$

$[10\times10 \cdot 10\times10 + 10\times10]$

(b) 20×10 for BC^3

$[20\times10][10\times10]$

(c) ~~20X10~~ for $I + BC^T$

$I + [20\times10][10\times10]$

$I + [20\times10]$

As Identity is a square matrix → Not possible

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d) for $B^T - [C\ I]$

$[10\times20] - [10\times10\ \ I]$ $\to 10\times10$

$[10\times20] - [10\times20]$

$\to \boxed{10\times20}$

e) $B\begin{bmatrix}A\\A\end{bmatrix}C$

$[20\times10]\begin{bmatrix}5\times10\\5\times10\end{bmatrix}[10\times10]$

$[20\times10]\,[10\times10]\,[10\times10]$

$[20\times10]\,[10\times10]$

$\boxed{[20\times10]}$