COT 5615 Math for Intelligent Systems Fall 2021 Homework #1

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Problem 1.7

Transforming between two encodings for Boolean vectors

Solution

Vector y in terms of x can be shown as follows: $y = 2x - 1$.
Ex: when $x_i = 1$ we get $y_i = 2 \cdot 1 - 1 = +1$; and when $x_i = 0$ we get $y_i = 2 \cdot 0 - 1 = -1$.
Similarly, x in terms of y can be shown as follows: $x = (1/2)(y + 1)$

Problem 1.13

Average age in a population

Solution

(a) The total population can be calculated as $1^T x$

(b) The total number of people aged 65 or over can be calculated as $a^T x$ where $a = (0_{65}, 1_{35})$

(c) The average age of the population can be calculated as

$$\frac{(0, 1, 2, ..., 99)^T x}{1^T x}$$

Problem 1.17

Linear combinations of cash flows

Solution

From the given details, c can be written as follows,

$$c = (1, 0, \ldots, 0, -(1 + r)^{T-1})$$

Single period loan, at time period t can be written as follows,

$$l_t = (0, \ldots, 0, 1, -(1 + r), 0, \ldots, 0), \quad t = 1, \ldots, T - 1$$

So, solution is to take a new loan each time period to cover the amount you owe up until that period. So, after taking \$1 in period 1, we take out a loan for \$(1+r) in period 2, and end up owing \$(1+r)^2 in period 3. Then we take out a loan for \$(1+r)^2 in period 3, and so on. Therefore, We can express the c vector using the above loan period definition as follows,

$$\begin{aligned}(1, 0, \ldots, 0, -(1 + r)^{T-1}) &= (1, -(1 + r), 0, 0, \ldots, 0, 0) + (0, 1, -(1 + r), 0, 0, \ldots, 0, 0) \cdot (1 + r) \\ &+ (0, 0, 1, -(1 + r), 0, 0, \ldots, 0, 0) \cdot (1 + r)^2 + \ldots + (0, 0, \ldots, 0, 1, -(1 + r)) \cdot (1 + r)^{T-2}\end{aligned}$$

This can be further simplified as follows,

$$c = l_1 + l_2 \cdot (1 + r) + l_3 \cdot (1 + r)^2 + \ldots + l_{T-1} \cdot (1 + r)^{T-2}$$

Here, the coefficients of linear combinations are $1, 1 + r, (1 + r)^2, \ldots, (1 + r)^{T-2}$.

Problem A1.2

Creating vectors in Julia

Solution

```
#a)
a = (1:10 .== 5)
x = rand(10)
print(transpose(a)*x,"\n")
#b)
a = [0.3, 0.4, 0.3]
x = rand(3)
print(transpose(a)*x,"\n")
#c)
x = rand(22)
a = zeros(22)
for i in 1:22
    if mod(i,4) == 0
        a[i] = 1
    elseif mod(i,7) == 0
        a[i] = -1
    end
end
print(transpose(a)*x,"\n")
#d)
x = rand(11)
a = zeros(11)
a[convert(Int32,((length(a)-5)//2))+1:convert(Int32,((length(a)+5)//2))] = ones(5)
print(transpose(a)*x/5)
```

Problem A1.6

Solution

For any inner product, we need n multiplications and $n-1$ additions, so in total we generate $2n-1$ flops. Thus for 10^6-vector inner product, we generate 11 flops which are calculated in 0.001 seconds. Now, for 10^7-vector inner product, we generate 13 flops which are thus calculated in $(13 \cdot 0.001)/11 = 0.0011\overline{81}$ seconds.

Problem 2.4

Linear function

Solution

ϕ cannot be linear. Here, the third point $(1, -1, -1)$ is the opposite of the second point $(-1, 1, 1)$. If ϕ were linear, the two values of ϕ would need to be opposite (negative) of each other. But they are both 1 and 1. Thus, the function is not linear.

Problem 2.8

Integral and derivative of polynomial

Solution

(a) Here the integration is calculated as follows

$$\int_{\alpha}^{\beta} p(x)\ dx = c_1(\beta-\alpha) + \frac{c_2}{2}(\beta^2-\alpha^2) + \ldots + \frac{c_n}{n}(\beta^n-\alpha^n)$$

Therefore,

$$\alpha = (\beta-\alpha, \frac{\beta^2-\alpha^2}{2}, \ldots, \frac{\beta^n-\alpha^n}{n})$$

(b) Here the derivation at $\hat{x}$ is calculated as follows

$$p'(\alpha) = c_2 + 2c_3\alpha + 3c_4\alpha^2 + \ldots + (n-1)c_n\alpha^{n-2}$$

Therefore,

$$b = (0, 1, 2\alpha, 3\alpha^2, \ldots, (n-1)\alpha^{n-2})$$