Department of Mathematics and Computer Science
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CMSC 141: Exercise 2

Provide complete and satisfactory solutions to each of the following items. Incomplete solutions will not be given full credit.

1. Construct (discuss) a pushdown automaton P that accepts the following languages on the binary alphabet $\Sigma = \{\texttt{0}, \texttt{1}\}$. Provide the formal description of P. *(5 points each)*

   (a) $L(R)$ where R is the RegEx $\texttt{11}^*\texttt{00}^*$

   (b) $L = \{\texttt{1}^a\texttt{0}^b\texttt{1}^c : a + c = b \text{ where } a, b, c \geq 0\}$

2. Use the procedure presented in class to construct a pushdown automaton P from the following context-free grammar. Give the formal description of P. Pick **ONE** only. *(5 points)*

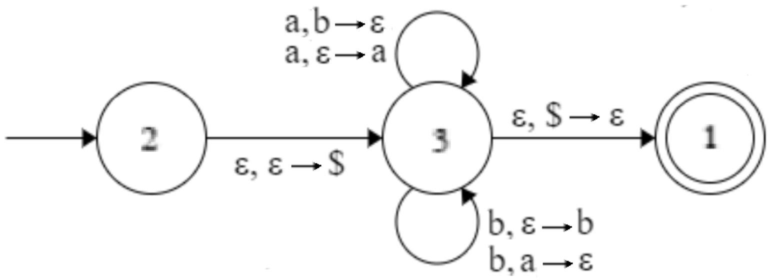
   (a) $G = (\{S, A, B, C\}, \{\texttt{0}, \texttt{1}, \texttt{2}\}, R, S)$ where R is

$$\begin{aligned} S &\rightarrow \texttt{0}A \\ A &\rightarrow \texttt{0}ABC \mid \texttt{1}B \mid \texttt{0} \\ B &\rightarrow \texttt{1} \\ C &\rightarrow \texttt{2} \end{aligned}$$

   (b) $G = (\{S, T, U\}, \{\texttt{a}, \texttt{b}\}, R, S)$ where R is

$$\begin{aligned} S &\rightarrow TT \mid U \\ T &\rightarrow \texttt{b}T \mid T\texttt{b} \mid \texttt{a} \\ U &\rightarrow \texttt{b}U\texttt{bb} \mid \texttt{a} \end{aligned}$$

3. Use the procedure presented in class to find a context-free grammar G generating the language accepted by the pushdown automata below. Give the formal description of G. *(5 points)*



   You may reduce the grammar by eliminating useless variables and productions.

4. Let A_1 and A_2 be context-free languages over Σ and G_1 and G_2, respectively, be the grammars generating the languages, that is, $L(G_1) = A_1$ and $L(G_2) = A_2$. *(5 points)*

   (a) Determine the grammar G that generates the language $A_2 \circ A_1$.

   (b) Support your answer in (a) by proving that $L(G) = A_2 \circ A_1$.

5. Let L be the language of all palindromes over $\{\texttt{a}, \texttt{b}\}$ containing equal numbers of $\texttt{a}$s and $\texttt{b}$s. Prove that L is *not* context-free. *(5 points)*

END of Exercise 2
Total Score: 30 points