reference line smoother

Quadratic programming + Spline interpolation

1. Objective function

1.1 Segment routing path

Segment routing path into **n** segments. each segment trajectory is defined by two polynomials:

$$x = f_i(t) = a_{i0} + a_{i1} * t + a_{i2} * t^2 + a_{i3} * t^3 + a_{i4} * t^4 + a_{i5} * t^5$$

$$y = g_i(t) = b_{i0} + b_{i1} * t + b_{i2} * t^2 + b_{i3} * t^3 + b_{i4} * t^4 + b_{i5} * t^5$$

1.2 Define objective function of optimization for each segment

$$cost = \sum_{i=1}^{n} \Big(\int\limits_{0}^{t_i} (f_i''')^2(t) dt + \int\limits_{0}^{t_i} (g_i''')^2(t) dt \Big)$$

1.3 Convert the cost function to QP formulation

QP formulation:

$$\begin{aligned} \frac{1}{2} \cdot x^T \cdot H \cdot x + f^T \cdot x \\ s.t. LB \leq x \leq UB \\ A_{eq}x = b_{eq} \\ Ax \leq b \end{aligned}$$

2 Constraints

2.1 Joint smoothness constraints

This constraint smoothes the spline joint. Let's assume two segments, seg_k and seg_{k+1}, are connected and the accumulated **s** of segment seg_k is s_k. Calculate the constraint equation as:

$$f_k(s_k) = f_{k+1}(s_0)$$

Similarly the formula works for the equality constraints, such as:

$$
\begin{aligned}
f'_k(s_k) &= f'_{k+1}(s_0)\\
f''_k(s_k) &= f''_{k+1}(s_0)\\
f'''_k(s_k) &= f'''_{k+1}(s_0)\\
g_k(s_k) &= g_{k+1}(s_0)\\
g'_k(s_k) &= g'_{k+1}(s_0)\\
g''_k(s_k) &= g''_{k+1}(s_0)\\
g'''_k(s_k) &= g'''_{k+1}(s_0)
\end{aligned}
$$

2.2 Sampled points for boundary constraint

Evenly sample **m** points along the path and check the predefined boundaries at those points.

$$
\begin{aligned}
f_i(t_l) - x_l &< boundary\\
g_i(t_l) - y_l &< boundary
\end{aligned}
$$