Algorithm Improvements 2017-08-25

Lu Yu

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1 LOAM track height compensation

Current LOAM algorithm produces a track in three dimensional space, with coordinates x, y and z. It we only use x and y coordinates, it is noted that LOAM track is noticeably shorter than GPS track. Thus we offset this discrepancy by projecting LOAM track onto x-y plane, while preserving step distance. This can be done at the same time when 3D LOAM track is calculated.

Algorithm 1: LOAM track height compensation

Input : 3D LOAM track $(x_i, y_i, z_i), i = 1, \ldots, N$
Output: 2D LOAM track $(\hat{x_i}, \hat{y_i}), i = 1, \ldots, N$

1 $\hat{x_1} = x_1, \hat{y_1} = y_1$
2 **for** $i = 2, \ldots, N$ **do**
3 $\quad d_i = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$
4 $\quad \hat{d_i} = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}$
5 $\quad \hat{x_i} = \frac{\hat{d_i}}{d_i}(x_i - x_{i-1}) + \hat{x_{i-1}}$
6 $\quad \hat{y_i} = \frac{\hat{d_i}}{d_i}(y_i - y_{i-1}) + \hat{y_{i-1}}$
7 **end**

2 Distance based LOAM segments with overlap

Current LOAM tracks use equal-time segments. which leads to considerable difference in segment length. To make equidistant LOAM segments, a cumulative distance counter is needed.
Due to uncertainty of GPS signal quality and availability, sometimes we need to use calibrated GPS track. Thus LOAM segments need to have some overlap. Naturally, we choose a length D and overlap length $D/2$.

Algorithm 2: Equidistant LOAM segments with overlap

Input : Point cloud time series, LOAM segment length D and overlap length $D/2$

Output: LOAM segments with length D and overlap $D/2$

```
while not end of point cloud series do
    ...
    while counter < D do
        Calculate next LOAM step
        counter += step size
        if counter > D/2 and next LOAM segment has not started then
            Start next LOAM segment
        end
    end
    ...
end
```

3 Weighted ICP

We improve the original ICP method by taking into account of weights at each timestamp. Weights are always non-negative. Registration of LOAM and GPS tracks are done by calculating covariance matrix and then doing singular value decomposition.

Algorithm 3: Weighted ICP

Input : LOAM track $\mathbf{p}_i$, GPS track $\mathbf{q}_i$, weights on timestamps w_i, $i = 1, \ldots, N$

Output: Rotation matrix R and translation vector $\mathbf{t}$ that minimizes $\sum_{i=1}^{N} w_i \|\mathbf{q}_i - (R\mathbf{p}_i + \mathbf{t})\|^2$

Centroids $\bar{\mathbf{p}} = (\sum_{i=1}^{N} w_i \mathbf{p}_i)/(\sum_{i=1}^{N} w_i), \bar{\mathbf{q}} = (\sum_{i=1}^{N} w_i \mathbf{q}_i)/(\sum_{i=1}^{N} w_i)$

Centered vectors $\mathbf{x}_i = \mathbf{q}_i - \bar{\mathbf{q}}, \mathbf{y}_i = \mathbf{p}_i - \bar{\mathbf{p}}$

Covariance matrix $S = XWY^{\mathrm{T}}$, where X and Y have $\mathbf{x}_i$ and $\mathbf{y}_i$ as columns respectively, and W is a diagonal matrix with w_i on diagonal.

Singular value decomposition $S = U\Sigma V^{\mathrm{T}}$

$R = UV^{\mathrm{T}}$ and $\mathbf{t} = \bar{\mathbf{q}} - R\bar{\mathbf{p}}$

LOAM track after calibration in global coordinates $\mathbf{p}'_i = R\mathbf{p}_i + \mathbf{t}$

4 Weight assignment

We take two factors into account for weight assignment. The first is speed based on LOAM track $s_i = \|\mathbf{p}_i - \mathbf{p}_{i-1}\|$ (can be normalized), which compensates for time oversampled points. With s_i, LOAM tracks are equally sampled along equal distances. The second factor is credibility of GPS signal c_i, which will be determined by registration result and after iteration.

5 Least absolute deviations

ICP method based on least squares is not robust against outliers, i.e. they tend to be affected by bad GPS signals. We use least absolute deviations to make our algorithm more robust. The problem can be solved by iteratively re-weighted least squares. During each iteration, we apply Algorithm 3, i.e. singular value decomposition of covariance matrix. The result of iteration converges to least absolute deviations, that is to minimize $\sum_{i=1}^{N} s_i \|\mathbf{q}_i - (R\mathbf{p}_i + \mathbf{t})\|$.
The final weights are used as a measurement of credibility, which is part of weight assigned to timestamps.

Algorithm 4: Least absolute deviations

Input : LOAM track $\mathbf{p}_i$, GPS track $\mathbf{q}_i$, maximum loop times m, credibility distance bound δ, error bound e
Output: Credibility for timestamps c_i

$c_i^{(0)} = 1, w_i^{(0)} = s_i c_i^{(0)}, n = 0$
while $n < m$ **do**
    Do Algorithm 3 with $w_i^{(n)}$ and obtain $R^{(n+1)}$ and $\mathbf{t}^{(n+1)}$
    Update $c_i^{(n+1)} = 1/\max(\delta, \|\mathbf{q}_i - (R^{(n+1)}\mathbf{p}_i + \mathbf{t}^{(n+1)})\|)$
    Update $w_i^{(n+1)} = s_i c_i^{(n+1)}$
    if $\sum_{i=1}^{N} w_i^{(n+1)} \|\mathbf{q}_i - (R^{(n+1)}\mathbf{p}_i + \mathbf{t}^{(n+1)})\|^2 < e$ **then**
        break
    end
    $n++$
end

6 Weighted moving average

When two overlapping tracks are to be fused, curve smoothing is needed to alleviate step effect. One simple method is weighted moving average. Window size is based on timestamps and can be customized. In this method, credibility of timestamps is not considered.

In order to avoid oversampling due to speed variance, a series of approximately equidistant timestamps are chosen first. Then moving average is applied at these timestamps. The other timestamps are processed by linear interpolation. The minimum distance is a customizable parameter.

Algorithm 5: Track fusion with weighted moving average

Input : Overlapping LOAM tracks $\mathbf{p}_i, \mathbf{p}'_i, i = 1, \ldots, N$ and minimum distance d_{min} in meters

Output: Fused track $\bar{\mathbf{p}}_i, i = 1, \ldots, N$

Distance counter $d = 0$

Point counter $i = 2$

$k_1 = 1$

for $j = 2, \ldots, N - 1$ **do**

  $d += \|\mathbf{p}_{j+1} - \mathbf{p}_j\|$

  if $d \geq d_{min}$ **then**

    $k_i = j$

    $i + +$

    $d = 0$

  end

end

$m = i$ and $k_m = N$

Window size $W = m/2$ (rounded down)

for $i = 1, \ldots, W$ **do**

  $\bar{\mathbf{p}}_{k_i} = (1 - \frac{i}{2W})\mathbf{p}_{k_i} + \frac{i}{2W}\mathbf{p}'_{k_i}$

end

for $i = W + 1, \ldots, m$ **do**

  $\bar{\mathbf{p}}_{k_i} = \frac{m-i+1}{2W}\mathbf{p}_{k_i} + (1 - \frac{m-i+1}{2W})\mathbf{p}'_{k_i}$

end

for $k_i < j < k_{i+1}$ and $i = 1, \ldots, m$ **do**

  Linear interpolation of $\bar{\mathbf{p}}_{k_i}$ and $\bar{\mathbf{p}}_{k_{i+1}}$ on timestamps to obtain $\bar{\mathbf{p}}_j$

end