OKVIS ImuError

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States

According to the two papers:

- Preintegration: $[p, R, v, b_g, b_a]$
- OKVIS: $[p, R, {}_Sv, b_g, b_a]$.

This leads to different measurement model:

- Preintegration:

$$\tilde{a} = C^T(\dot{v} - g) + b_a + \eta_a$$

- OKVIS:

$$\tilde{a} = C^T(\omega \times {}_Sv + C_S\dot{v} - g) + b_a + \eta_a$$

The OKVIS version is more troublesome.

However, in OKVIS code, they still use the same v as Preintegration.

The actual propogation:

$$v_1 = v_0 + C_0 \sum_k C_{0:k}(\tilde{a}_k - b_a)\Delta t_k - g\Delta t$$

Note, the vector g in OKVIS code is of the **opposite sign** of that in Preintegration. In OKVIS, $g = [0, 0, 9.8]$. In contrast, in Preintegration, ${}_Wg = [0, 0, -9.8]$, which is more physically accurate. Imagine that in still state, the measured ${}_W\tilde{a}$ should be $-{}_Wg$.

Never mind, we only need to ensure the computation correctness in the implementation.

The actually used measurement model in OKVIS code:

$$\tilde{a} = C^T(\dot{v} + g) + b_a + \eta_a$$

In still state, the IMU gives measurement of $[0, 0, 9.8]$; and when $a = [0, 0, -9.8]$ the measurement is 0. This equation is correct.

SE3 Update

Preintegration: $p \leftarrow p + C\delta p$.

OKVIS: $p \leftarrow p + \delta p$

Therefore,

$$\delta p_{\text{pre}} = C^T \delta p_{\text{okvis}}$$

Preintegration with right perturbation: $C \leftarrow C\,\text{Exp}(\delta\alpha)$

OKVIS with left perturbation: $C \leftarrow \text{Exp}(\delta\alpha)C$

$$\text{Exp}(\delta\alpha_{\text{pre}}) = C^T\text{Exp}(\delta\alpha_{\text{okvis}})C$$

$$\delta\alpha_{\text{pre}} = C^T \delta\alpha_{\text{okvis}}$$

Different forms of state incrementing would lead to different Jacobians.

Jacobians

Calculate some intermiate variables:

$$
\begin{aligned}
&{}_W\delta_p = t_0 - t_1 + v_0\Delta t - 0.5g\Delta t^2 \\
&{}_W\delta_v = v_0 - v_1 - g\Delta t \\
&\Delta\tilde{q} = \prod q((\tilde{w}_k - b_g)\Delta t_k) \\
&\Delta\tilde{q} \leftarrow \delta q(-\frac{d\alpha}{db_g}\Delta b_g) * \Delta\tilde{q} \\
&\int_a = \int_{a_{(0:1)}} = \sum_k C_{0:k}(\tilde{a}_k - b_a)\Delta t_k \\
&\iint_a = \sum_k \int_{a_{(0:k)}} \Delta t + 0.5C_{0:k}(\tilde{a}_k - b_a)\Delta t_k^2 \\
&\int_C = \int_{C_{(0:1)}} = \sum_k C_{0:k}\Delta t_k \\
&\iint_C = \sum_k \int_{C_{(0:k)}} \Delta t + 0.5C_{0:k}\Delta t_k^2
\end{aligned}
$$

Here

- $\int_a$ corresponds to $\Delta\tilde{v}_{ij}$ in Preintegration;
- $\iint_a$ correctness to $\Delta\tilde{p}_{ij}$ in Preintegration.

Note that the calculation of $\frac{d\alpha}{db_g}$ in the code drops a minus symbol, so an extra minus symbol appears in $\Delta\tilde{q}$ above. Strange.

Jacobians of residuals w.r.t. parameter 0:

$$
\begin{aligned}
&F_0 = I_{15} \\
&F_{0_{(0:2,0:2)}} = C_0^T \\
&F_{0_{(0:2,3:5)}} = C_0^T * [{}_W\delta_p]_\times \\
&F_{0_{(0:2,6:8)}} = C_0^T * \Delta t \\
&F_{0_{(0:2,9:11)}} = \frac{dp}{db_g}' \\
&F_{0_{(0:2,12:14)}} = -\iint_C \\
&F_{0_{(3:5,3:5)}} = (Q_l(\Delta\tilde{q} * q_1^{-1})Q_r(q_0))_{(0:2,0:2)} \\
&F_{0_{(3:5,9:11)}} = (Q_r(q_1^{-1} * q_0)Q_r(\Delta\tilde{q}))_{(0:2,0:2)} * -\frac{d\alpha}{db_g}' \\
&F_{0_{(6:8,3:5)}} = C_0^T * {}_W\delta_v \\
&F_{0_{(6:8,6:8)}} = C_0^T \\
&F_{0_{(6:8,9:11)}} = \frac{dv}{db_g} \\
&F_{0_{(6:8,12:14)}} = -\int_C
\end{aligned}
$$

Jacobians of residuals w.r.t. parameter 1:

$$
\begin{aligned}
&F_1 = -I_{15} \\
&F_{1_{(0:2,0:2)}} = -C_0^T \\
&F_{1_{(3:5,3:5)}} = -(Q_l(\Delta\tilde{q})Q_r(q_0)Q_l(q_1^{-1}))_{(0:2,0:2)} \\
&F_{1_{(6:8,6:8)}} = -C_0^T
\end{aligned}
$$

Residuals

$$
\begin{aligned}
&r_p = e_{(0:2)} = C_0^T {}_W\delta_p + \iint_a + F_{0_{(0:2,9:14)}} * \Delta b \\
&r_R = e_{(3:5)} = 2(\Delta\tilde{q} * (q_1^{-1} * q_0))_v \\
&r_v = e_{(6:8)} = C_0^T * {}_W\delta_v + \int_a + F_{0_{(6:8,9:14)}} * \Delta b \\
&r_b = e_{(9:14)} = b_0 - b_1
\end{aligned}
$$

All residuals happend to be the inverse/negative version of those in Preintegration. This does not affect the optimization process, but would invert the sign of the Jacobians.