

An EKF-SLAM toolbox in Matlab

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1 Quick start

Hi there! To start the toolbox, do the following:

1. Move **slamToolbox.zip** where you want the SLAM toolbox to be installed. Unzip it.
2. Rename the expanded directory if wanted (we'll call this directory **SLAMtoolbox/**).
3. Open Matlab. Add all directories and subdirectories in **SLAMtoolbox/** to the Matlab path.
4. Execute **slamtb** from the Matlab prompt.

Or, if you want to get some more insight:

5. Edit **userData.m**. Read the help lines. Explore options and create, by copying and modifying, new robots and sensors. You can modify the robots' initial positions and motions and the sensors' positions and parameters. You can also modify the default set of landmarks or 'World'.
6. Edit and run **slamtb.m**. Explore its code by debugging step-by-step. Explore the Map figure by zooming and rotating with the mouse.
7. Read the help contents of the following 4 functions: **frame**, **fromFrame**, **q2R**, **pinHole**. Follow some of the **See also** links.
8. Read '[guidelines.pdf](#)' before contributing your own code.¹

¹Available at <http://www.laas.fr/~jsola/Joan%20Sola/objectes/toolbox/guidelines.pdf>.

2 The SLAM toolbox presentation

In a typical SLAM problem, one or more robots navigate an environment, discovering and mapping landmarks on the way by means of their onboard sensors. Observe in Fig. 1 the existence of robots of different kinds, carrying a different number of sensors of different kinds, which gather raw data and, by processing it, are capable of observing landmarks of different kinds. All this variety of data is handled by the present toolbox in a way that is quite transparent.

In this toolbox, we organized the data into three main groups, see Table 1. The first group contains the objects of the SLAM problem itself, as they appear in Fig. 1. A second group contains objects for simulation. A third group is designated for graphics output, Fig. 2.

Apart from the data, we have of course the functions. Functions are organized in three levels, from most abstract and generic to the basic manipulations, as is sketched in Fig. 3. The highest level, called *High Level*, deals exclusively with the structured data we mentioned just above, and calls functions of an intermediate level called the *Interface Level*. The interface level functions split the data structures into more mathematically meaningful elements, check objects types to decide on the applicable methods, and call the basic functions that constitute the basic level, called the *Low Level Library*.

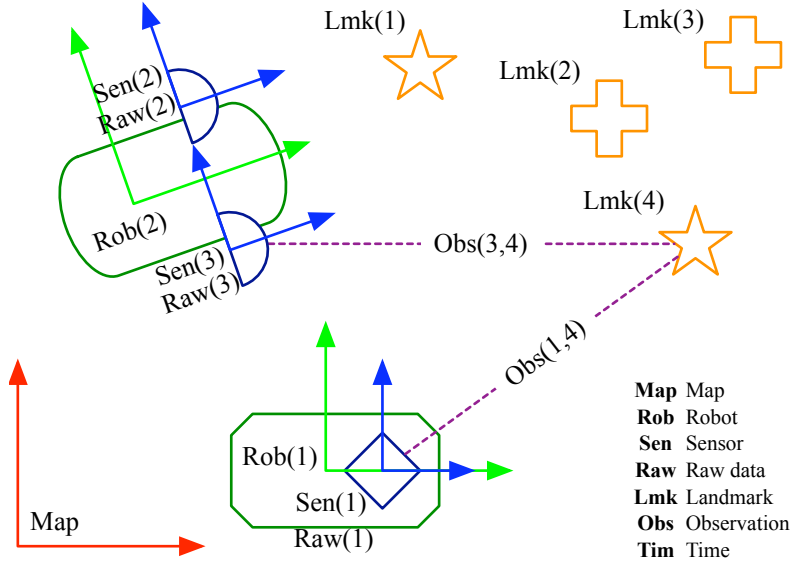


Figure 1: Overview of the SLAM problem with the principal data structures.

Table 1: All data structures.

Purpose	SLAM	Simulator	Graphics
Map	Map		MapFig
Robots	Rob	SimRob	
Sensors	Sen	SimSen	SenFig
Raw data	Raw		
Landmarks	Lmk	SimLmk	
Observations	Obs		
Time	Tim		



Figure 2: The set of figures. The structures **MapFig** and **SenFig(s)** contain the handles to all graphics objects drawn.

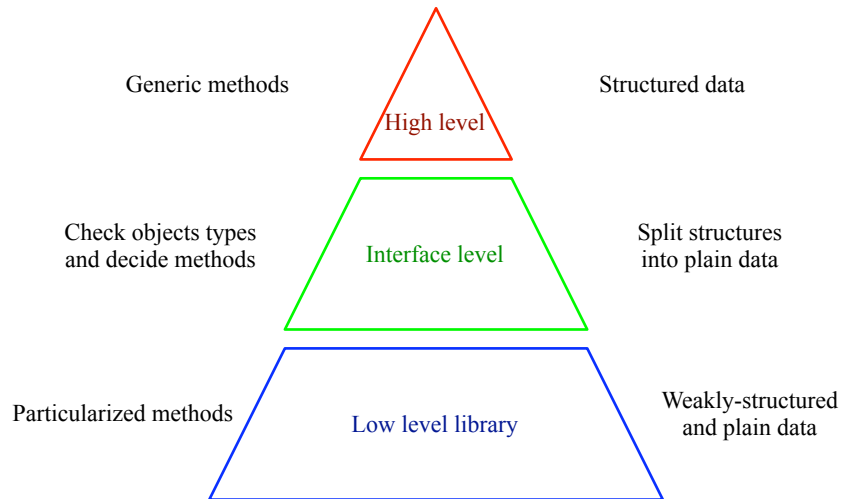


Figure 3: Overview of the levels of abstraction of the functions and their relation to data structuration. Functions and scripts in the High and Interface levels are in the **HighLevel/** and **InterfaceLevel/** directories. The Low Level library occupies all other directories.

3 Data organization

It follows a brief explanation of the SLAM data structures, the Simulation and Graphic structures, and the plain data types.

3.1 SLAM data

For a SLAM system to be complete, we need to consider the following parts:

Rob: A set of robots.

Sen: A set of sensors.

Raw: A set of raw data captures, one per sensor.

Lmk: A set of landmarks.

Map: A stochastic map containing the states of robots, landmarks, and eventually sensors.

Obs: The set of landmark observations made by processing **Raw** data.

Tim: A few time-related variables.

This toolbox considers these objects as the only existing data for SLAM. They are defined as structures holding a variety of fields (see Figs. 4 to 10 for reference). Structure arrays hold any number of such objects. For example, all the data related to robot number 2 is stored in **Rob(2)**. To access the rotation matrix defining the orientation of this robot we simply use **Rob(2).frame.R** (type **help frame** at the Matlab prompt for help on 3D reference frames). Observations require two indices because they relate sensors to landmarks. Thus, **Obs(sen, lmk)** stores the data associated to the observation of landmark **lmk** from sensor **sen**.

It would be wise, before reading on, to revisit Fig. 1 and see how simple things are.

It follows a reproduction of the arborescences of the principal structures in the SLAM data.

```

Rob(rob)      % Robot structure, containing:
  .rob        % index in Rob() array
  .id         % robot id
  .name       % robot name
  .type       % robot type
  .sensors    % list of installed sensors
  .motion     % motion model
  .con        % control structure
    .u        % control signals for the motion model
    .U        % covariance of u
  .frame      % frame structure, containing:
    .x        % 7-vector, position and orientation  $x = [t;q]$ 
    .P        % covariances matrix of  $x$ 
    .t        % position
    .q        % orientation quaternion
    .R        % rotation matrix,  $R = q2R(q)$ 
    .Rt       % transposed R
    .Pi       % PI matrix,  $Pi = q2Pi(q)$ 
    .Pc       % conjugate PI matrix,  $Pc = pi2pc(Pi)$ 
    .r        % range in the SLAM map Map
  .vel        % velocity stucture, containing
    .x        % 6-vector, linear and angular velocities
    .P        % covariances matrix of  $x$ 
    .r        % range in the SLAM map Map
  .state      % state structure, containing
    .x        % robot's state vector,  $x = [frame.x;vel.x]$ 
    .P        % covariances matrix of  $x$ 
    .size     % size of  $x$ 
    .r        % range in the SLAM map Map

```

Figure 4: The **Rob** structure array.

```

Sen(sen)      % Sensor structure, containing:
  .sen        % index in Sen() array
  .id         % sensor id
  .name       % sensor name
  .type       % sensor type
  .robot      % robot it is installed to
  .frameInMap % flag: is frame in Map?
  .frame      % frame structure, containing:
    .x        % 7-vector, position and orientation  $x = [t;q]$ 
    .P        % covariances matrix of  $x$ 
    .t        % position
    .q        % orientation quaternion
    .R        % rotation matrix,  $R = q2R(q)$ 
    .Rt       % transposed  $R$ 
    .Pi       % PI matrix,  $Pi = q2Pi(q)$ 
    .Pc       % conjugate PI matrix,  $Pc = pi2pc(Pi)$ 
    .r        % range in the SLAM map Map
  .par        % sensor parameters
    .k        % intrinsic params
    .d        % distortion vector
    .c        % correction vector
    .imSize   % image size
  .state      % state structure, containing
    .x        % sensor's state vector,  $x = frame.x$  or  $x = []$ 
    .P        % covariances matrix of  $x$ 
    .size     % size of  $x$ 
    .r        % range in the SLAM map Map

```

Figure 5: The **Sen** structure array.

```

Raw(sen)      % Raw data structure, containing:
  .type       % type of raw data
  .data       % raw data, containing
    .points   % a matrix of points (for simulated data)
    .ids      % a vector of points ids (for simulated data)
    .img      % a pixels image (for real images)

```

Figure 6: The **Raw** structure array.


```

Lmk(lmk)      % Landmark structure, containing:
    .lmk        % index in Lmk() array
    .id          % landmark id
    .type        % sensor type
    .sig         % landmark descriptor or signature
    .used        % flag: is landmark used in the map?
    .state       % state structure, containing
        .r        % range in the SLAM map Map
    .par         % other lmk parameters

```

Figure 7: The **Lmk** structure array.

```

Map           % Map structure, containing:
    .used        % vector of flags indicating non-free positions
    .x           % state vector's mean
    .P           % covariances matrix

```

Figure 8: The **Map** structure.

```

Tim           % Time structure, containing:
    .firstFrame % first frame to evaluate
    .lastFrame  % last frame to evaluate
    .dt         % Sampling period

```

Figure 9: The **Tim** structure.

```

Obs(sen, lmk) % Observation structure, containing:
  .sen        % index to sensor in Sen() array
  .lmk        % index to landmark in Lmk() array
  .sid        % sensor id
  .lid        % landmark id
  .stype      % sensor type
  .ltype      % landmark type
  .meas       % measurement
    .y        % mean
    .R        % covariance
  .nom        % non-measurable degrees of freedom
    .n        % mean
    .N        % covariance
  .exp        % expectation
    .e        % mean
    .E        % covariance
  .inn        % innovation
    .z        % mean
    .Z        % covariance
    .iZ       % inverse covariance
    .MD2      % squared Mahalanobis distance
  .app        % appearance
    .pred     % predicted appearance
    .curr     % current appearance
    .sc       % matching quality score
  .vis        % flag: is lmk visible from sensor?
  .measured   % flag: has lmk been measured?
  .matched    % flag: has lmk been matched?
  .updated    % flag: has Map been updated?
  .Jac        % Jacobians of observation function
    .E_r     % wrt robot pose
    .E_s     % wrt sensor
    .E_l     % wrt landmark

```

Figure 10: The **Obs** structure array.

3.2 Simulation data

This toolbox also includes simulated scenarios. We use for them the following objects, that come with 6-letter names to differentiate from the SLAM data:

SimRob: Virtual robots for simulation.

SimSen: Virtual sensors for simulation.

SimLmk: A virtual world of landmarks for simulation.

The simulation structures **SimXxx** are simplified versions of those existing in the SLAM data. Their arborescence is much smaller, and sometimes they may have absolutely different organization. It is important to understand that none of these structures is necessary if the toolbox is to be used with real data.

It follows a reproduction of the arborescences of the principal simulation data structures.

```

SimRob(rob)    % Simulated robot structure, containing:
  .rob         % index in SimRob() array
  .id          % robot id
  .name        % robot name
  .type        % robot type
  .motion      % motion model
  .sensors     % list of installed sensors
  .frame       % frame structure, containing:
    .x         % 7-vector, position and orientation  $x = [t;q]$ 
    .t         % position
    .q         % orientation quaternion
    .R         % rotation matrix,  $R = q2R(q)$ 
    .Rt        % transposed R
    .Pi        % PI matrix,  $Pi = q2Pi(q)$ 
    .Pc        % conjugate PI matrix,  $Pc = pi2pc(Pi)$ 
  .vel         % velocity structure, containing
    .x         % 6-vector, linear and angular velocities

```

Figure 11: The **SimRob** structure array.

```

SimSen(sen)    % Simulated Sensor structure, containing:
  .sen         % index in SimSen() array
  .id          % sensor id
  .name        % sensor name
  .type        % sensor type
  .robot       % robot it is installed to
  .frame       % frame structure, containing:
    .x         % 7-vector, position and orientation  $x = [t;q]$ 
    .t         % position
    .q         % orientation quaternion
    .R         % rotation matrix,  $R = q2R(q)$ 
    .Rt        % transposed R
    .Pi        % PI matrix,  $Pi = q2Pi(q)$ 
    .Pc        % conjugate PI matrix,  $Pc = pi2pc(Pi)$ 
  .par         % sensor parameters
    .k         % intrinsic params
    .d         % distortion vector
    .c         % correction vector
    .imSize    % image size

```

Figure 12: The **SimSen** structure array.

```

SimLmk      % Simulated landmarks structure, containing:
  .ids      % N-vector of landmark identifiers
  .points   % 3-by-N array of 3D points
  .lims     % limits of playground in X, Y and Z axes
    .xMin   % minimum X coordinate
    .xMax   % maximum X coordinate
    .yMin   % minimum Y coordinate
    .yMax   % maximum Y coordinate
    .zMin   % minimum Z coordinate
    .zMax   % maximum Z coordinate
  .dims     % dimensions of playground
    .l      % length in X
    .w      % width in Y
    .h      % height in Z
  .center   % central point
    .xMean  % central X
    .yMean  % central Y
    .zMean  % central Z

```

Figure 13: The **SimLmk** structure.

3.3 Graphics data

This toolbox also includes graphics output. We use for them the following objects, which come also with 6-letter names:

MapFig: A structure of handles to graphics objects in the 3D map figure. One Map figure showing the world, the robots, the sensors, and the current state of the estimated SLAM map (Figs. 14 and 15).

SenFig: A structure array of handles to graphics objects in the sensor figures. One figure per sensor, visualizing its *measurement space* (Figs. 16 and 17).

It follows a reproduction of the arborescences of the principal graphics structures.

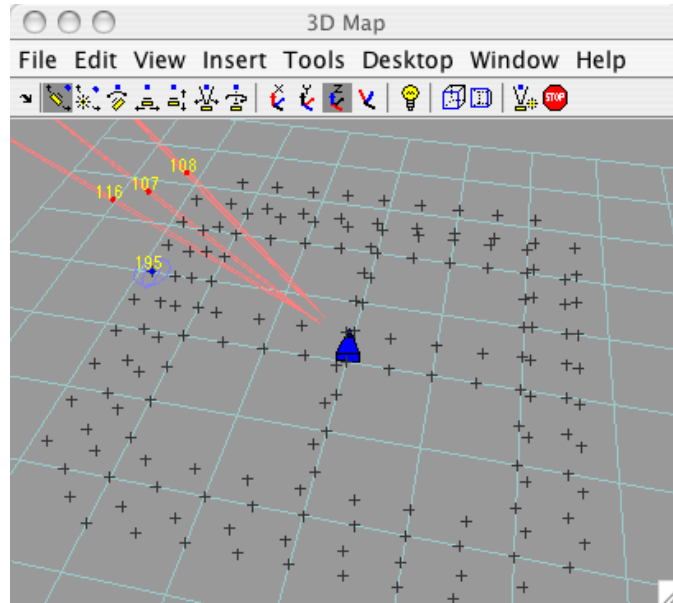


Figure 14: The 3D map figure. **MapFig** contains handles to all objects drawn.

```
MapFig          % Map figure structure, containing:
    .fig        % figure number and handle
    .axes       % axes handle
    .ground     % handle to floor object
    .simRob     % array of handles to simulated robots
    .simSen     % array of handles to simulated sensors
    .simLmk     % handle to simulated landmarks
    .estRob     % array of handles to SLAM robots
    .estSen     % array of handles to SLAM sensors
    .estLmk     % handles to SLAM landmarks, containing:
        .mean   % array of handles to landmarks means
        .ellipse % array of handles to landmarks ellipses
        .label  % array of handles to landmarks labels
```

Figure 15: The **MapFig** structure.

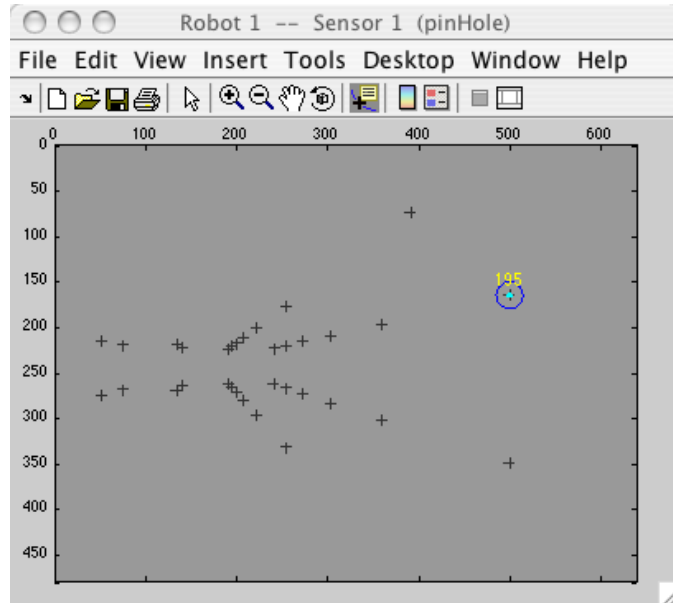


Figure 16: A pin-hole sensor view figure. **SenFig(1)** contains handles to all objects drawn.

```

SenFig(sen)    % Sensor figure structure, containing:
    .fig        % figure number and handle
    .axes       % axes handle
    .raw        % handle to raw data
    .measure    % array of handles to landmarks measurements
    .ellipse    % array of handles to landmarks ellipses
    .label      % array of handles to landmarks labels

```

Figure 17: The **SenFig** structure array.

3.4 Plain data

The structured data we have seen so far is composed of chunks of lower complexity structures and plain data. This plain data is the data that the low-level functions take as inputs and deliver as outputs.

For plain data we mean:

logicals and scalars: Any Matlab scalar value such as **a = 5** or **b = true**.

vectors and matrices: Any Matlab array such as **v = [1;2]**, **w = [1 2]**,
c = [true false] or **M = [1 2;3 4]**.

character strings: Any Matlab alphanumeric string such as **type = 'pinHole'**
or **dir = '%HOME/temp/'**.

frames: Frames are Matlab structures that we created to store data belonging to 3D frames (see Fig. 18 for an instance of the **frame** structure; type **help frame** at the Matlab prompt). We do this to avoid having to compute multiple times rotation matrices and other frame-related constructions.

A frame is specified by a 7-vector **frame.x** containing translation vector and an orientation quaternion (type **help quaternion** at the Matlab prompt). This is the essential frame information. After each setting or modification of the state **frame.x**, call the **updateFrame()** function to create/update the rest of the frame structure.

```
frame      % Frame structure, containing:
.x         % the state 7-vector
.t         % translation vector,      t = x(1:3)
.q         % orientation quaternion,  q = x(4:7)
.R         % rotation matrix,        R = q2R(q)
.Rt        % transposed R,           Rt = R'
.Pi        % PI matrix,              Pi = q2Pi(q)
.Pc        % conjugate PI matrix,    Pc = q2Pi(iq)
```

Figure 18: The **frame** structure.

4 Functions

The SLAM toolbox is composed of functions of different importance, defining three levels of abstraction (Fig. 3). They are stored in subdirectories according to their field of utility. There are two particular directories: **HighLevel**, with two scripts and a limited set of high-level functions; and **InterfaceLevel**, with a number of functions interfacing the high level data with the low-level library. All other directories contain low-level functions.

4.1 High level

The high level scripts and functions are located in the directory **SLAMtoolbox/HighLevel/**.

There are two main scripts that constitute the highest level:

slamtb.m: the main script. It initializes all data structures and figures, and performs the temporal loop by first simulating motions and measurements, second estimating the map and localization (the SLAM algorithm itself), and third visualizing all the data.

Here is a simplified version of this script:

```
% SLAMTB EKF-SLAM simulator. Main script.

% User-defined data.
userData;

% Create all data structures
[Rob, Sen, Lmk, Obs, Tim] = createSlamStructures(...
    Robot, Sensor, Landmark, Time, ...
    EstOpt);
[SimRob, SimSen, SimLmk] = createSimStructures(...
    Robot, Sensor, World);
[MapFig, SenFig] = createGraphicsStructures(...
    Rob, Sen, Lmk, Obs, ...
    SimRob, SimSen, SimLmk, ...
    FigOpt);

% Main loop
for currentFrame = Tim.firstFrame : Tim.lastFrame

    % 1. SIMULATION
    for rob = [Rob.rob]

        % Simulate robot motion
```

```

SimRob(rob) = simMotion(SimRob(rob), Tim);

for sen = SimRob(rob).sensors

    % Simulate measurements
    SimObs(sen) = simObservation(...
        SimRob(rob), SimSen(sen), SimLmk);
end

end

% 2. SLAM
for rob = [Rob.rob]

    % Robot motion
    Rob(rob) = motion(Rob(rob), Tim);

    for sen = Rob(rob).sensors

        % Observe known landmarks
        [Rob(rob), Sen(sen), Lmk, Obs(sen,:)] = ...
            correctKnownLmks(...
                Rob(rob), Sen(sen), Raw(sen), Lmk, Obs(sen,:));

        % Initialize new landmarks
        [Lmk, Obs(sen,:)] = initNewLmks(...
            Rob(rob), Sen(sen), Raw(sen), Lmk, Obs(sen,:));
    end

end

% 3. VISUALIZATION
drawMapFig(MapFig, Rob, Sen, Lmk, SimRob, SimSen);
for sen = [Sen.sen]
    drawSenFig(SenFig(sen), Sen(sen), Raw(sen), Obs(sen,:));
end
drawnow;
end

```

userData.m: a script containing the data the user must enter to configure the simulation. It is called by **slamtb.m** at the very first lines of code.

High-level functions exist to help initializing all the structured data. They are called by **slamtb** just after **userData**:

```

createSLAMstructures()      % Create SLAM structures
createSimStructures()      % Create simulation structures

```

```
createGraphicsStructures() % Create graphics structures
```

Other high-level functions create all graphics figures. They are called by `createGraphicsStructures.m`:

```
createMapFig() % Create 3D Map figure  
createSenFig() % Create all sensors' figures
```

4.2 Interface level

The interface level functions are located in the directory `SLAMtoolbox/InterfaceLevel/`.

The interface level functions interface the high-level scripts and structured data with the low-level functions and the plain data. These functions serve three purposes:

1. Check the type of structured data and select the appropriate methods to manipulate them.
2. Split the structured data into smaller parts of plain data.
3. Call the low-level functions with the plain data (see Section 3.4), and assign the outputs to the appropriate fields of structured data.

Interface-level functions perform the different simulation, SLAM, and redraw operations. They are called inside the main loop:

```
% Simulator  
simObservation() % Simulated observations  
simMotion() % Simulate motions  
% SLAM  
observeKnownLmks() % EKF-update of known landmarks  
initNewLmk() % Landmark initialization  
motion() % Robot motion  
% Visualization  
drawMapFig() % Redraw 3D Map figure  
drawSenFig() % Redraw sensors figures
```

A good example of interface function is `simMotion.m`, whose code is reproduced in Fig. 19.

```

function Rob = simMotion(Rob, Tim)

% SIMMOTION Simulated robot motion.
% Rob = SIMMOTION(Rob, Tim) performs one motion step to robot
% Rob, following the controls in Rob.con according to the motion
% model in Rob.motion. The time information Tim is used only if
% the motion model requires it, but it has to be provided because
% SIMMOTION is a generic method.
%
% See also CONSTVEL, ODO3, UPDATEFRAME.

switch Rob.motion      % check robot's motion model

    case 'constVel'
        Rob.state.x = constVel(Rob.state.x, Rob.con.u, Tim.dt);
        Rob.frame.x = Rob.state.x(1:7);
        Rob.vel.x   = Rob.state.x(8:13);
        Rob.frame   = updateFrame(Rob.frame);

    case 'odometry'
        Rob.frame = odo3(Rob.frame, Rob.con.u);

    otherwise
        error('??? Unknown motion model '%s'', Rob.motion);
end

```

Figure 19: The `simMotion.m` interface function. Observe that (1) the interface function checks data types and selects different low-level functions accordingly; (2) the structures are split into chunks of plain data before entering the low-level functions; (3) in `case 'constVel'`, `frame.x` is modified by the low-level motion functions, and we need a call to `updateFrame()` afterwards; (4) the low-level odometry function `odo3()` already performs frame update; (5) there is an error message for unknown motion models.

4.3 Low level library

There are different directories storing a lot of low-level functions. Although this directory arborescence is meant to be complete, you are free to add new functions and directories (do not forget to add these new directories to the Matlab path). The only reason for these directories to exist is to have the functions organized depending on their utility.

The toolbox is delivered with the following directories:

DataManagement/	% Certain data manipulations
DetectionMatching/	% Features detection and matching
EKF/	% Extended Kalman Filter
FrameTransforms/	% Frame transformations
rotations/	% Rotations (inside FrameTransforms/)
Graphics/	% Graphics creation and redrawing
Kinematics/	% Motion models
Math/	% Some math functions
Observations/	% Observation models
Simulation/	% Methods exclusive to simulation
Slam/	% Low-level functions for EKF-SLAM

The functions contained in this directories take plain data as input, and deliver plain data as output.

To explore the contents of the library, start by typing **help DirectoryName** at the Matlab prompt.

5 Developing new observation models

This section describes the necessary steps for creating new observation models any time a new type of sensor and/or a new type of landmark is considered. Please read ‘[guidelines.pdf](#)’ before contributing your own code.

Before you develop a new observation model, you must take care of the following facts:

1. You need a *direct observation model* for observing known landmarks and correcting the map, and an *inverse observation model* for landmark initialization.
2. The robot acts as a mere support for sensors. Normally, only its current frame is of any interest. In some (rare) special cases, the robot’s velocity may be of interest if the measurements are sensitive to it (for example, when considering a sonar sensor with Doppler-effect capabilities).
3. The sensor’s frame is specified in robot frame. It may be part of the SLAM state vector.
4. The sensor contains other parameters. The number and nature of these parameters depend on the type of sensor and cannot be generalized. We have not considered these parameters as part of the SLAM state vector.
5. The landmark has the main parameters in the SLAM vector, but it may have some other parameters out of it.
6. The sensor may provide full or partial measurements of the landmark state. In case of partial measurements, you have to provide a Gaussian prior of the non-measured part for initialization.

5.1 Steps to incorporate new models

Once you decide to incorporate new models, follow these steps:

1. Write direct and inverse observation models. See next sections.
2. Edit `userData.m`. Add a new landmark type `Landmark{L}`. Add a number of such landmarks in structure `World`.

3. Edit `createLandmarks.m` and `createSimLmk.m` to reflect the new landmark type in the SLAM and simulation structures. Follow the existing code for Euclidean and inverse-depth points as a guide.
4. Edit `initNewLmk.m` for landmark initialization. Add new case lines `case 'mySen'` and `case 'myLmk'`, and write the necessary code that calls the functions in your inverse model.
5. Edit `observeKnownLmks.m` for landmark corrections. Add new case lines `case 'mySen'` and `case 'myLmk'`, and write the necessary code that calls the functions in your direct model. See that these function calls request Jacobians. Name the return variable and Jacobians exactly as in the other existing models: they are used later.
6. Edit `simObservation.m` for simulating landmark measurements. Add new case lines `case 'mySen'` and `case 'myLmk'`, and write the necessary code that calls the functions in your direct model. These function calls do not request Jacobians.

5.2 Direct observation model for map corrections

The observation operations are split into three stages: transformation to robot frame, transformation to sensor frame, and projection into the sensor's measurement space. The model takes the general form $\mathbf{e} = \mathbf{h}(\mathbf{Rf}, \mathbf{Sf}, \mathbf{Sp}, \mathbf{l})$, with \mathbf{Rf} the robot frame, \mathbf{Sf} the sensor frame, \mathbf{Sp} the sensor parameters, \mathbf{l} the landmark parameters, and \mathbf{e} the expected measurement or projection (in the EKF argot, $\mathbf{e} = \mathbf{h}(\mathbf{x})$). Here is a simplified implementation:

```
function e = observationModel(Rf,Sf,Sp,l)
% Rf: robot frame
% Sf: sensor frame
% Sp: sensor parameters
% l : landmark in world frame
% e : projected magnitude
lr = toFrame(Rf,l);           % landmark in robot frame
ls = toFrame(Rs,lr);          % landmark in sensor frame
e = projectToSensor(Sp,ls);    % projection to sensor's space
```

This shows that we need to create three functions for a direct observation model: `toFrame`, `projectToSensor` and `observationModel`, whose names will be properly particularized for the types of sensor and landmark of the model.

This scheme must be enriched with two important capabilities, namely:

- Jacobian matrices computation.
- Vectorized operation for multiple landmarks.

The following code exemplifies the direct measurement model for a pin-hole camera mounted on a robot and observing Euclidean 3D points. Use it as a guide for writing your own models. Notice the systematic use of the chain rule for computing the Jacobians (see ‘[guidelines.pdf](#)’ for info on the chain rule).

```
function [u, s, U_r, U_s, U_k, U_d, U_l] = ...
    projEucPntIntoPinHoleOnRob(Rf, Sf, Spk, Spd, l)

if narginout ≤ 2 % No Jacobians requested
    lr = toFrame(Rf,l); % lmk to robot frame
    ls = toFrame(Sf,lr); % lmk to sensor frame
    [u,s] = pinHole(ls,Spk,Spd); % lmk into measurement space
else % Jacobians requested

    if size(l,2) == 1 % single point
        % Same functions with Jacobian output
        [lr, LR_r, LR_l] = toFrame(Rf,l);
        [ls, LS_s, LS_lr] = toFrame(Sf,lr);
        [u,s,U_ls,U_k,U_d] = pinHole(ls,Spk,Spd);

        % Apply the chain rule for Jacobians
        U_lr = U_ls*LS_lr;
        U_r = U_lr*LR_r;
        U_s = U_ls*LS_s;
        U_l = U_lr*LR_l;
    else
        error('??? Jacobians not available for multiple points.')
    end
end
```

The model makes use of the functions `toFrame()` and `pinHole()`. The first function is specific to the landmark type, while the second depends on both the landmark type and the sensor type. We reproduce them here:

```
function [pf, PF_f, PF_p] = toFrame(F, pw)

s = size(pw,2); % number of points in input matrix

if s==1 % one point
    pf = F.Rt*(pw - F.t);
```

```

    if nargout > 1 % Jacobians.
        PF_t = -F.Rt;
        sc = 2*F.Pc*(pw - F.t);
        PF_q = [...
            sc(2)  sc(1) -sc(4)  sc(3)
            sc(3)  sc(4)  sc(1) -sc(2)
            sc(4) -sc(3)  sc(2)  sc(1)];
        PF_p = F.Rt;
        PF_f = [PF_t PF_q];
    end

else % multiple points
    pf = F.Rt*(pw - repmat(F.t,1,s));
    if nargout > 1
        error('??? Jacobians not available for multiple points.');
```

```

function [u, s, U_p, U_k, U_d] = pinHole(p, k, d)

% Point's depths
s = p(3,:);

if nargin < 3, d = []; end % Default is no distortion

if nargout ≤ 2 % no Jacobians requested
    u = pixellise(distort(project(p),d),k);
else % Jacobians

    if size(p,2) == 1 % p is a single 3D point
        [up, UP_p] = project(p);
        [ud, UD_up, UD_d] = distort(up,d);
        [u, U_ud, U_k] = pixellise(ud,k);
        U_d = U_ud*UD_d;
        U_p = U_ud*UD_up*UP_p;

    else % p is a 3D points matrix – no Jacobians possible
        error('??? Jacobians not available for multiple points.')
```

5.3 Inverse observation model for landmark initialization

The inverse model works inversely to the direct one, with one important detail: for sensors providing partial landmark measurements, a prior is needed in order to provide the inverse function with the full necessary information.

The model takes the general form $\mathbf{l} = \mathbf{g}(\mathbf{Rf}, \mathbf{Sf}, \mathbf{Sp}, \mathbf{e}, \mathbf{n})$, with \mathbf{Rf} the robot frame, \mathbf{Sf} the sensor frame, \mathbf{Sp} the sensor parameters, \mathbf{e} the measurement, \mathbf{n} the non-measured prior, and \mathbf{l} the retro-projected landmark parameters. Here is a simplified implementation:

```
function l = invObsModel(Rf, Sf, Sp, e, n)
% Rf: robot frame
% Sf: sensor frame
% Sp: sensor parameters
% e : measurement
% n : non-measured prior
% l : obtained landmark
ls = retroProjectFromSensor(Sp, e, n); % lmk in sensor frame
lr = fromFrame(Sf, ls);               % lmk in robot frame
l  = fromFrame(Rf, lr);               % lmk in world frame
```

In this case, only Jacobians computation need to be added, as it is not likely that we need to retro-project several points at a time (contrary to what happens with the direct models).

The following code exemplifies the inverse measurement model for a pin-hole camera mounted on a robot and observing 3D points, rendering 3D landmarks parametrized as inverse-depth [1]. The inverse depth is precisely the non-measured part \mathbf{n} , and is provided as a prior with a separate input.²

```
function [idp, IDP_rf, IDP_sf, IDP_sk, IDP_sc, IDP_u, IDP_n] = ...
    retroProjIdpPntFromPinHoleOnRob(Rf, Sf, Sk, Sc, u, n)

if nargin == 1 % No Jacobians requested
    idps = invPinHoleIdp(u, n, Sk, Sc) ;
    idpr = fromFrameIdp(Sf, idps) ;
    idp  = fromFrameIdp(Rf, idpr) ;

else % Jacobians requested
    % function calls
    [idps, IDPS_u, IDPS_n, IDPS_sk, IDPS_sc] = ...
        invPinHoleIdp(u, n, Sk, Sc) ;
```

²For speed reasons, the function `retroProjIdpPntFromPinHoleOnRob` is implemented somewhat differently in the toolbox.

```

[idpr, IDPR_sf, IDPR_idps] = fromFrameIdp(Sf, idps) ;
[idp, IDP_rf, IDP_idpr] = fromFrameIdp(Rf, idpr) ;

% The chain rule
IDP_idps = IDP_idpr*IDPR_idps; % intermediate result, used hereafter
IDP_sk    = IDP_idps*IDPS_sk ;
IDP_sc    = IDP_idps*IDPS_sc ;
IDP_u     = IDP_idps*IDPS_u ;
IDP_n     = IDP_idps*IDPS_n ;
end

```

5.4 Landmark reparametrization

In case you are using landmarks with a parametrization that is specialized for initialization, such as Inverse Depth points, you must consider reparametrizing them to more economical forms, such as Euclidean points. Read [2] if you do not know what I am talking about. In this case, write reparametrization functions and include them in the code.

Here is an example:

```

function [p,P_idp] = idp2p(idp)

% IDP2P Inverse Depth to cartesian point conversion.
% IDP2P(IDP) returns the cartesian 3D representation of the point
% coded in Inverse depth.
%
% IDP is a 6-vector : IDP = [x0 y0 z0 el az rho]' where
%   x0, z0, y0: anchor: the 3D point P0 where where distance is
%               referred to.
%   el, az:     azimuth and elevation of the ray through P that
%               starts at P0.
%   rho:        inverse of the distance from point P to P0.
%
% [P,P_idp] = IDP2P(...) returns the Jacobian of the conversion.
%
% See also p2idp.

x0 = idp(1:3,:); % origin
py = idp(4:5,:); % pitch and roll
r  = idp(6,:);   % inverse depth

if size(idp,2) == 1 % one only Idp

    [v,Vpy] = py2vec(py); % unity vector
    p       = x0 + v/r;

```

```

    if nargout > 1 % jacobians

        Px = eye(3);
        Pv = eye(3)/r;
        Pr = -v/r^2;

        Ppy = Pv*Vpy;

        P_idp = [Px Ppy Pr];
    end

else % A matrix of Idps

    v = py2vec(py); % unity vector
    p = x0 + v./repmat(r,3,1);

    if nargout > 1
        error('??? Jacobians not available for multiple landmarks.')
    end
end
end

```

To include this function in the code, edit the function `reparametrizeLmk()`, create the `switch` – `case` tree and add your code as appropriated.

5.5 Landmark parameters out of the SLAM map

There are some landmark parameters that are not part of the stochastic state vector estimated by SLAM. The number and nature of these parameters depend on the landmark type and cannot be generalized.

The direct and inverse observation models must be complemented with the appropriate methods to initialize and update these parameters.

Examples of such parameters are:

- The landmark's appearance information, used in appearance-based feature matching. See [3, 6] for examples on setting and using these parameters. See [5] for an example of updating them.
- The endpoints of segment landmarks. See [4, 7] for examples of setting and updating.

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