

# An EKF-SLAM toolbox in Matlab

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# 1 Quick start

Hi there! To start the toolbox, do the following:

1. Visit [www.laas.fr/~jsola](http://www.laas.fr/~jsola) and download the toolbox package.
2. Move **slamToolbox.zip** where you want the SLAM toolbox to be installed. Unzip it.
3. Rename the expanded directory if wanted (we'll call this directory **SLAMTB/**).
4. Open Matlab. Add all directories and subdirectories in **SLAMTB/** to the Matlab path.
5. Execute **slamtb** from the Matlab prompt.

Or, if you want to get some more insight:

6. Edit **userData.m**. Read the help lines. Explore options and create, by copying and modifying, new robots and sensors. You can modify the robots' initial positions and motions and the sensors' positions and parameters. You can also modify the default set of landmarks or 'World'.
7. Edit and run **slamtb.m**. Explore its code by debugging step-by-step. Explore the Map figure by zooming and rotating with the mouse. Try using **userDataIdp**, **userDataHmg** and **userDataPlk** instead of the default **userData** (at the third line of code in **slamtb.m**).
8. Read the help contents of the following 4 functions: **frame**, **fromFrame**, **q2R**, **pinHole**. Follow some of the **See also** links.
9. Read '[guidelines.pdf](#)' before contributing your own code.<sup>1</sup>

---

<sup>1</sup>Also available at <http://www.laas.fr/~jsola/Joan%20Sola/objectes/toolbox/guidelines.pdf>.

## 2 The SLAM toolbox presentation

In a typical SLAM problem, one or more robots navigate an environment, discovering and mapping landmarks on the way by means of their onboard sensors. Observe in Fig. 1 the existence of robots of different kinds, carrying a different number of sensors of different kinds, which gather raw data and, by processing it, are capable of observing landmarks of different kinds. All this variety of data is handled by the present toolbox in a way that is quite transparent.

In this toolbox, we organized the data into three main groups, see Table 1. The first group contains the objects of the SLAM problem itself, as they appear in Fig. 1. A second group contains objects for simulation. A third group is designated for graphics output, Fig. 2.

Apart from the data, we have of course the functions. Functions are organized in three levels, from most abstract and generic to the basic manipulations, as is sketched in Fig. 3. The highest level, called *High Level*, deals exclusively with the structured data we mentioned just above, and calls functions of an intermediate level called the *Interface Level*. The interface level functions split the data structures into more mathematically meaningful elements, check objects types to decide on the applicable methods, and call the basic functions that constitute the basic level, called the *Low Level Library*.

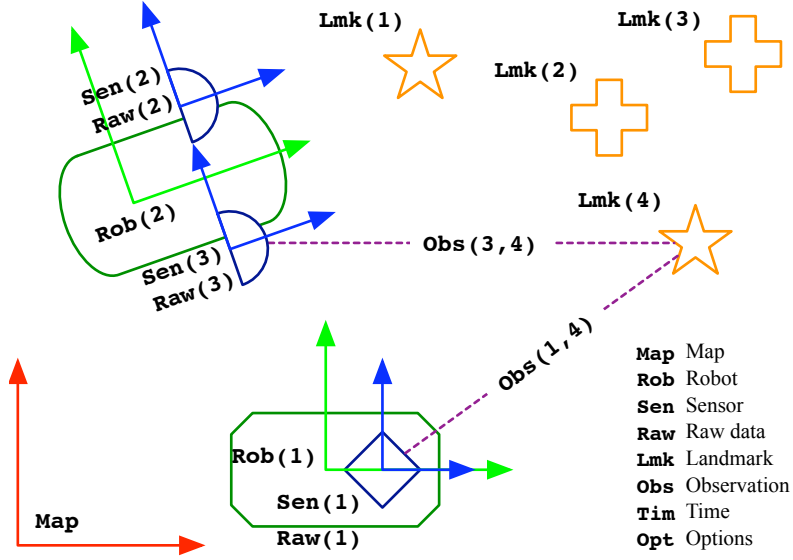


Figure 1: Overview of the SLAM problem with the principal data structures.

Table 1: All data structures.

Purpose	SLAM	Simulator	Graphics
Map	<b>Map</b>		<b>MapFig</b>
Robots	<b>Rob</b>	<b>SimRob</b>	
Sensors	<b>Sen</b>	<b>SimSen</b>	<b>SenFig</b>
Raw data	<b>Raw</b>		
Landmarks	<b>Lmk</b>	<b>SimLmk</b>	
Observations	<b>Obs</b>		
Time	<b>Tim</b>		
Options	<b>Opt</b>	<b>SimOpt</b>	<b>FigOpt</b>



Figure 2: The set of figures. The structures **MapFig** and **SenFig(s)** contain the handles to all graphics objects drawn.

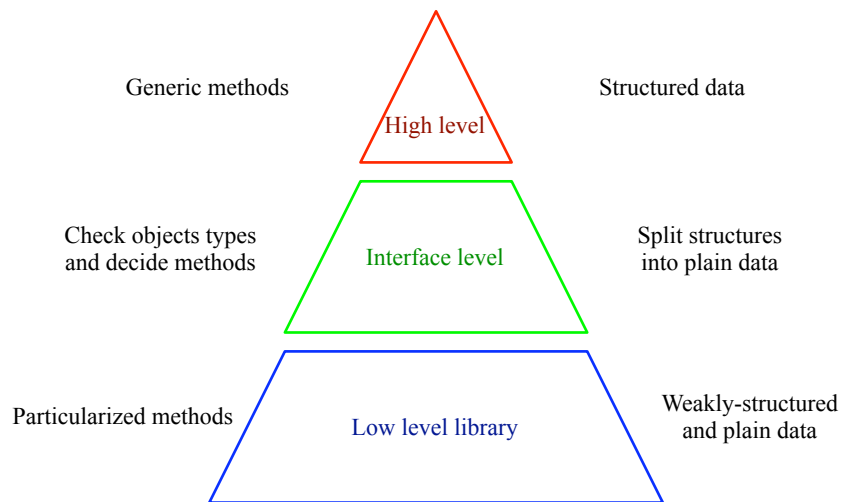


Figure 3: Overview of the levels of abstraction of the functions and their relation to data structuration. Functions and scripts in the High and Interface levels are in the **HighLevel/** and **InterfaceLevel/** directories. The Low Level library occupies all other directories.

### 3 Data organization

It follows a brief explanation of the SLAM data structures, the Simulation and Graphic structures, and the plain data types.

#### 3.1 SLAM data

For a SLAM system to be complete, we need to consider the following parts:

**Rob**: A set of robots.

**Sen**: A set of sensors.

**Raw**: A set of raw data captures, one per sensor.

**Lmk**: A set of landmarks.

**Map**: A stochastic map containing the states of robots, landmarks, and eventually sensors.

**Obs**: The set of landmark observations made by processing **Raw** data.

**Tim**: A few time-related variables.

**Opt**: Algorithm options.

This toolbox considers these objects as the only existing data for SLAM. They are defined as structures holding a variety of fields (see Figs. 4 to 11 for reference). Structure arrays hold any number of such objects. For example, all the data related to robot number 2 is stored in **Rob(2)**. To access the rotation matrix defining the orientation of this robot we simply use **Rob(2).frame.R** (type **help frame** at the Matlab prompt for help on 3D reference frames). Observations require two indices because they relate sensors to landmarks. Thus, **Obs(sen, lmk)** stores the data associated to the observation of landmark **lmk** from sensor **sen**.

It would be wise, before reading on, to revisit Fig. 1 and see how simple things are.

It follows a reproduction of the arborescences of the principal structures in the SLAM data.

```

Rob(rob)      % Robot structure, containing:
  .rob        % index in Rob() array
  .id         % robot id
  .name       % robot name
  .type       % robot type
  .sensors    % list of installed sensors
  .motion     % motion model
  .con        % control structure
    .u        % control signals for the motion model
    .uStd     % standard deviation of u
    .U        % covariance of u
  .frame      % frame structure, containing:
    .x        % 7-vector, position and orientation  $x = [t;q]$ 
    .P        % covariances matrix of  $x$ 
    .t        % position
    .q        % orientation quaternion
    .R        % rotation matrix,  $R = q2R(q)$ 
    .Rt       % transposed R
    .Pi       % PI matrix,  $Pi = q2Pi(q)$ 
    .Pc       % conjugate PI matrix,  $Pc = pi2pc(Pi)$ 
    .r        % range in the SLAM map Map
  .vel        % velocity structure, containing
    .x        % 6-vector, linear and angular velocities
    .P        % covariances matrix of  $x$ 
    .r        % range in the SLAM map Map
  .state      % state structure, containing
    .x        % robot's state vector,  $x = [frame.x;vel.x]$ 
    .P        % covariances matrix of  $x$ 
    .size     % size of  $x$ 
    .r        % range in the SLAM map Map

```

Figure 4: The **Rob** structure array.

```

Sen(sen)      % Sensor structure, containing:
  .sen        % index in Sen() array
  .id         % sensor id
  .name       % sensor name
  .type       % sensor type
  .robot      % robot it is installed to
  .frameInMap % flag: is frame in Map?
  .frame      % frame structure, containing:
    .x        % 7-vector, position and orientation  $x = [t;q]$ 
    .P        % covariances matrix of  $x$ 
    .t        % position
    .q        % orientation quaternion
    .R        % rotation matrix,  $R = q2R(q)$ 
    .Rt       % transposed R
    .Pi       % PI matrix,  $Pi = q2Pi(q)$ 
    .Pc       % conjugate PI matrix,  $Pc = pi2pc(Pi)$ 
    .r        % range in the SLAM map Map
  .par        % sensor parameters
    .k        % intrinsic params
    .d        % distortion vector
    .c        % correction vector
    .imSize   % image size
    .pixErr   % pixel error std.
    .pixCov   % pixel covariances matrix
  .state      % state structure, containing
    .x        % sensor's state vector,  $x = frame.x$  or  $x = []$ 
    .P        % covariances matrix of  $x$ 
    .size     % size of  $x$ 
    .r        % range in the SLAM map Map

```

Figure 5: The **Sen** structure array.

```

Raw(sen)      % Raw data structure, containing:
  .type       % type of raw data
  .data       % raw data, containing
    .points   % 3D point landmarks (for simulated data)
      .coord  % a matrix of points
      .app    % a vector of appearances
    .segments % 3D segment landmarks (for simulated data)
      .coord  % a matrix of segments (two endpoints, stacked)
      .app    % a vector of appearances
  .img        % a pixels image (for real images)

```

Figure 6: The **Raw** structure array.



```

Lmk(lmk)      % Landmark structure, containing:
    .lmk        % index in Lmk() array
    .id         % landmark id
    .type       % sensor type
    .sig        % landmark descriptor or signature
    .used       % flag: is landmark used in the map?
    .state      % state structure, containing
        .r       % range in the SLAM map Map
    .par        % other lmk parameters
        .p0      % Line origin
        .endp()  % 2 endpoints for segments
            .t    % abscissa
            .e    % endpoints mean
            .E    % endpoints covariances matrix
    .nSearch    % number of times searched
    .nMatch     % number of times matched
    .nInlier    % number of times declared inlier

```

Figure 7: The **Lmk** structure array.

```

Map          % Map structure, containing:
    .used      % vector of flags indicating non-free positions
    .x         % state vector's mean
    .P         % covariances matrix

```

Figure 8: The **Map** structure.

```

Obs(sen, lmk) % Observation structure, containing:
  .sen      % index to sensor in Sen() array
  .lmk      % index to landmark in Lmk() array
  .sid      % sensor id
  .lid      % landmark id
  .stype    % sensor type
  .ltype    % landmark type
  .meas     % measurement
    .y      % mean
    .R      % covariance
  .nom      % non-measurable degrees of freedom
    .n      % mean
    .N      % covariance
  .exp      % expectation
    .e      % mean
    .E      % covariance
    .um     % uncertainty measure, um = det(E)
  .inn      % innovation
    .z      % mean
    .Z      % covariance
    .iZ     % inverse covariance
    .MD2    % squared Mahalanobis distance, MD2 = z'*iZ*z
  .app      % appearance
    .pred   % predicted appearance
    .curr   % current appearance
    .sc     % matching quality score
  .par      % other parameters
    .endp() % two segment endpoints
      .e    % mean
      .E    % covariance
  .vis      % flag: is lmk visible from sensor?
  .measured % flag: has lmk been measured?
  .matched  % flag: has lmk been matched?
  .updated  % flag: has Map been updated?
  .Jac      % Jacobians
    .E_r    % expectation wrt robot frame vector
    .E_s    % expectation wrt sensor frame vector
    .E_l    % expectation wrt landmark parameters
    .Z_r    % innovation wrt robot frame vector
    .Z_s    % innovation wrt sensor frame vector
    .Z_l    % innovation wrt landmark parameters

```

Figure 9: The **Obs** structure array.

```

Tim           % Time structure, containing:
  .firstFrame % first frame to evaluate
  .lastFrame  % last frame to evaluate
  .dt         % Sampling period

```

Figure 10: The **Tim** structure.

```

Opt           % Options structure, containing:
  .map          % Options for the map
    .numLmks    % map capacity: number of 3d landmarks
    .lmkSize    % nominal lmk size (for map size estimation)
  .correct      % Options for lmk correction
    .reprojectLmks % reproject lmks after active search?
    .nUpdates    % maximum simultaneous updates
    .MD2th       % threshold on Mahalanobis distance
    .linTestIdp  % threshold on IDP linearity test
    .lines       % Options related to line landmarks correction
      .innType    % innovation type for lines
      .extPolicy  % endpoints extension policy
      .extSwitch  % policy switching threshold
  .init         % Options for initialization
    .initType    % Type of landmark to initialize
    .idpPnt      % options for IDP initialization
      .nonObsMean % mean of non-observable prior
      .nonObsStd  % std dev. of non-observable prior
    .hmgPnt      % options for homogeneous point
      .nonObsMean % mean of non-observable prior
      .nonObsStd  % std dev. of non-observable prior
    .plkLin      % options for Plucker line
      .nonObsMean % mean of non-observable prior
      .nonObsStd  % std dev. of non-observable prior
  .obs          % Observation options
    .lines       % options for lines or segments
    .minLength   % minimum segment length

```

Figure 11: The **Opt** structure.

### 3.2 Simulation data

This toolbox also includes simulated scenarios. We use for them the following objects, that come with 6-letter names to differentiate from the SLAM data:

**SimRob:** Virtual robots for simulation.

**SimSen:** Virtual sensors for simulation.

**SimLmk:** A virtual world of landmarks for simulation.

**SimOpt:** Options for the simulator.

The simulation structures **SimXxx** are simplified versions of those existing in the SLAM data. Their arborescence is much smaller, and sometimes they may have absolutely different organization. It is important to understand that none of these structures is necessary if the toolbox is to be used with real data.

It follows a reproduction of the arborescences of the principal simulation data structures.

```

SimRob(rob)    % Simulated robot structure, containing:
  .rob         % index in SimRob() array
  .id          % robot id
  .name        % robot name
  .type        % robot type
  .motion      % motion model
  .sensors     % list of installed sensors
  .frame       % frame structure, containing:
    .x         % 7-vector, position and orientation  $x = [t;q]$ 
    .t         % position
    .q         % orientation quaternion
    .R         % rotation matrix,  $R = q2R(q)$ 
    .Rt        % transposed R
    .Pi        % PI matrix,  $Pi = q2Pi(q)$ 
    .Pc        % conjugate PI matrix,  $Pc = pi2pc(Pi)$ 
  .vel         % velocity structure, containing
    .x         % 6-vector, linear and angular velocities
  .con         % Control vector
    .u         % control signals for the motion model
    .uStd      % standard deviation of u
    .U         % covariance of u

```

Figure 12: The **SimRob** structure array.

```

SimSen(sen)    % Simulated Sensor structure, containing:
    .sen       % index in SimSen() array
    .id        % sensor id
    .name      % sensor name
    .type      % sensor type
    .robot     % robot it is installed to
    .frame     % frame structure, containing:
        .x      % 7-vector, position and orientation  $x = [t;q]$ 
        .t      % position
        .q      % orientation quaternion
        .R      % rotation matrix,  $R = q2R(q)$ 
        .Rt     % transposed R
        .Pi     % PI matrix,  $Pi = q2Pi(q)$ 
        .Pc     % conjugate PI matrix,  $Pc = pi2pc(Pi)$ 
    .par       % sensor parameters
        .k      % intrinsic params
        .d      % distortion vector
        .c      % correction vector
        .imSize % image size

```

Figure 13: The **SimSen** structure array.

```

SimLmk          % Simulated landmarks structure, containing:
  .points        % Point landmarks
    .id          % N-vector of point identifiers
    .coord       % 3-by-N array of 3D points
  .segments      % segment landmarks
    .id          % M-vector of segment identifiers
    .coord       % 6-by-M array of 3D segments
  .lims          % limits of playground in X, Y and Z axes
    .xMin        % minimum X coordinate
    .xMax        % maximum X coordinate
    .yMin        % minimum Y coordinate
    .yMax        % maximum Y coordinate
    .zMin        % minimum Z coordinate
    .zMax        % maximum Z coordinate
  .dims          % dimensions of playground
    .l           % length in X
    .w           % width in Y
    .h           % height in Z
  .center        % central point
    .xMean       % central X
    .yMean       % central Y
    .zMean       % central Z

```

Figure 14: The **SimLmk** structure.

```

SimOpt          % Simulator options structure, containing:
  .random        % random generator options
    .active       % use true random generator?
    .fixedSeed    % random seed for non-random runs
    .seed         % actual seed
  .obs           % options for simulated observations.
                % (this is a hard-copy of Obs.obs)

```

Figure 15: The **SimOpt** structure

### 3.3 Graphics data

This toolbox also includes graphics output. We use for them the following objects, which come also with 6-letter names:

**MapFig:** A structure of handles to graphics objects in the 3D map figure. One Map figure showing the world, the robots, the sensors, and the current state of the estimated SLAM map (Figs. 16 and 17).

**SenFig:** A structure array of handles to graphics objects in the sensor figures. One figure per sensor, visualizing its *measurement space* (Figs. 18 and 19).

**FigOpt:** A structure with options for figures such as colors, views and projections.

It follows a reproduction of the arborescences of the principal graphics structures. See Section 5.6 for information about graphic functions.



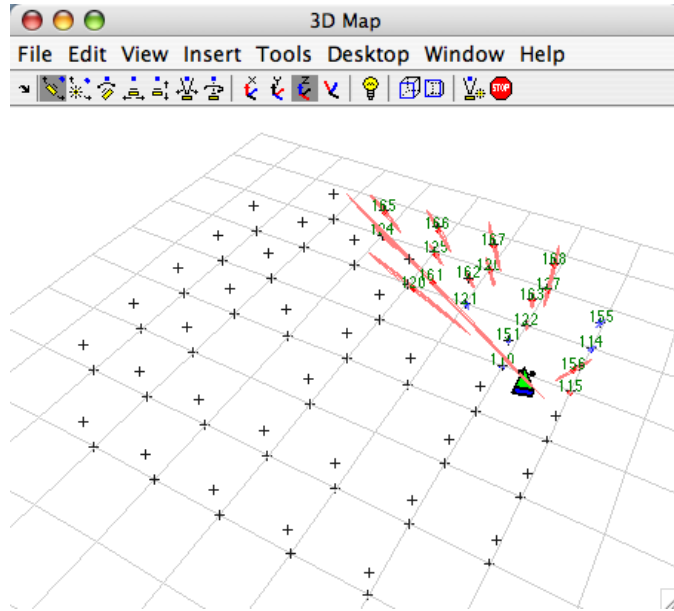


Figure 16: The 3D map figure. **MapFig** contains handles to all objects drawn.

```

MapFig          % Map figure structure, containing:
  .fig           % figure number and handle
  .axes          % axes handle
  .ground        % handle to floor object
  .Rob           % array of structures to SLAM robot handles
    .patch       % handle to robot graphics patch
    .ellipse     % handle to robot's uncertainty ellipsoid
  .Sen           % array of handles to SLAM sensors
  .Lmk           % handles to SLAM landmarks, containing:
    .drawn       % array of flags indicating drawn landmarks
    .mean        % array of handles to landmarks means
    .ellipse     % array of handles to landmarks ellipses
    .label       % array of handles to landmarks labels
  .simRob        % array of handles to simulated robots
  .simSen        % array of handles to simulated sensors
  .simLmk        % handle to simulated landmarks

```

Figure 17: The **MapFig** structure.

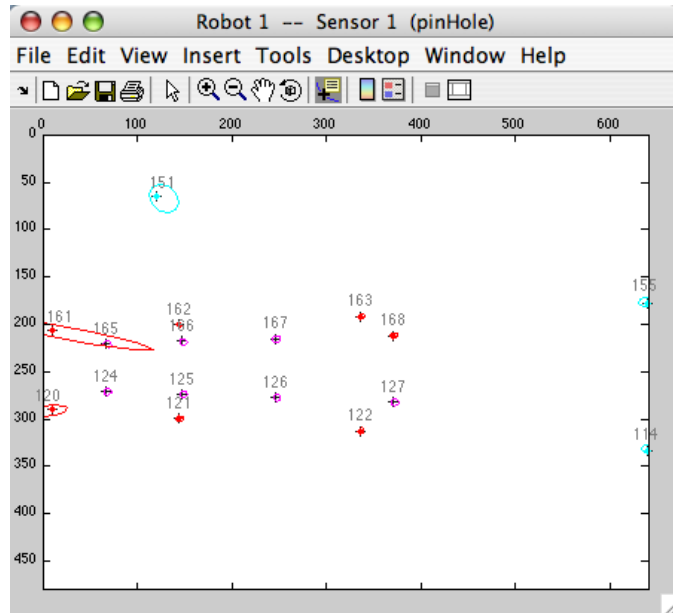


Figure 18: A pin-hole sensor view figure. **SenFig(1)** contains handles to all objects drawn.

```

SenFig(sen)    % Sensor figure structure, containing:
    .fig        % figure number and handle
    .axes       % axes handle
    .raw        % handles to raw data
        .points % handle to one line object for all raw points
        .segments % array of handles to line objects for raw segments
    .drawn      % vector of flags indicating drawn observations
    .measure    % array of handles to landmarks measurements
    .ellipse    % array of handles to landmarks ellipses
    .label      % array of handles to landmarks labels

```

Figure 19: The **SenFig** structure array.

```

FigOpt                % Figure options structure, containing:
    .renderer          % renderer
    .createVideo       % create video sequence?
    .skipFrames        % rendering period in frames
    .map               % map figure options
        .proj          % projection of the 3d figure
        .view          % viewpoint of the 3d figure
        .size          % map figure size
        .showEllip     % show uncertainty ellipsoids?
        .colors        % map figure colors
            .border    % border
            .axes      % axes, ticks and axes labels
            .bckgnd    % background
            .simLmks   % simulated landmarks
            .simu      % simulated robots and sensors
            .est       % estimated robots and sensors
            .ground    % ground
            .label     % landmark ID labels
    .sensor            % sensor figures options
        .size          % sensor figure size
        .showEllip     % show uncertainty ellipses?
        .colors        % Sensor figure colors:
            .border    % border
            .axes      % axes, ticks and axes labels
            .bckgnd    % background
            .raw       % raw data
            .label     % label

```

Figure 20: The **FigOpt** structure.

### 3.4 Plain data

The structured data we have seen so far is composed of chunks of lower complexity structures and plain data. This plain data is the data that the low-level functions take as inputs and deliver as outputs.

For plain data we mean:

**logicals and scalars:** Any Matlab scalar value such as `a = 5` or `b = true`.

**vectors and matrices:** Any Matlab array such as `v = [1;2]`, `w = [1 2]`,  
`c = [true false]` or `M = [1 2;3 4]`.

**character strings:** Any Matlab alphanumeric string such as `type = 'pinHole'`  
or `dir = '%HOME/temp/'`.

**frames:** Frames are Matlab structures that we created to store data belonging to 3D frames (see Fig. 21 for an instance of the `frame` structure; type `help frame` at the Matlab prompt). We do this to avoid having to compute multiple times rotation matrices and other frame-related constructions.

A frame is specified by a 7-vector `frame.x` containing translation vector and an orientation quaternion (type `help quaternion` at the Matlab prompt). This is the essential frame information. After each setting or modification of the state `frame.x`, call the `updateFrame()` function to create/update the rest of the frame structure.

```
frame      % Frame structure, containing:
.x         % the state 7-vector
.t         % translation vector,      t = x(1:3)
.q         % orientation quaternion,  q = x(4:7)
.R         % rotation matrix,         R = q2R(q)
.Rt        % transposed R,            Rt = R'
.Pi        % PI matrix,               Pi = q2Pi(q)
.Pc        % conjugate PI matrix,     Pc = q2Pi(iq)
```

Figure 21: The `frame` structure.

## 4 Functions

The SLAM toolbox is composed of functions of different importance, defining three levels of abstraction (Fig. 3). They are stored in subdirectories according to their field of utility. There are two particular directories: **HighLevel**, with two scripts and a limited set of high-level functions; and **InterfaceLevel**, with a number of functions interfacing the high level data with the low-level library. All other directories contain low-level functions.

### 4.1 High level

The high level scripts and functions are located in the directory **SLAMtoolbox/HighLevel/**.

There are two main scripts that constitute the highest level, one for the code and one for the data:

**slamtb.m**: the main script. It initializes all data structures and figures, and performs the temporal loop by first simulating motions and measurements, second estimating the map and localization (the SLAM algorithm itself), and third visualizing all the data.

Here is a simplified version of this script:

```
% SLAMTB EKF-SLAM simulator. Main script.

% User-defined data.
userData;

% Create all data structures
[Rob, Sen, Lmk, Obs, Tim] = createSlamStructures(...
    Robot, Sensor, Landmark, Time, ...
    Opt);
[SimRob, SimSen, SimLmk] = createSimStructures(...
    Robot, Sensor, World, ...
    SimOpt);
[MapFig, SenFig] = createGraphicsStructures(...
    Rob, Sen, Lmk, Obs, ...
    SimRob, SimSen, SimLmk, ...
    FigOpt);

% Main loop
for currentFrame = Tim.firstFrame : Tim.lastFrame

    % 1. SIMULATION
```

```

for rob = [SimRob.rob]

    % Simulate robot motion
    SimRob(rob) = simMotion(SimRob(rob), Tim);

    for sen = SimRob(rob).sensors

        % Simulate measurements
        SimObs(sen) = simObservation(...
            SimRob(rob), SimSen(sen), SimLmk) ;
    end

end

% 2. SLAM
for rob = [Rob.rob]

    % Robot motion
    Rob(rob) = motion(Rob(rob), Tim);

    for sen = Rob(rob).sensors

        % Correct known landmarks
        [Rob(rob), Sen(sen), Lmk, Obs(sen,:)] = ...
            correctKnownLmks(...
                Rob(rob), Sen(sen), Raw(sen), Lmk, Obs(sen,:));

        % Initialize new landmarks
        [Lmk, Obs(sen,:)] = initNewLmks(...
            Rob(rob), Sen(sen), Raw(sen), Lmk, Obs(sen,:));
    end

end

% 3. VISUALIZATION
drawMapFig(MapFig, Rob, Sen, Lmk, SimRob, SimSen);
for sen = [Sen.sen]
    drawSenFig(SenFig(sen), Sen(sen), Raw(sen), Obs(sen,:));
end
drawnow;
end

```

**userData.m**: a script containing the data the user must enter to configure the simulation. It is called by **slamtb.m** at the very first lines of code.

High-level functions exist to help initializing all the structured data. They are called by **slamtb** just after **userData**:

<code>createSLAMstructures()</code>	<code>% Create SLAM structures</code>
<code>createSimStructures()</code>	<code>% Create simulation structures</code>
<code>createGraphicsStructures()</code>	<code>% Create graphics structures</code>

## 4.2 Interface level

The interface level functions are located in the directory **SLAMtoolbox/InterfaceLevel/**.

The interface level functions interface the high-level scripts and structured data with the low-level functions and the plain data. These functions serve three purposes:

1. Check the type of structured data and select the appropriate methods to manipulate them.
2. Split the structured data into smaller parts of plain data.
3. Call the low-level functions with the plain data (see Section 3.4), and assign the outputs to the appropriate fields of structured data.

Interface-level functions perform the different simulation, SLAM, and redraw operations. They are called inside the main loop:

<code>% Simulator</code>	
<code>simMotion()</code>	<code>% Simulate motions</code>
<code>simObservation()</code>	<code>% Simulated observations</code>
<code>% SLAM</code>	
<code>motion()</code>	<code>% Robot motion</code>
<code>observeKnownLmks()</code>	<code>% EKF-update of known landmarks</code>
<code>initNewLmk()</code>	<code>% Landmark initialization</code>
<code>% Visualization</code>	
<code>drawMapFig()</code>	<code>% Redraw 3D Map figure</code>
<code>drawSenFig()</code>	<code>% Redraw sensors figures</code>

Other intermediate-level functions create all graphics figures. They are called by **createGraphicsStructures.m**:

<code>createMapFig()</code>	<code>% Create 3D Map figure</code>
<code>createSenFig()</code>	<code>% Create all sensors' figures</code>

A good example of interface function is **simMotion.m**, whose code is reproduced in Fig. 22.

```

function Rob = simMotion(Rob, Tim)

% SIMMOTION Simulated robot motion.
% Rob = SIMMOTION(Rob, Tim) performs one motion step to robot
% Rob, following the controls in Rob.con according to the motion
% model in Rob.motion. The time information Tim is used only if
% the motion model requires it, but it has to be provided because
% SIMMOTION is a generic method.
%
% See also CONSTVEL, ODO3, UPDATEFRAME.

switch Rob.motion      % check robot's motion model

    case 'constVel'
        Rob.state.x = constVel(Rob.state.x, Rob.con.u, Tim.dt);
        Rob.frame.x = Rob.state.x(1:7);
        Rob.vel.x   = Rob.state.x(8:13);
        Rob.frame   = updateFrame(Rob.frame);

    case 'odometry'
        Rob.frame = odo3(Rob.frame, Rob.con.u);

    otherwise
        error('??? Unknown motion model '%s'', Rob.motion);
end

```

Figure 22: The `simMotion.m` interface function. Observe that (1) the interface function checks data types and selects different low-level functions accordingly; (2) the structures are split into chunks of plain data before entering the low-level functions; (3) in `case 'constVel'`, `frame.x` is modified by the low-level motion functions, and we need a call to `updateFrame()` afterwards; (4) the low-level odometry function `odo3()` already performs frame update; (5) there is an error message for unknown motion models.



### 4.3 Low level library

There are different directories storing a lot of low-level functions. Although this directory arborescence is meant to be complete, you are free to add new functions and directories (do not forget to add these new directories to the Matlab path). The only reason for these directories to exist is to have the functions organized depending on their utility.

The toolbox is delivered with the following directories:

<b>DataManagement/</b>	<b>% Certain data manipulations</b>
<b>DetectionMatching/</b>	<b>% Features detection and matching</b>
<b>EKF/</b>	<b>% Extended Kalman Filter</b>
<b>FrameTransforms/</b>	<b>% Frame transformations</b>
<b>rotations/</b>	<b>% Rotations (inside FrameTransforms/)</b>
<b>Graphics/</b>	<b>% Graphics creation and redrawing</b>
<b>Kinematics/</b>	<b>% Motion models</b>
<b>Math/</b>	<b>% Some math functions</b>
<b>Observations/</b>	<b>% Observation models</b>
<b>Simulation/</b>	<b>% Methods exclusive to simulation</b>
<b>Slam/</b>	<b>% Low-level functions for EKF-SLAM</b>

The functions contained in this directories take plain data as input, and deliver plain data as output.

To explore the contents of the library, start by typing **help DirectoryName** at the Matlab prompt.

## 5 Developing new observation models

This section describes the necessary steps for creating new observation models any time a new type of sensor and/or a new type of landmark is considered. Please read ‘[guidelines.pdf](#)’ before contributing your own code.

Before you develop a new observation model, you must take care of the following facts:

1. You need a *direct observation model* for observing known landmarks and correcting the map, and an *inverse observation model* for landmark initialization.
2. The robot acts as a mere support for sensors. Normally, only its current frame is of any interest. In some (rare) special cases, the robot’s velocity may be of interest if the measurements are sensitive to it (for example, when considering a sonar sensor with Doppler-effect capabilities).
3. The sensor’s frame is specified in robot frame. It may be part of the SLAM state vector.
4. The sensor contains other parameters. The number and nature of these parameters depend on the type of sensor and cannot be generalized. We have not considered these parameters as part of the SLAM state vector, although this could be done. Most observation functions in the toolbox already return the Jacobians with respect to these parameters.
5. The landmark has the main parameters in the SLAM vector, but it may have some other parameters out of it.
6. The sensor may provide full or partial measurements of the landmark state. In case of partial measurements, you have to provide a Gaussian prior of the non-measured part for initialization.

### 5.1 Steps to incorporate new models

Once you decide to incorporate new models, follow these steps:

1. Write direct and inverse observation models. See Sections [5.2](#) and [5.3](#).
2. Edit `userData.m`. Add a number of new landmarks in structure `World`. Type `help userData` and explore the comments within its code to learn how to achieve this.

3. Edit `simObservation.m` for simulating landmark measurements. Add new case lines `case 'mySen'` and `case 'myLmk'`, and write the necessary code that calls the functions in your direct model. These function calls do not request Jacobians.
4. Edit `initNewLmk.m` for landmark initialization. Add new case lines `case 'mySen'` and `case 'myLmk'`, and write the necessary code that calls the functions in your inverse model.
5. Edit `correctKnownLmks.m` for landmark corrections. Add new case lines `case 'mySen'` and `case 'myLmk'`, and write the necessary code that calls the functions in your direct model. See that these function calls request Jacobians. Name the return variable and Jacobians exactly as in the other existing models: they are used later. See Sections 5.2, 5.4 and 5.5.
6. Edit `createMapFig.m` and `createSenFig` for map and sensor figures. Add new `switch-case` entries and the methods to create the desired graphics. See Section 5.6.
7. Edit `drawMapFig.m` and `drawSenFig.m` to redraw the new landmarks in the map and sensor figures. Add new `switch-case` entries and create the desired methods for showing the landmarks and associated observations. See Section 5.6.

## 5.2 Direct observation model for map corrections

The observation operations are split into three stages: transformation to robot frame, transformation to sensor frame, and projection into the sensor's measurement space. The model takes the general form  $\mathbf{e} = \mathbf{h}(\mathbf{Rf}, \mathbf{Sf}, \mathbf{Sp}, \mathbf{l})$ , with  $\mathbf{Rf}$  the robot frame,  $\mathbf{Sf}$  the sensor frame,  $\mathbf{Sp}$  the sensor parameters,  $\mathbf{l}$  the landmark parameters, and  $\mathbf{e}$  the expected measurement or projection (in the EKF argot,  $\mathbf{e} = \mathbf{h}(\mathbf{x})$ ). Here is a simplified implementation:

```
function e = observationModel(Rf,Sf,Sp,l)
% Rf: robot frame
% Sf: sensor frame
% Sp: sensor parameters
% l : landmark in world frame
% e : projected magnitude
lr = toFrame(Rf,l);           % landmark in robot frame
ls = toFrame(Rs,lr);          % landmark in sensor frame
e = projectToSensor(Sp,ls);    % projection to sensor's space
```

This shows that we need to create three functions for a direct observation model: **toFrame**, **projectToSensor** and **observationModel**, whose names will be properly particularized for the types of sensor and landmark of the model.

This scheme must be enriched with two important capabilities, namely:

- Jacobian matrices computation.
- Vectorized operation for multiple landmarks.

The following code exemplifies the direct measurement model for a pin-hole camera mounted on a robot and observing Euclidean 3D points. Use it as a guide for writing your own models. Notice the systematic use of the chain rule for computing the Jacobians (see ‘[guidelines.pdf](#)’ for info on the chain rule).

```
function [u, s, U_r, U_s, U_k, U_d, U_l] = ...
    projEucPntIntoPinHoleOnRob(Rf, Sf, Spk, Spd, l)

if nargin ≤ 2 % No Jacobians requested
    lr    = toFrame(Rf,l);      % lmk to robot frame
    ls    = toFrame(Sf,lr);     % lmk to sensor frame
    [u,s] = pinHole(ls,Spk,Spd); % lmk into measurement space
else % Jacobians requested

    if size(l,2) == 1 % single point
        % Same functions with Jacobian output
        [lr, LR_r, LR_l] = toFrame(Rf,l);
        [ls, LS_s, LS_lr] = toFrame(Sf,lr);
        [u,s,U_ls,U_k,U_d] = pinHole(ls,Spk,Spd);

        % Apply the chain rule for Jacobians
        U_lr = U_ls*LS_lr;
        U_r  = U_lr*LR_r;
        U_s  = U_ls*LS_s;
        U_l  = U_lr*LR_l;
    else
        error('??? Jacobians not available for multiple points.')
    end
end
```

The model makes use of the functions **toFrame()** and **pinHole()**. The first function is specific to the landmark type, while the second depends on both the landmark type and the sensor type. We reproduce them here:

```

function [pf, PF_f, PF_p] = toFrame(F, pw)

s = size(p_W,2); % number of points in input matrix

if s==1 % one point
    pf = F.Rt*(pw - F.t);

    if nargin > 1 % Jacobians.
        PF_t = -F.Rt;
        sc = 2*F.Pc*(pw - F.t);
        PF_q = [...
            sc(2)  sc(1) -sc(4)  sc(3)
            sc(3)  sc(4)  sc(1) -sc(2)
            sc(4) -sc(3)  sc(2)  sc(1)];
        PF_p = F.Rt;
        PF_f = [PF_t PF_q];
    end
end

else % multiple points
    pf = F.Rt*(pw - repmat(F.t,1,s));
    if nargin > 1
        error('??? Jacobians not available for multiple points. ');
    end
end
end

```

```

function [u, s, U_p, U_k, U_d] = pinHole(p, k, d)

% Point's depths
s = p(3,:);

if nargin < 3, d = []; end % Default is no distortion

if nargin ≤ 2 % no Jacobians requested
    u = pixellise(distort(project(p),d),k);
else % Jacobians
    if size(p,2) == 1 % p is a single 3D point
        [up, UP_p] = project(p);
        [ud, UD_up, UD_d] = distort(up,d);
        [u, U_ud, U_k] = pixellise(ud,k);
        U_d = U_ud*UD_d;
        U_p = U_ud*UD_up*UP_p;
    else % p is a 3D points matrix – no Jacobians possible
        error('??? Jacobians not available for multiple points. ')
    end
end

```

```

end
end

```

### 5.3 Inverse observation model for landmark initialization

The inverse model works inversely to the direct one, with one important detail: for sensors providing partial landmark measurements, a prior is needed in order to provide the inverse function with the full necessary information.

The model takes the general form  $\mathbf{l} = \mathbf{g}(\mathbf{Rf}, \mathbf{Sf}, \mathbf{Sp}, \mathbf{e}, \mathbf{n})$ , with  $\mathbf{Rf}$  the robot frame,  $\mathbf{Sf}$  the sensor frame,  $\mathbf{Sp}$  the sensor parameters,  $\mathbf{e}$  the measurement,  $\mathbf{n}$  the non-measured prior, and  $\mathbf{l}$  the retro-projected landmark parameters. Here is a simplified implementation:

```

function l = invObsModel(Rf, Sf, Sp, e, n)
% Rf: robot frame
% Sf: sensor frame
% Sp: sensor parameters
% e : measurement
% n : non-measured prior
% l : obtained landmark
ls = retroProjectFromSensor(Sp, e, n); % lmk in sensor frame
lr = fromFrame(Sf, ls);               % lmk in robot frame
l  = fromFrame(Rf, lr);               % lmk in world frame

```

In this case, only Jacobians computation need to be added, as it is not likely that we need to retro-project several points at a time (contrary to what happens with the direct models).

The following code exemplifies the inverse measurement model for a pin-hole camera mounted on a robot and observing 3D points, rendering 3D landmarks parametrized as inverse-depth [1]. The inverse depth is precisely the non-measured part  $\mathbf{n}$ , and is provided as a prior with a separate input.<sup>2</sup>

```

function [idp, IDP_rf, IDP_sf, IDP_sk, IDP_sc, IDP_u, IDP_n] = ...
    retroProjIdpPntFromPinHoleOnRob(Rf, Sf, Sk, Sc, u, n)

if nargin == 1 % No Jacobians requested
    idps = invPinHoleIdp(u,n,Sk,Sc) ;
    idpr = fromFrameIdp(Sf, idps) ;
    idp  = fromFrameIdp(Rf, idpr) ;

```

<sup>2</sup>For speed reasons, the function `retroProjIdpPntFromPinHoleOnRob` is implemented somewhat differently in the toolbox.

```

else % Jacobians requested
    % function calls
    [idps, IDPS_u, IDPS_n, IDPS_sk, IDPS_sc] = ...
        invPinHoleIdp(u, n, Sk, Sc) ;
    [idpr, IDPR_sf, IDPR_idps] = fromFrameIdp(Sf, idps) ;
    [idp, IDP_rf, IDP_idpr] = fromFrameIdp(Rf, idpr) ;

    % The chain rule
    IDP_idps = IDP_idpr*IDPR_idps; % intermediate result
    IDP_sk = IDP_idps*IDPS_sk ;
    IDP_sc = IDP_idps*IDPS_sc ;
    IDP_u = IDP_idps*IDPS_u ;
    IDP_n = IDP_idps*IDPS_n ;
end

```

## 5.4 Landmark reparametrization

In case you are using landmarks with a parametrization that is specialized for initialization, such as Inverse Depth points, you must consider reparametrizing them to more economical forms, such as Euclidean points. Read [2] if you do not know what I am talking about. In this case, write reparametrization functions and include them in the code.

Here is an example of the reparametrization function:

```

function [p,P_idp] = idp2p(idp)

% IDP2P Inverse Depth to cartesian point conversion.
% IDP2P(IDP) returns the cartesian 3D representation of the point
% coded in Inverse depth.
%
% IDP is a 6-vector : IDP = [x0 y0 z0 el az rho]' where
% x0,z0,y0: anchor: the 3D point P0 where where distance is
%           referred to.
% el,az: azimuth and elevation of the ray through P that
%         starts at P0.
% rho: inverse of the distance from point P to P0.
%
% [P,P_idp] = IDP2P(...) returns the Jacobian of the conversion.
%
% See also p2idp.

x0 = idp(1:3,:); % origin
py = idp(4:5,:); % pitch and roll
r = idp(6,:); % inverse depth

```

```

if size(idp,2) == 1 % one only Idp

    [v,Vpy] = py2vec(py); % unity vector
    p       = x0 + v/r;

    if nargout > 1 % jacobians

        Px = eye(3);
        Pv = eye(3)/r;
        Pr = -v/r^2;

        Ppy = Pv*Vpy;

        P_idp = [Px Ppy Pr];
    end

else % A matrix of Idps

    v = py2vec(py); % unity vector
    p = x0 + v./repmat(r,3,1);

    if nargout > 1
        error('??? Jacobians not available for multiple landmarks.')
    end
end
end

```

To include this function in the code, edit the function `reparametrizeLmk()`, create a new **case** in the landmark's type **switch**, and add your code as appropriated.

Another example of reparametrization is `hmg2euc()`.

## 5.5 Landmark parameters out of the SLAM map

There are some landmark parameters that are not part of the stochastic state vector estimated by SLAM. The number and nature of these parameters depend on the landmark type and cannot be generalized.

The direct and inverse observation models must be complemented with the appropriate methods to initialize and update these parameters. In the toolbox, we use the functions `initLmkParams` and `updateLmkParams`. Initialization and updating of segment endpoints for lines of the type Plucker are supported in this toolbox.

Examples of such parameters are:

- The landmark's appearance information, used in appearance-based feature matching. See [3, 11] for examples on setting and using these parameters. See [8] for an example of updating them.



- The endpoints of segment landmarks. See [7, 15] for examples of setting and updating. Check functions `retroProjPlkEndPnts` and `updatePlkLinEndPnts`.

## 5.6 Graphic functions

Each new landmark needs its own drawing methods and, possibly, dedicated graphic data structures.

Graphics are managed with Matlab *handles*. If you do not know about handles, we give here a basic approach. In brief, when you create a graphics object you assign it a handle that will allow you to manipulate the object.<sup>3</sup> After that, to redraw the object you only need to update the values in that handle. You update the values that have changed, and leave the rest unchanged. The alternative to plot at each frame the whole graphic is not time efficient. Here is an example of a moving graphic using handles:

```
% create a fancy figure for our demo
f1 = figure(1);
set(f1, 'doublebuffer', 'on', ...
      'renderer', 'opengl')
ax = gca ; cla ; axis equal ;
axis([-1.1 1.1 -1.1 1.1])

% we create here the graphics object: a single-point line
angle0 = 0;
objecthandle = line( ...
    cos(angle0), sin(angle0), ...
    'parent', ax, ...
    'marker', 'o', ...
    'color', 'r', ...
    'markersize', 10);

for angle = angle0:0.01:(angle0+2*pi)
    % we change only its position
    set(objecthandle, ...
        'xdata', cos(angle), ...
        'ydata', sin(angle));
    drawnow
end
```

---

<sup>3</sup>Yes, handles are there always to manipulate objects. As well as *handle* comes from the English root ‘*hand*’, the word *manipulate* comes from the Latin root ‘*manus*’ for *hand*, and means “*handle or control (a mechanism, tool, etc.), typically in a skillful manner*” (Oxford). With no abuse at all, we can perfectly say that a cup’s handle allows us to manipulate the cup without getting burned.

We basically use two types of graphics object: **line** and **patch**. To know the properties of each object you can access to, simply create a default object with *e.g.* **h = line()** and then type **set(h)**. You will get a list of all possible properties with the default values. To know the properties, type **get(h)**. To modify a particular property, type **set(h, 'propertyName', value)**. To read a value, use **get(h, 'propertyName')**.

In the toolbox, objects are created in **createMapFig** and **createSenFig**, before the main loop. They are updated in the third part of the loop, with **drawMapFig** and **drawSenFig**. Visit these functions, add new **switch-case** entries for your objects, and code the necessary methods.

Bear in mind that, while new landmark parametrizations require new drawing methods for the 3D part (the map figure), once they are projected into a particular sensor they may end up having the same structure as other existing landmarks. For instance, Euclidean, IDP and homogeneous points all project into 2D points in the image. This means that we will find 3D-drawing functions **drawEucLmk**, **drawIdpLmk** and **drawHmgLmk**, but we will find only one function **drawObsPnt** for all of their 2D projections.

The following two examples show how to redraw a 3D ellipsoid belonging to an Euclidean landmark, and how to redraw a projected 2D segment in a pin-hole image.

```
function drawEucPnt(MapFig, Lmk, color)
% DRAWEUCPNT Draw Euclidean point landmark in MapFig.

global Map
posOffset = [0;0;.2];

% Mean and covariance
x = Map.x( Lmk.state.r);
P = Map.P(Lmk.state.r,Lmk.state.r);

% the expectation's mean:
set(MapFig.Lmk(Lmk.lmk).mean,...
    'xdata',    x(1),...
    'ydata',    x(2),...
    'zdata',    x(3),...
    'color',    color.mean,...
    'visible',  'on');

% the expectation's ellipse
[X,Y,Z] = cov3elli(x, P, 3, 10) ;
set(MapFig.Lmk(Lmk.lmk).ellipse,...
    'xdata',    X,...
```

```

        'ydata',    Y,...
        'zdata',    Z,...
        'color',    color.ellip,...
        'visible',  'on');

% the label
set(MapFig.Lmk(Lmk.lmk).label,...
    'position', x + posOffset,...
    'string',   num2str(Lmk.id),...
    'visible',  'on');

```

```

function drawObsLin(SenFig, Obs, colors, imSize, SenOpt)
% DRAWOBSLIN Redraw a line landmark on the pinHole sensor figure.

posOffset = 8; % label position offset wrt line, in pixels

if Obs.measured
    % the measurement:
    y = Obs.meas.y;
    set(SenFig.measure(Obs.lmk), ...
        'xdata', [y(1);y(3)], ...
        'ydata', [y(2);y(4)], ...
        'color', colors(1,:), ...
        'vis',    'on');

    % the label
    c = (Obs.meas.y(1:2)+Obs.meas.y(3:4))*0.5; % segment's center
    v = (Obs.meas.y(1:2)-Obs.meas.y(3:4));      % segment's vector
    n = normvec([-v(2);v(1)]);                  % segment's normal vector
    pos = c + n*posOffset;
    set(SenFig.label(Obs.lmk), ...
        'position', pos, ...
        'string',   num2str(Obs.lid), ...
        'vis',      'on');
else
    set(SenFig.measure(Obs.lmk), 'vis', 'off');
    set(SenFig.label(Obs.lmk),   'vis', 'off');
end

% the expectation's mean
s = trimHmgLin(Obs.exp.e, imSize);
X = s([1 3]);
Y = s([2 4]);
set(SenFig.mean(Obs.lmk), ...
    'xdata', X, ...
    'ydata', Y, ...
    'color', colors(2,:), ...

```

```

        'vis',      'on');

% ellipses
if SenOpt.showEllip
    [X,Y] = cov2elli(Obs.par.endp(1).e, ...
        Obs.par.endp(1).E, 3, 10) ;
    set(SenFig.ellipse(Obs.lmk,1), ...
        'xdata', X, ...
        'ydata', Y, ...
        'color', colors(3,:), ...
        'vis',      'on');
    [X,Y] = cov2elli(Obs.par.endp(2).e, ...
        Obs.par.endp(2).E, 3, 10) ;
    set(SenFig.ellipse(Obs.lmk,2), ...
        'xdata', X, ...
        'ydata', Y, ...
        'color', colors(3,:), ...
        'vis',      'on');
end

```

## 6 Bibliography selection

The following publications list is of mandatory reading for anyone wishing to understand/use/contribute to this toolbox.

- My thesis [12], but not in all its extension. Read Chapter 6, and particularly section 6.5 on Active Search.
- Articles by myself about monocular SLAM and the extensions to multi-camera: [14, 13, 11, 15].
- Articles by Andrew J. Davison, the most important contributor to monocular EKF-SLAM, and his colleagues at Oxford, London and Cambridge: [3, 4, 5, 8, 6].
- Articles from the University of Zaragoza, mostly on EKF-SLAM. Landmark initialization using inverse-depth parametrization, and multi-map SLAM for large environments: [9, 2, 1, 10].

If this list is too long for your available time or patience, try this reduced version of just 4 titles:

1. Davison on monocular SLAM [3].
2. Solà on undelayed landmark initialization [14].
3. Civera on inverse depth parametrization [1].
4. Solà on multi-camera SLAM [11].

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- [3] A. J. Davison. Real-time simultaneous localisation and mapping with a single camera. In *Int. Conf. on Computer Vision*, volume 2, pages 1403–1410, Nice, October 2003.
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- [9] J. Montiel, J. Civera, and A. J. Davison. Unified inverse depth parametrization for monocular SLAM. In *Robotics: Science and Systems*, Philadelphia, USA, August 2006.
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- [11] J. Solà, André Monin, Michel Devy, and T. Vidal-Calleja. Fusing monocular information in multi-camera SLAM. *IEEE Trans. on Robotics*, 24(5):958–968, 2008.

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- [13] Joan Solà, André Monin, and Michel Devy. BiCamSLAM: Two times mono is more than stereo. In *IEEE Int. Conf. on Robotics and Automation*, pages 4795–4800, Rome, Italy, April 2007.
- [14] Joan Solà, André Monin, Michel Devy, and Thomas Lemaire. Undelayed initialization in bearing only SLAM. In *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pages 2499–2504, Edmonton, Canada, 2005.
- [15] Joan Solà, Teresa Vidal-Calleja, and Michel Devy. Undelayed initialization of line segments in monocular SLAM. In *IEEE Int. Conf. on Intelligent Robots and Systems*, Saint Louis, USA, 2009. To appear.