**Shusen Wang** 

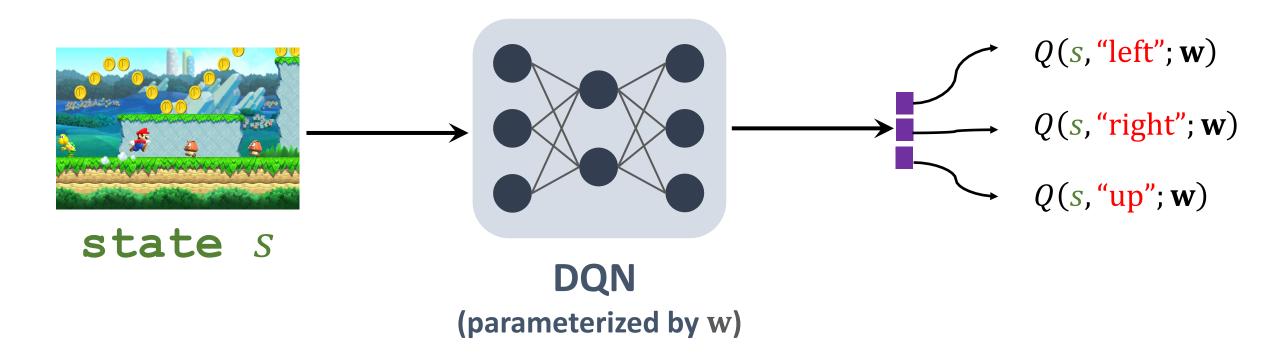
## **Revisiting DQN and TD Learning**

#### Deep Q Network (DQN)

Approximate the optimal action-value function  $Q^*(s, a)$  by Q(s, a; w).

#### Deep Q Network (DQN)

Approximate the optimal action-value function,  $Q^*(s, \mathbf{a})$ , by  $Q(s, \mathbf{a}; \mathbf{w})$ .



- Observe state  $s_t$  and perform action  $a_t$ .
- Environment provides new state  $s_{t+1}$  and reward  $r_t$ .
- TD target:  $y_t = r_t + \gamma \left( \max_a Q(s_{t+1}, a; \mathbf{w}) \right)$

- Observe state  $s_t$  and perform action  $a_t$ .
- Environment provides new state  $s_{t+1}$  and reward  $r_t$ .
- TD target:  $y_t = r_t + \gamma \cdot \max_a Q(s_{t+1}, a; \mathbf{w})$ .
- TD error:  $\delta_t = q_t y_t$  where  $q_t = Q(s_t, a_t; \mathbf{w})$ .
- Goal: Make  $q_t$  close to  $y_t$ , for all t. (Equivalently, make  $\delta_t^2$  small.)

- TD error:  $\delta_t = q_t y_t$ , where  $q_t = Q(s_t, a_t; \mathbf{w})$ .
- **TD learning:** Find **w** by minimizing  $L(\mathbf{w}) = \frac{1}{T} \sum_{t=1}^{T} \frac{\delta_t^2}{2}$ .
- Online gradient descent:
  - Observe  $(s_t, a_t, r_t, s_{t+1})$  and compute  $\delta_t$ .
  - Compute gradient:  $\mathbf{g}_t = \frac{\partial \delta_t^2/2}{\partial \mathbf{w}} = \delta_t \cdot \frac{\partial Q(s_t, \mathbf{a_t}; \mathbf{w})}{\partial \mathbf{w}}$
  - Gradient descent:  $\mathbf{w} \leftarrow \mathbf{w} \alpha \cdot \mathbf{g}_t$ .

- TD error:  $\delta_t = q_t y_t$ , where  $q_t = Q(s_t, a_t; \mathbf{w})$ .
- **TD learning:** Find **w** by minimizing  $L(\mathbf{w}) = \frac{1}{T} \sum_{t=1}^{T} \frac{\delta_t^2}{2}$ .
- Online gradient descent.
- Discard  $(s_t, a_t, r_t, s_{t+1})$  after using it.

#### **Shortcoming 1: Waste of Experience**

- A transition:  $(s_t, a_t, r_t, s_{t+1})$ .
- Experience: all the transitions, for  $t = 1, 2, \cdots$ .
- Previously, we discard  $(s_t, a_t, r_t, s_{t+1})$  after using it.
- It is a waste...

#### **Shortcoming 2: Correlated Updates**

- Previously, we use  $(s_t, a_t, r_t, s_{t+1})$  sequentially, for  $t = 1, 2, \cdots$ , to update **w**.
- Consecutive states,  $s_t$  and  $s_{t+1}$ , are strongly correlated (which is bad.)

- A transition:  $(s_t, a_t, r_t, s_{t+1})$ .
- Store recent n transitions in a replay buffer.

```
(S_t, \boldsymbol{a}_t, r_t, S_{t+1})
(S_{t+1}, a_{t+1}, r_{t+1}, S_{t+2})
(S_{t+2}, a_{t+2}, r_{t+2}, S_{t+3})
```

Replay Buffer (n transitions)

- A transition:  $(s_t, a_t, r_t, s_{t+1})$ .
- Store recent n transitions in a replay buffer.
- Remove old transitions so that the buffer has at most n transitions.
- Buffer capacity n is a tuning hyper-parameter [1, 2].
  - *n* is typically large, e.g.,  $10^5 \sim 10^6$ .
  - The setting of n depends on the specific application.

#### Reference:

- 1. Zhang & Sutton. A deeper look at experience replay. In NIPS workshop, 2017.
- 2. Fedus et al. Revisiting fundamentals of experience replay. In ICML, 2019.

### **TD** with Experience Replay

- Find w by minimizing  $L(\mathbf{w}) = \frac{1}{T} \sum_{t=1}^{T} \frac{\delta_t^2}{2}$ .
- Stochastic gradient descent (SGD):
  - Randomly sample a transition,  $(s_i, a_i, r_i, s_{i+1})$ , from the buffer.
  - Compute TD error,  $\delta_i$ .
  - Stochastic gradient:  $\mathbf{g}_i = \frac{\partial \delta_i^2/2}{\partial \mathbf{w}} = \delta_i \cdot \frac{\partial Q(s_i, \mathbf{a_i}; \mathbf{w})}{\partial \mathbf{w}}$
  - SGD:  $\mathbf{w} \leftarrow \mathbf{w} \alpha \cdot \mathbf{g}_i$ .

#### History

- Experience replay was proposed by Long-Ji Lin [1].
- The DQN paper [2] popularized experience replay.
- There many improvements, e.g., [3].

#### Reference:

- 1. Lin. Reinforcement Learning for Robots Using Neural Networks. PhD Dissertation, 1993.
- 2. Mnih et al. Human-level control through deep reinforcement learning. *Nature*, 2015.
- 3. Schaul et al. Prioritized experience replay. In ICLR, 2016.

## Prioritized Experience Replay

#### Reference:

1. Schaul, Quan, Antonoglou, & Silver. Prioritized experience replay. In ICLR, 2016.

#### **Basic Idea**

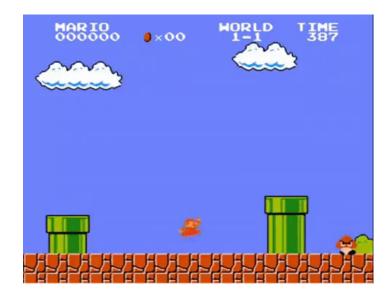
- Not all transitions are equally important.
- Which kinds of transitions are more important, left or right?





#### **Basic Idea**

- How do we know which transition is important?
- If a transition has high TD error  $|\delta_t|$ , it will be given high priority.





#### **Importance Sampling**

- Use importance sampling instead of uniform sampling.
- Option 1: Sampling probability  $p_t \propto |\delta_t| + \epsilon$ .
- Option 2: Sampling probability  $p_t \propto \frac{1}{\operatorname{rank}(t)}$ .
  - The transitions are sorted so that  $|\delta_t|$  is in the descending order.
  - rank(t) is the rank or the t-th transition.

### **Scaling Learning Rate**

- SGD:  $\mathbf{w} \leftarrow \mathbf{w} \boldsymbol{\alpha} \cdot \mathbf{g}$ , where  $\boldsymbol{\alpha}$  is the learning rate.
- If uniform sampling is used,  $\alpha$  is the same for all transitions.
- If importance sampling is used, the learning rate  $\alpha$  shall be adjusted.

### **Scaling Learning Rate**

- Scale the learning rate by  $(n p_t)^{-\beta}$ , where  $\beta \in (0,1)$ .
- If  $p_1 = \cdots = p_n = \frac{1}{n}$  (uniform sampling), the scaling factor is equal to 1.
- High-importance transitions (with high  $p_t$ ) have low learning rate.
- In the beginning, set  $\beta$  small; increase  $\beta$  to 1 over time.

#### **Update TD Error**

- Associate each transition,  $(s_t, a_t, r_t, s_{t+1})$ , a TD error,  $\delta_t$ .
- If the transition is newly collected, we do not know its  $\delta_t$ . Simply set its  $\delta_t$  to the maximum (i.e., most important.)
- Each time we use  $(s_t, a_t, r_t, s_{t+1})$ , we update its  $\delta_t$ .

#### **Transitions**

# Sampling Probabilities

# Learning Rates

• • •

• • •

• • •

$$(s_t, \boldsymbol{a_t}, r_t, s_{t+1}), \delta_t$$

$$p_t \propto |\delta_t| + \epsilon$$

$$\alpha \cdot (n p_t)^{-\beta}$$

$$(s_{t+1}, a_{t+1}, r_{t+1}, s_{t+2}), \delta_{t+1}$$

$$p_{t+1} \propto |\delta_{t+1}| + \epsilon$$

$$\alpha \cdot (n p_{t+1})^{-\beta}$$

$$(s_{t+2}, a_{t+2}, r_{t+2}, s_{t+3}), \delta_{t+2}$$

$$p_{t+2} \propto |\delta_{t+2}| + \epsilon$$

$$\alpha \cdot (n p_{t+2})^{-\beta}$$

• • •

• • •

• • •

Thank you!