Dueling Network

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Advantage Function

Return

Definition: Discounted return.

- $U_t = R_t + \gamma \cdot R_{t+1} + \gamma^2 \cdot R_{t+2} + \gamma^3 \cdot R_{t+3} + \cdots$

Value Functions

Definition: Discounted return.

- $U_t = R_t + \gamma \cdot R_{t+1} + \gamma^2 \cdot R_{t+2} + \gamma^3 \cdot R_{t+3} + \cdots$

Definition: Action-value function.

- $Q_\pi(s_t, a_t) = \mathbb{E}\left[U_t | S_t = s_t, A_t = a_t\right].$

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Definition: Action-value function.

- $Q_\pi(s_t, a_t) = \mathbb{E}\left[U_t | S_t = s_t, A_t = a_t\right]$.

Definition: State-value function.

- $V_\pi(s_t) = \mathbb{E}_A\left[Q_\pi(s_t, A)\right]$

Optimal Value Functions

Definition: Optimal action-value function.

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Definition: Optimal state-value function.

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Definition: Optimal advantage function.

- $A^{\star}(s, a) = \underline{Q^{\star}(s, a)} - \underline{V^{\star}(s)}.$

Properties of Advantage Function

Theorem 1: $V^\star(s) = \max_a Q^\star(s, a)$.

- Recall the definition of the optimal advantage function:

$$\boxed{A^\star(s, a)} = \boxed{Q^\star(s, a) - V^\star(s)}.$$

- It follows that

$$\max_a A^\star(s, a) = \max_a Q^\star(s, a) - V^\star(s).$$

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$$\max_a A^\star(s, a) = \max_a Q^\star(s, a) - V^\star(s).$$

- Using Theorem 1, we get $\max_a A^\star(s, a) = 0.$

Properties of Advantage Function

Definition of advantage: $A^{\star}(s, a) = Q^{\star}(s, a) - V^{\star}(s).$

Theorem 2: $Q^{\star}(s, a) = V^{\star}(s) + A^{\star}(s, a)$

Properties of Advantage Function

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Theorem 2: $Q^{\star}(s, a) = V^{\star}(s) + A^{\star}(s, a) - \max_{a} A^{\star}(s, a)$.

$= 0$

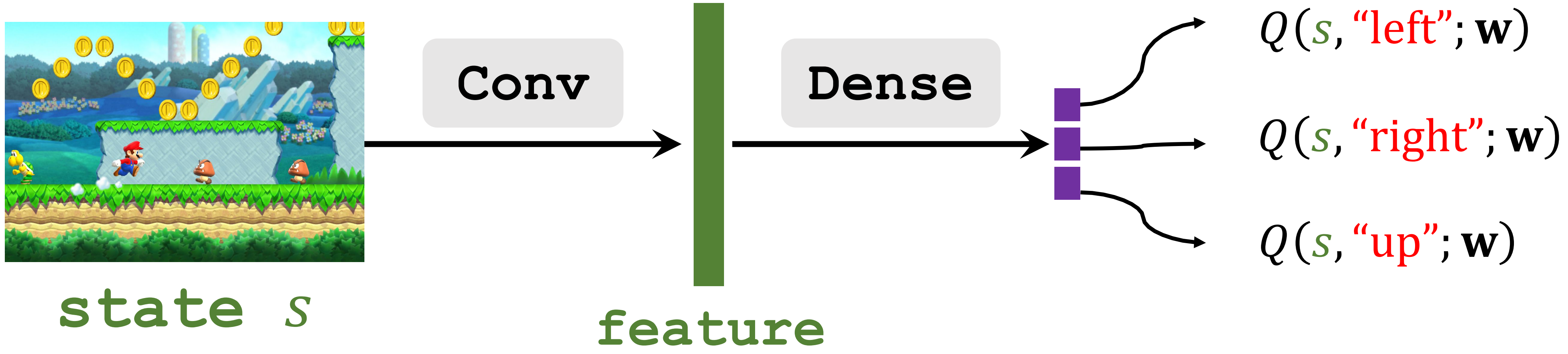
Dueling Network

Reference:

1. Wang et al. Dueling network architectures for deep reinforcement learning. In *ICML*, 2016.

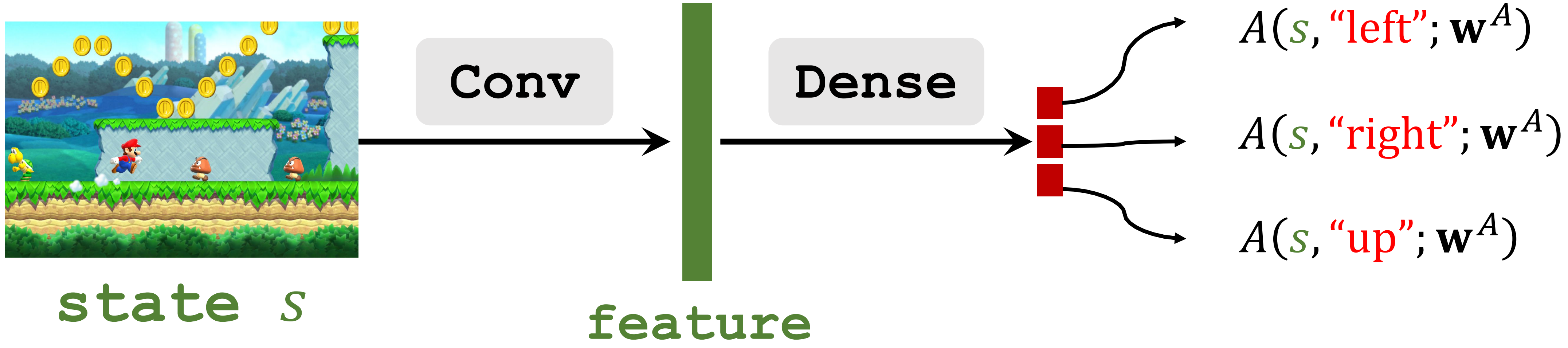
Revisiting DQN

- Approximate $Q^\star(s, a)$ by a neural network, $Q(s, a; \mathbf{w})$.



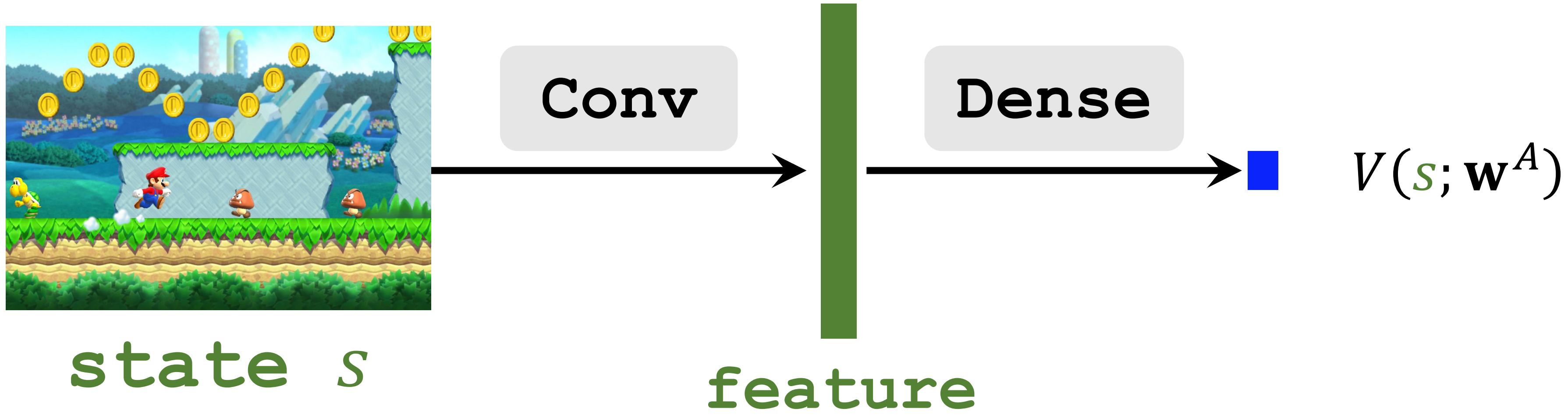
Approximating Advantage Function

- Approximate $A^{\star}(s, a)$ by a neural network, $A(s, a; \mathbf{w}^{A})$.



Approximating State-Value Function

- Approximate $V^{\star}(s)$ by a neural network, $V(s; \mathbf{w}^{V})$.



Dueling Network

Theorem 2: $Q^\star(s, a) = V^\star(s) + A^\star(s, a) - \max_{a} A^\star(s, a).$

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Dueling Network

Theorem 2: $Q^\star(s, a) = V^\star(s) + A^\star(s, a) - \max_a A^\star(s, a).$

- Approximate $V^\star(s)$ by a neural network, $V(s; \mathbf{w}^V)$.
- Approximate $A^\star(s, a)$ by a neural network, $A(s, a; \mathbf{w}^A)$.

Dueling Network

Theorem 2: $Q^\star(s, a) = V^\star(s) + A^\star(s, a) - \max_a A^\star(s, a).$

- Approximate $V^\star(s)$ by a neural network, $V(s; \mathbf{w}^V)$.
- Approximate $A^\star(s, a)$ by a neural network, $A(s, a; \mathbf{w}^A)$.
- Thus, approximate $Q^\star(s, a)$ by the dueling network:

$$Q(s, a; \mathbf{w}^A, \mathbf{w}^V) = V(s; \mathbf{w}^V) + A(s, a; \mathbf{w}^A) - \max_a A(s, a; \mathbf{w}^A).$$

Dueling Network

$$Q(s, a; \mathbf{w}) = V(s; \mathbf{w}^V) + A(s, a; \mathbf{w}^A) - \max_{a} A(s, a; \mathbf{w}^A).$$

Here, $\mathbf{w} = (\mathbf{w}^A, \mathbf{w}^V)$

Dueling Network

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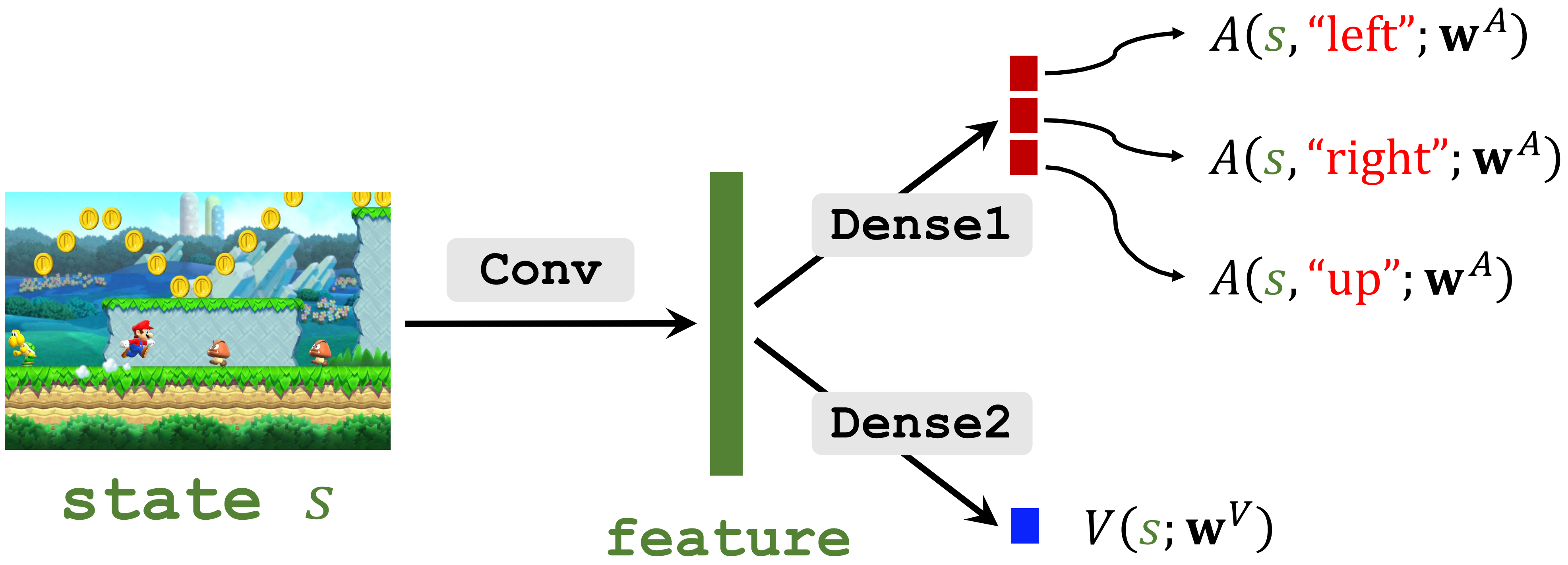
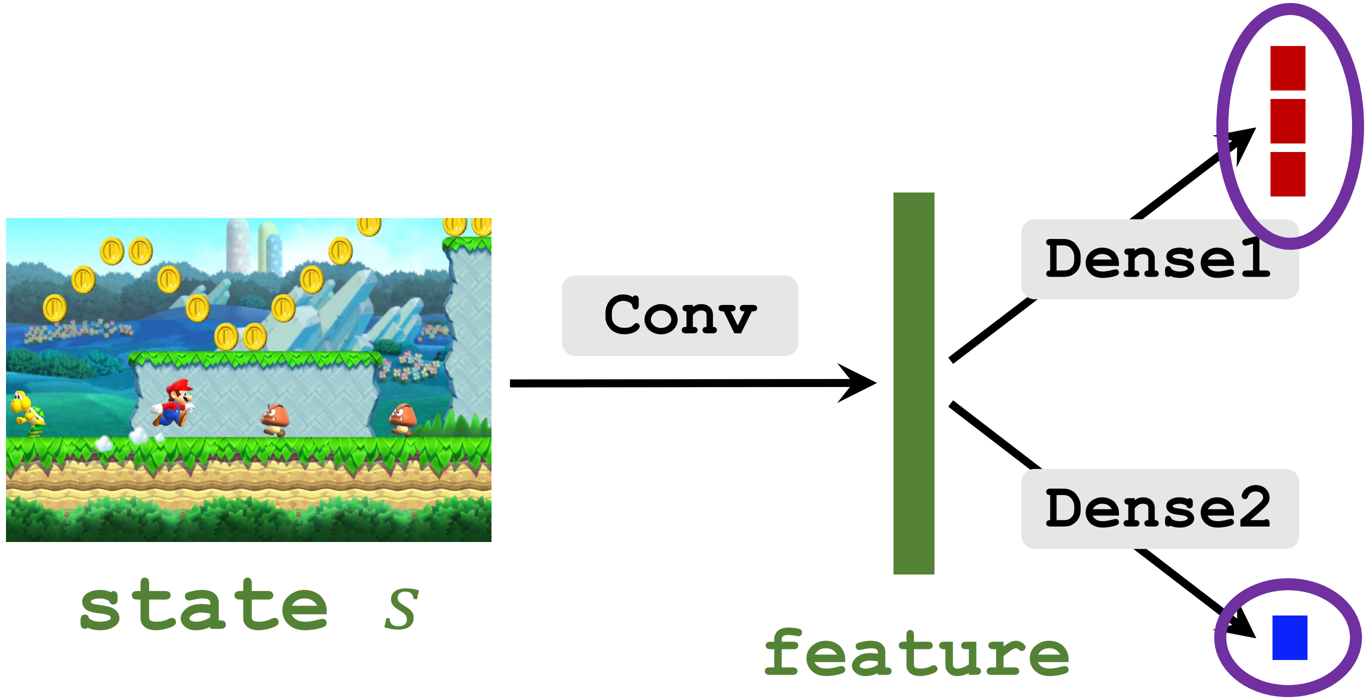


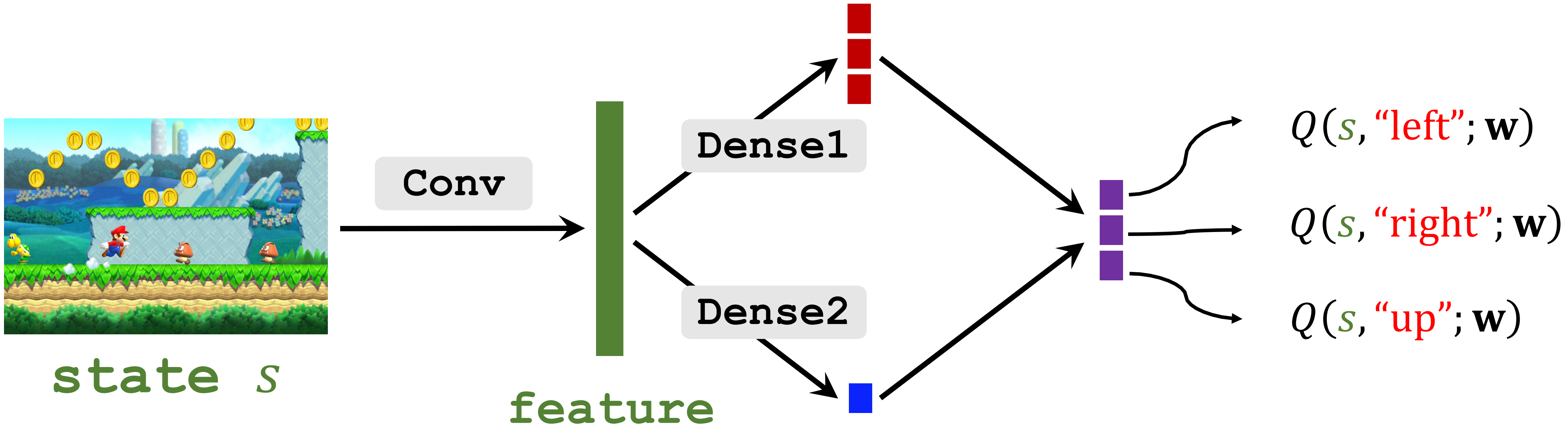
Dueling Network

$$Q(s, a; \mathbf{w}) = V(s; \mathbf{w}^V) + A(s, a; \mathbf{w}^A) - \max_{a} A(s, a; \mathbf{w}^A).$$



Dueling Network

$$Q(s, a; \mathbf{w}) = V(s; \mathbf{w}^V) + A(s, a; \mathbf{w}^A) - \max_{a} A(s, a; \mathbf{w}^A).$$



Training

- Dueling network, $Q(s, a; \mathbf{w})$, is an approximation to $Q^{\star}(s, a)$.
- Learn the parameter, $\mathbf{w} = (\mathbf{w}^{A}, \mathbf{w}^{V})$, in the same way as the other DQNs.
- Tricks can be used in the same way.
  - Prioritized experience replay.
  - Double DQN.
  - Multi-step TD target.

Overcome Non-identifiability

Problem of Non-identifiability

- **Equation 1:** $Q^{\star}(s,a) = V^{\star}(s) + A^{\star}(s,a).$
- **Equation 2:** $Q^{\star}(s,a) = V^{\star}(s) + A^{\star}(s,a) - \max_{a} A^{\star}(s,a)$

Question: Why is the zero term necessary?

Problem of Non-identifiability

- **Equation 1:** $Q^\star(s,a) = V^\star(s) + A^\star(s,a)$.
- Equation 1 has the problem of *non-identifiability*.
  - Let $V' = V^\star + 10$ and $A' = A^\star - 10$.
  - Then $Q^\star(s,a) = V^\star(s) + A^\star(s,a) = V'(s) + A'(s,a)$.
- Why is non-identifiability a problem?

Problem of Non-identifiability

- **Equation 2:** $Q^\star(s,a) = V^\star(s) + A^\star(s,a) - \boxed{\max_a A^\star(s,a)}.$
- Equation 2 does not have the problem.

Dueling Network

$$Q(s,a;\mathbf{w}) = V(s;\mathbf{w}^V) + A(s,a;\mathbf{w}^A) - \max_{a} A(s,a;\mathbf{w}^A).$$

Alternative:

$$Q(s,a;\mathbf{w}) = V(s;\mathbf{w}^V) + A(s,a;\mathbf{w}^A) - \operatorname*{mean}_{a} A(s,a;\mathbf{w}^A).$$

Thank you!