Q-Learning

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Sarsa VS Q-Learning

- Sarsa is for training action-value function, $Q_{\pi}(s, a)$.
- TD target: $y_t = r_t + \gamma \cdot Q_{\pi}(s_{t+1}, a_{t+1})$.
- We used Sarsa for updating value network (critic).

Sarsa VS Q-Learning

- Q-learning is for training the optimal action-value function, $Q^{\star}(s, a)$.
- TD target: $y_t = r_t + \gamma \cdot \max_{a} Q^{\star}(s_{t+1}, a)$.
- We used Q-learning for updating DQN.

Derive TD Target

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- We have proved that for all π,

$$Q_\pi(s_t, a_t) = \mathbb{E}\left[\boxed{R_t + \gamma \cdot Q_\pi(S_{t+1}, A_{t+1})}\right].$$

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- $Q_{\pi^{\star}}$ and $Q^{\star}$ both denote *the optimal action-value function*.

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- The action A_{t+1} is computed by

$$\boxed{A_{t+1} = \underset{a}{\operatorname{argmax}}\, Q^{\star}(S_{t+1}, a).}$$

- Thus $Q^{\star}(S_{t+1}, \underline{A_{t+1}}) = \max_{a} Q^{\star}(S_{t+1}, a)$.

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- Let (s_{t+1}, r_t) be an observation of (S_{t+1}, R_t).
- TD target: $\underline{y_t} = \underline{r_t} + \underline{\gamma \cdot \max_{a} Q^{\star}(s_{t+1}, a)}.$

Derive TD Target

Identity: $Q^{\star}(s_t, a_t) = \boxed{\mathbb{E}\left[R_t + \gamma \cdot \max_{a} Q^{\star}(S_{t+1}, a)\right]}.$

$\approx y_t$

- Let (s_{t+1}, r_t) be an observation of (S_{t+1}, R_t).
- TD target: $y_t = r_t + \gamma \cdot \max_{a} Q^{\star}(s_{t+1}, a)$.

Q-Learning: Tabular Version

Q-Learning (tabular version)

- Observe (s_t, a_t, r_t, s_{t+1}).
- TD target: $y_t = r_t + \gamma \cdot \max_{a} Q^{\star}(s_{t+1}, a)$.

Q-Learning (tabular version)

- Observe (s_t, a_t, r_t, s_{t+1}).
- TD target: $y_t = r_t + \gamma \cdot \boxed{\max_a Q^\star(s_{t+1}, a)}$.

	Action a_1	Action a_2	Action a_3	Action a_4	$\cdots$
State s_1					
State s_2					
State s_3					
$\vdots$					

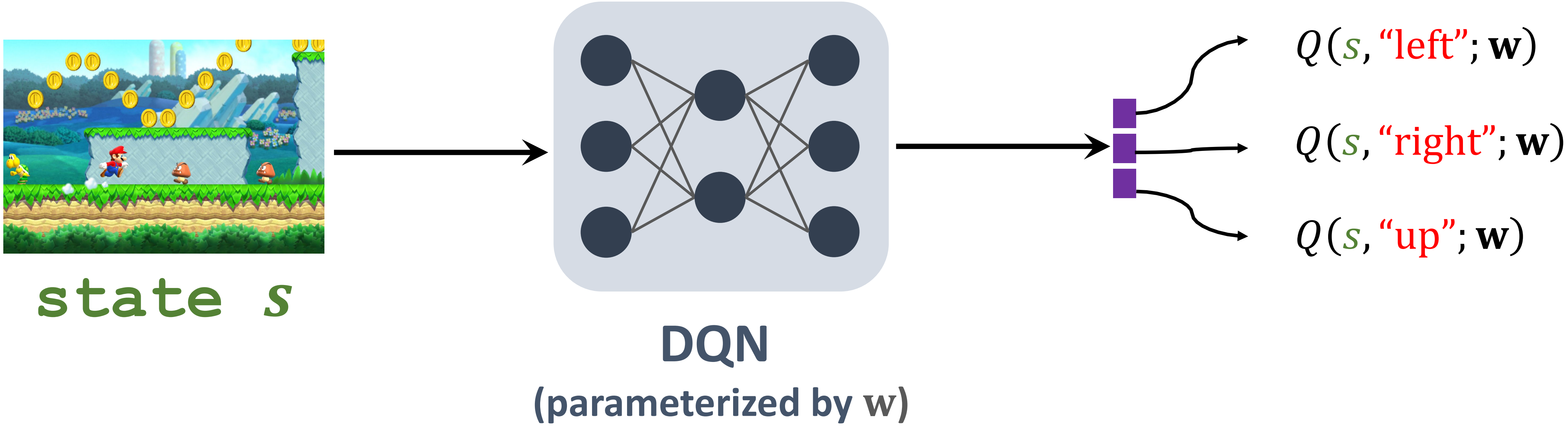
Q-Learning (tabular version)

- Observe (s_t, a_t, r_t, s_{t+1}).
- TD target: $y_t = r_t + \gamma \cdot \max_{a} Q^{\star}(s_{t+1}, a)$.
- TD error: $\delta_t = Q^{\star}(s_t, a_t) - y_t$.
- Update: $Q^{\star}(s_t, a_t) \leftarrow \underbrace{Q^{\star}(s_t, a_t) - \alpha \cdot \delta_t}_{\text{make } Q^{\star}(s_t, a_t) \text{ closer to } y_t}$.

Q-Learning: DQN Version

DQN Version

- Approximate $Q^{\star}(s, a)$ by DQN, $Q(s, a|\mathbf{w})$.



DQN Version

- Approximate $Q^\star(s, a)$ by DQN, $Q(s, a|\mathbf{w})$.
- DQN controls the agent by: $a_t = \underset{a}{\operatorname{argmax}}\, Q(s_t, a|\mathbf{w})$
- We seek to learn the parameter, $\mathbf{w}$.

Q-Learning (DQN Version)

- Observe (s_t, a_t, r_t, s_{t+1}).
- TD target: $y_t = r_t + \gamma \cdot \max_a Q(s_{t+1}, a \mid \mathbf{w})$.
- TD error: $\delta_t = Q(s_t, a_t \mid \mathbf{w}) - y_t$.
- Update: $\mathbf{w} \leftarrow \mathbf{w} - \alpha \cdot \delta_t \cdot \frac{\partial\, q(s_t, a_t \mid \mathbf{w})}{\partial\, \mathbf{w}}$.

Summary

- **Goal:** Learn the optimal action-value function $Q^\star$.
- **Tabular version** (directly learn $Q^\star$).
  - There are finite states and actions.
  - Draw a table, and update the table by Q-learning.
- **DQN version** (function approximation).
  - Approximate $Q^\star$ by the DQN, $Q(s, a|\mathbf{w})$.
  - Update the parameter, $\mathbf{w}$, by Q-learning.

Thank you!