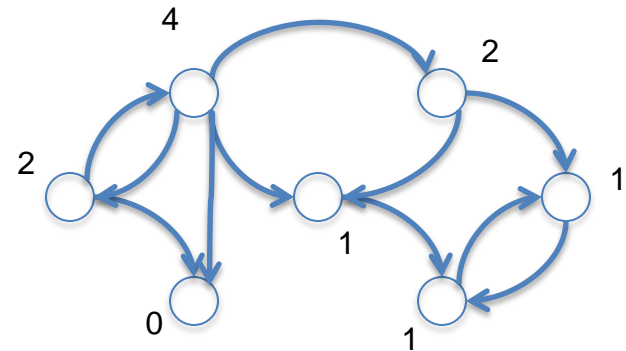
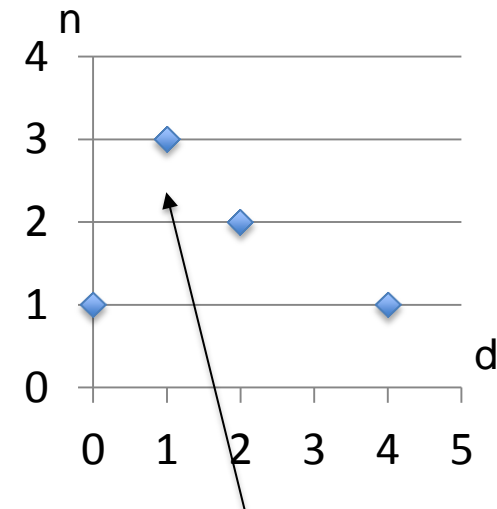


Structural Task: the Histogram of a Graph

- **Outdegree** of a vertex = number of outgoing edges
- For each integer d , let $n(d)$ = number of vertices with outdegree d
- The outdegree histogram of a graph = the **scatterplot** $(d, n(d))$

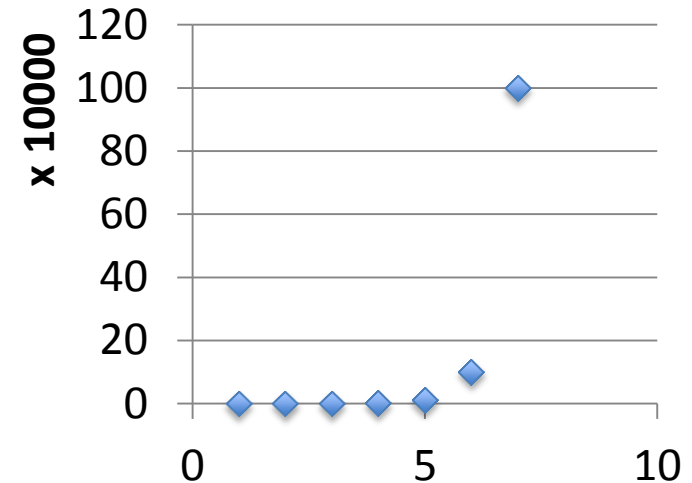
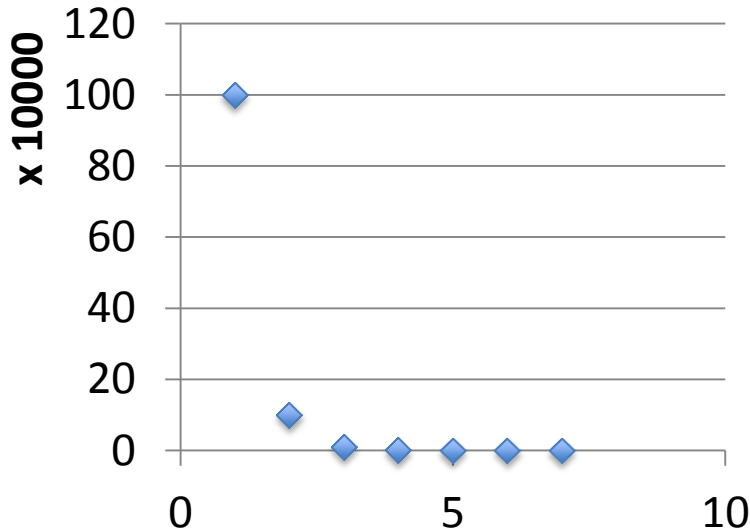


d	$n(d)$
0	1
1	3
2	2
3	0
4	1

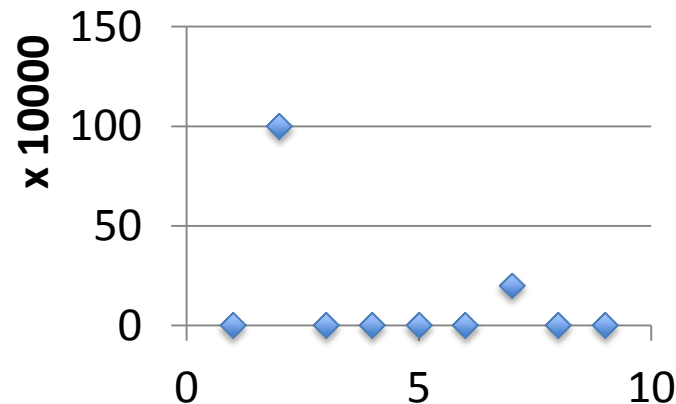


Outdegree 1 is
seen at 3 nodes

Histograms Tell Us Something About the Graph

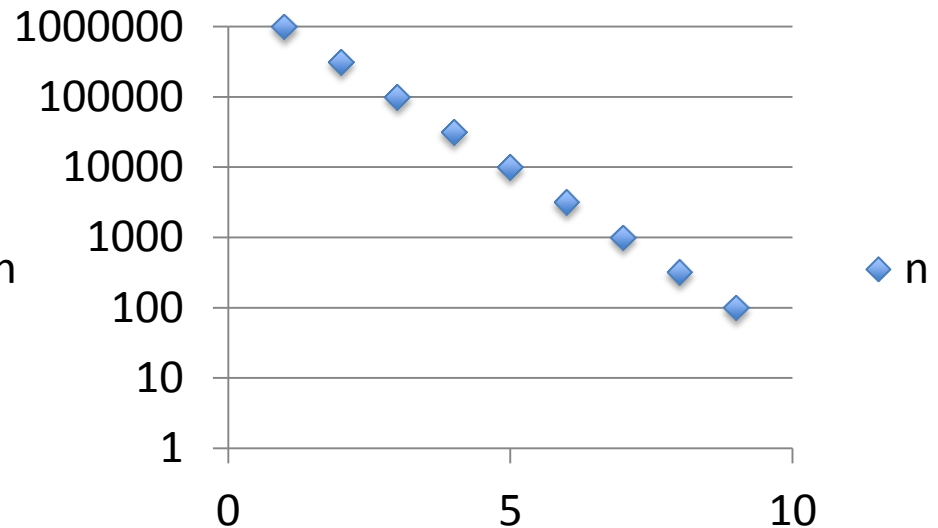
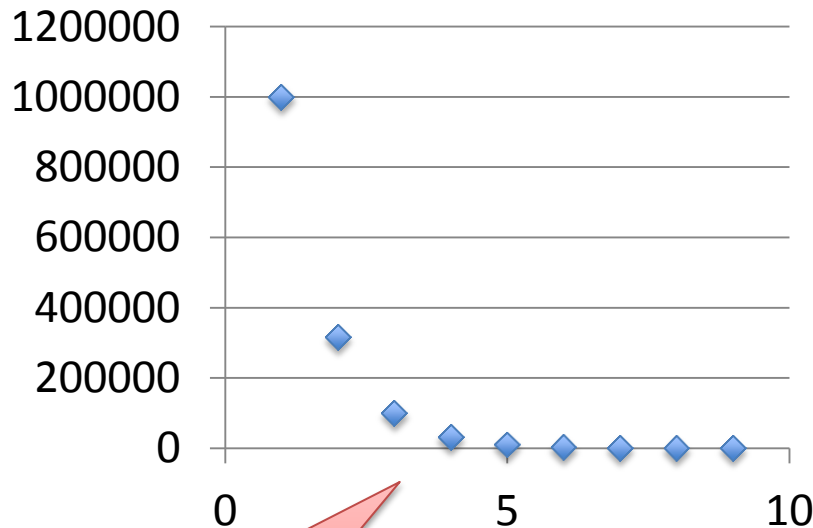


What can you say about these graphs?



Exponential Distribution

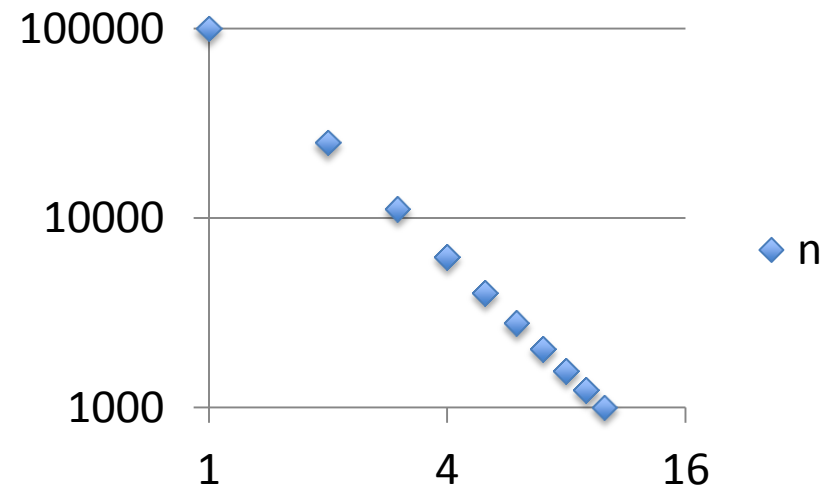
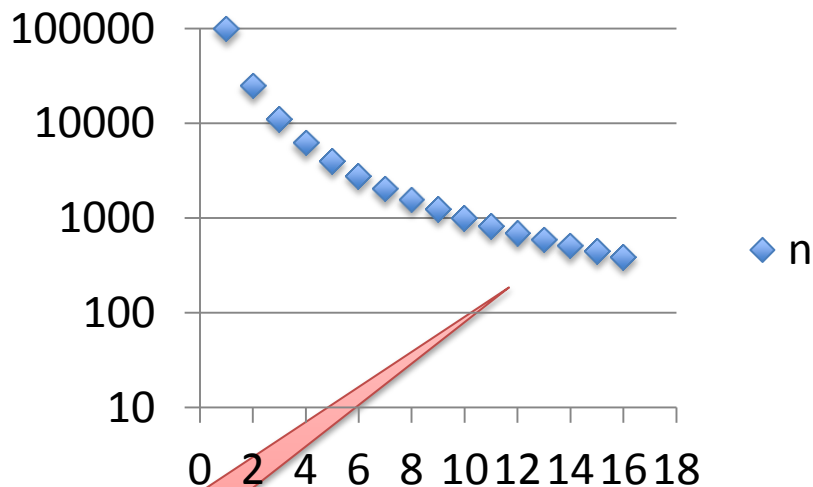
- $n(d) \simeq c \left(\frac{1}{2}\right)^d$ (generally, cx^d , for some $x < 1$)
- A *random graph* has exponential distribution
- Best seen when n is on a log scale



Quickly vanishing

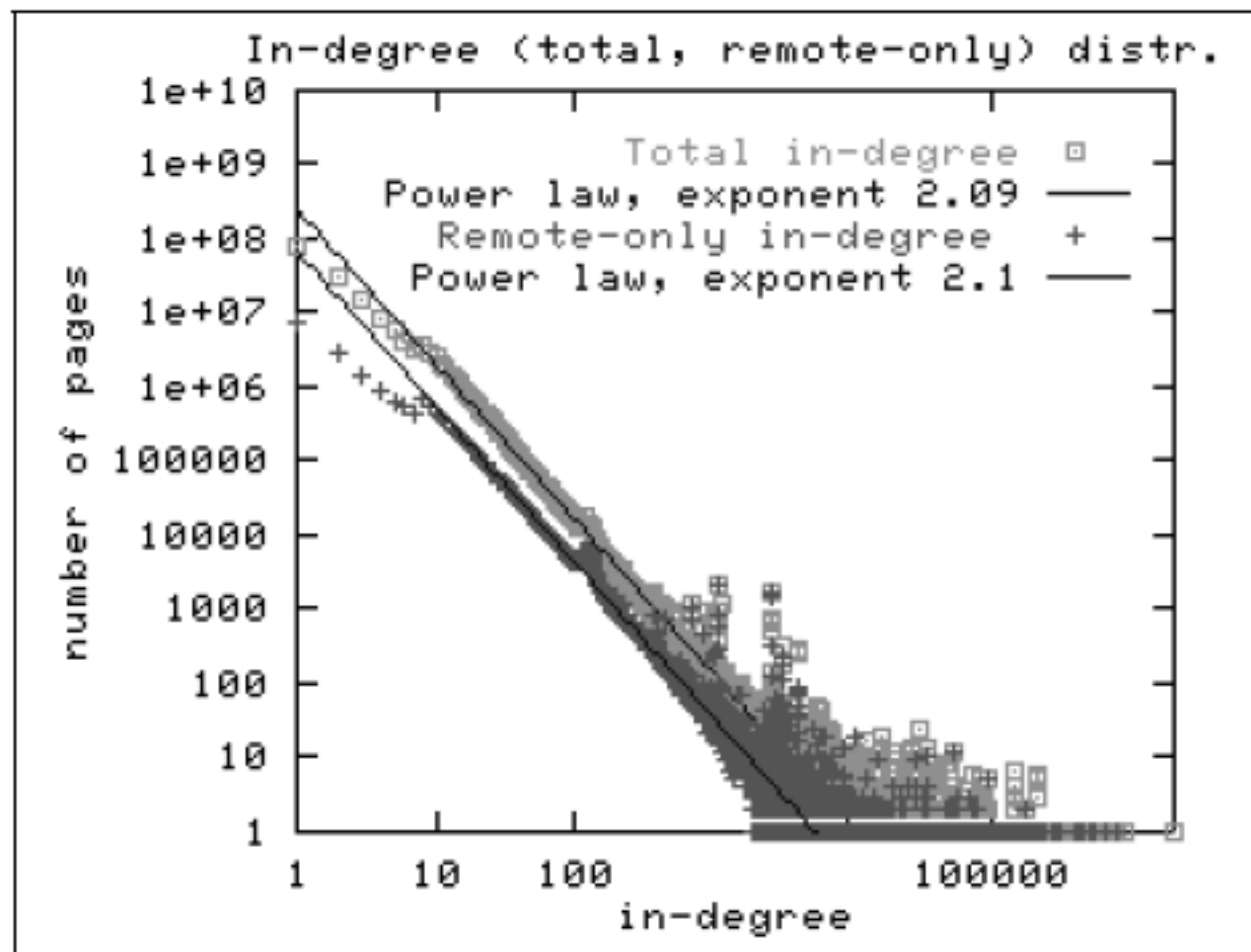
Zipf Distribution

- $n(d) \simeq \frac{1}{d^x}$ for some value $x > 0$
- Human-generated data has Zipf distribution: letters in alphabet, words in vocabulary, etc.
- Best seen in a log-log scale



Long tail

A Histogram of the Web



Late 1990's
200M Webpages

Exponential ?

Zipf ?

Figure 2: In-degree distribution.

The Bowtie Structure of the Web

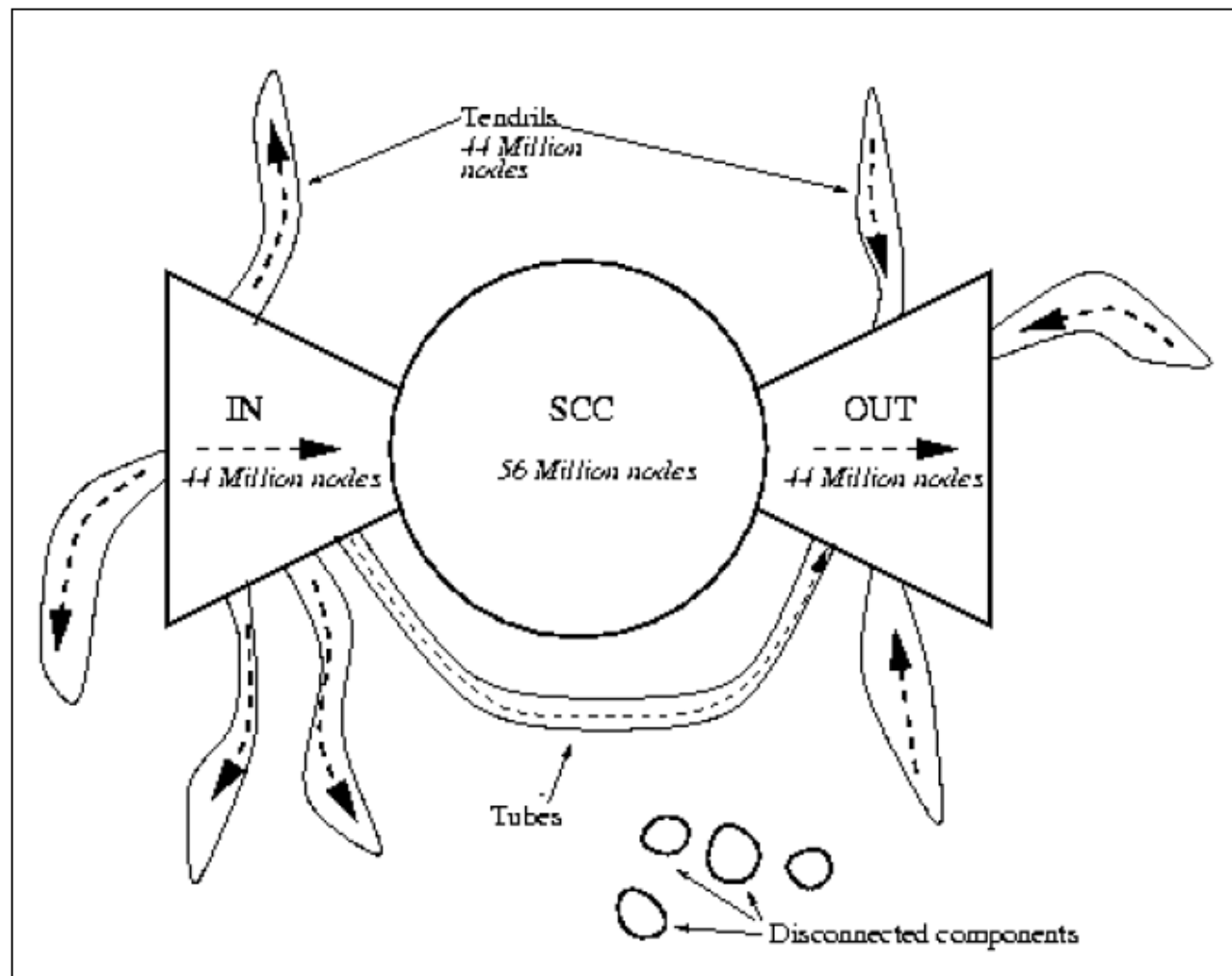


Figure 4: The web as a bowtie. *SCC* is a giant strongly connected component. *IN* consists of pages with paths to *SCC*, but no path from *SCC*. *OUT* consists of pages with paths from *SCC*, but no path to *SCC*. *TENDRILS* consists of pages that cannot surf to *SCC*, and which cannot be reached by surfing from *SCC*.