

Reason 3: Multiple Hypothesis Testing

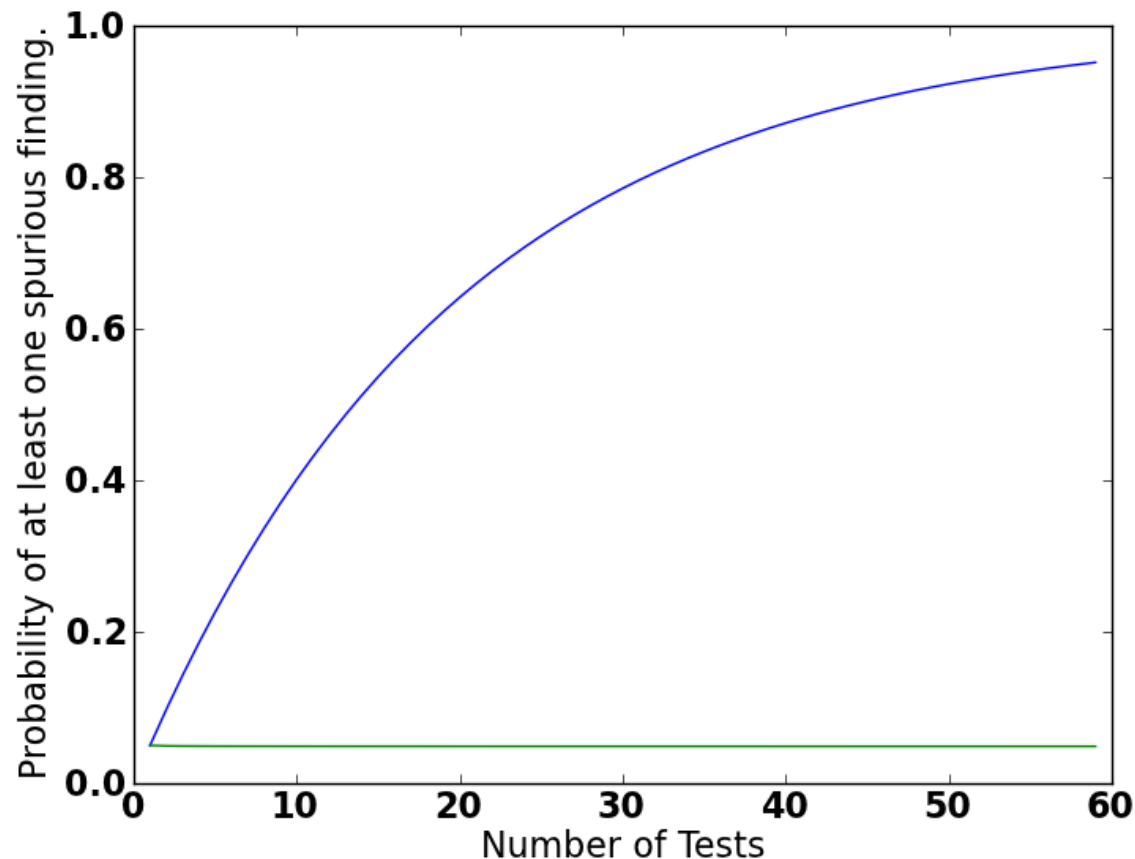
- If you perform experiments over and over, you're bound to find something
- This is a bit different than the publication bias problem: Same sample, different hypotheses
- Significance level must be adjusted down when performing multiple hypothesis tests

$P(\text{detecting an effect when there is none}) = \alpha = 0.05$

$P(\text{not detecting an effect when there is none}) = 1 - \alpha$

$P(\text{not detecting an effect when there is none on every experiment}) = (1 - \alpha)^k$

$P(\text{detecting an effect when there is none on at least one experiment}) = 1 - (1 - \alpha)^k$



$\alpha = 0.05$

“Familywise Error Rate”

Familywise Error Rate Corrections

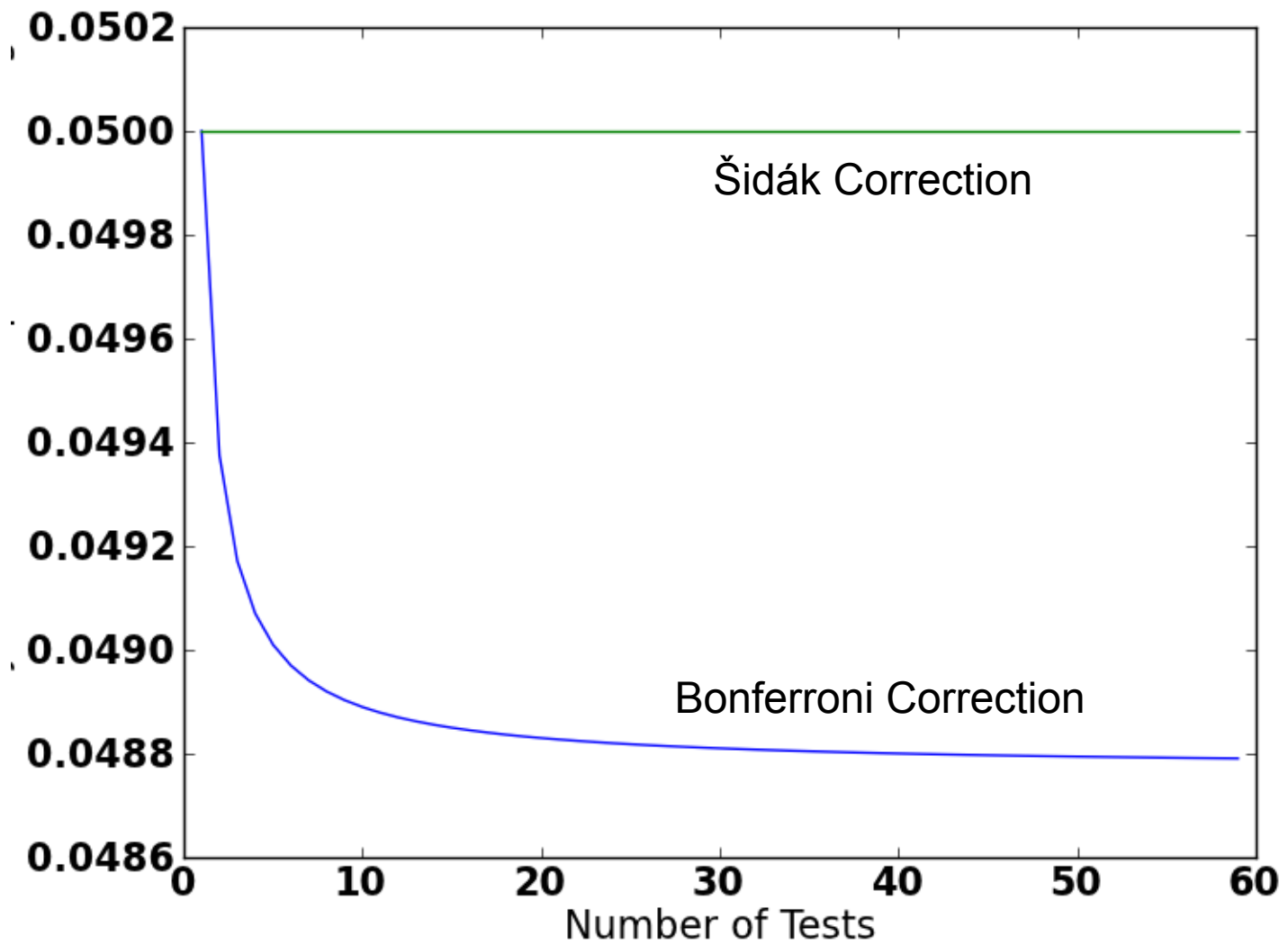
- Bonferroni Correction
 - Just divide by the number of hypotheses

$$\alpha_c = \frac{\alpha}{k}$$

- Šidák Correction
 - Asserts independence

$$\alpha = 1 - (1 - \alpha_c)^k$$

$$\alpha_c = 1 - (1 - \alpha)^{\frac{1}{k}}$$



False Discovery Rate

| | Reject H0 | Do Not Reject H0 | Total |
|-------------|-----------|------------------|--------|
| H0 is true | FD | TN | T |
| H0 is false | TD | FN | F |
| Total | D | N | $TFDN$ |

T/F = True/False

D/N = Discovery/Nondiscovery

$$Q = FDR = \frac{FD}{D}$$

FDR (2)

- Bonferroni correction and other FWER corrections tend to wipe out evidence of the most interesting effects; they suffer from low power.
- FDR control offers a way to increase power while maintaining a bound on the ratio of wrong conclusions
- Intuition:
 - 4 false discoveries out of 10 rejected null hypothesesis a more serious error than
 - 20 false discoveries out of 100 rejected null hypotheses.

adapted from a slide by Christopher Genovese

Benjamini-Hochberg Procedure

- Compute the p-value of m hypotheses
- Order them in increasing order of p-value
 - That is, most likely hypotheses are first

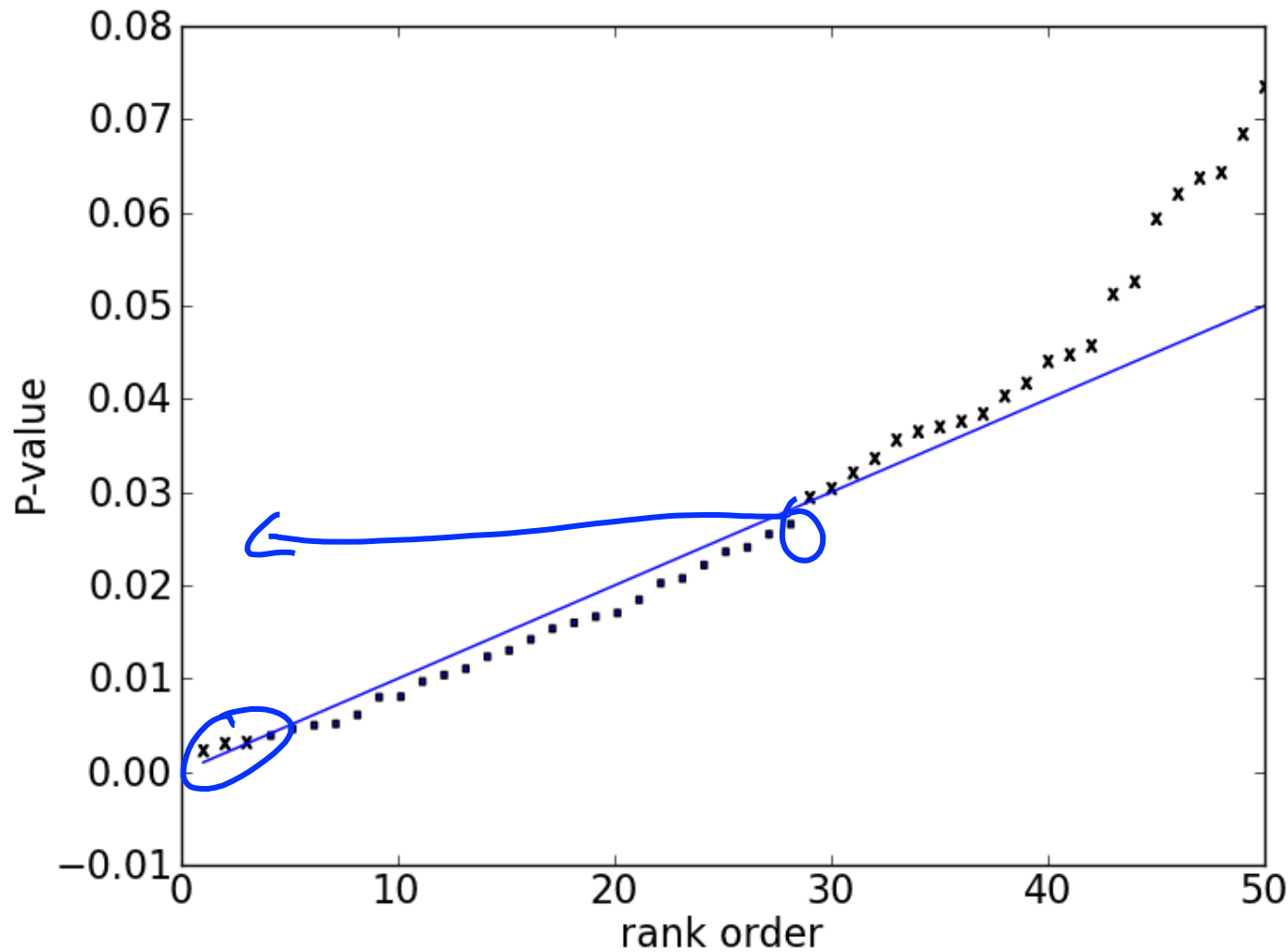
$$P_i \leq \frac{i}{m} \alpha$$

rank order
of hypotheses

| i | $\frac{i}{50} \alpha$ 0.05 |
|-----|----------------------------|
| 1 | 0.001 |
| 2 | 0.002 |
| 3 | 0.003 |
| ... | |
| 20 | 0.020 |
| ... | |

$$\underline{FDR} \leq \frac{T}{m} \alpha$$

Benjamini-Hochberg Procedure



$m = 50$
 $\alpha = 0.05$