

* Fourier Series

Let the function $f(x)$ be periodic function with period $\boxed{2\pi}$

$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

where

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin(nx) dx$$

بنطاق ال لو كانت $f(x)$ series فتره 2π

a_0, a_n, b_n are called Fourier coefficients

* Even and Odd Functions

1) If $f(x)$ is odd function $(f(-x) = -f(x))$

then $a_0 = 0, a_n = 0$ and $f(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$



2] If $f(x)$ is even function ($f(-x) = f(x)$)
 then $b_n = 0$ and $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x)$

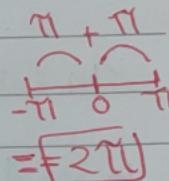
معلمات

$$* \sin(n\pi) = 0$$

$$* \cos(n\pi) = \cos(n\pi) = (-1)^n$$

ex

$$f(x) = \begin{cases} -1 & \text{when } -\pi \leq x \leq 0 \\ 1 & \text{when } 0 \leq x \leq \pi \end{cases}$$



find fourier series

الشونجية اسفل على اكمل

$$f(-x) = \begin{cases} -1 & -\pi \leq -x \leq 0 \\ 1 & 0 \leq -x \leq \pi \end{cases}$$

والحق

بالضرب ب -1 فوق وتحت (عند الضرب سايب الاكتوبر تتعكس)

$$f(x) = \begin{cases} +1 & \pi \geq x \geq 0 \\ -1 & 0 \geq x \geq -\pi \end{cases}$$

$$= \begin{cases} 1 & 0 \leq x \leq \pi \\ -1 & -\pi \leq x \leq 0 \end{cases}$$

$$f(-x) = f(x)$$

odd ns!



KAYAN

Date _____

Su Mo Tu We Th Fr Sa

$$a_0 = 0, a_n = 0 \Rightarrow f(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$$

bn تجريب

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^0 (-1) \sin(nx) dx + \int_0^{\pi} (1) \sin(nx) dx \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{n} \cos nx \Big|_0^{\pi} + \frac{-1}{n} \cos nx \Big|_0^{\pi} \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{n} \cos 0 - \frac{1}{n} \cos(-\pi n) - \frac{1}{n} \cos \pi n \right]$$

$$+ \frac{1}{n} \cos 0 \right] = \frac{1}{\pi} \left[\frac{2}{n} - 2(-1)^n \right]$$

$$= \frac{2}{n\pi} [1 - (-1)^n]$$