* Fourier series

Let the function $f(x)$ be periodic function with period $\boxed{2\pi}$

$$f(x) = \frac{1}{2}\boxed{a_0} + \sum_{n=1}^{\infty} \left(\boxed{a_n}\cos(nx) + \boxed{b_n}\sin(nx)\right)$$

where

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\sin(nx)\,dx$$

* بنطبق ال series دى لو كانت $f(x)$ ليها فترة 2π

a_0, a_n, b_n are called fourier cofficients

* Even and odd functions

1) If $f(x)$ is odd function $(f(-x) = -f(x))$, then $a_0 = 0$, $a_n = 0$ and $f(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$

2] If $f(x)$ is $\boxed{\text{even function}}$ $(f(-x) = f(x))$, then $b_n = 0$ and $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$

ملاحظات *

* $\sin(\pi n) = 0$
* $\cos(n\pi) = \cos(-n\pi) = (-1)^n$

ex

$$f(x) = \begin{cases} -1 & \text{when } -\pi \leq x \leq 0 \\ 1 & \text{when } 0 \leq x \leq \pi \end{cases}$$

$\pi + \pi$

$-\pi \quad 0 \quad \pi$

$= \boxed{2\pi}$

find fourier series

اشوف هي even ولا odd علشان اسهل عليا الحل

$$f(-x) = \begin{cases} -1 & -\pi \leq -x \leq 0 \\ 1 & 0 \leq -x \leq \pi \end{cases}$$

والعكس

بالضرب ب -1 فوق وتحت (عند الضرب في سالب الأكبر بيتعكس)

$$f(x) = \begin{cases} +1 & \pi \geq x \geq 0 \\ -1 & 0 \geq x \geq -\pi \end{cases}$$

$$= \begin{cases} 1 & 0 \leq x \leq \pi \\ -1 & -\pi \leq x \leq 0 \end{cases}$$

$f(-x) = f(x)$

odd إذن

KAYAN

Date ______ Su Mo Tu We Th Fr Sa

$a_0 = 0$, $a_n = 0$ و $f(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$ يبقى

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$$

عايزين b_n

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\, dx$$

$$= \cancel{\frac{1}{\pi}\int_{-\pi}^{\pi}} = \frac{1}{\pi}\left[\int_{-\pi}^{0} (-1) \sin(nx)\, dx + \int_{0}^{\pi} (1) \sin(nx)\, dx \right]$$

$$= \frac{1}{\pi}\left[\left[\frac{1}{n} \cos nx\right]_{-\pi}^{0} + \left[\frac{-1}{n} \cos nx\right]_{0}^{\pi} \right]$$

$$\frac{1}{\pi}\Big[\frac{1}{n} \overbrace{\cos 0}^{1} - \frac{1}{n} \overbrace{\cos(-\pi n)}^{(-1)^n} - \frac{1}{n} \overbrace{\cos n\pi}^{(-1)^n}$$

$$+ \frac{1}{n} \overbrace{\cos 0}^{1} \Big] = \frac{1}{\pi}\left[\frac{2}{n} - \frac{2}{n}(-1)^n \right]$$

$$= \frac{2}{n\pi}\left[1 - (-1)^n\right]$$