Homework 4

CSCI 4511W Spring 2018

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1. Two sentences in propositional calculus can be shown to be equivalent by proving that one entails the other and viceversa. Show that $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$ by doing following steps:

   (a) Prove by contradiction using resolution
   $\neg(P \wedge Q) \models \neg P \vee \neg Q$
   $\rightarrow$
   Put $\neg(P \wedge Q)$ in the Knowledge Base. And than, add the negation of the goal $\neg(\neg P \vee \neg Q)$, change them to CNF, and prove a contradiction.
   $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$ [1]
   $\neg(\neg P \vee \neg Q) \equiv P \wedge Q$ [2]
   [2] produces P [2a] and Q [2b]
   Resolve [1] with [2a] and results in $\neg Q$ [3]
   Resolve [2b] with [3] and prove a contradiction.

   (b) Prove by contradiction using resolution
   $\neg P \vee \neg Q \models \neg(P \wedge Q)$
   $\rightarrow$
   Put $\neg P \vee \neg Q$ in the Knowledge Base. And than, add the negation of the goal $\neg(\neg(P \wedge Q))$, change them to CNF, and prove a contradiction.
   $\neg P \vee \neg Q$ [1]
   $\neg(\neg(P \wedge Q)) \equiv P \wedge Q$ [2]
   [2] produces P [2a] and Q [2b]
   Resolve [1] with [2a], resulting in $\neg Q$ [3]
   Resolve [2v] with [3] and prove a contradiction.

2. Write each of the following sentences, in predicate calculus. Use the same predicates across the sentences. Use predicates such as House(x), Pet(x), In(x,y), Live(x,y), Big(x), and Cost(x,y).

   (a) All houses have at least one bathroom.
   $\rightarrow$ $\boxed{\forall x\ House(x) \Rightarrow \exists y\ Bathroom(y) \wedge In(x,y)}$

   (b) There is a house in Minneapolis which costs more than any other house.
   $\rightarrow$ $\boxed{\exists y\ House(y) \wedge In(Minneapolis,y) \wedge [\forall x\ House(x) \wedge \neg(x=y) \Rightarrow CostMore(x,y)]}$

(c) There is only one house in Minneapolis which is in the historical register.

→ $\boxed{\exists x\ House(x) \wedge In(Minneapolis,x) \wedge In(Historical\ Register,x) \wedge [\forall y\ House(y) \wedge In(Minneapolis,y) \wedge In(Historical\ Register,y) \Rightarrow x = y]}$

(d) There is a pet in each house.

→ $\boxed{\exists x\ \exists y\ Pet(x) \wedge House(y) \wedge Live(x,y)}$

(e) Rich people have big houses.

→ $\boxed{\forall x\ \forall y\ Person(x) \wedge Rich(x) \Rightarrow House(y) \wedge Own(x,y)}$

(f) All big houses are expensive

→ $\boxed{\forall x\ House(x) \wedge Big(x) \Rightarrow Expensive(x)}$

(g) All expensive houses are big

→ $\boxed{\forall x\ House(x) \wedge Expensive(x) \Rightarrow Big(x)}$

(h) A house is expensive if it is big

→ $\boxed{\forall x\ House(x) \wedge Big(x) \Rightarrow Expensive(x)}$

(i) Any small house costs less than any big house

→ $\boxed{\forall x\ \forall y\ House(x) \wedge Small(x) \Rightarrow House(y) \wedge Big(y) \wedge CostLess(x,y)}$

(j) There is only one house with a garden

→ $\boxed{\exists x\ House(x) \wedge In(x,Garden) \wedge \forall y[House(y) \wedge In(y,Garden) \Rightarrow (x{=}y)]}$

3. Transform to CNF the following knowledge base, where upper case arguments are constants, lower case arguments are variables. Do not forget to standardize variables apart.

   (a) $\forall$ G(x) ⇒ H(x) ≡ ¬G(x) ∨ H(x)

   (b) $\forall$ H(x) ⇒ I(x) ≡ ¬H(x) ∨ I(x)

   (c) $\forall$ H(x) ⇒ J(x,D) ≡ ¬H(x) ∨ J(x,D)

   (d) $\forall$ I(x) ⇒ J(C,x) ≡ ¬ I(x) ∨ J(C,x)

   (e) $\forall$ I(x) ⇒ ¬J(x,y) ≡ ¬I(x) ∨ ¬J(x,y)

   Prove by resolution with refutation "¬G(B)".

   →

   G(B), ¬G(x) ∨ H(x) (x/B)

   H(B)

   H(B), ¬H(B) ∨ J(x,D) (x/B)

   J(B,D)

   H(B), ¬H(x) ∨ I(x) (x/B)

   I(B)

   I(B), ¬ I(x) ∨ ¬J(C,x)

   ¬J(B,y)

   J(B,D), ¬J(B,y) (y/D)

   Empty Clause

   ∴ ¬G(B) is proven by using resolution with refutaion

4. You are give the following knowledge base:

   (a) No one is both a vegetarian and a butcher.

   (b) Everyone, except butchers, like vegetarians.

   (c) John is a vegetarian.

   Write the sentences in predicate calculus, transform to CNF, and use resolution to answer the question "Is there someone that John likes?"

   $\rightarrow$

   Predicate Calculus

   ∀x ¬(Vegetarian(x) ∧ Butchers(x))

   ∀x ∀y Vegetarian(y) ⇒ (Butchers(x) ∧ ¬Like(x,y)) ∨ (¬B(x) ∧ Like(x,y))

   Vegetarian(John)

   Goal: ∃x Like(John,x)

   CNF

   ∀x ¬(Vegetarian(x) ∧ Butchers(x))

   (¬Vegetarian(y) ∨ Butchers(x) ∨ ¬Butchers(x)) ∧ (¬Vegetarian(y) ∨ Butchers(x) ∨ Like(x,y)) ∧ (¬Vegetarian(y) ∨ ¬Like(x,y) ∨ ¬Butchers(x)) ∧ (¬Vegetarian(y) ∨ ¬Like(x,y) ∨Like(x,y))

   Vegetarian(John)

   Clauses

   ¬Vegetarian(x) ∨ Butchers(x)

   ¬Vegetarian(y) ∨ Butchers(x) ∨ ¬Butchers(x)

   ¬Vegetarian(y) ∨ Butchers(x) ∨ Like(x,y)

   ¬Vegetarian(y) ∨ ¬Like(x,y) ∨ ¬Butchers(x)

   ¬Vegetarian(y) ∨ ¬Like(x,y) ∨ Like(x,y)

   V(John)

   ¬Vegetarian(x) ∨ ¬Butchers(x), V(John) x/John

   ¬Butchers(John)

   ¬Butchers(John), ¬Vegetarian(y) ∨ Butchers(x) ∨ Like(x,y) x/John

   ¬Vegetarian(y) ∨ **Like(John,y)**

   ¬Vegetarian(y) ∨ Like(John,y), Vegetarian(John) y/John

   Like(John,John)

   ∴ By using resolution, the final answer was $Like(John, John)$, which means John likes himself and it also means that there is someone, which is John himself, that John like.

5. Programming questions

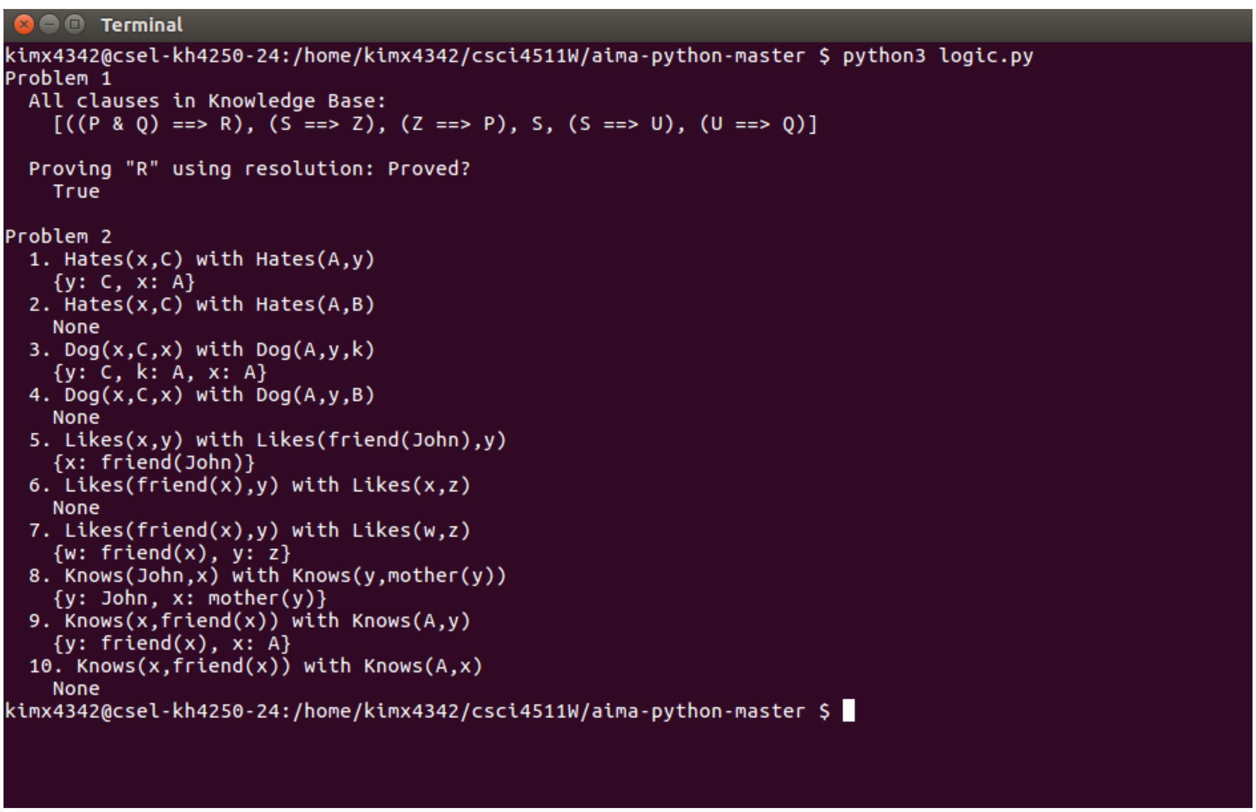
(a) Create a propositional data base with the following clauses:

i. $(P \wedge Q) \Rightarrow R$
ii. $S \Rightarrow Z$
iii. $Z \Rightarrow P$
iv. S
v. $S \Rightarrow U$
vi. $U \Rightarrow Q$

Prove "R" using resolution.
Create a definite clause knowledge base. Add to it the same clauses as before (they are all Horn clauses – disjunction of literals with at most one positive literal). Then use forward chaining to prove "R".

(b) Unify the following expressions:

i. Hates(x,C) with Hates(A,y)
ii. Hates(x,C) with Hates(A,B)
iii. Dog(x,C,x) with Dog(A,y,k)
iv. Dog(x,C,x) with Dog(A,y,B)
v. Likes(x,y) with Likes(friend(John),y)
vi. Likes(friend(x),y) with Likes(x,z)
vii. Likes(friend(x),y) with Likes(w,z)
viii. Knows(John,x) with Knows(y,mother(y))
ix. Knows(x,friend(x)) with Knows(A,y)
x. Knows(x,friend(x)) with Knows(A,x)



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Terminal
kimx4342@csel-kh4250-24:/home/kimx4342/csci4511W/aima-python-master $ python3 logic.py
Problem 1
  All clauses in Knowledge Base:
    [((P & Q) ==> R), (S ==> Z), (Z ==> P), S, (S ==> U), (U ==> Q)]

  Proving "R" using resolution: Proved?
    True

Problem 2
  1. Hates(x,C) with Hates(A,y)
    {y: C, x: A}
  2. Hates(x,C) with Hates(A,B)
    None
  3. Dog(x,C,x) with Dog(A,y,k)
    {y: C, k: A, x: A}
  4. Dog(x,C,x) with Dog(A,y,B)
    None
  5. Likes(x,y) with Likes(friend(John),y)
    {x: friend(John)}
  6. Likes(friend(x),y) with Likes(x,z)
    None
  7. Likes(friend(x),y) with Likes(w,z)
    {w: friend(x), y: z}
  8. Knows(John,x) with Knows(y,mother(y))
    {y: John, x: mother(y)}
  9. Knows(x,friend(x)) with Knows(A,y)
    {y: friend(x), x: A}
  10. Knows(x,friend(x)) with Knows(A,x)
    None
kimx4342@csel-kh4250-24:/home/kimx4342/csci4511W/aima-python-master $
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