Lecture 2

(Page 1) 证明欧几里得算法的时间复杂度为 O(loga):
不妨设 $r_k = r_{k+1}q_{k+1} + r_{k+2}$, 则

$$\frac{r_k}{r_{k+2}} = q_{k+2}q_{k+1} + q_{k+2} \cdot \frac{r_{k+2}}{r_{k+1}} + \frac{r_{k+3}}{r_{k+2}} \cdot q_{k+2} + \frac{r_{k+3}}{r_{k+1}}$$

有因为有 $r_{k+3} = r_{k+1} - r_{k+2}q_{k+2}$, 带入可得, 原式

$$= q_{k+2}q_{k+1} + q_{k+2} \cdot \frac{r_{k+2}}{r_{k+1}} + \frac{r_{k+1} - r_{k+2}q_{k+2}}{r_{k+2}} \cdot q_{k+2} + \frac{r_{k+1} - r_{k+2}q_{k+2}}{r_{k+1}}$$

$$= q_{k+2}(q_{k+1} + r_{k+1} - q_{k+2}) + 1 \geq 2$$

其中, 最后一步用到了 $r_{k+1} = r_{k+2}q_{k+2} + r_{k+3} \geq q_{k+2}$. 本质上来讲, 对于欧几里得算法更精确的时间复杂度的估计, 实际上就是把连续的两步欧几里得过程看成是一步.

(Page 4) 从欧拉的证明中推导出 $\sum_{p\ prime} \frac{1}{p}$ 收敛
考虑如下函数

$$\sum(n) = \prod_{k=1}^{n}(\sum_{i=0}^{\infty} \frac{1}{p_k^i})$$

由于 2 是最小的素数, 故有

$$\sum(n) < 1 + \sum_{i=1}^{n} \frac{1}{p_i} + \frac{1}{2}\sum_{i=1}^{n} \frac{1}{p_i} + ... + \frac{1}{2^n}\sum_{i=1}^{n} \frac{1}{p_i} + ...$$

$$= 1 + 2\sum_{i=1}^{n} \frac{1}{p_i}$$

令 $n \to \infty$, 即证.