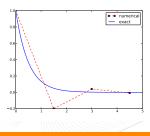
On Schemes for Exponential Decay

Hans Petter Langtangen^{1,2}

Center for Biomedical Computing, Simula Research Laboratory¹

Department of Informatics, University of Oslo²

Jan 8, 2014



Goal

The primary goal of this demo talk is to demonstrate how to write talks with Doconce and get them rendered in numerous HTML formats.

Layout.

This version utilizes beamer slides with the theme simula.

Mathematical problem

$$u'(t) = -au(t), \tag{1}$$

$$u(0) = I, (2)$$

- $t \in (0, T]$
- a, I, and T are prescribed parameters
- ightharpoonup u(t) is the unknown function



Numerical solution method

- ▶ Mesh in time: $0 = t_0 < t_1 \cdots < t_N = T$
- Assume constant $\Delta t = t_n t_{n-1}$
- \triangleright u^n : numerical approx to the exact solution at t_n

Numerical scheme:

$$u^{n+1} = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}u^n, \quad n = 0, 1, \dots, N - 1$$

Numerical solution method

- ▶ Mesh in time: $0 = t_0 < t_1 \cdots < t_N = T$
- Assume constant $\Delta t = t_n t_{n-1}$
- \triangleright u^n : numerical approx to the exact solution at t_n

Numerical scheme:

$$u^{n+1} = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}u^n, \quad n = 0, 1, \dots, N - 1$$

Numerical solution method

- ▶ Mesh in time: $0 = t_0 < t_1 \cdots < t_N = T$
- Assume constant $\Delta t = t_n t_{n-1}$
- \triangleright u^n : numerical approx to the exact solution at t_n

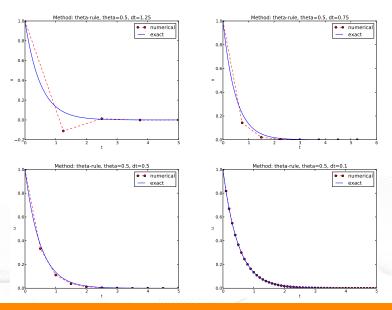
Numerical scheme:

$$u^{n+1} = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}u^n, \quad n = 0, 1, \dots, N - 1$$

Implementation

The numerical method is implemented in a Python function:

The Crank-Nicolson method



Exact solution of the scheme:

$$u^n = A^n$$
, $A = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}$

- Stability: |A| < 1</p>
- No oscillations: A > 0
- Always for Backward Euler ($\theta = 1$)
- ▶ $\Delta t < 1/a$ for Forward Euler ($\theta = 0$)
- ▶ $\Delta t < 2/a$ for Crank-Nicolson ($\theta = 1/2$)

Concluding remarks:

Exact solution of the scheme:

$$u^n = A^n$$
, $A = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}$.

- Stability: |A| < 1</p>
- No oscillations: A > 0
- ▶ Always for Backward Euler ($\theta = 1$)
- ▶ $\Delta t < 1/a$ for Forward Euler ($\theta = 0$)
- ▶ $\Delta t < 2/a$ for Crank-Nicolson ($\theta = 1/2$)

Concluding remarks:

Exact solution of the scheme:

$$u^n = A^n$$
, $A = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}$.

- ► Stability: |A| < 1
- ► No oscillations: *A* > 0
- ▶ Always for Backward Euler ($\theta = 1$)
- ▶ Δt < 1/a for Forward Euler (θ = 0)
- ▶ Δt < 2/a for Crank-Nicolson (θ = 1/2)

Concluding remarks:

Exact solution of the scheme:

$$u^n = A^n$$
, $A = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}$.

- ► Stability: |A| < 1
- No oscillations: A > 0
- ▶ Always for Backward Euler ($\theta = 1$)
- ▶ $\Delta t < 1/a$ for Forward Euler ($\theta = 0$)
- $\Delta t < 2/a$ for Crank-Nicolson ($\theta = 1/2$)

Concluding remarks: