

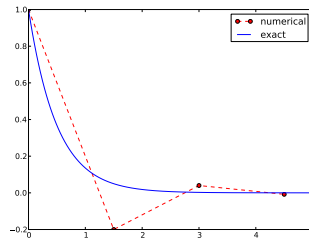
# On Schemes for Exponential Decay

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## 1 Goal

The primary goal of this demo talk is to demonstrate how to write talks with [Doconce](#) and get them rendered in numerous HTML formats.

### Layout.

This version utilizes latex document slides with the theme `no theme`.

The talk investigates the accuracy of three finite difference schemes for the ordinary differential equation  $u' = -au$  with the aid of numerical experiments. Numerical artifacts are in particular demonstrated.

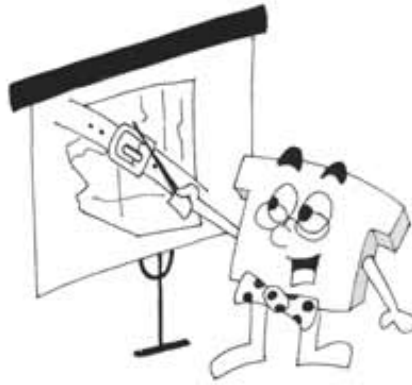
## 2 Mathematical problem

$$u'(t) = -au(t), \tag{1}$$

$$u(0) = I, \tag{2}$$

- $t \in (0, T]$
- $a$ ,  $I$ , and  $T$  are prescribed parameters

- $u(t)$  is the unknown function



### 3 Numerical solution method

- Mesh in time:  $0 = t_0 < t_1 < \dots < t_N = T$
- Assume constant  $\Delta t = t_n - t_{n-1}$
- $u^n$ : numerical approx to the exact solution at  $t_n$

Numerical scheme:

$$u^{n+1} = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t} u^n, \quad n = 0, 1, \dots, N-1$$

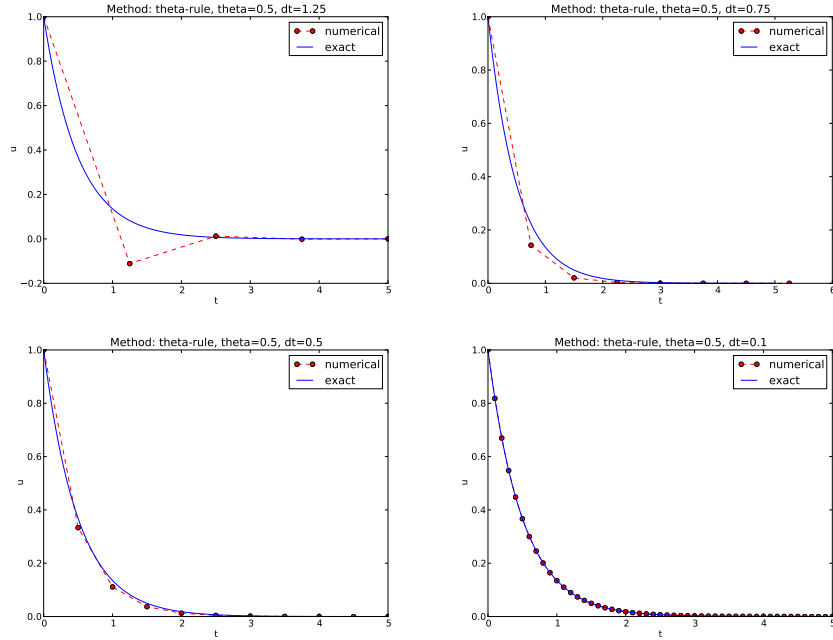
### 4 Implementation

The numerical method is implemented in a Python function:

```
def solver(I, a, T, dt, theta):
    """Solve u'=-a*u, u(0)=I, for t in (0,T] with steps of dt."""
    dt = float(dt) # avoid integer division
    N = int(round(T/dt)) # no of time intervals
    T = N*dt # adjust T to fit time step dt
    u = zeros(N+1) # array of u[n] values
    t = linspace(0, T, N+1) # time mesh

    u[0] = I # assign initial condition
    for n in range(0, N): # n=0,1,...,N-1
        u[n+1] = (1 - (1-theta)*a*dt)/(1 + theta*dt*a)*u[n]
    return u, t
```

## 4.1 The Crank-Nicolson method



## 4.2 The artifacts can be explained by some theory

Exact solution of the scheme:

$$u^n = A^n, \quad A = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}.$$

- Stability:  $|A| < 1$
- No oscillations:  $A > 0$
- Always for Backward Euler ( $\theta = 1$ )
- $\Delta t < 1/a$  for Forward Euler ( $\theta = 0$ )
- $\Delta t < 2/a$  for Crank-Nicolson ( $\theta = 1/2$ )

### Concluding remarks:

Only the Backward Euler scheme is guaranteed to always give qualitatively correct results.