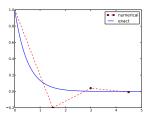
## On Schemes for Exponential Decay

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### Goal

The primary goal of this demo talk is to demonstrate how to write talks with DocOnce and get them rendered in numerous HTML formats.

#### Layout.

This version utilizes beamer slides with the theme blue\_shadow.

# Mathematical problem

$$u'(t) = -au(t), \qquad (1)$$

$$u(0)=I, \qquad (2)$$

- $t \in (0, T]$
- *a, I,* and *T* are prescribed parameters
- u(t) is the unknown function



## Numerical solution method

- Mesh in time:  $0 = t_0 < t_1 \cdots < t_N = T$
- Assume constant  $\Delta t = t_n t_{n-1}$
- $u^n$ : numerical approx to the exact solution at  $t_n$

#### Numerical scheme:

$$u^{n+1} = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}u^n, \quad n = 0, 1, \dots, N-1$$

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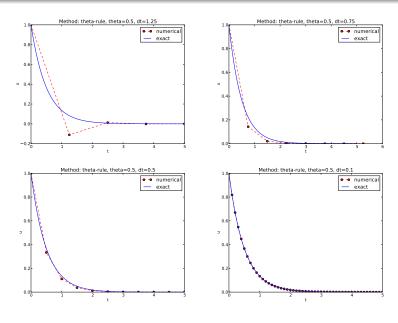
# Forward Euler explained

http://youtube.com/PtJrPEIHNJw

## **Implementation**

The numerical method is implemented in a Python function:

## The Crank-Nicolson method



Exact solution of the scheme:

$$u^n = A^n$$
,  $A = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}$ .

- Stability: |A| < 1
- No oscillations: A > 0
- Always for Backward Euler ( $\theta=1$ )
- $\Delta t < 1/a$  for Forward Euler ( $\theta = 0$ )
- $\Delta t < 2/a$  for Crank-Nicolson  $(\theta=1/2)$

### Concluding remarks

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