

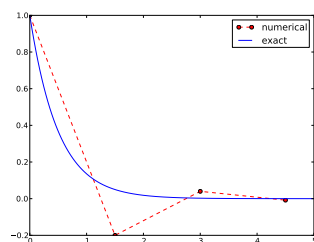
On Schemes for Exponential Decay

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1 Goal

The primary goal of this demo talk is to demonstrate how to write talks with [Doconce](#) and get them rendered in numerous HTML formats.

Layout.

This version utilizes latex document slides with the theme `no theme`.

The talk investigates the accuracy of three finite difference schemes for the ordinary differential equation $u' = -au$ with the aid of numerical experiments. Numerical artifacts are in particular demonstrated.

2 Mathematical problem

$$u'(t) = -au(t), \tag{1}$$

$$u(0) = I, \tag{2}$$

- $t \in (0, T]$
- a , I , and T are prescribed parameters
- $u(t)$ is the unknown function



3 Numerical solution method

- Mesh in time: $0 = t_0 < t_1 < \dots < t_N = T$
- Assume constant $\Delta t = t_n - t_{n-1}$
- u^n : numerical approx to the exact solution at t_n

Numerical scheme:

$$u^{n+1} = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t} u^n, \quad n = 0, 1, \dots, N - 1$$

3.1 Forward Euler explained

<http://youtube.com/PtJrPEIHJw>

4 Implementation

The numerical method is implemented in a Python function:

```
def solver(I, a, T, dt, theta):
    """Solve  $u' = -a*u$ ,  $u(0) = I$ , for  $t$  in  $(0, T]$  with steps of  $dt$ ."""
    dt = float(dt)          # avoid integer division
    N = int(round(T/dt))     # no of time intervals
```

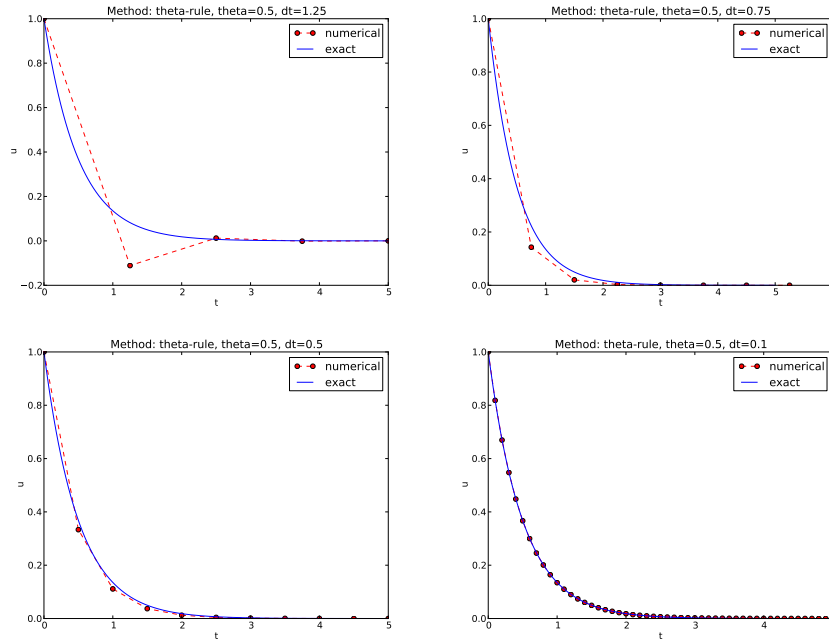
```

T = N*dt          # adjust T to fit time step dt
u = zeros(N+1)    # array of u[n] values
t = linspace(0, T, N+1) # time mesh

u[0] = I          # assign initial condition
for n in range(0, N): # n=0,1,...,N-1
    u[n+1] = (1 - (1-theta)*a*dt)/(1 + theta*dt*a)*u[n]
return u, t

```

4.1 The Crank-Nicolson method



4.2 The artifacts can be explained by some theory

Exact solution of the scheme:

$$u^n = A^n, \quad A = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}.$$

- Stability: $|A| < 1$
- No oscillations: $A > 0$
- Always for Backward Euler ($\theta = 1$)
- $\Delta t < 1/a$ for Forward Euler ($\theta = 0$)
- $\Delta t < 2/a$ for Crank-Nicolson ($\theta = 1/2$)

Concluding remarks:

Only the Backward Euler scheme is guaranteed to always give qualitatively correct results.