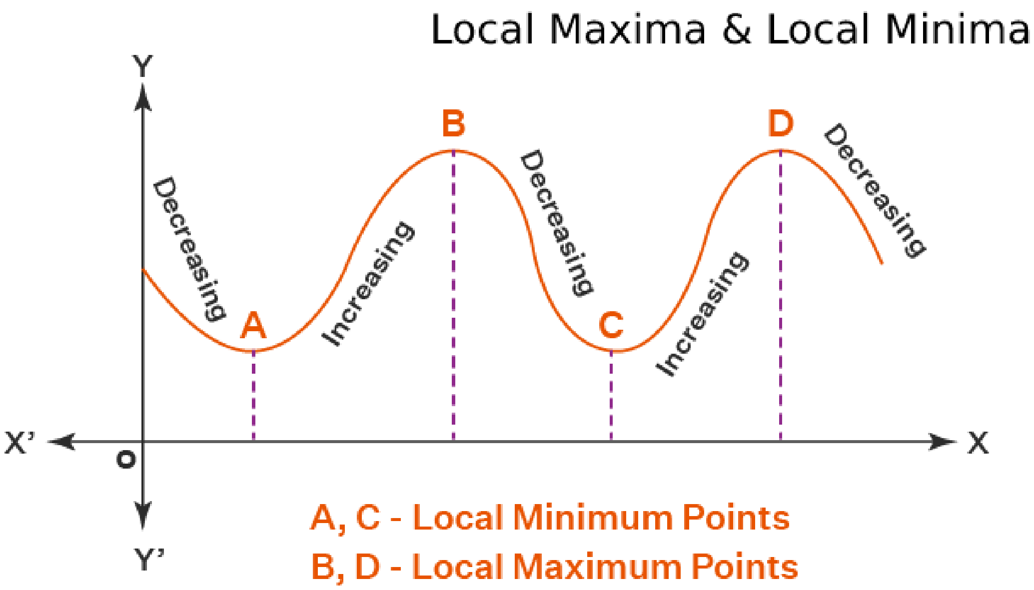
UNIT 2

CH- 1

APPLICATIONS OF DIFFERNTIATION

Related rates: finding a rate at which one quantity changes by relating that quantity to ither quantities whose rate of change is known (especially w.r.t time).

Local Maxima and minima:

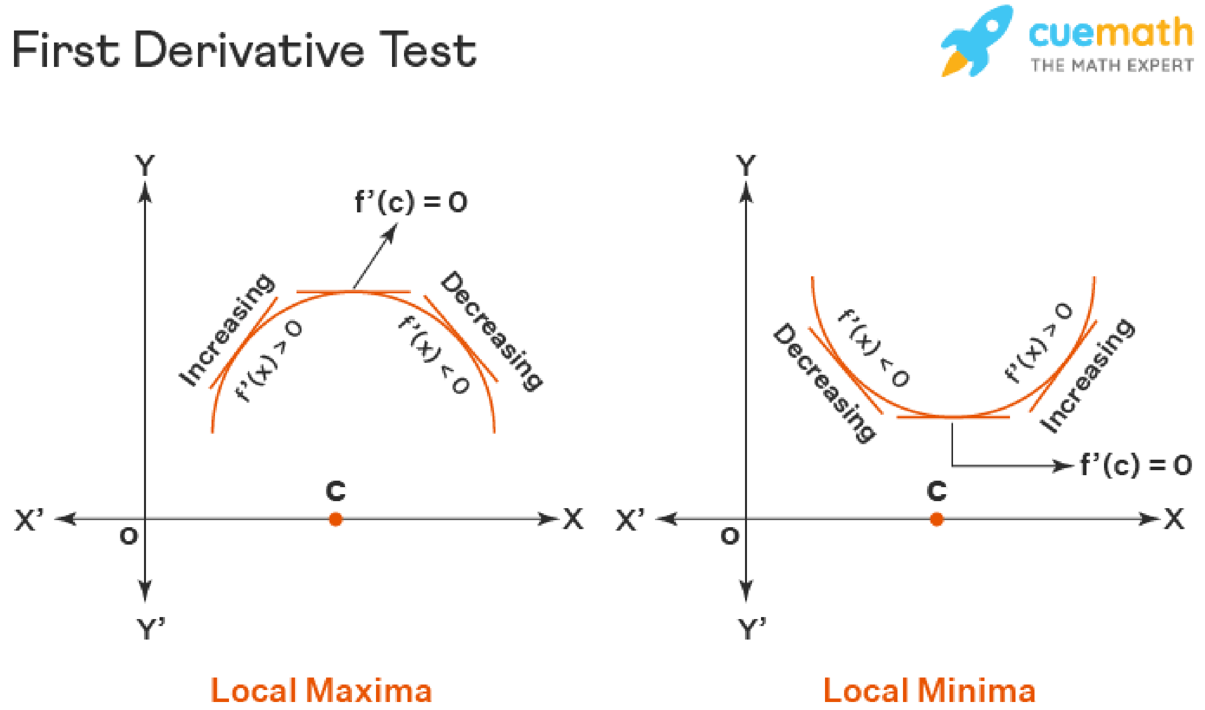


A function 'f' has a local maximum or relative maximum at x=c if f(c)>=f(x), when x is in neighbourhood of c, where c is critical point.

Similarly, 'f' has a local minimum or relative minimum at x = c if

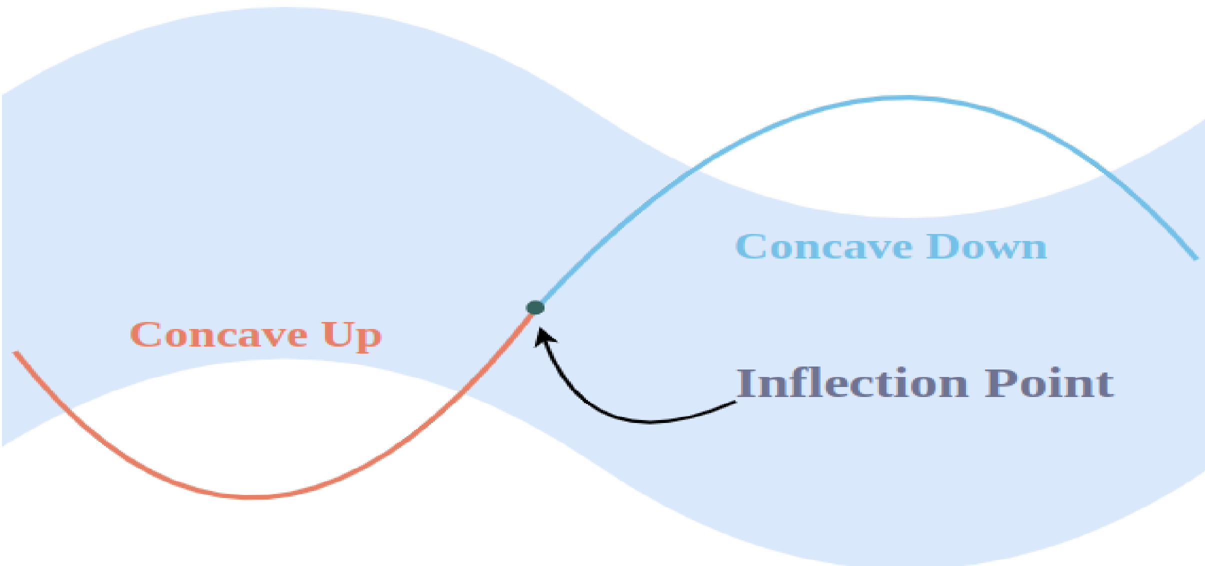
f(c)<=f(x), when x is in neighbourhood of c, where c is critical point.

1st derivative test:



- if f' changes from +ve to-ve at c => c will be local maximum.
- if f' changes from -ve to +ve at c => c will be local minimum.
- If f' does not change sign at c, then f has no local maximum nor minimum at c.

Concavity and point of inflection:



Let f be a function that is differentiable over an interval i.

- If f' is increasing over i, we say f is concave up over i.
- If f' is decreasing over i, we say f is concave down over i.

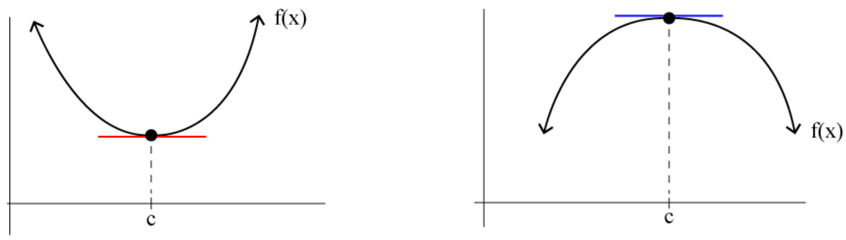
Test for concavity:

Let f be a function that is twice differentiable over an interval i.

- If f" (x)>0 for all x belongs to i, then f is concave up over i.
- If f" (x)<0 for all x belongs to i then f is concave down over i.

2nd derivative test:

Let f be a function that is twice differentiable over an interval i. Let c is a point which is neighbourhood of x.



$f'(c)=0$ **and** $f''(c)>0$
$f(c)$ is a local minimum

$f'(c)=0$ **and** $f''(c)<0$
$f(c)$ is a local maximum

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- If f'(c)=0 & f"(c)>0, then f has local minimum at c
- If f'(c)=0 & f"(c)<0, then f has local maximum at c

Closed interval method:

To find absolute maxima or absolute minima, we use closed interval method.

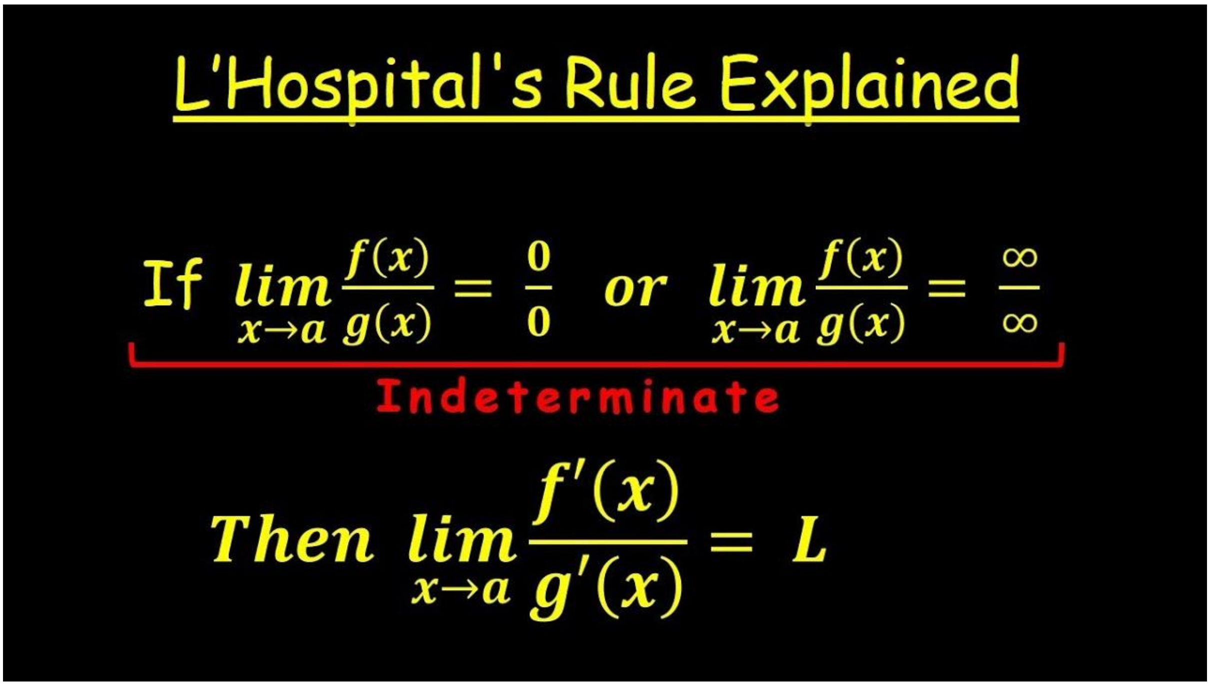
Procedure:

→ find values of f at the critical numbers of f.

→ find value of f at end points of given interval

→ the largest of values from stept & step 2 is the absolute maximum value & smallest is absolute minimum value.

<u>L – Hospital's rule:</u>



If we get intermediate forms like 0/0 or ∞/ ∞ we use L-Hospital's rule till we get a finite value.

Let f and g be two differentiable functions,

If f(a)=0 & g(a)=0, then lim(x->a) [f(a)/g(a) = lim(x->a) [f'(a)/g'(a)], such that lim(x->a) f'(x) and lim(x->a) g(x) do not vanish simultaneously (i.e, finite value esists).

Note:

- When a limit of a given function takes the form 0 X ∞ or ∞- ∞, reduce limiting form to 0/0 or ∞/ ∞.
- When a limit of a given function takes the form 1^∞, reduce limiting form to 0/0 or ∞/ ∞ by taking log on both sides.

Numerical methods:

→methods used to solve real valued functions by using basic operations.

Transcendental equations:
which involve trigonometric, exponential logarithmic terms,
ex:
1 Sinx+ x ^ 3> =0
e^x+ 2cosx = 0
log(x) + 2x = 0

Intermediate value property (IVP):

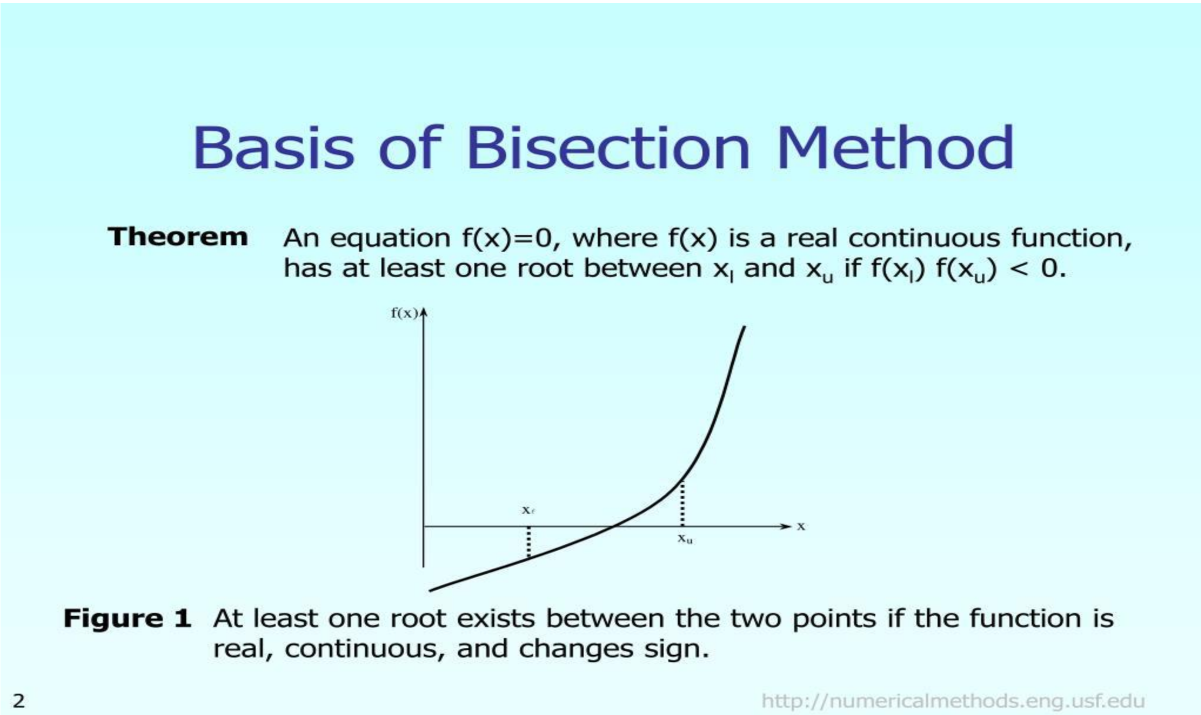
if f(x) is continuous real valued function on a closed interval [a,b], then there exists a number 'c' in [a,b] such that A(c) = N

if f(a) * f(b) <0, then there exists a root 'c' in [a,b]

I] Bisection methad

II] Newton-Raphson method

1) **Bisection method**



→ dividing an interval in half & then by selecting the sub interval by using IVP we find the roots.

Algorithm:

• Find two points, a & b such that a<b & f(a)*f(b)<0.

• find mid pt of a & b, r1.

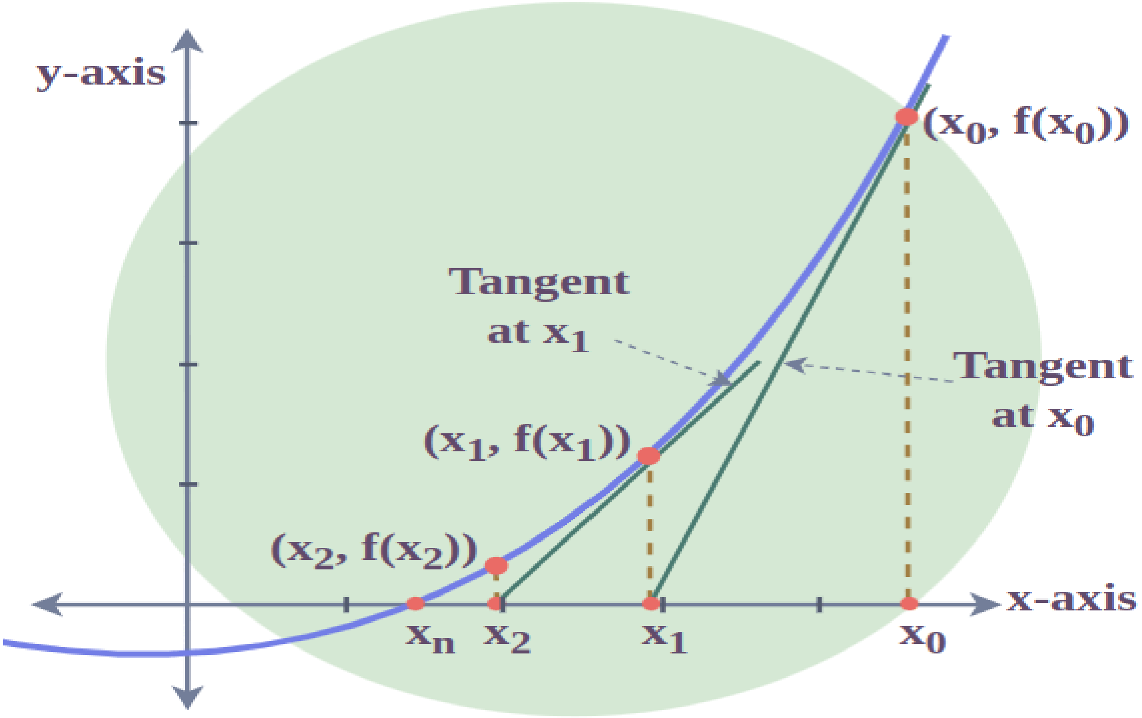
If f(r1) = 0, then root is r1.

else divide interval [a, b] such that –

- if f(r1)*f(a)<0, then root belongs to (r1, a)
- else if f(r1)*f(b)<0, then root belongs to (r1, b)
- Repeat above steps until f(r) = 0.

2] Newton-Raphson method

The Newton-Raphson method leverages the linear approximation provided by the tangent line to rapidly converge to a function's root. By iteratively applying this method, the approximations become successively closer to the actual root. This approach, visualized through tangents, makes the method both intuitive and powerful.



➔ X(n+1) = x(n) - f[x(n)] / f[x'(n)]

Steps:

1. **Initial Guess**: Start with an initial guess x(0) for the root.
2. **Evaluate the Function and its Derivative**: Calculate f[x(n)] and f′[x(n)]
3. **Update the Guess**: Use the formula to find the next approximation x(n+1).
4. **Iterate**: Repeat the process until the change in x (i.e., ∣x(n+1)−x(n)∣ is smaller than a predefined tolerance or until a maximum number of iterations is reached.

CH – 2

Integrals

WHAT IS INTEGRATION??

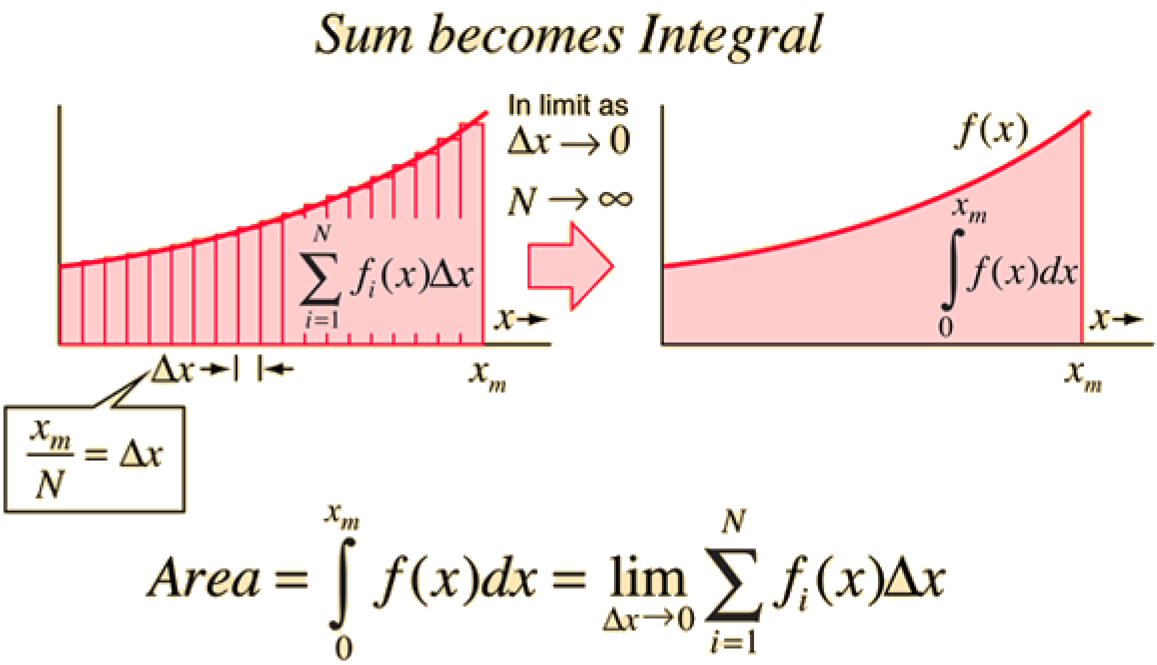
Integration is a fundamental concept in calculus and mathematical analysis. It is the process of finding the integral of a function, which can be understood as the reverse operation of differentiation. While differentiation gives us the rate of change or the slope of a function, integration helps us find the accumulation of quantities, such as areas under curves.

integration as the area

In mathematics, integration can be understood as a process to find the area under a curve. This interpretation is particularly intuitive and helps in visualizing what integration achieves. Let's explore this concept in more detail:

Definite Integral as Area Under the Curve:

The definite integral of a function $f(x)$ over an interval $[a, b]$ can be interpreted as the area under the curve of $f(x)$ from $x = a$ to $x = b$.

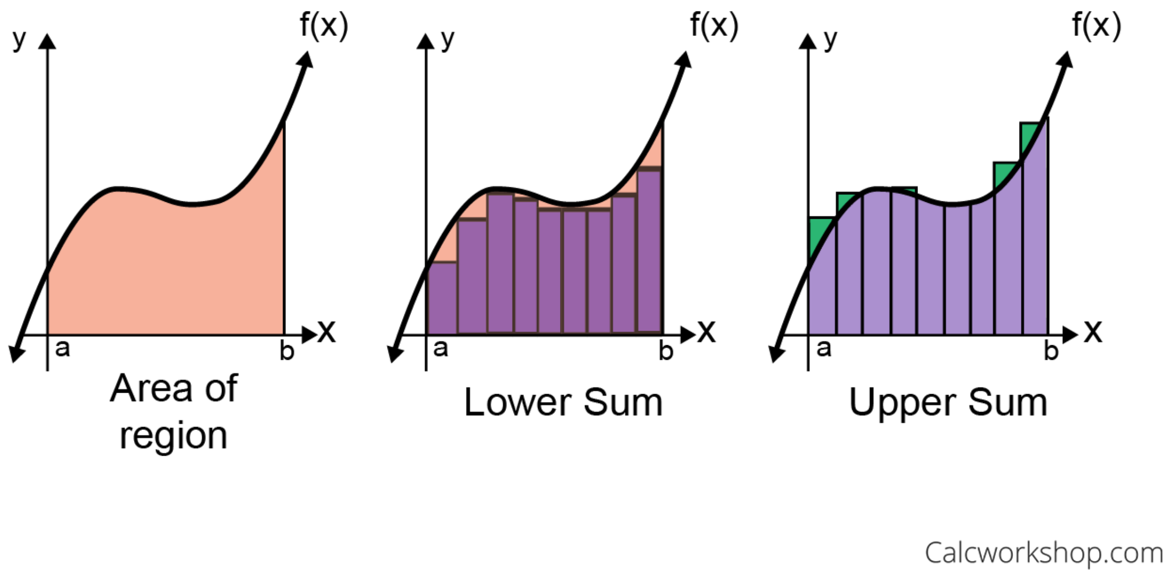


Steps to Understand:

1. **Function Graph**:
   - Plot the function f(x)f(x) on a graph.
   - Identify the interval [a,b][a, b] on the x-axis.
2. **Area Calculation**:
   - The area under the curve f(x)f(x) from x=ax = a to x=bx = b is represented by the definite integral ∫abf(x) dx\int_{a}^{b} f(x) \, dx.
   - If f(x)f(x) is positive in this interval, the integral represents the total area between the curve and the x-axis.
3. **Subdividing the Area**:
   - Divide the interval [a, b] into small subintervals.

- Approximate the area under the curve using rectangles (or trapezoids for a better approximation).

4. **Summing the Areas**:
    - Sum the areas of these rectangles to get an approximate value of the integral.
    - As the width of these subintervals approaches zero, the sum of the areas of the rectangles approaches the exact area under the curve.



Numerical methods:

To find area under curve using numerical integration we have:

1) Trapezoidal rule
2) Simpson's $1/3^{rd}$ rule

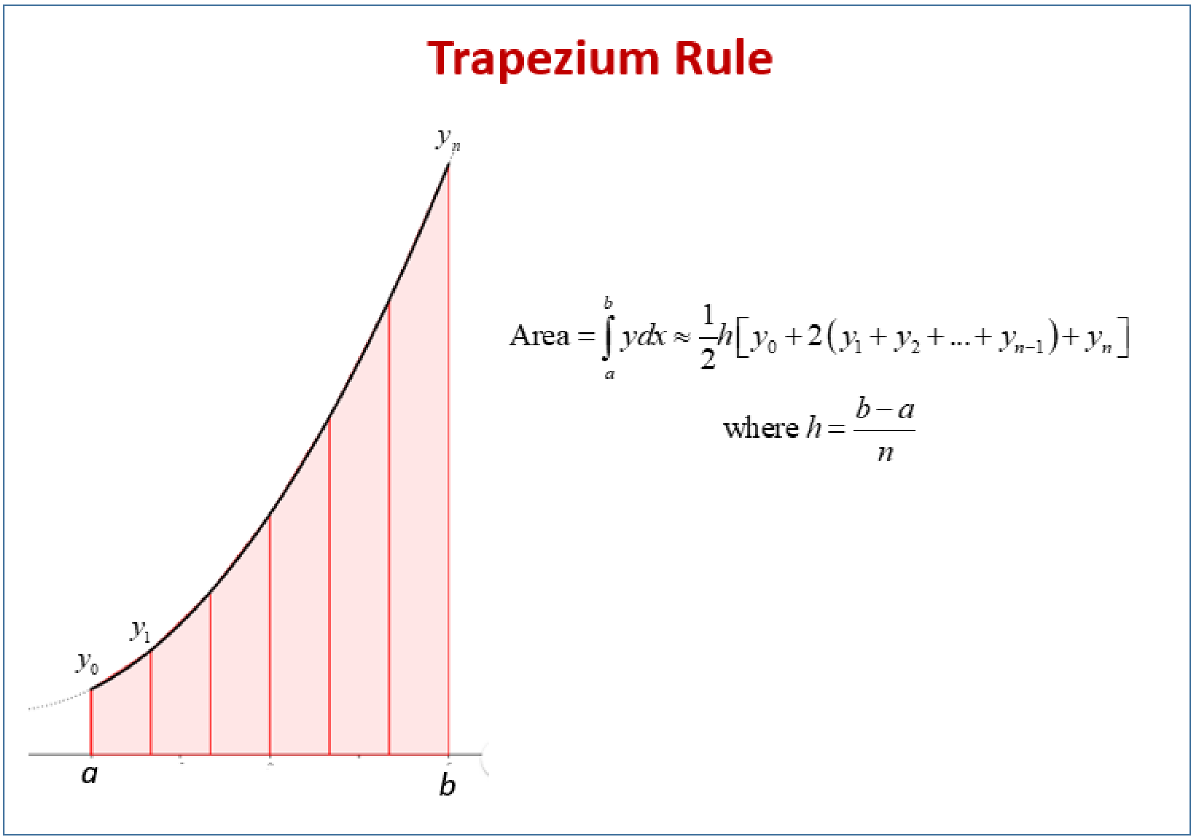
1) <u>Trapezoidal rule:</u>

The Trapezoidal Rule is a numerical method used to approximate the definite integral of a function. It is particularly useful when an analytical solution to the integral is difficult or impossible to obtain. The idea is to approximate the area under the curve by dividing it into a series of trapezoids and summing their areas.

Steps:

1. **Divide the Interval**: Divide the interval [a,b] into n equal parts.
2. **Calculate Endpoints**: Compute the function values at each of these endpoints.
3. **Sum Areas of Trapezoids**: Calculate the sum of the areas of the trapezoids formed by these endpoints.

Given a function f(x) and an interval [a,b][a, b] divided into nn equal subintervals, the Trapezoidal Rule is given by:



Where, yn is associated function with xn.
h is step length
n is number of sub intervals
(note : x0=a, xn=b)

2) Simpson’s 1/3rd rule

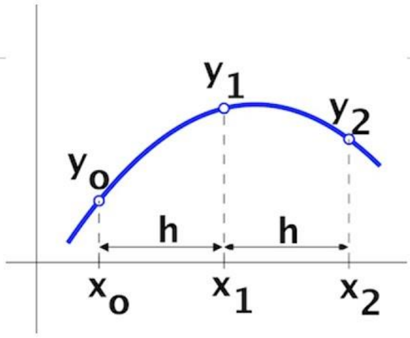
Simpson’s 1/3rd Rule is another numerical method used to approximate the definite integral of a function. It is more accurate than the Trapezoidal Rule for most functions, especially when the function is smooth.

Steps:

1. **Divide the Interval**: Divide the interval [a,b][a, b] into nn equal parts, where nn is even.
2. **Calculate Endpoints**: Compute the function values at each of these endpoints.
3. **Apply the Simpson's Rule Formula**: Use the formula to approximate the integral by summing the weighted function values.

Simpson's 1/3rd Rule is a powerful numerical integration technique that provides more accurate results by using parabolic segments to approximate the function, rather than straight lines as in the Trapezoidal Rule. It works exceptionally well for smooth functions and is widely used in scientific and engineering applications.

Numerical Integration Simpson’s 1/3 Rule

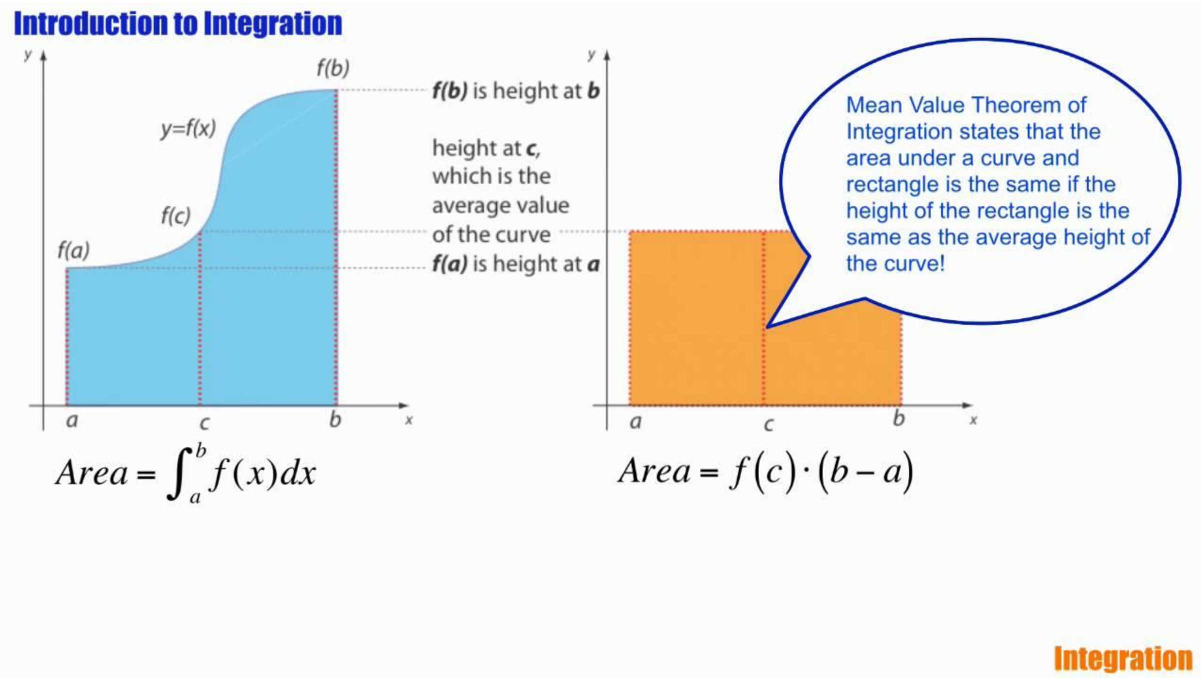


$$A = \frac{h}{3}\left[(y_0 + y_n) + 2(y_2 + y_4 + y_6 \cdots) + 4(y_1 + y_3 + y_5 \cdots) \right]$$

MEAN VALUE THEROEM :

The Mean Value Theorem for Integrals states that if f(x)f(x) is continuous on the closed interval [a,b][a, b], then there exists at least one point cc in (a,b)(a, b) such that:

$$\int_a^b f(x)\,dx = f(c)\cdot(b-a)$$



Interpretation:

This theorem tells us that the definite integral of a continuous function over [a, b] is equal to the function value at some point cc within (a, b), multiplied by the length of the interval (b−a). Essentially, f(c)f(c) represents the average value of the function over that interval.

Geometric Interpretation:

- **Integral as Area**: The definite integral ∫(a to b) f(x) dx represents the area under the curve f(x) from x = a to x= b.
- **Rectangle Representation**: According to the Mean Value Theorem for Integrals, there is a point c in (a, b) such that the area under the curve is the same as the area of a rectangle with width (b−a) and height f(c).

fundamental of calculus

The Fundamental Theorem of Calculus (FTC) is a central theorem in calculus that links the concept of differentiation and integration. It has two main parts, which together establish the relationship between these two operations:

The First Fundamental Theorem of Calculus

This part provides a way to compute derivatives of integrals. It states that if f is a continuous function on an interval [a, b], then the function G defined by:

$$G(x) = \int_a^x f(t)\,dt$$

is continuous on $[a, b]$, differentiable on (a, b), and its derivative is:

$$G'(x) = f(x)$$