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DIV- K

ENGINEERING MATHEMATICS -1

FUNCTIONS AND GRAPHS

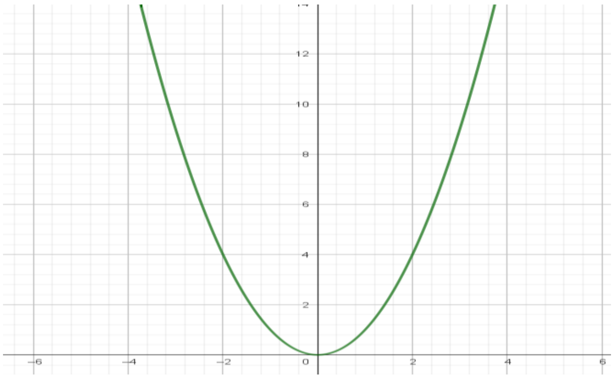
Key points:

Domain – set of possible inputs for a function.

Range – set of possible outputs for the function.

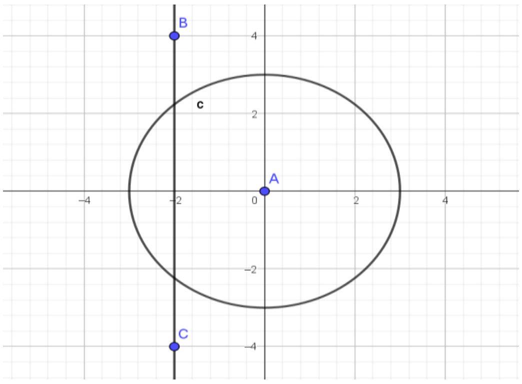
FUNCTIONS – are correspond with sets called domain and range where every element of domain is related with only one member of the range, but member of the range can be related to two or more members of the domain.

Note- domain consists of independent values whereas range contains values correspond to members of domain through the defined function



EX- Here in this example of x^2, every element of domain (i.e, x axis) is related to only one value of range (i.e, y axis).

For ex, 2 is related to 4 and -2 is also related with 4 but 2 (or -2) is not related to any another element. [range member can be connected with multiple members of domain but domain member should be related to only one member of the range.]

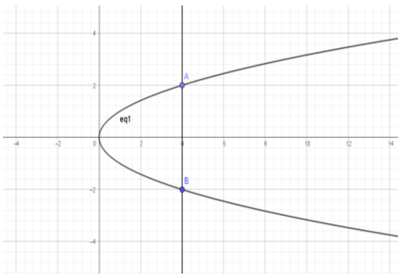


Here, it cannot be consider as a function because x=-2 has two outputs.

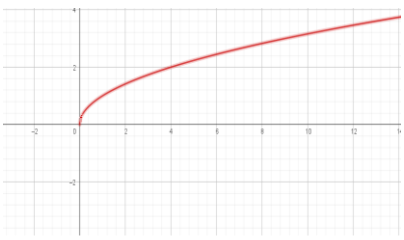
METHODS TO IDENTIFY FUNCTIONS

A curve on any plane is the graph of the function of x iff no vertical line intersects the curve at two points.

VERTICAL LINE TEST



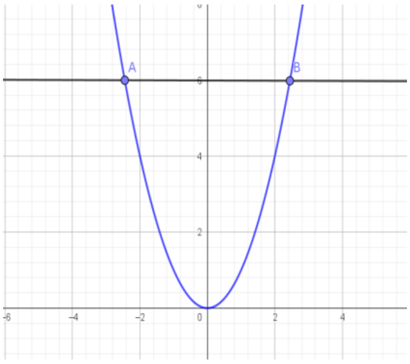
it is not a function.



it is a function.

HORIZONTAL LINE TEST [TO CHECK ONE-ONE FUNCTION]

If a function is said to be 1-1 iff no horizontal line is intersecting the curve at two points.



f(x) = x^2 is a function but not one-one function, since line intersects at A and B.

INVERSE OF A FUNCTION

If f is a one-one function having domain D and range R, then f inverse is also a one-one function with domain R and range D.

f(x)=y then f inverse y=x.

COMPOSITION OF FUNCTIONS

If f and g are functions then composite of f with g is denoted by fog

fog is defined by f(g(x)).

TYPES OF FUNCTIONS

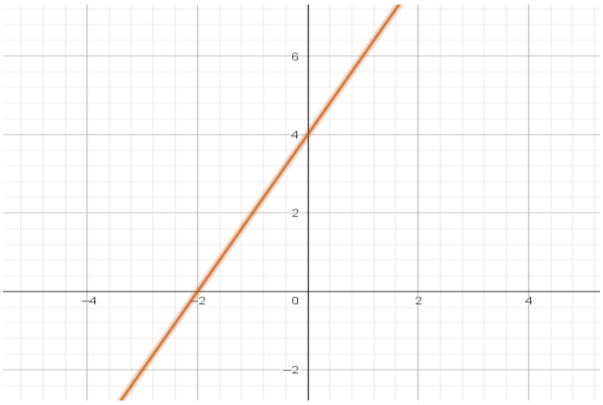
1) **Linear functions**

A function with degree 1 is called as linear.

key signs to identify:

a. Constant rate of change (constant slope)
b. examine the mathematical form (y=mx+b)
c. Analyse data by plotting, (st. line graph)

Eqn= > y=mx+c, where m=slope; c=y intercept



2) Quadratic functions (Paraboloid in nature)

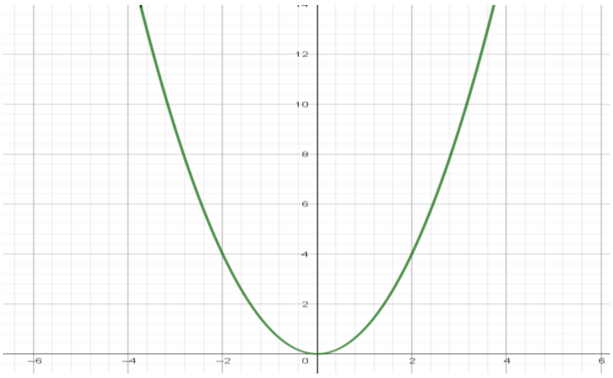
Eqn is of degree 2,

Key Signs to identify.

a. Examine the mathematical form (ax^2+bx+c)

b. Analyse data by plotting (U shaped curve)

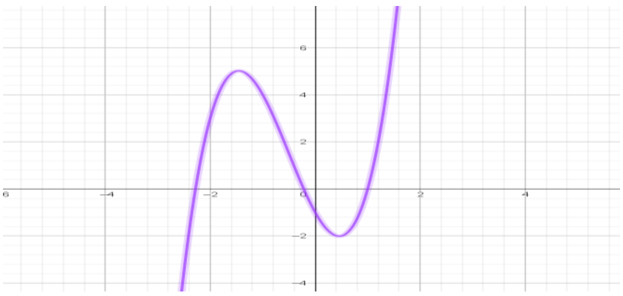
c. Axis of symmetry



3) Cubic functions

* degree of polynomial is 3

$\rightarrow ax^3+bx^2+cx+d$,



4) **Exponential function**

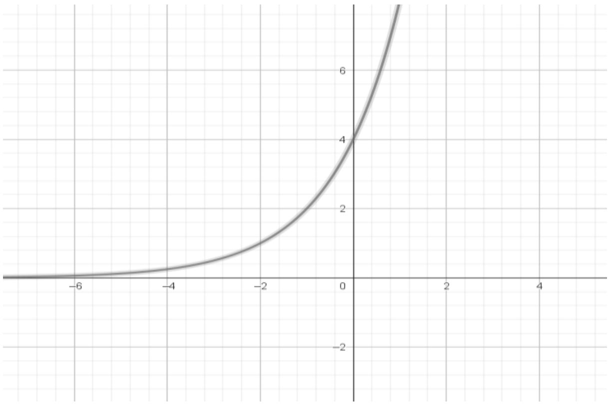
Function whose exponent is a variable.

Key Signs to identify.

a. Constant % change over certain interval of time.
b. Examine mathematical form (y=ab^x), a=>y intercept
c. Analyse data by plotting
d. Ratio of slopes is constant

Note= if b>1, growth function

If 0<b<1, decay function

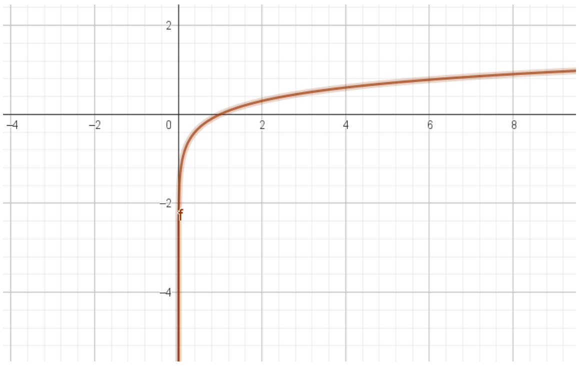


5) Logarithmic functions

Y=log(b)x, increasing if b>1 and decreasing if 0<b<1

Key Signs to identify.

a. Slow growth or decay
b. As x increases rate of Y decreases
c. Vertical asymptote



6) Trignometric functions

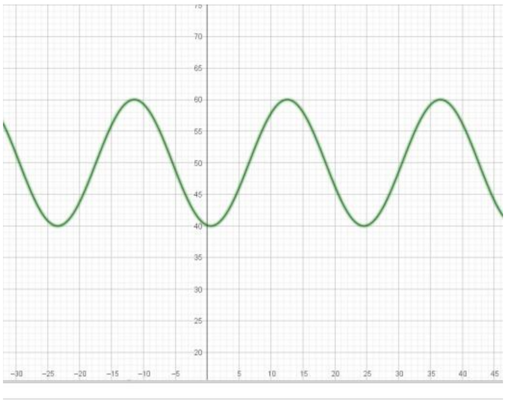
Trigonometric model eqn is given by

Y=Asin(B(x+h))+D

Where |A|= amplitude, B=frequency, h=phase shift

Time period=2(3.142)/B

Range=[D-A,D+A]



TRANSFORMATION OF FUNCTIONS

VERTICAL AND HORIZONTAL SHIFTS

1) Y=f(x)+c : shift the graph of y=f(x) c units upward
2) Y=f(x)-c : shift the graph of y=f(x) c units downward
3) Y=f(x-c) : shift the graph of y=f(x) c units right

STRETCHING AND REFLECTING

1) Y=cf(x) : stretch the graph of y=f(x) vertically by factor c.
2) Y=f(x)/c : compress the graph of y=f(x) vertically by factor c.
3) Y=f(cx) : compress the graph of y=f(x) horizontally by factor c.
4) Y=f(x/c) : stretch the graph of y=f(x) horizontally by factor c.
5) Y=-f(x) :reflect the graph of y=f(x) above x axis.
6) Y=f(-x) :reflect the graph of y=f(x) above y axis.

LIMITS AND DERIVATIVES

LIMITS: value of the function when x is tending to certain value a but not equals to a.

CONTINUOUS FUNCTION: a function is said to be continuous at x=a if

1) Limit exists at x=a
2) f(a) is defined
3) limit of f(a) as x->a = f(a)

a function is said to be discontinuous at x=a if

1) limit does not exist at a
2) lim f(a) is not equal to f(a)
3) f(a) is not defined at x=a

Discontinuities functions can be due to:

1) jump at x=a
2) f(a) is not defined at a

3) removable discontinuity RHL is not equal to f(a)

LIMIT AT INFINITY (x->infinity)

Horizontal asymptote :

Line y =L is called a horizontal asymptote of curve y=f(x) if either limf(x)=L as x-> infinity or limf(x)=L as x-> -(infinity), where L is a finite value.

The horizontal asymptote is a limit for long term behaviour.

INFINITE LIMIT (limit tends to infinity)

Veretical asymptote :

The line x= a is called vertical asymptote of curve y=f(x) if limf(x) as x-> a =infinity or - (infinity).

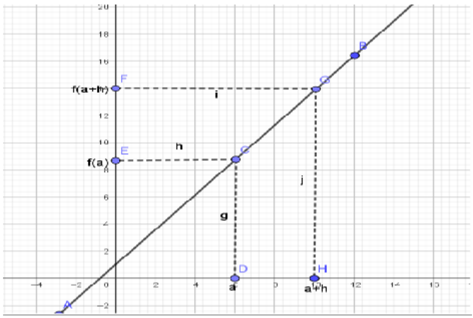
Vertical asymptote is an unimaginable or undefined state in the real world.

To get Vertical asymptote, the denominator of the given function must be zero.

DERIVATIVES : can be defined as

- instantaneous rate of change of independent variable w.r.t dependent variable
- slope of tangent line at that point

FIRST PRINCIPLE OF DERIVATIVES:



$$f'(a) = \lim_{h \to 0} \left\{ \frac{[\ f(a+h)-f(a)\ ]}{a+h-a} \right\}$$

as h->0 , a+h -> a