DFA

→ state, → Transition

$Q, \Sigma, \delta, q_0, F$

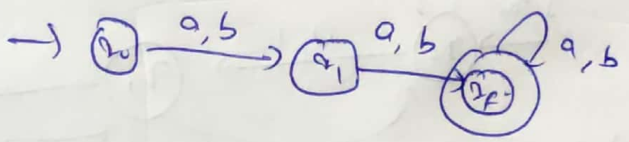
Input alphabet, initial, final

1) Construct a DFA over $\Sigma = \{a, b\}$ where the length of the string is atleast 2 ie

$|w| \geq 2$

$\Sigma = \{a, b\}$

$L = \{aa, ab, ba, bb, abb, aba, baa, \ldots\}$




| state | a | b |
|---|---|---|
| $q_0$ | $q_1$ | $q_1$ |
| $q_1$ | $q_f$ | $q_f$ |
| $q_f$ | $q_f$ | $q_f$ |

Transition table

# NFA

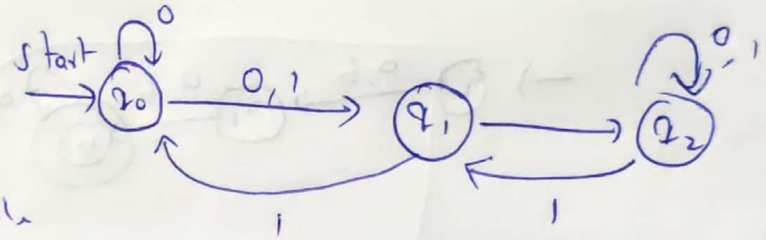
$Q = (q_0, q_1, q_2)$

$\Sigma = (0, 1)$

$q_0 = (q_0)$

$f = (q_2)$.

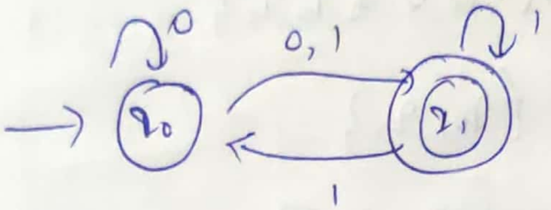
Transition diagram




Transition table

| Present state | next state for Input 0 | Next state of Input 1 |
|---|---|---|
| $\rightarrow q_0$ | $q_0, q_1$ | $q_1$ |
| $q_1$ | $q_2$ | $q_0$ |
| $*q_2$ | $q_2$ | $q_1, q_2$ |

NFA → DFA




| state | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $(q_0, q_1)$ | $(q_1)$ |
| $*q_1$ | $\phi$ | $(q_0, q_1)$ |

$\delta'([q_0], 0) = \{q_0, q_1\}$

$= \{q_0, q_1\}$

$\delta'([q_0], 1) = [q_1] = [q_1]$

$\delta'$ transition for $q_1$

$\delta'([q_1], 0) = \phi$

$\delta'([q_1], 1) = [q_0, q_1]$

$\delta'([q_0, q_1], 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$

$= (q_0, q_1) \cancel{\phi} \cup \phi$

$= \{q_0, q_1\}$

$= [q_0, q_1]$

$$\delta'([q_0, q_1], 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$$

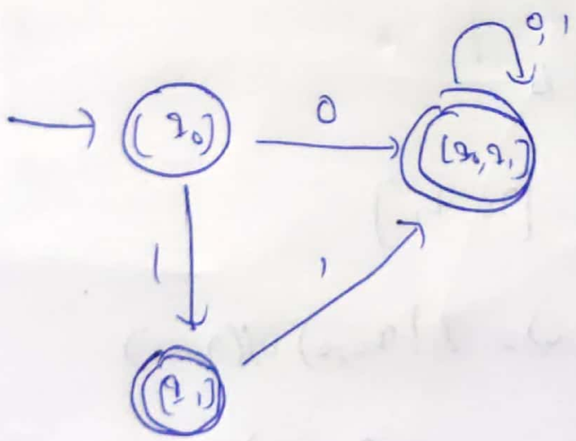
$$= \delta(q_1) \cup \{q_0, q_1\}$$

$$= \{q_0, q_1\}$$

$$= [q_0, q_1]$$

| State | Input | Output |
|---|---|---|
| $\rightarrow [q_0]$ | $[q_0, q_1]$ | $[q_1]$ |
| $* [q_1]$ | $\phi$ | $[q_0, q_1]$ |
| $* [q_0, q_1]$ | $[q_0, q_1]$ | $[q_0, q_1]$ |

Transition diagram



# Moore machine

Q: finite set of states

$q_0$: initial state of machine

$\Sigma$: finite set of input symbols

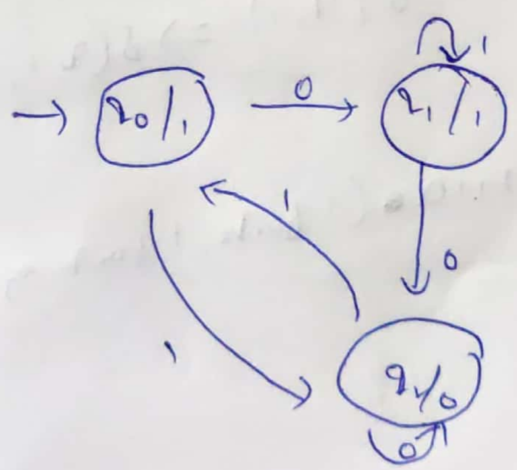
O: output alphabet.

$\delta$: Transition function where $Q \times \Sigma \to Q$

$\lambda$: Output function where $Q \to O$

Ex-1

state diagram of Moore machine is



Transition table

| Current state | Nextstate 0 | Nextstate 1 | Output |
|---|---|---|---|
| $q_0$ | $q_1$ | $q_2$ | 1 |
| $q_1$ | $q_2$ | $q_1$ | 1 |
| $q_2$ | $q_2$ | $q_0$ | 0 |

input : 010

transition : $\delta(q_0, 0)$

$\Rightarrow \delta(q_1, 1) \Rightarrow \delta(q_1, 0) \Rightarrow q_2$

Output: 1110 (1 for $q_0$, 1 for $q_1$ again 1 for $q_1$, 0 for $q_2$)

## Mealy machine

Q : finite set of states

$q_0$ : initial state of machine

$\Sigma$ : finite set of input alphabet

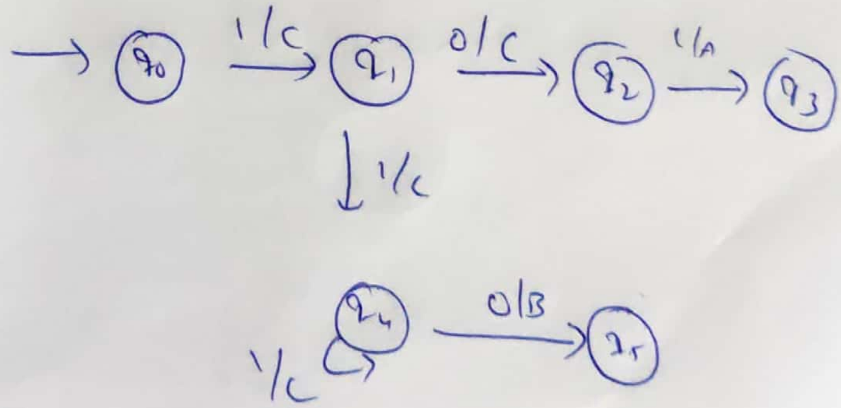
O : output alphabet

$\delta$ : transition function where $Q \times \Sigma \rightarrow Q$

$\lambda'$ : output function where $Q \times \Sigma \rightarrow O$

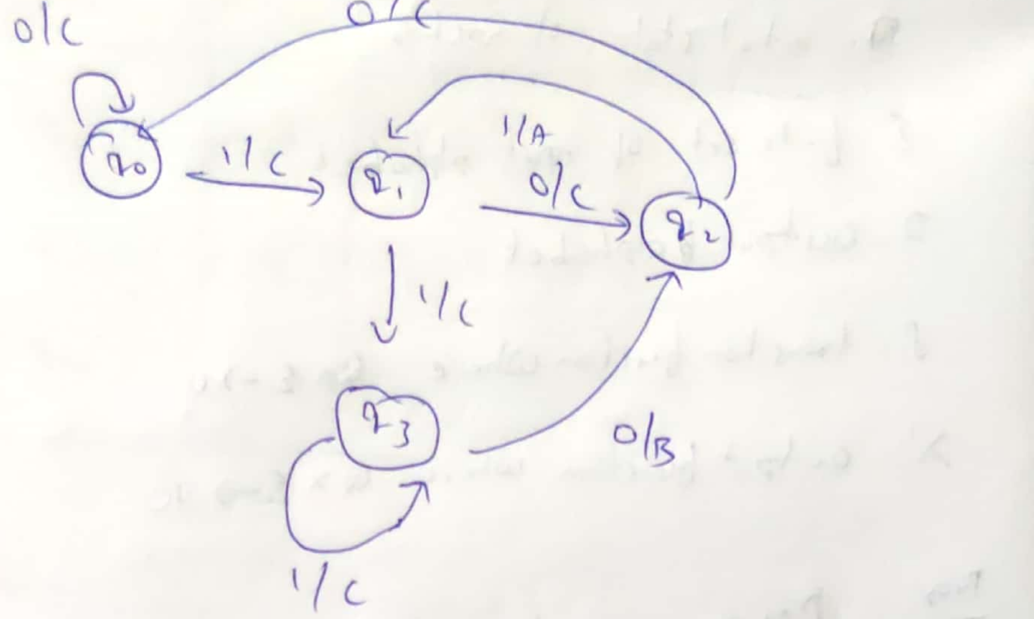
Prog: Design a Mealy machine for a binary input sequence such that if it has a substring 101, the machine output A. if the input has substring 110, it outputs B otherwise it outputs C.

Ans :

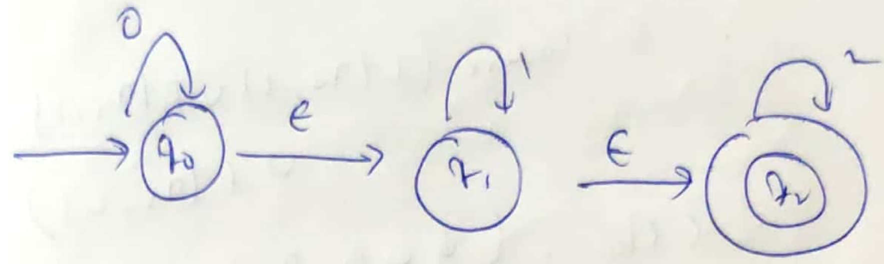


Now we will insert the possibilities of 0's and 1's

for each state. Thus the mealy machine becomes



## NFA-$\epsilon$ to NFA




$\delta'(q_0, 0) = \epsilon\text{-closure}(\delta(\delta'(q_0, \epsilon), 0))$

$= \epsilon\text{-closure}(\delta(q_0, q_1, q_2), 0))$

$= \epsilon\text{-closure}(q_0 \cup \emptyset \cup \emptyset)$

$= \epsilon\text{-closure}(q_0)$

$= \{q_0, q_1, q_2\}$

$\delta'(q_0, 1) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), 1))$

$= \epsilon\text{-closure}(\delta(q_0, q_1, q_2), 1))$

$= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$

$= \epsilon\text{-closure}(\emptyset \cup q_1 \cup \emptyset)$

$= \epsilon\text{-closure}\{q_1\}$

$= \{q_1, q_2\}$

$\delta'(q_0, 2)$

$= \epsilon\text{-closure}(\delta(\delta'(q_0, \epsilon), 2))$

$= \epsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2)$
$\cup \delta(q_2, 2))$

$= \epsilon\text{-closure}(\phi \cup \phi \cup q_2)$

$= \epsilon\text{-closure}(q_2)$

$= \{q_2\}$

$\delta'(q_1, 0)$

$= \epsilon\text{-closure}(\delta(\delta'(q_1, \epsilon), 0))$

$= \epsilon\text{-closure}(\delta(q_1 q_2), 0))$

$= \epsilon\text{-closure}(\delta(q_1, 0) \cup \delta(q_2, 0))$

$= \epsilon\text{-closure}(\phi)$

$= \phi$ //

$\delta'(q_1, 1) = \epsilon\text{-closure}(\delta(\delta'(q_1, \epsilon), 1))$

$= \epsilon\text{-closure}(\delta(q_1, q_2), 1))$

$= \epsilon\text{-closure}(\delta(q_1, 1) \cup \delta(q_2, 1))$

$= \epsilon\text{-closure}(q_1)$

$= \{q_1, q_2\}$

$\delta'(q_1, 2) = \epsilon\text{-closure}(\delta(\delta'(q_1,\epsilon), 2))$

$= \epsilon\text{-closure}(\delta(q_1, 2) \cup \delta(q_2, 2))$

$= \epsilon\text{-closure}(\phi \cup q_2)$

$= \epsilon\text{-closure}(q_2)$

$= \{q_2\}$

$\delta'(q_2, 0) = \epsilon\text{-closure}(\delta(\delta'(q_2,\epsilon), 0))$

$= \epsilon\text{-closure}(\delta(q_2), 0))$

$= \epsilon\text{-closure}(\delta(q_2), 0))$

$= \epsilon\text{-closure}(\phi)$

$= \phi$ //

$\delta'(q_2, 1)$

$= \epsilon\text{-closure}(\delta(\delta'(q_2,\epsilon), 1))$

$= \epsilon\text{-closure}(\delta(q_2, 1))$

$= \epsilon\text{-closure}(\phi)$

$= \phi$ //

$\delta'(q_2, 2)$

$= \epsilon\text{-closure}(\delta(\delta'(q_2,\epsilon), ))$

$= \epsilon\text{-closure}(\delta(q_2), 2))$

$= \{q_2\}$

$\delta'(q_0, 0) = \{q_0, q_1, q_2\}$

$\delta'(q_0, 1) = \{q_1, q_2\}$

$\delta'(q_0, 2) = \{q_2\}$

$\delta'(q_1, 0) = \{\phi\}$

$\delta'(q_1, 1) = \{q_1, q_2\}$

$\delta'(q_1, 2) = \{q_2\}$

$\delta'(q_2, 0) = \{\phi\}$

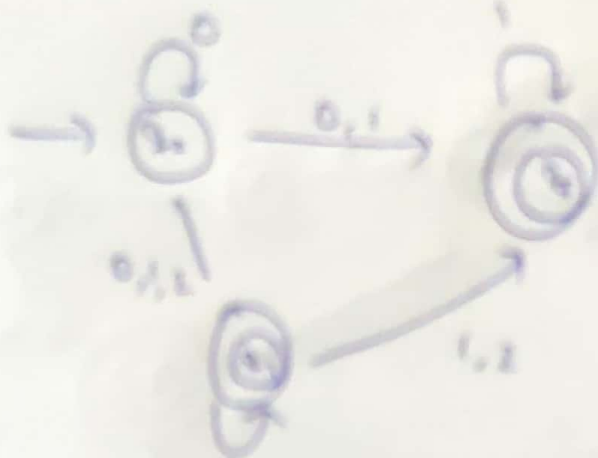
$\delta'(q_2, 1) = \{\phi\}$

$\delta'(q_2, 2) = \{q_2\}$

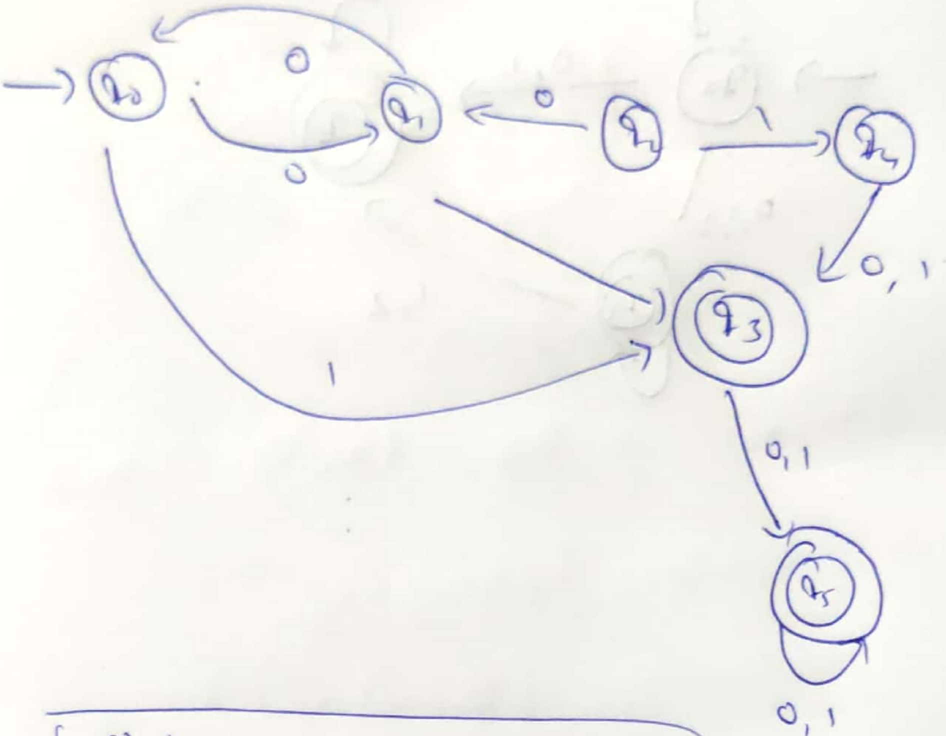
Transition table

| state | 0 | 1 | 2 |
|---|---|---|---|
| $q_0$ | $\{q_0, q_1, q_2\}$ | $\{q_1, q_2\}$ | $\{q_2\}$ |
| $q_1$ | $\phi$ | $\{q_1, q_2\}$ | $\{q_2\}$ |
| $q_2$ | $\phi$ | 0 | $\{q_2\}$ |

NFA without Epsilon



# Minimization of DFA




| State | 0 | 1 |
|---|---|---|
| $q_0$ | $q_1$ | $q_3$ |
| $q_1$ | $q_0$ | $q_3$ |
| * $q_3$ | $q_5$ | $q_5$ |
| * $q_5$ | $q_5$ | $q_5$ |

| State | 0 | 1 |
|---|---|---|
| $q_0$ | $q_1$ | $q_3$ |
| $q_1$ | $q_0$ | $q_3$ |

| State | 0 | 1 |
| --- | --- | --- |
| $q_3$ | $q_5$ | $q_5$ |
| $q_5$ | $q_5$ | $q_5$ |

| state | 0 | 1 |
| --- | --- | --- |
| $q_3$ | $q_3$ | $q_3$ |

| State | 0 | 1 |
| --- | --- | --- |
| $q_0$ | $q_1$ | $q_3$ |
| $q_1$ | $q_0$ | $q_3$ |
| $q_3$ | $q_3$ | $q_3$ |

transition diagram of minimized fA.