Traffic Operations

L3: Fundamental Diagrams

Konstantinos Gkiotsalitis

Q1 — 2018-19

Center for Transport Studies - University of Twente

Lecture Outline

Topics

- Representation of Macroscopic and Microscopic Data
- Fundamental Diagram
- Speed-density, flow-density and speed-flow diagrams

Related Material

- Chapter 4, Traffic Flow Dynamics — Data, Models and Simulation, M. Treiber, A. Kesting
- Malachy Carey & Michael Bowers (2011), A Review of Properties of Flow-Density Functions, Transport Reviews, 32:1, 49-73

TIME SERIES DATA FROM A CROSS-SECTIONAL DETECTOR

Time Series of Macroscopic Quantities

Consider the speed and the traffic flow[1] measurements at a location x_0.

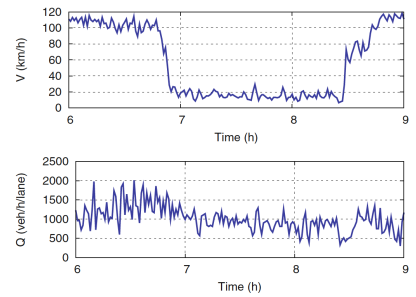
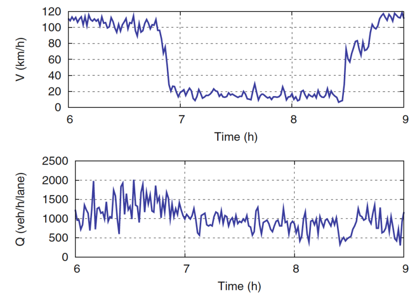


Figure 1: Time series of speed and flow measurements at a cross-section x_0 (from 6 AM to 9 AM)

[1] typically the traffic flow at a location x_0 is measured over a time period T and is expressed in veh/h even if time period T is very short.

Time Series of Macroscopic Quantities

One might first assume that a traffic breakdown occurred at $\sim$ 7 AM because the vehicle speed dropped from $\simeq$120km/h to $\simeq$20km/h.



Question

Why is it wrong to conclude from the time series that

- the traffic breakdown occurred at around 7 AM?
- the breakdown occurred at location x_0?

Answer

The exact location and timing of the breakdown **cannot be established using only time series data** from one cross-section x_0.

As presented in fig.2, a breakdown can start at a downstream[2] location before the time the significant speed drop at the cross-section x_0 is observed.

[2] similarly to hydraulics, in traffic operations a downstream location of a fixed location x_0 is the location after location x_0 in the direction of traffic flow

Time Series of Macroscopic Quantities

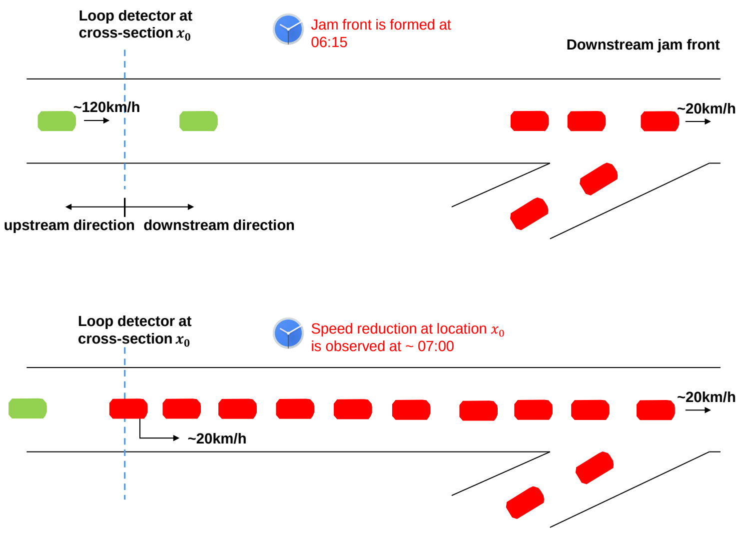


Figure 2: Traffic conditions and observed speeds at different time instances

Upstream and Downstream

There are two very common terms in traffic operations that indicate direction (the terms *upstream* and *downstream*).

Upstream and Downstream

There are two very common terms in traffic operations that indicate direction (the terms *upstream* and *downstream*).

- the upstream section of a cross-section x_0 is the section that is in the **opposite direction of traffic**

Upstream and Downstream

There are two very common terms in traffic operations that indicate direction (the terms *upstream* and *downstream*).

- the upstream section of a cross-section x_0 is the section that is in the **opposite direction of traffic**
- the downstream section of a cross-section x_0 is the section that is in the **direction of traffic**

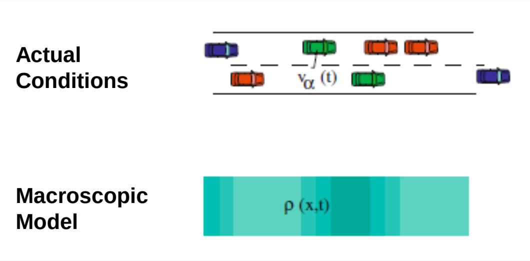
The terms *upstream* and *downstream* are very important for defining the formation and the movement of shockwaves as it will be described in the following lectures.

HYDRODYNAMIC RELATION

Hydrodynamic Relation

Macroscopic models describe traffic flow analogously to liquids or gases in motion. Hence they are sometimes called **hydrodynamic models**.

The dynamical variables are locally aggregated quantities such as the **traffic density** k, **flow** q, and **space mean speed** v.



Hydrodynamic Relation

Let k denote the **traffic density**, that is, the number of vehicles occupying a unit length of road in veh/km. If x vehicles are uniformly distributed on a road of length L, then the density is $k = x/L$.

The **flow rate** (q) denotes the number of vehicles which pass a point on a road during a specified time interval in veh/h.

The **space mean speed** (v) is the length of a road segment divided by the average time taken to traverse it (we hereafter refer to this as the speed).

Hydrodynamic Relation

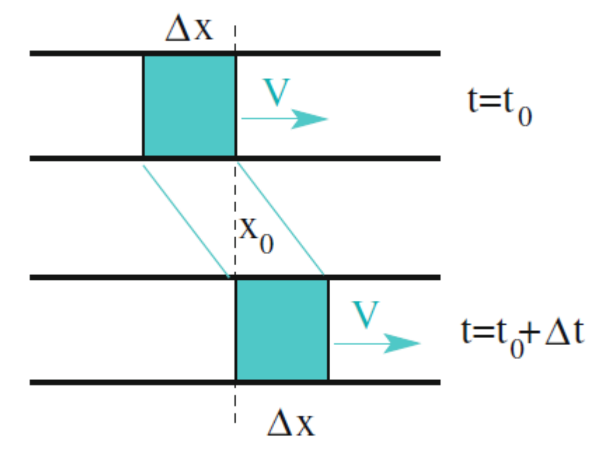
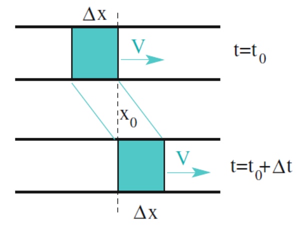


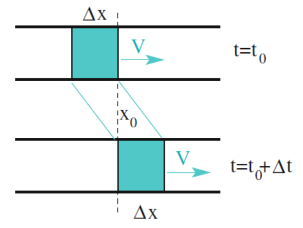
Figure 3: Movement of vehicles over time at the macroscopic level

Hydrodynamic Relation



If the section Δx belongs to the road of length L which has a density k, then it contains $k\Delta x$ vehicles.

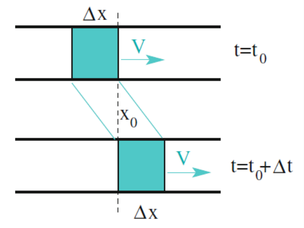
Hydrodynamic Relation



If the section Δx belongs to the road of length L which has a density k, then it contains $k\Delta x$ vehicles.

If v is the space-mean-speed within the road of length L, all vehicles $\Delta n = k\Delta x$ have completely passed the fixed location x_0 within the time interval $\Delta t = \frac{\Delta x}{v}$. Thus, at this location we have a traffic flow:

Hydrodynamic Relation



If the section Δx belongs to the road of length L which has a density k, then it contains $k\Delta x$ vehicles.

If v is the space-mean-speed within the road of length L, all vehicles $\Delta n = k\Delta x$ have completely passed the fixed location x_0 within the time interval $\Delta t = \frac{\Delta x}{v}$. Thus, at this location we have a traffic flow:

$$q(x_0, \Delta t) = \frac{\Delta n}{\Delta t} = \frac{k\Delta x}{\Delta x / v} = kv$$

This is known as the **hydrodynamic relation**.

Hydrodynamic Relation - Role of Space Mean Speed

The **hydrodynamic relation** connects the traffic flow, the traffic density and the space mean speed.

Hydrodynamic Relation - Role of Space Mean Speed

The **hydrodynamic relation** connects the traffic flow, the traffic density and the space mean speed.

It should be noted here that several works use (not completely correctly) the **time-mean-speed** instead of the space-mean-speed at the hydrodynamic relation since the former is **easier to compute**.

Hydrodynamic Relation - Role of Space Mean Speed

The **hydrodynamic relation** connects the traffic flow, the traffic density and the space mean speed.

It should be noted here that several works use (not completely correctly) the **time-mean-speed** instead of the space-mean-speed at the hydrodynamic relation since the former is **easier to compute**.

However, using the time-mean-speed leads to larger representation errors. If only cross-sectional data is available, it is advised to use the **harmonic mean speed** instead.

FUNDAMENTAL DIAGRAMS

Fundamental Diagram(s)

In traffic flow theory the **relations between the macroscopic characteristics of a flow** are called ”fundamental diagram(s)”. Three are in use, namely:

- flow - density $q = q(k)$
- speed - density $u = u(k)$
- speed - flow $u = u(q)$

It is important to understand that these three relations represent the same information and, using the hydrodynamic relation, one can deduce from one relation the other two.

Fundamental Diagram(s)

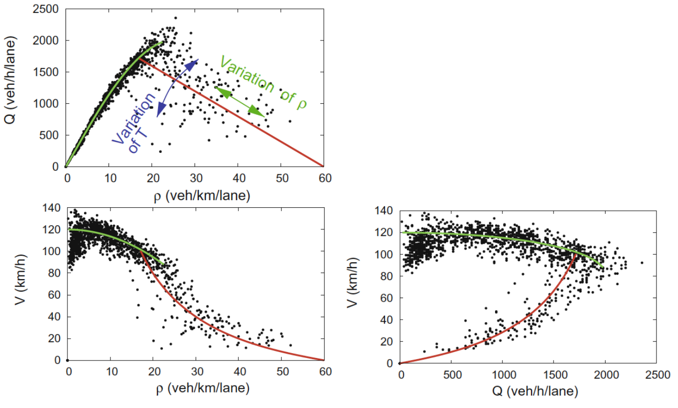


Figure 4: Flow-density, speed-density, and **speed-flow** diagrams of the 1-minute data captured on the Autobahn A5 near Frankfurt, Germany.

What do you notice from the empirical data?

Speed-Density Fundamental Diagram of Greenshields

The exact relationship between speed and density **can deviate significantly from road segment to road segment**.

Greenshields[3] conducted several empirical experiments and proposed the use of a **linear function** for the speed-density relationship (fig.5).

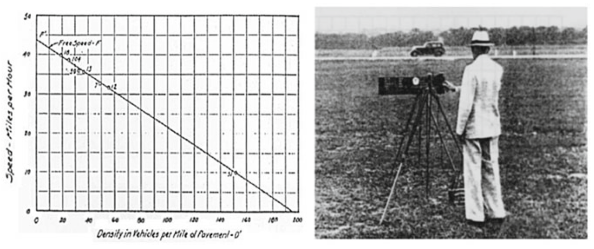


Figure 5: Historical speed-density diagram of Greenshields

[3] Greenshields, B. D. 1935. A study of traffic capacity. Proceedings of Highway Research Board, 14: 448-477.

Speed-Density Fundamental Diagram of Greenshields

$k(x, t_0)$ is the density of a road section x at a time instance t_0;

$v(x, t_0)$ is the space-mean-speed at the road section x at time instance t_0. For simplification purposes, we set $k \leftarrow k(x, t_0)$ and $v \leftarrow v(x, t_0)$.

Greenshields' linear function of the speed-density relationship at a specific road section x is:

$$v(k) = v_f(1 - \frac{k}{k_j})$$

This function is known as the **speed-density fundamental diagram of Greenshields**.

Speed-Density Fundamental Diagram of Greenshields

The **speed-density fundamental diagram of Greenshields**

$$v(k) = v_f(1 - \frac{k}{k_j})$$

can be completely determined by knowing two points on the line:

Speed-Density Fundamental Diagram of Greenshields

The **speed-density fundamental diagram of Greenshields**

$$v(k) = v_f(1 - \frac{k}{k_j})$$

can be completely determined by knowing two points on the line:

- The free flow speed, v_f, when the traffic density is zero $(k = 0; v = v_f)$

Speed-Density Fundamental Diagram of Greenshields

The **speed-density fundamental diagram of Greenshields**

$$v(k) = v_f(1 - \frac{k}{k_j})$$

can be completely determined by knowing two points on the line:

- The free flow speed, v_f, when the traffic density is zero ($k = 0; v = v_f$)
- The traffic density[4], k_j, when the speed is zero ($k = k_j; v = 0$) which is known as jam density.

[4] k_j is also known as jam density

Speed-Density Fundamental Diagram of Greenshields

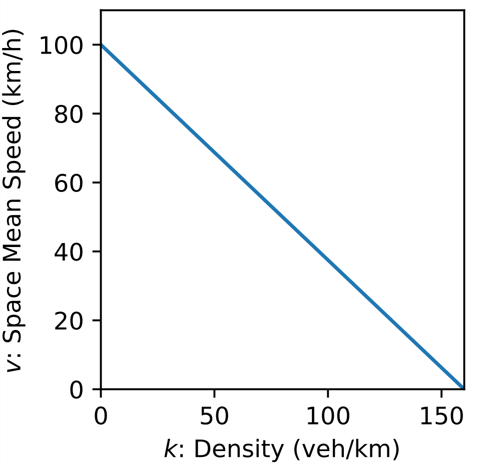


Figure 6: Greenshields Speed-Density fundamental diagrams for $v_f = 100$ km/h and $k_j = 160$ veh/km

Speed-Density Fundamental Diagram of Greenshields

Greenshields' linear function of the speed-density relationship implies that the **space mean speed of vehicles** on a road segment **reduces linearly with the increase of density, until reaching the jam density condition**, k_j, where all vehicles are in a standstill.

The free flow speed, v_f, is the minimum of

Speed-Density Fundamental Diagram of Greenshields

Greenshields' linear function of the speed-density relationship implies that the **space mean speed of vehicles** on a road segment **reduces linearly with the increase of density, until reaching the jam density condition**, k_j, where all vehicles are in a standstill.

The free flow speed, v_f, is the minimum of

1. the actual desired speed of the drivers,

Speed-Density Fundamental Diagram of Greenshields

Greenshields' linear function of the speed-density relationship implies that the **space mean speed of vehicles** on a road segment **reduces linearly with the increase of density, until reaching the jam density condition**, k_j, where all vehicles are in a standstill.

The free flow speed, v_f, is the minimum of

1. the actual desired speed of the drivers,
2. the physically possible attainable speed (especially relevant for trucks on uphill slopes),

Speed-Density Fundamental Diagram of Greenshields

Greenshields' linear function of the speed-density relationship implies that the **space mean speed of vehicles** on a road segment **reduces linearly with the increase of density, until reaching the jam density condition**, k_j, where all vehicles are in a standstill.

The free flow speed, v_f, is the minimum of

1. the actual desired speed of the drivers,
2. the physically possible attainable speed (especially relevant for trucks on uphill slopes),
3. and, possibly an administrated speed limit (plus the drivers' average speeding).

However, v_f is often directly referred to as the desired speed.

Speed-density diagram of the aggregated vehicle speed over traffic density from measurements on Autobahn A9 near Munich.

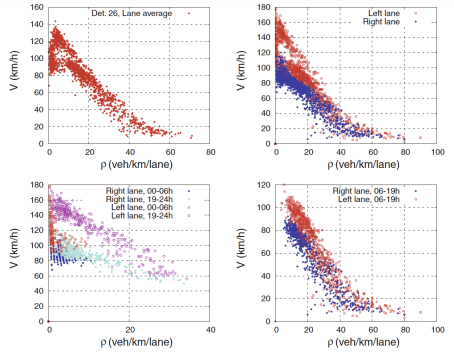


Figure 7: Speed-density relation obtained from one-minute data using the average over both lanes (top left), individual averages of both lanes (top right), individual averages conditioned on night (bottom left) and day hours (bottom right)

Speed-density diagram from empirical data

In very low traffic, the drivers are not influenced by other vehicles and we obtain the average free flow speed v_f for density $\rho \to 0$.

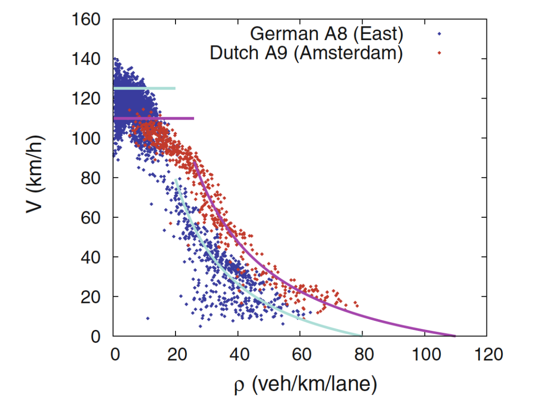


Figure 8: A8 Munich-Salzburg and A9 Amsterdam

Speed-density diagram from empirical data

In denser traffic, the speed difference towards lanes tends towards zero, leading to a speed synchronization of lanes as it is depicted in diagram 9 from Autobahn A9-South near Munich.

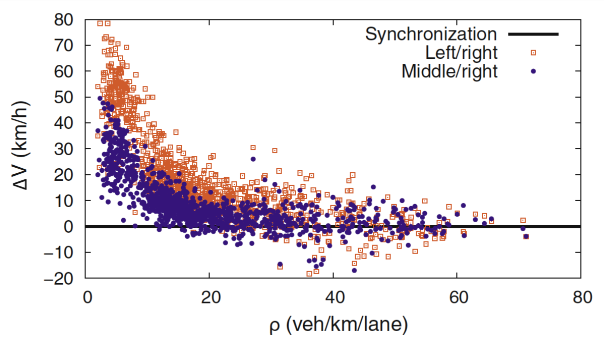


Figure 9: In denser traffic, the speed difference among lanes tends towards zero, leading to a speed synchronization of lanes

Time gaps from single-vehicle data

Using single-vehicle data, we can also obtain the distributions of time gaps as depicted in diagram 10.

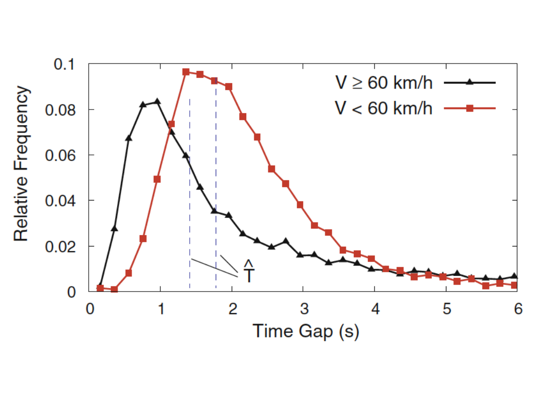
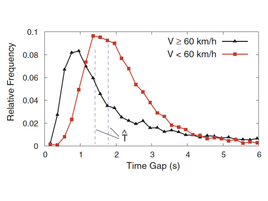


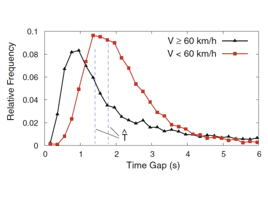
Figure 10: Distribution of time gaps measured on the Dutch A9

Time gaps from single-vehicle data



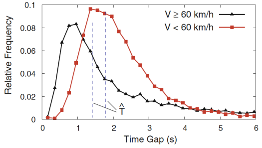
- Time gaps are broadly scattered with large standard deviations

Time gaps from single-vehicle data



- Time gaps are broadly scattered with large standard deviations
- In free traffic (with speeds larger than some critical speed V_c) the most probable time gap $\hat{T}$ (the statistical mode) is significantly smaller than in congested traffic (Why?)

Time gaps from single-vehicle data



- Time gaps are broadly scattered with large standard deviations
- In free traffic (with speeds larger than some critical speed V_c) the most probable time gap $\hat{T}$ (the statistical mode) is significantly smaller than in congested traffic (Why?)
- $\hat{T}$ can be smaller than the recommended safe time gap in Europe (safety distance (in meters) equals speed (in km/h) divided by two, corresponding to 1.8 s)

Flow-Density Fundamental diagram

The flow-density *fundamental diagram* describes the **theoretical relation between density and flow** in **stationary homogeneous traffic**, i.e., the steady state equilibrium of identical driver-vehicle units.

Flow-Density Fundamental diagram

The flow-density *fundamental diagram* describes the **theoretical relation between density and flow** in **stationary homogeneous traffic**, i.e., the steady state equilibrium of identical driver-vehicle units.

In contrary, the **flow-density diagram** represents aggregated empirical data observations and generally describes non-stationary heterogeneous traffic, i.e., different driver-vehicle units far from equilibrium (Fig.11).

Flow-Density Diagram

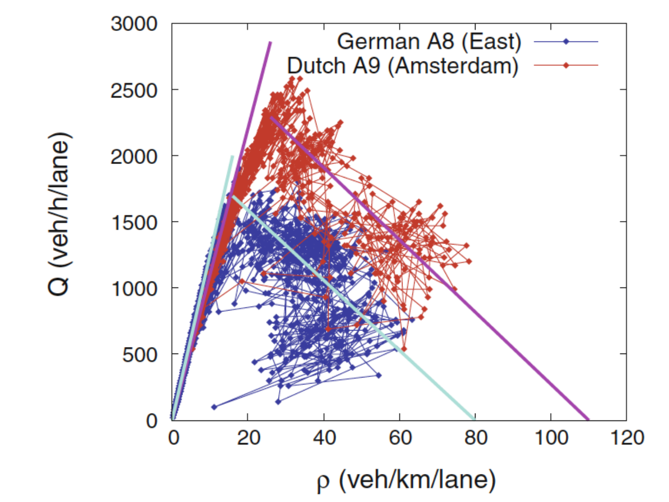


Figure 11: Flow-density diagram

Differences between Fundamental and Flow-Density Diagram

There are several reasons that can explain the differences between the *fundamental* flow-density diagram and the flow-density diagram:

Differences between Fundamental and Flow-Density Diagram

There are several reasons that can explain the differences between the *fundamental* flow-density diagram and the flow-density diagram:

- The measurement process induces systematic errors;

Differences between Fundamental and Flow-Density Diagram

There are several reasons that can explain the differences between the *fundamental* flow-density diagram and the flow-density diagram:

- The measurement process induces systematic errors;
- The traffic flow is not at equilibrium (transient states);

Differences between Fundamental and Flow-Density Diagram

There are several reasons that can explain the differences between the *fundamental* flow-density diagram and the flow-density diagram:

- The measurement process induces systematic errors;
- The traffic flow is not at equilibrium (transient states);
- The traffic flow has spatial inhomogeneities or contains non-identical driver-vehicle units.

Typical Parameters of Fundamental Diagrams

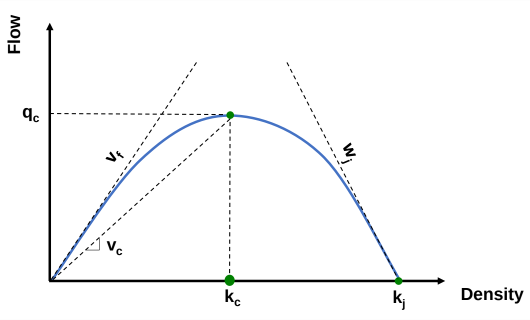


Figure 12: Example of Important Parameters of a Flow-Density Fundamental Diagram

Typical Parameters of Fundamental Diagrams

- Free-flow speed, v_f, corresponds to sufficiently low traffic densities where there is little interaction between vehicles and is the initial gradient of the fitted function $q = f(k)$;

Typical Parameters of Fundamental Diagrams

- Free-flow speed, v_f, corresponds to sufficiently low traffic densities where there is little interaction between vehicles and is the initial gradient of the fitted function $q = f(k)$;
- Jam density, k_j, is the maximum traffic density when a speed of 0 is imposed (as in a traffic jam or at a traffic stop signal);

Typical Parameters of Fundamental Diagrams

- Free-flow speed, v_f, corresponds to sufficiently low traffic densities where there is little interaction between vehicles and is the initial gradient of the fitted function $q = f(k)$;
- Jam density, k_j, is the maximum traffic density when a speed of 0 is imposed (as in a traffic jam or at a traffic stop signal);
- Wave speed at jam density, c_j (or w_j), is the gradient of the flow-density function of the fundamental diagram, $q = f(k)$, as density approaches jam density;

Typical Parameters of Fundamental Diagrams

- Free-flow speed, v_f, corresponds to sufficiently low traffic densities where there is little interaction between vehicles and is the initial gradient of the fitted function $q = f(k)$;
- Jam density, k_j, is the maximum traffic density when a speed of 0 is imposed (as in a traffic jam or at a traffic stop signal);
- Wave speed at jam density, c_j (or w_j), is the gradient of the flow-density function of the fundamental diagram, $q = f(k)$, as density approaches jam density;
- Maximum flow rate, q_c, which is usually referred to as the capacity;

Typical Parameters of Fundamental Diagrams

- Free-flow speed, v_f, corresponds to sufficiently low traffic densities where there is little interaction between vehicles and is the initial gradient of the fitted function $q = f(k)$;
- Jam density, k_j, is the maximum traffic density when a speed of 0 is imposed (as in a traffic jam or at a traffic stop signal);
- Wave speed at jam density, c_j (or w_j), is the gradient of the flow-density function of the fundamental diagram, $q = f(k)$, as density approaches jam density;
- Maximum flow rate, q_c, which is usually referred to as the capacity;
- k_c the corresponding density to the maximum flow rate (known also as critical density);

Typical Parameters of Fundamental Diagrams

- Free-flow speed, v_f, corresponds to sufficiently low traffic densities where there is little interaction between vehicles and is the initial gradient of the fitted function $q = f(k)$;
- Jam density, k_j, is the maximum traffic density when a speed of 0 is imposed (as in a traffic jam or at a traffic stop signal);
- Wave speed at jam density, c_j (or w_j), is the gradient of the flow-density function of the fundamental diagram, $q = f(k)$, as density approaches jam density;
- Maximum flow rate, q_c, which is usually referred to as the capacity;
- k_c the corresponding density to the maximum flow rate (known also as critical density);
- v_c the corresponding space mean speed to the maximum flow rate given by the slope of the secant through (0,0) and (k_c, q_c).

General characteristics of a flow-density fundamental diagram

Any **point** of the flow-density fundamental diagram describes **the traffic state** at this point in terms of traffic density, traffic flow and space mean speed.

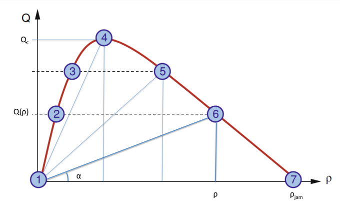


Figure 13: Flow-Density Fundamental diagram

SINGLE-REGIME FUNDAMENTAL DIAGRAMS

Single-regime Fundamental diagrams (Greenshields)

Recall: Greenshields proposed the use of a linear function for the **speed-density fundamental diagram**:

$$v(k) = v_f(1 - \frac{k}{k_j})$$

Using the hydrodynamic relation $q = kv$, one can derive the **flow-density fundamental diagram** implied by the Greenshields model:

$$q(k) = v_f(k - \frac{k^2}{k_j})$$

Single-regime Fundamental diagrams (Greenshields)

$$q = v_f(k - \frac{k^2}{k_j})$$

- the wave speed at the jam density w_j is equal to the negative of the free flow speed because $q'(k) = v_f - \frac{2kv_f}{k_j}$, thus $w_j = q'(k_j) = v_f - \frac{2k_jv_f}{k_j} = -v_f$.

Single-regime Fundamental diagrams (Greenshields)

$$q = v_f(k - \frac{k^2}{k_j})$$

- the wave speed at the jam density w_j is equal to the negative of the free flow speed because $q'(k) = v_f - \frac{2kv_f}{k_j}$, thus $w_j = q'(k_j) = v_f - \frac{2k_jv_f}{k_j} = -v_f$.
- the flow $q(k)$ takes its maximum value at a critical density k_c where $q'(k_c) = 0$ or $v_f - 2v_f\frac{k_c}{k_j} = 0 \Rightarrow k_c = \frac{k_j}{2}$.

Single-regime Fundamental diagrams (Greenshields)

$$q = v_f(k - \frac{k^2}{k_j})$$

- the wave speed at the jam density w_j is equal to the negative of the free flow speed because $q'(k) = v_f - \frac{2kv_f}{k_j}$, thus $w_j = q'(k_j) = v_f - \frac{2k_jv_f}{k_j} = -v_f$.
- the flow $q(k)$ takes its maximum value at a critical density k_c where $q'(k_c) = 0$ or $v_f - 2v_f\frac{k_c}{k_j} = 0 \Rightarrow k_c = \frac{k_j}{2}$.
- the maximum flow $q_c = v_f(\frac{k_j}{2} - \frac{1/4k_j^2}{k_j})$ is always 1/4 times k_jv_f, which will not always match the empirical observations.

Single-regime Fundamental diagrams (Greenshields)

$$q = v_f(k - \frac{k^2}{k_j})$$

- the wave speed at the jam density w_j is equal to the negative of the free flow speed because $q'(k) = v_f - \frac{2kv_f}{k_j}$, thus $w_j = q'(k_j) = v_f - \frac{2k_j v_f}{k_j} = -v_f$.
- the flow $q(k)$ takes its maximum value at a critical density k_c where $q'(k_c) = 0$ or $v_f - 2v_f \frac{k_c}{k_j} = 0 \Rightarrow k_c = \frac{k_j}{2}$.
- the maximum flow $q_c = v_f(\frac{k_j}{2} - \frac{1/4k_j^2}{k_j})$ is always 1/4 times $k_j v_f$, which will not always match the empirical observations.
- the critical space mean speed, v_c, is $q_c = v_c k_c \Rightarrow \frac{k_j v_f}{4} = v_c \frac{k_j}{2} \Rightarrow v_c = \frac{v_f}{2}$

Single-regime Fundamental diagrams (Greenshields)

Greenshields' FD is a single-regime model because it uses a single flow-density function for both the congested and the uncongested region.

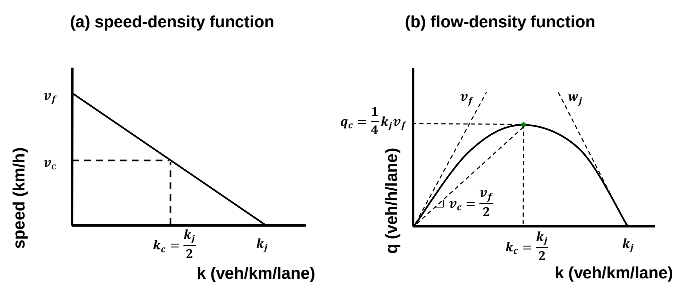


Figure 14: Speed-density and Flow-density Fundamental Diagrams implied by Greenshields

Single-regime Fundamental diagrams (Greenberg and Underwood)

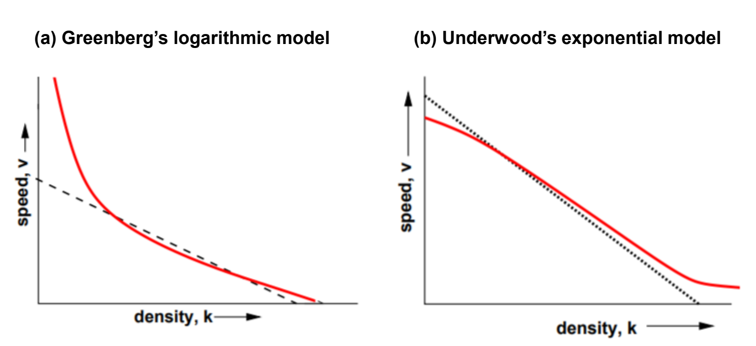
Author	Model	Parameters
Greenshields	$v = v_f(1 - \frac{k}{k_j})$	v_f, k_j
Greenberg	$v = v_c \ln(\frac{k_j}{k})$	v_c, k_j
Underwood	$v = v_f e^{-\frac{k}{k_c}}$	v_f, k_c

The above models are one-equation models, meaning that the models apply to the entire range of density.

Single-regime Fundamental diagrams (Greenberg)

Greenberg assumed a **logarithmic relation** between speed and density.

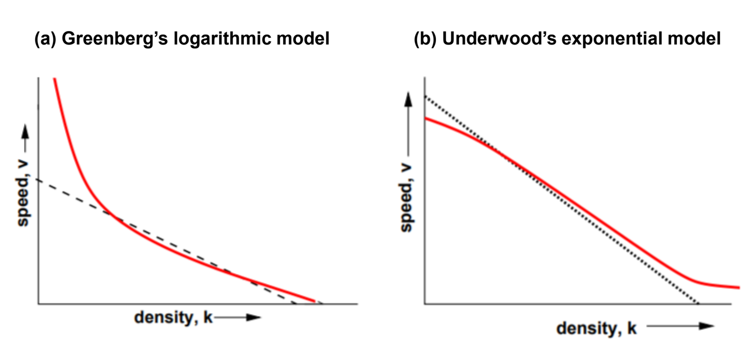
Main drawbacks of this model are that **as density tends to zero, speed tends to infinity** and the model is unable to predict the speeds at lower densities.



Single-regime Fundamental diagrams (Underwood)

Trying to overcome the limitation of Greenberg's model, Underwood puts forward an **exponential model** $v = v_f e^{-\frac{k}{k_c}}$.

In this model, **speed becomes zero only when density reaches infinity** which is the drawback of this model. Hence this cannot be used for predicting speeds at high densities.



Exercise 1

Derive and sketch the speed-density and the flow-density fundamental diagrams using the single-regime models of:

1. Greenshields when the free flow speed is $v_f = 100$km/h and the jam density $k_j = 160$veh/km
2. Greenberg when the critical speed is $v_c = 75$km/h and the jam density $k_j = 160$veh/km
3. Underwood when the free flow speed is $v_f = 100$km/h and the critical density $k_c = 75$veh/km

MULTI-REGIME FUNDAMENTAL DIAGRAMS

Multi-regime fundamental diagrams

All the above models are based on the assumption that the same speed-density relation is valid for the entire range of densities seen in traffic streams (**single-regime models**).

Multi-regime fundamental diagrams

All the above models are based on the assumption that the same speed-density relation is valid for the entire range of densities seen in traffic streams (**single-regime models**).

However, human behavior will be different at different densities. Several works have tried to capture those changes with the use of **multi-regime models**.

Triangular Fundamental Diagram

Newell[5] was the first to propose a **triangular flow-density fundamental diagram** with two regimes (congested and uncongested):

$$q(k) = \begin{cases} u_f k, \text{ for } 0 \leq k \leq k_c \\ q_c - \frac{k-k_c}{k_j-k_c} q_c, \text{ for } k_c < k \leq k_j \end{cases} \tag{1}$$

[5] Newell, G. F. 1993. A simplified theory of kinematic waves in highway traffic. Transportation Research Part B, 27(4): 281-313.

Triangular Fundamental Diagram

This relation yields the following fundamental diagram:

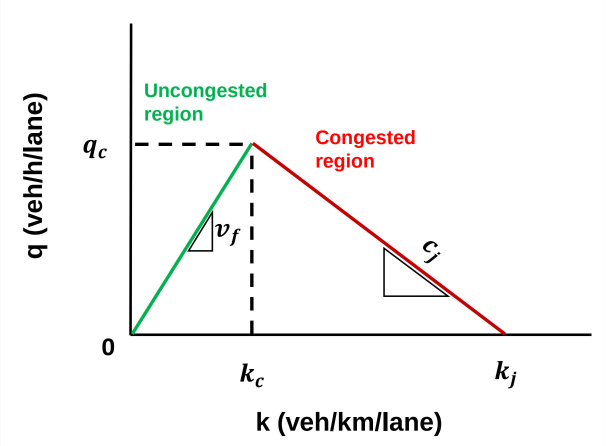
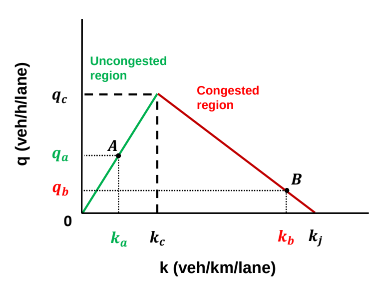


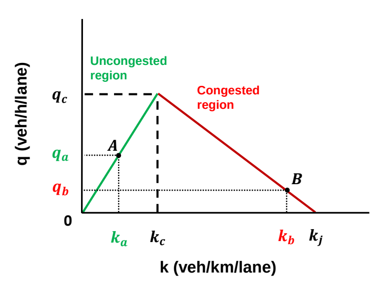
Figure 15: Triangular flow-density Fundamental Diagram

Triangular Fundamental Diagram



In the triangular approximation, vehicles are assumed to move with the free flow speed u_f below the critical density k_c. I.e., in traffic state A the slope of the secant through (0,0) and (k_a, q_a) is $v_a = \frac{q_a}{k_a} = \frac{v_f k_a}{k_a} = v_f$.

Triangular Fundamental Diagram



In the triangular approximation, vehicles are assumed to move with the free flow speed u_f below the critical density k_c. I.e., in traffic state A the slope of the secant through (0,0) and (k_a, q_a) is $v_a = \frac{q_a}{k_a} = \frac{v_f k_a}{k_a} = v_f$.

When the density k is greater than k_c, a vehicle queue forms, resulting in lower speeds for the traveling vehicles. I.e., in traffic state B the slope of the secant through (0,0) and (k_b, q_b) is:

$$v_b = \frac{q_b}{k_b} = \frac{1}{k_b}(q_c - \frac{k_b - k_c}{k_j - k_c} q_c) = v_f \frac{k_c}{k_b}(1 - \frac{k_b - k_c}{k_j - k_c}) < v_f.$$

Inverted λ Fundamental Diagram

Analyzing in more detail the empirical data from traffic measurements one can note that there is a significant capacity drop when we are entering the congested region.

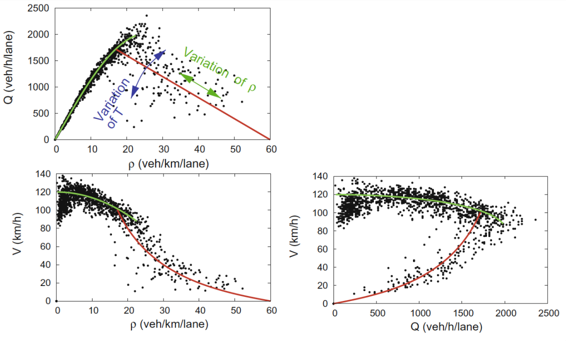


Figure 16: Flow-density, speed-density, and speed-flow diagrams of the 1-minute data captured on the Autobahn A5 near Frankfurt, Germany.

Inverted λ Fundamental Diagram

Several observations from empirical data have shown that there are in fact two capacities:

Inverted λ Fundamental Diagram

Several observations from empirical data have shown that there are in fact two capacities:

Free flow (or pre-queue) capacity: is the **maximum flow rate** just before the **on-set of congestion**. These maximum flows are characterized by the absence of queues or congestion upstream the bottleneck.

Inverted λ Fundamental Diagram

Several observations from empirical data have shown that there are in fact two capacities:

Free flow (or pre-queue) capacity: is the **maximum flow rate** just before the **on-set of congestion.** These maximum flows are characterized by the absence of queues or congestion upstream the bottleneck.

Queue discharge capacity: is the **maximum flow rate** that can be observed **as long as congestion exists.** That is, once there is a congested state upstream of the bottleneck and no influence from bottlenecks farther downstream.

Inverted λ Fundamental Diagram

Both free flow (or pre-queue) capacity and queue discharge capacity are measured downstream the bottleneck. The difference between the pre-queue and the queue discharge capacity is known as **capacity drop** and is usually in the range of 1% to 15%.

This implies that once a traffic has emerged, the traffic demand has to fall to a much lower value to dissolve the jam. This capacity drop describes the fact that, **once congestion has formed, drivers are not maintaining a time headway that is as close as it was before the speed breakdown.**

Inverted λ Fundamental Diagram

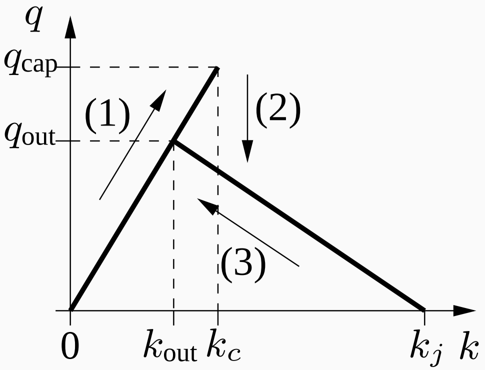


Figure 17: the typical inverted λ shape of the (k, q) fundamental diagram, showing a capacity drop from q_{cap} to below $q_{out} < q_{cap}$ (the queue discharge flow). The hysteresis effect occurs when going from the congested to the free-flow branch, as indicated by the three arrows (1) - (3).

Inverted λ Fundamental Diagram

The inverted λ diagram has a flow-density fundamental relation of the form:

$$q(k) = \begin{cases} v_f k, \text{ for } 0 \leq k \leq k_c \\ v_f k_1 - \frac{k-k_1}{k_j-k_1} v_f k_1, \text{ for } k \geq k_1 \end{cases} \tag{2}$$

where $k \geq k_1$ means that traffic is congested ($k_1 = k_{out}$ from Fig.17). This fundamental diagram shape allows two traffic states to have the same flow but different densities.

Exercise 2

Derive and sketch the flow-density fundamental diagrams using the multi-regime triangular flow-density model when the free flow speed is $u_f = 100$km/h, the capacity is $q_c = 1900$veh/h and the traffic density $k_j = 150$veh/km

Exercise 3

1. A highway section has an average spacing of 8 m under jam conditions and a free-flow speed of 100 km/h. Assuming that the relationship between speed and density is linear (hint: follows Greenshields' model), determine the jam density, the maximum flow, the density at maximum flow, and the speed at maximum flow.
2. A section of highway is known to have a free-flow speed of 100 km/h and a capacity of 3300 vehicles/h. In a given hour, 2100 vehicles were counted at a point along the road. If Greenshields' model applies, what would be the space mean speed of these 2100 vehicles?