ENGR 652 Advanced Optical Image Processing Project 4
Image Restroation Using The Pseudo-Inverse and Wiener Filters
Due Date Nov 21

1. The problem

Restoring an image corrupted by an LSI blur operator and with additive Gaussian noise is the classic problem in image restoration. Here we study the use of the pseudo-inverse filter and two forms of the Wiener filter in combating these degradatios. In particular, we will deifne an impulse invariant system to model simple uniform linear motion blur. We will artificially degrade an image using our model and then we will attemot to restore the image. We will quantitatively and subjectively compare various image restoration methods and we investigate the performance of the filters as a function of their free tuning parameters.

2. Background

When relative motion exists between the scene and the focal plane array during the temporal integration of the array, motion blur results. Let's consider a purely horizontal blur resulting from uniform linear motion such that the scene spans exactly 9 detector spacings during temporal integration. The continuous LSI impulse response of such a blur process is given by

$$h(x,y) = \frac{1}{9} rect\left(\frac{x}{9A}, \frac{y}{A}\right) \tag{1}$$

where A is the square detector width. The continuous-space Foureir transform of this impulse response is

$$H(u,v) = A^2 \, sinc(9Au, Av). \tag{2}$$

This blur function is not band-limited, and therefore, cannot be perfeclty represented by an impulse invariant system. However, if one combines the diffraction-limited frequency response with the motion blur caused by the spatio-temporal integration of the detectors, the combined system is in fact band-limited and can be perfectly modeled by an impulse invariant system (provided that the image is sampled at the Nyquist rate). However, in many cases, good results can be obtained by tracking the motion blur alone, because it is the dominant degradation (compared with the diffraction blur) and because the motion blur sinc "dies" out quickly. Thus, we can argue that it is nealry band-limited. In fact, as we discussed in class, most imaging systems sample at twice the frequency of the first detector sinc zero-crossing, and not twice the diffraction-limited cut-off frequency (which would be technically required). Sampling the continuous impulse response at the detector spacing A (e.g., 100% fill factor detectors), the resulting impulse invariant system becomes a simple discrete 9x1 moving average filter.

Note that since the continuous system is not band-limited, a different filter would result using the frequency doamin mapping of (2) to obtain the impulse invariant system. Here we are assuming that the discrepency is negligible and the simpler impulse invariant system is obtainrd in the space domain. Again, this problem could be resolved by incorporating the diffraction-limited blur into the degradation model.

3. Procedure

- Load the kett.mat image. Generate a 9x1 (horizontal) moving average impulse response and convolve this with the kett.mat image (use softpad and the valid option to keep the output the same size as the input). Next add Gaussian noise with standard deviation 2 to the blurred image to create a more realistic scenario (see runpseudo.m).

- Begin by restoring the degraded image using a pseudo-inverse (pseudoinv2d.m). Pad the image with softpad.m with 100 pixels on all sides prior to the filtering and then crop the image back to the same size as the original so that it perfectly lines up with the original. This can be done simply by setting the parameter border=100 in pseudoinv2d.m. Use 20 logarithmically spaced thresholds ranging from 10^{-2} to 10^{0} and compute the mean absolute error (MAE) between each and the original. Generate a properly labeled sampled plot (use semilogx.m) showing MAE vs. the threshold value. Use a solid line with circles where the data points lie. Use logspace.m to generate the logarithmically space values.

- Write a function modeled after pseudoinv2d.m which implements the constant noise to signal ration (NSR) version of the Wiener filter implemented with DFTs.

- Use a border padding of 100 on all sides (as above) and use 20 logarithmically spaced NSRs ranging from 10^{-3} to 10^{-1} and compute the mean absolute error (MAE) between each and the original. Generate a properly labeled semilog plot (use semilogx.m) showing MAE vs. the threshold value. Use a solid line with circles where the data points lie.

- Write a function modeled after pseudoinve2d.m which implements the parametric version of the Wiener filter implemented with DFT's which uses a NSR power spectrum ratio of

$$NSR(\omega_1,\omega_2) = \gamma\left|P(\omega_1,\omega_2)\right|^2, \tag{3}$$

  where,

$$P(\omega_1,\omega_2) = DSFT\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}. \tag{4}$$

  Note that here the NSR increases with frequency. This makes sense if the signal power of your underlying image is higher at lower frequencies and the noise is white. Use the DFT to evaluate samples of (4) at the same resolution as the other components of the Wiener frequency response (i.e., use a DFT of the size of the image to be filtered).

- Use a border padding of 100 on all sides (as above) and use 20 logarithmically spaced gamma values ranging from 10^{-3} to 10^{-1} and compute the mean absolute error (MAE) between each and the original. Generate a properly labeled semilog plot (use semilogx.m) showing MAE vs. the threshold value. Use a solid line with circles where the data points lie.

- You might be curious to see what the magnitude frequency responses of the three types of filters look like. You will notice that the pseudo inverse goes to zero cleanly when the forward filter response gets too small. The constant NSR Wiener filter more smoothly transitions to zero. The parametric Wiener filter emphasizes the signal at lower frequencies where the NSR is the smallest (highest signal-to-noise ratio SNR).

4. What to turn in

In this project I require from you to write the theoretical part as well as the simulation results in a report. This report should include the following sections: Introduction, theoretical study of the

filters, experimental results, conclusion. Also, I ask that you work independently on the project and not discuss it with anyone other than myself. Further, please do not compare results or computer codes with others.
Here are some specific items that you should include in an appropriate place in your report:

- Equations of the theoretical frequency response of the three filters used here with some discussion about their development, parameters, and their interrelationship.

- Use dsftl.m to show the magnitude frequency response of the 9x1MA filter.

- Show the original and the corrupted images.

- Show the magnitude spectrum of the original and corrupted images using imspecxy.m. Notice that the magnitude spectrum of the corrupted image will have 8 dark vertical stripes corresponding to the notches in the MA filter. In the case of a 7x1 MA filter you will have 6 stripes, with a 5x1 MA you get 4, and you get 2 with a 3x1 MA filter. The number of stripes in the spectrum provides the clue needed to determine the appropriate MA filter length to use to model uniform linear motion when the exact motion and integration times are unknown (i.e., in most realistic applications).

- Semilog plot of MAE vs. pseudo-inverse filter threshold.

- Semilog plot of MAE vs. constant NSR for the Wiener filter.

- Semilog plot of MAE vs. gamma for the parametric Wiener filter.

- Show the restored image using the best (in an MAE sense) of the pseudo-inverse filters.

- Show the restored image using the best(in an MAE sense) of the constant NSR Wiener filters.

- Show the restored image using the best(in an MAE sense) of the parametric Wiener filters.

- Table of the minimum MAE's for the three filters along with the MAE for the unfiltered image (the observed image with no restoration).

- Plot the magnitude frequency responses of the three types of filters.

- Discussion of the results.

- MATLAB Code.