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Abstract

Owning to similar business nature, it should be possible to directly migrate successful airline revenue management techniques to the hotel domain. However, one of the salient differences between airlines and hotels is rarely highlighted—the network structure of length of stay or the displacement effect. The hotel patrons go from a first stay-over night to a last stay-over night in consecutive night stays. The arrival demands for multi-night stays and the lengths of stay are stochastic in nature.

In this paper, we propose a network optimization model for hotel revenue management under an uncertain environment. The network optimization is in a stochastic programming formulation so as to capture the randomness of the unknown demand (unknown number of arrivals and length of stays). A novel approach of robust optimization techniques for stochastic programming is applied to solve the problem. We also discuss the strategies for hotel management to take into account of risk trade-off; different pricing policies; cancellations and no-show; early check-outs; extended stay and over-booking are discussed. We showed that our proposed model can be modified to adopt these strategic considerations.
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1. Introduction

Revenue management (also known as yield management) is used to find optimal inventory allocation and scheduling strategies as well as price setting for perishable assets so as to maximize revenue within the planning horizon. Revenue management is rooted in the airline industry, in which revenue management systems have been applied for over 40 years [1]. Many successful revenue management systems have led to hundreds of million of dollars of improvement in revenue e.g. [2,3].

Other industries with similar characteristics to airlines are in the midst of developing their revenue management systems. The hotel business has the highest potential for application of revenue

management techniques as hotels share very close characteristics with airlines. For example: (i) both hotel rooms and air-seats are perishable and cannot be stored for future sale; (ii) capacity is usually fixed and the cost of instant expansion is very high (loss of goodwill and high costs in moving customers to other competitor hotels); and (iii) advance booking is allowed (and thus cancellations, no-shows and overbooking problems exist).

It would appear possible to directly migrate successful airline revenue management tools, such as overbooking models [3], inventory allocation for nested or non-nested models [2] to the hotel domain. However, one of the salient differences between airlines and hotels is rarely highlighted— the network structure of length of stay [4] or the displacement effect [5]. Hotel patrons go from a first stay-over night to a last stay-over night on consecutive night stays. The arrival demands for multi-night stays and the lengths of stay are unfortunately stochastic in nature.

There is a very rich literature on revenue management for the airline industry. For the hotel business, quantitative tools for solving revenue management problem are relatively limited when the displacement effect of multi-night stays is taken into consideration. However, Weatherford [6] reported that taking the length of stay into account in hotel revenue management can increase revenue by as much as 2.94%. In the hotel business, this means a saving in of millions of US dollars each year.

To our best knowledge from a review of the literature, simulation is the main tool currently used for the investigation of hotel revenue management problems of multi-night stays. Weatherford [6] and Birtan and Modschin's [7] used simulation models with data from hotels to test their heuristic approaches of accept/reject decisions for booking. Baker extended the above studies and compared them with some more of his heuristic models for overbooking and allocation [4].

In this paper, we propose a network optimization model for hotel revenue management under an uncertain environment. The network optimization is in a stochastic programming formulation so as to capture the randomness of the unknown demand (unknown number of arrivals and unknown length of stays). A novel approach of robust optimization techniques is applied to solve the problem.

The paper is organized as follows. Notations and parameters used in this paper will be introduced in Section 2, and the basic mathematical model formulation presented in Section 3. The importance of stochastic programming and the solution scheme for robust optimization will be discussed in Section 4. Some numerical illustrative examples are presented in Section 5. Section 6 shows some other considerations for hotel revenue management systems with managerial implications for parameter setting in our proposed mathematical models. Finally, some concluding remarks and future research recommendations are given in Section 7.

2. Notations and assumptions

The following are the major notations for parameters and variables used in this paper:

- $x_{i,j}$ is the number of bookings accepted (decision variables) for check-in on day i and check out on day j where $0 \leqslant i < j \leqslant T$. $i = \{0, 1, 2, \ldots, T-1\}$ is time index for check-in and $j = \{1, 2, 3, \ldots, T\}$ is index for check-out,
- C is total capacity,
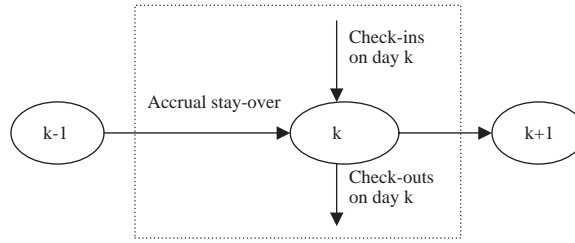
Fig. 1. Flows of check-ins and check-outs for day k.

- $R_{i,j}$ is revenue gained per booking with check-in on day i and check-out on day j,
- $U_{i,j}$ is booking demand for check-in on day i and check-out on day j.

Note that

- $\sum_{j=i+1}^{T} x_{i,j}$ is number of check-ins on day i,
- $\sum_{i=0}^{j-1} x_{i,j}$ is number of check-outs on day j.

We assume there are no customers staying before day 0 and all customers have to leave the hotel on or before day T. It is also assumed that any customer who has checked-in has to stay at least one night.

3. Stochastic network formulation

The check-ins and check-outs can be viewed as the flows in and out of the nodes in a network. We consider a particular day, day k ($k = \{1, 2, \ldots, (T-1)\}$), in the planning period (Fig. 1).

The following equation models the hotel's occupation status on day k for $k = 1, 2, 3, \ldots, (T-1)$:

$$\sum_{i=0}^{k-1} \sum_{j=k+1}^{T} x_{i,j} + \sum_{j=k+1}^{T} x_{k,j} - \sum_{i=0}^{k-1} x_{i,k}. \tag{1}$$

With limited capacity, we should have the following constraints for day $k = 1, 2, 3, \ldots, (T-1)$:

$$\sum_{i=0}^{k-1} \sum_{j=k+1}^{T} x_{i,j} + \sum_{j=k+1}^{T} x_{k,j} - \sum_{i=0}^{k-1} x_{i,k} \leqslant C. \tag{2}$$

On day 0, we assume there are no check-outs and no stay-over accrued. We have the following equation for day 0:

$$\sum_{j=1}^{T} x_{0,j} \leqslant C. \tag{3}$$

We follow the basic idea of the mathematical model given in Raeside and Windle [8].

$$\text{Max} \quad \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} R_{i,j} x_{i,j}$$

$$\text{S.t.} \quad \sum_{i=0}^{k-1} \sum_{j=k+1}^{T} x_{i,j} + \sum_{j=k+1}^{T} x_{k,j} - \sum_{i=0}^{k-1} x_{i,k} \leqslant C,$$

$$\sum_{j=1}^{T} x_{0,j} \leqslant C, \tag{4}$$

$$x_{i,j} \leqslant U_{i,j},$$

$$x_{i,j} \geqslant 0,$$

$$\text{for all } 0 \leqslant i < j \leqslant T.$$

4. Stochastic formulation and robust optimization solution scheme

The problem looks like a linear integer programming problem. Unfortunately, the parameters $U_{i,j}$ in (4) are usually uncertain at the beginning of planning period. Moreover, the revenues may not be fixed, as the decision-maker would like to set different pricing, which in turn results in different demands. One may want to solve this by replacing the parameters by their best point estimator, for instance, using expected value $E(U_{i,j})$ to replace the uncertain parameter of $U_{i,j}$. Although sometimes practitioners can obtain reasonably success by using expected value approaches, a drawback of this approach is that we may not always guarantee the solution is feasible. One may then carry out sensitivity analysis for some corrective action. Such an approach is commented as a reactive one [9]. We believe decision-makers would prefer to use proactive tools to obtain their solutions.

While it is impossible to remove the uncertainty fully, the best way to make decisions under an uncertain environment is to accept uncertainty first, and then understand uncertainty and put it into the planning decision model. Stochastic programming tools are based on this idea. Robust optimization [9,10] is one of the proactive approaches used to solve stochastic problems, and it represents an integration of goal programming and the scenario-based description of unknown data.

We define the following measurements of robustness:

Definition 4.1 (Solution robustness). An optimal solution is *solution robust* with respect to optimality if it remains "close" to optimal for any scenario $s \in \Omega$.

Definition 4.2 (Model robustness). An optimal solution is *model robust* with respect to feasibility if it remains "almost" feasible for any scenario $s \in \Omega$.

It is assumed that the decision-maker has a set of scenarios $s \in \Omega = \{1, 2, \ldots, S\}$ associated with unknown parameters. For each scenario, the corresponding probability is P_s such that $P_s \geqslant 0$ and $\sum_{s=1}^{S} p_s = 1$.

The philosophy of robust optimization is built on the trade-off between solution robustness and model robustness.

We can then transform our formulation into a robust optimization model as the following.

$$
\text{Max} \quad \sum_{s=1}^{S} p_s \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} R_{i,j}^s x_{i,j} - \lambda \sum_{s=1}^{S} p_s \left| \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} R_{i,j}^s x_{i,j} - \left(\sum_{s=1}^{S} p_s \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} R_{i,j}^s x_{i,j} \right) \right|
$$

$$
- \sum_{s=1}^{S} p_s \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} w_{i,j} |U_{i,j}^s - x_{i,j}|
$$

$$
\text{S.t.} \quad \sum_{i=0}^{k-1} \sum_{j=k+1}^{T} x_{i,j} + \sum_{j=k+1}^{T} x_{k,j} - \sum_{i=0}^{k-1} x_{i,k} \leqslant C,
$$

$$
\sum_{j=1}^{T} x_{0,j} \leqslant C,
$$

$$
x_{i,j} \leqslant \max\{U_{i,j}^s\},
$$

$$
x_{i,j} \geqslant 0,
$$

$$
\text{for all } s \in \Omega, 0 \leqslant i < j \leqslant T,
$$

(5)

where λ and $w_{i,j}$ are non-negative weighting parameters. The first term in the objective function is the expected revenue, while the second term is the mean absolute deviation of the revenue. When the revenue of different scenarios are wildly spread, it will result a larger value of mean absolute values, the penalty will then be increased. Together, these two terms can be viewed as a measurement of trade-off of solution robustness. The parameter λ can be regarded as a risk trade-off factor, between expected revenue and deviation, for the decision-maker. The absolute deviation in the third term is a model robustness measurement while the parameters $w_{i,j}$ are the penalty weights for the constraints violations. By using the mean absolute values as penalties, the model can generate solutions which are robust in all scenarios.

The mean absolute deviation terms, however, introduce some complexity owing to increasing number of artificial variables when the model is solved using linear programming. Yu and Li [11] propose an improvement in computational efficiency of the robust optimization with mean absolute deviation like formulation (5). The mean absolute value is transformed in a linear terms by a linearization method below.

Theorem 4.1 (Yu and Li [11]). *A goal programming*

$$
\begin{aligned}
\text{Minimize} \quad & Z = |f(X) - g|, \\
\text{Subject to} \quad & X \in F,
\end{aligned}
$$

(6)

where F is a feasible set and can be linearized using the following form:

$$\begin{aligned}
\text{Minimize} \quad & Z' = f(X) - g + 2\delta \\
\text{Subject to} \quad & g - f(X) - \delta \leqslant 0, \\
& \delta \geqslant 0 \\
& X \in F.
\end{aligned} \tag{7}$$

Proof. The new variable δ has a positive coefficient in objective function. The minimization will thus force the variable to take its possible minimum value. Notice that from the associated constraints, we have $\delta \geqslant g - f(X)$ and $\delta \geqslant 0$. In other words, the minimum possible value of $\delta = \min\{g - f(X), 0\}$.

If $f(X) - g \geqslant 0$, then the constraint $g - f(X) - \delta \leqslant 0$ will always be satisfied for all values of $\delta \geqslant 0$. So δ will be forced to take its possible minimum value, i.e. $\delta = 0$. Hence, $Z' = Z$. On the other hand, if $f(X) - g < 0$, then the minimum possible value of δ is $g - f(X)$. Then $Z' = g - f(X) = Z$.

In other words, the two formulations (6) and (7) are equivalent. The proof is completed. $\square$

To apply Theorem 4.1 to our problem (5), we introduce a set of non-negative variables z^s and $y_{i,j}^s$ for all scenarios where $s \in \Omega$. Our robust optimization model is now converted into the following:

$$\begin{aligned}
\text{Max} \quad & \sum_{s=1}^{S} p_s \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} R_{i,j}^s x_{i,j} - \lambda \sum_{s=1}^{S} p_s \left[\sum_{i=0}^{T-1} \sum_{j=i+1}^{T} R_{i,j}^s x_{i,j} - \left(\sum_{s=1}^{S} p_s \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} R_{i,j}^s x_{i,j} \right) + 2z^s \right] \\
& - \sum_{s=1}^{S} p_s \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} w_{i,j} [U_{i,j}^s - x_{i,j} + 2y_{i,j}^s] \\[4pt]
\text{S.t.} \quad & \sum_{i=0}^{k-1} \sum_{j=k+1}^{T} x_{i,j} + \sum_{j=k+1}^{T} x_{k,j} - \sum_{i=0}^{k-1} x_{i,k} \leqslant C, \\
& \sum_{j=1}^{T} x_{0,j} \leqslant C, \\
& \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} R_{i,j}^s x_{i,j} - \left(\sum_{s=1}^{S} p_s \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} R_{i,j}^s x_{i,j} \right) + z^s \leqslant 0, \\
& x_{i,j} - y_{i,j}^s \leqslant U_{i,j}^s, \\
& x_{i,j} \leqslant \max\{U_{i,j}^s\}, \\
& x_{i,j}, z^s, y_{i,j}^s \geqslant 0, \\
& \text{for all } s \in \Omega, 0 \leqslant i < j \leqslant T.
\end{aligned} \tag{8}$$

The prominent feature of formulation (8) is that it is now in a linear programming form and ready to be solved by popular linear modelling packages like LINDO [12] when the weighting parameters are assigned by the decision maker.

Table 1
Demands for single scenario example

i	(j)									
	1	2	3	4	5	6	7	8	9	10
0	13	24	68	92	72	11	12	6	4	3
1		11	15	88	110	12	7	7	4	4
2			11	20	91	8	6	14	6	6
3				34	96	11	15	17	8	7
4					23	90	15	7	7	7
5						7	10	11	7	5
6							8	13	14	5
7								8	25	29
8									11	19
9										24

5. Illustrative examples

5.1. Single scenario (deterministic)

We consider first a single scenario (deterministic) example for a revenue management problem for a business hotel under certain demand. The planning horizon is set to be 10 days (starting from a Sunday). The hotel has a maximum of 400 rooms. For simplicity, the unit rate for each room night is kept constant at 0.84. The revenue for any single or multiple night stay is linearly proportional to the length of stay. Since the parameters are all deterministic, there is no deviation. The problem is a standard integer programming model of formulation (4).

Demands for all pairs of (i, j), $U_{i,j}$ are forecast as shown in Table 1. The hotel's customers are mainly business travellers. The demand is usually higher for median length stays (e.g. 3-nights or 4-nights) while the demand for short stays (1-night, 2-nights) are lower. Check-out rates on Thursday and Friday are usually high, whereas demands for stays over the weekend are low. Long stays (over 5-nights) are low as well.

Optimal results obtained are summarized in Table 2.

5.2. Multiple scenario example I

We now consider the case where a hotel would like to take different future demand scenarios into its planning, although the hotel management will fix their prices (0.84) for all scenarios. In this case we have no variation for the revenue coefficients. So the only stochastic variable is the demand $U_{i,j}$. Suppose there are four scenarios with probability of 0.1, 0.5, 0.3 and 0.1, respectively.

The demands $U_{i,j}$ are shown as the following Tables 3–6 for the four scenarios.

For simplicity, all weights $w_{i,j}$ are set to be equal to 1. The optimal solutions obtained are summarized in Table 7.

Table 2
Optimal solution for single scenario example

i	(j)									
	1	2	3	4	5	6	7	8	9	10
0	13	24	68	92	72	11	12	6	4	3
1		11	15	24	37	12	7	7	4	4
2			11	5	1	8	6	14	6	6
3				0	95	11	15	17	8	7
4					22	90	15	7	7	7
5						7	10	11	7	5
6							8	13	14	5
7								8	25	29
8									11	19
9										24

Table 3
Demands for multiple scenario example I (scenario 1)

i	(j)									
	1	2	3	4	5	6	7	8	9	10
0	20	35	90	120	100	20	18	10	8	8
1		20	25	120	140	20	10	10	8	5
2			17	30	120	10	8	20	10	10
3				50	140	15	22	25	12	11
4					26	12	25	10	10	8
5						8	15	15	8	8
6							12	20	20	10
7								15	35	40
8									18	28
9										40

5.3. Multiple scenario example II

In this subsection, we consider the situation where the hotel management sets different price levels to cope with uncertainty. The different strategic price settings can simulate demands into different levels. The price per room night for each scenario is 0.7, 0.8, 0.9 and 1, respectively. The demand is forecast as being the same as in the above example. The risk trade-off factor λ is set to be 1. The optimal solution obtained is then summarized in Table 8.

5.4. Hotel revenue management strategies

The discussions in above sections mainly focus on the computational aspect of hotel revenue management. Indeed, in addition to the uncertain data that will cause different strategies, management's

Table 4
Demands for multiple scenario example I (scenario 2)

i	(j)									
	1	2	3	4	5	6	7	8	9	10
0	15	30	75	100	80	15	15	8	5	4
1		14	16	100	120	15	10	10	5	5
2			13	25	100	10	8	16	8	8
3				40	120	15	20	20	10	10
4					25	10	20	8	8	8
5						8	12	12	8	6
6							10	15	15	8
7								8	30	35
8									12	20
9										25

Table 5
Demands for multiple scenario example I (scenario 3)

i	(j)									
	1	2	3	4	5	6	7	8	9	10
0	10	15	60	80	60	6	8	3	4	3
1		8	12	70	100	9	5	5	3	3
2			8	12	80	8	5	12	4	4
3				25	60	7	8	15	5	5
4					22	8	7	6	6	8
5						6	8	10	8	5
6							6	12	15	3
7								8	20	25
8									10	18
9										20

attitudes will be also important in strategy setting. The weight parameters in the model are used to capture these hotel management considerations.

5.4.1. Risk trade-off factor

The different values of the parameter of risk trade-off factor λ represent different degrees of management's risk aversion. The following graph presents the relationship between the risk trade-off factor and the expected revenue generated.

We can observe from the graph in Fig. 2 that in general, the expected revenue decreases as the risk trading-off factor increases.

When the risk trade-off factor is very large, the model gives all values of the decision variable as zero and results in zero expected revenue. In other words, if management is very conservative toward risk, the model will suggest that he get rid of all business risks by not running a business.

Table 6
Demands for multiple scenario example I (scenario 4)

i	(j)									
	1	2	3	4	5	6	7	8	9	10
0	5	15	40	60	40	5	5	2	2	2
1		5	12	50	60	5	3	3	2	2
2			5	12	50	5	5	5	2	2
3				20	40	5	5	5	3	3
4					20	5	5	2	3	3
5						4	5	5	2	1
6							4	5	5	0
7								4	5	5
8									5	15
9										15

Table 7
Optimal solutions for multiple scenario example I

i	(j)									
	1	2	3	4	5	6	7	8	9	10
0	20	35	90	80	80	15	18	10	8	8
1		20	16	0	0	10	10	10	8	5
2			14	6	12	10	8	20	10	10
3				19	81	15	22	25	12	11
4					25	12	25	10	10	8
5						8	15	15	8	8
6							12	20	20	10
7								15	35	40
8									18	28
9										40

Another measurement of risk is the ratio of head-counts per room night. Consider the examples of accepting a 3-night stay customer and three 1-night stay customers. Although the expected revenues for two cases are the same (as we assume the room-night rate is the same for the whole period of planning horizon), the head-count per room-night ratio is higher in the latter case. This means the hotel would not like to tie-up available capacity as it would like to accept more short-staying customers. We can see from the Fig. 3 that in general, the ratio increases as the risk trade-off factor increases.

5.4.2. Penalty weights for feasibility robustness

The penalty weights for feasibility robustness are other decision controls used by management. For example, if management would like to accept more business travellers (probably their loyal

Table 8
Optimal solutions for multiple scenario example II

i	(j)									
	1	2	3	4	5	6	7	8	9	10
0	20	35	90	80	0	15	18	10	8	8
1		20	16	13	77	0	10	10	8	5
2			14	6	12	10	8	20	10	10
3				6	99	15	22	20	12	11
4					25	12	25	10	10	8
5						8	15	15	8	8
6							10	20	20	10
7								8	35	40
8									12	28
9										25
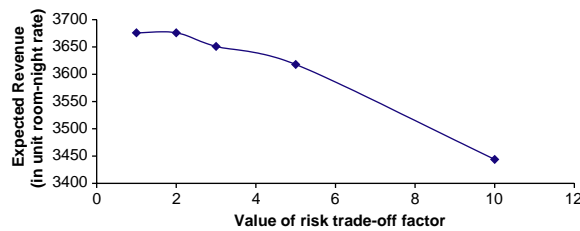
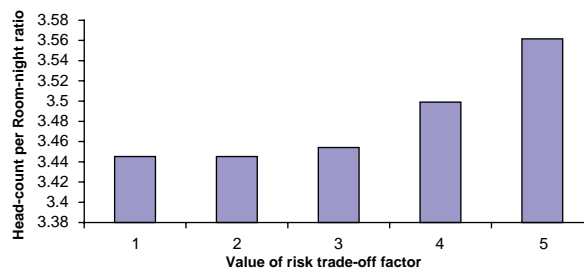Fig. 2. Relationship between expected revenue and risk trade-off factor.

Fig. 3. Relationship between head-count per room-night ratio and risk trade-off factor.

customers) who have 3-night to 4-night stays, or the management would like to accept more intakes on a particular day, he can release the corresponding weights or add more weights for other stays. We illustrate this by re-calculating the example in Section 5.2 while decreasing the weights relating to checking-in on day 0 and checking-out on day 4. By adjusting the weights, more customers are accepted for such period (an increase from 80 headcounts to 100 headcounts).

6. Other considerations in hotel revenue management system

In our model detailed in the above sections, the demand for any feasible length of stay is an unknown random variable of randomness due to an unknown number of arrivals and unknown lengths of stay. Denote the probability of $U_{i,j}$ by $f_{u_{i,j}}(U_{i,j})$.

Assume demand for arrival d_i follows a probability distribution (e.g. Poisson distribution) $f_{d_i}(d_i)$ for day i, and length of stay, $L_{i,j}$, can be assumed to follow another probability distribution (e.g. Geometric distribution) $f_{L_i}(L_i)$ for day i. It is further assumed that the two distributions are statistically independent.

$$f_{u_{i,j}}(U_{i,j}) = f_{d_i}(d_i) \times f_{L_i}(L_i). \tag{9}$$

We extend our formulated model to cover more realistic cases including, (i) cancellation and no-show; (ii) early-check-out; (iii) stay-over-extension; and (iv) over-booking.

6.1. Cancellation and no-show

The customer arriving to check-in will be changed due to a cancellation or no show. We denote C_i as number of cancellations/no-shows for a booking (originally checking-in on day i) and check-out on day j ($0 \leqslant i < j \leqslant T$). Suppose $C_{,i}$ follows a probability distribution of $f_{c_i}(C_i)$. Our "actual" demand $U_{i,j}$ (after taking into account cancellations/no-shows) should follow the revised distribution:

$$f_{d_i}(d_i) \leftarrow f_{d_i}(d_i) \times (1 - f_{c_i}(C_i)) \quad \text{for all } 0 \leqslant i < j \leqslant T. \tag{10}$$

6.2. Early check out

Denote, $E_{i,e,j}$ as the number of early check-outs on day k (checked-in on day i) originally checking-out day on day j ($0 \leqslant i < e < j \leqslant T$). Suppose $E_{i,e,j}$ follows a probability distribution of $f_{E_{i,e,j}}(E_{i,e,j})$. The "actual" length-of-stay will be changed to follow the distribution

$$f_{L_{i,j}}(L_{i,j}) \leftarrow (1 - f_{E_{i,e,j}}(E_{i,e,j})) \times f_{L_{i,j}}(L_{i,j}). \tag{11}$$

While

$$f_{L_{i,e}}(L_{i,e}) \leftarrow (1 + f_{E_{i,e,j}}(E_{i,e,j})) \times f_{L_{i,e}}(L_{i,e}) \quad \text{for all } 0 \leqslant i < e < j \leqslant T. \tag{12}$$

6.3. Extension of stay

Denote $V_{i,j,v}$ as number of extended check-outs (checked-in on day i) from day j to day v. Suppose $V_{i,j,v}$ follows a probability distribution of $f_v(V_{i,j,v})$. The "actual" length-of-stay will be changed to follow the distribution

$$f_{L_{i,j}}(L_{i,j}) \leftarrow (1 - f_v(V_{i,j,v})) \times f_{L_{i,j}}(L_{i,j}) \tag{13}$$

while

$$f_{L_{i,v}}(L_{i,v}) \leftarrow (1 + f_v(V_{i,j,v})) \times f_{L_{i,v}}(L_{i,v}) \quad \text{for all } 0 \leqslant i < j < v \leqslant T. \tag{14}$$

The early check out and extend stay are considered as two independent events. When a decision maker tries to consider both cases, a joint distribution is needed to model the likelihood of

length-of-stay. The distribution of stay periods can be obtained by multiplying the relative probability of early check out and extended stay owning to the assumption of statistically independence.

6.4. Over-booking

Over-booking is a widely adopted strategy for airlines to solve the cancellation or no shows problems. The overbooking level for a hotel is complicated, however, owing to the complexity of multiple-night-stays as well as early check-outs and extensions of stay. The booking level under our consideration is not fixed as hotel overbooking is affected by the demand. If demand is low, the hotel can accept a high overbooking level. If the hotel is full, it is easier to find a room for a customer in a sister hotel or another hotel of same class in the city. On the other hand, if demand were high, the hotel would like to lower overbooking to reduce that risk. Define O_i as the overbooking level for day i check-in. Then the limitation of booking capacity will be $C + O_i$ for day i. If O_i is a non-increasing function of demand of arrival on day i, $O_i = h(d_i)$. Then the constraints for our model becomes

$$\sum_{i=0}^{k-1} \sum_{j=k+1}^{T} x_{i,j} + \sum_{j=k+1}^{T} x_{k,j} - \sum_{i=0}^{k-1} x_{i,k} \leqslant C + h\left(\sum_{j=k+1}^{T} U_{k,j}\right),$$
(15)

$$\sum_{j=1}^{T} x_{0,j} \leqslant C + h\left(\sum_{j=1}^{T} U_{0,j}\right).$$
(16)

7. Conclusions and future study

This paper develops a stochastic network optimization model for the hotel revenue management problem with uncertain demand arrivals and uncertain length of stays. A novel approach of robust optimization is applied to solve the problem on a scenario-basis. The decision-maker's risk aversion is considered in the objective function. Mean absolute value is used to measure risk of the deviation of revenue from its expected value. A linearlization technique is applied to transform the absolute value into a linear form so that widely available linear modelling packages can be applied directly to the model.

In this paper, we also discuss the strategies for hotel management to take into account of risk trade-off and different pricing policies. Other considerations such as cancellations and no-show; early check-outs; extended stay and over-booking are discussed. We showed that our proposed model can be modified to adopt these strategic considerations.

From a management point of view, an area for future studies is to consider the situation when rooms are damaged or planned for maintenance. While from mathematically modelling point of view, the mean absolute deviation is only a special form of risk measurement for solution robustness in robust optimization. The robust optimization based on the idea of a trade-off between solution robustness and model robustness can be applied to capture a higher degree of measurements such as variance or high moments of probability distributions. A parameter embedding technique is under studied [13] to transform the higher order objective function into a bi-level programming model.

References

[1] McGill J, Van Ryzin G. Revenue management: research overview and prospects. Transportation Sciences 1999;33(2):233–56.

[2] Belobaba PP. Application of a probabilistic decision model to airline set inventory control. Operations Research 1989;37(2):183–97.

[3] Chatwin RE. Multiperiod airline overbooking with a single fare class. Operations Research 1998;46(6):805–19.

[4] Baker TK. New approaches to yield management: comprehensive overbooking/allocation heuristics for the hotel industry. Unpublished Doctoral dissertation, Fisher College of Business, The Ohio State University, Columbus, Ohio, 1994.

[5] Jones P. Defining yield management and measuring its impact on hotel performance. In: Ingold, et al., editors. Yield management: strategies for service industries, 2nd ed. London: Cassell; 2000.

[6] Weatherford LR. Length of stay heuristics: do they really make a difference? Cornell Hotel and Restaurant Administration Quarterly 1995;36(6):70–9.

[7] Birtan GR, Modschin SV. An application of yield management to the hotel industry considering multiple day stays. Operations Research 1995;43(3):427–43.

[8] Raeside R, Windle D. Quantitative aspects of yield management. In: Ingold, et al., editors. Yield management: strategies for service industries, 2nd ed. London: Cassell; 2000.

[9] Mulvey JM, Vanderbei RJ, Zenios SA. Robust optimization of large scale systems. Operations Research 1995;43(2):264–81.

[10] Mulvey JM, Ruszczynski A. A new scenario decomposition method for large scale stochastic optimization. Operations Research 1995;43(3):477–91.

[11] Yu CS, Li HL. A robust optimization model for stochastic logistic problems. International Journal of Production Economics 2000;64:385–97.

[12] Shrage LE. Optimization modelling with LINDO. Duxbury Press, Pacific Grove, CA; 1997.

[13] Ng WL. Iterative parametric separation scheme for robust optimization in two-stage stochastic program. Proceedings in the 2002 Fall National Conference of ORSJ, Hokkaido, Japan, 2002. p. 272–8.