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REGULAR ARTICLE

Model-based decision support for optimal brochure pricing: applying advanced analytics in the tour operating industry

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Abstract The research presented in this paper is motivated by an industry project conducted with TUI Deutschland, Germany's leading tour operator. We consider the decision problem of optimally determining hotel room prices to be published in the tour operator's brochure, which is usually valid for a half-year period. In practice, this task is performed manually by a large number of pricing specialists, each of whom is in charge of setting up to 100,000 prices. In this paper, we develop an advanced analytics approach to provide decision support for this task. More precisely, we propose a mixed integer linear programming-based approach, involving state-of-the-art methods from data analysis and optimization. In this context, we formally introduce the brochure pricing problem as a new optimization problem and present several alternative mathematical model formulations. The problem incorporates demand-side behavior by including a general attraction model whose parameters can be obtained from past booking data. Furthermore, we present different real-world scenarios of model-based decision support, showing how the brochure pricing problem and some variants thereof can be integrated into the manual decision making process, given the requirement of using standard optimization software. For example, the model-based approach can help the pricing specialist balance the objective of profit maximization and the disadvantage of a very complicated pricing structure.

Keywords Optimization · Analytics · Integer programming · Choice modeling · Decision support · Tour operator

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1 Introduction

Since the seminal paper "Competing on Analytics" by Davenport (2006) appeared in the Harvard Business Review, the application of quantitative methods from data analysis and optimization theory has generated great interest in Operations Research methods in industries that had until then not considered these methods appropriate for solving their real-world problems. Addressing such problems successfully often requires combining data analysis methods ("predictive analytics") with optimization models and methods ("prescriptive analytics"). Another requirement is the use of standard software for reasons like rapid development, lean integration into an existing IT landscape, and easy maintenance. Finally, and perhaps most importantly, the solutions to be developed are usually not intended to be a black box that will replace human decision makers, but to help their users make faster and smarter decisions. From an Operations Research perspective, this trend requires examining how well established data analysis methods can be efficiently combined with optimization methods to become part of an analytics toolkit within decision support frameworks (see Liberatore and Luo 2011). These frameworks should not only generate a single solution for a (perhaps greatly) simplified model of a real-world problem, but should also help decision makers answer different questions while systematically and iteratively developing a solution which incorporates their personal expertise.

1.1 Brochure pricing at TUI Deutschland

This paper examines how the aforementioned tasks can be addressed when solving a real-world decision problem in the tour operating industry. The decision problem is motivated from a joint project that the Department of Analytics and Optimization at the University of Augsburg and IBM Global Business Services conducted with TUI Deutschland. TUI Deutschland is Germany's leading tour operator; in 2011, it had a market share of 18.4 percent and a turnover of 4.2 billion euros (see FVW 2011; DRV 2012). The project was specifically conducted with TUI Deutschland's business intelligence and planning and pricing department and considered decision support approaches in the context of hotel room pricing. An outline of the project is given in IBM (2011).

In Germany, the majority of holiday packages is sold via brochures that potential customers can obtain gratis from their local travel agency and from which they select their holiday. TUI Deutschland produced and distributed more than 19 million copies of their brochures for the German market in the year 2011 (see TUI 2011). Together with the brochure, the relevant hotel room prices, as well as prices of, for example, flights, transfers, and circular tours are usually published in a separate booklet, which is valid for the entire brochure season. A significant number of pricing specialists in the tour operator's pricing department are specifically responsible for determining hotel room prices. In this context, each pricing specialist at TUI Deutschland is in charge of setting up to 100,000 prices for the period for which a brochure is valid, like winter or summer (see IBM 2011). When setting the prices, the specialist has to take factors like purchase prices, hotel ratings, competitor offerings, as well as

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Reisetermine										
	von	1.11.	7.11.	30.11.	24.12.	7.1.	1.2.	27.2.	26.3.	15.4.
	bis	6.11.	29.11.	23.12.	6.1.	31.1.	26.2.	25.3.	14.4.	6.5.
		E	B	S	F	A	D	C	D	S

TFS34040	max. Belegung	Verpflegung	Belegung	S	A	B	C	D	E	F
				Hotelpreis Paket						
FZX1 BAD/WC/BKTE/AC/GS/PO	2+2	H	2	58	59	69	74	76	80	124
FZM1 BAD/WC/BKTE/AC/MB	2+2	H	2	77	78	88	93	96	99	144
DZX1 BAD/WC/BKTE/AC/GS/PO	2	H	2	59	60	70	75	76	81	124
		H	1	97	98	108	113	114	119	162
DZM1 BAD/WC/BKTE/AC/MB	2	H	2	70	71	90	95	96	101	144
		H	1	106	107	126	131	132	137	180
DZM2 BAD/WC/BKTE/AC/MB	2+1	H	2	89	90	100	105	106	111	154
		H	1	125	126	136	141	142	147	190
JSM1 BAD/WC/BKTE/AC/WO/MB	3/2+2	H	2	95	96	106	111	112	117	160

Fig. 1 Brochure excerpt of a typical hotel price structure

customer preferences into account that are, for example, influenced by school holiday dates.

Figure 1 shows an excerpt from TUI Deutschland's 2010 winter brochure showing a specific hotel's *price structure*. The table "Reisetermine" at the top of Fig. 1 shows that the brochure includes the prices for half a year, which is divided into different seasons. The seasons are denoted by the letters A to F and, additionally, S. The table defines the *season pattern* for the hotel; that is, a season is assigned to each day of the period in which the brochure is valid. Note that one season can be used repeatedly over time with other seasons appearing in between (in the example, seasons D and S are used twice). This leads to a certain number of season changes that come with a price change of (at least one of) the offered room types. The number of seasons and the number of season changes are restricted by the brochure size and for clarity's sake (*seasonality requirements*). Note that this kind of pricing on the basis of a number of hotel-specific seasons is not TUI Deutschland-specific but standard in the tour operating industry in Germany.

The prices of the different seasons are illustrated in the lower table of Fig. 1. Every season (columns) has its corresponding *hotel price pattern*, which describes the combination of prices of all the room types (rows) in the hotel. The room types are abbreviated, using a four-letter combination. For example, FZX1 and FZM1 are family rooms; DZX1, DZX2, and DZM1 are double rooms; JSM1 is a junior suite. The prices of all room types rise monotonically from season A to season F, with S as additional "saver" season offered at the lowest price. Given the relevant season, the favored room type, and occupancy (the column "Belegung"), customers can use the table to find the price per night and calculate the price of their total stay.

For each brochure to be published, pricing specialists have to define the following for each of the hotels of which they are in charge: (a) the number of seasons to be used, (b) the price of each room type in each season, and (c) which season to assign to each day of the planning horizon. A key challenge lies in finding a trade-off between a purely profit-maximizing, but potentially complex, price structure and an easy-to-read price brochure. This trade-off is represented by selecting a reasonable *pricing policy*; that is, an adequate number of seasons and season changes.

Prior to our project, the high number of prices a pricing specialist has to determine usually made only a great deal of effort possible for the most important destinations and hotels. Furthermore, demand substitution and customer choice effects regarding a given hotel's different room types or those between hotels at the same destination were not well considered. Therefore, there was a general need for a new, more customer-centric pricing approach (see Cross and Dixit 2005 for a discussion) incorporating these demand-side effects, which could be applied to the tour operator's entire portfolio—even to minor destinations and hotels.

1.2 Outline and positioning of our model-based decision support approach

In the real-world solution, on whose implementation TUI Deutschland has been working since February 2011, particular emphasis has been given on features which allow the pricing specialists to efficiently model their market knowledge concerning customer preferences and demand. In this paper, we choose a different approach by complementing and partly replacing the specialists' expertise with a customer choice model, which can be estimated on the basis of past data. Based upon this demand model, we introduce a (general) brochure pricing optimization problem and derive scenarios that are relevant for providing decision support. We develop appropriate integer programming models for both the brochure pricing problem and the different scenarios, as well as evaluate their usability in practice settings by, for example, analyzing their performances when using a standard solver.

The approach we present in this paper to model and solve the brochure pricing problem and its variants combines predictive and prescriptive analytics.

The predictive analytics part of our approach concerns the customers' demand model. We aim to accurately model customers' choice behavior and the demand interdependencies regarding the different hotel and room options with which customers are faced, and to consider this behavioral model in the price optimization process. In this context, we assume that customers base their choices on an arbitrary attraction choice model (see, e.g. Cooper and Nakanishi 1988). This means that the demand for a product is determined by its attractiveness divided by the sum of the attractiveness of all the options available to customers, including the option not to purchase at all. The class of attraction choice models is quite general and includes several other well-known behavioral models, such as the (extended) Bradley–Terry–Luce model, the multiplicative competitive interaction model, and (asymptotically) the first choice model (see Schön 2010a). One of the most prominent models falling under the class of attraction models is the multinomial logit (MNL) model, which, from an econometric point of view, also belongs to the class of discrete choice models (see, e.g. Train 2009 for a comprehensive introduction). These models are consistent with random utility theory; that is, they are based on the assumption that customers' utilities regarding the alternatives are random variables, and that customers maximize their utility. Discrete choice models are well established in Econometrics and Marketing and their parameters can be consistently estimated from either real-world or survey data. Our approaches incorporate a generalization of the standard attraction choice model in the sense that multiple customer segments are considered simultaneously, with each of these segments following a separate attraction choice model. If each segment behaves according to a MNL model, we arrive at a mixed multinomial logit (MMNL) model with a discrete mixing distribution, which is similar to the latent class model and is quite popular in different fields (see, e.g. Greene and Hensher 2003).

The focus of our work is on the prescriptive analytics part; that is, the optimization models and techniques. We introduce a (general) brochure pricing optimization problem, which we model as a generalized fractional (or hyperbolic) mixed integer programming problem optimizing a sum-of-ratios. We also present reformulations leading to standard mixed integer linear programming problems. Therefore, from a methodological point of view, the prescriptive analytics part of our work is generally related to integer programming and fractional programming theory [see Wolsey (1998); Bajalinov (2003) for comprehensive introductions to these two fields, respectively]. With regard to the modeled decision problem, the prescriptive part of our work is closest related to work on product line design [see Kraus and Yano (2003), Sect. 2 for a compact review of product line optimization models and Schön (2010b) for references to a number of relevant review papers]. If we ignore seasonality requirements, our problem reduces to a standard product line pricing problem with probabilistic choice rule, where different prices have to be simultaneously determined for different hotels' room types. In this context, the optimization problem proposed by Schön (2010a) is particularly related to our work, as she considers several customer segments simultaneously and each segment's probabilistic customer behavior is modeled by an attraction model. It should be noted that, contrary to our work, her model allows the segments to be individually priced, while our brochure pricing setting does not allow price discrimination. Other authors that include probabilistic customer behavior in product line design problems are, for example, Hanson and Martin (1996), Chen and Hausman (2000), and Kraus and Yano (2003).

1.3 Contribution and outline of the paper

The contribution of this paper is twofold: On the one hand, we identify and introduce the brochure pricing problem as a new mathematical optimization problem. The problem incorporates state-of-the-art modeling techniques from data analysis and optimization, and is motivated by a real-world setting which has not been considered in theory before. We derive a general mixed integer linear programming formulation of the problem and show its NP-hardness. Furthermore, we propose several model reformulations that are, for example, of potential advantage when using standard software solvers like ILOG CPLEX. On the other hand, we examine how a model-based approach can support brochure pricing decisions in practice. In particular, based upon the project with TUI Deutschland, we identify a number of relevant decision support scenarios and derive appropriate model-based approaches for each of them. While one of the scenarios immediately includes the brochure pricing problem as introduced before, the others require variants of the problem in order to optimally support decisions. We investigate the different scenarios in a common setting that is based on real-world data. This setting corresponds directly to a specific destination of which a TUI Deutschland pricing specialist is in charge. In each scenario, we show how the pricing specialist's decisions can be supported by the model-based approach, and examine the properties of the corresponding models and their performance if they are solved by a standard solver.

The remainder of the paper is structured as follows: In Sect. 2, we introduce the brochure pricing problem. In Sect. 3, we derive a mixed integer linear programming formulation of the problem and propose several reformulations. Section 4 presents and investigates different scenarios concerning decision support. The paper concludes in Sect. 5 with summary and outlook.

2 The brochure pricing problem

As outlined in the previous section, we consider a tour operator's decision problem regarding how to optimally determine brochure prices for the different hotels and room types in its portfolio such that its total profit is maximized and seasonality requirements are taken into consideration. The latter are given by maximum constraints regarding the number of seasons and the number of season changes allowed over time. In Sects. 2.1 and 2.2, we formalize this decision problem by defining the assumptions and the setting considered in detail, as well as by introducing a formal notation with respect to the supply-side and the demand-side. In Sect. 2.3, we then formulate a mathematical optimization model that can be used to solve the decision problem.

2.1 Modeling the supply-side

We assume a planning horizon of $T = \{1, \ldots, \bar{T}\}$ days. The tour operator's portfolio contains a given number of hotels $h \in H$ with a hotel-specific number of room types $r \in R_h$. The set of possible price levels for room type r in hotel h is denoted by $L_{hr} = \{1, \ldots, \bar{L}_{hr}\}$. Each price level $l \in L_{hr}$ corresponds to a nonnegative price value p_{hrl} with $p_{hr1} < \ldots < p_{hr\bar{L}_{hr}}$ and refers to a single day/night. The daily cost of room type r in hotel h for a specific day $t \in T$ that the tour operator needs to pay the hotel is given by c_{hrt}.

The decision problem now consists of three parts (see Sect. 1.1). First and second, in each hotel h, the number of seasons has to be determined along with the related price level for each season. A given maximum number of seasons $\bar{Q}_h$ can be used throughout the planning horizon for each hotel h; $Q_h = \{1, \ldots, \bar{Q}_h\}$ is the corresponding set of possible seasons. The assignment of prices to seasons is then achieved through the binary decision variables sp_{hrql} with

$$sp_{hrql} := \begin{cases} 1 & \text{if in hotel } h \in H, \text{ room type } r \in R_h \text{ in season } q \in Q_h \\ & \text{is offered at price level } l \in L_{hr} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Third, for each hotel h, a season $q \in Q_h$ needs to be assigned to each day t of the planning horizon. Therefore, we define binary decision variables se_{hqt} with

$$se_{hqt} := \begin{cases} 1 & \text{if in hotel } h \in H\text{, season } q \in Q_h \text{ is assigned to day } t \in T \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Furthermore, let $\bar{K}_h$ denote the maximum number of season changes allowed for hotel h throughout the planning horizon. In this context, we define auxiliary variables k_{ht} with

$$k_{ht} := \begin{cases} 1 & \text{if there is a season change in hotel } h \in H \\ & \text{from day } t-1 \in T \text{ to day } t \in T \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Finally, for modeling purposes, we define decision variables x_{hrtl}—whose values directly follow from the above assignments sp_{hrql} and se_{hqt}—as follows:

$$x_{hrtl} := \begin{cases} 1 & \text{if in hotel } h \in H\text{, room type } r \in R_h \text{ is offered} \\ & \text{at price level } l \in L_{hr} \text{ on day } t \in T \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Remark 1 Note that in keeping with the proceedings in practice, we do not incorporate any capacity constraints regarding the availability of different room types in each hotel, because these limitations are usually not relevant in reality. However, in the case of an overseas destination this is, from a theoretical point of view, a simplification, as interdependencies with potentially limited flight capacities are ignored. We will discuss this issue as a potential for future research in Sect. 5.

2.2 Modeling the demand-side

We model demand by using an appropriate customer choice modeling approach. Therefore, the total market is split into a number of customer segments $s \in S$. We assume that customers in each segment choose their hotel and room type according to an attraction choice model (see Sect. 1.2); that is, the purchase probability of room type r in hotel h is equal to its relative attraction with proportional substitution patterns. Thus, given that a price level is selected for each room type in each hotel on a specific day t—that is, $\sum_{l \in L_{hr}} x_{hrtl} = 1$ for all h, r—the purchase probability of segment s of a specific room type r' in hotel h' priced at level l' can be calculated as

$$\frac{A_{sh'r'l'} x_{h'r'tl'}}{C_{st} + \sum_{h \in H} \sum_{r \in R_h} \sum_{l \in L_{hr}} A_{shrl} x_{hrtl}}, \tag{5}$$

with $A_{shrl} \geq 0$ being the attraction value of room type r in hotel h at price level l for segment s, and $C_{st} > 0$ denoting the segment-specific, day-dependent attraction value of outside alternatives, aggregated over all competitors and including the option not to purchase at all. Note that this modeling approach is consistent with probability theory, as it implies that each alternative's purchase probability is non-negative and that the probabilities of all the alternatives, including the competitor and the no-purchase alternative, sum to one.

Now, let d_{st} denote the size of segment s on day t. We can then construct the aggregated demand share of room type r' in hotel h' priced at level l' on day t incorporating all segments $s \in S$ as

$$\sum_{s\in S} d_{st} \cdot \frac{A_{sh'r'l'} x_{h'r'tl'}}{C_{st} + \sum_{h\in H} \sum_{r\in R_h} \sum_{l\in L_{hr}} A_{shrl} x_{hrtl}}. \tag{6}$$

Remark 2 Note that within the demand model, we assume that competitors cannot respond to the tour operator's price moves in the short run as they also publish their price structure in a brochure once per half-year. Competing products may be explicitly included in our model at fixed prices if the corresponding demand data is available to calibrate such an extended model. Different scenarios with respect to the competitors' brochure prices can then be considered via sensitivity analysis, which could be required if the own brochure is published first. Furthermore, note that we model demand on a per-day basis, which is a simplification that ignores network effects that might occur due to differing length-of-stays. We will discuss this limitation in Sect. 5.

2.3 Optimization model

The brochure pricing problem can now be formulated as the following general fractional mixed integer programming model (Model **BPP**):

$$\text{Maximize} \sum_{t\in T} \sum_{s\in S} d_{st} \cdot \frac{\sum_{h\in H} \sum_{r\in R_h} \sum_{l\in L_{hr}} (p_{hrl} - c_{hrt}) A_{shrl} x_{hrtl}}{C_{st} + \sum_{h\in H} \sum_{r\in R_h} \sum_{l\in L_{hr}} A_{shrl} x_{hrtl}} \tag{7}$$

subject to

$$\sum_{l\in L_{hr}} x_{hrtl} = 1 \qquad \forall h \in H, r \in R_h, t \in T \tag{8}$$

$$\sum_{q\in Q_h} se_{hqt} = 1 \qquad \forall h \in H, t \in T \tag{9}$$

$$se_{hqt} - se_{hq(t-1)} \le k_{ht} \qquad \forall h \in H, q \in Q_h, t \in \{2, \ldots, \bar{T}\} \tag{10}$$

$$\sum_{t\in\{2,\ldots,\bar{T}\}} k_{ht} \le \bar{K}_h \qquad \forall h \in H \tag{11}$$

$$\sum_{l\in L_{hr}} sp_{hrql} = 1 \qquad \forall h \in H, r \in R_h, q \in Q_h \tag{12}$$

$$sp_{hrql} + se_{hqt} - 1 \le x_{hrtl} \qquad \forall h \in H, r \in R_h, t \in T, q \in Q_h, l \in L_{hr} \tag{13}$$

$$se_{hqt} \in \{0, 1\} \qquad \forall h \in H, q \in Q_h, t \in T \tag{14}$$

$$sp_{hrql} \in \{0, 1\} \qquad \forall h \in H, r \in R_h, q \in Q_h, l \in L_{hr} \tag{15}$$

$$k_{ht} \ge 0 \qquad \forall h \in H, t \in \{2, \ldots, \bar{T}\} \tag{16}$$

$$x_{hrtl} \in \{0, 1\} \qquad \forall h \in H, r \in R_h, t \in T, l \in L_{hr} \tag{17}$$

The objective function (7) maximizes the total achieved profit by multiplying for each day t each alternative's demand share as defined by (6) with the corresponding net contribution $(p_{hrtl} - c_{hrt})$ and summing the resulting profits. Constraints (8) ensure that exactly one price level l is selected for each room type r in each hotel h on each day t of the planning horizon. Constraints (9)–(13) model the seasonality requirements. Constraints (9) ensure that a season is assigned to each hotel h for each day t, implicitly limiting the maximum number of seasons that can be used to $\bar{Q}_h$. Constraints (10) and (11) refer to the number of season changes. Constraints (10) force the variable k_{ht} to a value of at least 1 if there is a season change in hotel h from day $t-1$ to day t. Note that if there is a season change, there will always be one season q for which the difference on the right hand side of constraint (10) will become 1, such that it is not necessary to also consider the difference with the exchanged terms. Constraints (11) restrict the total number of season changes for each hotel h to a maximum of $\bar{K}_h$. Constraints (12)–(13) refer to the season pricing. Constraints (12) ensure that—for each hotel h and room type r—exactly one price level l is assigned to each season q. Constraints (13) then link these season prices to the daily prices that we need within the objective function by compelling a specific price level l for a room type r on day t to be selected if the season assigned to day t for the specific hotel h is linked to price level l for room type r. Note that it is not necessary to add additional constraints to force the x_{hrtl} variables to 0 in all other cases, as this is already implicitly ensured by the combination of constraints (8), (9), and (12). Constraints (14)–(17) define the decision variables of the model. In this context, please note that the auxiliary variables k_{ht} need not be specified as binary, as constraint (11) would anyway force greater values to 0 or 1 if the number of season changes becomes binding.

Remark 3 Note that the BPP could be extended by a number of additional aspects motivated by practical considerations. For example, the set of possible price levels L_{hr} could be defined to differ from day to day. Furthermore, as there is usually a hierarchy of different hotels and of different room types with regard to quality, certain price monotonicities are sometimes desirable. These can be modeled as additional constraints, which thus force higher quality hotels and room types to be consistently higher priced. In addition, it is often desirable for the prices of all room types in a hotel to simultaneously rise or fall from one season to another. For example, in one season, all room types in a hotel are priced at least as high as in another season. This can be achieved by adding the following constraints:

$$\sum_{l \in L_{hr}} p_{hrl} sp_{hrq_2l} \geq \sum_{l \in L_{hr}} p_{hrl} sp_{hrq_1l} \quad \forall h \in H, r \in R_h,\ q_1, q_2 \in Q_h, q_1 < q_2 \tag{18}$$

Note that from a technical perspective, the latter constraints also help to break the symmetry inherent in the model, as there are no longer multiple similar solutions which only differ in terms of permutations of the assigned season numbers.

Remark 4 The presented BPP is a complex optimization problem which is NP-hard (see Appendix A for the proof). Furthermore, remember that the objective function of the BPP contains a sum-of-ratios such that the formulation is not linear, but instead represents a general fractional mixed integer programming problem.

3 Model reformulations

In Sect. 3.1, we show that the BPP can be transformed into a standard mixed integer linear programming formulation. The advantage of this reformulation is that any standard software package that allows mixed integer linear problems to be solved, like ILOG CPLEX, can be used to solve the problem. In Sect. 3.2, we propose two different model reformulations with respect to the seasonality requirements. Depending on the specific problem instance, they are more compact than the original formulation and are potentially easier to solve with a standard solver.

3.1 Transformation into a mixed integer linear programming formulation

We first show how the model formulation of the BPP can be transformed into a general mixed integer programming (MIP) formulation (see, e.g. Li 1994). We define additional variables y_{st} with $y_{st} \geq 0$ for all $s \in S$ and $t \in T$. Each of these variables is thought of as to take the reciprocal of the corresponding denominator of the objective function as value; that is,

$$y_{st} = \frac{1}{C_{st} + \sum_{h\in H} \sum_{r\in R_h} \sum_{l\in L_{hr}} A_{shrl} x_{hrtl}}. \tag{19}$$

This can be directly ensured by adding the constraints $C_{st} y_{st} + \sum_{h\in H} \sum_{r\in R_h} \sum_{l\in L_{hr}} A_{shrl} x_{hrtl} y_{st} = 1$ to the model for each $s \in S$ and $t \in T$. The resulting non-linear MIP formulation is as follows:

$$\text{Maximize} \sum_{t\in T} \sum_{s\in S} d_{st} \cdot \sum_{h\in H} \sum_{r\in R_h} \sum_{l\in L_{hr}} (p_{hrl} - k_{hrt}) A_{shrl} x_{hrtl} y_{st} \tag{20}$$

subject to

$$C_{st} y_{st} + \sum_{h\in H} \sum_{r\in R_h} \sum_{l\in L_{hr}} A_{shrl} x_{hrtl} y_{st} = 1 \qquad \forall s \in S, t \in T \tag{21}$$

$$y_{st} \geq 0 \qquad \forall s \in S, t \in T \tag{22}$$

and the constraints (8)–(17) from the BPP.

Furthermore, in order to obtain a linear MIP (MILP) formulation, we need to linearize the non-linear term $x_{hrtl} y_{st}$. We follow the approach by Wu (1997), who proposes a set of constraints to linearize a polynomial binary term like xy, where x is a binary variable and y takes any non-negative value. Wu introduces a new variable $\hat{x}$, which represents the term xy. Equivalence is then ensured by the following set of inequalities: (a) $y - \hat{x} \leq K - Kx$; (b) $\hat{x} \leq y$; (c) $\hat{x} \leq Kx$; (d) $\hat{x} \geq 0$, where K is a large number greater than y. In the context of our problem, we define the variable $\hat{x}_{shrtl} = x_{hrtl} y_{st}$ and add the above constraints, leading to the following MILP formulation (Model **BPP-L1**):

$$\text{Maximize} \sum_{t\in T}\sum_{s\in S} d_{st}\cdot\sum_{h\in H}\sum_{r\in R_h}\sum_{l\in L_{hr}}(p_{hrl}-k_{hrt})A_{shrl}\hat{x}_{shrtl} \tag{23}$$

subject to

$$C_{st}y_{st}+\sum_{h\in H}\sum_{r\in R_h}\sum_{l\in L_{hr}}A_{shrl}\hat{x}_{shrtl}=1 \qquad \forall s\in S, t\in T \tag{24}$$

$$y_{st}-\hat{x}_{shrtl}\le K_{st}-K_{st}x_{hrtl} \qquad \forall s\in S, h\in H, r\in R_h,\ t\in T, l\in L_{hr} \tag{25}$$

$$\hat{x}_{shrtl}\le y_{st} \qquad \forall s\in S, h\in H, r\in R_h,\ t\in T, l\in L_{hr} \tag{26}$$

$$\hat{x}_{shrtl}\le K_{srtl}x_{hrtl} \qquad \forall s\in S, h\in H, r\in R_h,\ t\in T, l\in L_{hr} \tag{27}$$

$$\hat{x}_{shrtl}\ge 0 \qquad \forall s\in S, h\in H, r\in R_h, t\in T, l\in L_{hr} \tag{28}$$

$$y_{st}\ge 0 \qquad \forall s\in S, t\in T \tag{29}$$

and the constraints (8)–(17) from the BPP.

An adequate value for the upper bound K_{st} in constraint (25) can be calculated on the basis of the following considerations: If $x_{hrtl}=1$, K_{st} cancels out of the constraint. If $x_{hrtl}=0$, the inequality reduces to $y_{st}\le K_{st}$, as $\hat{x}_{shrtl}$ is forced to 0 by constraint (27) in this case. Therefore, a valid upper bound for y_{st} is

$$K_{st}:=1\Bigg/\left(C_{st}+\sum_{h\in H}\sum_{r\in R_h}\min_{l\in L_{hr}}A_{shrl}\right) \tag{30}$$

which results from finding the minimum value that the denominator in definition (19) of y_{st} can potentially take. More precisely, for every segment, the competitors' attraction value C_{st} is summed with the smallest values of A_{shrl} for all own alternatives. This can be thought of as charging the highest price for each room type under the assumption that attraction values fall monotonically with rising prices. In constraint (27), we can determine K_{srtl} as an upper bound for $\hat{x}_{shrtl}$ in a similar way, leading to

$$K_{srtl}:=1\Bigg/\left(C_{st}+\sum_{h\in H}\left(A_{shrl}+\sum_{d\in\{R_h\backslash r\}}\min_{m\in L_{hd}}A_{shdm}\right)\right). \tag{31}$$

The difference is that the bound can now be tightened even further by recognizing that for the specific considered alternative r to which $\hat{x}_{shrtl}$ refers, we can use the actual attraction value belonging to the specific considered price level l instead of using the minimal value in the sum.

3.2 Reformulations of seasonality requirements

While the BPP and BPP-L1 model the seasonality requirements perhaps most intuitively by using the variables se_{hqt} and sp_{hrql}, as well as the constraints (9)–(13), we now present two alternative formulations. These formulations potentially include a reduction of the overall number of variables, depending on the specific problem instance; that is, the number of hotels, room types, days, seasons, and price levels. In Sect. 4.4, we compare the performance of the reformulations with the original formulation BPP-L1 when solved with ILOG CPLEX.

3.2.1 Model reformulation 1

The idea of the first reformulation is to eliminate the binary variables sp_{hrql}, indicating the season prices defined for each price level, from the model and to instead use a number of non-negative continuous decision variables sp_{hrq} with one index dimension less. By replacing the BPP-L1 constraints (12), (13), and (15), the following model formulation is obtained (**BPP-L2**):

$$\text{Maximize} \sum_{t\in T}\sum_{s\in S} d_{st} \cdot \sum_{h\in H}\sum_{r\in R_h}\sum_{l\in L_{hr}} (p_{hrl} - k_{hrt}) A_{shrl} \hat{x}_{shrtl} \tag{32}$$

subject to

$$\sum_{l\in L_{hr}} p_{hrl} x_{hrtl} - sp_{hrq} \leq M_{hr}(1 - se_{hqt}) \quad \forall h \in H, r \in R_h, t \in T, q \in Q_h \tag{33}$$

$$\sum_{l\in L_{hr}} p_{hrl} x_{hrtl} - sp_{hrq} \geq -M_{hr}(1 - se_{hqt}) \quad \forall h \in H, r \in R_h, t \in T, q \in Q_h \tag{34}$$

$$sp_{hrq} \geq 0 \quad \forall h \in H, r \in R_h, q \in Q_h \tag{35}$$

and the constraints (8)–(11), (14), (16), (17), (24)–(29).

Constraints (33) and (34) ensure that, for each day, each continuous season price variable sp_{hrq} is set to the price corresponding to the selected level $l(\sum_{l\in L_{hr}} p_{hrl} x_{hrtl})$ if the day t belongs to season q; that is, if $se_{hqt} = 1$. Thus, the price level will be the same for all days belonging to the same season over the whole planning horizon. If $se_{hqt} = 0$, the Big-M parameter M_{hr} becomes relevant. In order to ensure that the differences on the left-hand side are not bounded in this case, we set $M_{hr} := \max_{l\in L_{hr}} p_{hrl} - \min_{l\in L_{hr}} p_{hrl}$. Constraints (35) define the additional decision variables. Furthermore, note that the following constraints, which, like (18), enforce season price monotonicity can also be introduced:

$$sp_{hrq_1} \leq sp_{hrq_2} \quad \forall h \in H, r \in R_h, q_1, q_2 \in Q_h, q_1 < q_2 \tag{36}$$

3.2.2 Model reformulation 2

The second reformulation is based on the idea not to explicitly use variables indicating the season prices, but—based on the daily price assignments given by x_{hrtl}—to check how many different prices are used and whether this fits within the maximum allowed number of seasons $\bar{Q}_h$ for each hotel. Therefore, we define the binary decision variables $w_{ht_1t_2}$ that indicate whether two days t_1 and t_2 must belong to different seasons. Then, by replacing the BPP-L1 constraints (12), (13), and (15), the following model formulation is obtained (**BPP-L3a**):

$$\text{Maximize} \sum_{t\in T}\sum_{s\in S} d_{st}\cdot \sum_{h\in H}\sum_{r\in R_h}\sum_{l\in L_{hr}} (p_{hrl}-k_{hrt})A_{shrl}\hat{x}_{shrtl} \tag{37}$$

subject to

$$\sum_{l\in L_{hr}} lx_{hrt_1l} - \sum_{l\in L_{hr}} lx_{hrt_2l} \leq M_{hr}w_{ht_1t_2} \qquad \forall h\in H, r\in R_h, t_1,t_2\in T, t_1\neq t_2 \tag{38}$$

$$\sum_{q\in Q_h} q\cdot se_{hqt_1} + M_h(1-w_{ht_1t_2}) \geq \sum_{q\in Q_h} q\cdot se_{hqt_2}+1 \qquad \forall h\in H, t_1,t_2\in T, t_1\neq t_2 \tag{39}$$

$$w_{ht_1t_2}\in\{0,1\} \qquad \forall h\in H, t_1,t_2\in T, t_1\neq t_2 \tag{40}$$

and the constraints (8)–(11), (14), (16), (17), (24)–(29).

Constraints (38) force the variables $w_{ht_1t_2}$ to 1 if there is a room type in hotel h where the assigned price (level) on day t_1 is higher than that on day t_2. We set $M_{hr} := \bar{L}_{hr}-1$. Constraints (39) consider the season numbers of each pair of days t_1 and t_2, ensuring that the season number assigned to day t_1 is higher than that of day t_2 if at least one room type's price is higher on day t_1; that is, if $w_{ht_1t_2} = 1$. If $w_{ht_1t_2} = 0$, the parameter M_h is used to deactivate the inequality and we can set $M_h := \bar{Q}_h$. Finally, constraints (40) define the additional decision variables. Note that, similar to (18) and (36), the formulation implicitly enforces season price monotonicity and, in this sense, it differs from the basic versions of the BPP-L1 and BPP-L2.

Note that, intuitively, we can further reduce the number of decision variables by defining $w_{ht_1t_2}$ only for $t_1 < t_2$. The resulting model formulation, **BPP-L3b**, which requires a slightly more complicated system of constraints, is given in Appendix B.

4 Decision support

As Geoffrion (1976) already pointed out, the job of decision makers, i.e. pricing specialists, does not only consist of a "one-and-done" run of an optimization model. Instead, they will usually begin an iterative process of model validation and what-if analysis (see Sharda and Steiger 1996; Little 2004). During this process, they will gradually develop their understanding of the problem by considering a number of slightly modified models, defining suitable instances, and analyzing the instances' (optimal) solutions. In our case, this allows them to contribute their implicit knowledge about the destinations and the target market they are responsible for and their expectations regarding the development of demand drivers, which cannot be learned automatically from past data.

In this section, we define an example setting (Sect. 4.1), in which we then illustrate some typical scenarios with different scopes that pricing specialists experience, introduce and examine corresponding variants of the brochure pricing problem (including the initial one; that is, the brochure pricing problem introduced in Sects. 2 and 3), and analyze the models' performance by means of standard software (Sects. 4.2–4.4). Following the practical requirement of using a standard solver, the models were solved using ILOG CPLEX 12.4 on an Intel Core i7 2.8 GHz computer with 8 GB of RAM. The maximum time limit examined in each of the scenarios was 30 min, which has been identified as the maximum time span that pricing specialists accept in a real-world setting.

4.1 Setting and model parameters

We illustrate the different scenarios of decision support with a setting based on real-world assumptions, as well as on the experiences we gained throughout the pricing project with TUI Deutschland. In this context, please note that, in order to protect the interest of TUI Deutschland and to keep sensitive data confidential, we modify and withhold certain details.

We consider a popular European holiday destination, most of whose tourists come from Germany. At this destination, we consider a tour operator's portfolio comprising a total of six different hotels. The hotels' classifications range from 3 to 5 stars, and all of them focus on leisure customers. On average, each hotel offers five different room types. This means that the tour operator needs to price about 30 different alternatives from which customers can choose. The brochure for which the prices are determined is for one brochure season; that is, $\bar{T} = 180$ days.

The number of price levels $\bar{L}_{hr}$ used in the model, and which can be chosen from for a given room type, lies between 66 and 150, depending on the hotel. These price points along with the corresponding prices are an input from market experts that include cost and competition considerations. Furthermore, the maximum numbers of seasons $\bar{Q}_h$ that can be used to build the price structure and the maximum numbers of season changes $\bar{K}_h$ are exogenously given and are the result of industry-wide standards and strategic management decisions. In our illustration, we will show how different values of these parameters influence the optimal pricing results.

The remaining parameters A_{shrl}, C_{st}, and d_{st} refer to the attraction choice model used on the demand-side (see Sect. 2.2). In our setting, we use an instance of the well-known MNL model (see Sect. 1.2) to model a specific segment's choice behavior. Following the general MNL model specification, the attraction values A_{shrl} are given by $e^{V_{shrl}}$, where V_{shrl} can be interpreted as a segment's s customer's (deterministic) utility with respect to room type r in hotel h priced at level l. The utility V_{shrl} is given by a function of the attributes of the segment and the corresponding choice alternative, which also includes the price. The other attributes that we use include the information available in the brochure to all potential customers, such as the type of room (double room, family room, junior suite, or suite), the view from the room (sea view, land view, unknown), as well as the current customer ratings obtained from different rating platforms such as holidaycheck. We use a definition of V_{shrl} that is linear in the parameters. Plugging the attraction values' definitions $A_{shrl} = e^{V_{shrl}}$ into term (5), we obtain the well-known logit probabilities (see, e.g. Train 2009).

In order to determine the values of the utility functions' parameters, we used a generated data set consisting of 50,000 fictive records of bookings for the corresponding market. The generation of the dataset was based on our experience with the TUI Deutschland project and, as in reality, only includes the tour operator's own bookings. In practice, the preceding year's seasonal booking data are used for this purpose. Based on the dataset, we then estimated a (simplified) demand model with a single segment modeling market behavior as a whole. In order to obtain estimates of the model parameters, we applied the standard maximum likelihood approach using BIOGEME, a scientific software package designed for the estimation of a number of discrete choice models (see Bierlaire 2003). As desired, the results of the estimation show that the price has a significant negative influence on customers' utility. We then calculated the competitors' attraction value C_{st} by using data which we obtained from the census bureau in charge of the relevant destination.

4.2 Single price vs. complete price variation

As already mentioned, pricing specialists are usually responsible for the pricing of many destinations, resulting in several thousands of prices that have to be set (see Sect. 1.1). Therefore, it is of major importance to have an idea of how much effort should be invested in manually creating a customized price structure for a specific market. A model-based approach, which allows for estimating the gain to be achieved through manual pricing efforts rather than a simple policy using a single price for each hotel room type not changed throughout the season (*single price*), can be used to give support in this scenario. A single price policy would be the easiest possible price structure by far and could be favorable if the potential improvements from introducing seasons and season changes could be shown to be limited. On the other hand, pricing specialists can never perform better than in a scenario without seasonality requirements; that is, without a maximum number of seasons or a given maximum number of season changes (*complete price variation*), which can also be calculated by means of a corresponding optimization model.

Apart from knowing the resulting profit range between the single price policy and the complete price variation, it is also beneficial for pricing specialists to perform sensitivity analysis on this range; that is, analyzing how exogenous parameters, like costs, strength of competitors, market size, etc. influence it. This is especially important if pricing specialists have knowledge not included in the prior data. For example, if they know that a competitor offers a new hotel, changes his price structure, or even does not longer exist, they might want to perform a rapid analysis of this situational change's possible effects.

4.2.1 Modeling approach

In order to calculate an optimal *single price* strategy for all hotels' room types, we introduce a new model **SBPP** using binary decision variables x_{hrl} with one dimension less than in the BPP [see (4)] denoting which price level l is selected for room type r of hotel h throughout the planning horizon. Furthermore, compared to the BPP, all constraints and variables needed to enforce seasonality requirements can be omitted, due to the single price requirement. The formulation can be linearized rather similar to the procedure described in Sect. 3.1, leading to a mixed integer linear program (**SBPP-L**).

With respect to *complete price variation*, we keep the decision variables of the BPP, but the constraints and variables regarding seasonality requirements can be omitted as well. The new formulation (**CPV**) only consists of the objective function (7) and the constraints (8) and (17) and thus is a relaxation of the BPP. The CPV can be linearized rather similar to the procedure described in Sect. 3.1, resulting in a mixed integer linear program (**CPV-L**). Note that the problem is separable in time, because the different days are no longer connected by the remaining constraints. Moreover, in the special case of $|S| = 1$, each day's problem is similar to the fractional programming model considered by Chen and Hausman (2000). Using the properties derived in their work, we can relax the binary constraints on the decision variables. In order to linearize the objective function of the resulting model, we then apply a Charnes–Cooper transformation (Charnes and Cooper 1962, also see Schön 2010a), ending up with the following linear programming formulation which can be easily solved by means of standard simplex methods (note that the index s is no longer necessary because of $|S| = 1$ and thus is omitted):

$$\text{Maximize} \sum_{t\in T} d_t \cdot \sum_{h\in H} \sum_{r\in R_h} \sum_{l\in L_{hr}} (p_{hrl} - k_{hrt})\, A_{hrl}\hat{x}_{hrtl} \tag{41}$$

subject to

$$C_t y_t + \sum_{h\in H} \sum_{r\in R_h} \sum_{l\in L_{hr}} A_{hrl}\hat{x}_{hrtl} = 1 \qquad \forall t \in T \tag{42}$$

$$\sum_{l\in L_{hr}} \hat{x}_{hrtl} = y_t \qquad \forall h \in H, r \in R_h, t \in T, l \in L_{hr} \tag{43}$$

$$\hat{x}_{hrtl} \geq 0 \qquad \forall h \in H, r \in R_h, t \in T, l \in L_{hr} \tag{44}$$

$$y_t \geq 0 \qquad \forall t \in T \tag{45}$$

4.2.2 Illustration of application

Table 1 shows the results when the models SBPP-L and CPV-L are applied in the example setting introduced in Sect. 4.1. The results are given for different competitors' attraction values (first row). A value of 1 means that competitors' attraction C_{st} is similar to the one contained in the data, which is usually calculated from the preceding year's seasonal bookings in practice (see Sect. 4.1). A value of, for example, 1.1 means that competitors' attraction is proportionally scaled up with the factor 1.1 on all days. Thus, the table shows how pricing specialists can perform sensitivity analysis regarding the influence of competitors' market power on the available profit range. The second and third row of Table 1 lists the profits obtained by applying the CPV-L and the corresponding computation times. As we can use model (41)–(45) because of $|S| = 1$, the computation times are quite fast. The fourth row reports the obtained profit of the SBPP-L. Note that these values cannot be calculated to optimality within the given time limit of 30 min (the corresponding integrality gaps are listed in Table 4 in Appendix C). The fifth row gives an upper bound on the relative potential for profit improvements, which is calculated by

$$\text{Profit potential} := \frac{\text{CPV-L profit} - \text{SBPP-L profit}}{\text{SBPP-L profit}} \cdot 100\,\%. \tag{46}$$

For example, we see that in case of a competitors' attraction factor of 1.0, the profit potential is bounded at a maximum of 16.06 %, which pricing efforts could exploit.

Table 1 Performance of CPV-L vs. SBPP-L

Intensity of competitors' attraction		0.8	0.9	1.0	1.1	1.2
CPV-L	Profit	2,949,210	2,649,772	2,406,079	2,204,050	2,033,460
	Comp. time	0.17 s	0.16 s	0.18 s	0.17 s	0.15 s
SBPP-L	Profit	2,559,166	2,290,918	2,073,144	1,893,581	1,742,788
Profit potential		15.2 %	15.7 %	16.1 %	16.4 %	16.7 %

Based on the profits obtained from the single price policy, as well as the complete price variation, we define the following two measures that will be used throughout the examples in the following subsections in order to judge the performance of the proposed pricing approaches. The first measure is the percental difference between the profit obtained from the price structure under consideration and the profit of the complete price variation CPV-L (ΔCPV):

$$\Delta\text{CPV} := \frac{\text{CPV-L profit} - \text{Obtained profit}}{\text{CPV-L profit}} \cdot 100\,\% \qquad (47)$$

The second measure indicates how much of the potential profit range between CPV-L and SBPP-L—as given in Table 1—can be exploited by the price structure under consideration; that is, the level of achievement (LA):

$$\text{LA} := \frac{\text{Obtained profit} - \text{SBPP-L profit}}{\text{CPV-L profit} - \text{SBPP-L profit}} \cdot 100\,\% \qquad (48)$$

4.3 Exogenous hotel price patterns

It is a rather common scenario for pricing specialists to have a number of possible price combinations regarding the different room types in mind for each hotel that they want to stick to throughout the planning horizon. These combinations, which we call *hotel price patterns* (see Sect. 1.1), can, for example, be the result of long-term business experience or strategic management decisions. Furthermore, such an approach is often followed because it reduces the problem complexity in a manual pricing process by a kind of decomposition. Having defined the potential hotel price patterns in a first step, pricing specialists' remaining decision problem in the second step only comprises arranging these patterns "optimally" over the days, such that the resulting solution is feasible with regard to the number of seasons and season changes. As we will demonstrate in this scenario, a model-based approach can directly support, automate, and improve the latter step. Even pricing specialists with a great deal of expert knowledge about the target market can benefit from such automation, as they no longer have to worry about sticking to the rules that the pricing policy determines, but can concentrate their effort on thinking about the best hotel price patterns in the first step.

4.3.1 Modeling approach

In the case of exogenously given *hotel price patterns* the problem consists of selecting for each hotel h on each day t a specific hotel price pattern v from a given set of patterns V_h. Let v_r denote the price of room type $r \in R_h$ in hotel price pattern v. We can then calculate the aggregated attraction values A_{shv} that, for each segment s, represent the total attraction of hotel h when the hotel price pattern $v \in V_h$ is selected:

$$A_{shv} := \sum_{r \in R_h} A_{shrv_r} \tag{49}$$

Owing to the proportional substitution property that attraction choice models imply, we can also calculate the aggregated weighted price values $\bar{p}_{hv}$, as well as the aggregated weighted cost values $\bar{c}_{htv}$ that come with a given hotel price pattern v:

$$\bar{p}_{hv} := \sum_{r \in R_h} \frac{A_{shrv_r}}{A_{shv}} p_{hrv_r} \tag{50}$$

$$\bar{c}_{htv} := \sum_{r \in R_h} \frac{A_{shrv_r}}{A_{shv}} c_{hrt} \tag{51}$$

We define the binary decision variables x_{htv} indicating whether the hotel price pattern v is selected for hotel h on day t. Furthermore, the binary decision variables z_{hv} are introduced with

$$z_{hv} := \begin{cases} 1 & \text{if hotel price pattern } v \in V_h \text{ of hotel } h \in H \text{ is used at} \\ & \text{at least one day throughout the planning horizon} \\ 0 & \text{otherwise} \end{cases} \tag{52}$$

The hotel price pattern-based brochure pricing problem can thereafter be formulated as follows (**HP-BPP**):

$$\text{Maximize} \sum_{t \in T} \sum_{s \in S} d_{st} \cdot \frac{\sum_{h \in H} \sum_{v \in V_h} (\bar{p}_{hv} - \bar{c}_{htv}) A_{shv} x_{htv}}{C_{st} + \sum_{h \in H} \sum_{v \in V_h} A_{shv} x_{htv}} \tag{53}$$

subject to

$$\sum_{v \in V_h} x_{htv} = 1 \qquad \forall h \in H, t \in T \tag{54}$$

$$\sum_{t \in T} x_{htv} \leq M z_{hv} \qquad \forall h \in H, t \in \{2, \ldots, \bar{T}\}, v \in V_h \tag{55}$$

$$\sum_{v \in V_h} z_{hv} \leq \bar{Q}_h \qquad \forall h \in H \tag{56}$$

$$x_{htv} - x_{h(t-1)v} \leq k_{ht} \qquad \forall h \in H, t \in \{2, \ldots, \bar{T}\}, v \in V_h \tag{57}$$

$$\sum_{t\in\{2,\dots,T\}} k_{ht} \le \bar{K}_h \qquad \forall h \in H \tag{58}$$

$$z_{hv} \in \{0, 1\} \qquad \forall h \in H, v \in V_h \tag{59}$$

$$x_{htv} \in \{0, 1\} \qquad \forall h \in H, t \in \{2, \dots, \bar{T}\}, v \in V_h \tag{60}$$

$$k_{ht} \ge 0 \qquad \forall h \in H, t \in \{2, \dots, \bar{T}\} \tag{61}$$

The objective function (53) incorporates the aggregations given by (49)–(51), such that, compared to the BPP, the consideration of individual room types can be eliminated from the model. Constraints (54) correspond to constraints (8), ensuring that a hotel price pattern is selected for each day. The remaining constraints enforce the seasonality requirements. Constraints (55) and (56) refer to the maximum number of allowed seasons. More precisely, constraints (55) force the z_{hv} variable to 1 as soon as the hotel price pattern v is used on at least one day (we can set the Big-M parameter to $M := \bar{T}$), and constraints (56) simply calculate the total number of used hotel price patterns for each hotel h and restricts it by $\bar{Q}_h$. Constraints (57) and (58) are respectively directly adapted from constraints (10) and (11) and restrict the number of allowed season changes over time. Constraints (59)–(61) define the model's decision variables. The formulation can be linearized rather similar to the procedure described in Sect. 3.1, leading to a mixed integer linear program (**HP-BPP-L**).

4.3.2 Illustration of application

We now apply the HP-BPP-L in the example setting introduced in Sect. 4.1. Instead of relying on expert knowledge, we use the output from the model with complete price variation (CPV) in Sect. 4.2 in order to obtain the predefined sets of possible hotel price patterns as input for the optimization model. More precisely, we select the maximum number of price patterns occurring in the corresponding optimal CPV solution which fulfill the season price monotonicity as defined, for example, by constraint (18). The value C_{st} is not scaled as in Sect. 4.2.2, but is similar to the one contained in the data.

Table 2 shows the obtained results of several pricing policies defined by different combinations of the maximum allowed seasons $\bar{Q}_h$ (rows) and the maximum allowed season changes $\bar{K}_h$ (columns). Identical values are set for all $h \in H$. The empty fields in the table represent combinations that are obviously not reasonable. For example, season changes cannot take place at all if just one season is allowed, and it makes no sense to allow more seasons than the maximum number of season changes, as they could not all be used. The obtained profit of each potential pricing policy is listed in the row "profit". Furthermore, the row "ΔCPV" shows how good the current solution is compared to the hypothetical solution obtained under complete price variation [see (47)]. In addition, the row "LA" reports the level of achievement with respect to the available profit range [see (48)]. Note that the LA takes a negative value for $\bar{Q}_h = 1$ and $\bar{K}_h = 0$, which is in line with what can be expected. Specifically, this policy means that a single price needs to be selected, which the SBPP-L can accomplish better as it is not based on predefined hotel price patterns.

The decision support in this context is threefold. First, with respect to price specialists who want or need to follow a specific pricing policy, the corresponding table entry

Table 2 Performance of HP-BPP-L

		Number of season changes $\bar{K}_h$					
		0	1	3	5	7	9
Number of seasons $\bar{Q}_h$							
1	Profit	2,027,739	–	–	–	–	–
	ΔCPV	15.7 %	–	–	–	–	–
	LA	−13.6 %	–	–	–	–	–
2	Profit	–	2,185,845	2,298,930	2,325,739	2,327,724	2,327,724
	ΔCPV	–	9.2 %	4.5 %	3.3 %	3.3 %	3.2 %
	LA	–	33.9 %	67.8 %	75.9 %	76.5 %	76.5 %
3	Profit	–	–	2,316,418	2,358,763	2,377,743	2,377,744
	ΔCPV	–	–	3.7 %	2.0 %	1.2 %	1.2 %
	LA	–	–	73.1 %	85.8 %	91.5 %	91.5 %
4	Profit	–	–	2,320,176	2,368,115	2,389,939	2,391,837
	ΔCPV	–	–	3.6 %	1.6 %	0.7 %	0.6 %
	LA	–	–	74.2 %	88.6 %	95.2 %	95.7 %
5	Profit	–	–	–	2,373,413	2,396,918	2,399,437
	ΔCPV	–	–	–	1.4 %	0.4 %	0.3 %
	LA	–	–	–	90.2 %	97.3 %	98.0 %
6	Profit	–	–	–	2,374,212	2,399,521	2,402,929
	ΔCPV	–	–	–	1.3 %	0.3 %	0.1 %
	LA	–	–	–	90.4 %	98.0 %	99.1 %
7	Profit	–	–	–	–	2,400,577	2,405,210
	ΔCPV	–	–	–	–	0.2 %	0.0 %
	LA	–	–	–	–	98.4 %	99.7 %

delivers a quick estimation of the relative performance of the hotel price patterns they have generated as input. Second, the model-based approach can also help decide which policy to choose. In particular, pricing specialists could calculate and then analyze the table as a whole. Thus, they can balance the advantage of an easily readable price structure—thus fewer different seasons and fewer season changes—with the advantage of profit maximization. In the example, it can be seen that, as expected, the profit rises monotonously with a higher number of seasons and/or a higher number of season changes, although not all instances could be solved provably to optimality within the given time limit (see Table 5 in Appendix C for the integrality gaps). The combination of seven seasons and nine season changes almost reaches the value of the CPV. Finally, pricing specialists can, of course, use the model's calculated solution (not displayed here)—that is, the assignment of given hotel price patterns to days in the planning horizon—directly for a specific pricing policy in order to set their brochure prices by taking the solution as a suggestion and starting point for additional considerations.

Remark 5 Besides exogenously given hotel price patterns, another scenario often occurring in practice is that season patterns (see Sect. 1.1) are already fixed in advance.

In such a scenario, pricing specialists' remaining task is to find optimal prices for each season and each room type, such that the overall profit is maximized. As this decision problem is a subproblem of the original BPP, the corresponding model formulation can straightforwardly be obtained by adding a single type of constraints to the BPP, predefining the season assignments. As the observations and the relative performance results of different pricing policies are similar to those with given hotel price patterns, we do not consider this scenario in the paper.

4.4 Application of the BPP model

Finally, we return to the BPP and its reformulations (see Sects. 2 and 3) which can potentially be applied in practice if pricing specialists do not need or want to incorporate restrictions on the optimal pricing patterns, such as those that lead to the model presented in Sect. 4.3.

Table 3 shows the performance of the original model formulation BPP-L1, as well as of the reformulations BPP-L2, BPP-L3a, and BPP-L3b presented in Sect. 3.2. Note that we respectively added the monotonicity constraints (18) and (36) to the BPP-L1 and BPP-L2, in order to make the results comparable to the BPP-L3a and BPP-L3b. The results are given for different meaningful pricing policies; that is, the maximum number of allowed seasons $\bar{Q}_h$ (first column) and the maximum allowed season changes $\bar{K}_h$ (second column), which are identical to those considered in Sect. 4.3. We report the profit obtained within the given time limit of each pricing policy and each model formulation (see Table 6 in Appendix C for the corresponding integrality gaps), as well as the value of the LA. Furthermore, note that we use the single price solution obtained from the SBPP-L as a warm start solution for the solver, which is reasonable and guarantees that LA ≥ 0 for all pricing policies.

A comparison of the results obtained from the different formulations shows that the reformulation BPP-L3a outperforms the BPP-L1, BPP-L2, and BPP-L3b in nearly all the considered pricing policies. The formulation BPP-L1 suffers from a high number of variables which influence the performance negatively, with the exception of two seasons and a low number of season changes. By construction, the formulation BPP-L2 requires a lower number of variables, which lead to a better overall performance than the BPP-L1. Interestingly, even though the BPP-L3b is a variation of the BPP-L3a with fewer variables, its performance is generally worse than the BPP-L3a with the exception of two pricing policies. With respect to the models BPP-L2 and BPP-L3b, in 11 of the 22 pricing policies that we consider, the BPP-L3b yields a higher performance, in 10 pricing policies the BPP-L2 performs better, and in one the performance is the same. Note that the BPP-L3b performs better if the number of seasons and season changes are high, whereas the strength of the BPP-L2 lies in pricing policies in which these values are lower.

Note that the relative performance of the different formulations we observe in this example is very similar to what we observed in most examples from the TUI Deutschland pricing project. Overall, the BPP-L3a mostly leads to the highest profit within the given time limit. However, as we have seen in the example, we also find that with regard to specific model parameter settings, other formulations can lead to equally

Table 3 Performance of BPP

$\bar{Q}_h$	$\bar{K}_h$	BPP-L1		BPP-L2		BPP-L3a		BPP-L3b	
		Profit	LA (%)	Profit	LA (%)	Profit	LA (%)	Profit	LA (%)
1	0	2,073,182	0.0	2,073,144	0.0	2,073,277	0.0	2,073,144	0.0
2	1	2,131,333	17.5	2,086,695	4.1	2,106,558	10.0	2,100,042	8.1
	3	2,231,053	47.4	2,226,380	46.0	2,177,640	31.4	2,151,919	23.7
	5	2,231,504	47.6	2,171,693	29.6	2,201,410	38.5	2,157,576	25.4
	7	2,243,367	51.1	2,297,999	67.5	2,299,582	68.0	2,150,618	23.3
	9	2,233,240	48.1	2,237,642	49.4	2,179,571	32.0	2,147,383	22.3
3	3	2,126,983	16.2	2,165,858	27.8	2,283,608	63.2	2,267,923	58.5
	5	2,135,031	18.6	2,284,681	63.5	2,311,069	71.5	2,272,749	60.0
	7	2,227,085	46.2	2,290,228	65.2	2,363,894	87.3	2,290,432	65.3
	9	2,237,491	49.4	2,271,272	59.5	2,295,363	66.7	2,261,404	56.5
4	3	2,231,602	47.6	2,216,238	43.0	2,263,424	57.2	2,216,480	43.1
	5	2,201,181	38.5	2,273,187	60.1	2,320,143	74.2	2,235,776	48.8
	7	2,265,112	57.7	2,241,855	50.7	2,359,014	85.9	2,338,057	79.6
	9	2,257,653	55.4	2,277,713	61.4	2,353,814	84.3	2,262,795	57.0
5	5	2,186,911	34.2	2,252,925	54.0	2,288,669	64.7	2,222,212	44.8
	7	2,276,436	61.1	2,274,964	60.6	2,386,598	94.1	2,354,088	84.4
	9	2,281,339	62.5	2,309,849	71.1	2,366,179	88.0	2,343,028	81.1
6	5	2,243,567	51.2	2,259,907	56.1	2,221,426	44.5	2,212,610	41.9
	7	2,141,644	20.6	2,255,139	54.7	2,321,605	74.6	2,368,259	88.6
	9	2,319,435	74.0	2,246,979	52.2	2,393,237	96.1	2,393,660	96.3
7	7	2,174,953	30.6	2,237,122	49.3	2,375,515	90.8	2,348,954	82.8
	9	2,308,343	70.6	2,292,086	65.8	2,395,280	96.8	2,393,936	96.4

good and even better results. The same is true when other values of the given time limit are considered. Therefore, with respect to application in practice, we conclude that although it could make sense to exclusively concentrate on the reformulation BPP-L3a, the recommendation is that, given the necessary computational power, all the formulations should be solved in parallel, and then the best solution obtained within the given time limit can be used.

Similar to what we discussed in Sect. 4.3, the decision support by this model for the pricing specialists is also threefold, as they can estimate the profit of a specific pricing policy, compare different policies, and finally directly use the model's solution—that is, the obtained season assignments and season prices—in the brochure. Interestingly, if we compare the obtained LA values in Table 3 with those obtained with predefined hotel price patterns (HP-BPP) as illustrated in Sect. 4.3 (see Table 2), it turns out that the performance of the general BPP within the given time limit is worse in respect of all the considered pricing policies with multiple seasons ($\bar{Q}_h > 1$), even though we use the single price as warm start solution. If we do not use a warm start solution, the performance is even worse, with the LA being negative regarding most pricing policies. This implies that, even if pricing specialists generally have full flexibility with respect to the room types' relative prices within each hotel, they might do better by predefining meaningful hotel price patterns and solving the model HP-BPP instead of BPP. Furthermore, our experiments with real-world data from the TUI Deutschland project also showed that with a much higher number of hotels, room types, and season length, all the BPP formulations are often insolvable due to computer memory issues. Therefore, there are markets where the BPP can no longer be solved at all, thus requiring predefining hotel price patterns or season patterns (see Sect. 4.3) anyway.

Remark 6 Note that contrary to what we observed in the model from Sect. 4.3 with respect to the solution obtained within the given time limit, in this example, profit obviously does not rise monotonously with a higher number of seasons and/or a higher number of season changes. We could enforce monotonicity by sequentially calculating the profit for the different policies, using found solutions as a warm start input for pricing policies with one more season and/or one more season change. However, this would inevitably lead to the necessity to always calculate the table as a whole.

5 Summary and outlook

In this paper, we have considered a tour operator's decision problem regarding optimally determining hotel room prices to be published in his price brochures. A team of pricing specialists usually undertake this task. We have formalized this problem by following an advanced analytics approach that incorporates a demand model whose parameters can be estimated by using methods from data analysis. The seasonality requirements are the key element of the problem; that is, the prices need to follow a certain price structure due to the brochure layout. This structure sets a framework for a possible pricing policy, which is in turn limited by a given maximum number of seasons with specific prices, as well as by a maximum number of season changes over time. Defining a pricing policy thus requires resolving the trade-off between a purely profit-maximizing, but potentially complex, price structure and an easy-to-read price brochure. For this purpose, we have introduced and formulated the brochure pricing problem as an integrated mathematical optimization model. This model determines the number of seasons to be used for each hotel, the price of each room type in each season, as well as the assignment of seasons to days in the planning horizon simultaneously.

Furthermore, we have investigated several real-world scenarios regarding providing pricing specialists with decision support, using an example setting based on a project that we conducted with TUI Deutschland. Since tourism markets are rather volatile, one basic idea has been to allow the specialists' expert knowledge to be contributed as input in order to generate more robust pricing policies. Such expert knowledge can, for example, comprise proven previously successful hotel price patterns or season patterns. However, in the real-world setting, creating such possibilities immediately leads to the specialists' need to be able to interact with the model in order to explore the effects of the different hotel price and season patterns. Hence, we have proposed and examined corresponding model formulations. A practical requirement in this context was to use a standard solver like ILOG CPLEX and to restrict the computation times, as our project experience showed that the specialists do not find a proven optimal solution as important as the possibility to make a decision of sufficient quality in a reasonable length of time. Interestingly, using predefined patterns was found to potentially lead to better results in terms of the profit than solving the full general brochure pricing problem within the given time limit.

In general, the computational experiments in the example setting and the scenarios examined show that it is possible to successfully provide decision support through our model-based approach. It is thus a meaningful way of enabling interaction between the pricing specialists and the models proposed. Through this support, pricing specialists no longer need to make minor decisions and can concentrate on their key capabilities. In all scenarios, the models' output can, for example, be used as a starting point for further considerations requiring manual changes. Furthermore, the models also allow pricing specialists to find a reasonable pricing policy by solving the trade-off between maximizing profit and an easy-to-read brochure.

In our opinion, the results are promising and encourage future work. Even though this is not in line with the current practical setup, one could, first, develop approaches that integrate pricing decisions regarding hotel rooms and flights. Since capacity constraints may exist on flights, there could currently be insufficient flight seats to transport the calculated number of customers to a destination. It would therefore be reasonable to integrate information on the expected flight load factor when pricing hotel room types. Second, it could be promising to develop demand models that explicitly consider several lengths-of-stay in order to make the model more accurate. A comparable development—ranging from single-leg-based approximations to network models—is known from revenue management. Nevertheless, our current experience is that, in our setting, such approaches are difficult to handle due to the large number of different resulting options and the difficulties regarding the amount of data required to accurately estimate the demand models. Third, one could use a different type of econometric demand model with different properties. With respect to application in practice, we believe that the results presented in this paper can serve as a successful example for other real-world implementations of advanced analytics approaches that allow a fruitful interaction of model-based decision support with industry specialists' expert knowledge.

Appendix A: Proof of NP-hardness of the BPP

In the following, we show that the column generation subproblem used to solve the well-known choice-based deterministic linear programming problem (CDLP) in revenue management, which is known to be NP-hard (see, e.g. Miranda Bront et al. 2009), can be reduced to the BPP. From that, it immediately follows that the BPP in its general form is also NP-hard.

The column generation subproblem of the CDLP is defined as follows (**CGS**):

$$\text{Maximize} \quad \sum_{j=1}^{n} w_j y_j \left(\sum_{l=1}^{\mathcal{L}} \frac{\lambda_l v_{lj}}{\sum_{i=1}^{n} v_{li} y_i + v_{l0}} \right) \tag{62}$$

subject to

$$y_j \in \{0, 1\} \qquad j = 1, \dots, n \tag{63}$$

with the model parameters $\mathcal{L} > 0, n > 0, w_j > 0, v_{lj} \geq 0, v_{l0} > 0, 0 \leq \lambda_l \leq 1$ for all $l = 1, \dots, \mathcal{L}$ and $j = 1, \dots, n$.

Let I be an arbitrary instance of CGS. We will construct an instance J of the BPP corresponding to I as follows: Regarding the supply-side, we consider a single hotel $H = \{1\}$ with n different room types $R_1 = \{1, \dots, n\}$. The planning horizon contains a single day $\bar{T} = 1$ such that $\bar{K}_1$ as well as $\bar{Q}_1$ can be set to arbitrary positive integers ($\bar{K}_1 > 0$, $\bar{Q}_1 > 0$). For each room type $r \in R_1$, we define two potential price levels $L_{1r} = \{1, 2\}$ with $p_{1r1} = 0$, $p_{1r2} = w_r$, and $c_{1r1} = 0$ for $r = 1, \dots, n$. Regarding the demand-side, we define $\mathcal{L}$ different segments $S = \{1, \dots, \mathcal{L}\}$ with segment size $d_{s1} = \lambda_s$ for all $s \in S$. The attraction values are defined by $A_{s1r1} = 0$ and $A_{s1r2} = v_{sr}$ for all $s \in S$ and $r \in R_1$. The competitors' attraction values are defined by $C_{s1} = v_{s0}$ for all $s \in S$. In total, we end up with $2n$ binary decision variables $x_{1111}, x_{1112}, \dots, x_{1n11}, x_{1n12}$. Thus, if in a solution to J price level 1 is selected for a room type $r \in R_1$, that is $x_{1r11} = 1$ and $x_{1r12} = 0$, this corresponds to $y_r = 0$ in the corresponding solution to I. Vice versa, if price level 2 is selected, that is $x_{1r11} = 0$ and $x_{1r12} = 1$, this corresponds to $y_r = 1$. Overall, we have constructed a polynomial transformation from instance I to instance J, such that a solution to J implies a solution to I. $\square$

Appendix B: Model reformulation BPP-L3b

By replacing constraints (12), (13), and (15) from the BPP-L1, the following model formulation is obtained (**BPP-L3b**):

$$\text{Maximize} \quad \sum_{t\in T}\sum_{s\in S} d_{st} \cdot \sum_{h\in H}\sum_{r\in R_h}\sum_{l\in L_{hr}} (p_{hrl} - k_{hrt}) A_{shrl} \hat{x}_{shrtl} \tag{64}$$

subject to

$$\sum_{l\in L_{hr}} l x_{hrt_1 l} - \sum_{l\in L_{hr}} l x_{hrt_2 l} \leq M_{hr} w_{ht_1 t_2} \qquad \forall h\in H, r\in R_h, t_1, t_2 \in T, t_1 < t_2 \tag{65}$$

$$\sum_{l\in L_{hr}} l x_{hrt_2 l} - \sum_{l\in L_{hr}} l x_{hrt_1 l} \leq M_{hr} (1 - w_{ht_1 t_2}) \qquad \forall h\in H, r\in R_h, t_1, t_2 \in T, t_1 < t_2 \tag{66}$$

$$\begin{aligned} &M_h \left(\sum_{q\in Q_h} q se_{hqt_1} - \sum_{q\in Q_h} q se_{hqt_2} \right) + N_h (1 - w_{ht_1 t_2}) \\ &\quad \geq \sum_{r\in R_h}\sum_{l\in L_{hr}} l x_{hrt_1 l} - \sum_{r\in R_h}\sum_{l\in L_{hr}} l x_{hrt_2 l} \qquad \forall h\in H, t_1, t_2\in T, t_1 < t_2 \end{aligned} \tag{67}$$

$$M_h\left(\sum_{q\in Q_h} qse_{hqt_2}-\sum_{q\in Q_h} qse_{hqt_1}\right)+N_h w_{ht_1t_2}$$
$$\geq \sum_{r\in R_h}\sum_{l\in L_{hr}} lx_{hrt_2l}-\sum_{r\in R_h}\sum_{l\in L_{hr}} lx_{hrt_1l} \qquad \forall h\in H, t_1, t_2\in T, t_1<t_2 \tag{68}$$
$$w_{ht_1t_2}\in\{0,1\} \qquad \forall h\in H, t_1, t_2\in T, t_1<t_2 \tag{69}$$

and the constraints (8)–(11), (14), (16), (17), (24)–(29).

Contrary to the BPP-L3a, the decision variables $w_{ht_1t_2}$ are now only defined for $t_1 < t_2$. Constraints (65) and (66) force the variables $w_{ht_1t_2}$ to 1 if there is a room type in hotel h where the assigned price (level) on day t_1 is higher than that on day t_2 and, vice versa, force them to 0 if there is a room type in hotel h where the assigned price (level) on day t_1 is lower than that on day t_2. Similar to the BPP-L3a, we set $M_{hr} := \bar{L}_{hr} - 1$. Constraints (67) and (68) concern the assignment of the season number. We define $M_h := \sum_{r\in R_h}(\bar{L}_{hr}-1)$ and $N_h := M_h(\bar{Q}_h-1)$. Three cases can then be distinguished:

1. If at least one room type's price is higher on day t_1 than on day t_2, that is, $\sum_{r\in R_h}\sum_{l\in L_{hr}} lx_{hrt_1l} > \sum_{r\in R_h}\sum_{l\in L_{hr}} lx_{hrt_2l}$, it follows from constraints (65) and (66) that $w_{ht_1t_2} = 1$. Thus, the term $N_h(1-w_{ht_1t_2})$ cancels out in constraint (67) and the remaining inequality ensures that $\sum_{q\in Q_h} qse_{hqt_1} > \sum_{q\in Q_h} qse_{hqt_2}$; that is, day t_1 will have a higher season index than day t_2. Constraint (68) is deactivated due to the term $N_h w_{ht_1t_2}$.
2. If at least one room type's price is higher on day t_2 than on day t_1, that is, $\sum_{r\in R_h}\sum_{l\in L_{hr}} lx_{hrt_2l} > \sum_{r\in R_h}\sum_{l\in L_{hr}} lx_{hrt_1l}$, it follows from constraints (65) and (66) that $w_{ht_1t_2} = 0$. Thus, the term $N_h w_{ht_1t_2}$ cancels out in constraint (68) and the remaining inequality ensures that $\sum_{q\in Q_h} qse_{hqt_2} > \sum_{q\in Q_h} qse_{hqt_1}$; that is, day t_2 will have a higher season index than day t_1. Constraint (67) is deactivated due to the term $N_h(1-w_{ht_1t_2})$.
3. If all the room types' prices are identical on day t_1 and day t_2, that is, $\sum_{r\in R_h}\sum_{l\in L_{hr}} lx_{hrt_1l} = \sum_{r\in R_h}\sum_{l\in L_{hr}} lx_{hrt_2l}$, it follows from constraints (65) and (66) that either $w_{ht_1t_2} = 0$ or $w_{ht_1t_2} = 1$. Let us assume that $w_{ht_1t_2} = 0$. Then, constraint (67) is deactivated due to the term $N_h(1-w_{ht_1t_2})$. In constraint (68), the term $N_h w_{ht_1t_2}$ cancels out and the remaining inequality ensures that $\sum_{q\in Q_h} qse_{hqt_2} \geq \sum_{q\in Q_h} qse_{hqt_1}$. Thus, it is possible that both days have the same season assigned as required. The argumentation for $w_{ht_1t_2} = 1$ is the same. Constraint (68) is deactivated and from constraint (67) it follows that $\sum_{q\in Q_h} qse_{hqt_1} \geq \sum_{q\in Q_h} qse_{hqt_2}$, which again allows for identical season assignments.

Finally, constraints (69) define the additional decision variables.

Appendix C: Integrality gaps

Tables 4, 5 and 6 provide the integrality gaps in the example and the different scenarios considered in Sect. 4 for the given time limit of 30 min. If a model can be solved to

Table 4 Integrality gaps (IG) for the single price case

Intensity of competitors' attraction		0.8	0.9	1.0	1.1	1.2
SBPP-L	Profit	2,559,166	2,290,918	2,073,144	1,893,581	1,742,788
	IG	4.5 %	4.2 %	3.2 %	3.3 %	2.9 %

Table 5 Integrality gaps for the exogenous hotel price patterns

	Number of season changes $\|K\|$					
	0	1	3	5	7	9
Number of seasons $\|Q\|$						
1	0 %/6 s	–	–	–	–	–
2	–	0 %/323 s	0 %/1,637 s	0 %/393 s	0 %/992 s	0 %/900 s
3	–	–	0.2 %	0.5 %	0.3 %	0.2 %
4	–	–	0.3 %	1.1 %	0.7 %	0.6 %*
5	–	–	–	0.6 %	0.4 %*	0.3 %*
6	–	–	–	0.4 %	0.3 %*	0.1 %*
7	–	–	-	–	0.2 %*	0.0 %*

Table 6 Integrality gaps for the brochure pricing problem

$\bar{Q}_h$	$\bar{K}_h$	BPP-L1 (%)	BPP-L2 (%)	BPP-L3a (%)	BPP-L3b (%)	$\bar{Q}_h$	$\bar{K}_h$	BPP-L1 (%)	BPP-L2 (%)	BPP-L3a (%)	BPP-L3b (%)
1	0	3.8	9.6	6.8	6.6	4	5	8.5	5.5	3.6	7.1
2	1	11.4*	13.3*	12.4*	12.7*		7	5.9*	6.8*	2.0*	2.8*
	3	7.3*	7.5*	9.5*	10.6*		9	6.2*	5.3*	2.2*	6.0*
	5	7.3*	9.7*	8.5*	10.3*	5	5	9.1*	6.4*	4.9*	7.6*
	7	6.8*	4.5*	4.4*	10.6*		7	5.4*	5.4*	0.8*	2.2*
	9	7.2*	7.0*	9.4*	10.8*		9	5.2*	4.0*	1.7*	2.6*
3	3	11.6*	10.0*	5.1*	5.7*	6	5	6.8*	6.1*	7.7*	8.0*
	5	11.3*	5.0*	3.9*	5.5*		7	11.0*	6.3*	3.5*	1.6*
	7	7.4*	4.8*	1.8*	4.8*		9	3.6*	6.6*	0.5*	0.5*
	9	7.0*	5.6*	4.6*	6.0*	7	7	9.6*	7.0*	1.3*	2.4*
4	3	7.3*	7.9*	5.9*	7.9*		9	4.1*	4.7*	0.4*	0.5*

optimality, we also give the computation time. If the value obtained by solving CPV is lower than the LP relaxation, we calculate the gap from the CPV solution (marked with *).

References

Bajalinov EB (2003) Linear-fractional programming: theory, methods, applications and software. Kluwer Academic Publishers, Dordrecht

Bierlaire M (2003) BIOGEME: a free package for the estimation of discrete choice models. In: Proceedings of the 3rd Swiss transportation research conference, Ascona
Charnes A, Cooper WW (1962) Programming with linear fractional functionals. Nav Res Logist Q 9: 181–186
Chen KD, Hausman WH (2000) Technical note: mathematical properties of the optimal product line selection problem using choice-based conjoint analysis. Manag Sci 46:327–332
Cooper LG, Nakanishi M (1988) Market-share analysis: evaluating competitive marketing effectiveness. Kluwer Academic Publishers, Boston
Cross RG, Dixit A (2005) Customer-centric pricing: the surprising secret for profitability. Bus Horiz 48: 483–491
Davenport T (2006) Competing on analytics. Harv Bus Rev 84:98–107
DRV (2012) Fakten und Zahlen zum deutschen Reisemarkt 2011. Deutscher ReiseVerband. Tourism report. http://www.drv.de/fileadmin/user_upload/Fachbereiche/Statistik_und_Marktforschung/Fakten_und_Zahlen_zum_deutschen_Reisemarkt_2011.pdf
FVW (2011) Deutsche Veranstalter 2011. FVW Dossier 25-26/11. Verlag Dieter Niedecken, Wiesbaden
Geoffrion AM (1976) The purpose of mathematical programming is insight, not numbers. Interfaces 7(1):81–92
Greene WH, Hensher DA (2003) A latent class model for discrete choice analysis: contrasts with mixed logit. Transp Res Part B 37:681–698
Hanson W, Martin K (1996) Optimizing multinomial logit profit functions. Manag Sci 42:992–1003
IBM (2011) Getting the price right: IBM business analytics provides the technological foundation for building customer-focused pricing. Business analytics and optimization report. http://public.dhe.ibm.com/common/ssi/ecm/en/ytc03354deen/YTC03354DEEN.PDF
Kraus UG, Yano CA (2003) Product line selection and pricing under a share-of-surplus choice model. Eur J Oper Res 150:653–671
Li H (1994) A global approach for general 0-1 fractional programming. Eur J Oper Res 73:590–596
Liberatore M, Luo W (2011) INFORMS and the analytics movement: the view of the membership. Interfaces 41:578–589
Little JDC (2004) Models and managers: the concept of a decision calculus. Manag Sci 50:1841–1853
Miranda Bront JJ, Méndez-Díaz I, Vulcano G (2009) A column generation algorithm for choice-based network revenue management. Oper Res 57:769–784
Schön C (2010a) On the optimal product line selection problem with price discrimination. Manag Sci 56:896–902
Schön C (2010b) On the product line selection problem under attraction choice models of consumer behavior. Eur J Oper Res 206:260–264
Sharda R, Steiger DM (1996) Inductive model analysis systems: enhancing model analysis in decision support systems. Inf Syst Res 7:328–341
Train KE (2009) Discrete choice methods with simulation, 2nd edn. Cambridge University Press, Cambridge
TUI (2011) Mediadaten 2011. http://www.tui-deutschland.de/media/td/das_unternehmen/TUI+Mediadaten+2011.pdf
Wolsey LA (1998) Integer programming. Wiley, New York
Wu T (1997) A note on a global approach for general 0-1 fractional programming. Eur J Oper Res 101: 220–223