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Slot allocation with minimum quantity commitment in container liner revenue management: A robust optimization approach

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Slot allocation with minimum quantity commitment in container liner revenue management

A robust optimization approach

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Abstract

Purpose – The purpose of this paper is to propose robust optimization models addressing the container slot allocation problem with minimum quantity commitment (MQC) under uncertain demand, which is faced by international companies export to USA.

Design/methodology/approach – A novel robust optimization approach handling linear programming (LP) with right-hand-side uncertainty is developed by incorporating new parameters: uncertainty level, infeasibility tolerance and reliability level. Two types of uncertainty, namely, bounded uncertainty and symmetric uncertainty are considered, respectively.

Findings – The present work finds that the expected revenue increases as the uncertainty level and the MQC decrease, as well as the infeasibility tolerance and the reliability level increase, no matter which type of uncertainty is considered.

Research limitations/implications – Typically, the capacity constraints in a container shipping model should include two major restrictions: (1) number of slots and (2) total weight of loaded and empty containers. However, this study only addresses the first restriction for simplicity. It is recommended that future research explore the optimal solutions with additional restriction (2).

Originality/value – This paper fills a theoretical and practical gap for the problem of slot allocation with MQC in container liner revenue management. Deterministic and tractable mixed integer LP is formulated to derive robust solutions which immunes to demand uncertainty. Illustrative examples are presented to test the proposed models. The present work provides practical and solid advice and examples which demonstrates the application of the proposed robust optimization approach for logistics managers.

Keywords Uncertainty management, Transportation management

Paper type Research paper

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Nomenclature

- C available capacity on the container ship
- I index for loading port, $i = 0, 1, \ldots, P-1$
- j index for unloading port, $j = 1, 2, \ldots, P$
- $r_{i,j}$ predetermined average revenue gained per container that moves from port i to port j
- q the minimum quantity of cargos (in terms of the quantity of containers in this study) for a container ship that stipulated by MQC
- y_k $\begin{cases} 1, & \text{if the occupation status of the container ship at port } k \text{ is more than a fixed, minimum quantity,} \\ 0, & \text{if the occupation status of the container ship at port } k \text{ is } 0, \end{cases}$ where $k = 1, 2, \ldots, P$
- $U_{i,j}$ uncertain booking demand containers move that from port i to port j

Introduction

Revenue management, alternatively known as yield management, aims to find optimal inventory allocation and scheduling strategies, as well as setting prices so as to maximize revenue over the planning horizon. By predicting consumer behavior at the micro-market level and optimizing product availability and pricing, effective revenue management systems significantly convert market uncertainty into probability, and probability into revenue (Cross, 1997). Revenue management came into vogue after deregulation of the US airline industry in the late 1970s when revenue management systems were implemented to optimize capacity utilization of flights by offering early bird discounts (McGill and Van Ryzin, 1999). Revenue management is concerned with demand-management decisions and addresses the following three categories of problems (Talluri and Ryzin, 2006): structural decisions: selling format, segmentations or differentiation mechanisms, terms of trade, etc.; price decisions: setting posted price, pricing across product categories, pricing overtime; and quantity decisions: accepting or rejecting offer, capacity allocation. Boyed and Bilegan (2003) propose four elements of a traditional revenue management system: the inventory control mechanism; optimization; demand modeling and forecasting; and interaction with users of the revenue management system. Nowadays, revenue management is a widely accepted discipline which focuses on providing enhancement of revenue and profitability in the airlines, hospitability, car rental, cruise lines, railroad and television broadcast industries (Kalyan and Garret, 2004).

In today's fiercely competitive market, container liner companies are finding that it is increasingly difficult to generate reasonable profits and some are even running deficits, due to the limited potential for increasing freight rates and the high cost of capacity expansion. Proven to be an effective tool in the aforementioned industries, revenue management has the considerable potential for application in the container liner industry. Several special characteristics of container liner industry make application of revenue management techniques desirable. For instance: space on container ships is perishable and cannot be stored for future sales; container capacity is usually fixed and the cost of immediate capacity expansion is very high; and most container ships usually allocate the fixed capacity according to clients' advance booking, implying cancellations and overbooking problems exist. Consequently, container liner companies need to build revenue management systems to achieve the highest amount of revenue by allocating fixed container capacity to the right customers, at the right time and at the right price.

Despite the similarity between airline and container liner industries, the salient difference between the two businesses is the configuration of capacity utilization in terms of the combination of space required and ports of loading and unloading (occupation length). For example, consider a simple case, as shown in Figure 1.

Assuming only one slot is left on the container ship, which can be assigned to a container (say Container A) that would be loaded at port 1, and unloaded at port 3. Assigning this slot to Container A definitely means that when the ship reaches port 2, this space would remain occupied and will not be available. There may be a case where another container (say Container B) can be loaded at port 2, for unloading at port 10. Obviously, Container B will generate more revenue than Container A. Hence we have to consider future opportunity costs. In particular, the multi-port arrival demand and configuration of space utilization are essentially uncertain in nature.

To address the problem of parameter uncertainty, there is a trend of development of new robust optimization approaches. Robust optimization handles parameter uncertainty by assuming that uncertain parameters belong to a bounded, convex uncertainty set and minimizing the negative impact on the objective value, while guaranteeing the solution is feasible (Düzgün and Thile, 2010). The fundamental idea behind robust optimization is to consider the worst-case scenario without a specific distribution assumption. Soyster (1973) pioneered the work on robust optimization by requiring that all uncertain parameters reach their worst-case value, which was deemed to be over-conservative in practical implementation. Ben-Tal and Nemirovski (1998, 1999, 2000) made some improvement by employing an ellipsoidal uncertainty set to adjust the level of conservatism, and presented tractable mathematical reformulations. Bertsimas and Sim (2003, 2004) addressed uncertainty by defining a polyhedron for each parameter, and then proposed a concept of "budget-of-uncertainty" to control conservatism. Düzgün and Thile (2010) pointed out that a single range for each parameter still leads to overly conservative results, and then described uncertain parameters using multiple ranges. Another stream of work followed scenario-based framework, in which the uncertainty was modeled through the use of a number of scenarios and then stochastic programming formulations were presented (Mulvey *et al.*, 1995). However, accurate probability data are extremely hard to obtain in practice, particularly for distributions varying over time. Ben-Tal and Nemirovski (2000) and Lin *et al.* (2004) formulated a deterministic robust counterpart (RC), given the magnitude of uncertainty of data, infeasibility tolerance and reliability level when a probabilistic measurement is applied. The appealing advantages of their approach are linear, applicable, and deterministically solvable and ease controlling of the level of conservatism. Our robust optimization model presented in this paper is developed on basis of the work of Ben-Tal and Nemirovski (2000) and Lin *et al.* (2004), and particularly applied in the domain of container liner revenue management (CLRM).

Revenue management traditionally assumes that customer demand is unknown, but can be represented by a stochastic process and a probability distribution (Perakis

Note: A simple illustration of network structure of occupation length in container liner slot allocation

Figure 1. Network structure in container liner slot allocation

and Roels, 2010), which is extremely difficult to solve. Robust optimization emerges as a promising methodology to address a large range of management problems subject to uncertainty, which makes few assumptions on the underlying probabilities, remains numerical tractable and incorporates the decision-maker's risk aversion (Thiele, 2004). Several representative papers have analyzed robust booking limits on a single leg, which is of significant similarity to our investigated container slot allocation problem. Ball and Queyranne (2006) employed the competitive ratio to study nested booking limits, without precise information about the demand. Birbil *et al.* (2009) presented efficient algorithms to derive the maximum booking limits under ellipsoidal uncertainty.

The robust optimization approach proposed in this work produces robust solutions which are in a sense immune against uncertainties in the right-hand-side parameters of the inequality, which differs from the conventional revenue management techniques by introducing the concept of uncertainty level, infeasibility tolerance and reliability level to describe uncertain demand, and then derive tractable formulations. The present work investigates the container slot allocation problem with minimum quantity commitment (MQC) under uncertain demand, which is faced by international companies export to USA. The US Federal Maritime Commission stipulates that the total quantity of cargos delivered by each container ship to US cities must either be none or at least as large as a fixed minimum quantity (Lim *et al.*, 2006). In this framework, managers determine whether or not to accept a specific booking request for a specific port, given the status of accepted booking and forecast future requests.

The contributions to the literature can be summarized as below:

(1) We consider multi-port stays and creatively describe the container ship's occupancy status on a calling port in a network formulation, in which loading and unloading are viewed as the flows in and out of the nodes in the network.

(2) We develop two linear, applicable RCs of the generic linear programming (LP) in which the right hand sides of the constraints are uncertain, while the uncertain data are modeled as bounded uncertainty and symmetric uncertainty, respectively.

(3) We introduce the concepts of uncertainty level, infeasibility tolerance and reliability level into the robust formulations. In practical application, these parameters are determined from the decision-maker's limited knowledge about the uncertainty distribution, conservativeness degree of the constraints feasibility.

(4) We apply these two tractable RCs to derive robust revenue management reformulations when the market demand is uncertain, and thus derive meaningful implications.

The remainder of this paper proceeds as follows. Problem description and notations are introduced in the next section. The robust optimization approach to the generic LP with uncertain right-hand-side constraints is presented in the following section. The next section shows the RCs of CLRM under demand uncertainty. Numerical illustrative examples with managerial implications are given in the penultimate section. Concluding remarks and future research recommendations are discussed in the final section.

Problem description and notations

In this study, we consider the case where specified containers are loaded at a port at a designated time and delivered to another port according to booking requests. MQC is executed in all ports. The decision makers need to determine the right number of booking request to be accepted or rejected, so as to maximize revenue while meet MQC. To address this issue, the container liner managers can incorporate MQC constraints into the mathematical model. Container liner managers do not know the exact information about demand at different ports and occupation length of containers.

The major notations for relative variables and parameters used in this paper are given in the nomenclature.

Note that the total number of out-bound containers departing from port i is $\sum_{j=i+1}^{P} x_{i,j}$, while the number of containers arriving at and dispatched from port j is $\sum_{i=0}^{j-1} x_{i,j}$.

For simplicity, we assume there is no container on the ship before port 0 and all containers have to be unloaded from the container ship at or before the last port P. We also assume that any container has to be transported at least one port forward.

The container loading and unloading can be viewed as the flows in and out of the nodes in a network, which is depicted in Figure 2.

We consider a particular port k, $k\in\{1, 2, \ldots, P-1\}$ in the planning horizon. The following equations model the container ship's occupancy status on port k, $k\in\{1, 2, \ldots, P-1\}$:

$$\sum_{i=0}^{k-1} \sum_{j=k+1}^{P} x_{i,j} - \sum_{i=0}^{k-1} x_{i,k} + \sum_{j=k+1}^{P} x_{k,j}. \tag{1}$$

The first term is the container capacity on the ship that has been occupied and that continues to be occupied; the second term is the space previously occupied but released at port k. The last term represents the newly loaded containers that are to be transported to forward ports.

With limited capacity and MQC, we should have the following constraints for container ship at port k ($k=1, 2, \ldots, (P-1)$):

$$y_{k+1} \cdot q \leqslant \sum_{i=0}^{k-1} \sum_{j=k+1}^{P} x_{i,j} - \sum_{i=0}^{k-1} x_{i,k} + \sum_{j=k+1}^{P} x_{k,j} \leqslant y_{k+1} \cdot C. \tag{2}$$

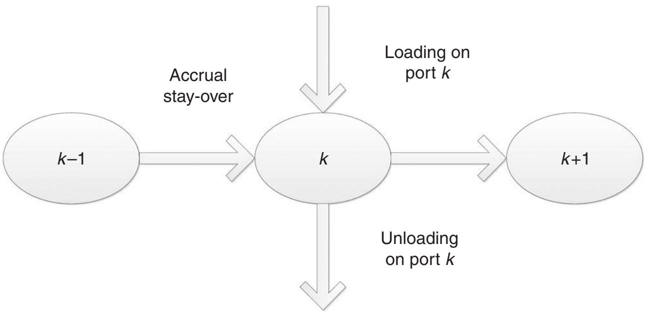

Figure 2.
Flows of loading and unloading for a container ship at port k

For port 0, we assume no containers have been unloaded and no containers are waiting to be unloaded. We have the following inequation for container ship at port 0:

$$y_1 \cdot q \leqslant \sum_{j=1}^{P} x_{0,j} \leqslant y_1 \cdot C. \tag{3}$$

Under the stipulation of MQC, the container ship will determine whether to accept booking request that delivering containers to certain port or not, considering the revenue generated from the delivery. The ship may reject previous delivery requests which are not considered to be profitable or violate MQC.

Total revenue during a voyage can be described as:

$$\sum_{i=1}^{P-1} \sum_{j=i+1}^{P} r_{i,j} x_{i,j}. \tag{4}$$

Consequently, the mathematical programming model for CLRM can be described as:

$$\begin{gathered}
\max \quad \sum_{i=1}^{P-1} \sum_{j=i+1}^{P} r_{i,j} x_{i,j} \\
\text{s.t. } y_{k+1} \cdot q \leqslant \sum_{i=0}^{k-1} \sum_{j=k+1}^{P} x_{i,j} - \sum_{i=0}^{k-1} x_{i,k} + \sum_{j=k+1}^{P} x_{k,j},\ k = 1, 2, \ldots,\ (P-1) \\
\sum_{i=0}^{k-1} \sum_{j=k+1}^{P} x_{i,j} - \sum_{i=0}^{k-1} x_{i,k} + \sum_{j=k+1}^{P} x_{k,j} \leqslant y_{k+1} \cdot C,\ k = 1, 2, \ldots,\ (P-1) \\
y_1 \cdot q \leqslant \sum_{j=1}^{P} x_{0,j} \\
\sum_{j=1}^{P} x_{0,j} \leqslant y_1 \cdot C \\
x_{i,j} \leqslant U_{i,j}, 0 \leqslant i < j \leqslant P \\
x_{i,j} \geqslant 0, 0 \leqslant i < j \leqslant P \\
y_k \in \{0, 1\}, k = 1, 2, \ldots,\ P
\end{gathered} \tag{5}$$

This model seems to be a mixed integer LP (MILP) problem. However, the parameters $U_{i,j}$ in (5) are always uncertain in nature. Lai and Ng (2005) handled this issue by employing an integration of goal programming and the scenario-based description of unknown data, which can be defined as a stochastic approach. In this work, we present an alternative approach to deal with uncertainty, and thus provide two RCs of the ori0ginal models.

Since the uncertainty exists in the right-hand-side constraints, we are interested in investigating the methodology that handles MILP with uncertain right hand sides. This work is left for the following section.

Robust optimization for general integer LP with right-hand-side uncertainty
In this section, we study general LP in which right hand sides is uncertain. We are concerned about developing a robust optimization methodology to generate "reliable" solutions to LP, which are immune against data uncertainty. It is plausible to consider employing classical duality relationship to transfer the uncertain right hand sides to the objective function coefficients, and then use existing robust optimization approaches to derive robust solutions. However, Minoux (2009) has studied this issue and has showed that the classical duality relationships cannot be applied on the robust version. Beck and Ben-Tal (2009) demonstrated that the primal worst equals to dual best.

We consider the following generic LP with $m \times n$ variables and m constraints:

$$\begin{cases} \max cx \\ \text{s.t. } Ax \leqslant b, x \geqslant 0, \end{cases} \tag{6}$$

with A being an $m \times n$ integer matrix of rank m and $b \in \mathfrak{R}^m$. Uncertainty arises from the right-hand-side parameters of the inequality constraints, namely, b_i, $i = 1, 2, \ldots, m$. In a robust optimization framework, we are concerned about the feasibility of the following constraints:

$$\sum_{j \in J} a_{i,j} x_{i,j} \leqslant b_i. \tag{7}$$

Ben-Tal and Nemirovski (2000) stated that even if the nominal data are slightly perturbed, one or more constraints may be violated substantially. The objective of this section is to produce robust solutions to the generic LP problem which are in a sense immune against uncertainty. The robust optimization methodology presented in this paper was first introduced for LP problem with uncertain coefficients by Ben-Tal and Nemirovski (2000), and then was extended by Lin *et al.* (2004) to deal with the MILP problem with uncertain coefficients and right-hand-side parameters of the inequality constraints. The way of introducing robustness into the original model in this research closely resembles the one used by Lin *et al.* (2004). This paper extends the aforementioned work to LP problem with uncertain right-hand-side parameters of the inequality constraints to derive robust solutions to revenue management problem facing uncertain market demand. Two types of uncertainty set are addressed: bounded uncertainty and symmetric uncertainty.

Bounded uncertainty
Suppose that uncertainty data are ranged in the following interval:

$$\left|\tilde{b}_i - \overline{b_i}\right| \leqslant \varepsilon \left|\overline{b_i}\right|, \tag{8}$$

where $\tilde{b}_i$ are the "true" values, $\overline{b}_i$ are nominal values and ε is defined as the uncertainty level.

We provide the following definition of "robust" solution to the LP problem with bounded uncertain right-hand-side parameters:

Definition 3.1 (Lin *et al.*, 2004) When the right-hand-side uncertainty is described in a bounded manner, we call a solution x robust if it satisfies the following conditions:

(i) x is feasible for the nominal problem; and

(ii) whatever the true values (say $\tilde{b}_i$) of the right-hand-side parameters from the intervals (7), x must satisfy the ith inequality constraint with an error of at most $\delta \max\{1, |\overline{b}_i|\}$, where δ is interpreted as a given infeasibility level.

More specifically, condition (ii) can be expressed as:

$$\forall i\left(\left|\tilde{b}_i - \overline{b}_i\right| \leqslant \varepsilon\left|\overline{b}_i\right|\right): \sum_{j \in J} a_{i,j} x_{i,j} \leqslant \tilde{b}_i + \delta \cdot \max\{1, |\overline{b}_i|\}. \tag{9}$$

For the sake of deriving robust solutions, we use the worst-case values of the uncertain parameters:

$$\tilde{b}_i \geqslant \overline{b}_i - \varepsilon|\overline{b}_i|, \tag{10}$$

and substitute (10) into (9).

Therefore, it is definitely clear that x is robust if and only if x is a feasible solution to the following optimization problem:

$$\begin{aligned}
&\max \ cx \\
&\text{s.t.} \sum_{j \in J} a_{i,j} x_{i,j} \leqslant \overline{b}_i \\
&\sum_{j \in J} a_{i,j} x_{i,j} \leqslant \overline{b}_i - \varepsilon|\overline{b}_i| + \delta \max\{1, |\overline{b}_i|\} \\
&x_{i,j} \geqslant 0, \forall i, j.
\end{aligned} \tag{11}$$

The derived formulation (11) is called as "(ε, δ) – interval RC (IRC(ε, δ))" of the original ILP problem with right-hand-side uncertainty.

Symmetric uncertainty

In this subsection, the uncertain data $\tilde{b}_i$ are assumed to be distributed around the nominal values $\overline{b}_i$ randomly and symmetrically as follows:

$$\tilde{b}_i = (1 + \varepsilon\varsigma_i)\overline{b}_i, \tag{12}$$

where the perturbations ς_i are independent variables symmetrically distributed in the interval [−1, 1].

For the purpose of providing a similar definition of "robust" solution to the ILP problem with bounded uncertain right-hand-side parameters, it is of great significance to transfer the deterministic version (ii) to its probabilistic version. Therefore, the definition of "robust" solution to the LP problem with symmetric uncertain right-hand-side parameters is presented as follows:

Definition 3.2 (Lin *et al.*, 2004) When the right-hand-side uncertainty is symmetric, we call a solution x robust if it satisfies the following conditions:

(i) x is feasible for the nominal problem; and

(ii) for every i, the probability of the event of constraint violation, i.e.:

$$\sum_{j \in J} a_{i,j} x_{i,j} > \tilde{b}_i + \delta \cdot \max\{1, |\overline{b}_i|\},$$

is at most κ, in which $\delta > 0$ is a given infeasibility tolerance, and $\kappa > 0$ is a given "reliability level."

Therefore, a robust solution to LP problem with symmetric uncertain right-hand-side parameters will be derived by finding solutions to the following ((ε, δ, κ) – RC(ε, δ, κ)):

$$\max \ cx$$

$$\text{s.t.} \sum_{j \in J} a_{i,j} x_{i,j} \leqslant \bar{b}_i$$

$$\sum_{j \in J} a_{i,j} x_{i,j} + \varepsilon\Omega \left|\bar{b}_i\right| \leqslant \bar{b}_i + \delta \max\left\{1, \left|\bar{b}_i\right|\right\}$$

$$x_{i,j} \geqslant 0, \forall i, j, \tag{13}$$

where Ω is a positive parameter with $\kappa = \exp\{-\Omega^2/2\}$. The case investigated here is a special form of the work of Lin *et al.* (2004), thus the process to derive RC (ε, δ, κ) can be directly referred to Lemma 1 and Theorem 2 in the paper of Lin *et al.* (2004), which is eliminated in this paper for the sake of brevity.

Note that in the aforementioned clarification of relative robust formulations, relative parameters of uncertainty level (ε), infeasibility tolerance (δ) and reliability level (κ) are assumed to be single and common, for the sake of simplicity. However, the proposed robust optimization techniques can be easily extended to account for more general cases in which these parameters are dependent on the constraint of interest.

Robust CLRM models under uncertain demand

The robust optimization models proposed in the previous section can be directly applied to the container slot allocation problem with MQC under uncertain demand. Recalling the basic model (5), we consider the application of IRC(ε, δ) and RC(ε, δ, κ) to this basic formulation, respectively. Note that in the context of CLRM, the parameters $\bar{b}_i$ in (11) and (13) are definitely positive. It is absolutely rational to assume that there exists at least one booking request arrives each day. Therefore, $\max\left\{1, \left|\bar{b}_i\right|\right\} = \bar{b}_i$.

Consequently, the proposed robust optimization models of CLRM are now converted into the following:

$$\text{Max} \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} r_{i,j} x_{i,j}$$

$$\text{s.t. } y_{k+1} \cdot q \leqslant \sum_{i=0}^{k-1} \sum_{j=k+1}^{P} x_{i,j} - \sum_{i=0}^{k-1} x_{i,k} + \sum_{j=k+1}^{P} x_{k,j}, \ k = 1, 2, \ \ldots, \ (P-1)$$

$$\sum_{i=0}^{k-1} \sum_{j=k+1}^{P} x_{i,j} - \sum_{i=0}^{k-1} x_{i,k} + \sum_{j=k+1}^{P} x_{k,j} \leqslant y_{k+1} \cdot C, \ k = 1, 2, \ \ldots, \ (P-1)$$

$$y_1 \cdot q \leqslant \sum_{j=1}^{P} x_{0,j}$$

$$\sum_{j=1}^{P} x_{0,j} \leqslant y_1 \cdot C$$

$$x_{i,j} \leqslant \overline{U}_{i,j}$$

$$x_{i,j} \leqslant (1-\varepsilon+\delta)\overline{U}_{i,j}$$

$$x_{i,j} \geqslant 0,\ 0 \leqslant i < j \leqslant P$$

$$y_k \in \{0,1\}, k = 1, 2,\ \ldots,\ P \tag{14}$$

and:

$$\text{Max} \sum_{i=0}^{T-1} \sum_{j=i+1}^{T} r_{i,j} x_{i,j}$$

$$\text{s.t. } y_{k+1} \cdot q \leqslant \sum_{i=0}^{k-1} \sum_{j=k+1}^{P} x_{i,j} - \sum_{i=0}^{k-1} x_{i,k} + \sum_{j=k+1}^{P} x_{k,j},\ k = 1, 2,\ \ldots,\ (P-1)$$

$$\sum_{i=0}^{k-1} \sum_{j=k+1}^{P} x_{i,j} - \sum_{i=0}^{k-1} x_{i,k} + \sum_{j=k+1}^{P} x_{k,j} \leqslant y_{k+1} \cdot C,\ k = 1, 2,\ \ldots,\ (P-1)$$

$$y_1 \cdot q \leqslant \sum_{j=1}^{P} x_{0,j}$$

$$\sum_{j=1}^{P} x_{0,j} \leqslant y_1 \cdot C$$

$$x_{i,j} \leqslant \overline{U}_{i,j}$$

$$x_{i,j} \leqslant (1-\varepsilon \cdot \Omega+\delta)\overline{U}_{i,j}$$

$$x_{i,j} \geqslant 0$$

$$y_k \in \{0,1\}, k = 1, 2,\ \ldots,\ P$$

$$\text{for all } 0 \leqslant i < j \leqslant T, \tag{15}$$

where Ω is a positive parameter that satisfies $\kappa = \exp\{-\Omega^2/2\}$, respectively.

Notice that the characteristic structure of models (14) and (15) implies that when parameters satisfy $\delta \geqslant \varepsilon$ and $\delta \geqslant \varepsilon\Omega$, respectively, the optimal solutions derived from our robust optimization models equal to the case with nominal values. Therefore, in order to study the performance of our proposed robust optimization models, we are of significant interest to investigate the cases where $\varepsilon \geqslant \delta$ and $\varepsilon\Omega \geqslant \delta$.

The prominent feature of the above formulations (14) and (15) is that both are now in an integer LP (ILP) form and ready to be solved by popular linear modeling packages, i.e. LINGO (Shrage, 1997) when relative parameters uncertainty level (ε), infeasibility tolerance (δ) and reliability level (κ) are assigned by the decision-maker.

Numerical illustrations

In this section, we investigate the practical performance of our robust optimization models for container slot allocation problem with MQC under uncertain demand. For simplicity, the unit rate for each stop is constant at 1, thus the revenue for any single or multiple stop is linearly proportional to the number-of-stop. All data employed in this study are provided by a third-part logistics company in Hong Kong. For business security, we eliminate the name of the company and normalize relative data. The container ship has a maximum capacity of 8,000 twenty-foot equivalent units (TEUs), and the stipulation of MQC is denoted by $b = 4{,}000$ TEUs. Nominal values of all pairs of (i, j), $\overline{U}_{i,j}$ are shown in Table I.

Bounded uncertain demand

In this subsection, we test the performance of our proposed robust model handling bounded uncertainty. Recalling the aforementioned model (14) presented in fourth section, we assume that the uncertainty level (ε) is 10 percent, and the infeasibility tolerance level (δ) is 1 percent. By solving the proposed formulation (14), a "robust" container slot allocation strategy is summarized as shown in Table II, while the nominal solution obtained at the nominal values of demand is given in square brackets.

Figure 3 indicates the relationship between expected revenue and the uncertainty level ε as well as the infeasibility tolerance δ, respectively.

The different values of the parameter of uncertainty level ε and infeasibility tolerance δ represent different degrees of risk aversion of the manager. More specifically, a larger value of uncertainty level ε and a smaller value of infeasibility tolerance δ indicate higher degree of risk aversion. As shown in Figure 3, at a given infeasibility tolerance δ, the expected revenue decreases as the uncertainty level ε increases, while with a given uncertainty level ε, the expected revenue increases with an increase of infeasibility tolerance δ. This means that more revenue can be earned if the manager is less conservative toward risk.

Recalling that the common parameters, namely, uncertainty level ε and infeasibility tolerance δ can be easily extended to investigate more general and dedicated cases, i.e. $\varepsilon_{i,j}$ and $\delta_{i,j}$, in which these parameters are dependent on the constraint of interest. Different values of these parameters can be deemed as decision control tools used by the manager. For instance, if the manager is less conservative toward typical

Table I.
Nominal values of demand $(\overline{U}_{i,j})$

					j					
i	1	2	3	4	5	6	7	8	9	10
0	260	480	1,360	1,840	1,440	220	240	120	80	60
1		220	300	1,760	2,200	240	140	140	80	80
2			220	400	1,820	160	120	280	120	120
3				680	1,920	220	300	340	160	140
4					460	1,800	300	140	140	140
5						140	200	220	140	100
6							160	260	280	100
7								160	500	580
8									220	380
9										480

Table II. Robust container slot allocation strategy with bounded uncertainty

	j									
i	1	2	3	4	5	6	7	8	9	10
0	236 (260)	436 (480)	1,237 (1,360)	1,674 (1,840)	1,310 (1,440)	200 (220)	218 (240)	109 (120)	72 (80)	54 (0)
1		200 (220)	273 (300)	121 (240)	1,788 (1,320)	218 (240)	127 (140)	127 (0)	72 (80)	0 (0)
2			200 (220)	0 (0)	219 (260)	145 (160)	109 (120)	254 (280)	109 (100)	0 (0)
3				618 (340)	1,239 (1,920)	200 (220)	273 (300)	309 (280)	145 (0)	0 (0)
4					418 (460)	1,638 (1,800)	273 (300)	0 (140)	13 (140)	0 (0)
5						127 (140)	182 (200)	0 (0)	0 (100)	0 (0)
6							82 (160)	0 (0)	0 (0)	0 (0)
7								0 (140)	0 (0)	0 (0)
8									200 (220)	134 (240)
9										423 (480)
y_k, $k = 1, \ldots, 10$	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
Revenue					51,943 (54,560)					

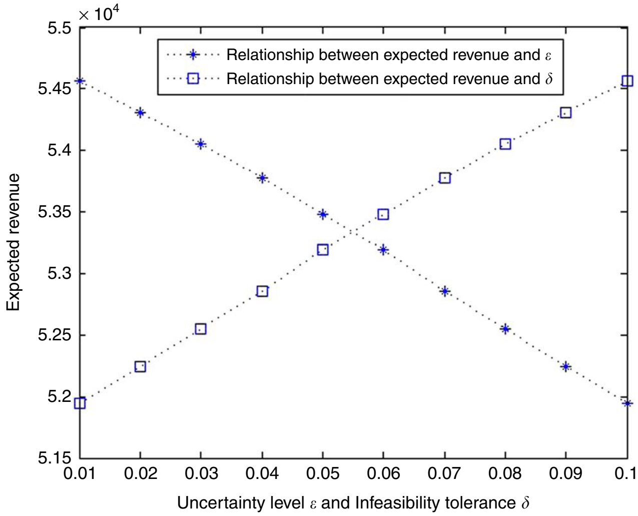

Figure 3.
Relationship between expected revenue and uncertainty level ε as well as infeasibility tolerance δ with bounded uncertainty

reservation request, or would like to accept more booking request at a particular port, she can decrease the corresponding $\varepsilon_{i,j}$ and increase $\delta_{i,j}$, or change the values of other parameters in a reverse direction. We illustrate this by re-calculating the above example while controlling the values related to container loading at port 0 and unloading at port 5. By adjusting the values of these parameters, more booking request are accepted for such delivery, i.e. an increase from 1,310 to 1,324.

The effect of MQC on expected revenue with bounded uncertain demand is demonstrated in Figure 4, in which the uncertainty level (ε) is assumed to be 10 percent, and the infeasibility tolerance level (δ) is 1 percent.

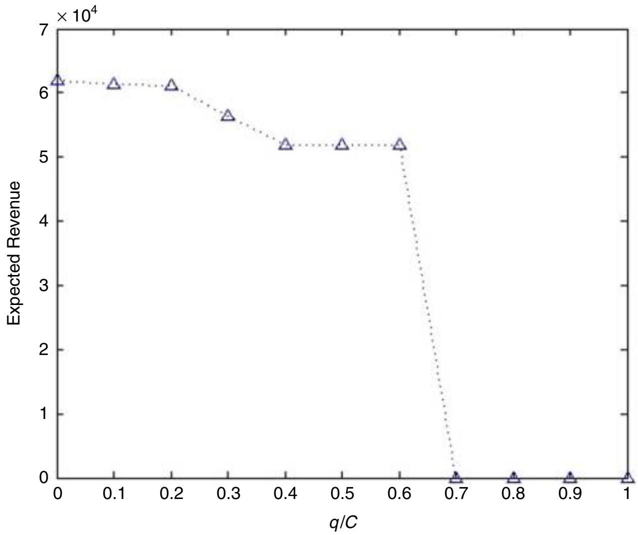

Figure 4.
Effect of MQC on expected revenue with bounded uncertain demand

It is observed that as the committed quantity increases, the expected revenue decreases. When the committed quantity is "large enough," the expected revenue unfortunately reaches 0.

Symmetric uncertain demand

In this subsection, we conduct a computational study to test the performance of our proposed robust models when the uncertainty set is symmetric. The robust optimization formulation (15) is employed to address this issue. We assume that the uncertainty level (ε) is 10 percent, the infeasibility tolerance level (δ) is 2 percent and the probability of constraint violation (κ) is 10 percent. The robust solution under this framework is summarized in Table III, while the nominal solution obtained at the nominal values of demand is still given in square brackets.

Similar to the work conducted in the bounded uncertainty case, the relationship between expected revenue and the uncertainty level ε, the infeasibility tolerance δ as well as the probability of constraint violation κ is demonstrated in Figures 5-7, respectively.

Figure 5 demonstrates that with given infeasibility tolerance δ and probability of constraint violation κ with symmetric uncertainty, the expected revenue decreases as the uncertainty level ε increases. Figure 6 shows that at given uncertainty level ε and probability of constraint violation κ with symmetric uncertainty, the expected revenue increases as infeasibility tolerance δ is increased. Figure 7 indicates that with fixed uncertainty level ε and given infeasibility tolerance δ with symmetric uncertainty, the expected revenue increases with an increase of probability of constraint violation κ.

Detailed decisions on booking accepting/rejecting strategy in general cases when we consider different values of $\varepsilon_{i,j}$ and $\delta_{i,j}$ is similar between bounded uncertainty and symmetric uncertainty cases, which are presented in the analysis in previous subsection. We reconsider the computational test presented in this subsection by increasing the values of $\kappa_{0,5}$, i.e. from 10 to 20 percent, more booking request are accepted for such delivery (an increase from 1,159 to 1,211).

The effect of MQC on expected revenue with symmetric uncertain demand is demonstrated in Figure 8, in which the uncertainty level (ε) is assumed to be 10 percent, and the infeasibility tolerance level (δ) is 2 percent and the probability of constraint violation (κ) is 10 percent.

It is observed that the behavior of expected revenue with symmetric uncertain demand is analogous to the bounded case. More specifically, as the committed quantity increases, the expected revenue decreases. When the committed quantity is "large enough," the expected revenue unfortunately reaches 0. Moreover, it is of significant interest to notice that when q/C reaches 0.7 in both cases, the expected revenue suddenly becomes 0, which provides meaningful insights for decision making on the value of MQC.

Conclusions

In this paper, we have presented a novel robust optimization approach to address the problem of CLRM with uncertain demand in terms of unknown arrivals and length of stays. A network structure is employed to describe the status of uncertain demand. Within the framework of bounded and symmetric uncertainty, two tractable mixed integer programming formulations are derived to investigate the robust CLRM

Table III. Robust container slot allocation strategy with symmetric uncertainty

	j									
i	1	2	3	4	5	6	7	8	9	10
0	209 (260)	386 (480)	1,095 (1,360)	1,481 (1,840)	1,159 (1,440)	177 (220)	193 (240)	96 (120)	64 (80)	48 (0)
1		177 (220)	241 (300)	1,288 (240)	1,323 (1,320)	193 (240)	112 (140)	112 (0)	64 (80)	0 (0)
2			177 (220)	291 (0)	0 (260)	128 (160)	96 (120)	225 (280)	0 (100)	0 (0)
3				213 (340)	1,546 (1,920)	177 (220)	241 (300)	158 (280)	128 (0)	0 (0)
4					370 (460)	1,449 (1,800)	241 (300)	112 (140)	111 (140)	0 (0)
5						112 (140)	161 (200)	0 (0)	0 (100)	0 (0)
6							74 (160)	0 (0)	0 (0)	0 (0)
7								0 (140)	0 (0)	0 (0)
8									177 (220)	111 (240)
9										385 (480)
y_k, $k=1, \ldots, 10$	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
Revenue					48,715 (54,560)					

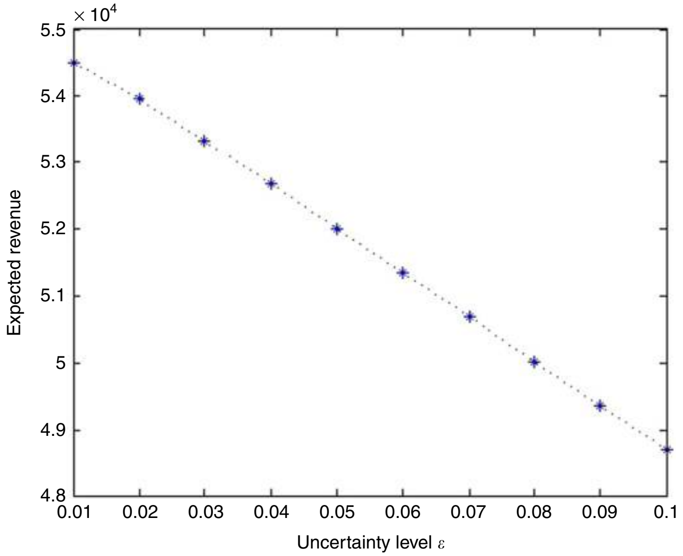

Figure 5. Relationship between expected revenue and uncertainty level ε with symmetric uncertainty

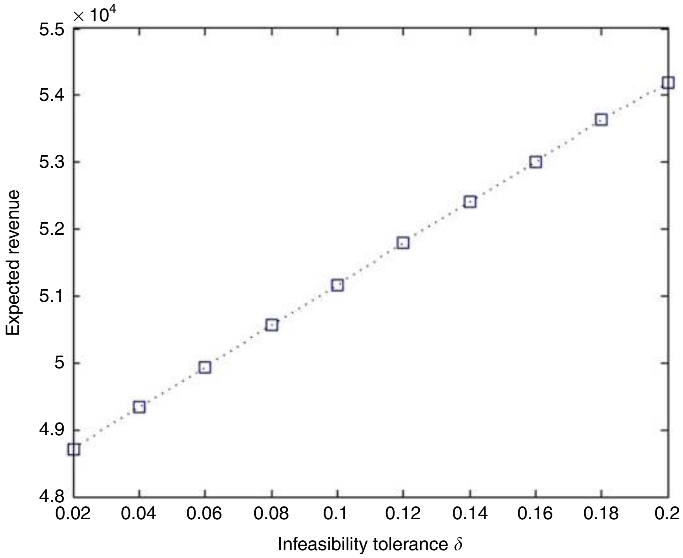

Figure 6. Relationship between expected revenue and infeasibility tolerance δ with symmetric uncertainty

problem. Meaningful managerial implications are presented when considering the relationship between expected revenue and uncertainty level, infeasibility tolerance as well as probability of constraint violation. The relationship between expected revenue and MQC has also been investigated in the study.

Future research could also consider the occurrence of cancellation and no-show, early check-out, extension of stay and overbooking.

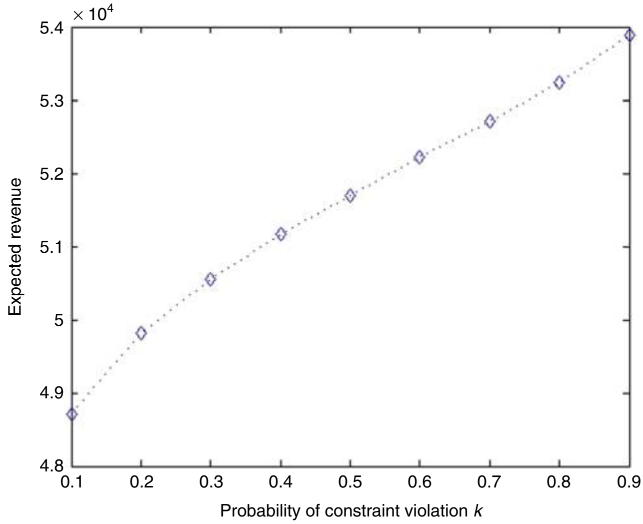

Figure 7.
Relationship between expected revenue and probability of constraint violation κ with symmetric uncertainty

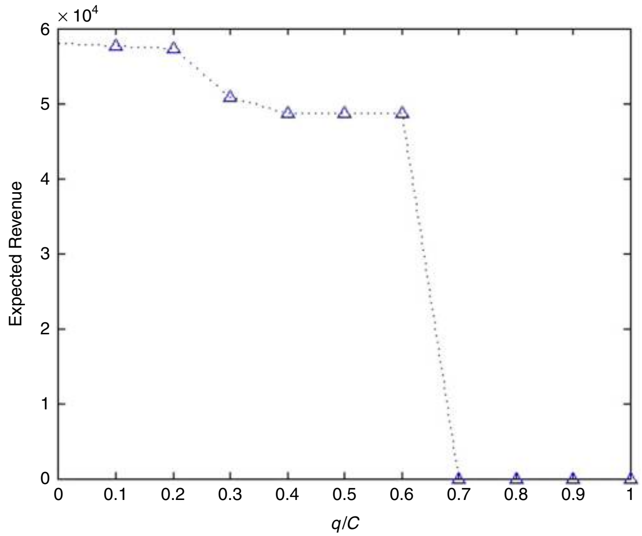

Figure 8.
Effect of MQC on expected revenue with symmetric uncertain demand

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