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An Integer Linear Programming Based Optimization for Home Demand-side Management in Smart Grid

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***Abstract*—We propose a consumption scheduling mechanism for home area load management in smart grid using integer linear programming (ILP) technique. The aim of the proposed scheduling is to minimise the peak hourly load in order to achieve an optimal (balanced) daily load schedule. The proposed mechanism is able to schedule both the optimal power and the optimal operation time for power-shiftable appliances and time-shiftable appliances respectively according to the power consumption patterns of all the individual appliances. Simulation results based on home and neighbourhood area scenarios have been presented to demonstrate the effectiveness of the proposed technique.**

***Index Terms*—Demand-side management, load scheduling, integer linear programming**

I. Introduction

Ever since the first time people were able to light their rooms using a switch, the electric power system has been in place for more than 100 years. However, the infrastructure of the existing system has been little changed by far [1]. In the recent years, the energy industry has been facing significant challenges in terms of environmental issues, security and utility management. Many believe that a fundamental evolution of the electric power system is needed. The goal of the transformation is to make the next generation power grid a green, reliable and intelligent system, which is generally known as the *Smart Grid*.

In smart grid, advanced electric technologies will improve the efficiency of the central bulk generation and transmission network. Medium/low voltage Distributed Generation (DG) system using various environment-friendly sources will be seamlessly integrated. Information and communication technologies (ICT) will be widely applied to the grid to increase its operation efficiency, reliability and flexibility [2], [3]. Eventually, consumers will get the benefit of lower bills while proactively managing their electricity consumption.

Smart grid has become an attractive research area. Currently, the main research directions are on designing workable ICT infrastructure and energy management applications such as distributed energy resource management, load management and demand response techniques, as discussed in [4]–[11].

The focus of the work in this paper is on power consumption scheduling known as demand-side management. In [4], a powerful convex optimization technique was proposed to schedule the power of individual appliances. Both the centralised optimization based on linear programming and the decentralised approach based on game theory have been proposed. However, the optimization framework in [4] might not be suitable for all appliances in practice. This is because some appliances have a fixed power consumption pattern, which means that once the appliance is turned on, it has to work according to its own power consumption pattern until the job is finished. In this case, we could optimize only the starting time, but the power consumption during the operation of the appliance is not under the control of the optimizer, hence it is not an optimization parameter. In addition for consumer's comfort, we wish to introduce a feature that once certain appliance such as washing machine and dish washer is scheduled to start, it should be allowed to finish the task without any break.

In this paper, inspired by the scheduling mechanism in [4], we propose a demand-side management method using integer linear programming technique. The proposed mechanism will be able to minimise the peak hourly load and satisfy both the user preference and specific requirements of all individual appliances. The mechanism can be firstly applied to the home energy management unit/gateway embedded in the smart meters, which will probably work as a load control unit as well, in the home area network (HAN). Provided the consumption patterns of all the appliances and the user's own load plan, the meter will be able to make an optimal scheduling for all the connected appliances in the household. The mechanism can also be applied to the distribution stations (central control node) in the neighbourhood/local areas to achieve centralised load management. In the future, when real-time communications and various smart grid services such as flexible demand response technique become widely available, the mechanism may be further applied to distributed load control for every individual consumer in the grid.

The rest of the paper is organised as follows. The proposed

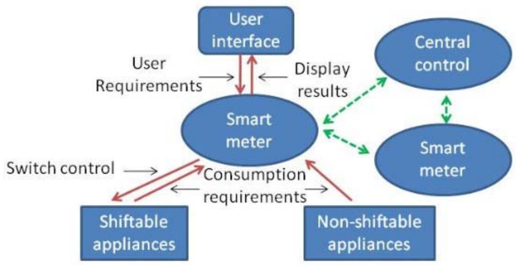

Fig. 1. System components.

demand-side load management mechanism is described in Section II. Section III presents system simulation results and conclusions are drawn in Section IV.

II. Demand-side consumption scheduling

A. System description

Fig. 1 depicts the overall structure of the load management system. As shown in the figure, the smart meter is the key component in the system. It connects with the user interface to collect the user's own power consumption plan and preference and display the scheduling information. On the other side, the smart meter connects with all the home appliances, not only to provide electricity for the appliances but also to determine the total requirements and power consumption patterns of all individual appliances. Based on all the collected information, the meter will globally optimize the hourly consumption and schedule all appliances. For non-shiftable appliances such as TV and fridge which have fixed power requirement and operation period, the optimization will ensure continuous supply of power. The scheduling optimization will be carried out mainly for the shiftable appliances. For time-shiftable appliances, such as washing machine, the smart meter will be able to control the switch and provide sufficient electricity corresponding to the power pattern during the scheduled periods. For power-shiftable appliances, such as water boiler and electric vehicle chargers, the smart meter will schedule flexible power and ensure the total supply.

The system can be further extended to multiple users' scenario where many smart meters are connected together and they agree to achieve a cooperative scheduling. The central control node will take the overall responsibility of scheduling the whole network and assigning individual meters their corresponding tasks.

Effective communication networks are required for the system. Wireless sensor networks (WSN) combines sensing and communications together, and provides low-cost and low-power information gathering, processing and communication with flexible self-organising network deployments [12], [13]. It is believed that WSN will be one of the promising technologies for HAN communications. For communications beyond the home environment, i.e. among the meters/gateways, control nodes and operators, the obvious candidates are wireless cellular technologies and various broadband solutions (e.g. GPRS, WiFi, or LTE). However, as for non-real-time scheduling in our system, the information exchange may not require communications with high data rate and low delay.

B. Consumption scheduling optimization

The consumption scheduling mechanism can be described as a linear optimization problem which aims to minimise the hourly load, as shown below

$$\min_{L, x_{a,h} \in \mathbb{R}} \quad L \tag{1}$$

$$\text{s.t.} \quad \sum_{a \in \mathbb{A}} x_{a,h} \leq L, \ \forall h \in \mathbb{H}, \tag{2}$$

$$\mathbf{1}^T \mathbf{x}_a = l_a, \ \forall a \in \mathbb{A}, \tag{3}$$

$$x_{a,h} \geq 0, \tag{4}$$

where $\mathbf{1} = [1, 1, \ldots 1]^T$. We define a as an individual appliance in a set of appliances, $\mathbb{A}$. $\mathbf{x}_a = [x_{a,1}, x_{a,2}, \ldots, x_{a,24}]^T$ denotes the schedule plan for appliance a and $x_{a,h}$ is the scheduling variable which represents the power consumption of appliance a in the particular hour $h \in \mathbb{H}$, $\mathbb{H} = [1, 2, \ldots, 24]$ of the day. The variable L represents the peak hourly load. The cost function of the above optimization problem is to minimise the hourly load L subject to the constraints in (2)-(4). The hourly load should be greater than or equal to the sum of the scheduled power for all appliances in that time. For each appliance, the total daily supply has to meet the requirement l_a. The power consumption $x_{a,h}$ must be non-negative value, i.e., $x_{a,h} \geq 0$. We use a time resolution of one hour for hourly scheduling in this paper, hence $\mathbf{x}_a$ contains 24 elements. This resolution can be increased by increasing the length of $\mathbf{x}_a$.

We formulate the requirements for all the individual appliances and the user preferences as constraints and add them into the optimization problem (1)-(4). For non-shiftable appliances, the hourly power requirement is fixed at δ_a during its working period from h_{as} to h_{af} where $h_{as}, h_{af} \in \mathbb{H}$. This constraint can be written as

$$x_{a,h} \geq \delta_a, \forall h \in [h_{as}, h_{a(s+1)}, \ldots, h_{af}]. \tag{5}$$

Note that a similar constraint can also be applied to some of the shiftable appliances if the consumer has a specific requirement in terms of starting and finishing time for the appliance. For example, people usually control the electric heater using a programmable timer or a thermostat. This kind of heater can be treated as a non-shiftable appliance.

For power-shiftable appliances with a standby power α_a

and a maximum working power β_a, together with a possibly preferred working period, the consumption requirement can be written as

$$\alpha_a \leq x_{a,h} \leq \beta_a, \forall h \in [h_{as}, h_{a(s+1)}, \ldots, h_{af}]. \quad (6)$$

Now consider a time-shiftable appliance that consumes power l_a. If we impose only the constraint (3), the optimization will ensure supply of a total power of l_a over the 24 hours. For example, if a washing machine requires 1.5kWh for its operation, the constraint in (3) could ensure 1.5kWh power supply over 24 hours. This total power can be distributed in any possible ways. For example, the vector $\mathbf{x}_a$ can have 0.25 for six hours and zeros for the rest. However, a washing machine cannot operate according to this power schedule. It might require 1kW for the first hour of operation and 0.5kW for the second hour of operation. This kind of appliances will need to operate according to their own power consumption pattern for correct operation. Therefore, only a constraint in the form of (3) is inadequate to ensure feasible operation of certain appliances. Considering this, we model this kind of requirement as follows.

Suppose, an appliance a has a fixed consumption pattern $\mathbf{p}_a = [p_{a,1}, \ldots, p_{a,24}]^T$, the schedule result $\mathbf{x}_a$ has to be exactly the same as one of the cyclic shifts of that pattern. All the possible shifts for the vector $\mathbf{p}_a$ and hence all the possible $\mathbf{x}_a$ can be put in a matrix form as

$$\begin{aligned} \mathbf{P}_a &= \begin{bmatrix} \mathbf{c}_{a1} & \mathbf{c}_{a2} & \ldots & \mathbf{c}_{a24} \end{bmatrix} \\ &= \begin{bmatrix} p_{a,1} & p_{a,24} & \cdots & p_{a,3} & p_{a,2} \\ p_{a,2} & p_{a,1} & \cdots & p_{a,4} & p_{a,3} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{a,24} & p_{a,23} & \cdots & p_{a,2} & p_{a,1} \end{bmatrix}. \end{aligned} \quad (7)$$

The above constraint cannot be directly formulated using linear programming. Hence, we apply integer linear programming (ILP) framework to formulate the constraint.

Firstly, we define the schedule plan as $\mathbf{x}_t = [x_{t,1}, \ x_{t,2}, \ \ldots, \ x_{t,24}]^T \geq 0$, and the appliance set $\mathbb{T} \subset \mathbb{A}$ for the time-shiftable appliance $t \in \mathbb{T}$. Then, we define a binary integer vector $\mathbf{s}_t = [s_{t,1}, \ s_{t,2}, \ \ldots, \ s_{t,24}]^T$ as the switch control for the time-shiftable appliance. There is only one non-zero element in the vector $\mathbf{s}_t$ which is equal to one. This property is written as follows

$$\mathbf{1}^T \mathbf{s}_t = 1, \mathbf{s}_t \in \{0, 1\}^{24 \times 1}. \quad (8)$$

Now the schedule plan can be written as

$$\mathbf{x}_t = \mathbf{P}_t^T \mathbf{s}_t. \quad (9)$$

Then, we rewrite the optimization problem in (1) by adding $\mathbf{s}_t$ as an integer optimization variable, together with other constraints as

$$\begin{aligned} \min_{L, x_{a,h} \in \mathbb{R}, \mathbf{s}_t \in \mathbb{Z}^{24 \times 1}} \ & L \\ \text{s.t.} \ & \sum_{a \in \mathbb{A} - \mathbb{T}} x_{a,h} + \sum_{t \in \mathbb{T}} x_{t,h} \leq L, \ \forall h \in \mathbb{H}, \\ & \mathbf{1}^T \mathbf{x}_a = l_a, \ \forall a \in \mathbb{A} - \mathbb{T}, \\ & x_{a,h} \geq 0, \\ & x_{a,h} \geq \delta_a, \forall h \in [h_{as}, \ldots, h_{af}], \\ & \alpha_a \leq x_{a,h} \leq \beta_a, \forall h \in [h_{as}, \ldots, h_{af}], \\ & 0 \leq \mathbf{s}_t \leq 1, \ \mathbf{1}^T \mathbf{s}_t = 1, \ \forall t \in \mathbb{T}, \\ & \mathbf{x}_t = \mathbf{P}_t^T \mathbf{s}_t, \ \forall t \in \mathbb{T}. \end{aligned} \quad (10)$$

Note that at this stage, the scheduling objective for the time-shiftable appliance is the switch vector $\mathbf{s}_a$, not the power $x_{a,h}$. Therefore, we have to remove the corresponding appliance from the appliance set $\mathbb{A}$. The set $\mathbb{A} - \mathbb{T}$ now contains non-shiftable and power-shiftable appliances, whose power allocations are still needed to be scheduled by $x_{a,h}$.

The optimization problem above is a mixed integer linear programming problem which contains both integer variables and non-integer variables. It can be solved using Branch and Bound method [14]. Note that if there are more than one optimal solution (with equal cost value) to the problem, the method will provide one of them.

The above scheduling scheme is suitable for fixed price scenario where the unit price of the electricity consumption is fixed at any time. In this case, consumers have little incentive to shift their consumption. However, operators who want to reduce the peak load will be able to achieve direct load control without hurting the consumers' preferences by applying this scheduling scheme. If the user is on a particular tariff, e.g. the UK Eco 7 which provides much lower price for late night consumption [15], the users may change their preference and the mechanism will also be able to provide optimal schedule.

The mechanism can also be further applied to distributed control systems or real-time scheduling with flexible demand response and pricing policies. The main difference is the distributed control system may probably need a perfect communication system to support the exchange of information.

III. Simulation Results

A set of system simulations has been carried out using Matlab to implement the proposed scheduling mechanism in home area and neighbourhood area scenarios. In the simulation, we first define a set of home appliances and their individual requirements as listed in Table I.

According to the settings, we are able to formulate the total power requirement and the individual consumption patterns to

Name	Type	User preference and power requirement
1.Hob and oven 2.heater	Non-shiftable	Operating period: 7pm-8pm(Hob/oven) 9pm-10pm, 3am-5am(heater) hourly consumption: 1kWh
3.Fridge and freezer	Non-shiftable	Operating 24hrs hourly consumption: 0.12kWh
4.Water boiler	Power-shiftable	Hourly consumption: 0-1.5kWh daily requirement: 3kWh
5.Electric vehicle (15 miles daily driving [16])	Power-shiftable	Preferred charging period: 8pm-8am charging power: 0.1kW-3kW daily requirement: 5kWh
6.Washing machine	Time-shiftable	Operating 2hrs, once per day 1kWh for the 1st hr 0.5kWh for the 2nd hr
7.Dish washer	Time-shiftable	power: 0.8kWh for 1 hour daily requirement: 0.8kWh

TABLE I
APPLIANCES AND POWER CONSUMPTION PATTERNS

the corresponding constraint formats as in (5), (6), (8) and (9). For example for appliance 1, which is the hob and oven, we can use the expression in (5) and make the constraint as

$$x_{1,h} \geq 1, \ \forall h \in [19, 20]. \tag{11}$$

Similarly for the electric vehicle battery charger we can write

$$0.1 \leq x_{5,h} \leq 3, \ \forall h \in [1, 8] \cup [20, 24]. \tag{12}$$

For the time-shiftable washing machine, we defined the power pattern as $\mathbf{p}_6 = [1, 0.5, 0, \cdots, 0]^T$ and the switch vector $\mathbf{s}_6$ to formulate the constraint as

$$\mathbf{x}_6 = \mathbf{P}_6^T \mathbf{s}_6. \tag{13}$$

After incorporating all the constraints together, we can solve the ILP problem in (10) and achieve the optimal scheduling. Fig. 2 depicts the scheduled hourly load. It can be seen that the maximum hourly load of 1.22kWh appears during the hours of 3-5, 14 and 19-22. The minimum load of 0.3kWh appears during the hours of 9-13 and 17-18. From the appliances consumption requirement, we can see that when the heater is turned on between 9-10pm, and considering the power of the fridge and the charging of electric vehicle battery, the required load is 1+0.12+0.1=1.22 (kWh). For any time of the day, the scheduled load is less than or equal to 1.22kWh. Hence we claim that the optimal hourly scheduling with a minimum possible peak is achieved. When the number of appliances is increased, a more balanced hourly power consumption schedule can be expected.

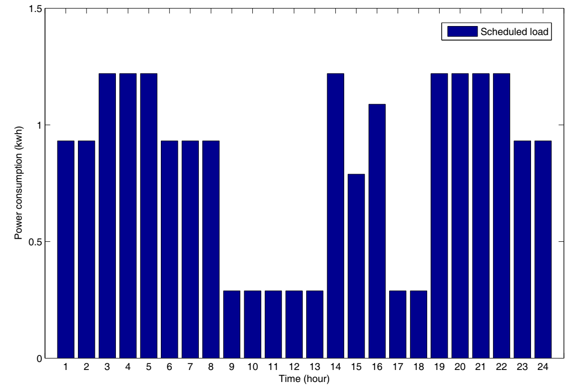

Fig. 2. Hourly power consumption schedule.

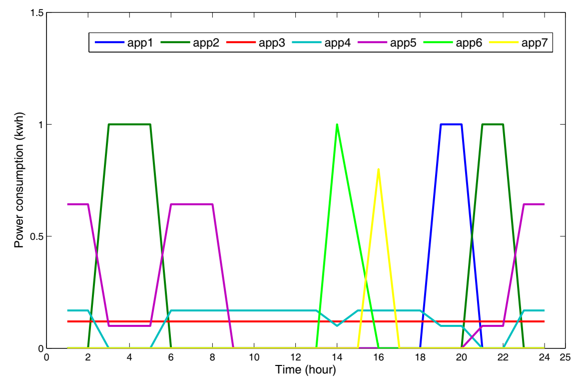

Fig. 3. Optimal schedule for individual appliances.

Fig. 3 depicts the scheduling result for the individual appliances. Clearly, we can see that during 9-10pm, the heater (app2) is consuming fixed 1kWh and the fridge (app3) is consuming 0.12kWh. The water boiler (app4) has been totally shut down and only 0.1kWh which is the minimum charging power requirement for the electric vehicle (app5) is provided. Time-shiftable appliances were not scheduled to operate during the period. Indeed, the washing machine (app6) and the dish washer (app7) are both scheduled in the day time between 2pm-5pm where there is lower demand from other appliances.

We further carry out a simulation for the scenario of a small neighbourhood area with multiple households. Assume there are 4 houses and the total daily load requirement is 43 units. Each house has similar appliances as in the previous case but with different consumer preferences and power patterns for some of the appliances. All the required information is collected from the smart meter of each household. Now a centralised scheduling which will meet all the requirements

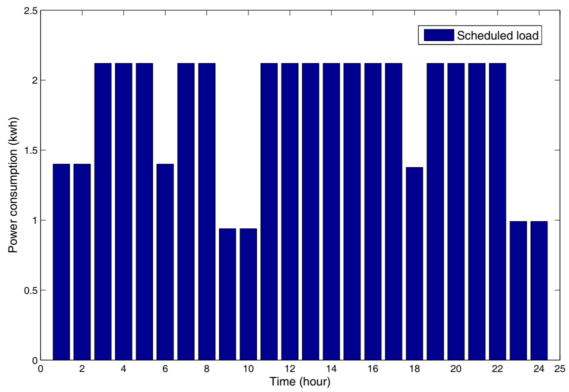

Fig. 4. Hourly power consumption schedule for multiple households.

is achievable.

Fig. 4 depicts the overall scheduled hourly load. As seen, the optimized hourly peak load is 2.14kWh which is just around one unit higher than that of the previous case. Besides, the overall load allocation is more balanced over the 24 hours. The peak to average load ratio is $2.14/(43/24) = 1.19$, which means the peak load is just 19% higher than the daily average. It can be observed that the scheduling mechanism is able to reduce the maximum load and improve the performance and the reliability of the power grid due to reducing the peak to average load.

IV. Conclusion

We have proposed an integer linear programming based optimization mechanism for the home demand-side management in smart grid. The proposed mechanism is able to schedule both the optimal power and the optimal operation time for power-shiftable appliances and time-shiftable appliances respectively according to user preference and the power consumption patterns of individual appliances. Simulation results demonstrated the effectiveness of the mechanism. When multiple households participate in the scheduling, a more balanced hourly load is achieved.

V. Acknowledgements

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VI. Biographies

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