University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

MIS40520 Assignment 2: M(I)LP Applications and Formulations

This assignment can be completed as an individual or in a team of two. Teams should be notified to Dr Carroll by Monday 27th February 17.00. All

The Problems:

You'll find a list on Blackboard assigning one of the problems below to you/your team.

What you have to do:

1. Part 1: Academic Papers:
   - Locate and summarise at least one academic article relevant to your M(I)LP application area;
2. Part 2: M(I)LP formulation and implementation:
   - Formulate an appropriate algebraic M(I)LP model of the problem;
   - Implement the model in Mosel using good practices (eg. separate the data into a data file, use the sum and forall constructs, your model should be commented fully throughout).
3. Part 3: Report
   - Write a short technical report (max 8 pages) to explain:
     - How the research articles relate to your specific problem;
     - Any assumptions in your modelling approach;
     - Your algebraic formulation;
     - A complete interpretation of the optimal solution(s) and recommendation(s);
     - Any conclusions or other insights.
     - Note: Details of your Mosel implementation should not appear in the body of the report but can be included in an appendix which doesn't contribute to the page count.
4. Teams of two should also address the following question: How does your model perform on larger problem instances? To answer this question you may wish to use additional data (and variables) to empirically test the performance of your implementation. You may wish to use open source or simulated data.

What's the point of this assignment:

This assignment addresses the module learning outcomes as follows:

Index	Learning Outcome	Papers	Formulation	Report
1	discuss a portfolio of important business and other optimisation problems;	x		x
2	describe the main principles of mathematical modelling as they apply to decision problems and optimisation;		x	x
3	explain the concepts of a suite of optimisation techniques such as linear, integer linear and non-linear programming and network approaches;			
4	apply optimisation techniques to improve the quality of analysis and decision-making;		x	
5	formulate and solve appropriate mathematical models to solve a variety of real world problems.		x	
6	use mathematical computer packages and information technology as an aid in decision making;		x	x

What you have to submit:

- Upload your report through the Blackboard SafeAssign link.
- Upload a single Windows zipped folder (*.zip) containing your Mosel model, data and any execution instructions on Blackboard.

In the case of teams, the team nominee is responsible for a single upload from the team. Both team members should keep an archive copy of all work.

Regulations:

All UCD policies on plagiarism and integrity apply. These policies apply equally to code. Submissions for this module are expected to exhibit academic integrity, whereby appropriate acknowledgement is given to the work of others through citation and references. Plagiarism is academic dishonesty and will attract the strongest penalties at university level. Less serious cases, where submitted work which is derivative from the work of others, may attract reduced marks or a requirement for resubmission. This policy applies equally to code submitted as coursework.

You can include your team agreement and statement of authorship as an appendix in your report. The appendix does not count toward the page count limit. You'll find suggestions on how to write a good technical report in the Assignments folder on Blackboard.

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Problem List

1. Security Guards

A company offers a security service by placing guards within the grid squares shown in the Table below. Their customer requires at least two guards per row and at least two guards per column. The Table below shows the maintenance pay for each location.

	1	2	3	4
A	6	10	10	9
B	5	7	5	10
C	3	7	6	9
D	9	4	7	8

Table: Security guard maintenance pay

The customer is willing to increase pay by 10% if the firm will provide an even number of guards per row and per column. An even number is an integer which yields no remainder when divided by two. Show how your (I)LP model for could be adjusted to accommodate the amended requirements.

2. Computer Production

A computer manufacturer can produce at most 400 computers per month. Production costs per computer in January are €120. Costs increase by 10% for each following month of a six month planning horizon. The company can store inventory at a cost of €10 per computer per month but has a maximum storage capacity of 250. Inventory can be used in any month but must be used by the end of the planning horizon. You can assume that inventory is empty at the start of the planning horizon. The Table below shows the data. Formulate an (I)LP model to determine how many computers to produce and store each month to meet demand and capacity constraints at minimum cost.

Month	Demand (No. units)	Production Cost (€)
Jan	180	120.00
Feb	260	132.00
Mar	340	145.20
Apr	210	159.72
May	400	175.69
Jun	320	193.26

Table: Computer production data

3. Nurse Scheduling

The manager of a nursing home wishes to determine a schedule for nursing and nurse's aid staff to make sure there are enough on duty for the time periods shown below. All staff report for duty at the start of a four-hour period and work eight consecutive hours. At most 12 nurses and at most 12 nurses's aid can start any shift to minimise disruption to patient care.

Shift start time	6.00	10.00	14.00	18.00	22.00	2.00
Nursing Requirement	12	8	8	12	8	8
Nurse's Aid Requirement	18	12	10	18	4	4

Formulate an (I)LP model to determine a schedule to meet minimum requirements using the minimum number of staff. How is the solution affected if health services regulations are implemented that at least two nurse's aids are scheduled for each nurse in all shifts?

4. Electricity Management

Efficient management of power generation is the focus of much current research. Data for four generating units and a six hour planning horizon are shown below. Each generator has to work between its min and max specified Mega Watt (MW) power levels. Demand in MW for each hour must be met.

	Unit 1	Unit 2	Unit 3	Unit 4		
Pmax (MW)	455	455	130	130		
Pmin (MW)	150	150	20	20		
cost (€/MW hr)	90	100	110	112		
Hour	1	2	3	4	5	6
Demand MW	700	750	850	950	1,000	1,100

Formulate an (I)LP model to determine the output power level of each generator in each hour that minimises costs. How is the solution affected if demand drops to one tenth in all hours?

5. Breakfast Cereal

A health food company makes three varieties of muesli, called Everyday, Ownbrand and Luxury. The muesli varieties sell for €2, €2.5 and €3 per kg and marketing estimate demand is 2,500, 3,000 and 4,000 kg respectively. These mueslis are made from Oats, raisins, coconut flakes and crushed almond nuts of which 10,000 4,000 2,000 and 2,000 kg are available. These ingredients cost €1.00, €1.20, €1.10 and €2.00/kg respectively. The product development team have specified that Everyday muesli should have a 50:5 mix of oats and raisins. Ownbrand should have a 55:2 mix of oats and coconut. Luxury should have a mix of 60:3 oats and almonds. They have not specified recipe constraints for the other ingredients in the cereal. Formulate an (I)LP model to determine the optimal production mix of the cereals and the associated amounts of ingredients. How does your model perform if scaled up for a company that makes three hundred breakfast cereal brands?

6. A Cargo Balancing Problem

A freight company is shipping goods to the Aran Islands and wants to maximise its profits while adhering to Marine safety rules. Its ships have 4 compartments A to the front of the ship, B and C in the middle of the ship and D to the rear. The space available in the compartments is 125, 512, 512 and 216 m^3 respectively. The weight allowed in the compartments is 9, 15, 15 and 7 tonnes respectively.

There are currently four categories of goods available that could be shipped:

	Profit (€/tonne)	Volume (m^3)	Weight (tonnes)
Grain Feed	250	59	15
Fuel (turf)	310	100	17
Food Stuffs	450	60	13
Household Goods	525	70	16

Formulate and solve an appropriate (I)LP model to maximise profits. If Department of Health regulations say that food stuffs (for human consumption) cannot be packed with any other types of goods, how does this impact your optimal solution?

7. Woodturner Problem

A small company makes wood turned bowls, platters, urns and ring boxes. Each item is turned, decorated and finished at sequential workstations, processing times and profits for the items are shown in the table below along with the amount of beech wood required for each item. There is 1.1 m^3 of beech available per week. All workstations requires daily maintenance to clear shavings and paint etc which takes up 10%, 14% and 3% respectively of the daily time available at each type of workstation. There are 12 turners, 12 decorators and 3 finishers, all working 40 hours per week. They have 12 orders on the books for urns this week. Formulate an (I)LP model to assist the owner in determining the optimal product mix that minimises the idle time this week. Is this very different than the mix if the owner seeks to maximise profit?

Workstation	Bowl	Platter	Urn	Ring-box
Turning (hrs)	5	3	6	2
Decorating (hrs)	1.5	3.5	2	0.5
Finishing (Hrs)	1	1	1.25	0.25
Beech (m^3)	0.024	0.004	0.003	0.00011
Profit	€ 75.00	€ 85.00	€ 50.00	€ 20.00

8. WakeMeUp Coffee

The WakeMeUp Coffee company produce three types of espresso coffee mix, Bronze, Silver and Gold. Each type is made from a blend of arabica and robusto coffee beans. The Table below gives, for each type,

- the quantities of arabica and robusto required per unit,
- the time in hours needed to produce a unit,
- the minimum demand in units as determined by Marketing,
- the profit per unit in €.

The rightmost column of the Table below gives the quantities of arabica and robusto, and total factory time, available per week.

Unit requirements				
Raw material	Bronze	Silver	Gold	Available
Arabica	4	7	10	65,000
Robusto	6	3	0	30,000
Time per unit (hrs)	2	3	4	42,000
Minimum Demand	2,800	2,000	1,100	
Profit per unit (€)	30	50	80	

Table of requirements and constraints

In addition to these requirements, Marketing state that:

- the number of units of Bronze produced must be between 3 and 5 times the number of units of Gold;
- the number of units of Silver produced must be between 2 and 4 times the number of units of Gold.

The company wants to maximise profit, while meeting the availability constraints of arabica, robusto and factory time, as well as meeting the minimum demands and Marketing requirements. Formulate and solve this business problem as an (I)LP. Should the company focus on getting more supply of beans or more factory time?

9. Project Selection Problem

A consulting company is involved in a number of projects but only has 20 analysts and 5 specialists available and a fund of €300,000. The company wants to determine which projects to focus on to maximise its profits. The staff can work on multiple projects at the same time. If they focus on project F, they must also focus on project C. The table shows the project requirements in terms of cost, the number of analysists and specialists. The profit from the projects is also shown:

Project	Cost (€'000s)	Num. Analyst	Num. Specialist	Profit (€'M)
A	50	7	2	0.4
B	100	6	0	0.8
C	53	8	0	0.3
D	47	4	2	0.16
E	90	7	3	0.56
F	85	6	0	0.61
G	73	8	0	0.48

Subsequently, two of the specialists resign, how does this impact your proposed solution?

10. Staff Allocation Problem

A busy electrical retailer is trying to decide how best to assign its five staff to four departments so as to maximise sales. The estimated sales revenues per day for each staff member are shown and reflect their individual area of expertise. Assume at least one employee must be allocated to each department and that staff can be part-allocated to more than one department.

Suppose management decide to make the least productive employee redundant, how could your model be changed to solve the modified problem?

Staff No	PCs	TV	Small Kitchen Appliance	Large Kitchen Appliance
1	€ 550	€ 1,000	€ 400	€ 1,200
2	€ 850	€ 1,200	€ 300	€ 950
3	€ 1,200	€ 1,100	€ 325	€ 500
4	€ 950	€ 900	€ 450	€ 600
5	€ 550	€ 1,100	€ 425	€ 750

How does your model perform if scaled up for a company that employs five hundred staff across forty departments?

11. Warehouse Problem

A supermarket has warehouses in Ballymun and Citywest. It is running a promotional offer of high quality laptops for €250 to customers who spend over €800 in a four week period in one of their 6 outlets. Each warehouse has a supply of 300. Estimated demand in the outlets and shipping costs are shown.

	Clonee	Dundrum	Airport	Bray	Raheny	Stillorgan
Ballymun	3.75	5.5	1	6.5	3	4.5
Citywest	4.5	3	2.5	4	5	3.25

Shipping Costs

Clonee	Dundrum	Airport	Bray	Raheny	Stillorgan
175	105	85	65	75	95

Demand

Formulate an (I)LP model to minimise costs subject to supply and demand constraints. What impact is there on the solution if the marketing team have estimate the demand in Clonee is 175 plus or minus 25 Clonee? The company are reviewing their warehouse locations. What advice can you give?

12. Worker Assignment Problem

A crystal glass cutting company is reviewing its operations to weather out the current economic climate. All 100 staff currently work a 40 hour week. 50 are skilled workers who can operate all 3 production processes, 30 are semi-skilled workers who can operate processes 1 and 2 only while the remaining unskilled staff can operate process 1 only. Research finds the following in relation to demand for their products, processing time, costs and availability of workers:

Products	Demand	Process 1 (mins)	Process 2 (mins)	Process 3 (mins)
Goblet	100	200	120	160
Small Vase	125	100	400	200
Large Vase	75	140	640	220
Candle holder	40	240	160	180

Workers	Skilled	semi-skilled	unskilled
Cost	28	18	10
# on payroll	50	30	20

Formulate an (I)LP model to help the company assign workers of differing skill grades to the production processes. Management need to know if all workers are currently required or should some groups be place on a short working week.

13. Project assignment Problem

A student coordinator wishes to assign 5 students to 3 projects, called Project A, B and C. Each student must be assigned to exactly one project and each project may have no more than 2 students assigned to it. Students have also indicated who they would like to be partnered with. Students 1 would like to work with Student 2. Student 3 would like to work with student 4.

Students state their preferences a follows:

	Student 1	Student 2	Student 3	Student 4	Student 5
1st Choice	A	B	A	C	A
2nd Choice	C	A	B	B	C
3rd Choice	B	C	C	A	B

Formulate an (I)LP model to solve this problem. In this instance, is it possible to give each student their first preference?

14. The Relish Maker problem

A small farmer grows cabbage, tomatoes and onions to make two types of relish under the label *Farmer Joes' Country Relish*. The profit per kilo and recipe for a 400g jar are shown:

	type 1	type 2
profit (€/kg)	4.25	3.95
cabbage (g)	230	170
tomato (g)	85	170
onion (g)	**85**	**60**

	cabbage	tomato	onion
availability (g)	54,500	40,000	20,000

Farmer Joe has the capacity in the kitchen to produce 110kg of relish in total. He produces the relish but does not bottle it. Determine the optimal quantity (in kg) of each type of relish to produce. Which of farmer Joes' crops are surplus to requirements?

15. Animal Feed Mix Problem

An animal feed company makes feed mix from oats, corn, soybean and a vitamin supplement. The availability (in Kg) and price (in €/kg) of the resources is as shown below:

	Kg	€
oats	135	0.50
corn	180	1.20
soybean	90	0.60
vitamin	45	2.00

It uses the following recipe for a its feed:

- At least 30% of the mix must be soybean,
- At least 20% of the mix must be vitamin supplement.

Determine the quantity of each ingredient to use in the mix so as to minimise costs subject to demand of at least 225kg of feed mix. The quality team are considering changing the recipe, which of the blend constraints should it focus its effort on?

16. Production planning problem

Under normal working conditions a factory can produce up to 120 units of a certain product in each of four consecutive time periods at costs which vary from period to period as shown in the table below.

Time Period	Demand	Normal Production Costs €	Additional overtime capacity	Overtime production cost €
1	205	8	50	11
2	80	9	50	11
3	125	10	20	13
4	200	8	50	11

Additional units can be produced by working overtime. The maximum quantity and costs are shown in the table above, together with the forecast demands for the product in each of the four time periods.
Storing inventory costs €2 / unit / time period. Maximum inventory level is 100, and 70 units must be in stock at the end of period 4.
It is required to determine the production and storage schedule which will meet the stated demands over the four time periods at minimum cost given that at the start of period 1 we have 35 units in stock. Formulate and solve this problem as an (I)LP problem.

17. Help-line shift management

The manager of a round the clock help-line must create shift patterns every week for his or her workforce. Each day of every working week is divided into three eight-hour shift periods (Day shift, Evening shift & Night Shift). The call centre must be manned at all times and the minimum number of workers required for each of these shifts over any working week is as below:

	Mo	Tu	We	Th	Fr	Sa	Su
Day	7	8	9	5	7	2	5
Evening	5	6	4	8	9	3	4
Night	7	9	11	5	7	2	3

Union agreement stipulates that each worker is assigned to work either a night shift or a day shift or an evening shift and once a worker has been assigned to a shift they must remain on the same shift every day that they work. Each worker works four consecutive days during any seven day period. In total there are currently 50 workers. Formulate the manager's problem as an (I)LP program. Comment upon the advantages/disadvantages you foresee of formulating and solving this problem as a **linear** programming problem.

18. Java Coffee Problem

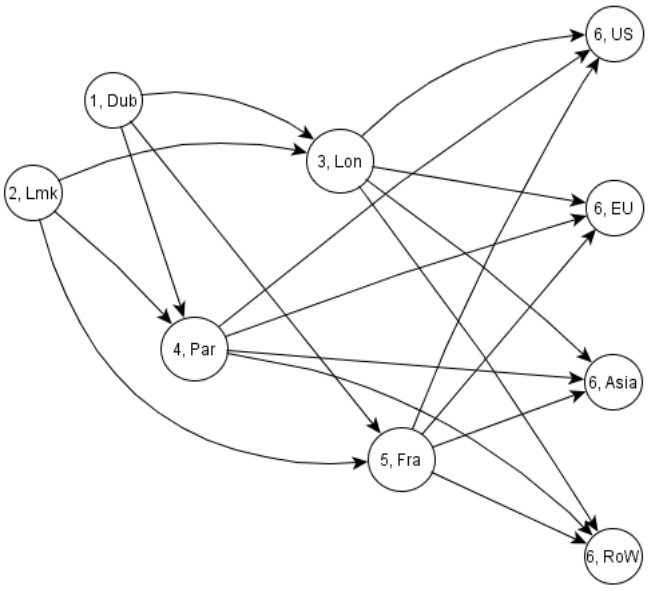
The Java Coffee Company is a commodity trading company and buys and selling coffee for cash. It owns a warehouse with a capacity of 5,000 kgs. At the start of the year, it has an initial stock of 1,000 kgs, and a cash balance of €20,000. The quantitative forecasters estimate that prices for the next quarter as:

	Buying Price	Selling Price
Jan	2.85	3.10
Feb	2.80	3.25
Mar	2.90	2.95

Coffee purchased in one month cannot be sold until the next month, and purchases are all cash on delivery (no credit given). Java management would like to have 2,000 kgs in stock at the end of the first quarter. Formulate as an (I)LP and maximise the total net return.

19. Supply chain Problem

A food producer has production plants in Dublin (Dub) and Limerick (Lmk). They have storage and processing depots in London (Lon), Paris (Par) and Frankfurt (Fra). Markets for end products are the US, EU, Asia and Rest of World (RoW).
6,000 kg can be supplied from each of Dub and Lmk. Demand at US, EU, Asia and Row is estimated at 2,840, 2,800, 2,600 and 2,820 kg respectively. Capacity (kg) and cost (€/kg) for the depot nodes at Lon, Par and Fra are (6,000, 1,700), (7,000, 1,600) and (8,000, 2000) respectively. The supply network is shown below, the table shows the arc capacities and costs.



i	DUB	DUB	DUB	LMK	LMK	LMK	LON	LON	LON	LON	PAR	PAR	PAR	PAR	FRA	FRA	FRA	FRA
j	LON	PAR	FRA	LON	PAR	FRA	US	EU	ASIA	RoW	US	EU	ASIA	RoW	US	EU	ASIA	RoW
Capacity ('000s kg)	2.5	2.5	2.5	3.0	5.3	5.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
Cost (€/kg)	4	5	6	1	2	3	4	4	4	4	4	4	4	4	4	4	4	4

Formulate an (I)LP to determine how the goods should be routed to minimise costs.

20. Conference Management

You have been asked to help organise hotel accommodation for 150 delegates attending a conference in UCD. You use a hotel booking website to extract the data shown in the Table below for ten four star hotels in Dublin city centre. The Table shows the price for a single room per person per night, the customer satisfaction rating and the number of rooms available for each hotel. The conference organisers want to allocate the delegates to hotels at minimum cost while achieving an average customer satisfaction rate of at least 8.3. Formulate an (I)LP model of this problem.

Hotel Index	1	2	3	4	5	6	7	8	9	10
Price (€)	89	99	119	112	143	94	130	98	155	152
Customer rating	7.8	8.3	8	8.7	8	8.1	8.6	8.9	8.9	8.4
Room Availability	35	30	15	15	15	20	15	10	10	20

Table: Hotel data

21. Management Consultancy

A management consultancy firm wishes to assign employees to project teams. Employees can work on at most one project. Their skill capabilities, salaries and a creativity score are shown in the Table below. A *Y* indicates an employee has a qualification in that skill, an *N* indicates they do not have that skill. The table shows the budget availability and number of employees of each skill set required for each project. You may assume that each project will last one year.

Employee	1	2	3	4	5	6	7	8	9	10	11	12
Creativity index	0.87	0.53	0.53	0.53	0.87	0.80	0.73	0.67	0.40	0.47	0.53	0.93
Skill 1	*Y*	*Y*	*Y*	*N*	*Y*	*Y*	*N*	*Y*	*Y*	*N*	*Y*	*Y*
Skill 2	*Y*	*N*	*Y*	*Y*	*Y*	*Y*	*N*	*N*	*Y*	*Y*	*Y*	*Y*
Skill 3	*Y*	*Y*	*N*	*Y*	*N*	*N*	*Y*	*Y*	*Y*	*Y*	*Y*	*Y*
Annual Salary ('000s €)	65	45	48	51	62	58	48	52	55	41	54	68

Table 4: Employee profile

Project	1	2	3	4
Skill 1 requirement	3	1	2	1
Skill 2 requirement	2	2	1	2
Skill 3 requirement	1	1	1	2
Budget ('000s €)	209	340	150	175

Table: Project data

The consultancy firm wishes to assign employees at minimum cost subject to the skills and budget requirements shown in the tables. Formulate an (I)LP model to solve the problem. The consultancy firm are considering assigning employees at minimum cost but in addition wish to maximise creativity of each project team. How could your model be amended to capture the creativity index?

22. Urban renewal project

An urban redevelopment scheme has received €1.2M to spend on community projects on a plot of 2 hectares. The estimate usage (people/day), costs (€'000s) and space requirements (ha) of five projects are as follows;

- 25 m Swimming pool: 250 people/day, €350 and 0.25 ha,
- Tennis court: 75 people/day, €170 and 0.5 ha,
- Running Track: 180 people/day, €155 and 0.3 ha,
- Soccer pitch: 175 people/day, €170 and 0.75 ha
- Function Rooms: 80 people/day, €95 and 0.15 ha
- Coffee Shop: 375 people/day, €130 and 0.1 ha

A focus group has indicated that if function rooms are selected then either a running track or soccer pitches should also be selected not both. In addition, at least two tennis courts should be built for every running track. Common sense suggests there should be at most one swimming pool and at most one coffee shop.

Formulate an (I)LP model to select the mix of projects that maximises usage. Summarise your recommendations for the redevelopment scheme, including a reflection on how the focus group concerns have shaped the solution.

23. Bike Part Manufacturer

A company makes bike parts: wheels and chains. Market demand holds the ratio of wheels to chains to at least 2:1. Weekly demand is for at least 1000 wheels and at least 600 chains. The company runs a single day shift but can use overtime and call on a subcontractor for additional units. Production capacity and costs are as shown. Formulate the problem as a linear programming problem. Management are particularly concerned about the use of subcontractors because of quality concerns. They have a tender from another sub-contractor to make wheels at €1.80/unit and chains at €2.05/unit. They wonder if it is necessary to subcontract out work and if they should include quality constraints in their optimisation models. Include a recommendation with regard to the use of sub-contacting in your report.

Part	Production	Production capacity	Unit Cost €
Wheel	Day Shift	800	2.00
	Overtime	100	2.50
	Subcontract	∞	3.00
Chain	Day Shift	450	2.10
	Overtime	120	2.80
	Subcontract	∞	3.50

Table: Bike Manufacturing data

24. Bus Route problem

A private bus company leases 25- and 50 seater buses to operate on three routes. They estimate that daily passenger demand on route 1 will be between 75 and 350, on route 2 demand is estimated to be between 100 and 700 and on route 2: demand is estimated between 50 and 150.
The company can avail of an EU BMW grant of up to €750,000 toward leasing busses. A 25-seater bus can be leased for €20,000 while a 50-seater can be leased for €32,000 annually. The daily costs of running the buses on the routes are given as follows:

costs (€)	route 1	route 2	route 3
25 – seater	20	40	50
50 – seater	50	60	70

Table: Bus Routing Data

How many busses of each type should the company lease to minimise costs but meet demand? Route 2 passes a narrow bridge, the county council are considering placing a weight restriction on the bridge which means that the 50-seater bus cannot be used on that route. How would your solution be affected if the county council go ahead with their plan? Is minimising costs the best way to solve this problem?

25. Resource Problem

The manager of a fast food restaurant which makes its own sausages and burgers wants to decide how many sausages and burgers to prepare. The requirements and profit are shown below. There are also 7 hours of labour available and a local butcher is contracted to supply 10 kg of minced beef and 10 kg of sausage mix each morning. The manager always holds 8kg of flour in stock. In addition, the manager can buy extra sausage mix from the butcher at a cost of €2.00/kg and extra minced beef at a cost of €3.00/kg. Formulate a model to maximise profits and determine how many sausages and burgers to make and how much, if any, additional meat should be purchased. Note: any fractional sausage or burger in the optimal solution is fed to the restaurant guard dog!

	Labour (hr)	Sausage mix (kg)	Minced Beef (kg)	Flour (kg)	Profit (€)
Sausage	0.01	0.05		0.02	0.6
Beef Burger	0.024		0.08	0.02	0.8

Table: Sausage and Burger data