Computer Lab 5: Models of Short-Term Interest Rates

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1. Background

A Monte Carlo simulation is a simulation that can include randomness, which means that there will be different results for each simulation. In finance, a Monte Carlo simulation can be used to forecast interest rates. By doing multiple simulations, multiple scenarios are created and the probability of different outcomes can be calculated.

A Monte Carlo simulation can be based on an interest rate model to simulate a possible scenario of future daily values of interest rates. The Chan-Karolyi-Longstaff-Sanders process, CKLS, is a stochastic process that can be used as an interest rate model, see Equation 1. In the CKLS process the dZ is a Wiener process and in this lab, $\sqrt{\Delta t}u_{t+1}$ is used as dZ. The CKLS process is a general process with many methods using adaptations of this equation. The conditional mean of CKLS can be seen in Equation 2 and the conditional variances of the CKLS can be seen in Equation 3.

$$dr = (\alpha + \beta r_t)dt + \sigma r_t^{\gamma} dZ \qquad \text{Equation 1}$$

$$E[dr] = (\alpha + \beta r_t)\Delta t \qquad \text{Equation 2}$$

$$var[dr] = \sigma^2 r_t^{2\gamma} \Delta t \qquad \text{Equation 3}$$

The Merton model is a special case of the CKLS process where β and Ɣ are both zero, see Equation 4. The Merton model is used by many to decide how well a company can meet financial obligations. The conditional volatility in the Merton model does not change over time.

$$dr = \alpha dt + \sigma dZ \qquad \text{Equation 4}$$

2. Results

2.1 CKLS and Merton Model

First, the potential future daily values were simulated for the short-term interest rate, with the initial values of the parameters, using the CKLS model and the Merton model, see Figures 1 and 2. These values were calculated using the start value according to Table 1 and Equations 1 and 4, respectively. All the constants used are found in Table 1.

Table 1: The constants used in the calculations of the CKLS and the Merton model

Rate r0 at t=0	**8,00%**
$\Delta \tau$	**0,0028**
k	**0,5**
μ	**6%**
α	**0,03**
β	**-0,5**
σ	**0,05**
γ	**0,5**

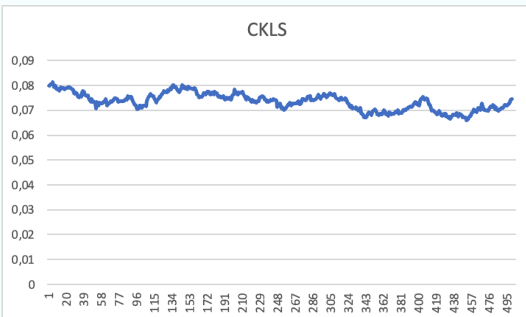

Figure 1: The interest rate series for the CKLS

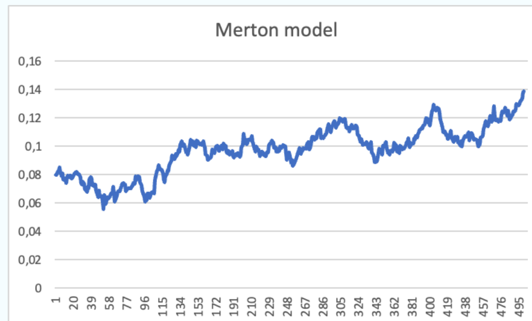

Figure 2: The interest rate series for the Merton model

The first difference of the interest rates was then calculated based on the interest rate series by subtracting the previous value from the current value. The sample mean and the sample variance of the difference were then calculated. The values for the CKLS and the Merton model can be found in Tables 2 and 3, respectively.

Table 2: The sample mean and sample variance for the difference of the interest rates of the CKLS

	CKLS
Sample return mean	-0,00112%
Sample return variance	0,0000005159

Table 3: The sample mean and sample variance for the difference of the interest rates of the Merton model

	Merton
Sample return mean	0,01180%
Sample return variance	0,000007035

After this, the conditional mean and conditional variance were calculated according to Equations 2 and 3. They can be observed in Figures 3 and 4.

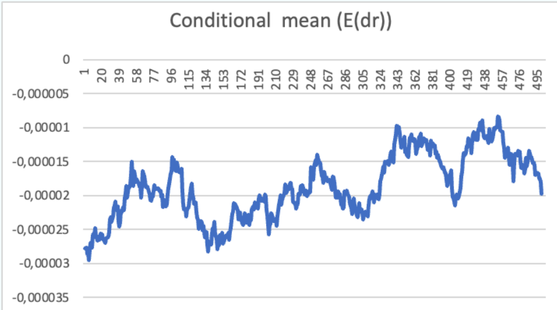

Figure 3: The conditional mean of the CLKS

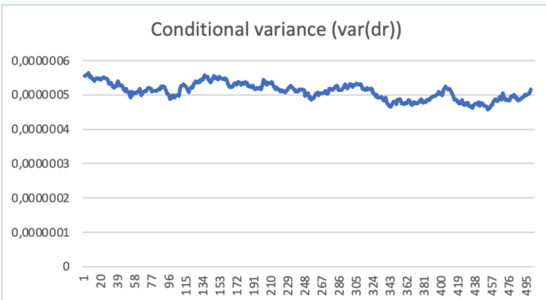

Figure 4: The conditional variance of the CLKS

After these values had been determined, the constants, Δt, γ, μ, α, β, σ and k, were altered one at a time to see what effects this would have on the CKLS model.

First, the gamma was changed from 0.5 to 0. The altered CKLS interest time series, conditional mean, and variance can be observed in Figures 5–7. When it was set to zero, the biggest change was that the conditional variance became a flat line, which can be seen in Figure 7.

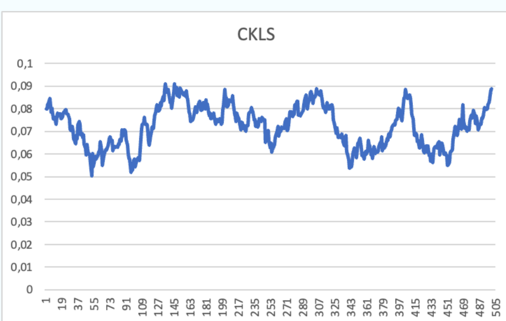

Figure 5: The CKLS interest rate series with $\gamma = 0$

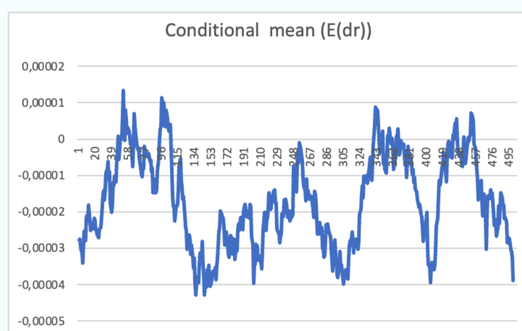

Figure 6: The conditional mean of the CKLS interest rate series with $\gamma = 0$

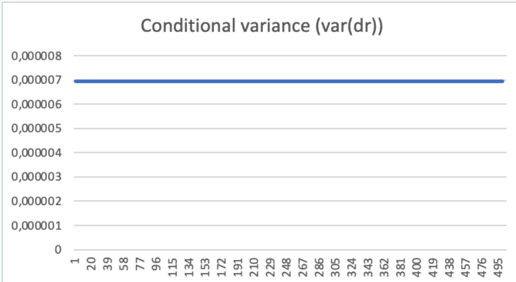

Figure 7: The conditional variance of the CKLS interest rate series with $\gamma = 0$

Next, gamma was changed to two. This can be observed in Figures 8 through 10. With this change, all three of the plots became more or less linear and all of the volatility and waviness disappeared.

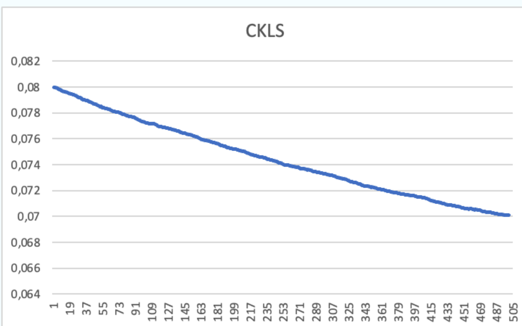

Figure 8: The CKLS interest rate series with $\gamma = 2$

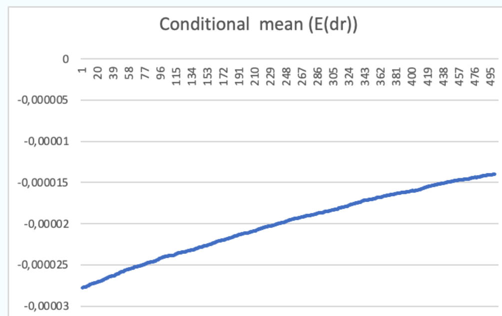

Figure 9: The conditional mean of the CKLS interest rate series with $\gamma = 2$

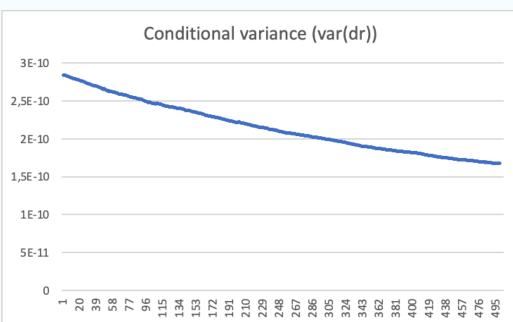

Figure 10: The conditional variance of the CKLS interest rate series with $\gamma = 2$

After this, the sigma value was changed from 0.05 to 0.1. This alteration can be observed in Figure 11–13. This change had a less drastic impact than the changes in gamma.

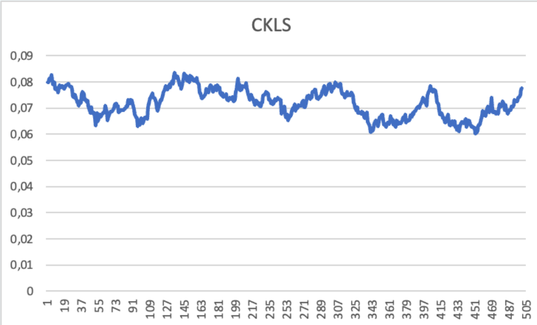

Figure 11: The CKLS interest rate series with $\sigma = 0.1$

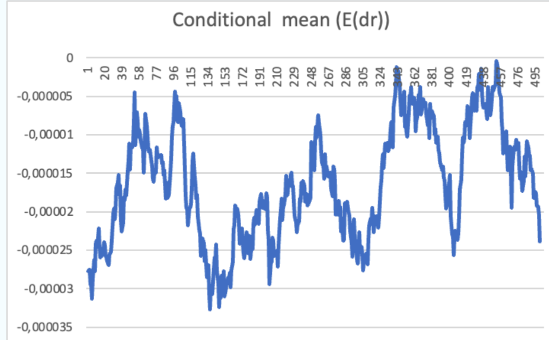

Figure 12: The conditional mean of the CKLS interest rate series with $\sigma = 0.1$

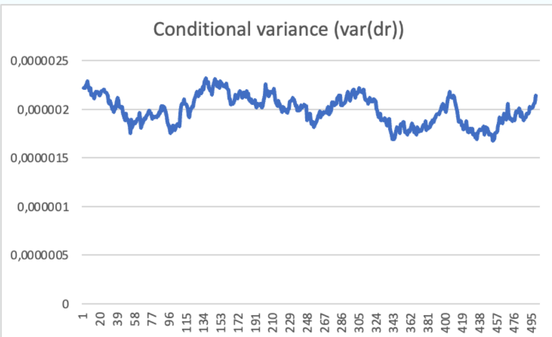

Figure 13: The conditional variance of the CKLS interest rate series with $\sigma = 0.1$

Next, the k-value was changed from 0.5 to 0.9. The k-value cannot be found in Equation 1. Instead, it impacts both the alpha and the beta values. The beta value is the negative equivalent of k, whereas the alpha value is described by Equation 5. The plots are found in Figures 14 through 16.

$$\alpha = k * \mu \qquad \text{Equation 5}$$

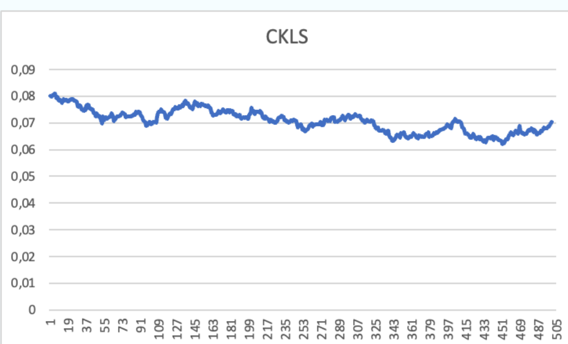

Figure 14: The CKLS interest rate series with $k = 0.9$

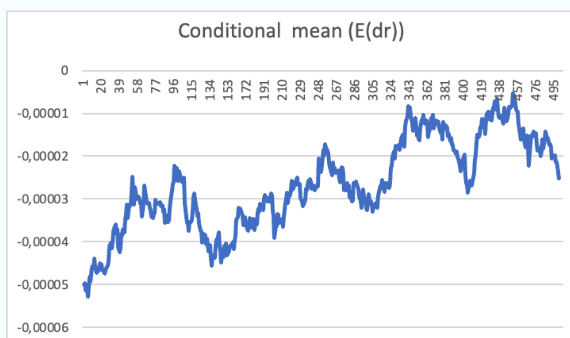

Figure 15: The conditional mean of the CKLS interest rate series with $k = 0.9$

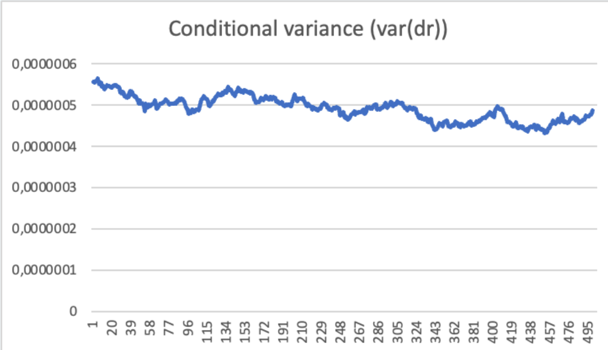

Figure 16: The conditional variance of the CKLS interest rate series with $k\ =\ 0.9$

Then the mu parameter was changed from 6% to 10%. This change is displayed in Figures 17–19. Just like the k, the mu parameter is not present in Equation 1. Instead, as can be seen in Equation 5, it is an input into the alpha value.

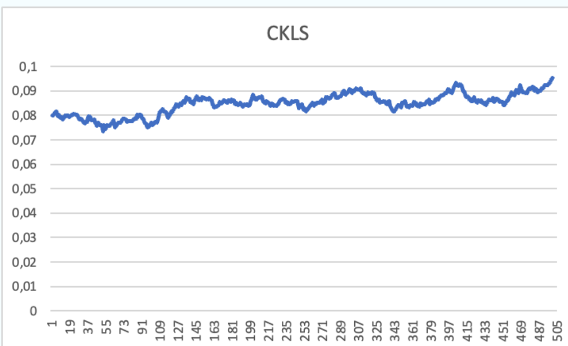

Figure 17: The CKLS interest rate series with $\mu\ =\ 10\%$

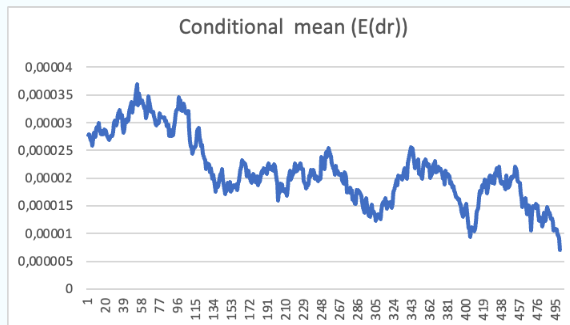

Figure 18: The conditional mean of the CKLS interest rate series with $\mu\ =\ 10\%$

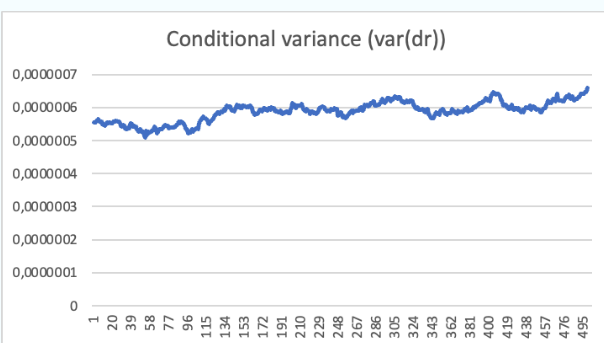

Figure 19: The conditional variance of the CKLS interest rate series with $\mu\ =\ 10\%$

2.2 Empirical Distribution of Bond Price Using the CKLS Model

In this step, ten random standard normal distributions were created using the Excel Analysis Toolbox. These values then replaced the u_t that was used in 2.1 and created ten different CKLS interest rate series. The interest rate series are plotted together in Figure 20.

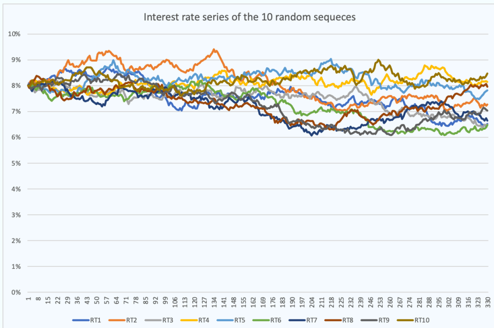

Figure 20: The interest rate series from the CKLS interest rate series with the ten random normally distributed sequences

Equation 6 was used to estimate the bond price. These prices are found in Figure 21.

$$P_{330} = 1/(1 + r_{1,330}/12) \qquad \text{Equation 6}$$

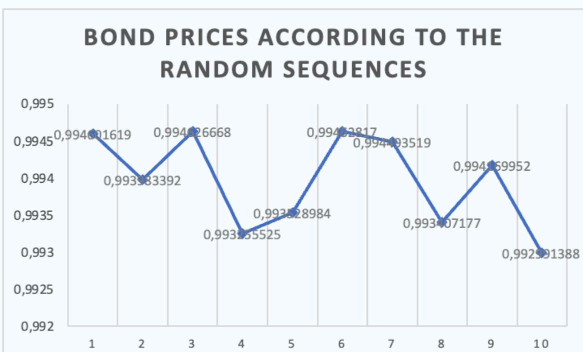

Figure 21: The estimated bond prices from the CKLS interest rate series with the ten random normally distributed sequences

3. *Description of the estimation approach*

The estimation approach has been in accordance with the theories that have been used. The instructions for Excel in the lab instructions were followed thoroughly. Since very small numbers were calculated the amount of decimals visible in Excel was increased. It was determined that at least four significant digits should be observable in the tables that were visible in the report.

4. *Discussion*

Using the initial parameter values and observing the graphs of the interest rates it can be noted that the interest rate when using CKLS is not changing as drastically as when using the Merton model. The interest rate is also decreasing using the CKLS and increasing using the Merton model. By looking at the sample return mean and variance for the difference of the interest rates of the CKLS and Merton model more differences between the models can be observed. The mean for CKLS is negative while the mean for the Merton model is positive. This corresponds to what was noted on the graphs, that the interest rate using CKLS is decreasing while it is increasing using the Merton model. The variance using the Merton model is higher than when using the CKLS which is also something noted in the graphs. This might conclude that nonzero gamma and beta values make the forecasted interest rates more stable.

When changing the gamma in the CKLS process to zero it can be observed that the interest rate is changing more drastically. This can also be noted by looking at the conditional variance which now is higher and constant, meaning there are more fluctuations. The conditional mean increased when changing gamma to zero. This is in line with the reasoning that the Merton model is less stable, as setting the gamma to zero is a way to make the CKLS more similar to the Merton model. When changing the gamma to two the interest rate is steadily decreasing. The conditional variance also decreases and is almost zero which is reasonable since the interest rate graph is quite smooth. An increase in the conditional mean is also visible when increasing gamma as an increase in r_t is now exponential.

An increase in sigma gave an increase in the conditional variance which can also be noted in the interest rate graph. The conditional mean changes more at each timestamp than when the sigma was 0.05. A larger sigma means that the error term is affecting the interest rate more.

When changing the k-value to a larger value the conditional variance slightly decreased and the conditional mean was in a smaller interval and didn't change as much. Increasing the mu-value to 10% makes the interest rate increase instead of decreasing, which can also be observed in the conditional mean since this is positive. The conditional variance is stable and slowly increasing which concludes that the forecasted interest rates further away in time differ from the previous value more than the ones close in time.

When computing the bond prices for the ten random normally distributed sequences, the prices derived from the ten different calculations were quite similar, but they still had some differences. The widest spread was almost 0.02, which can become a quite big difference when owning a lot of these bonds. This stochastic impact translates into risk for the investor as the price can vary a little bit.

5. Appendix

The Excel sheet "DataLab52024" is attached in Canvas.

5.1 Use of AI-based tools

AI-based tools have not been used in this lab.