Computer Lab 3: Performance Measures and Test for Unconditional CAPM

Table of contents:

Victoria Lagerstedt & Kajsa Hansson Willis

Fall 2024
EXTQ30: Empirical Finance

1. *Background*

This lab aims to test different performance measures, specifically the Sharpe ratio, Treynor's performance index, Jensen's performance index, and the information ratio, as well as test for the unconditional CAPM. This is done by examining monthly excess returns of a broad stock index and five constructed portfolios for the period 199201–200807 and testing the aforementioned concepts on the portfolios.

1.1 The Sharpe ratio

The Sharpe ratio, which can be observed in Equation 1, is a measure of the relationship between expected return and risk. Assuming risk aversion, one would expect that the expected return increases with increasing risk, and thus, the Sharpe ratio is a measure of the slope of this relationship. If it is possible to lend money at the risk-free rate, one can construct a portfolio with any amount of risk, and the return for that portfolio is maximized when the only risky asset held is the one with the maximal Sharpe ratio. Therefore, it is assumed that the market portfolio is a portfolio with the maximal Sharpe ratio since if another portfolio had a higher Sharpe ratio, investors would invest in that portfolio instead, making it the new market portfolio.

$$sr_i = \frac{E(r_i) - R_f}{\sigma_i} \qquad \text{Equation 1}$$

Additionally, the ratio can be aggregated over time. If the periods are uncorrelated the ratio on a yearly basis is obtained through Equation 2, where 12 represents the 12 months that are put together to translate the ratio to a yearly basis.

$$sr_i(12) = \sqrt{12} * sr_i \qquad \text{Equation 2}$$

1.2 Treynor's performance index

Treynor's performance index is another portfolio measurement that measures excess return per unit of risk. The difference between the Sharpe ratio and Treynor's performance index is that Treynor's performance index measures the risk through the beta of the portfolio instead of the variance. The portfolio beta is a measure of how the portfolio fluctuates with the market portfolio, which has a beta of one. The same principle holds that maximizing the performance index is a good sign of outperforming the market. Treynor's performance index is derived through the CAPM and pictured in Equation 3.

$$tr_i = \frac{E[R_i] - R_f}{\beta_i} \qquad \text{Equation 3}$$

1.3 Jensen's performance index

Jensen's performance index is similar to the other performance measurements as it measures the abnormal return of the portfolio, known as the average residual return, compared to what

the return should be according to the CAPM. It is given by the alpha value from Equation 4 below. If the CAPM holds, the alpha value should be zero.

$$E[R_i] - R_f = \alpha_i + \beta_i * (E[R_m] - R_f) \quad \text{Equation 4}$$

If the alpha value is not zero, it indicates that it is possible to outperform the market in a way that should not be possible if the CAPM were to hold and that other factors could affect the return of a portfolio. This is the same principle that could allow a higher Sharpe ratio than the market portfolio or a high Treynor's performance index.

1.4 Information ratio

The information ratio portrays the ratio between the average residual return, or Jensen's performance index, and the standard deviation of the mean zero random component of residual return. The mean zero random component of the residual return is calculated by a regression comparing the alpha values to the past return, and it should be viewed as an error term. The information ratio is given by Equation 5.

$$IR_i = \frac{\alpha_i}{\sigma_{\varepsilon i}} \quad \text{Equation 5}$$

1.5 Sharpe Linter CAPM

The Sharpe Linter CAPM is an unconditional model that checks the pricing ability of pricing models. To do this Equation 6 is tested and if all the alphas are zero the CAPM holds. This means the model is explaining all the excess returns.

$$E[Z_i] = \alpha_i + \beta_{im} E[Z_m] \quad \text{Equation 6}$$

2. Results

2.1 Evaluating the performance of all the portfolios

2.1.1 Sharpe ratio

When evaluating the performance of the five portfolios using the Sharpe ratio, the average of each portfolio per period was computed along with the standard deviation of each portfolio. These values were then inserted into Equation 1 to obtain the Sharpe ratio. The risk-free rate is assumed to be zero as it is not specified. The Sharpe ratios are presented in Table 1.

Sharpe ratio:
0.116236512
0.04517088
0.141840389
0.001514707
0.069076622
0.105050113

Table 1: The Sharpe ratios per period of the five assets from the first to the fifth and the market portfolio below

The aggregated Sharpe ratios were also computed according to Equation 2. These are presented in Table 2.

Aggregated Sharpe ratio:
0.402655089
0.156476519
0.491349521
0.005247098
0.239288437
0.363904265

Table 2: The annualized Sharpe ratios of the five assets from the first to the fifth and the market portfolio below

2.1.2 Treynor's performance index

The Treynor's performance index was calculated using the beta values of each portfolio, which was calculated using the intercept function in Excel. The performance index was then calculated by inserting the values into Equation 3, and can be observed in Table 3.

Treynors performance index:
0.007758831
0.003265038
0.013342534
0.000120641
0.004825604

Table 3: Treynor's performance index of the five portfolios from the first to the fifth

2.1.3 Jensen's performance index

Jensen's performance index was calculated in Excel using the intercept function, based on Equation 4. This gave the alpha values that constitute Jensen's performance index.
The obtained alpha values, or intercepts, for the portfolios can be observed in Table 4.

Intercept (alpha):
0.001281773
-0.003910414
0.005052727
-0.007552293
-0.001767151

Table 4: Jensen's performance index of the five portfolios from the first to the fifth

2.1.4 Information ratio

The information ratio was obtained by calculating the standard deviation of the error for every portfolio using the formulas index and linest in Excel. This value was then inserted into Equation 5. The information ratios for the portfolios can be observed in Table 5.

Information ratio
0.051072678
-0.092179288
0.096095468
-0.131358096
-0.057074414

Table 5: Information ratio of the five portfolios from the first to the fifth

2.2 Testing the Sharpe-Lintner CAPM using the Likelihood ratio approach

To test the Sharpe-Lintner CAPM with the Likelihood ratio approach, initial values for alpha and beta were used. Both the unrestricted model and the restricted model were used.

2.2.1 Unrestricted model

The initial values for all the alphas were set to zero and the initial values for all the betas were set to one. The log-likelihood for all the portfolios for each of the periods was calculated and the sum of them was calculated. The sum of all the log-likelihood values was 1794. To find the optimal log-likelihood this sum was maximized using Excel's solver with the alphas and betas as variables. The optimal log-likelihood was 1825, see Table 6. The optimal values of the alphas and betas are seen in Table 7.

Sum of log values
1825.4062

Table 6: Optimal log-likelihood with unrestricted model

	Values:
Alpha1	0.001282
Alpha2	-0.00391
Alpha3	0.005052
Alpha4	-0.00755
Alpha5	-0.00177
Beta1	1.094205
Beta2	1.176997
Beta3	0.747982
Beta4	1.167862
Beta5	1.003034

Table 7: Optimal values for alpha and beta with unrestricted model

2.2.2 Restricted model

The optimal log-likelihood value with the restricted model was calculated in the same way as the optimal log-likelihood value with the unrestricted model with the only difference being that only the beta values were variables when maximizing. The alphas were fixed to be zero. When calculating the optimal log-likelihood with the restricted model the optimal value was 1822, see Table 8. The optimal values for beta and alpha using the restricted model are seen in Table 9.

Sum of log values
1821.6689

Table 8: Optimal log-likelihood with restricted model

	Values:
Alpha1	0
Alpha2	0
Alpha3	0
Alpha4	0
Alpha5	0
Beta1	1.096338
Beta2	1.170482
Beta3	0.756398
Beta4	1.155285
Beta5	1.00009

Table 9: Optimal values for alpha and beta with restricted model

2.2.3 Test statistics

The likelihood ratio was then calculated between the optimal log-likelihood with the unrestricted model and the optimal log-likelihood with the restricted model, see Table 10. With this, the p-value for the test statistics was calculated to be 0.1877, see Table 10.

LR	7.4746
p-value	0.187670804

Table 10: Optimal values for alpha and beta with restricted model

3. Description of the estimation approach

The estimation approach has been in accordance with the theories that have been used. These are primarily the Capital Asset Pricing Model and the portfolio valuation measurements used, as well as the likelihood ratio approach. The instructions for Excel in the lab instructions were followed thoroughly. It was decided to use four significant digits as it was deemed appropriate after observing the results to be able to describe everything accurately but not have too many. It was not specified in the assignment what the risk-free rate was, which was needed for the calculations in the computer lab. As the calculations did not include it in formulas where it should have been preset, the risk-free rate was assumed to be zero.

4. Discussion

4.1 Discussion of the performance measures

4.1.1 Sharpe ratio

The Sharpe ratio of the market portfolio was 0.1051. For the other portfolios, the values were 0.1162, 0.04517, 0.1418, 0.001515, and 0.06908 respectively. This shows that the market portfolio does not have the highest Sharpe ratio. This is not in line with theory, since it would be possible to outperform the market portfolio with a combination of a risk-free asset and the third portfolio. All the investors should then move their assets to that combination and it should become the new market portfolio or the demand should make the price increase and thus the excess return decrease. Since theory shows otherwise, there must be some obstacle in the way, such as transaction costs or that investors do not act rationally. It should be noted that two of the portfolios outperform the market portfolio and that two of the three others are behind by a very large margin.

When observing the aggregated Sharpe ratio, the same relationship holds although the values are different. The third portfolio still vastly outperforms the market portfolio, the first portfolio is a little bit better, the fifth portfolio is a little bit worse, and the second and fourth portfolios are quite a bit worse than the market portfolio.

4.1.2 Treynor's performance index

The results of Treynor's performance index are above zero for all of the assets. It is worth mentioning that the beta values obtained were all positive, which means that the performance

index should act normally. The highest, for the third portfolio, is as high as 0.01334. This indicates that an excess return is present that the CAPM cannot explain. This is present for all of the portfolios but in varying degrees. This suggests that the CAPM does not give a complete explanation of a portfolio's return and that other factors are at large.

4.1.3 Jensen's performance index

Jensen's performance index consists of the alpha values that represent the intercept for the market correlation return function. The values obtained were a mix of positive and negative values, with the third portfolio having the highest at 0.005053 and the fourth portfolio having the lowest at -0.007552. If CAPM holds, the alpha values should be zero. Although the values are low, they are not zero for the portfolios. These alpha values can be contrasted to the average return values of the portfolios which are of a similar size. Thus, these findings indicate that the CAPM does not hold for these portfolios, and more factors than market correlation have an effect on the return. This is in line with the findings in 4.1.2.

4.1.4 Information ratio

The information ratio gave both positive and negative values, with the highest being 0.09610 for the third portfolio and the lowest being -0.1314 for the fourth portfolio. This is in line with the positive and negative alpha values discussed in 4.1.3. A negative value displays a negative risk-adjusted excess return compared to the volatility. According to the CAPM, the alpha value should be zero and thus the information ratio should be zero. This is not the case. Adjusting the alpha value for the standard error also increases the value, which indicates that it is a meaningful difference from zero, and thus shows evidence against the CAPM.

4.2 Discussion of the Sharpe-Lintner CAPM

The optimal log-likelihood value for the unrestricted model was slightly larger than the optimal log-likelihood value for the restricted model. This could indicate that the optimal values for alpha, the mispricing, are not zero. To check this a p-value on the likelihood ratio was calculated. The null hypothesis is that all alphas are zero, so a small p-value would mean the null hypothesis could be rejected and the optimal alphas are not zero. A p-value smaller than 0.05 would mean the null hypothesis could be rejected with a 95% probability. The calculated p-value was 0.1877 which is a relatively large p-value and higher than 0.05. This means the null hypothesis cannot be rejected and the optimal alpha values could all be zero. If all the alphas are zero the CAPM model explains all the expected excess returns and the market portfolio is efficient.

4.3 Comparison

The performance measures provide a different conclusion than the Likelihood ratio approach despite using the same data set. One thing that supports this difference is the fact that the performance measures do not take the likelihood that something is pure coincidence into account or have confidence intervals. While the CAPM may hold on a general level, it must not in all small data sets. Therefore, it is possible that the ratios and performance indexes indicate that the CAPM does not hold, even if it holds. It is important to note that the likelihood ratio approach only says that we cannot discard the hypothesis that the alpha can

be zero with our confidence interval. It does not confirm that the data follows the CAPM. The results from the performance measurements may very well lie within the confidence intervals that they could be zero, but this is not a measure that is used. Therefore, the difference between the two testing methods is that they take different approaches to when the hypothesis should be discarded.

5. Appendix

The Excel sheet “EmpiricalDataLab3” is attached in Canvas.

5.1 Use of AI-based tools

No AI-based tools were used in the completion of this assignment.