Computer Lab 4: Principal Components

Table of contents:

1. *Background*
2. *Results*
3. *Description of the estimation approach*
4. *Discussion*
5. *Appendix*

Victoria Lagerstedt & Kajsa Hansson Willis

Fall 2024

EXTQ30: Empirical Finance

1. Background

This lab aims to calculate the principal components of interest rates and compare these with the inflation rate.

1.1 The yield curve

The yield curve is a way to display different interest rates, or yields, across different maturities, i.e. contract lengths, at a given time. The yield for a certain maturity can be calculated from the price, P, of a bond that makes a single payment of one unit at the same maturity through Equation 1.

$$Y_n = P_n^{-(1/n)} - 1 \qquad \text{Equation 1}$$

The set of yields at one time for different maturities makes up the term structure of the interest rates. The yield curve is the graphical interpretation of the term structure of the interest rates.

The properties of the yield curve can be leveraged to draw conclusions about the expectations of the future state of the economy. If the yield grows as maturity increases, i.e. the curve slopes upward, the expectation is that yields on longer-term bonds will continue to grow, which is a sign of economic growth and stable conditions. High inflation is also expected. If the yield curve instead slopes downward, it is expected that yields on future bonds will decrease, which is a sign of recession. If the yield curve is flat, the expectation is either that the future yields will be the same as today or that the future is uncertain.

1.2 Maximized weight vectors

In this lab, the maximized weight vectors were calculated, and the foundation of determining the principal components. These values are a measure of the exposure that the interest rates have against the factors that the principal components represent, as the values have their basis in multiple regression of the interest rates of the five different interest rates on the principal components.

In general, empirical findings are that the first principal component typically represents the factors that affect the level of the yield curve. It is found that the second principal component generally represents the factors that affect the slope of the yield curve. Further, it has been established that the third principal component largely represents the factors that affect the curvature, i.e. convexity or concavity, of the yield curve.

1.3 Principal Components Analysis

Principal components analysis is a statistical approach to establish factors that explain the covariance structure of returns. It exploits the idea that higher variance means more meaningful information can be captured. Instead of analyzing many factors, the principal components analysis slims it down to a few and selects those that account for the highest variance using the maximized weight vectors. To counteract any dependencies between factors, the selected factors, often two or three, are required to be orthogonal. The result of this is that the problem is simplified to ensure that as little information as possible is lost. The first principal component is given by Equation 2 where the x-vector is derived in the variance-maximizing Equation 3 and R_t is the vector of asset returns.

$$PC_{1,t} = x'R_t \qquad \text{Equation 2}$$

$$max_x \; x'\widehat{V}x \text{ subject to } x'x = 1 \qquad \text{Equation 3}$$

The later principal components are required to be orthogonal components so the later principal components are determined through Equation 4, subject to the conditions in Equation 3 as well.

$$PC_{k,t} = x_k'R_t \text{ subject to } x_j'x = 0 \text{ for all } j<k \qquad \text{Equation 4}$$

If the weights are scaled so that they sum to one, the principal component can be interpreted as the return on the portfolio.

2. *Results*

To compare the principal components with the inflation rate, first, the three principal components for the interest rates were calculated. Each principal component used the earlier components' results as a constraint. The principal components were computed by first calculating the covariance matrix for the returns, see Table 1. The x-vector was initialized with five values of 0.2 each. To get each of the principal components the variance of the portfolios was maximized by changing the five values in x. The five values in the x-vector after each maximization, see Table 2, were used to compute the principal components by multiplying them with the asset returns.

Table 1: The covariance matrix for the returns

	Kolumn 1	*Kolumn 2*	*Kolumn 3*	*Kolumn 4*	*Kolumn 5*
Kolumn 1	1,2317917	0,95641609	0,55644565	0,47659278	0,3356751
Kolumn 2	0,95641609	1,36534763	1,10699278	1,03867783	0,88810221
Kolumn 3	0,55644565	1,10699278	1,05294729	1,02704923	0,92060984
Kolumn 4	0,47659278	1,03867783	1,02704923	1,02593788	0,92937052
Kolumn 5	0,3356751	0,88810221	0,92060984	0,92937052	0,85638339

Table 2: The maximized weight vectors for each principal component

PC1 (x1)	***PC2 (x2)***	***PC3 (x3)***
0,34619253	0,83916162	0,41464735
0,53800249	0,17101627	-0,7589595
0,47240464	-0,2171491	-0,0682295
0,45605363	-0,2955037	0,31192046
0,39944107	-0,363435	0,38742568

These maximized weight vectors were plotted together, see Figure 1. These values show the interest rate of different maturities on the principal components.

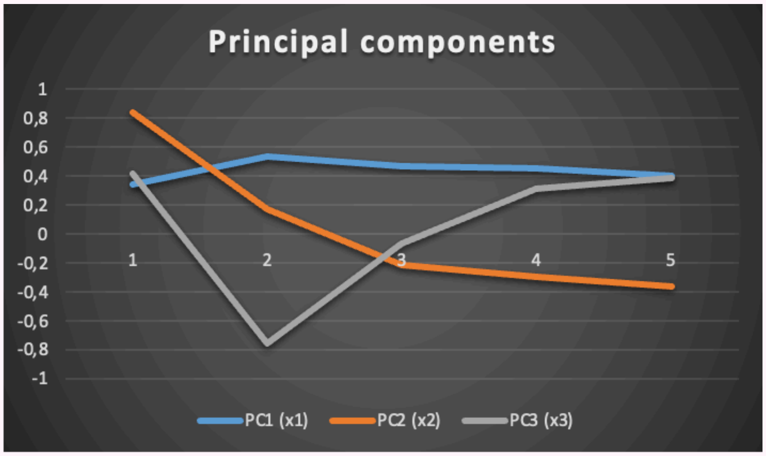

Figure 1: The maximized weight vectors for each principal component

Lastly, the first two principal components were plotted against the inflation rate, see Figures 2 and 3.

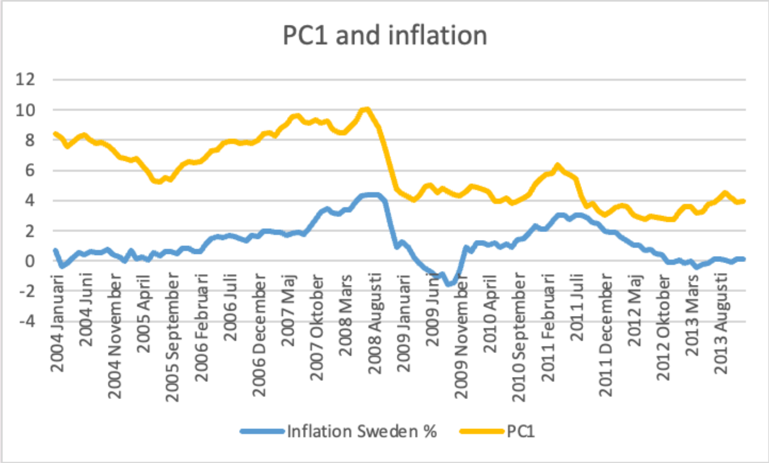

Figure 2: First principal component and inflation rate

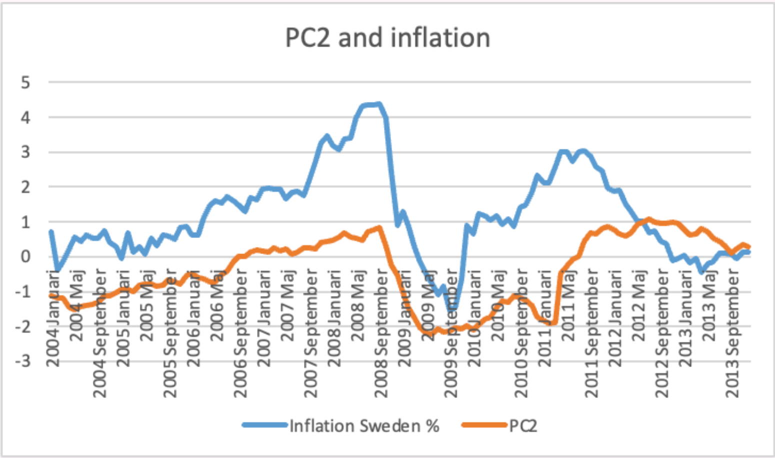

Figure 3: Second principal component and inflation rate

3. *Description of the estimation approach*

The estimation approach has been in accordance with the theories used. The instructions for Excel in the lab instructions were followed thoroughly. The data analysis add-on in Excel was used to calculate the covariances in Table 1.

4. *Discussion*

As specified in the background, the weight vectors show how much, at a certain maturity, the three principal components affect the changes in interest rate.

The first principal component represents how factors affect the level of the yield curve. In Figure 1 it can be seen that the weight vector for the first principal component is relatively stable and always positive. This means the average interest rate, the level on the yield curve, is contributing to the change of interest rate positively and almost the same for all maturities. This weight vector is further from zero than the other two at all maturities but one and two, this means that the level of the yield curve is what is contributing to interest rate changes the most for maturities higher than two.

The second principal component represents how the factors affect the slope of the yield curve. In Figure 1 it can be seen that the weight vector for the second principal component is decreasing for increasing maturities and for higher maturities it is negative. This means the difference between long-term and short-term rates is contributing to the change in interest rate less for higher maturities. At maturity one this is the weight vector with the highest value. This means that the slope of the yield curve is what is affecting the changes in interest rates the most at maturity one.

The third principal component represents how the factors affect the curvature of the yield curve. In Figure 1 it can be seen that the weight vector for the third principal component that is the closest to zero is at maturity three, which means the curvature of the yield curve is contributing to the change of interest rate the least at maturity three. The weight vector furthest from zero is at maturity two which means that is where the curvature affects the changes in interest rates the most.

In Figures 2 and 3, it can be seen that the first and second principal components follow the inflation rate fairly well. One can make an argument that the first principal component captures the inflation curve a little bit better though. This indicates that the first principal component may incorporate a factor that is strongly correlated with inflation well. As the first principal component generally affects the level of the yield curve and the level of the yield curve is heavily tied to the (expected) inflation, this is not surprising. The second principal component also follows the inflation rate well, and since they are both tied to the slope of the yield curve, this result is not surprising either. The movements between PC2 and the inflation rate are quite similar, but the scales are a little bit different, which is not unreasonable.

5. *Appendix*

The Excel sheet “EmpiricalDataLab4” is attached in Canvas.

5.1 Use of AI-based tools

ChatGPT has been used in a very limited amount. This AI-based tool has only been used to further develop our understanding of financial and statistical concepts, such as the principal components theory. The information generated by ChatGPT was compared against the course literature to check its validity. This use of ChatGPT was very helpful since it simplified some concepts and helped us understand them better, which made the analysis in this report better.