Short-Term Scientific Mission Grant
1. APPLICATION FORM[1] -

Action number: CA20111

Applicant name: Maribel Fernandez

Details of the STSM

Title: Hierarchical Higher-Order Port Graphs for the representation and analysis of proofs

Start and end date: 12/09/2022 to 18/09/2022

Goals of the STSM

Purpose and summary of the STSM. *(max.200 word)*

Our long-term goal is to develop a graph-based tool for the representation, analysis and management of proofs using graph rewriting techniques. As a first step, we aim to develop representations of proofs using hierarchical port graphs, which we hope can serve as a common language for the encoding of proofs generated by different proof systems. Port graphs have been successfully used as a modelling tool in a variety of domains (e.g., biochemistry, finance, social networks): port graph rewriting tools, such as PORGY, provide a visual representation of the dynamics of the system modelled. We expect the same techniques can be applied to build visual representations of proofs and to study the dynamics of proof management systems (e.g., proof simplification processes).
The representation of proofs via graphs is not a new idea (cf proof nets for linear logic), however, so far there is no general graph-based language that can be used to represent proofs from different systems in a uniform way. By designing a general proof representation formalism based on hierarchical port graphs, we hope to contribute to the general aim of this COST Action: boosting the interoperability and usability of proof systems.

Working Plan

Description of the work to be carried out by the applicant. *(max.500 word)*

To achieve our long-term goal of developing a graph-based tool for the representation, analysis and management of proofs, we will build on existing work on graph-based representations of proofs. Recently G. Guerrieri and co-authors ("Gluing resource proof-structures: inhabitation and inverting the Taylor expansion", LMCS 2022) used hierarchical nets to represent proofs in linear logic, and D. Ghica, K. Muroya and collaborators

1 This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

proposed the use of hierarchical hypernets (FSCD 2022). The applicant and host (in collaboration with I. Mackie and S. Maulat) studied graphical representations of intuitionistic logic proofs using higher-order extensions of interaction nets (TERMGRAPH 2011). Each of these works focuses on specific logics (intuitionistic logic, linear logic), however, in all of them a notion of hierarchical graph appears prominently.

We believe that hierarchical port graphs are a good candidate to unify previous work on graphical formalisms for proof representation: port graphs have already proven to be a good foundation for modelling tools (they are the basis of modelling environments such as PORGY and BioNetGen), and hierarchical port graphs inherit the properties of port graphs while providing additional structuring and abstraction features. Hierarchical port graphs have been successfully used to specify financial processes and computation models (see e.g. Ene, Fernandez and Pinaud's papers at WPTE 2018 and AMINSE 2019), but have not yet been applied in the area of proof representation. On this basis, we plan first to check the suitability of hierarchical port graphs and rewriting rules as a modelling tool for proof systems by using them to specify proofs in some paradigmatic logics (e.g., intuitionistic logic, linear logic, nominal logic). We will also consider the use of hierarchical port graph rewriting rules to specify operations on proofs (e.g., proof normalisation), and we will define strategies to control the rules to achieve expected results (e.g., to ensure the uniqueness of normal forms). We expect that by using hierarchical port graphs as a representation language for paradigmatic logics, we will be able to distil general principles for the encoding of proofs, which can guide the design of a domain-specific language for proof representation. In a later stage, we will consider the representation of proofs in powerful logical frameworks, such as Dedukti.

We believe the first part of this research programme can be done more effectively with in-person meetings over one week, which is why we are requesting funds for a short visit.

We plan to continue this work after the visit, with the aim of developing the foundations for a graph-based proof management environment (in the style of PORGY but specifically tailored to the management of proofs). This will require some engineering work as well as more research on proof formats and proof management. This research can also shed light on closely related problems, such as, proof visualisation, proof search, proof maintenance (e.g. proof updates and extensions).

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables. *(max.500 words)*

Below we list the main expected results, their relevance for the Action, and contribution towards the deliverables of the Action.

1 A comparison of existing graph-based proof representation languages, focusing on the features of graphs used in the encoding of proofs. We have already started reviewing existing work on graph-based proof representations, and we plan to summarise and compare their main features during the visit. This output contributes towards the Action objectives, in particular Objective 8 (develop the use of natural or controlled languages in proof systems) since graph-based representations of proofs can suggest new graphical languages for proof systems.
2. Encodings of proofs in paradigmatic logics (intuitionistic logic, linear logic, nominal logic) using hierarchical higher-order port graphs, and analysis of their properties

via graph rewriting rules. We expect to start the encodings during the visit and complete them in the following months (this will be reported in a paper describing encodings of proofs and proof transformations using hierarchical port graph rewriting). This will also contribute towards Action Objective 8 and it is a necessary step towards output number 3.

3. Encodings of proofs in powerful logical frameworks using hierarchical port graphs. Analysis of the suitability of graph representations of proofs as a tool to communicate and exchange proofs between different proof assistants, and as a visual tool to facilitate proof construction, extension and update. The latter is relevant to the work done in WG4 (Libraries of formal proofs), whose aim is to investigate approaches to efficiently maintain libraries of proofs, so that they can be modified and queried by users without expert knowledge of the system used to develop proofs. We believe that graphs, which provide a visual representation, can facilitate the understanding of proofs and contribute towards the aims of WG4. We expect this will give rise to collaborations with other teams (in particular the Dedukteam) and contribute towards the deliverables of the Action (in particular: description of proof formats, translation of proofs from different systems, tools to manage proofs and for searching and querying collections of proofs).
4. Identification of the main features required for a general graph-based language for proof representation and manipulation. Design of a domain-specific version of a graph-based modelling tool, such as PORGY, to model/analyse proofs in various systems of interest, and link the modelling tool with proof assistants. For this, we will seek input from members of the PORGY team in the University of Bordeaux, with whom we have a long-standing collaboration, as well as from members of the Dedukteam.