The Rapid Software Verification Framework

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What are we solving?

```
func main() {
 const Int[] a;
 Int[] b, c;
 Int i, j, k = 0;
 while (i < a.length) {
  if (a[i] ≥ 0) {
   b[j] = a[i];
   j++;
  } else {
   c[k] = a[i];
   k++;
  }
  i++;
 }
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```

Partial correctness:

b is initialized by elements of a

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Partial correctness:

b is initialized by elements of a

$$\forall pos_{\mathbb{I}}.\exists pos'_{\mathbb{I}}.(\\ 0 \leq pos < j \land a.length \geq 0 \\ \rightarrow \\ 0 \leq pos' < a.length \\ \land b(pos) = a(pos'))$$

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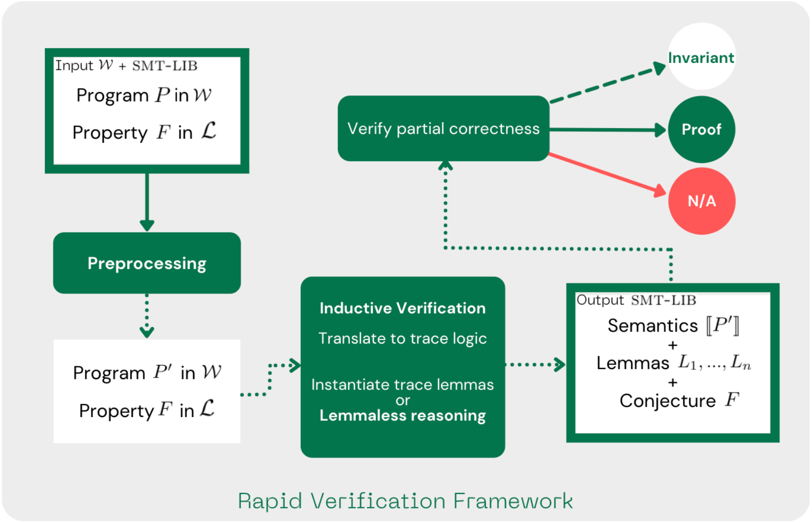
```

- Partial correctness
- Programs containing integer and integer-array variables
- Arbitrary amount of loops
- Arbitrarily quantified program properties
- Automated first-order theorem proving

The Rapid Verification Framework



Input $\mathcal{W}$ + SMT-LIB
Program P in $\mathcal{W}$
Property F in $\mathcal{L}$
Preprocessing
Program P' in $\mathcal{W}$
Property F in $\mathcal{L}$
Inductive Verification
Translate to trace logic
Instantiate trace lemmas
or
Lemmaless reasoning
Output SMT-LIB
Semantics $[\![P']\!]$
+
Lemmas $L_1, ..., L_n$
+
Conjecture F
Verify partial correctness
Invariant
Proof
N/A
Rapid Verification Framework

Trace Logic in a Nutshell

- **Full first-order logic** with equality (over UFDTLIA)
- Program values: standard theory of integers
- Loop iterations: theory of natural numbers $(0, s, p, <)$ (no arithmetic!)
- Reasoning over **timepoints**
  - allows to express induction directly in the language
  - reason about properties of *all loop iterations*
  - reason about the *existence of certain loop iterations*

Semantics based on Trace Logic

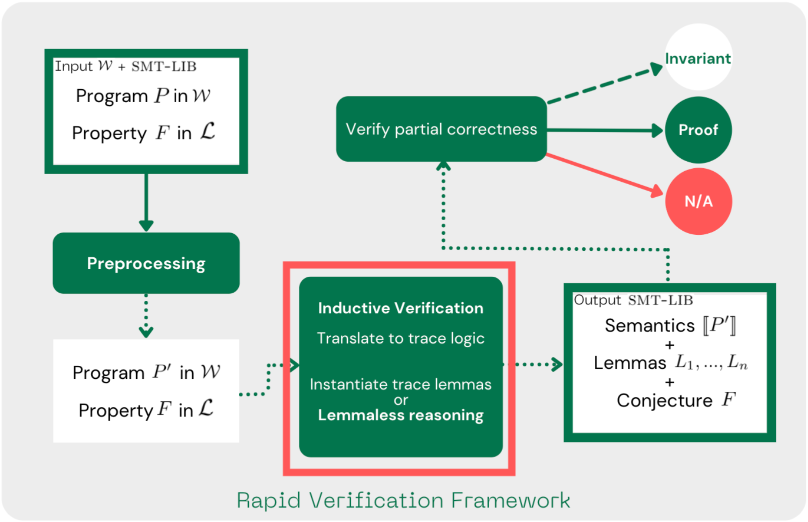
```
1    func main() {
2     const Int[] a;
3     Int[] b, c;
4     Int i, j, k = 0;
5     while (i < a.length) {
6      if (a[i] ≥ 0) {
7       b[j] = a[i];
8       j++;
9      } else {
10      c[k] = a[i];
11      k++;
12     }
13     i++;
14    }
15   }
16
```

Conjunction of all statements

$$i(l_4) \simeq 0$$

$$\forall it_{\mathbb{N}}. \Big(it < n_5 \rightarrow$$
$$i(l_5(s(it))) \simeq i(l_{13}(it)) + 1$$
$$\wedge$$
$$\big(a(i(l_5(it)) \geq 0) \rightarrow$$
$$b(l_9(it), j(l_5(it))) \simeq a(i(l_5(it)))\big)\Big)$$

What about induction?



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What about induction?

- Trace Lemma Reasoning
- Lemmaless Induction

Trace Lemma Reasoning

- valid formulas, derivable from *instances of the induction scheme*
- *manually identified* set of useful lemmas
- can include *quantifier alternations*
- can include *quantification over loop iterations and variable values*
- can’t be automatically generated by state-of-the-art techniques

Trace Lemma Example

```
1    func main() {
2     const Int[] a;
3     Int[] b, c;
4     Int i, j, k = 0;
5     while (i < a.length) {
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Value Evolution Theorem

Apply bounded induction over loop iterations to derive equality of variable values throughout time

$$\forall p^{\mathbb{I}}.\ \forall it_L^{\mathbb{N}}.\ \forall it_R^{\mathbb{N}}.\ \Big(\big(\forall it^{\mathbb{N}}.(it_L \le it < it_R \wedge b(l_7(it_L),p) = b(l_7(it),p)) \rightarrow b(l_7(it_L),p) = b(l_7(s(it)),p)\big) \rightarrow \big(it_L \le it_R \rightarrow b(l_7(it_L),p) = b(l_7(it_R),p)\big)\Big)$$

Trace Lemma Reasoning

- automatically instantiated for all (relevant) program variables
- many unnecessary lemmas generated that don’t help to find a proof
- search space *blow up*
- can’t be automatically generated by state-of-the-art techniques

Lemmaless Induction

- inbuilt *inductive inference rules* in first-order theorem proving
- specialized for trace logic
- uses clauses that contain interesting timepoints
- goal-directed reasoning: *multi-clause goal induction*
- program semantics reasoning: *array-mapping induction*

Multi-clause Goal Induction

$$\frac{C_1[nl_{\mathtt{w}}] \qquad C_2[nl_{\mathtt{w}}] \quad \dots \quad C_n[nl_{\mathtt{w}}]}{\mathrm{CNF}\left(\begin{array}{c}\left(\begin{array}{c}\neg(C_1[0] \land C_2[0] \land \dots \land C_n[0]) \land \\ \forall it_{\mathbb{N}}.\begin{pmatrix}((it < nl_{\mathtt{w}}) \land \neg(C_1[it] \land C_2[it] \land \dots \land C_n[it])) \to \\ \neg(C_1[\mathtt{suc}(it)] \land C_2[\mathtt{suc}(it)] \land \dots \land C_n[\mathtt{suc}(it)]))\end{pmatrix}\end{array}\right) \\ \to (\forall it_{\mathbb{N}}.(it < nl_{\mathtt{w}}) \to \neg(C_1[it] \land C_2[it] \land \dots \land C_n[it]))\end{array}\right)}$$

- clauses are *derived from safety assertion*
- works well for *properties that are structurally close to the required invariant*

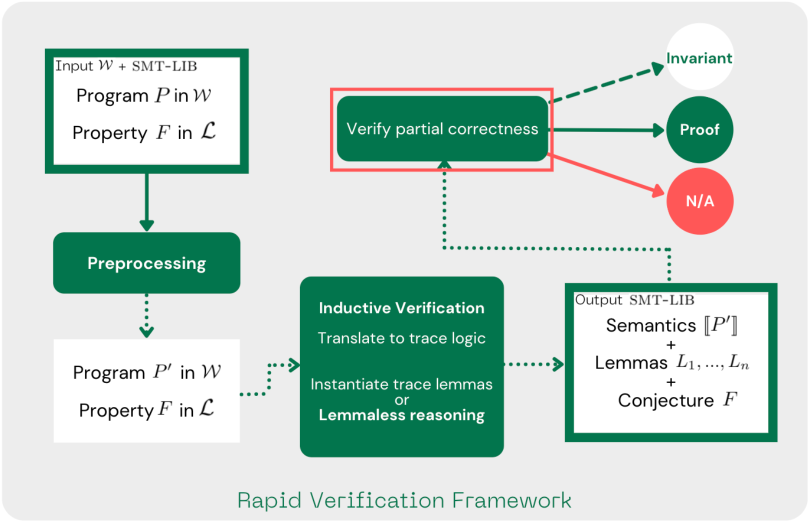
Array-mapping Induction

```
func main(){
 Int[] a;
 Int i, j = 0;
 const Int n;
 while(i < a.length) {
  a[i] = a[i] + n;
  i = i + 1;
 }
 while(j < a.length) {
  a[j] = a[j] - n;
  j = j + 1;
 }
}

```

- clauses are *derived from program semantics*
- necessary when required invariant(s) depend on program behavior rather than safety assertion
- *consecutive loops!*

How to discharge verification conditions?



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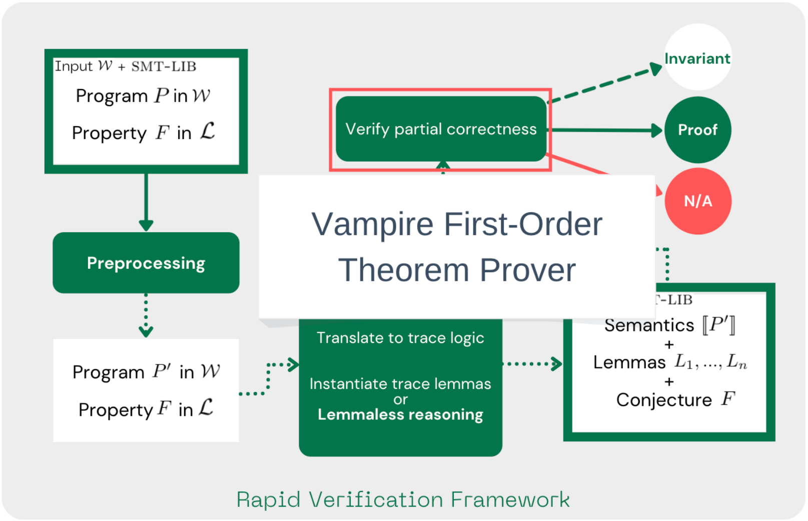

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Vampire First-Order Theorem Prover

Results

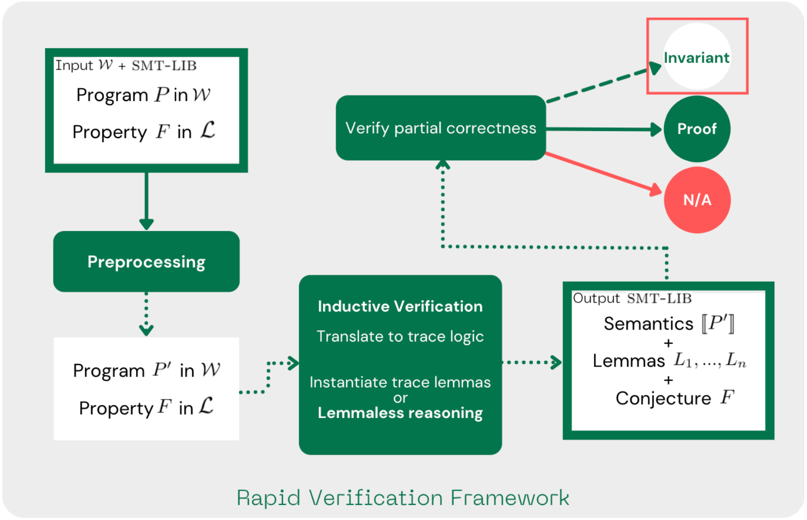
TABLE I: Experimental Results

Total	Rapid$_{\mathrm{std}}$	Rapid$_{\mathrm{lemmaless}}$	Diffy	SeaHorn
140	91 (5)	103 (10)	61 (1)	17 (0)

Fin

Thank you for your attention!

What about invariants?



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Invariant Generation

- *consequence finding* from semantics and trace lemmas until a conjunction of clauses proves a postcondition
- allows integration with other tools
- works (at most) for already found proofs

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```

Timepoint reasoning

$$l_4 : Timepoint$$

$$l_5(0) : Nat \mapsto Timepoint$$

$$l_5(s(0))$$

$$l_5(n_5)$$

$$l_5(it)$$

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```

Program variables

$i(l_5(0))$ $j(l_5(n_5))$

$a(i(l_5(0)))$ $b(l_5(it), j(l_5(it)))$

$$i : Timepoint \mapsto Int$$

$$a : Int \mapsto Int$$

$$b : Timepoint \times Int \mapsto Int$$