Qorpi AI TM

Quantum Programming

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Introduction to Qutip

QuTiP is open-source software for **simulating the dynamics of open quantum systems**. The QuTiP library depends on the excellent Numpy, Scipy, and Cython numerical packages. In addition, graphical output is provided by Matplotlib. QuTiP aims to provide user-friendly and efficient numerical simulations of a wide variety of Hamiltonians, including those with arbitrary time-dependence, commonly found in a wide range of physics applications. https://qutip.org

Qorpi

ket $|\psi\rangle = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \vdots \\ \psi_n \end{pmatrix}$

$\langle A| = (A_1^* \ A_2^* \ \dots \ A_n^*)$ bra

$\boxed{|\psi\rangle^\dagger = \langle\psi|}$

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Longrightarrow$ هرمیتی

$A = A^\dagger \longrightarrow$ transpose + Conjucate

$ts = transpose(\sigma_y) = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \Rightarrow ts^* = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix}$

$A = A^\dagger$
$B = B^\dagger$

$(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger = \hat{B}\hat{A}$

$\boxed{|0\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle := \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$

Qubit

basis (N, n)

روش مشخص کردن state

$N \longleftarrow$ تعداد حالت در فضای هیلبرت

$n \longleftarrow$ کدوم حالت مدنظرت هست

fock states $\longrightarrow$ second quantization

N تعداد ذرات	Bosonic basis state
0	$\|0,0,\dots,\dots\rangle$
1	$\|1,0,\dots\rangle, \|0,1,\dots\rangle$
2	$\|2,0,0,\dots\rangle, \|1,1,\dots\rangle \dots \|0,1,1,\dots\rangle$
n	$\|n_{k_1}, n_{k_2}, \dots n_{k_i}, \dots\rangle$

$$\alpha|0\rangle + \beta|1\rangle =$$
$$|\alpha|^2 + |\beta|^2 = 1$$

Qorpi AI™

A = Annihilation operator

C = Creation operator

$\Longrightarrow A^{+} = C$ or $C^{+} = A$

Position

$$X = \sqrt{\frac{\hbar}{2m\omega}}\,(A + A^{+})$$

پیدا کردن مکان

از روی عملگر خلق و فنا

$$P = i\sqrt{\frac{m\hbar\omega}{2}}\,(A - A^{+})$$

پیدا کردن تکانه

از روش عملگر خلق و فنا

Number operation

$$\hat{N}_i \equiv a^{+}(\varphi_i)\,a(\varphi_i)$$

Commutator

$$[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \quad , \quad [\hat{A},\hat{B}] + [\hat{B},A] = 0$$

$$[\hat{A},\hat{A}] = 0 \quad , \quad [\hat{A},\hat{B}+\hat{C}] = [\hat{A},\hat{B}] + [\hat{A},\hat{C}]$$

$$[\hat{A},\hat{B}\hat{C}] = [\hat{A},\hat{B}]\hat{C} + \hat{B}[\hat{A},\hat{C}]$$

$$[\hat{A}\hat{B},C] = [\hat{A},\hat{C}]\hat{B} + \hat{A}[\hat{B},\hat{C}]$$

Normalization $\xrightarrow{\text{بهنجارش}}$ پایسته بودن احتمال $\longrightarrow \langle\psi|\psi\rangle = 1$ $\qquad \int_{-\infty}^{+\infty} P(x)\,dx = 1$

$\psi_{pure} \leftarrow 1$ $\qquad \rightarrow 1$

$$\rho = \sum_{j=1}^{n} P_j |\psi_j\rangle\langle\psi_j| = \sum_{j=1} P_j |\psi_j\rangle\langle\psi_j| = |\psi\rangle\langle\psi| \Longrightarrow \text{pure states density matrix}$$

$$\rho \geq 0 \,,\quad \rho^{\dagger} = \rho \,,\quad \rho^2 = \rho \Longrightarrow \text{pure} \qquad tr(\rho) = 1$$

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mix State density matrix

$$\{|\psi_j\rangle\}_{j=1}^{n} = \{|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_n\rangle\}$$

$$\rho \equiv \sum_{j=1}^{n} P_j |\psi_j\rangle\langle\psi_j|$$

expectation value ⟶ مقدار چشم داشتی

$$\langle x\rangle = \int x P(x)\, dx$$

$$\langle\psi|A|\psi\rangle = \langle A\rangle = \int \psi^* A \psi \, dx = Tr(\rho_A)$$

Tensor product

$$\begin{bmatrix}1\\2\end{bmatrix} \otimes \begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}1\begin{bmatrix}3\\4\end{bmatrix}\\2\begin{bmatrix}3\\4\end{bmatrix}\end{bmatrix} = \begin{bmatrix}3\\4\\6\\8\end{bmatrix}$$

Qorpi AI™

$$\begin{bmatrix} 1 & \alpha \\ \alpha^* & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & \beta \\ \beta^* & 1 \end{bmatrix} = \begin{bmatrix} 1\begin{bmatrix} 1 & \beta \\ \beta^* & 1 \end{bmatrix} & \alpha\begin{bmatrix} 1 & \beta \\ \beta^* & 1 \end{bmatrix} \\ \alpha^*\begin{bmatrix} 1 & \beta \\ \beta^* & 1 \end{bmatrix} & 1\begin{bmatrix} 1 & \beta \\ \beta^* & 1 \end{bmatrix} \end{bmatrix}$$

$$\alpha \otimes \beta \neq \beta \otimes \alpha$$

Jaynes-Cumming (light-matter interaction)

$$H = W_c\, a^\dagger a - \frac{1}{2} w_a\, \sigma_z + g\,(a\sigma_+ + a^\dagger \sigma_-)$$

$$a\sigma = (\underbrace{a}_{5d} \otimes \underbrace{I}_{2d})(\underbrace{I}_{5d} \otimes \underbrace{\sigma}_{2d})$$

$w_c = 1$, $w_a = 1$, $g = 0.1$

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partial trace $\Rightarrow \rho_A = Tr_B(\rho_{AB})$

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle \Rightarrow \rho_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}| =$$

$$= \frac{1}{2}[\,|0_A 0_B\rangle\langle 0_A 0_B| + |0_A 0_B\rangle\langle 1_A 1_B| + |1_A 1_B\rangle\langle 0_A 0_B| + |1_A 1_B\rangle\langle 1_A 1_B|$$

$\rho_B = Tr_A(\rho_{AB})$

$$= \frac{1}{2}[\,Tr_A(|0_A 0_B\rangle\langle 0_A 0_B|) + Tr_A(|0_A 0_B\rangle\langle 1_A 1_B|) + Tr_A(|1_A 1_B\rangle\langle 0_A 0_B|)$$

$$+ Tr_A(|1_A 1_B\rangle\langle 1_A 1_B|) = \frac{1}{2}[\,Tr(|0_A\rangle\langle 0_A|)(|0_B\rangle\langle 0_B|) + Tr(|0_A\rangle\langle 1_A|)(|0_B\rangle\langle 1_B$$

$$+ Tr_A(|1_A\rangle\langle 0_A|)(|1_B\rangle\langle 0_B|) + Tr_A(|1_A\rangle\langle 1_A|)(|1_B\rangle\langle 1_B|) =$$

$$= \frac{1}{2}\Big[\langle 0_A|0_A\rangle\, |0_B\rangle\langle 0_B| + \langle 1_A|0_A\rangle\, |0_B\rangle\langle 1_B| +$$

$$\langle 0_A|1_B\rangle\, |1_B\rangle\langle 0_B| + \langle 1_A|1_A\rangle\, |1_B\rangle\langle 1_B|\Big] =$$

$$= \frac{1}{2}\Big[\, |0_B\rangle\langle 0_B| + |1_B\rangle\langle 1_B| \,\Big] \Rightarrow \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

the evolution and Quantum systems Dynamics

$$|\psi'\rangle = U|\psi\rangle \Rightarrow i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle \Rightarrow H = H^{\dagger}$$

$$H = \sum_E E|E\rangle\langle E|$$

$E \leftarrow$ ویژه مقدار

$|E\rangle \leftarrow$ ویژه بردار نرمال شده

$$|E\rangle \rightarrow e^{\left(-\frac{iEt}{\hbar}\right)}|E\rangle$$

Qubit

$$H = \hbar w X$$

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} , \quad \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$\frac{|0\rangle - |1\rangle}{\sqrt{2}} \quad \alpha \; - \hbar w$$

$$\psi(t_2) = e^{-iH(t_2 - t_1)/\hbar} |\psi(t_1)\rangle = U(t_1, t_2) |\psi(t_1)\rangle$$

$$U(t_1, t_2) \equiv e^{-iH(t_2 - t_1)/\hbar} \xrightarrow{\text{یعنی}} \text{unitary} \xrightarrow{\text{دلخواه}} U = e^{iK}$$

$$\Rightarrow K = K^{\dagger}$$

$$i\hbar \frac{\partial}{\partial t} \psi = \hat{H} \psi \Rightarrow \psi(t) = U(t, t_0)\, \psi(t_0)$$

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the Von Neumann equation fo time evolution

Time-evolution of the density matrix

$$\frac{\partial}{\partial t}|\psi\rangle = -\frac{i}{\hbar}\hat{H}|\psi\rangle \quad ; \quad \frac{\partial}{\partial t}\langle\psi| = \frac{i}{\hbar}\langle\psi|$$

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t}\left[|\psi\rangle\langle\psi|\right] = \left[\frac{\partial}{\partial t}|\psi\rangle\right]\langle\psi| \frac{i}{\hbar} |\psi\rangle \frac{\partial}{\partial t}\langle\psi|$$

$$= -\frac{i}{\hbar} H |\psi\rangle\langle\psi| \frac{i}{\hbar} |\psi\rangle \langle\psi| H = \frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho]$$

$$\Longrightarrow \rho(t) = U \rho(t_0)\, U^{+}$$

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Time-dependent Hamiltonians

$$H = [H0 ,\ [H1 ,\ Py\_coef1],\ [H_2 ,\ Py\_coef2], \ldots]$$

$$f_t = A \exp\left(-\left(\tfrac{t}{2}\right)^2\right) \qquad H = H_0 - f(t)\, H_1$$

non-Unitary systems

$\psi \rightleftarrows$ Env

$$H_{total} = H_{sys} + H_{Env} + H_{Interaction}$$

$$\frac{\partial \rho_{total}(t)}{\partial t} = \rho_{total}(t) = -\frac{i}{\hbar}\left[H_{total} , \rho_{total}(t)\right]$$

$$\rho_{sys} = Tr_{Env}(\rho_{total})$$

Master equation

$$\dot{\rho}(t) = \underbrace{-\frac{i}{\hbar}\left[H(t), \rho(t)\right]}_{\text{unitary}} + \underbrace{\left[\sum_n \frac{1}{2}\, C_n \rho(t) C^{\dagger}_{n} - \rho(t) C^{\dagger}_n C_n - C^{\dagger}_n C_n \rho(t)\right]}_{\text{non-unitary}}$$

$$C_n = \sqrt{\gamma_n}\,\hat{A}_n \longrightarrow \gamma_n = \text{collapsing rate}$$

Tran

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