DC CIRCUIT ANALYSIS –:

① Active Elements – which supply energy to Network. Eg – Voltage source

② Passive Elements – which dissipate or store Energy. Eg – capacitor, Inductor, Resistor

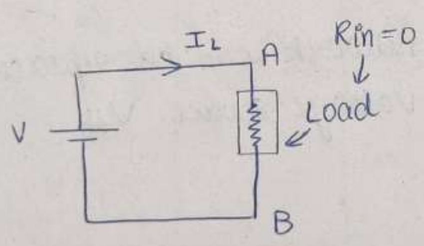
③ Unilateral Elements – whose properties depend upon the direction of current. Eg. – Diode, Transistor.

④ Bilateral Elements – whose properties doesn't depend upon the direction of current. Eg. – Resistance, Inductor, Capacitor

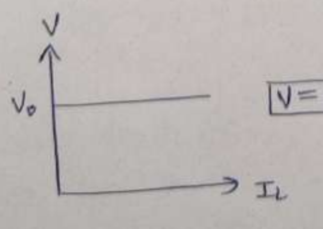
⇒ Active and Passive Network –

* A Network is said to be passive if it contains no source of emf in it.
* When a network contains one or more sources of emf or current then it's said to be passive.

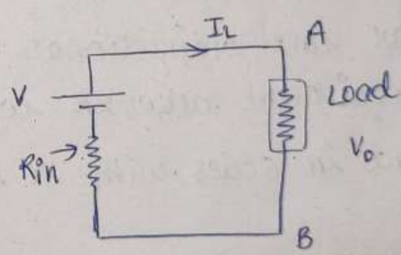
⑤ Ideal and Practical Voltage Source –



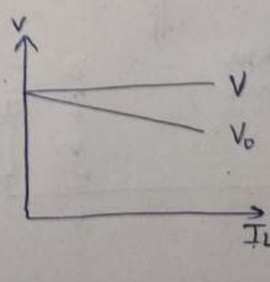
⇒ The source which maintains a constant voltage across the load, irrespective of the load current.



$\boxed{V = V_0}$

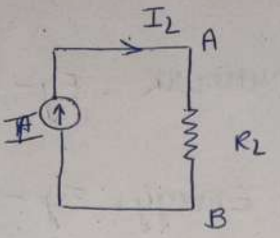


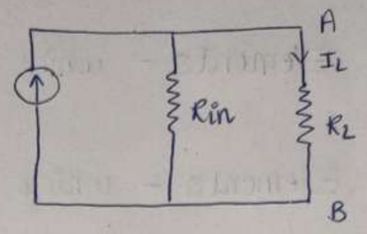
⇒ The source whose output terminal voltage decreases as we increase the load resistance.



$\boxed{V_0 = V - I_L R_{in}}$

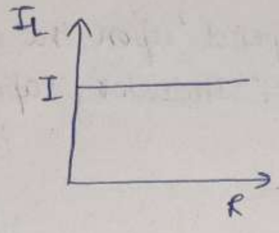
⑥ Ideal and Practical current source.



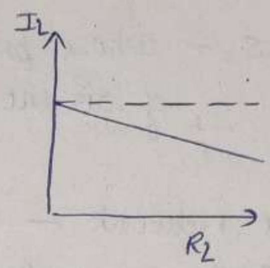


⇒ The source which delivers constant current to the load irrespective of load resistances.

⇒ The source whose output current decreases as we increase the Load Resistance.



$\boxed{I_L = I}$



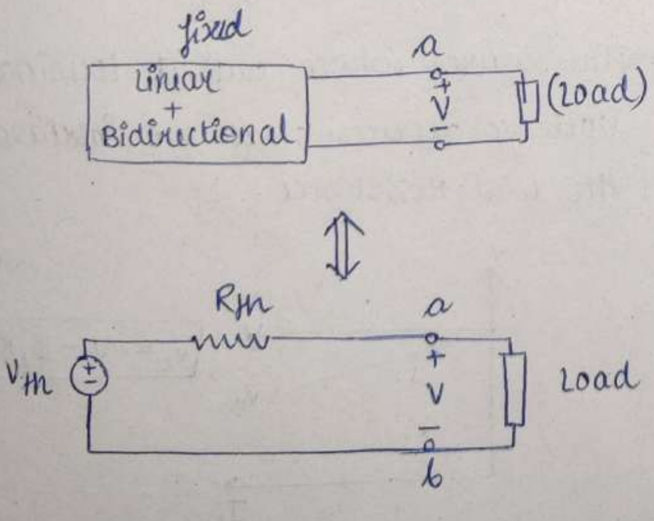
$I_L = \dfrac{I R_{in}}{R_{in} + R_L}$

★ Internal Resistance of Ideal source is ∞.

Source Transformation

THEVENIN'S THEOREM –:

A linear and bidirectional two-terminal network can be replaced by an equivalent network consisting of a voltage source V_{th} connected in series with a resistor R_{th}.

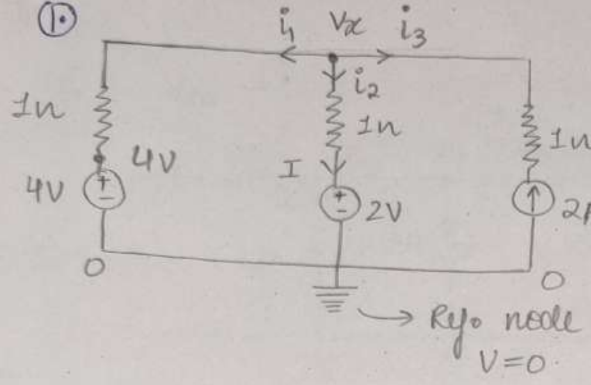


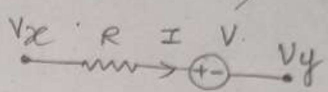
Nodal Analysis

① Assign voltage at every node and one node is taken as reference. (with Pot. = 0V)
② Develop KCL Eq. and solve.

Sample Problem -:

①





→ opp the net current so (-)

$V_x - iR - V = V_y$

$i = \_\_\_$

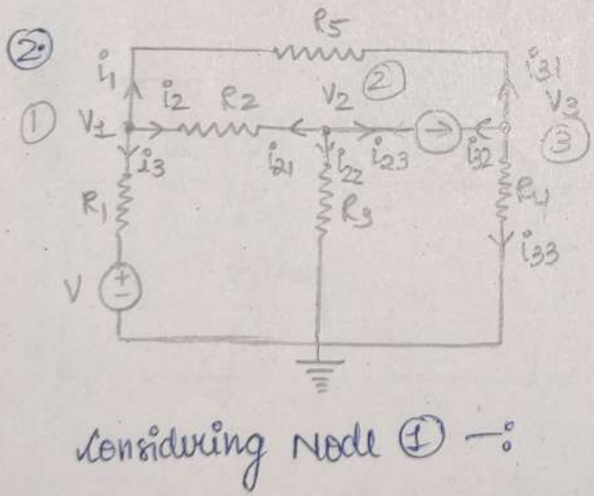
$i = \frac{(V_x - V_y) - V}{R}$

$I_1 + I_2 + I_3 = 0$ $\qquad$ $i_3 = -2A$

$$\frac{(V_x - 0) - 4}{1} + \frac{(V_x - 0) - 2}{1} + \frac{\boxed{V_x - 0}}{1}^{-2} = 0$$

$V_x = \underline{4V}$

②



considering Node ① -:

$i_1 + i_2 + i_3 = 0$

$$\frac{V_1 - V_3}{R_5} + \frac{(V_1 - V_2)}{R_2} + \frac{(V_1 - V)}{R_1} = 0$$

considering Node ② -:

$i_{21} + i_{22} + i_{23} = 0$

$$\frac{(V_2 - V_1)}{R_2} + \frac{V_2}{R_3} + I = 0$$

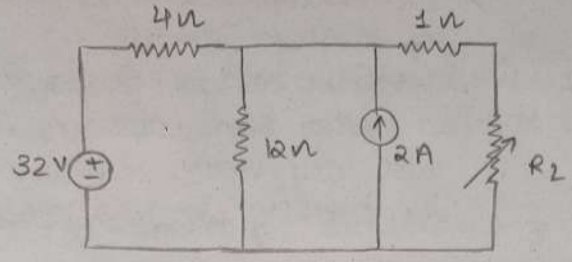
considering Node ③ -:

$i_{31} + i_{32} + i_{33} = 0$

$$\frac{(V_3 - V_1)}{R_5} + \cancel{V_3 - V_2}\,(-I) + \frac{(V_3 - 0)}{R_4} = 0$$

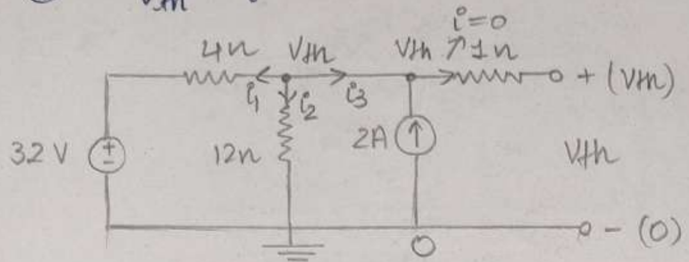
Sample Problem Thevinen's Theorem :-

(1)



Find V_{th} and R_{th} for the circuit.

(a) V_{th} :-

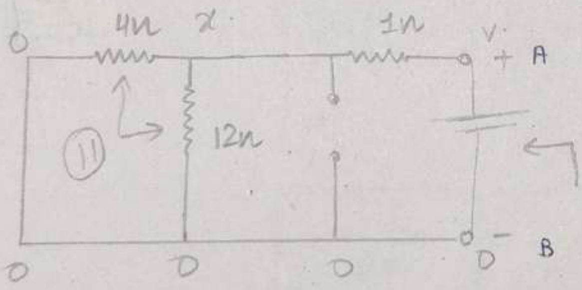


$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_x - 0 - 32}{4} + \frac{V_x - 0}{12} + (-2) = 0$$

$V_x = 30\ V$ $\quad$ $(V_x = V_{th})$

(b) R_{th} :-



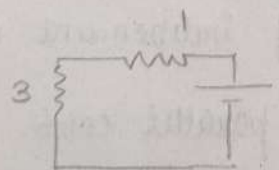
Steps :-

* Remove all the independent sources (turning off).
* Calculate Req looked from AB.

$R_{eq} = 4\Omega$

$R_{th} = 4\Omega$

$\frac{\cancel{48}}{\cancel{18}}$ 3

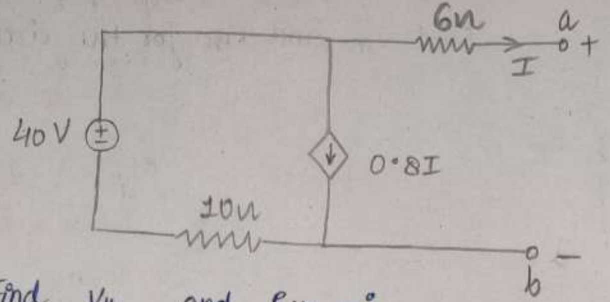


(4)

Imp.

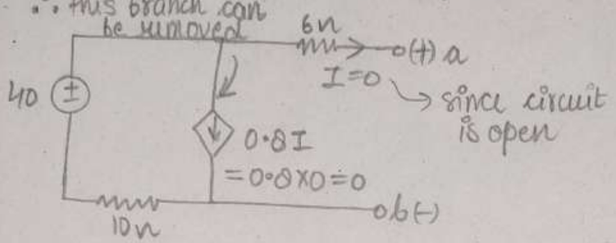
★ Indi. Voltage source turning off → short circuit
★ Indi. current source turning off → open circuit

② Sample Problem with dependent sources —:



★ In this case we can't find out eq. Resistance directly. Hence, we have to take a current or voltage source across a & b, with any value, in this case for simplicity we have taken 1 V.

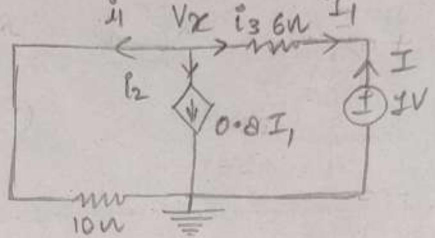
Find V_{th} and R_{th} —:



No current will flow, hence there will be no voltage drop across resistance –

∴ $\boxed{V_{th} = 40\ V}$

$(i_3 = I_1)$



$R_{th} = \frac{1}{I}$

~~(10+6)Ω = 16Ω~~

$\frac{V_x}{10} + 0{\cdot}8I_1 + I_1 = 0$

$V_x + 18I_1 = 0 \longrightarrow ①$

$I_1 = \frac{V_x - 1}{6} \longrightarrow ②$

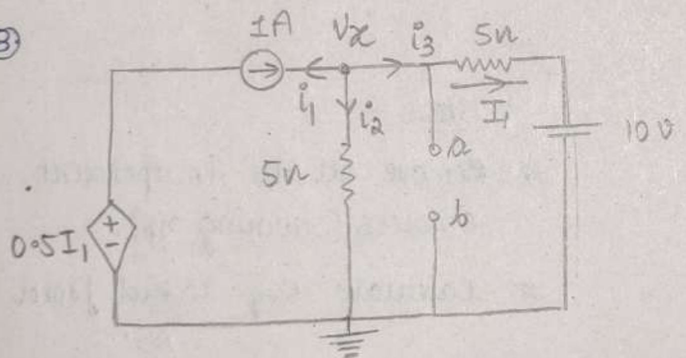
$V_x = \frac{3}{4}\ V$

$I = -I_1$

$I = \frac{1 - V_x}{6} = \frac{1}{24}\ A$

$\boxed{R_{th} = 24\ \Omega}$

③



Find R_{th} & V_{th}.

[GATE ECE 2005]

(i) $V_x = V_{th}$ $\quad i_1 = -1$

$i_1 + i_2 + i_3 = 0$

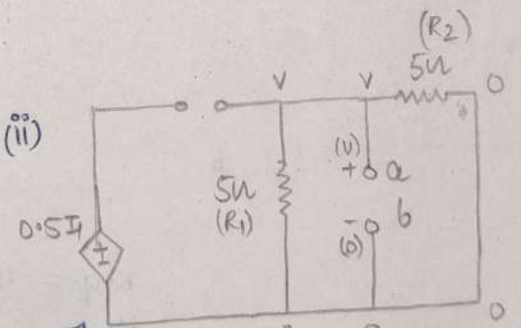
$-1 + \frac{V_x}{5} + \frac{V_x - 10}{5} = 0$

$2V_x = 15$

$V_x = 7{\cdot}5\ V$

$\boxed{V_{th} = 7{\cdot}5\ V}$

(ii)



After turning off independent sources

R_1 & R_2 are in parallel comb.

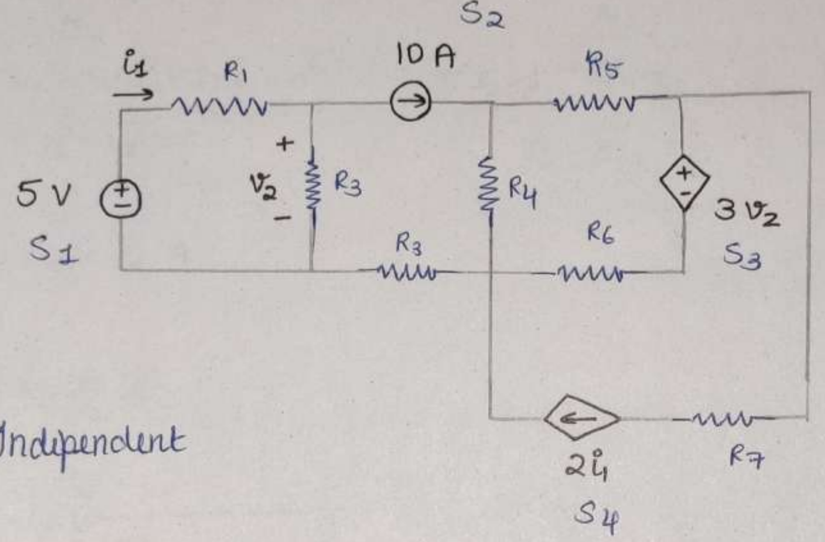
$R_{eq} = \frac{5}{2} = 2{\cdot}5\ \Omega$

$\boxed{R_{th} = 2{\cdot}5\ \Omega}$

Dependent and Independent Sources –

① Independent source – The element for which both voltage and current don't depend on the voltage or current elsewhere in the circuit.

② Dependent source – The element for which either the voltage and current depends on the voltage or current elsewhere in the circuit.



S1 } → Independent
S2 }

S4 → Dependent upon current in other part of circuit.

S3 → Dependent

$$V_x + \frac{\cancel{18}^{3} V_x}{\cancel{6}} - \frac{\cancel{18}^{3}}{\cancel{6}} = 0$$

$$4V_x - 3 = 0$$

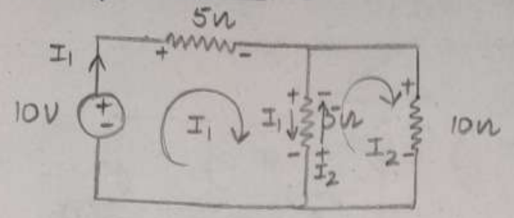
$$V_x = 3/4$$

$$R_{th} = \frac{5}{I} = \frac{5}{\cancel{29}} \times 24$$

Mesh Analysis

Norton → Source Transf.

Sample Problem ①



* Jis mesh ke equation rahe uss mesh ke current ko greater manege.

M_1

$$10 - 5I_1 - 5(I_1 - I_2) = 0$$

$$2I_1 - I_2 = 0$$

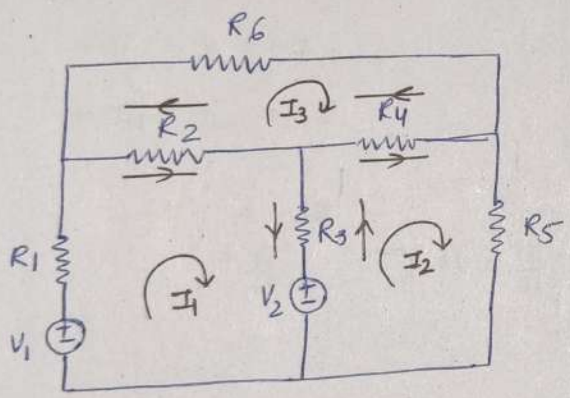
M_2

$$-5(I_2 - I_1) - 10I_2 = 0$$

$$I_1 - 3I_2 = 0$$

$$I_2 = 2/5 \text{ A}$$

Sample Problem ②



Mesh 1

$$V_1 - I_1R_1 - R_2(I_1 - I_3) - R_3(I_1 - I_2) - V_2 = 0$$

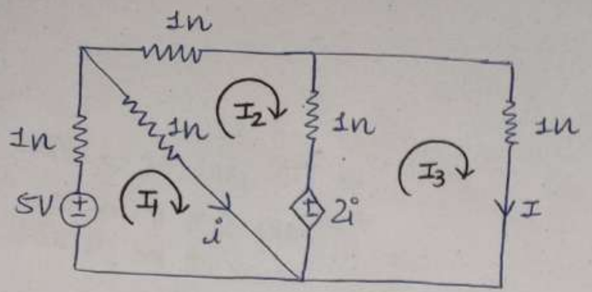
Mesh 2

$$V_2 - R_3(I_2 - I_1) - R_4(I_2 - I_3) - R_5(I_2) = 0$$

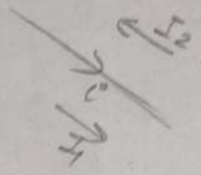
Mesh 3

$$-R_2(I_3 - I_1) - I_3R_3 - R_4(I_3 - I_2) = 0$$

Sample Problem (3)



$I = I_3$



$i = I_1 - I_2$ $\longrightarrow$ (4)

M1

$5 - I_1 - (I_1 - I_2) = 0$ $\longrightarrow$ (1)

$5 - 2I_2 + I_2 = 0$ $\longrightarrow$ (7)

M2

$-I_2 - (I_2 - I_3) - 2i - (I_2 - I_1) = 0$ $\longrightarrow$ (2)

M3

$-I_3 + 2i - (I_3 - I_2) = 0$ $\longrightarrow$ (3)

$-\cancel{I_2} - \cancel{I_2} + I_3 - 2I_1 + 2\cancel{I_2} - I_2 + I_1 = 0$

$-I_1 + I_3 - I_2 = 0$ $\longrightarrow$ (5)

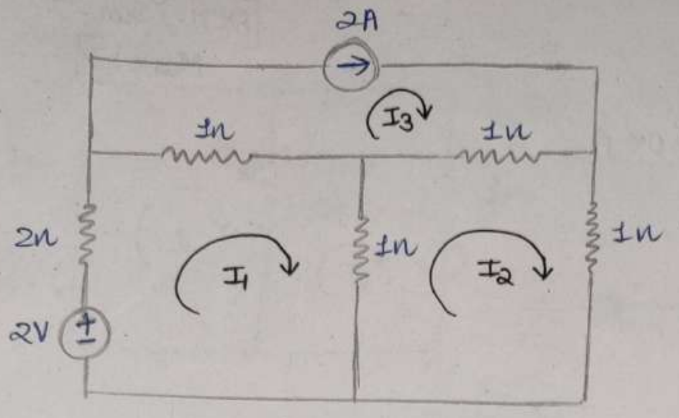
$-I_3 + 2I_1 - 2I_2 - I_3 + I_2 = 0$

$-2I_3 + 2I_1 - I_2 = 0$ $\longrightarrow$ (6)

$-2I_3 + 5 = 0$

$$\boxed{I_3 = \frac{5}{2}}$$

Mesh Analysis – with current source



$I_3 = 2A \longrightarrow ③$

mesh 1

$2 - 2i_1 - (i_1 - i_3) - (i_1 - i_2) = 0$

$-4i_1 + i_2 + i_3 = -2 \longrightarrow ①$

$-4i_1 + i_2 = -4$

$-12i_1 + 3i_2 = -12$

$-11i_1 = -14$

$i_1 = \frac{14}{11}$ $i_2 = \frac{12}{11}$

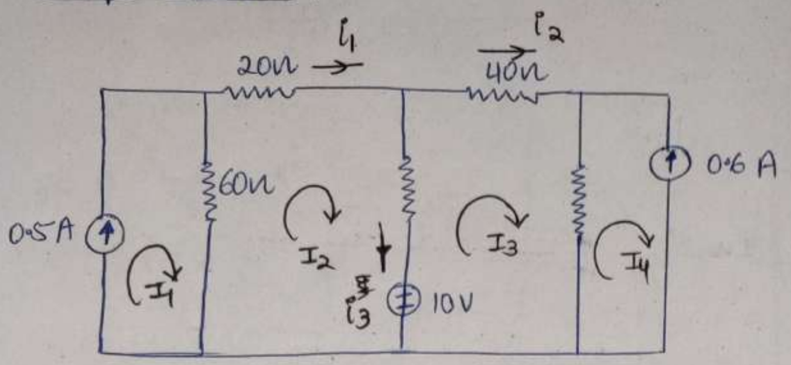
mesh 2

$-(i_2 - i_1) - (i_2 - i_3) - i_2 = 0$

$i_1 - 3i_2 + i_3 = 0 \longrightarrow ②$

$-i_1 + 3i_2 = 2$

Sample Problem



[AKTU Sem - I
Mark : 7]

$I_1 = 0.5A$

$I_4 = -0.6A$

mesh ②

$$-60(I_2 - I_1) - 20I_2 - 15(I_2 - I_3) - 10 = 0$$

$$-60I_2 + 60I_1 - 20I_2 - 15I_2 + 15I_3 - 10 = 0$$

$$-95I_2 + 60I_1 + 15I_3 - 10 = 0$$

$$-95I_2 + 15I_3 + 20 = 0$$

$$19I_2 - 3I_3 + 4 = 0 \longrightarrow ①$$

mesh ③

$$10 - 15(I_3 - I_2) - 40I_3 - 100(I_3 - I_4) = 0$$

$$10 - 15I_3 + 15I_2 - 40I_3 - 100I_3 + 100I_4 = 0$$

$$-155I_3 + 15I_2 - 70 = 0$$

$$-31I_3 + 3I_2 - 14 = 0 \longrightarrow ②$$

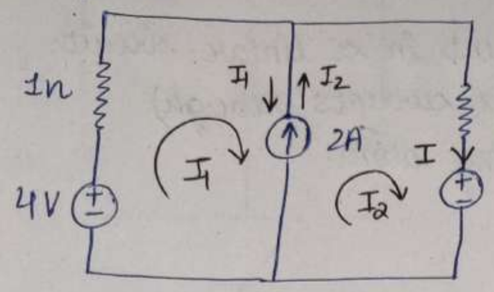
$I_2 =$ — $I_3 =$ ___

$i_1 = I_2$ $i_3 = I_2 - I_3$

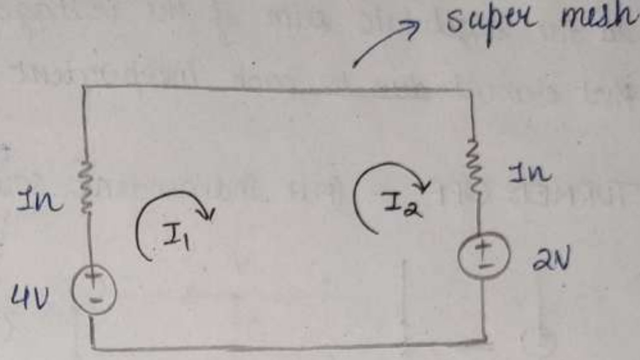
$i_2 = I_3$

Supermesh Analysis

Sample ①



Find I



$$4 - I_1 - I_2 - 2 = 0$$

$$I_1 + I_2 = 2 \longrightarrow ①$$

$$I_2 - I_1 = 2 \longrightarrow ②$$

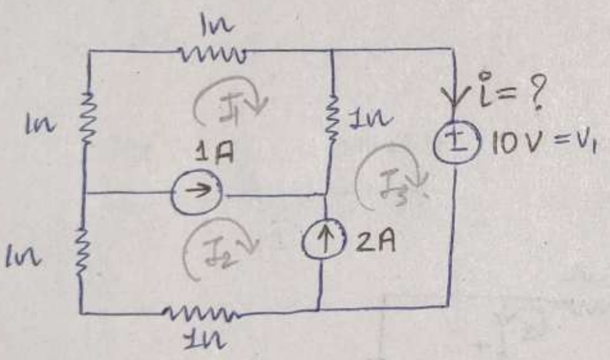
Add ① + ②

$$I_2 = I = 2A$$

Sample ②

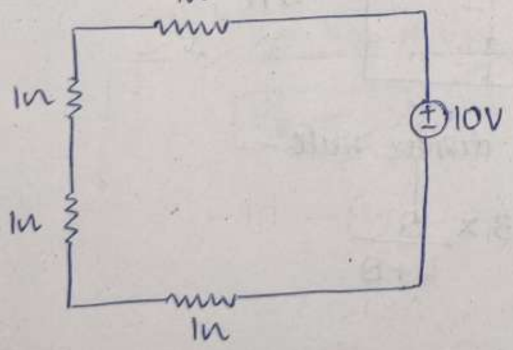
Find Power delivered by V_1.

[GATE 2010]



$$I_2 - I_1 = 1A \longrightarrow ②$$

$$I_3 - I_2 = 2A \longrightarrow ③$$



$$-I_1 - I_1 - 10 - 2I_2 = 0$$

$$I_1 + I_2 = -5 \longrightarrow ①$$

$$I_2 = -2$$

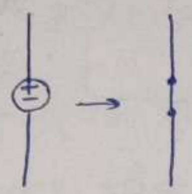
Therefore $I_3 = 0$

Power delivered by voltage source $= 0$ W

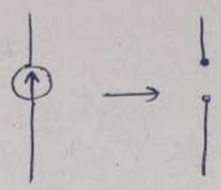
Superposition Theorem -

The voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

TURNED OFF - (All independent sources)



Voltage source

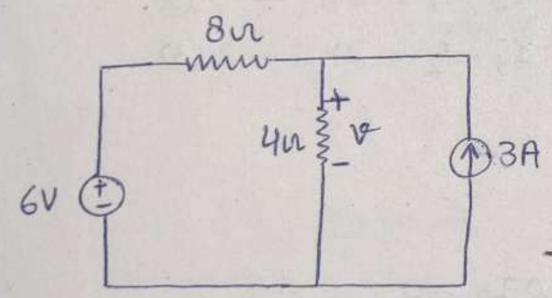


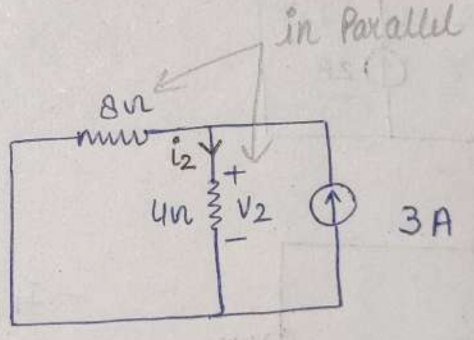
Current source

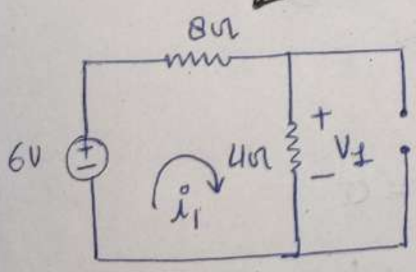
☆ The dependent sources are left as it is.

Sample Problem (1)

Find 'V' by Superposition theorem







$i_1 = \frac{6}{8+4}$

$i_1 = 0{\cdot}5\,A$

$V_1 = 0{\cdot}5 \times 4 = 2\,V$

By current divider rule -

$i_2 = 3 \times \frac{8}{4+8}$

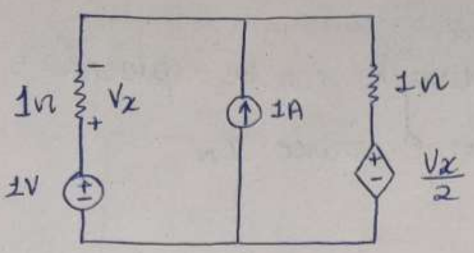
$i_2 = 2A$

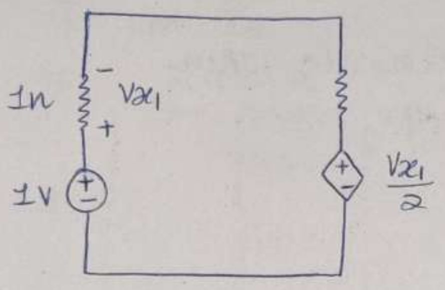
$V_2 = 2 \times 4 = 8V$

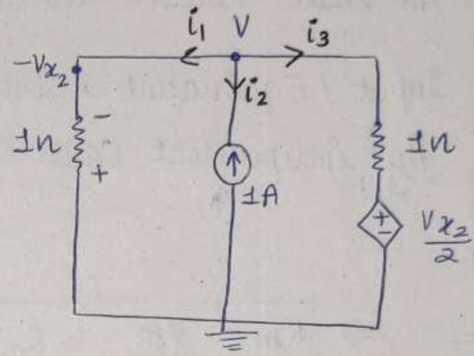
$V = V_1 + V_2$

$\boxed{V = 10V}$

Sample Problem - ② [with Dependent sources]







$$1 - 2i - \frac{V_{x_1}}{2} = 0 \quad \left(i = \frac{V_{x_1}}{1}\right)$$

$$1 - 2V_{x_1} - \frac{V_{x_1}}{2} = 0$$

$$V_{x_1} = \frac{2}{5} = 0.4\,V$$

$$V = -V_{x_2}$$

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V}{1} + (-1) + \frac{V - \frac{V_{x_2}}{2}}{1} = 0$$

$$-V_{x_2} - V_{x_2} - \frac{V_{x_2}}{2} = 1$$

$$V_{x_2} = \frac{-2}{5}$$

$$V_x = V_{x_1} + V_{x_2}$$

$$\boxed{V_x = 0}$$

Norton's Theorem –:

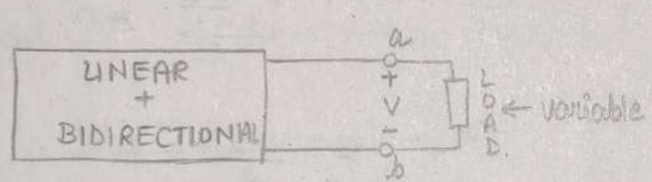
A linear and bi-directional two-terminal network can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N.

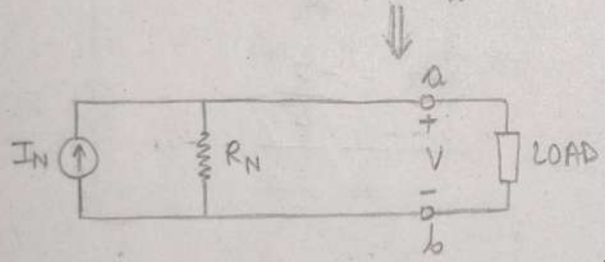
$I_N \longrightarrow$ The short-circuit current through the terminals.

$R_N \longrightarrow$ Input / Equivalent resistance at the terminals when the independent sources are turned off.

$R_N = R_{th}$

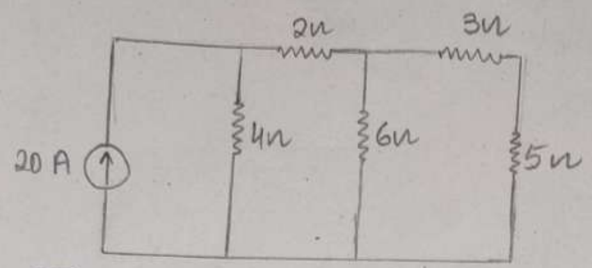
$$I_N = \frac{V_{th}}{R_{th}} \Rightarrow \underbrace{\boxed{R_{th} = \frac{V_{th}}{I_n} = R_N}}_{\text{source transformation.}}$$





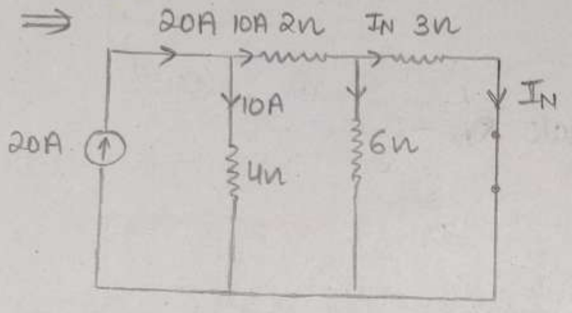
Sample Problem (1)

Find the current flowing through 5Ω resistor.



To find I_N —:

⟹



$(3 \& 6)\Omega \rightarrow \parallel$

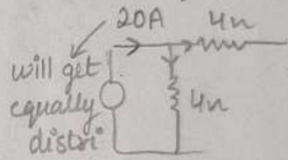
$\downarrow$

$R_{eq} = 2\Omega$

2Ω & $2\Omega \rightarrow$ series

$\downarrow$

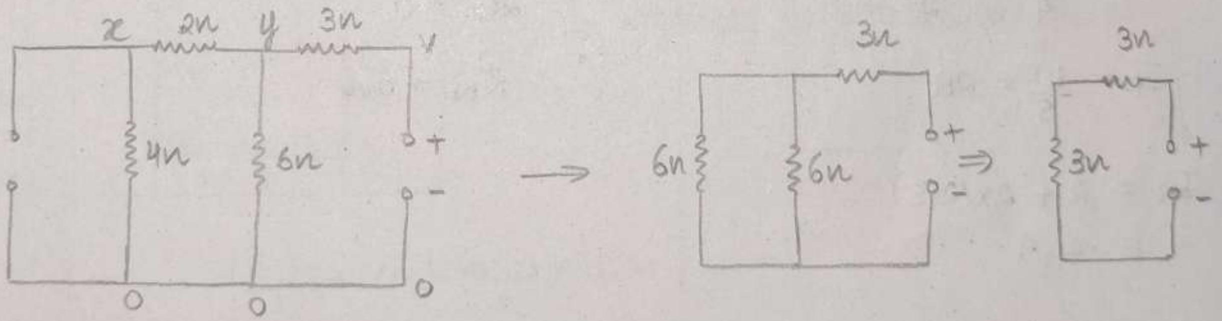
$R_{eq} = 4\Omega$.

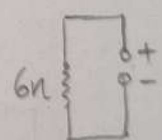


$$I_N = 10 \times \frac{6}{6+3}$$ } By current divider rule

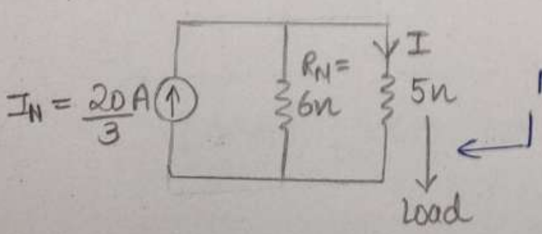
$$I_N = \frac{20}{3}\ A$$

To find R_N —:





$$R_N = 6\Omega$$



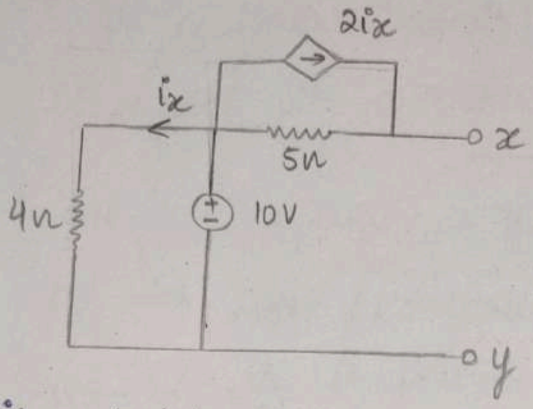
Norton's eq. circuit

$$I = \frac{20}{3} \times \frac{6}{6+5}$$

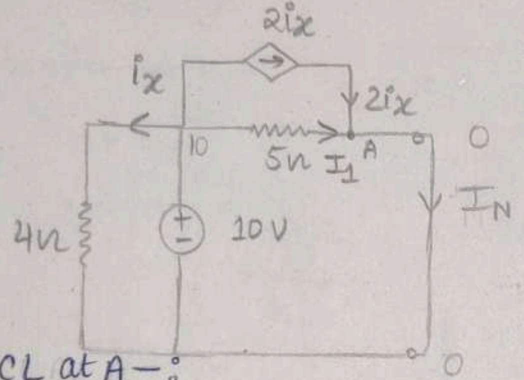
$$I = \frac{40}{11}$$

Sample Problem 2 → (with Dependent source)

Develop the Norton's equivalent circuit between the terminals x and y.



(i) calculate I_N :-



KCL at A :-

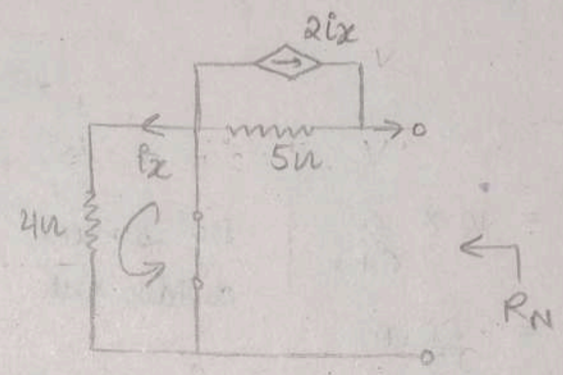
$$I_N = I_1 + 2i_x \longrightarrow (1)$$

$$i_x = \frac{10}{4} = \frac{5}{2} = 2.5\,A$$

$$I_1 = \frac{10}{5} = 2A$$

$$I_N = 2 + 2\times(2.5)$$
$$= 7\,A$$

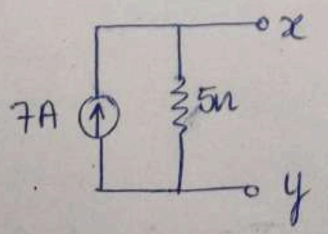
(ii) calculate R_N :-



$$i_x = 0\,A$$

$$2i_x = 0$$

$$\therefore\ R_N = 5\Omega$$



Norton's equ circuit

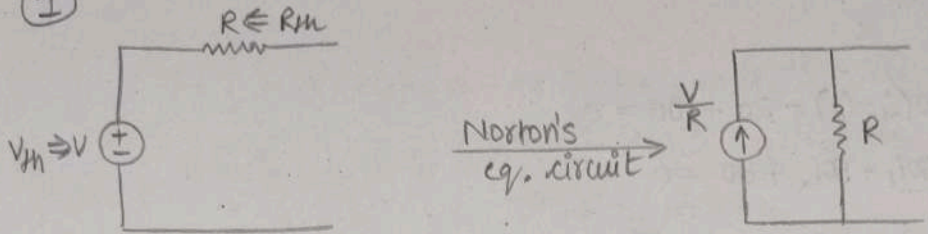
SOURCE TRANSFORMATION

$$R_{th} = R_N = \frac{V_{th}}{I_N}$$

T ⟷ N

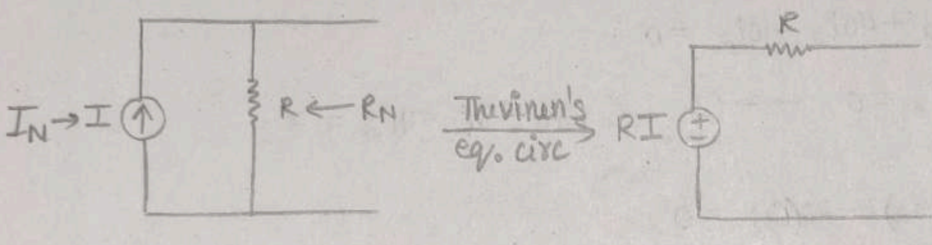
VS ⟷ CS

(I)



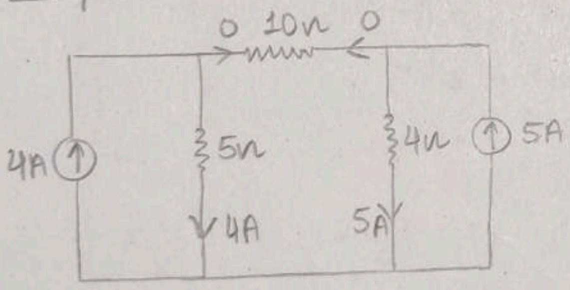
Thevinen's circuit

(II)



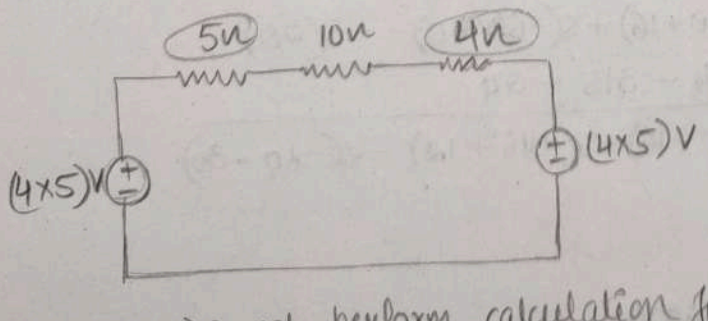
Norton's circuit

Example



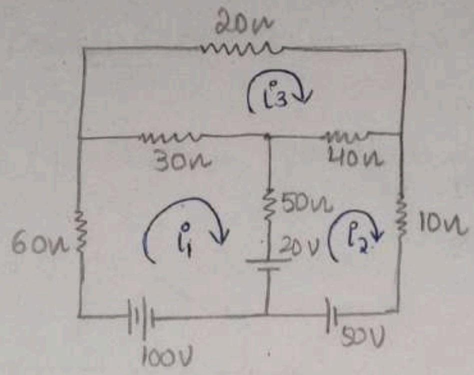
⇒ currents in branches are given

↓ By source transformation



Do not perform calculation for transformed resistors.

Ques.)
Mesh Analysis



$$-60i_1 - 30(i_1 - i_3) - 50(i_1 - i_2) - 20 + 100 = 0.$$

$$-60i_1 - 30i_1 + 30i_3 - 50i_1 + 50i_2 + 80 = 0$$

$$+14\cancel{0}i_1 - 3\cancel{0}i_3 - 5\cancel{0}i_2 = 8\cancel{0} \longrightarrow ①$$

$$20 - 50(i_2 - i_1) - 40(i_2 - i_3) - 10i_2 + 50 = 0$$

$$70 - 50i_2 + 50i_1 - 40i_2 + 40i_3 - 10i_2 = 0$$

$$7\cancel{0} - 10\cancel{0}i_2 + 5\cancel{0}i_1 + 4\cancel{0}i_3 = 0 \longrightarrow ② \qquad 10i_2 - 5i_1 - 4i_3 = 7$$

$$-30(i_3 - i_1) - 40(i_3 - i_2) - 20(i_3) = 0$$

$$-30i_3 + 30i_1 - 40i_3 + 40i_2 - 20i_3 = 0.$$

$$-\cancel{+}9\cancel{0}i_3 + 3\cancel{0}i_1 + 4\cancel{0}i_2 = 0 \longrightarrow ③$$

$$\begin{bmatrix} 14 & -5 & -3 \\ -5 & 10 & -4 \\ 3 & 4 & -9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 0 \end{bmatrix}$$

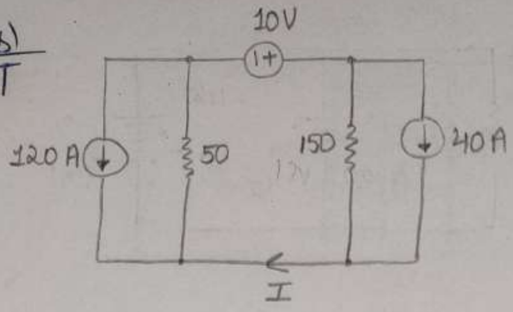
$$i_1 = \frac{Dx}{D}$$

$$i_1 = \frac{\begin{vmatrix} 8 & -5 & -3 \\ 7 & 10 & -4 \\ 0 & 4 & -9 \end{vmatrix}}{\begin{vmatrix} 14 & -5 & -3 \\ -5 & 10 & -4 \\ 3 & 4 & -9 \end{vmatrix}} = \frac{8(-90+16) + 5(-63+0) - 3(28) = -8\times 74 - 315 - 84}{14(-90+16) + 5(+45+12) - 3(-20-30)}$$

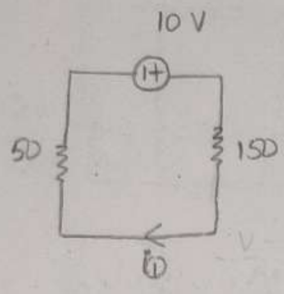
Similarly :- $\qquad = 1.65A$

$i_2 = 2.12A$ & $i_3 = 1.5A$

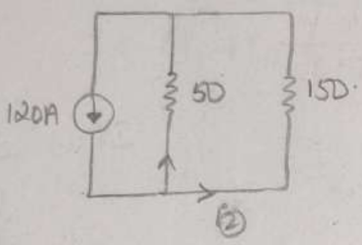
Ques)
SPT



calculate current load I

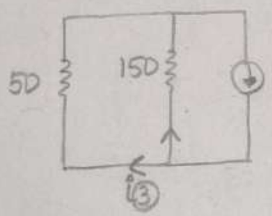


$$i_1 = \frac{1\cancel{0}}{20\cancel{0}} = 0.05\,A$$



$$i_2 = \left(\frac{50}{50+150}\right) \times 120$$ (By current divider rule)

$$= \frac{\cancel{120}}{\cancel{4}} \quad 30\,A$$



$$i_3 = \left(\frac{150}{50 \times 150}\right)40$$

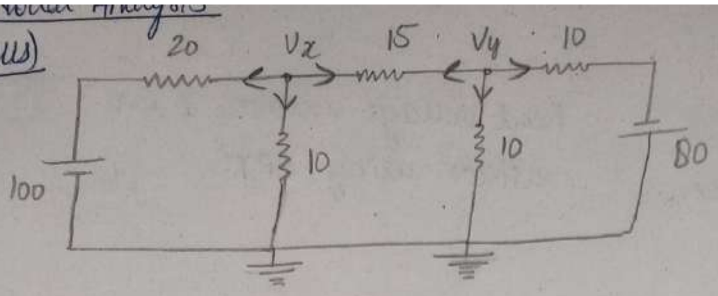
$$i_3 = \frac{3}{4} \times 40 = 30\,A$$

$$i_1 + i_3 - i_2 = 0.05 + 30 - 30$$

$$= 0.05\,A$$

Nodal Analysis

Q iii)



$$\frac{V_x-100}{\cancel{20}\,4}+\frac{V_x}{\cancel{10}\,2}+\frac{V_x-V_y}{\cancel{15}\,3}=0$$

$$\frac{V_y-V_x}{\cancel{15}\,3}+\frac{V_y}{\cancel{10}\,2}+\frac{V_y+80}{\cancel{10}\,2}=0$$

$$\Rightarrow 2V_y-2V_x+3V_y+3V_y+240=0.$$

$$8V_y-2V_x+240=0$$

$$\boxed{4V_y-V_x+120=0}$$

$$\Rightarrow 3V_x-300-6V_x+4V_x-4V_y=0.$$

$$\boxed{13V_x-4V_y=300}$$

$$13(4V_y+120)-4V_y=300$$

$$52V_y+1560-4V_y=300$$

$$48V_y=-1260$$

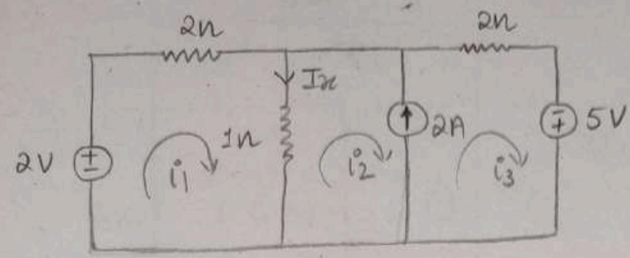
$$V_y=\frac{-1260}{48}=-26.25 \qquad V_x=15\,V$$

$$V_x-V_y/15$$

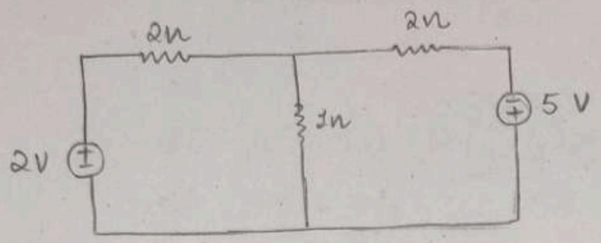
$$i=\frac{(+26.25)+(15)}{15}=2.75$$

⇒ assumed that current through 15Ω flows from V_x ∴ which comes out to be true!

Ques)
MA



Find I_x



$i_3 - i_2 = 2$

$2 - 3i_1 - 1(i_1 - i_2) = 0$

$2 - 3i_1 - i_1 + i_2 = 0$

$2 - 4i_1 + i_2 = 0$

$-i_2 + i_1 - 2i_3 + 5 = 0$

$-i_2 + i_1 - 2(2 + i_2) + 5 = 0$

$-i_2 + i_1 - 4 - 2i_2 + 5 = 0$

$-3i_2 + i_1 + 1 = 0$

$i_2 = \frac{i_1}{3} + \frac{1}{3}$

$+2 - 4i_1 + \frac{i_1}{3} + \frac{1}{3} = 0$

$\frac{7}{3} = \frac{11 i_1}{3}$

$i_1 = \frac{7}{11}$ A

$i_2 = \frac{7}{33} + \frac{1}{3}$

$i_2 = \frac{18}{33}$ A

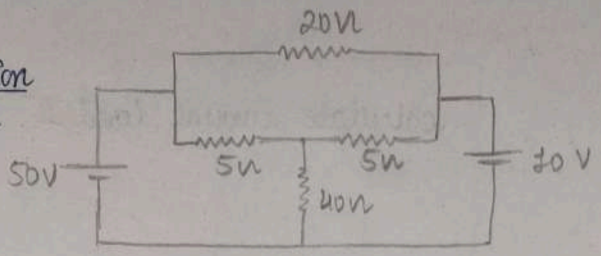
$I_x = i_1 - i_2$

$= \frac{7}{11} - \frac{18}{33}$

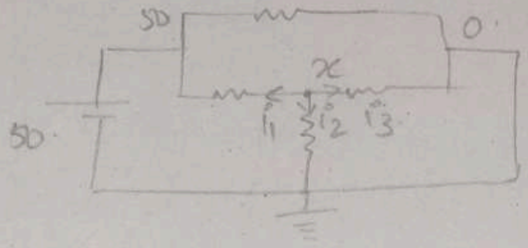
$= \frac{3}{33}$

$= \frac{1}{11}$ A

Ques)
Superposition theorem



→ current 40Ω branch.



$$\frac{V_x}{\cancel{40}} + \frac{8(V_x - 50)}{\cancel{5}} + \frac{8V_x}{\cancel{5}} = 0$$

$$V_x + 8V_x - 400 + 8V_x = 0$$

$$17V_x = 400$$

$$V_x = \frac{400}{17} \qquad i_1 = \frac{\cancel{400}10}{17\times\cancel{40}}$$

$$\frac{V_x}{\cancel{40}} + \frac{8(V_x - 10)}{\cancel{5}} + \frac{8(V_x)}{\cancel{5}} = 0 \qquad i_2 = \frac{\cancel{80}2}{17\times\cancel{40}}$$

$$17V_x = 80$$

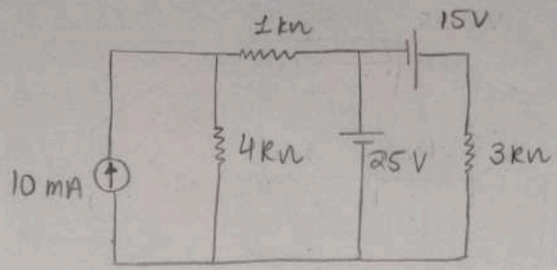
$$V_x = \frac{80}{17}$$

$$\frac{12}{17}$$

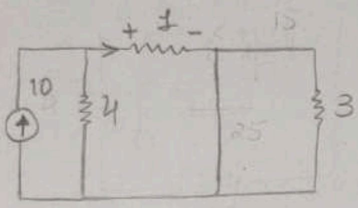
$$V_x \quad \frac{480}{17}$$

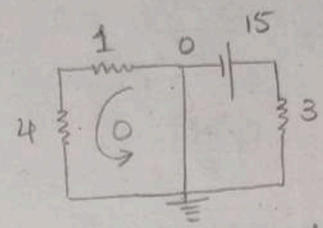
$$I = \frac{\overset{12}{\cancel{480}}}{17\times\cancel{40}} = 0.705$$

Ques)
SPT



Find voltage across 1 kΩ resistor using SPT.



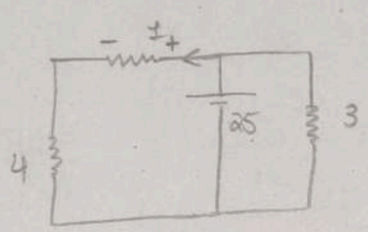


i across 1Ω resistor = 0

$I_{③} = 0$

$I = 10 \times \frac{4}{4+1}$ (By current divider rule)

$I_{①} = 8 \text{ mA}$



$V = \frac{1}{4+1} \times 25$ (By voltage divider rule)

$V = 5$

$I_{②} = 5 \text{mA}$

Since direction of current $I_{①}$ & $I_{②}$ are different

$I = (8-5) \text{mA}$

$I = 3 \text{ mA}$

$V = R \times I$

$V = 1 k\Omega \times 3 \text{mA}$

$V = 3V$