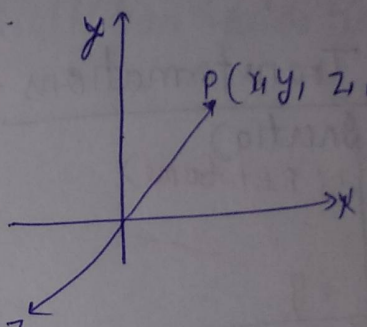
RELATIVISTIC MECHANICS

*Event ÷ Occurance of something at one point at an instant in space. Ex ÷ shot of a bullet, swing of a pendulam when it comes in the mean position.

*Observer ÷ who observes the occurence of event in the space. (Rest or moving together)

*frame of Reference ÷ Geometrical frame work (normal Cartesian System) required to describe the occurance of event in the space.

(x, y, z, t) → space-lines
↓
Special Coordinate



*Position Vector ÷ The position vector of the moving object (at any instant) →

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Velocity of the moving object ÷

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

Acceleration ÷

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

→ Classification of frame of Reference ÷

Inertial frame
* The frame which follows the law of inertia (Newton's Ist law)
* In inertial frame

$$a = \frac{d^2r}{dt^2} = 0, \text{ or}$$

$$\frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} = \frac{d^2z}{dt^2} = 0$$

Non Inertial frame
* Do not follow law of inertia
* In non Inertial frame

$$a = \frac{d^2r}{dt^2} \neq 0 \text{ or}$$

$$\frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} = \frac{d^2z}{dt^2} = 0$$

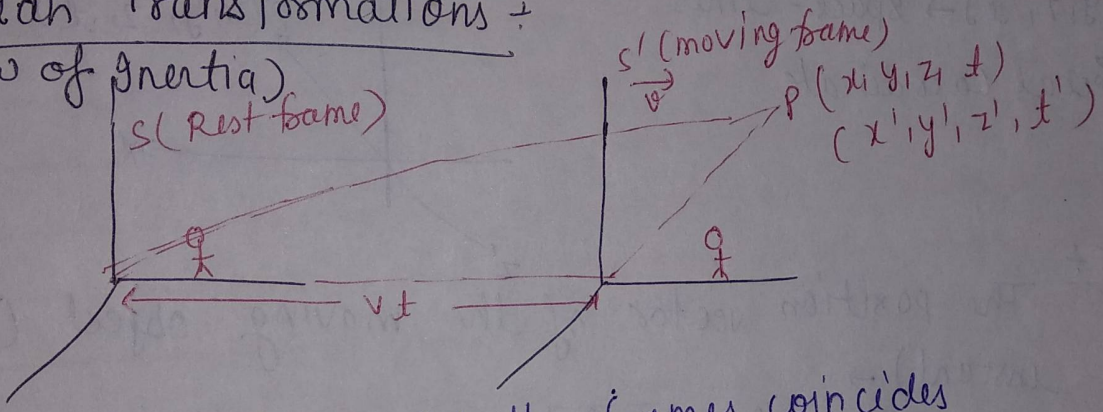
it is called as Non accelerated frame.

Theory of relativity which is applicable to inertial frame of Reference known as special theory of relativity.

* Particle will experience some force due to accelerated frame of reference.
* Particle will experience force, because of acc of FOR known as Fictitious force or pshudo force.
* The relative which is applicable to non inertial frame is known as General theory of relativity.

Galilean Transformations :

(Law of Inertia)



at $t = t' = 0$ origin of both frames coincides

$$r = r' + vt$$

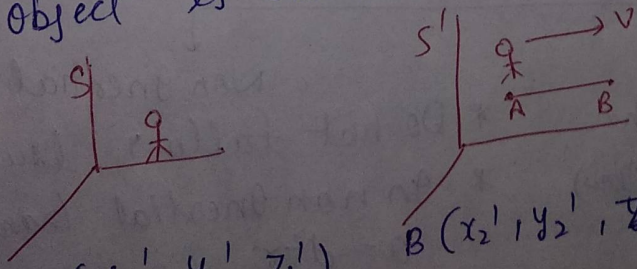
Component form

$$x = x' + vt$$
$$z' = z$$
$$y' = y$$
$$t' = t$$
$$\boxed{x' = x - vt}$$

Galilean transformation

Component of G.I. :

Length of object is absolute (invariant)



S' Frame $A(x_1', y_1', z_1')$ $B(x_2', y_2', z_2')$

S Frame $A(x_1, y_1, z_1)$ $B(x_2, y_2, z_2)$

$$(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2$$

Let us transform (x_1', y_1', z_1') using G.T

$$L = \sqrt{(x_2 - vt - x_1 + vt)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\boxed{L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

↓ w.r.t S frame

$\boxed{L' = L}$ → Length is in-variant under Gallilian Tranformation

Velocity of moving object :

$$r' = r - vt \quad - (i)$$

$$\frac{dr'}{dt} = \frac{dr}{dt} - v,$$

$$\boxed{v' = V - v} \rightarrow \text{F.O.R.} \qquad \boxed{V = v' + v}$$ Gallilian addition of velocities

↓ moving object

Acceleration :

$$\frac{d^2r'}{dt^2} = \frac{d^2r}{dt^2} - 0$$

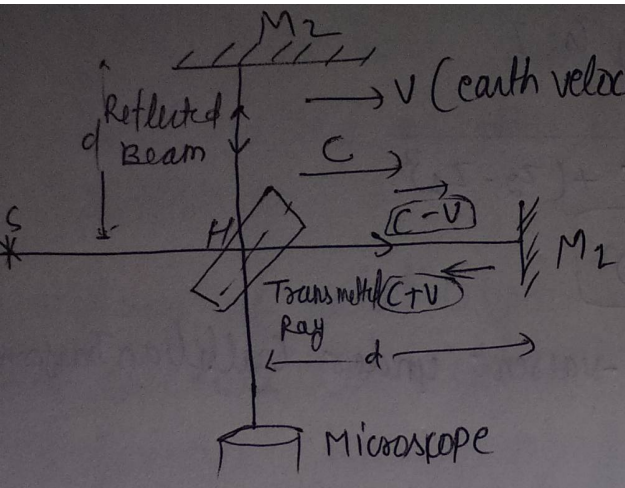
$$\boxed{a' = a} \Rightarrow \boxed{F' = F}$$

All the law of physics are identical for all the observer for inertial frame → Gallilian hypothesis of invarience

Michelson Morley experiment :

Objectives :

- whether speed of light 'c' get modified in accordance with G.T.
- Aether - Material medium which was supposed to be present throughout the universe
  * Perfectly elastic
  * highly transparent
  * Negligible density
- existance of aether was assumed as absolute frame of Reference Relative to which the motion of bodies can be detected
- To justify the aether hypothesis.



$\Delta t = \frac{2t}{c} - \frac{2t}{c}$

$\boxed{t_2 - t_1 = 0}$

When velocity of earth at aether frame is zero.

$\because \Delta t = 0 \Rightarrow \Delta x = 0$

no frinze pattern is visible

When earth is moving

$\Delta t = t_2 - t_1 \neq 0$

Some path diff^n will definitely occure b/w transmitted & reflected ray

$\boxed{\text{Path diff}^n = \frac{dv^2}{c^2}}$

frinze pattern should appear & visible

for prove that to shift frinze (interference) this turn into 90°

but no frinze shift was expermintly observed.

so $\boxed{v = 0}$

therefore

→ Motion of earth could not be detected relative to aether.

→ Earth is absolutly rest in aether

→ This is known as negative or null result.

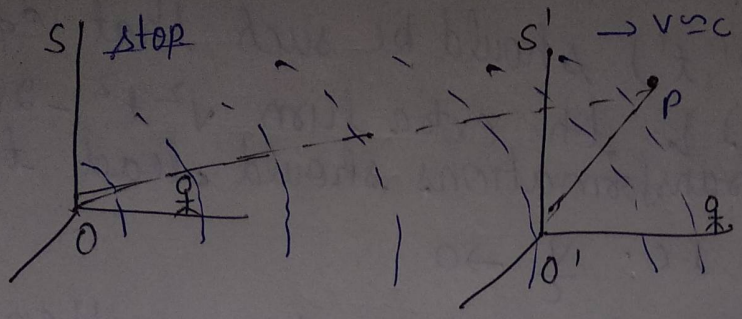
→ The speed of light is universal constant identical for all the observes of inertial frame.

Basic Postulates of Relativity :

i) All the laws of physics are identical for all the observer of the initial ~~figure~~ frame that move with a constant velocity relative to one-another.

ii) The velocity of light (c) is a universal constant and is identical for all the observers of the initial frame. This is also known as constancy of velocity of light.

Lorentz transformation :



Initially at $t = t' = 0$ origions coinciding

When S & S' are coinciding, a light signal emits from O

P co-ordenate from :

Rest frame observer (S) :- (x, y, z, t)

S' observer :- (x', y', z', t')

time taken by signal to reach P For S frame observer

$$\boxed{t = \frac{OP}{c} = \frac{\sqrt{x^2+y^2+z^2}}{c}}$$

$$\boxed{x^2+y^2+z^2 = c^2t^2} \quad -(i)$$

For S' frame observer

$$t' = \frac{O'P}{c} = \frac{\sqrt{x'^2+y'^2+z'^2}}{c}$$

$$\boxed{x'^2+y'^2+z'^2 = c^2t'^2} \quad -(ii)$$

∴ c is identical for all observer

The new transfomation will be such that eqⁿ (ii) transform into eqⁿ (i)

Let's apply G.T. again in eqⁿ (2)

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

$$(x-vt)^2 + y^2 + z^2 = c^2t^2$$

$$x^2+y^2+z^2 + v^2t^2 - 2vtx = c^2t^2 \quad -(iii)$$

Compairing eqⁿ (1) & (3)

Extra term → $v^2t^2 - 2vxt$

⇒ G-T. fail or need some modification.

⟹

∴ New transformation b/w (x, y, z, t) and (x', y', z', t') should be such that eq^n(iii) transforms into eq^n(i) & the extra term $v^2t^2 - 2vxt$ should cancel. Also new transformations should lead to G.T. for smaller velocities i.e. $\frac{v}{c} \to 0$.

Let the new or modified eq^n's can be written as

$x' = \alpha(x - vt)$, $t' = \alpha(t + f(x))$

where α, α' & f are Constant.

substuting these in eq^n (ii)