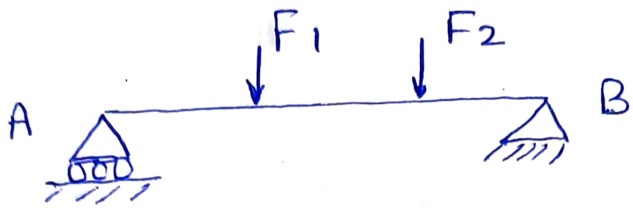
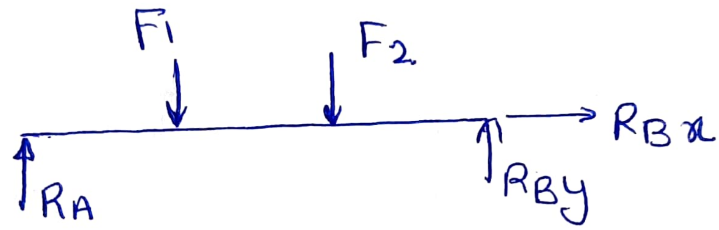
Shear Force and Bending Moment

Types of Beams

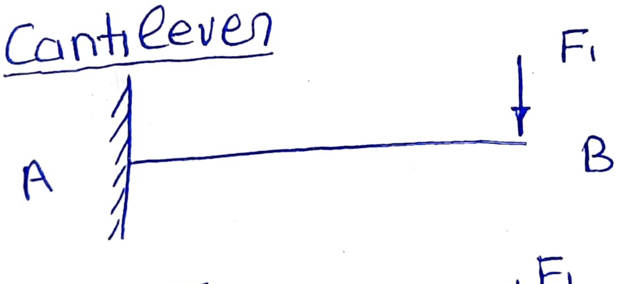
(i) Simply Supported Beam.

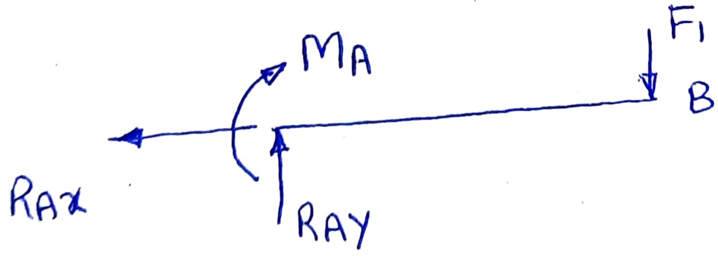


FBD

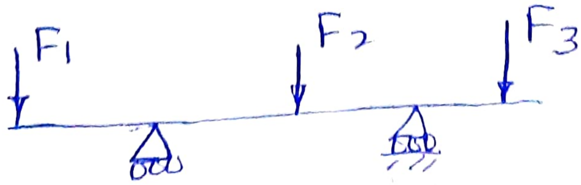


(ii) Cantilever



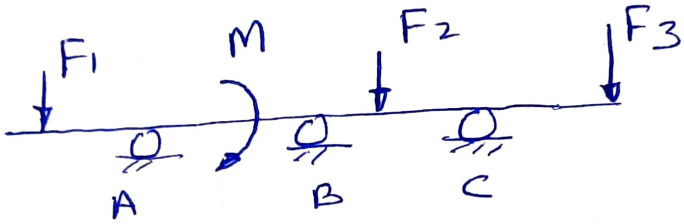


(iii) Overhanging Beam



(IV) <u>Fixed Beam</u>

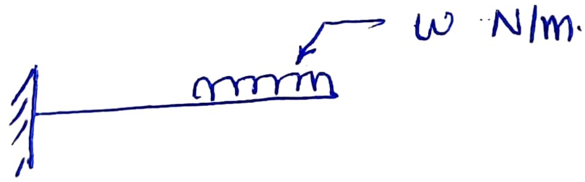
(V) Continuous Beam



Types of Loading

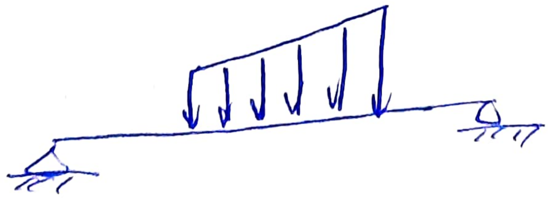
(1) Concentrated Force
(2) Concentrated Moment
(3) Uniformly Distributed Load

Ex



(4) Variable Intensity Loading (UVL)

Ex

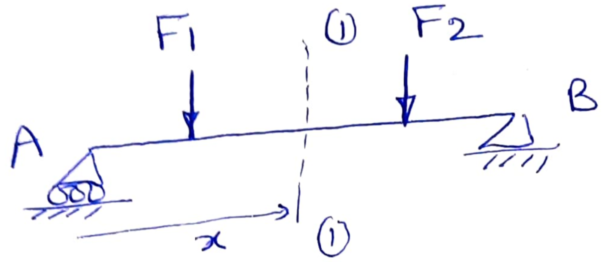


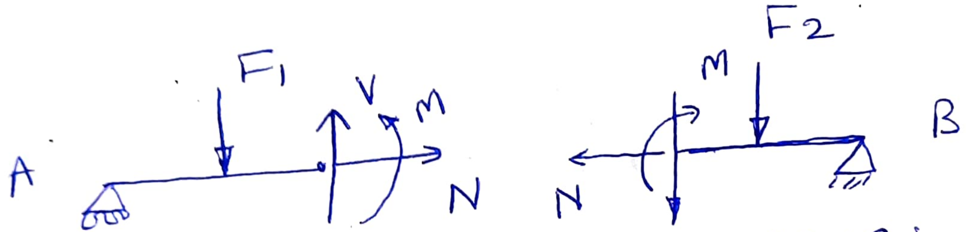
Determining of Support Reaction

For statically determinate beam.

$$\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma M_z = 0$$

Shear Force & Bending Moment

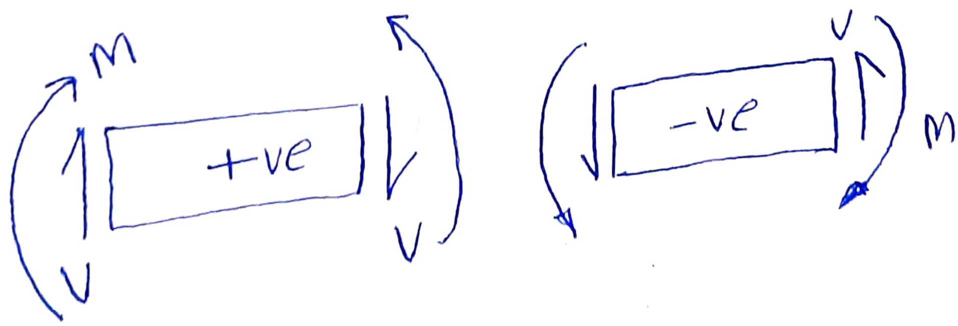




FBD of L.H.S (i) FBD of R.H.S. (ii)

According to Newtons Third Law

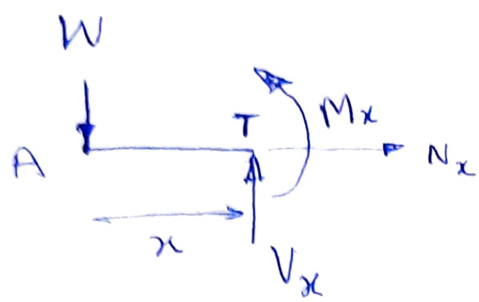
Sign Convention

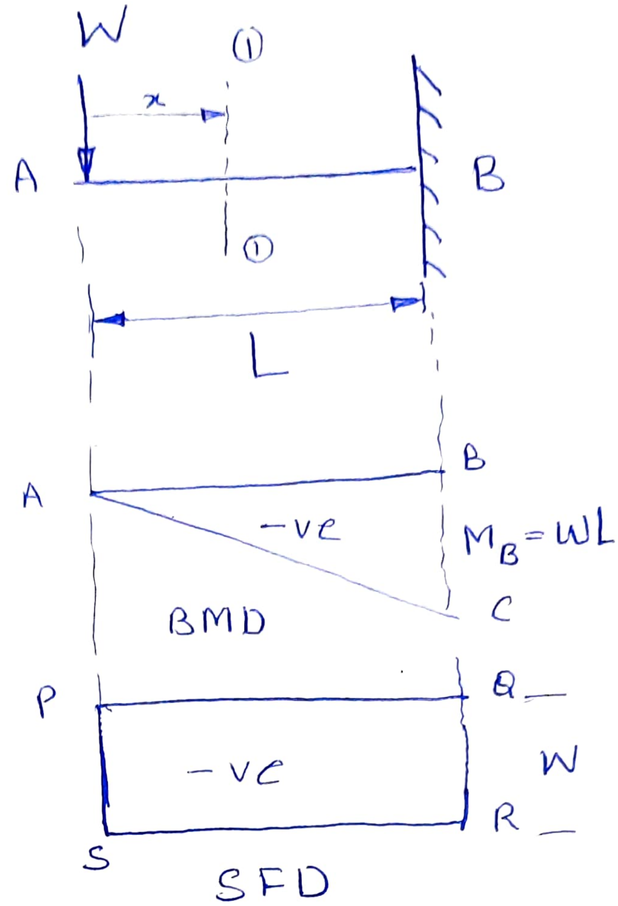


Example

$0 \leq x \leq L$

FBD of LHS





(1) $\Sigma F_x = 0$

$$N_x = 0 \quad \text{— (4)}$$

(2) $\Sigma F_y = 0$

$$W - V_x = 0 \quad \text{— [illegible]}$$

$$V_x = W \quad \text{-- (5)}$$

(3) ⇒ $\Sigma M_x = 0$

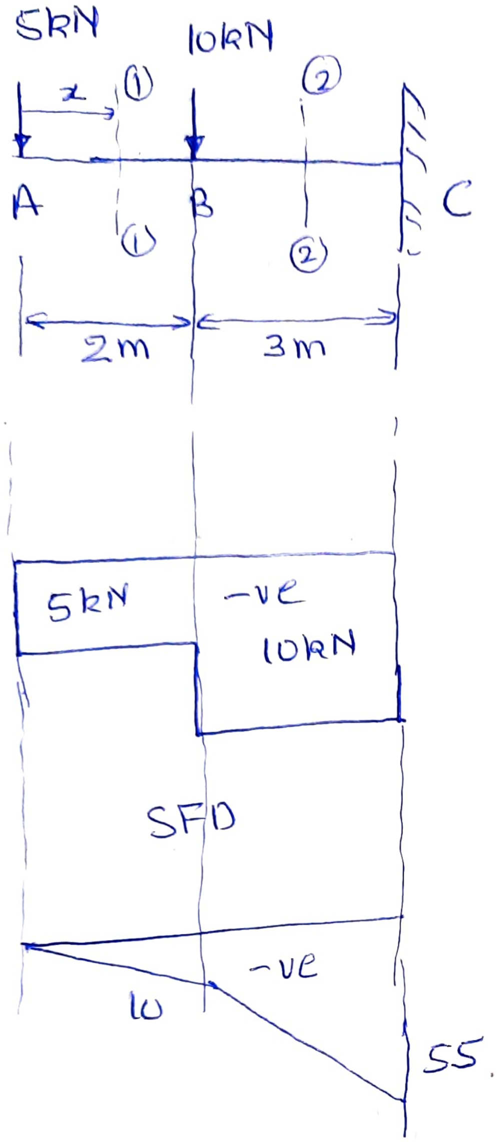
$$Wx + M_x = 0$$

$$M_x = -Wx \quad \text{— (6)}$$

$$\frac{dM_x}{dx} = -W$$

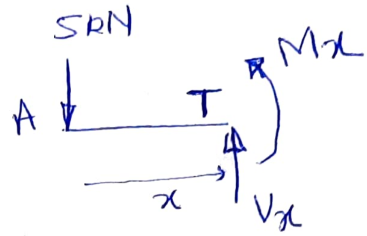
$$\frac{dN_x}{dx} = 0$$

Example



Taking section 1-1 at a distance x from A. Draw FBD

$0 \le x \le 2$



(2) ⇒ $V_x - 5 = 0$

$V_x = 5$ --- (4)

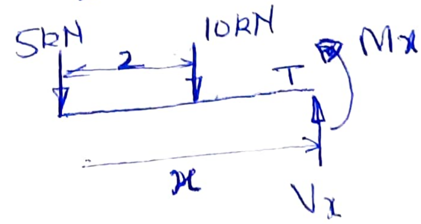
(3) ⇒ $\Sigma M_T = 0$

$5x + M_x = 0$

$M_x = -5x$ — (5)

section 2-2

$2 \le x \le 5$



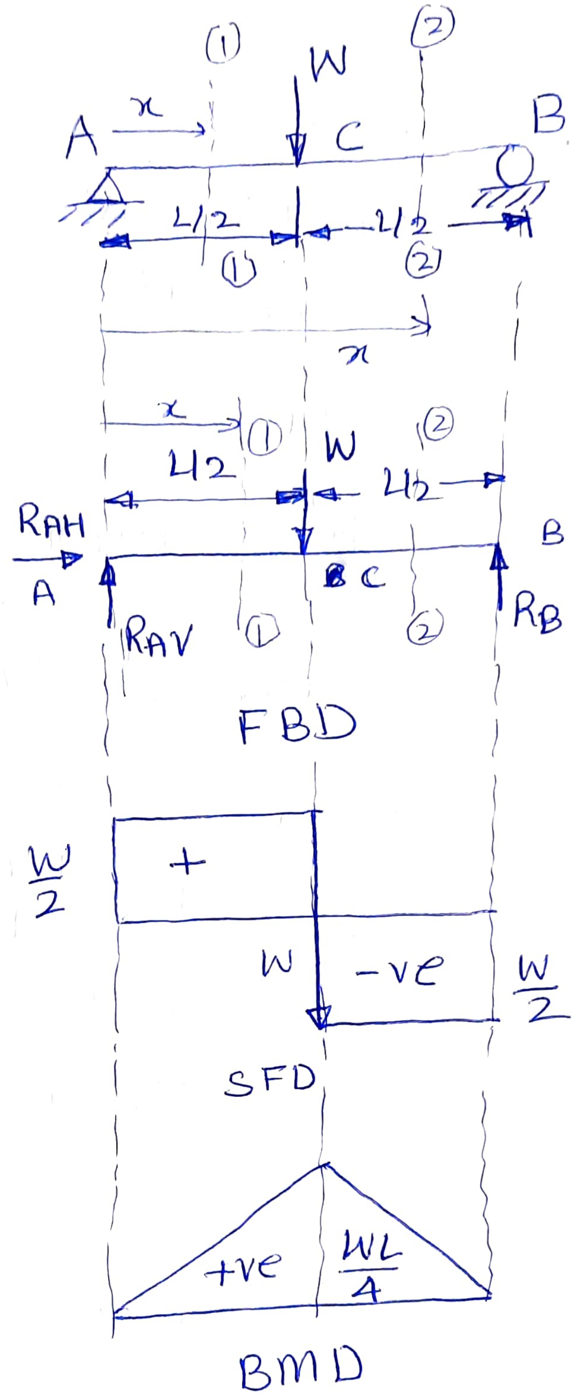
(2) ⇒ $V_x = 15$ — (6)

(3) ⇒ $\Sigma M_T = 0$

$5x + 10(x-2) + M_x = 0$

$M_x = -5x - 10(x-2)$ — (7)

Example: Simply supported Beam carrying A concentrated Load.



① Reaction

$\Sigma M_A = 0$

$R_B \times L - \frac{WL}{2} = 0$

$R_B = \frac{W}{2}$ — (4)

② ⇒

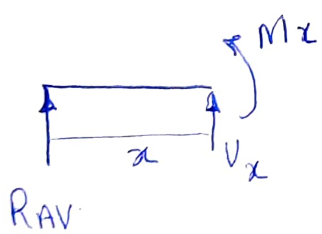
$R_{AV} + R_B = W$

$R_{AV} = \frac{W}{2}$ — (5)

① ⇒ $\Sigma F_x = 0$

$R_{AH} = 0$ (6)

Section ①-①



$0 \le x \le L/2$

$\Sigma M_{①-①} = 0$

$R_{AV}\, x - M_x = 0$

$M_x = \frac{W}{2} x$ — (7)

$\Sigma F_y = 0$

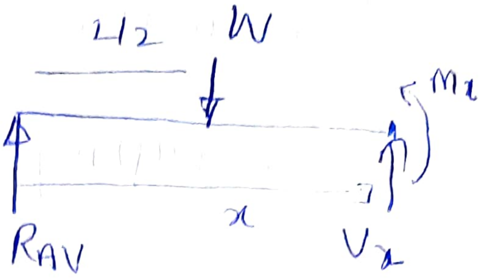
$R_{AV} + V_x = 0$

$V_x = -\frac{W}{2}$ — (8)

P.T.O

Section (2)-(2)

$\frac{L}{2} < x \le L$



$\Sigma F_y = 0$

$R_{AV} + V_x = W$

$V_x = \frac{W}{2}$

$\Sigma M_{(2)-(2)} = 0$

$\frac{W}{2} x - W\left(x - \frac{L}{2}\right) - M_x = 0$

$M_x = \frac{W}{2}(L - x)$

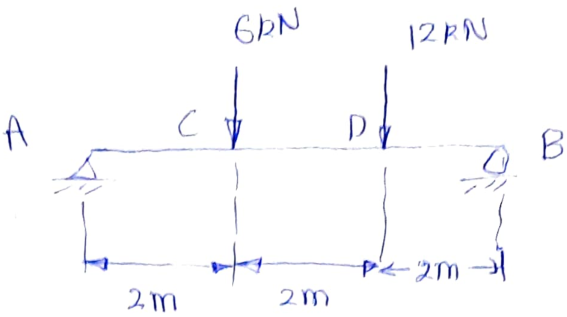
at $x = L/2$

$M_x = \frac{WL}{4}$

at $x = L$

$M_x = 0$

Example



Sol.

$\Sigma M_B = 0$

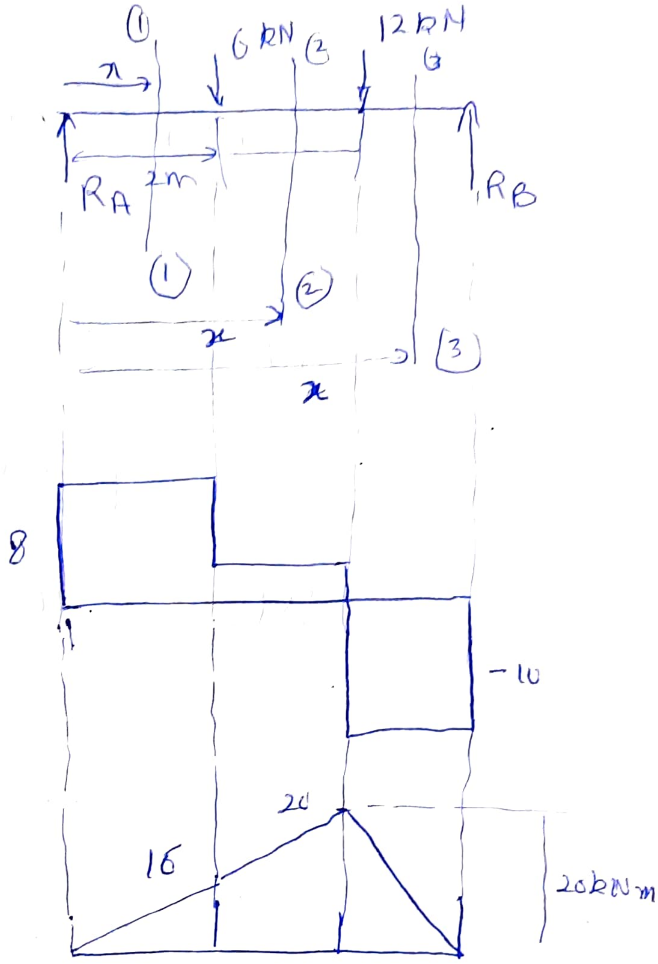
$R_A \times 6 - 6 \times 4 - 12 \times 2 = 0$

$R_A = 8$

$\Sigma F_y = 0$

$R_A + R_B = 6 + 12$

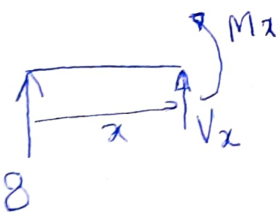
$R_B = 10$



BMD

Section 1-1

$0 \leq x \leq 2$



$\Sigma F_y = 0$

$8 + V_x = 0$

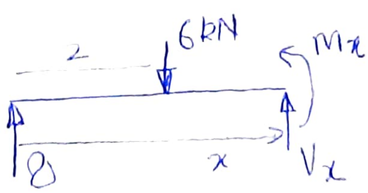
$V_x = -8$

$\Sigma M_{1-1} = 0$

$8x - M_x = 0$

$M_x = 8x$

Section 2-2



(2) $\Rightarrow$ $8 + V_x - 6 = 0$

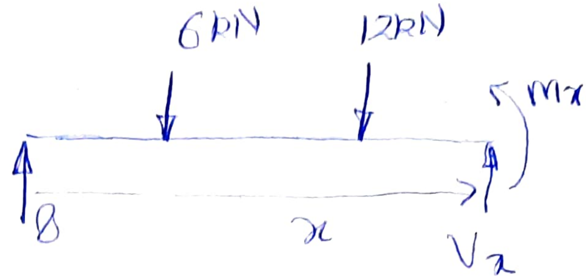
$V_x = -2$

(3) $\Sigma M_{2-2} = 0$

$8x - 6(x-2) - M_x = 0$

$M_x = 2x + 12$

Section (3)-(3)



$4 \le x \le 6$

$8 + V_x - 6 - 12 = 0$

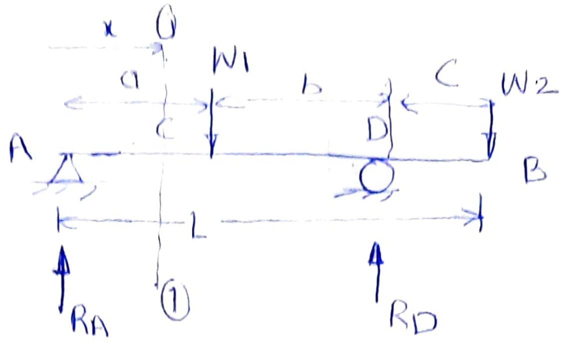
$V_x = 10 \quad (-ve)$

$\Sigma M_{(2)-(3)} = 0$

$8x - 6(x-2) - 12(x-4) - M_x = 0$

$M_x = -10x + 60$

SFD & BMD of Overhanging Beam



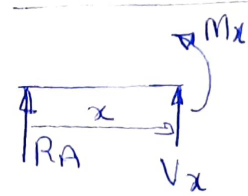
$\Sigma M_A = 0$

$W_1 a + W_2 L = R_D (a+b)$

$$R_D = \frac{W_1 a + W_2 L}{a+b} \quad \text{--- (1)}$$

$$R_A = W_1 + W_2 - R_D \quad \text{--- (2)}$$

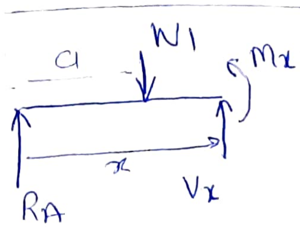
Section (1)-(1)



$0 \le x \le a$

$V_x = -R_A$

$M_x = R_A x$



section (2)-(2)

For Sec (2)-(2)

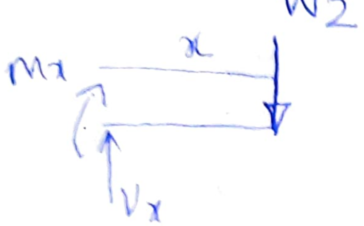
$R_A - W_1 + V_x = 0$

$V_x = -R_A + W_1$

$\Sigma M_{(2)-(2)} = 0$

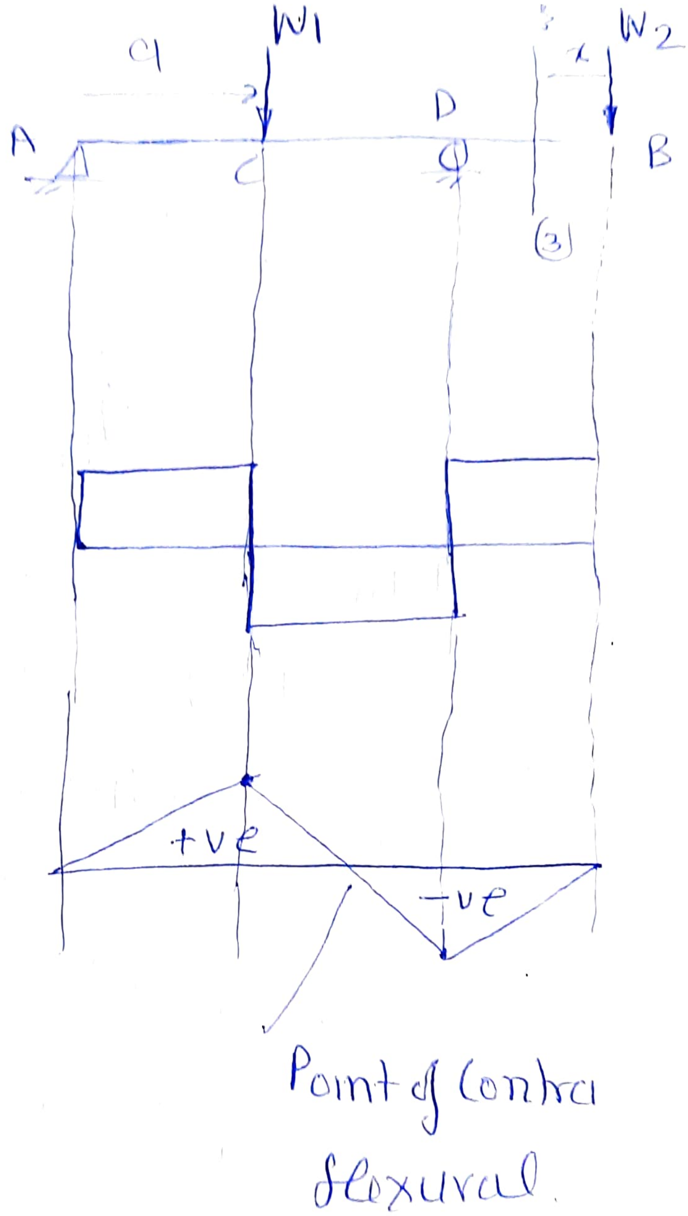
$M_x - R_A x + W_1 (x - a) = 0$

RHS Section 3-3

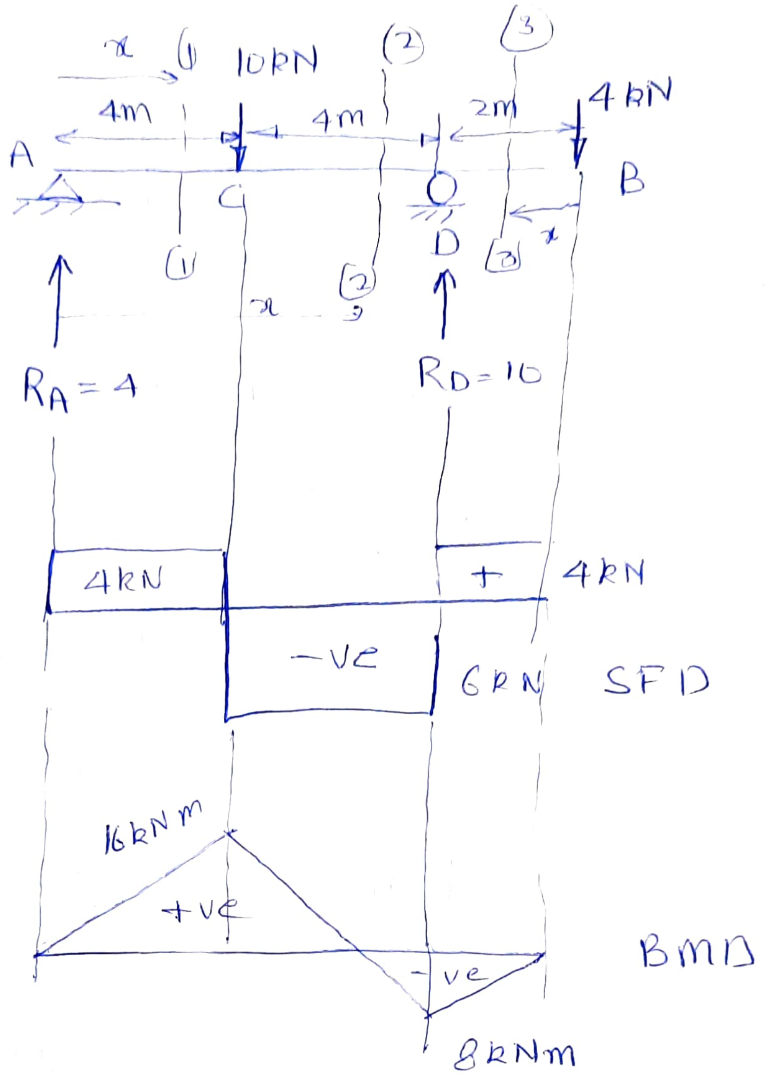


$V_x = W_2$

$M_x = W_2 C.$



Example



$\Sigma M_A = 0$

$10 \times 4 + 4 \times 10$

$= 8 R_D$

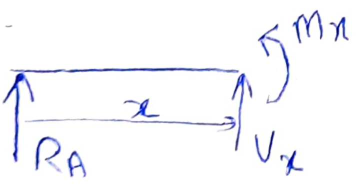
$R_D = 10\,kN$

$R_A + R_D = 14$

$R_A = 4\,kN$

Section (1) – (1)

LHS $0 \leq x \leq 4$



$R_A + V_x = 0$

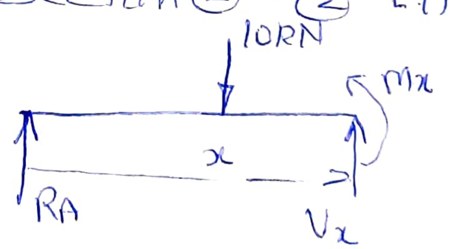
$V_x = -4\,kN$ (+ve)

$\Sigma M_{(1)-(1)} = 0$

$R_A x - M_x = 0$

$M_x = 4x$

Section (2) – (2) L.H.S



$R_A + V_x = 10$

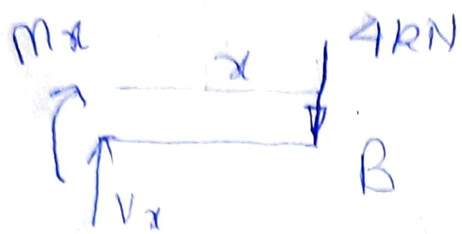
$V_x = 6$ — (A)

$\Sigma M_{(2)-(2)} = 0$

$R_A x - 10(x-4) - M_x = 0$

$M_x = -6x + 40$ — (B)

Section (3) - (3)



$V_x - 4 = 0$

$V_x = 4\,kN$ (+ve)

$M_x = 4x$ (-ve)

$M_x|_{x=4} = 16$

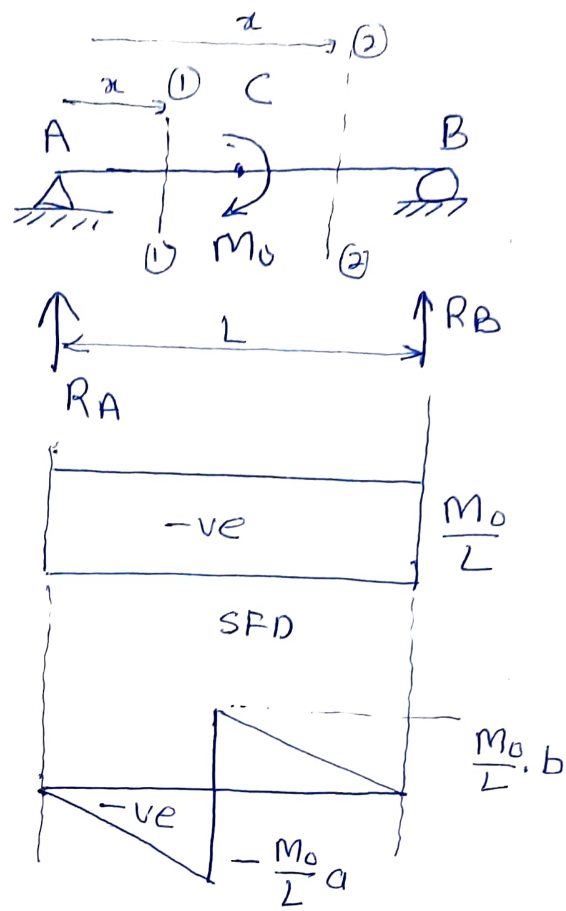
Point of contraflexure in portion CD.

From Eq (B) put $M_x = 0$

$-6x + 40 = 0$

$x = 6.667\,m$

Beam Subjected to Couple



$$\Sigma M_B = 0$$

$$R_A L + M_0 = 0$$

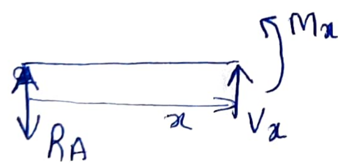
$$R_A = -\frac{M_0}{L}$$

$$R_A + R_B = 0$$

$$R_B = \frac{M_0}{L}$$

Section (1)-(1)

L.H.S, FBD



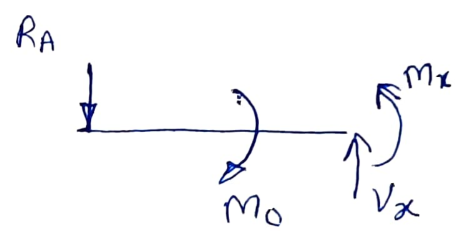
$$V_x - R_A = 0$$

$$V_x = R_A$$

$$\Sigma M_x = 0$$

$$R_A x + M_x = 0$$

$$M_x = -R_A x$$



$$V_x = R_A$$

$$\Sigma M_x = 0$$

$$R_A x + M_x - M_0 = 0$$

$$M_x = -R_A x + M_0$$

at $x = L$ $\quad M_x = 0$

at $x = a$

$$M_x = \frac{M_0 b}{L}$$

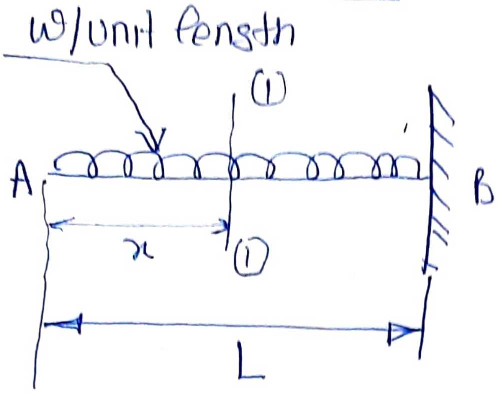
Uniformly distributed Load

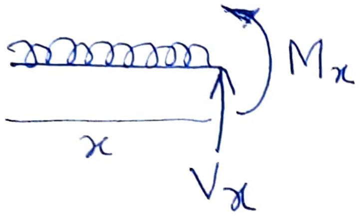
Section (1)-(1)

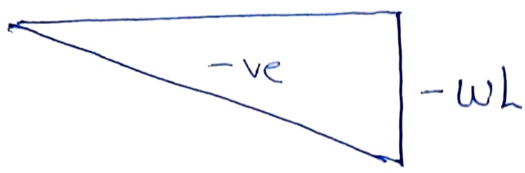
L.H.S. of beam

FBD

$0 \leq x \leq L$





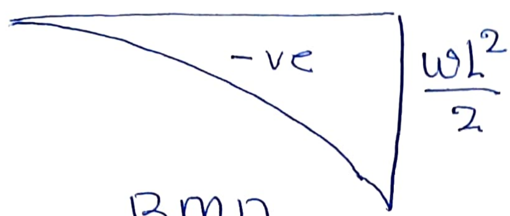


SFD

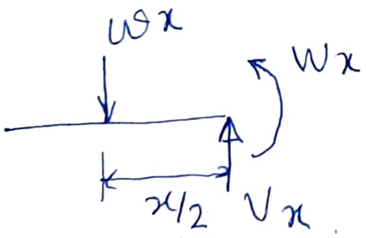
$$V_x - wx = 0$$

$$V_x = wx$$

(-ve)



BMD

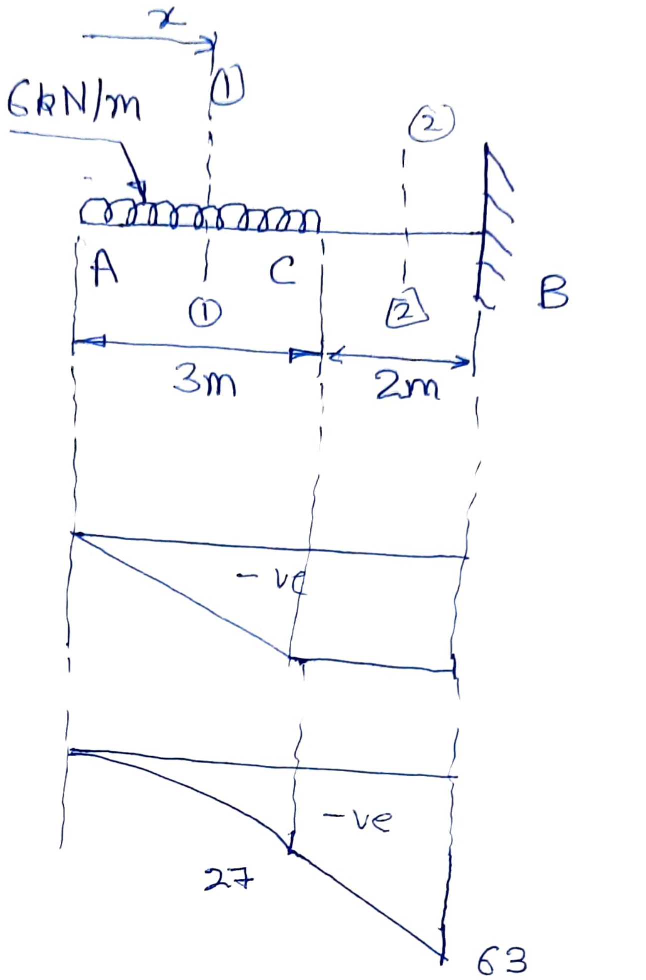


$$\Sigma M_{(1)-(1)} = 0$$

$$wx \cdot \frac{x}{2} + M_x = 0$$

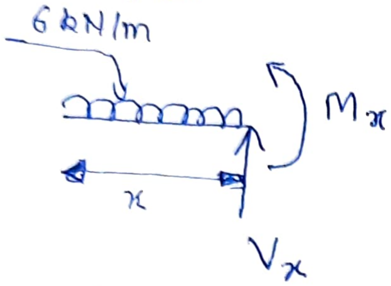
$$M_x = -\frac{wx^2}{2}$$

Example



Section 1-1

L.H.S.



$0 \leq x \leq 3$

$\Sigma F_y = 0$

$6x - V_x = 0$

$V_x = 6x$ — (4)

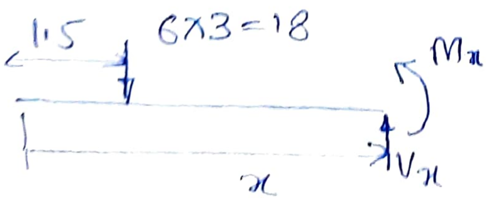
$\Sigma M_{1-1} = 0$

$6x \cdot \frac{x}{2} + M_x = 0$

$M_x = 3x^2$ — (5)

Section 2-2 LHS

$3 \leq x \leq 5$



$18 - V_x = 0$

$V_x = 18 (-ve)$

$M_x = -18(x - 1.5)$

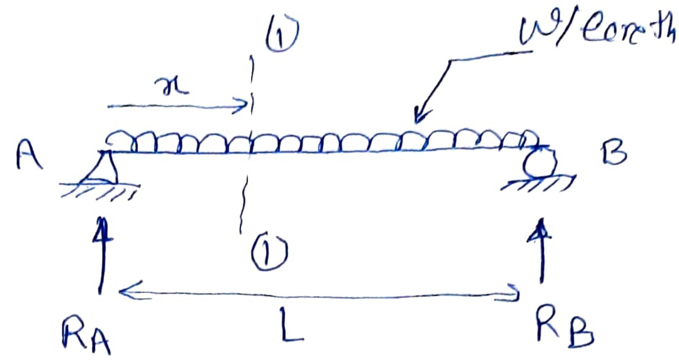
where $x = 3$

$M_x = -27$ N.m

$x = 5$

$M_x = -63$

Simply Supported Beam Carrying UDL



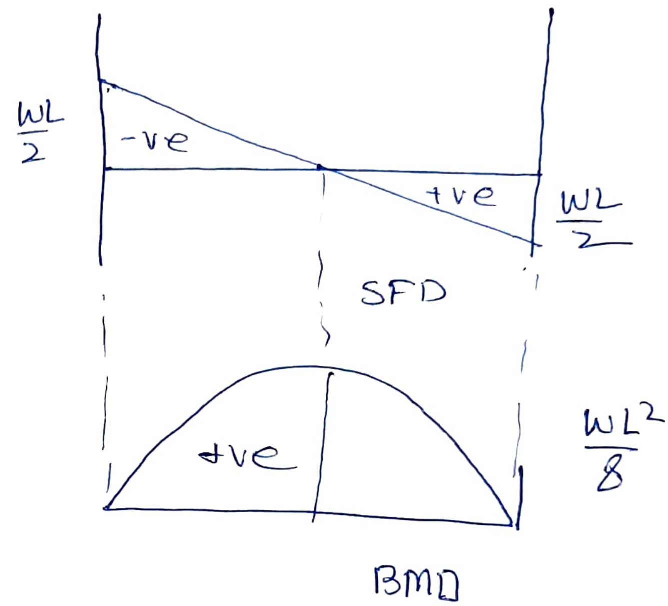
Reaction

$$R_A + R_B = wL$$

$$\Sigma M_B = 0$$

$$R_A L - wL \cdot \frac{L}{2} = 0$$

$$R_A = \frac{wL}{2}$$



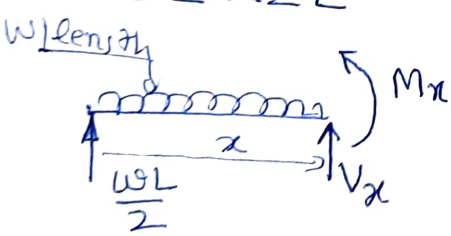
$$R_A + R_B = wL$$

$$R_B = \frac{wL}{2}$$

Section ① – ①

FBD of L.H.S.

$$0 \leq x \leq L$$



$$\frac{wL}{2} - wx + V_x = 0$$

$$V_x = wx - \frac{wL}{2}$$

at $x = 0$

$$V_x = -\frac{wL}{2} \quad (+ve)$$

at $x = L$

$$V_x = \frac{wL}{2} \quad (-ve)$$

$$\Sigma M_{①-①} = 0$$

$$\frac{wL}{2} \cdot x - wx \cdot \frac{x}{2} - M_x = 0$$

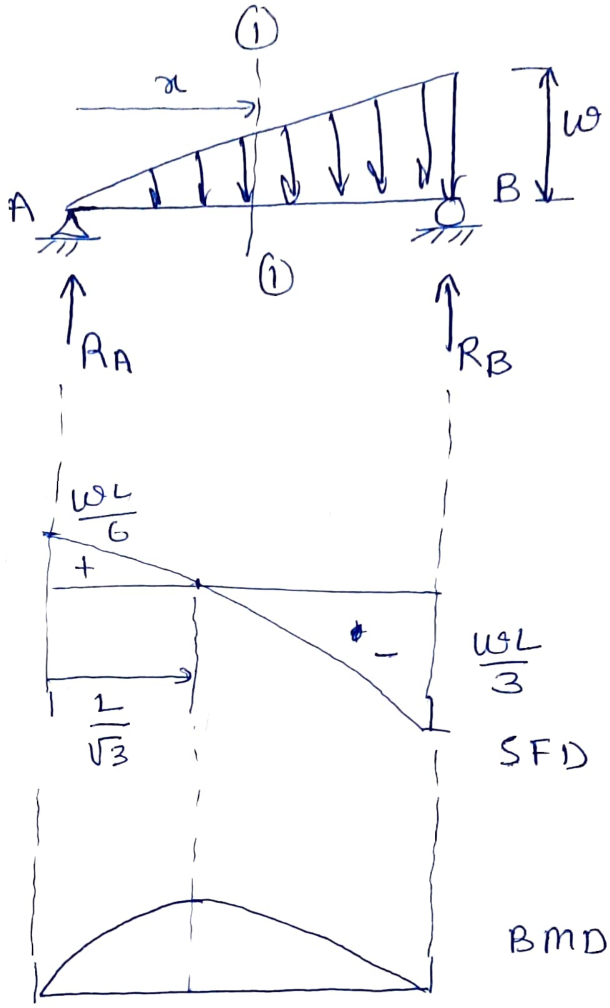
$$M_x = \frac{wL}{2} x - w\frac{x^2}{2}$$

at $x = 0, L$ $\quad M_x = 0$

at $x = \frac{L}{2}$

$$M_x = \frac{wL^2}{8}$$

Linearly Varying Load



Reaction

$\Sigma M_B = 0$

$R_A L - \frac{wL}{2} \times \frac{L}{3} = 0$

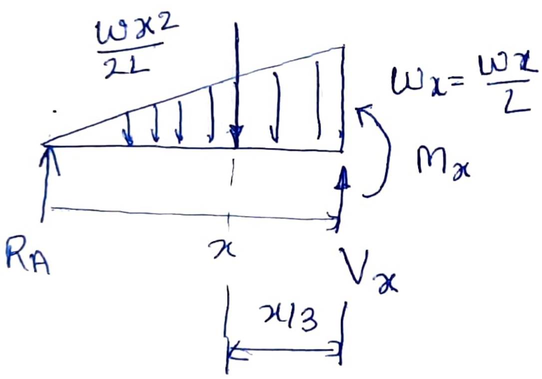
$R_A = \frac{wL}{6}$ —(4)

$R_A + R_B = \frac{wL}{2}$

$R_B = \frac{wL}{3}$ —(5)

Section ① − ①

LHS, FBD



② ⟹ $R_A + V_x - \frac{wx^2}{2L} = 0$

$V_x = \frac{wx^2}{2L} - \frac{wL}{6}$

at $x = 0$, $V_x = -\frac{wL}{6}$ (+ve)

at $x = L$ $V_x = \frac{wL}{3}$

$V_x = 0 \Rightarrow$

$x = L/\sqrt{3}$

③ ⟹

$M_x + \frac{wx^2}{2L} \cdot \frac{x}{3} - \frac{wL}{6} x = 0$

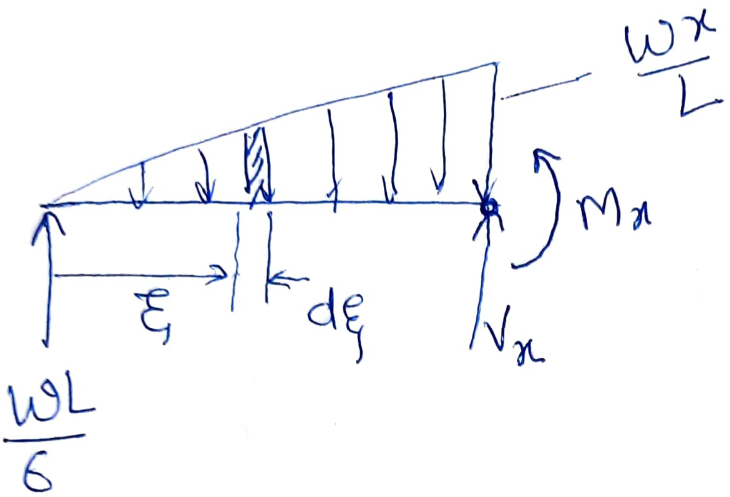
$M_x = \frac{wL}{6} x - \frac{wx^3}{6L}$

at $x = 0, L$

$M_x = 0$

at $x = L/\sqrt{3}$

$M_x = \frac{wL^2}{9\sqrt{3}}$



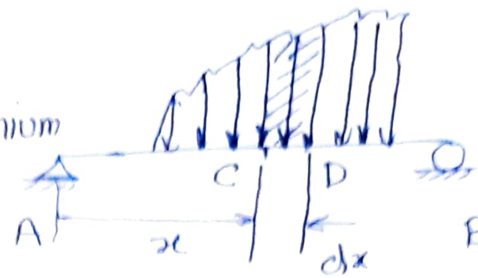
$$R_A x - \int_0^x \frac{w\xi}{L} \cdot d\xi \, (x-\xi) - M_x = 0$$

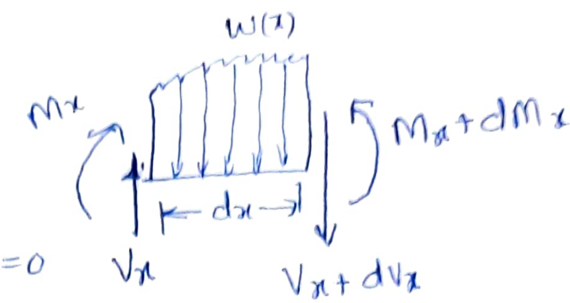
$$M_x = \frac{wL}{6} x - \frac{wx^3}{6L}$$

Relations between Rate of loading & SF & BM

$w(x)$

Considering Equilibrium of section CD Elements





$\Sigma F_y = 0$

$$V_x - (V_x + dV_x) - w\,dx = 0$$

$$dV_x = -w\,dx$$

$$\boxed{\frac{dV_x}{dx} = -w} \quad \text{—(1)}$$

$\Sigma M_D = 0$

$$M_x + dM_x - M_x - V_x \cdot dx + w(x) \cdot \cancel{dx \cdot \frac{dx}{2}}^{\,0} = 0$$

$$\boxed{\frac{dM_x}{dx} = V_x} \quad \text{—(2)}$$

From (1) is $w = 0$ $\quad F = \text{Constant}$

$$\int dM_x = \int F\,dx$$

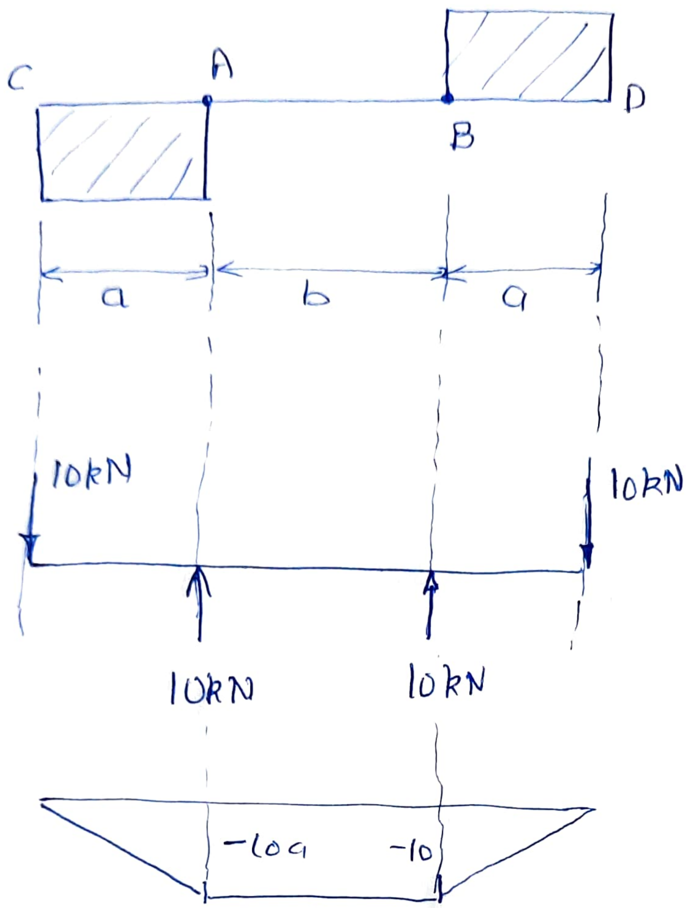
$$M = Fx + C$$

$$\frac{d^2M_x}{dx^2} = \frac{dV_x}{dx} = -w$$

$$F = -wx + C_1$$

$$M = -\frac{wx^2}{2} + C_1x + C_2$$

Ex.



$R_A = 10kN$ $\qquad$ $R_B = 10\,kN$

$M = Fx + C$