Module (3) – Radiation.

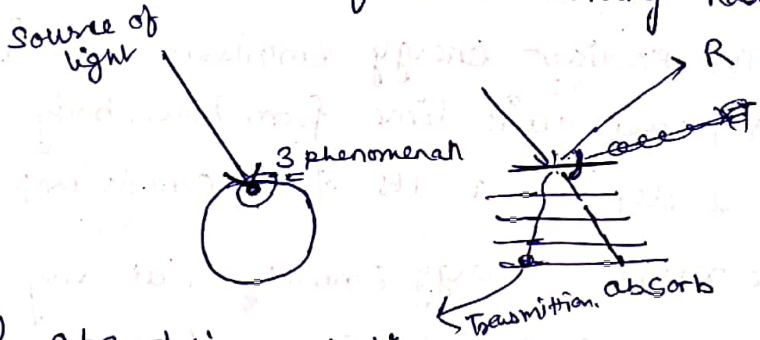
* **Radiation** :- Energy streaming through space at the speed of light, may originate in various ways.

Some types of material will emit radiation when they are treated by External agencies, such as electron bombardment, electric discharge, or radiation of definite wavelengths.

Radiation due to these effects will not be explain now.

In this topic we only discuss the classification / ~~of~~ mode of Radiation.

Fundamental facts concerning Radiation — "The process by which heat is transferred from a body by virtue of its temperature, without the aid of any intervening medium, is called thermal radiation."



1. absorbtion of light (α)
2. Reflection of light (γ)
3. transmittance of light (τ)

$$\boxed{\alpha + \gamma + \tau = 1} \quad \text{--- (1)}$$

Transmittivity depends on surface of object e.g. surface smoothing, absorbtion capability.

Radiation moves through space in straight lines, or beams and only substances in sight of a radiating body can intercept radiation from that body.

* The fraction of radiation falling onto a body that is reflected is called reflectivity (r).
* The fraction that is absorbed is called absorptivity (α). ~~and~~ and

* The fraction that is transmitted is called the transmissibility (τ).

$$\boxed{\tau + \alpha + r = 1}$$

Radiation as such is not heat, and when transferred into heat on absorption, it is no longer radiation.
In practicle sense, reflected or transmitted radiation usually falls on other absorptive bodies and is eventually converted into heat, perhaps after many successive reflection.

" The maximum possible absorptivity is unity, attained only if the body absorbs all radiation ~~incidente~~ incident upto it and reflected or transmitted none. A body that absorbs all incident radiation is called a block body."
This is a concept and ~~A.O~~ no such body exists in nature.
~~Black body~~

* Black body Radiation — The Radiant energy emmission per unit area and unit time from black body over all the wavelengths is defined as its total emmissive power and is denoted as E_b. The radiant energy emmission at any wavelength λ is known as monocromatic emmissive power and denoted as $E_{b\lambda}$. "Radiation of a single wavelength is called monochromatic." An actual beam of radiation consists of many monocromatic beams. Although radiation of any wavelength from zero to infinity is, in principle, convertible into heat on absorption by matter, the portion of electromagnetic spectrum that is of importance in heat flow lies in ~~m~~ the wave length range between 0.5 to 50 um.

The variation of monocromatic emmisive power with wavelength is given by plank's law (derived in 1900 using quantum theory).

monocromatic Emmisive power at any given wavelength and temp. is given by

$$E_{b\lambda}(T) = C_1 / \left(\lambda^5 \left(e^{C_2/\lambda T} - 1\right)\right) = \frac{C_1 \lambda^{-5}}{\left(e^{C_2/\lambda T} - 1\right)}$$

Where $E_{b\lambda}$ = monocromatic Emmisive poweat at wavelength λ and Temp T.

$E_{b\lambda}$ = $(W/m^2/m)$, λ = wavelength m.

C_1 = first radiation costant = 3.7415×10^{-16} $(W \cdot m^2)$

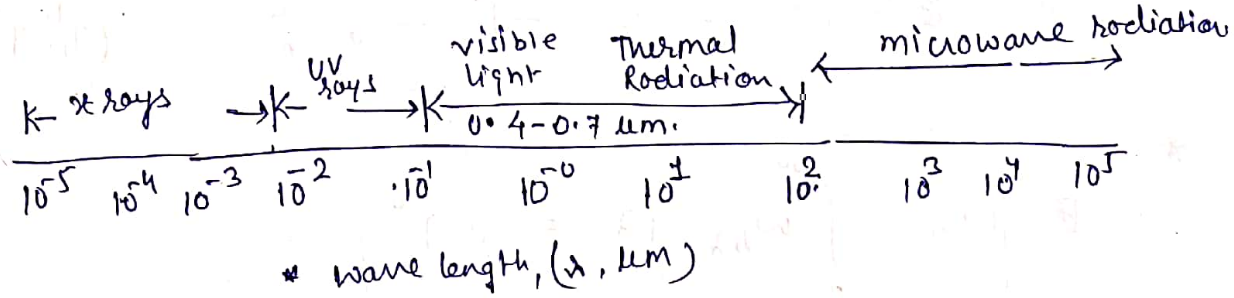
C_2 = second radiation costant = 1.4388×10^{-2} (mK)

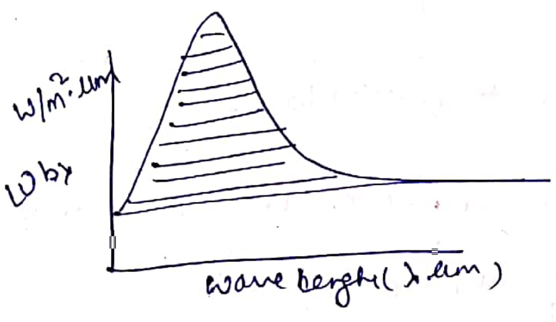
The monocromatic radiating power of a gray body of emmisivity 0.9 at 200°f.

At any given temperature, the maximum monochromatic Radiating Power is attained at a definite wavelength (λ_{max}). wien's displacement law states that λ_{max} is inversely proportional to the absolute temperature or.

$\boxed{\lambda_{max} \cdot T = C}$; C is 2890 when λ_{max} in (μm)

T is in Kelvines.





$$W_b = \int_0^\infty W_{b\lambda} \cdot d\lambda$$

Stefan-Boltzman Law – Law of Blackbody Radiation –

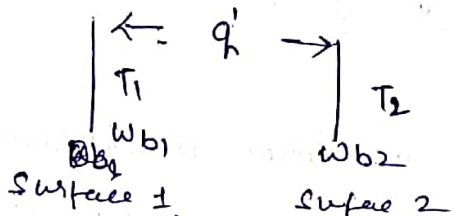
A basic relationship for blackbody Radiation is the Stefan-Boltzman law, which states that the total emmisive power of a blackbody is proportional to the fourth power of the absolute temperature –

$$\boxed{W_b = \sigma T_{abs}^4}$$

where σ is a universal constant depending on the units used to measure T & W_b; $\sigma = 5.67 \times 10^{-8} \left(W/m^2 K^4 \right)$

The Stefan-Boltzman law is ~~an~~ ~~Exact loading~~ an Exact Consequence of the laws of thermodynamics & electromagnetism.

*



~~The heat flux will be the net radiation~~

Consider two infinite plane surfaces, both facing each other. idea surfaces. T_1 & T_2 Temp. of both surface. both temperatures are at ~~abo~~ above absolute zero., both will Radiate energy as described by Stefan-Boltzman law.

The heat flux will be the net radiant flow as given by :-

$$\boxed{Q'' = W_{b1} - W_{b2} = \sigma T_1^4 - \sigma T_2^4 = \sigma \left(T_1^4 - T_2^4 \right)}$$

*

$$\boxed{W_b = \int_0^\infty W_{b\lambda} \cdot d\lambda = \sigma T^4}$$

after integrating Stefan-Boltzman & plank's law.

Relation between Emmisive power, monochromatic emmisive power and Temp. during Radiation in black body.

Real surface emmisivity. –

S-B. law – $W_b = \sigma . T^4$. —(1) for ideal surface.

if real surface having Emmisive power W, $\bar{\omega}$ is less than that obtained theoritically by boltzman W_b. To account for this reduction we introduce Emissivity (ε).

$$\cancel{\boxed{\varepsilon = \frac{W_b}{\varepsilon W_b}}} \quad \text{—}\cancel{(2)} \qquad \boxed{\varepsilon = \frac{W}{W_b}} \quad \text{—}(2)$$

So that the Emmisive power from any real surface is given by

$$\boxed{W = \varepsilon . \sigma T^4} \quad \text{—}(3)$$

* Real surface emmisivity. –

S-B. law – $W_b = \sigma . T^4$. —(1) for ideal surface.

If real surface having Emmisive power W, $\bar{\omega}$ is less than that obtained theoritically by boltzman W_b. To account for this reduction we introduce Emissivity (ε).

$$\boxed{\cancel{\varepsilon = \frac{W_b}{\varepsilon W_b}}} \quad \cancel{—(2)} \qquad \boxed{\varepsilon = \frac{W}{W_b}} \quad —(2)$$

So that the Emmisive power from any real surface is given by

$$\boxed{W = \varepsilon . \sigma T^4} \quad —(3)$$

* Black body – with in visual band of radiation, any material which absorbs all visible light, appears as black.

Extending this concept to much boarder thermal band. and surface with zero transmittance & zero reflection is called "black" or "thermally black body."

* we, know that for Radiation over a surface –

$$\alpha + r + \tau = 1 ;$$

$\tau = 0, \gamma = 0$ then

$\boxed{\alpha = 1}$ black body.

if system with some transmittivity & reflectivity or any one than it is called Grey body.

Scanned with CamScanner

* View-factor from one to other surface:-

Concept of solar Radiation and view-factor–

The magnitude of the Energy the sun varies with time and is closely associated with such factors as solar flares and sunspots. The energy leaving the sun is emitted outward in all directions, so that at any particular distance from the sun we may imagine the energy being dispersed over an imaginary spherical area.

At average distance between Earth & sun having heat flux 1353 W/m^2.

* Surface to sun view angle –

$$G_{so} = S_c \cdot f \cdot \cos\theta \quad \text{—(1)}$$

S_c = Solar constant, 1353 W/m^2

f = correction factor for eccentricity in Earth orbit ($0.97 < f < 1.03$)

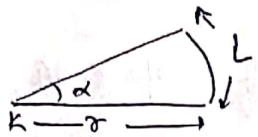
θ = Angle of surface from sun normal to sun.

Because of reflection and absorption in Earth's atmosphere, this no. is significantly reduced at ground level.

This value eqn (1) also give us the some different point of view to estimate the potential for solar energy also, such as in photovoltaic cells.

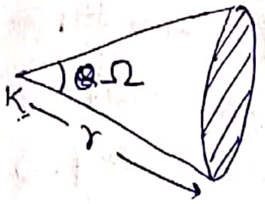
Scanned with CamScanner

Angle & arc length — ① The angle as two dimentional object



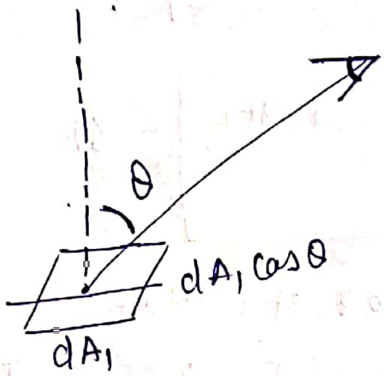
$\boxed{L = r \cdot \alpha}$ —①

The arc length is fined as eqⁿ ①



② Solid angle — An angle and an arc length to three dimensions and define a solid angle Ω, with no direction. ~~The solid angle~~. Area

$$\boxed{A = r^2 \cdot d\Omega} \quad \text{—②}$$



③ projected area — The area dA_1, from prospective of viewer. Angle θ from normal to the surface. This smaller area is termed the projected area.

$$\boxed{A_{projected} = \cos\theta \cdot A_{normal}} \quad \text{—③}$$

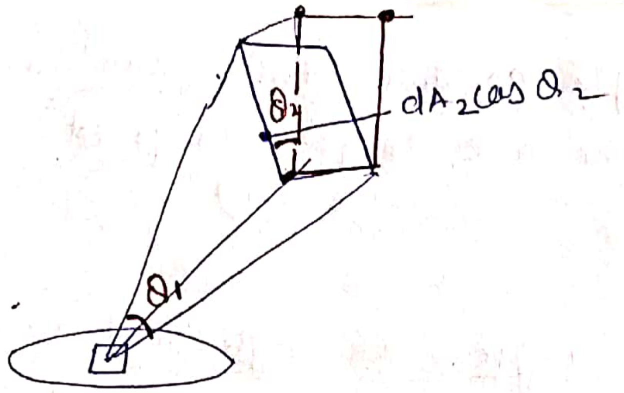
* Intensity & Radiation Exchange — The ideal intensity I_b, may be defined as the Energy emitted from an ideal body per unit projected area per unit time, per unit solid angle:

$$\therefore \quad \boxed{I = \frac{dq}{\cos\theta \cdot dA_1 \cdot d\Omega}} \quad \text{—④}$$

rearrange eqⁿ ④ $\boxed{dq = I \cdot \cos\theta \cdot dA_1 \cdot d\Omega}$

*

Scanned with CamScanner



Differential areas for Radiation.

* Project receiving surface onto the hemisphere surrounding the source.

Projected area of surface dA_2 = $dA_2 \cdot \cos\theta_2$

Projected area of surface dA_1 = $dA_1 \cdot \cos\theta_1$

Heat flow in hemisphere surrounding ⇒

$$\boxed{dq = \frac{I \cdot \cos\theta_1 \, dA_1 \cdot \cos\theta_2 \cdot dA_2}{R^2}} \quad - (5)$$

after integrating

$$\boxed{q_{1-2} = \int_{A_2}\int_{A_1} \frac{I \cdot \cos\theta_1 \cdot dA_1 \cdot \cos\theta_2 \, dA_2}{R^2}} \quad - (6)$$

To express the total energy emitted from surface 1, It will be recall the ~~beta~~ relations betⁿ emissive power E & intensity I

$$\boxed{q_{emitted} = E_1 \cdot A_1 = I_1 \cdot A_1 \cdot \pi} \quad — (7)$$

view factors — Define the view factor, F_{1-2} as the fraction of Energy Emitted from surface 1. to striked surface by eqⁿ (6) & (7)

$$F_{1-2} = \frac{q_{1-2}}{q_{emitted}} = \frac{\int_{A_2}\int_{A_1} \frac{I \cdot \cos\theta_1 \, dA_1 \cos\theta_2 \cdot dA_2}{R^2}}{\pi \cdot I \cdot A_1}$$

$$\boxed{F_{12} = \frac{1}{A_1}\int_{A_2}\int_{A_1} \frac{\cos\theta_1 \cos\theta_2 \, dA_2 \, dA_1}{\pi R^2}} \quad — (8)$$

MODULE I

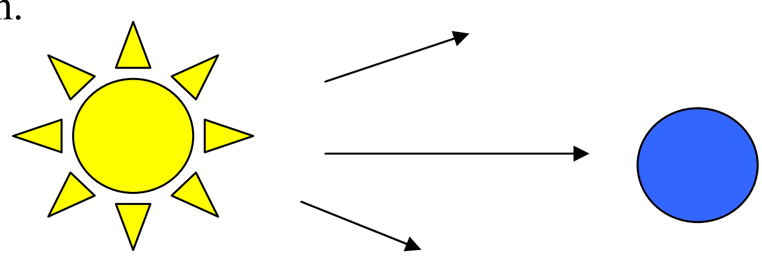
RADIATION HEAT TRANSFER

Radiation

Definition

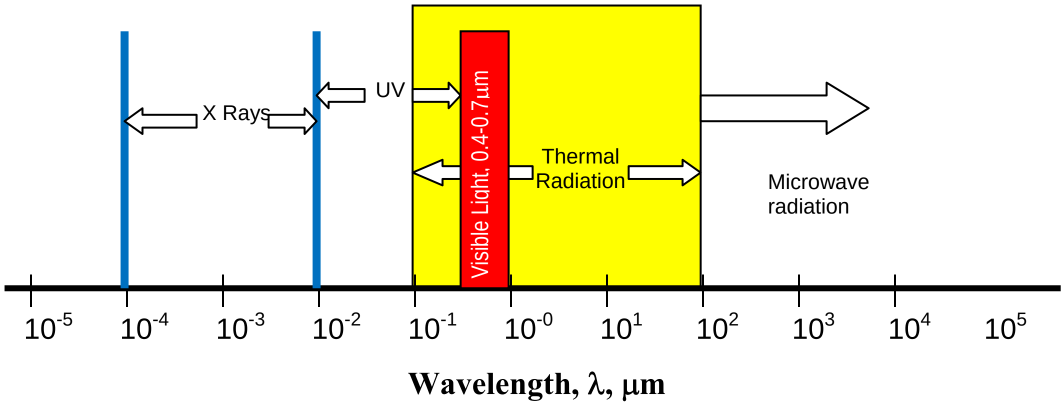
Radiation, energy transfer across a system boundary due to a ΔT, by the mechanism of photon emission or electromagnetic wave emission.

Because the mechanism of transmission is photon emission, unlike conduction and convection, there need be no intermediate matter to enable transmission.



The significance of this is that radiation will be the only mechanism for heat transfer whenever a vacuum is present.

Electromagnetic Phenomena.
We are well acquainted with a wide range of electromagnetic phenomena in modern life. These phenomena are sometimes thought of as wave phenomena and are, consequently, often described in terms of electromagnetic wave length, λ. Examples are given in terms of the wave distribution shown below:



One aspect of electromagnetic radiation is that the related topics are more closely associated with optics and electronics than with those normally found in mechanical engineering courses. Nevertheless, these are widely encountered topics and the student is familiar with them through every day life experiences.

From a viewpoint of previously studied topics students, particularly those with a background in mechanical or chemical engineering, will find the subject of Radiation Heat Transfer a little unusual. The physics background differs fundamentally from that found in the areas of continuum mechanics. Much of the related material is found in courses more closely identified with quantum physics or electrical engineering, i.e. Fields and Waves. At this point, it is important for us to recognize that since the subject arises from a different area of physics, it will be important that we study these concepts with extra care.

Stefan-Boltzman Law

Both Stefan and Boltzman were physicists; any student taking a course in quantum physics will become well acquainted with Boltzman's work as he made a number of important contributions to the field. Both were contemporaries of Einstein so we see that the subject is of fairly recent vintage. (Recall that the basic equation for convection heat transfer is attributed to Newton.)

$$E_b = \sigma \cdot T_{abs}^4$$

where: E_b = Emissive Power, the gross energy emitted from an ideal surface per unit area, time.

σ = Stefan Boltzman constant, $5.67 \cdot 10^{-8}$ W/m$^2 \cdot$K^4

T_{abs} = Absolute temperature of the emitting surface, K.

Take particular note of the fact that absolute temperatures are used in Radiation. It is suggested, as a matter of good practice, to convert all temperatures to the absolute scale as an initial step in all radiation problems.

You will notice that the equation does not include any heat flux term, q". Instead we have a term the emissive power. The relationship between these terms is as follows. Consider two infinite plane surfaces, both facing one another. Both surfaces are ideal surfaces. One surface is found to be at temperature, T_1, the other at temperature, T_2. Since both temperatures are at temperatures above absolute zero, both will radiate energy as described by the Stefan-Boltzman law. The heat flux will be the net radiant flow as given by:

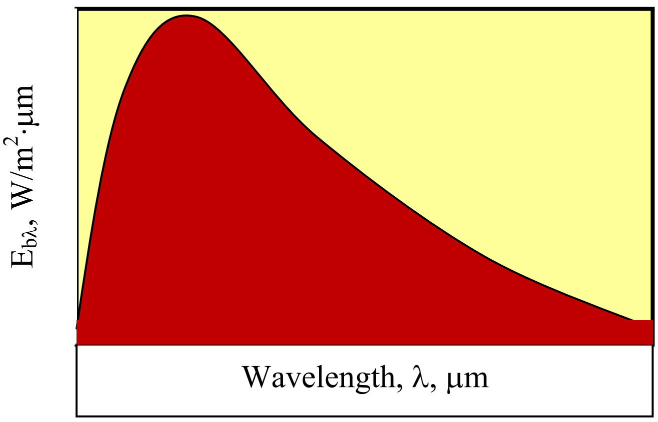
$$q'' = E_{b1} - E_{b2} = \sigma \cdot T_1^4 - \sigma \cdot T_2^4$$

Plank's Law

While the Stefan-Boltzman law is useful for studying overall energy emissions, it does not allow us to treat those interactions, which deal specifically with wavelength, λ. This problem was overcome by another of the modern physicists, Max Plank, who developed a relationship for wave-based emissions.

$$E_{b\lambda} = f(\lambda)$$

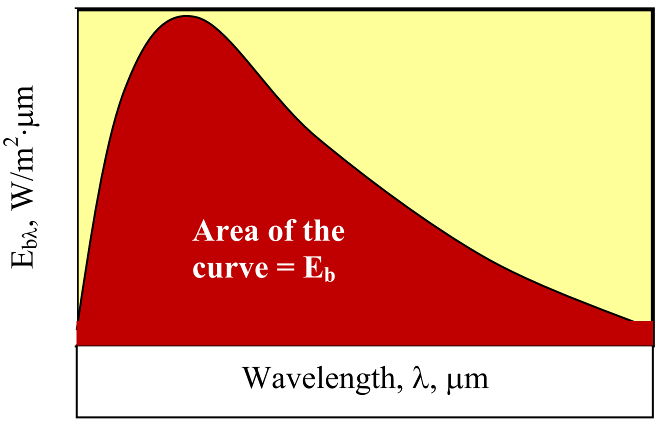
We plot a suitable functional relationship below:



We haven't yet defined the Monochromatic Emissive Power, $E_{b\lambda}$. An implicit definition is provided by the following equation:

$$E_b = \int_0^{\infty} E_{b\lambda} \cdot d\lambda$$

We may view this equation graphically as follows:



A definition of monochromatic Emissive Power would be obtained by differentiating the integral equation:

$$E_{b\lambda} \equiv \frac{dE_b}{d\lambda}$$

The actual form of Plank's law is:

$$E_{b\lambda} = \frac{C_1}{\lambda^5 \cdot \left[e^{C_2 / \lambda \cdot T} - 1 \right]}$$

where: $C_1 = 2 \cdot \pi \cdot h \cdot c_o^{\,2} = 3.742 \cdot 10^8\ \mathrm{W \cdot \mu m^4 / m^2}$
$C_2 = h \cdot c_o / k = 1.439 \cdot 10^4\ \mathrm{\mu m \cdot K}$

Where: h, c_o, k are all parameters from quantum physics. We need not worry about their precise definition here.

This equation may be solved at any T, λ to give the value of the monochromatic emissivity at that condition. Alternatively, the function may be substituted into the integral $E_b = \int_0^\infty E_{b\lambda} \cdot d\lambda$ to find the Emissive power for any temperature. While performing this integral by hand is difficult, students may readily evaluate the integral through one of several computer programs, i.e. MathCad, Maple, Mathmatica, etc.

$$E_b = \int_0^\infty E_{b\lambda} \cdot d\lambda = \sigma \cdot T^4$$

Emission Over Specific Wave Length Bands

Consider the problem of designing a tanning machine. As a part of the machine, we will need to design a very powerful incandescent light source. We may wish to know how much energy is being emitted over the ultraviolet band (10^{-4} to 0.4 μm), known to be particularly dangerous.

$$E_b(0.0001 \rightarrow 0.4) = \int_{0.001 \cdot \mu m}^{0.4 \cdot \mu m} E_{b\lambda} \cdot d\lambda$$

With a computer available, evaluation of this integral is rather trivial. Alternatively, the text books provide a table of integrals. The format used is as follows:

$$\frac{E_b(0.001 \to 0.4)}{E_b} = \frac{\int_{0.001\cdot\mu m}^{0.4\cdot\mu m} E_{b\lambda}\cdot d\lambda}{\int_0^\infty E_{b\lambda}\cdot d\lambda} = \frac{\int_0^{0.4\cdot\mu m} E_{b\lambda}\cdot d\lambda}{\int_0^\infty E_{b\lambda}\cdot d\lambda} - \frac{\int_{0.}^{0.0001\cdot\mu m} E_{b\lambda}\cdot d\lambda}{\int_0^\infty E_{b\lambda}\cdot d\lambda} = F(0 \to 0.4) - F(0 \to 0.0001)$$

Referring to such tables, we see the last two functions listed in the second column. In the first column is a parameter, $\lambda\cdot T$. This is found by taking the product of the absolute temperature of the emitting surface, T, and the upper limit wave length, λ. In our example, suppose that the incandescent bulb is designed to operate at a temperature of 2000K. Reading from the table:

λ, μm	T, K	$\lambda\cdot T$, $\mu m\cdot K$	$F(0\to\lambda)$
0.0001	2000	0.2	0
0.4	2000	600	0.000014
F(0.4→0.0001μm) = F(0→0.4μm)- F(0→0.0001μm)			**0.000014**

This is the fraction of the total energy emitted which falls within the IR band. To find the absolute energy emitted multiply this value times the total energy emitted:

$$E_{bIR} = F(0.4\to 0.0001\mu m)\cdot\sigma\cdot T^4 = 0.000014\cdot 5.67\cdot 10^{-8}\cdot 2000^4 = \mathbf{12.7\ W/m^2}$$

Solar Radiation

The magnitude of the energy leaving the Sun varies with time and is closely associated with such factors as solar flares and sunspots. Nevertheless, we often choose to work with an average value. The energy leaving the sun is emitted outward in all directions so that at any particular distance from the Sun we may imagine the energy being dispersed over an imaginary spherical area. Because this area increases with the distance squared, the solar flux also decreases with the distance squared. At the average distance between Earth and Sun this heat flux is 1353 W/m^2, so that the average heat flux on any object in Earth orbit is found as:

$$G_{s,o} = S_c \cdot f \cdot \cos\theta$$

Where S_c = Solar Constant, 1353 W/m^2

f = correction factor for eccentricity in Earth Orbit, ($0.97<f<1.03$)

θ = Angle of surface from normal to Sun.

Because of reflection and absorption in the Earth's atmosphere, this number is significantly reduced at ground level. Nevertheless, this value gives us some opportunity to estimate the potential for using solar energy, such as in photovoltaic cells.

Some Definitions

In the previous section we introduced the Stefan-Boltzman Equation to describe radiation from an ideal surface.

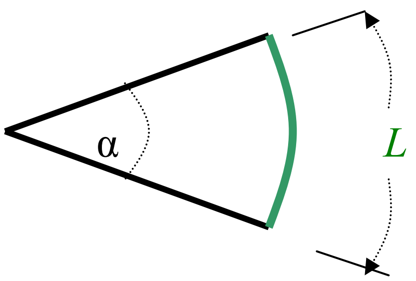
$$E_b = \sigma \cdot T_{abs}^{\ 4}$$

This equation provides a method of determining the total energy leaving a surface, but gives no indication of the direction in which it travels. As we continue our study, we will want to be able to calculate how heat is distributed among various objects.

For this purpose, we will introduce the radiation intensity, I, defined as the energy emitted per unit area, per unit time, per unit solid angle. Before writing an equation for this new property, we will need to define some of the terms we will be using.

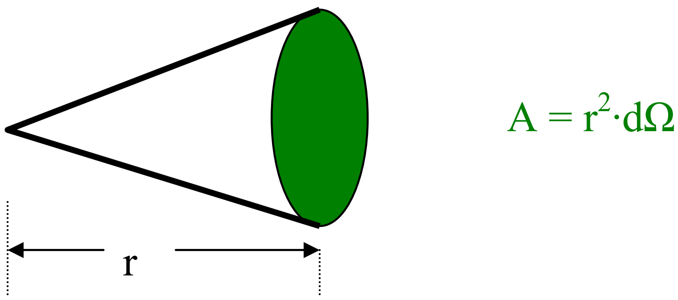
Angles and Arc Length

We are well accustomed to thinking of an angle as a two dimensional object. It may be used to find an arc length:



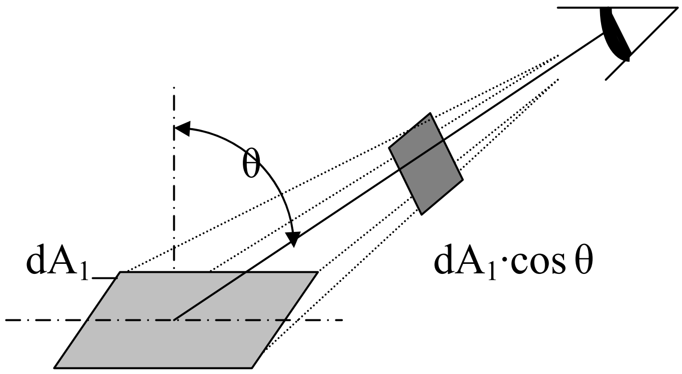
$$L = r \cdot \alpha$$

Solid Angle



We generalize the idea of an angle and an arc length to three dimensions and define a solid angle, Ω, which like the standard angle has no dimensions. The solid angle, when multiplied by the radius squared will have dimensions of length squared, or area, and will have the magnitude of the encompassed area.

Projected Area



The area, dA_1, as seen from the prospective of a viewer, situated at an angle θ from the normal to the surface, will appear somewhat smaller, as $\cos\theta \cdot dA_1$. This smaller area is termed the projected area.

$$A_{projected} = \cos\theta \cdot A_{normal}$$

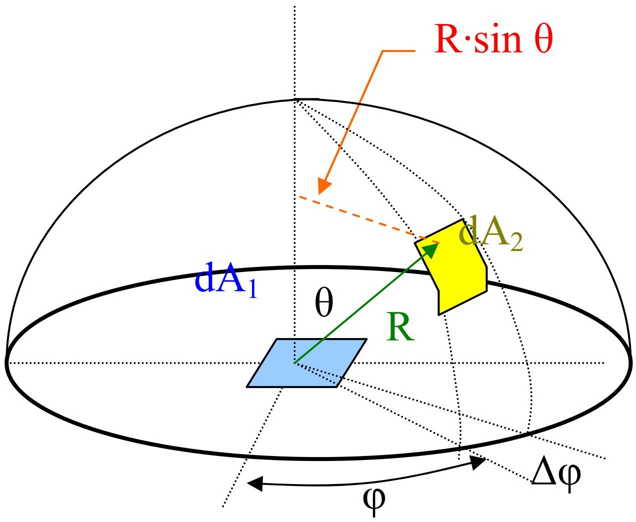
Intensity

The ideal intensity, I_b, may now be defined as the energy emitted from an ideal body, per unit projected area, per unit time, per unit solid angle.

$$I = \frac{dq}{\cos\theta \cdot dA_1 \cdot d\Omega}$$

Spherical Geometry

Since any surface will emit radiation outward in all directions above the surface, the spherical coordinate system provides a convenient tool for analysis. The three basic coordinates shown are R, φ, and θ, representing the radial, azimuthal and zenith directions.



In general dA_1 will correspond to the emitting surface or the source. The surface dA_2 will correspond to the receiving surface or the target. Note that the area proscribed on the hemisphere, dA_2, may be written as:

$$dA_2 = [(R \cdot \sin\theta) \cdot d\varphi] \cdot [R \cdot d\theta]$$

or, more simply as:

$$dA_2 = R^2 \cdot \sin\theta \cdot d\varphi \cdot d\theta]$$

Recalling the definition of the solid angle,

$$dA = R^2 \cdot d\Omega$$

we find that:

$$d\Omega = R^2 \cdot \sin\theta \cdot d\theta \cdot d\varphi$$

Real Surfaces

Thus far we have spoken of ideal surfaces, i.e. those that emit energy according to the Stefan-Boltzman law:

$$E_b = \sigma \cdot T_{abs}^4$$

Real surfaces have emissive powers, E, which are somewhat less than that obtained theoretically by Boltzman. To account for this reduction, we introduce the emissivity, ε.

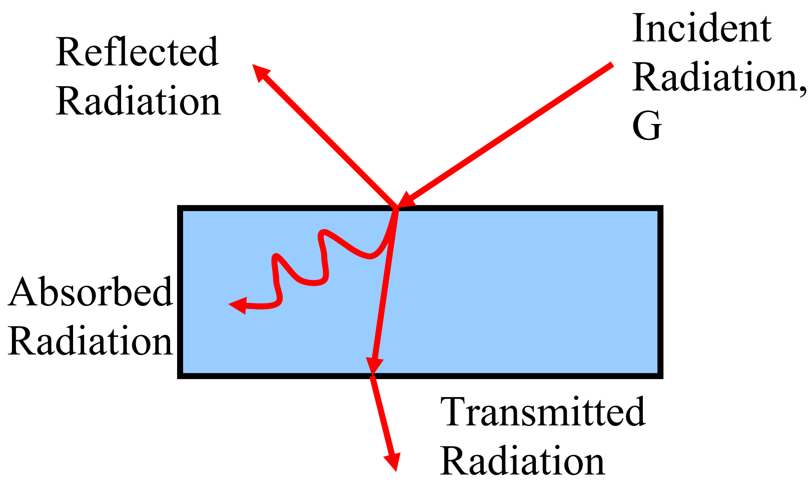
$$\varepsilon \equiv \frac{E}{E_b}$$

so that the emissive power from any real surface is given by:

$$\boxed{E = \varepsilon \cdot \sigma \cdot T_{abs}^{4}}$$

Receiving Properties

Targets receive radiation in one of three ways; they absorption, reflection or transmission. To account for these characteristics, we introduce three additional properties:



- Absorptivity, α, the fraction of incident radiation absorbed.
- Reflectivity, ρ, the fraction of incident radiation reflected.
- Transmissivity, τ, the fraction of incident radiation transmitted.

We see, from Conservation of Energy, that:

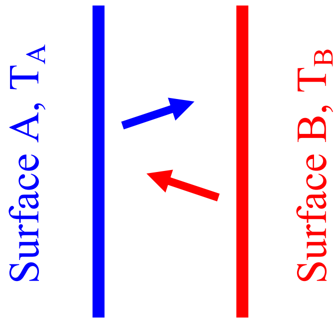
$$\alpha + \rho + \tau = 1$$

In this course, we will deal with only opaque surfaces, $\tau = 0$, so that:

$$\alpha + \rho = 1$$

Opaque Surfaces

Relationship Between Absorptivity,α, and Emissivity,ε



Consider two flat, infinite planes, surface A and surface B, both emitting radiation toward one another. Surface B is assumed to be an ideal emitter, i.e. $\varepsilon_B = 1.0$. Surface A will emit radiation according to the Stefan-Boltzman law as:

$$E_A = \varepsilon_A \cdot \sigma \cdot T_A^4$$

and will receive radiation as:

$$G_A = \alpha_A \cdot \sigma \cdot T_B^4$$

The net heat flow from surface A will be:

$$q'' = \varepsilon_A \cdot \sigma \cdot T_A^4 - \alpha_A \cdot \sigma \cdot T_B^4$$

Now suppose that the two surfaces are at exactly the same temperature. The heat flow must be zero according to the 2nd law. If follows then that:

$$\boxed{\alpha_A = \varepsilon_A}$$

Because of this close relation between emissivity, ε, and absorptivity, α, only one property is normally measured and this value may be used alternatively for either property.

Let's not lose sight of the fact that, as thermodynamic properties of the material, α and ε may depend on temperature. In general, this will be the case as radiative properties will depend on wavelength, λ. The wave length of radiation will, in turn, depend on the temperature of the source of radiation.

The emissivity, ε, of surface A will depend on the material of which surface A is composed, i.e. aluminum, brass, steel, etc. and on the temperature of surface A.

The absorptivity, α, of surface A will depend on the material of which surface A is composed, i.e. aluminum, brass, steel, etc. and on the temperature of surface B.

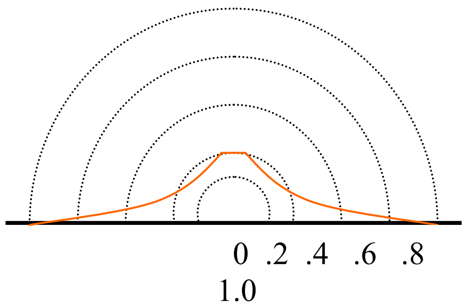
In the design of solar collectors, engineers have long sought a material which would absorb all solar radiation, ($\alpha = 1$, $T_{sun} \sim 5600K$) but would not re-radiate energy as it came to temperature ($\varepsilon << 1$, $T_{collector} \sim 400K$). NASA developed an anodized chrome, commonly called "black chrome" as a result of this research.

Black Surfaces

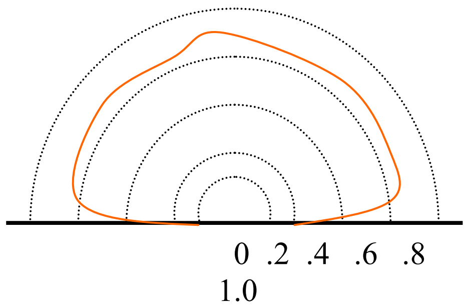
Within the visual band of radiation, any material, which absorbs all visible light, appears as black. Extending this concept to the much broader thermal band, we speak of surfaces with $\alpha = 1$ as also being "black" or "thermally black". It follows that for such a surface, $\varepsilon = 1$ and the surface will behave as an ideal emitter. The terms ideal surface and black surface are used interchangeably.

Lambert's Cosine Law:

A surface is said to obey Lambert's cosine law if the intensity, I, is uniform in all directions. This is an idealization of real surfaces as seen by the emissivity at different zenith angles:



Dependence of Emissivity on Zenith Angle, Typical Metal.



Dependence of Emissivity on Zenith Angle, Typical Non-Metal.

The sketches shown are intended to show is that metals typically have a very low emissivity, ε, which also remain nearly constant, expect at very high zenith angles, θ. Conversely, non-metals will have a relatively high emissivity, ε, except at very high zenith angles. Treating the emissivity as a constant over all angles is generally a good approximation and greatly simplifies engineering calculations.

Relationship Between Emissive Power and Intensity

By definition of the two terms, emissive power for an ideal surface, E_b, and intensity for an ideal surface, I_b.

$$E_b = \int_{hemisphere} I_b \cdot \cos\theta \cdot d\Omega$$

Replacing the solid angle by its equivalent in spherical angles:

$$E_b = \int_0^{2\cdot\pi} \int_0^{\pi/2} I_b \cdot \cos\theta \cdot \sin\theta \cdot d\theta \cdot d\varphi$$

Integrate once, holding I_b constant:

$$E_b = 2\cdot\pi\cdot I_b \cdot \int_0^{\pi/2} \cos\theta \cdot \sin\theta \cdot d\theta$$

Integrate a second time. (Note that the derivative of sin θ is cos θ·dθ.)

$$E_b = 2\cdot\pi\cdot I_b \cdot \left.\frac{\sin^2\theta}{2}\right|_0^{\pi/2} = \pi\cdot I_b$$

$$\boxed{E_b = \pi\cdot I_b}$$

Radiation Exchange

During the previous lecture we introduced the intensity, I, to describe radiation within a particular solid angle.

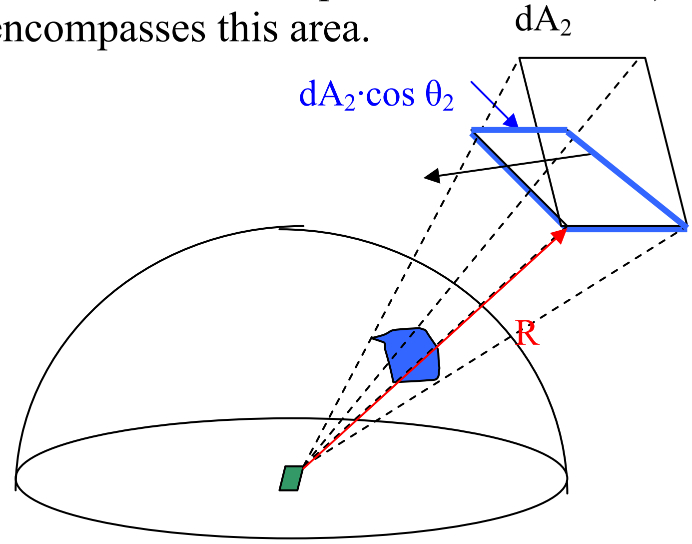
$$I = \frac{dq}{\cos\theta \cdot dA_1 \cdot d\Omega}$$

This will now be used to determine the fraction of radiation leaving a given surface and striking a second surface.

Rearranging the above equation to express the heat radiated:

$$dq = I \cdot \cos\theta \cdot dA_1 \cdot d\Omega$$

Next we will project the receiving surface onto the hemisphere surrounding the source. First find the projected area of surface dA_2, $dA_2 \cdot \cos\theta_2$. (θ_2 is the angle between the normal to surface 2 and the position vector, R.) Then find the solid angle, Ω, which encompasses this area.



Substituting into the heat flow equation above:

$$dq = \frac{I \cdot \cos\theta_1 \cdot dA_1 \cdot \cos\theta_2 dA_2}{R^2}$$

To obtain the entire heat transferred from a finite area, dA_1, to a finite area, dA_2, we integrate over both surfaces:

$$q_{1\to2} = \int_{A_2}\int_{A_1} \frac{I \cdot \cos\theta_1 \cdot dA_1 \cdot \cos\theta_2 dA_2}{R^2}$$

To express the total energy emitted from surface 1, we recall the relation between emissive power, E, and intensity, I.

$$q_{emitted} = E_1 \cdot A_1 = \pi \cdot I_1 \cdot A_1$$

View Factors-Integral Method

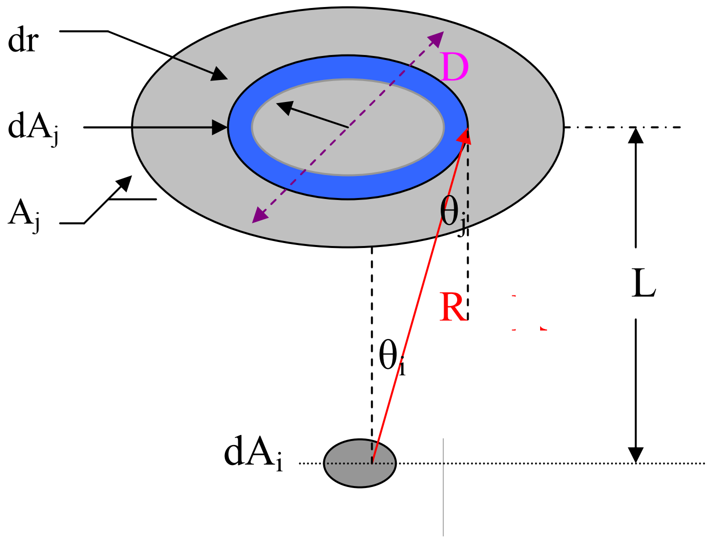
Define the view factor, $F_{1\text{-}2}$, as the fraction of energy emitted from surface 1, which directly strikes surface 2.

$$F_{1\to 2} = \frac{q_{1\to 2}}{q_{emitted}} = \frac{\int_{A_2}\int_{A_1} \frac{I \cdot \cos\theta_1 \cdot dA_1 \cdot \cos\theta_2 dA_2}{R^2}}{\pi \cdot I \cdot A_1}$$

after algebraic simplification this becomes:

$$F_{1\to 2} = \frac{1}{A_1} \cdot \int_{A_2}\int_{A_1} \frac{\cos\theta_1 \cdot \cos\theta_2 \cdot dA_1 \cdot dA_2}{\pi \cdot R^2}$$

Example Consider a diffuse circular disk of diameter D and area A_j and a plane diffuse surface of area $A_i \ll A_j$. The surfaces are parallel, and A_i is located at a distance L from the center of A_j. Obtain an expression for the view factor F_{ij}.



The view factor may be obtained from:

$$F_{1\to 2} = \frac{1}{A_1} \cdot \int_{A_2} \int_{A_1} \frac{\cos\theta_1 \cdot \cos\theta_2 \cdot dA_1 \cdot dA_2}{\pi \cdot R^2}$$

Since dA_i is a differential area

$$F_{1\to 2} = \int_{A_1} \frac{\cos\theta_1 \cdot \cos\theta_2 \cdot dA_1}{\pi \cdot R^2}$$

Substituting for the cosines and the differential area:

$$F_{1\to 2} = \int_{A_1} \frac{\left(L/R \right)^2 \cdot 2\pi \cdot r \cdot dr}{\pi \cdot R^2}$$

After simplifying:

$$F_{1\to 2} = \int_{A_1} \frac{L^2 \cdot 2 \cdot r \cdot dr}{R^4}$$

Let $\rho^2 \equiv L^2 + r^2 = R^2$. Then $2 \cdot \rho \cdot d\rho = 2 \cdot r \cdot dr$.

$$F_{1\to 2} = \int_{A_1} \frac{L^2 \cdot 2 \cdot \rho \cdot d\rho}{\rho^4}$$

After integrating,

$$F_{1\to 2} = -2 \cdot L^2 \cdot \left. \frac{\rho^{-2}}{2} \right|_{A_2} = -L^2 \cdot \left[\frac{1}{L^2 + \rho^2} \right]_0^{D/2}$$

Substituting the upper & lower limits

$$F_{1\to 2} = -L^2 \cdot \left[\frac{4}{4 \cdot L^2 + D^2} - \frac{1}{L^2} \right]_0^{D/2} = \frac{D^2}{4 \cdot L^2 + D^2}$$

This is but one example of how the view factor may be evaluated using the integral method. The approach used here is conceptually quite straight forward; evaluating the integrals and algebraically simplifying the resulting equations can be quite lengthy.

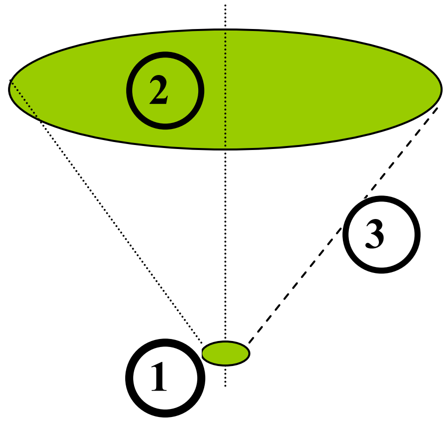
Enclosures

In order that we might apply conservation of energy to the radiation process, we must account for all energy leaving a surface. We imagine that the surrounding surfaces act as an enclosure about the heat source which receive all emitted energy. Should there be an opening in this enclosure through which energy might be lost, we place an imaginary surface across this opening to intercept this portion of the emitted energy. For an N surfaced enclosure, we can then see that:

$$\boxed{\sum_{j=1}^{N} F_{i,j} = 1}$$

This relationship is known as the "Conservation Rule".

Example: Consider the previous problem of a small disk radiating to a larger disk placed directly above at a distance L.



The view factor was shown to be given by the relationship:

$$F_{1\to 2} = \frac{D^2}{4 \cdot L^2 + D^2}$$

Here, in order to provide an enclosure, we will define an imaginary surface 3, a truncated cone intersecting circles 1 and 2.

From our conservation rule we have:

$$\sum_{j=1}^{N} F_{i,j} = F_{1,1} + F_{1,2} + F_{1,3}$$

Since surface 1 is not convex $F_{1,1} = 0$. Then:

$$F_{1\to 3} = 1 - \frac{D^2}{4 \cdot L^2 + D^2}$$

Reciprocity

We may write the view factor from surface i to surface j as:

$$A_i \cdot F_{i\to j} = \int_{A_j}\int_{A_i} \frac{\cos\theta_i \cdot \cos\theta_j \cdot dA_i \cdot dA_j}{\pi \cdot R^2}$$

Similarly, between surfaces j and i:

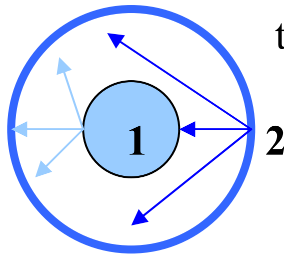
$$A_j \cdot F_{j\to i} = \int_{A_j}\int_{A_i} \frac{\cos\theta_j \cdot \cos\theta_i \cdot dA_j \cdot dA_i}{\pi \cdot R^2}$$

Comparing the integrals we see that they are identical so that:

$$\boxed{A_i \cdot F_{i\to j} = A_j \cdot F_{j\to i}}$$

This relationship is known as "Reciprocity".

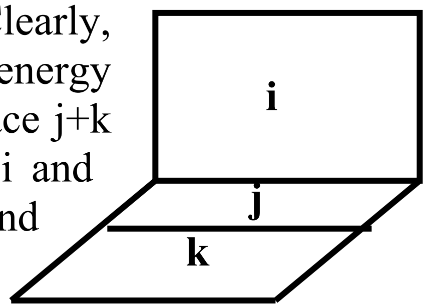
Example: Consider two concentric spheres shown to the right. All radiation leaving the outside of surface 1 will strike surface 2. Part of the radiant energy leaving the inside surface of object 2 will strike surface 1, part will return to surface 2. To find the fraction of energy leaving surface 2 which strikes surface 1, we apply reciprocity:



$$A_2 \cdot F_{2,1} = A_1 \cdot F_{1,2} \Rightarrow F_{2,1} = \frac{A_1}{A_2} \cdot F_{1,2} = \frac{A_1}{A_2} = \frac{D_1}{D_2}$$

Associative Rule

Consider the set of surfaces shown to the right: Clearly, from conservation of energy, the fraction of energy leaving surface i and striking the combined surface j+k will equal the fraction of energy emitted from i and striking j plus the fraction leaving surface i and striking k.



$$F_{i \Rightarrow (j+k)} = F_{i \Rightarrow j} + F_{i \Rightarrow k}$$

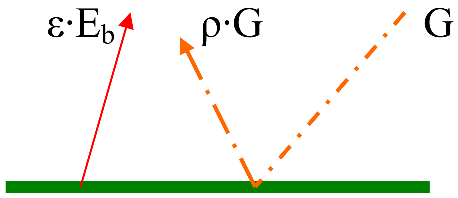
This relationship is known as the "Associative Rule".

Radiosity

We have developed the concept of intensity, I, which let to the concept of the view factor. We have discussed various methods of finding view factors. There remains one additional concept to introduce before we can consider the solution of radiation problems.

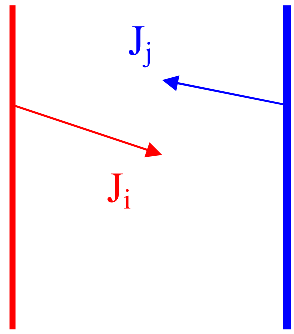
Radiosity, J, is defined as the total energy leaving a surface per unit area and per unit time. This may initially sound much like the definition of emissive power, but the sketch below will help to clarify the concept.

$$J \equiv \varepsilon \cdot E_b + \rho \cdot G$$



Net Exchange Between Surfaces

Consider the two surfaces shown. Radiation will travel from surface i to surface j and will also travel from j to i.



$$q_{i\rightarrow j} = J_i \cdot A_i \cdot F_{i\rightarrow j}$$

likewise,

$$q_{j\rightarrow i} = J_j \cdot A_j \cdot F_{j\rightarrow j}$$

The net heat transfer is then:

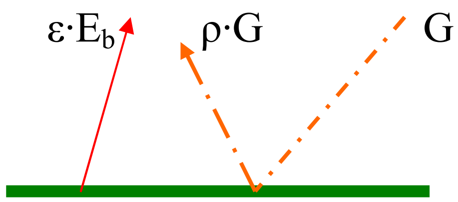
$$q_{j\rightarrow i\ (net)} = J_i \cdot A_i \cdot F_{i\rightarrow j} - J_j \cdot A_j \cdot F_{j\rightarrow j}$$

From reciprocity we note that $F_{1\rightarrow 2} \cdot A_1 = F_{2\rightarrow 1} \cdot A_2$ so that

$$q_{j\rightarrow i\ (net)} = J_i \cdot A_i \cdot F_{i\rightarrow j} - J_j \cdot A_i \cdot F_{i\rightarrow j} = A_i \cdot F_{i\rightarrow j} \cdot (J_i - J_j)$$

Net Energy Leaving a Surface

The net energy leaving a surface will be the difference between the energy leaving a surface and the energy received by a surface:



$$q_{1\rightarrow} = [\varepsilon \cdot E_b - \alpha \cdot G] \cdot A_1$$

Combine this relationship with the definition of Radiosity to eliminate G.

$$J \equiv \varepsilon \cdot E_b + \rho \cdot G \Rightarrow G = [J - \varepsilon \cdot E_b]/\rho$$

$$q_{1\rightarrow} = \{\varepsilon \cdot E_b - \alpha \cdot [J - \varepsilon \cdot E_b]/\rho\} \cdot A_1$$

Assume opaque surfaces so that $\alpha + \rho = 1 \Rightarrow \rho = 1 - \alpha$, and substitute for ρ.

$$q_{1\rightarrow} = \{\varepsilon \cdot E_b - \alpha \cdot [J - \varepsilon \cdot E_b]/(1 - \alpha)\} \cdot A_1$$

Put the equation over a common denominator:

$$q_{1\rightarrow} = \left[\frac{(1-\alpha)\cdot\varepsilon\cdot E_b - \alpha\cdot J + \alpha\cdot\varepsilon\cdot E_b}{1-\alpha}\right]\cdot A_1 = \left[\frac{\varepsilon\cdot E_b - \alpha\cdot J}{1-\alpha}\right]\cdot A_1$$

If we assume that $\alpha = \varepsilon$ then the equation reduces to:

$$q_{1\rightarrow} = \left[\frac{\varepsilon\cdot E_b - \varepsilon\cdot J}{1-\varepsilon}\right]\cdot A_1 = \left[\frac{\varepsilon\cdot A_1}{1-\varepsilon}\right]\cdot(E_b - J)$$

Electrical Analogy for Radiation

We may develop an electrical analogy for radiation, similar to that produced for conduction. The two analogies should not be mixed: they have different dimensions on the potential differences, resistance and current flows.

	Equivalent Current	Equivalent Resistance	Potential Difference
Ohms Law	I	R	ΔV
Net Energy Leaving Surface	$q_{1\rightarrow}$	$\left[\frac{1-\varepsilon}{\varepsilon\cdot A}\right]$	E_b - J
Net Exchange Between Surfaces	$q_{i\rightarrow j}$	$\frac{1}{A_1\cdot F_{1\rightarrow 2}}$	$J_1 - J_2$

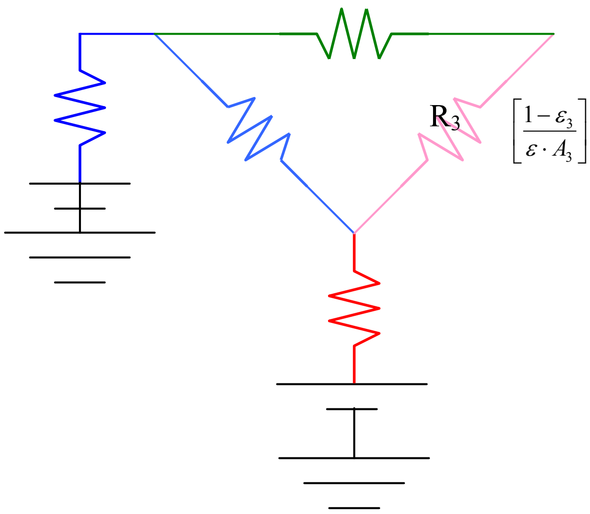
Alternate Procedure for Developing Networks

- Count the number of surfaces. (A surface must be at a "uniform" temperature and have uniform properties, i.e. ε, α, ρ.)
- Draw a radiosity node for each surface.
- Connect the Radiosity nodes using view factor resistances, $1/A_i \cdot F_{i \to j}$.
- Connect each Radiosity node to a grounded battery, through a surface resistance, $\left[1-\varepsilon \big/ \varepsilon \cdot A\right]$.

This procedure should lead to exactly the same circuit as we obtain previously.

Simplifications to the Electrical Network

- Insulated surfaces. In steady state heat transfer, a surface cannot receive net energy if it is insulated. Because the energy cannot be stored by a surface in steady state, all energy must be re-radiated back into the enclosure. *Insulated surfaces are often termed as re-radiating surfaces.*



  Electrically cannot flow through a battery if it is not grounded.

  Surface 3 is not grounded so that the battery and surface resistance serve no purpose and are removed from the drawing.

- Black surfaces: A black, or ideal surface, will have no surface resistance:

$$\left[\frac{1-\varepsilon}{\varepsilon \cdot A}\right] = \left[\frac{1-1}{1 \cdot A}\right] = 0$$

In this case the nodal Radiosity and emissive power will be equal.

This result gives some insight into the physical meaning of a black surface. Ideal surfaces radiate at the maximum possible level. Non-black surfaces will have a reduced potential, somewhat like a battery with a corroded terminal. They therefore have a reduced potential to cause heat/current flow.

- Large surfaces: Surfaces having a large surface area will behave as black surfaces, irrespective of the actual surface properties:

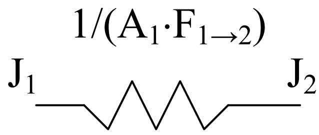
$$\left[\frac{1-\varepsilon}{\varepsilon \cdot A}\right] = \left[\frac{1-\varepsilon}{\varepsilon \cdot \infty}\right] = 0$$

Physically, this corresponds to the characteristic of large surfaces that as they reflect energy, there is very little chance that energy will strike the smaller surfaces; most of the energy is reflected back to another part of the same large surface. After several partial absorptions most of the energy received is absorbed.

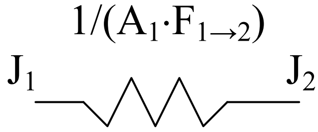
Solution of Analogous Electrical Circuits.

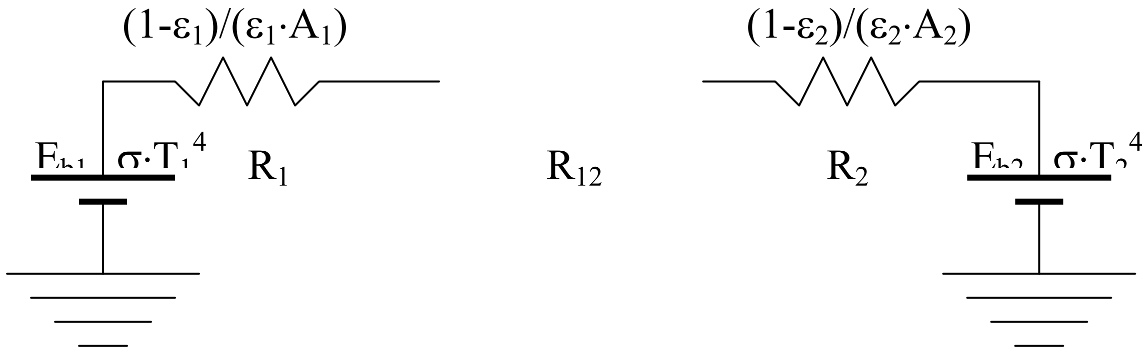
- Large Enclosures

Consider the case of an object, 1, placed inside a large enclosure, 2. The system will consist of two objects, so we proceed to construct a circuit with two radiosity nodes.

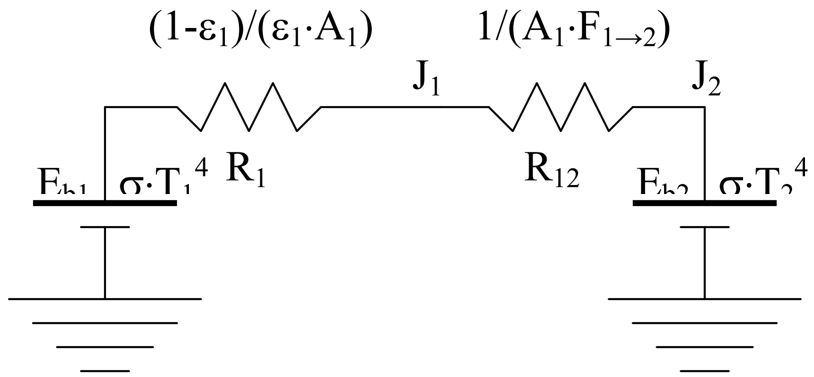


Now we ground both Radiosity nodes through a surface resistance.





Since A_2 is large, $R_2 = 0$. The view factor, $F_{1\rightarrow 2} = 1$



Sum the series resistances:

$$R_{Series} = (1\text{-}\varepsilon_1)/(\varepsilon_1\cdot A_1) + 1/A_1 = 1/(\varepsilon_1\cdot A_1)$$

Ohm's law:

$$i = \Delta V/R$$

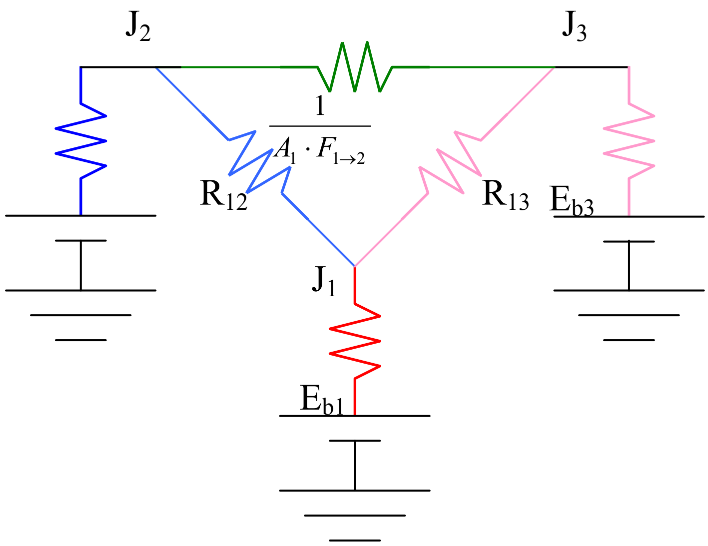
or by analogy:

$$q = \Delta E_b/R_{Series} = \varepsilon_1\cdot A_1\cdot\sigma\cdot(T_1^4 - T_2^4)$$

You may recall this result from Thermo I, where it was introduced to solve this type of radiation problem.

- Networks with Multiple Potentials

Systems with 3 or more grounded potentials will require a slightly different solution, but one which students have previously encountered in the Circuits course.



The procedure will be to apply Kirchoff's law to each of the Radiosity junctions.

$$\sum_{i=1}^{3} q_i = 0$$

In this example there are three junctions, so we will obtain three equations. This will allow us to solve for three unknowns.

Radiation problems will generally be presented on one of two ways:

- The surface net heat flow is given and the surface temperature is to be found.
- The surface temperature is given and the net heat flow is to be found.

Returning for a moment to the coal grate furnace, let us assume that we know (a) the total heat being produced by the coal bed, (b) the temperatures of the water walls and (c) the temperature of the super heater sections.

Apply Kirchoff's law about node 1, for the coal bed:

$$q_1 + q_{2\to 1} + q_{3\to 1} = q_1 + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0$$

Similarly, for node 2:

$$q_2 + q_{1\to 2} + q_{3\to 2} = \frac{E_{b2} - J_2}{R_2} + \frac{J_1 - J_2}{R_{12}} + \frac{J_3 - J_2}{R_{23}} = 0$$

(Note how node 1, with a specified heat input, is handled differently than node 2, with a specified temperature.

And for node 3:

$$q_3 + q_{1\to 3} + q_{2\to 3} = \frac{E_{b3} - J_3}{R_3} + \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} = 0$$

The three equations must be solved simultaneously. Since they are each linear in J, matrix methods may be used:

$$\begin{bmatrix} -\frac{1}{R_{12}} - \frac{1}{R_{13}} & \frac{1}{R_{12}} & \frac{1}{R_{13}} \\ \frac{1}{R_{12}} & -\frac{1}{R_2} - \frac{1}{R_{12}} - \frac{1}{R_{13}} & \frac{1}{R_{23}} \\ \frac{1}{R_{13}} & \frac{1}{R_{23}} & -\frac{1}{R_3} - \frac{1}{R_{13}} - \frac{1}{R_{23}} \end{bmatrix} \cdot \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} -q_1 \\ -\frac{E_{b2}}{R_2} \\ -\frac{E_{b3}}{R_3} \end{bmatrix}$$

The matrix may be solved for the individual Radiosity. Once these are known, we return to the electrical analogy to find the temperature of surface 1, and the heat flows to surfaces 2 and 3.

Surface 1: Find the coal bed temperature, given the heat flow:

$$q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{\sigma \cdot T_1^4 - J_1}{R_1} \Rightarrow T_1 = \left[\frac{q_1 \cdot R_1 + J_1}{\sigma}\right]^{0.25}$$

Surface 2: Find the water wall heat input, given the water wall temperature:

$$q_2 = \frac{E_{b2} - J_2}{R_2} = \frac{\sigma \cdot T_2^4 - J_2}{R_2}$$

Surface 3: (Similar to surface 2) Find the water wall heat input, given the water wall temperature:

$$q_3 = \frac{E_{b3} - J_3}{R_3} = \frac{\sigma \cdot T_3^4 - J_3}{R_3}$$

HEAT TRANSFER OPERATION

MODULE 3:

RADIATIVE HEAT TRANSFER

Thermal radiation is referred when a body is heated or exhibits the loss of energy by radiation. However, more general form "radiation energy" is used to cover all the other forms. The emission of other form of radiant energy may be caused when a body is excited by oscillating electrical current, electronic bombardment, chemical reaction etc. Moreover, when radiation energy strikes a body and is absorbed, it may manifest itself in the form of thermal internal energy, a chemical reaction, an electromotive force, etc. depending on the nature of the incident radiation and the substance of which the body is composed.

In this chapter, it will concentrate on thermal radiation (emission or absorption) that on radiation produced by or while produces thermal excitation of a body.

There are many theories available in literature which explains the transport of energy by radiation. However, a dual theory is generally accepted which enables to explain the radiant energy in the characterization of a wave motion (electromagnetic wave motion) and discontinuous emission (discrete packets or quanta of energy).

An electromagnetic wave propagates at the speed of light (3×10^8 m/s). It is characterized by its wavelength λ or its frequency ν related by

$$c = \lambda\nu \tag{7.1}$$

Emission of radiation is not continuous, but occurs only in the form of discrete quanta. Each quantum has energy

$$E = h\nu \tag{7.2}$$

where, = 6.6246×10^{-34} J.s, is known as Planck's constant.

Table 7.1 shows the electromagnetic radiation covering the entire spectrum of wavelength

Table 7.1: Electromagnetic radiation for entire spectrum of wavelength

Type	**Band of wavelength (μm)**
Cosmic rays	upto 4×10^{-7}
Gamma rays	4×10^{-7} to 1.4×10^{-4}

X-rays	1×10^{-5} to 2×10^{-2}
Ultraviolet rays	5×10^{-3} to 3.9×10^{-1}
Visible light	$3.9\times10^{-1}$to 7.8×10^{-1}
Infrared rays	7.8×10^{-1} to 1×10^{3}
Thermal radiation	1×10^{-1} to 1×10^{2}
Microwave, radar, radio waves	1×10^{3} to 5×10^{10}

It is to be noted that the above band is in approximate values and do not have any sharp boundary.

7.1 Basic definition pertaining to radiation

Before we further study about the radiation it would be better to get familiarised with the basic terminology and properties of the radiant energy and how to characterise it.As observed in the table 7.1 that the thermal radiation is defined between wavelength of about 1×10^{-1} and 1×10^{2} μm of the electromagnetic radiation. If the thermal radiation is emitted by a surface, which is divided into its spectrum over the wavelength band, it would be found that the radiation is not equally distributed over all wavelength. Similarly, radiation incident on a system, reflected by a system, absorbed by a system, etc. may be wavelength dependent. The dependence on the wavelength is generally different from case to case, system to system, etc. The wavelength dependency of any radiative quantity or surface property will be referred to as a spectral dependency. The radiation quantity may be monochromatic (applicable at a single wavelength) or total (applicable at entire thermal radiation spectrum). It is to be noted that radiation quantity may be dependent on the direction and wavelength both but we will not consider any directional dependency. This chapter will not consider directional effect and the emissive power will always used to be (hemispherical) summed overall direction in the hemisphere above the surface.

7.1.1 Emissive power

It is the emitted thermal radiation leaving a system per unit time, per unit area of surface. The total emissive power of a surface is all the emitted energy, summed over all the direction and all wavelengths, and is usually denoted as E. The total emissive power is found to be dependent upon the temperature of the emitting surface, the subsystem which this system is composed, and the nature of the surface structure or texture.

The monochromatic emissive power E_λ, is defined as the rate, per unit area, at which the surface emits thermal radiation at a particular wavelength λ. Thus the total and monochromatic hemispherical emissive power are related by

$$E = \int_0^\infty E_\lambda d\lambda \tag{7.3}$$

and the functional dependency of E_λ on λ must be known to evaluate E.

7.1.2 Radiosity

It is the term used to indicate all the radiation leaving a surface, per unit time and unit area.

$$J = \int_0^\infty J_\lambda \, d\lambda \tag{7.4}$$

where, J and J_λ are the total and monochromatic radiosity. The radiosity includes reflected energy as well as original emission whereas emissive power consists of only original emission leaving the system. The emissive power does not include any energy leaving a system that is the result of the reflection of any incident radiation.

7.1.3 Irradiation

It is the term used to denote the rate, per unit area, at which thermal radiation is incident upon a surface (from all the directions). The irradiative incident upon a surface is the result of emission and reflection from other surfaces and may thus be spectrally dependent.

$$G = \int_0^\infty G_\lambda d\lambda \tag{7.5}$$

where, G and G_λ are the total and monochromatic irradiation. Reflection from a surface may be of two types specular or diffusive as shown in fig.7.1.

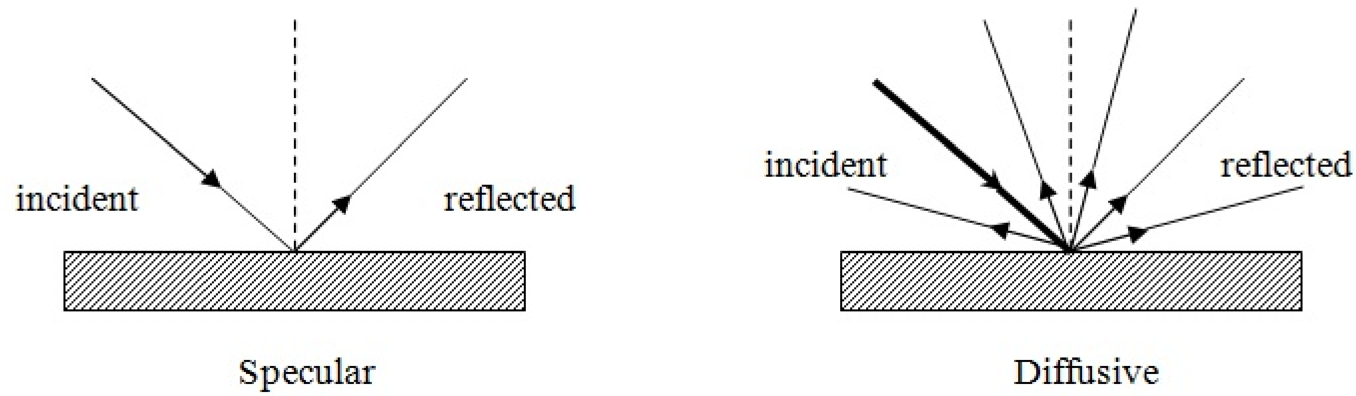


Fig. 7.1: (a) Specular, and (b) diffusive radiation

Thus,

$$J = E + \rho G \tag{7.6}$$

7.1.4 Absorptivity, reflectivity, and transmitting

The emissive power, radiosity, and irradiation of a surface are inter-related by the reflective, absorptive, and transmissive properties of the system. When thermal radiation is incident on a surface, a part of the radiation may be reflected by the surface, a part may be absorbed by the surface and a part may be transmitted through the surface as shown in fig.7.2. These fractions of reflected, absorbed, and transmitted energy are interpreted as system properties called reflectivity, absorptivity, and transmissivity, respectively.

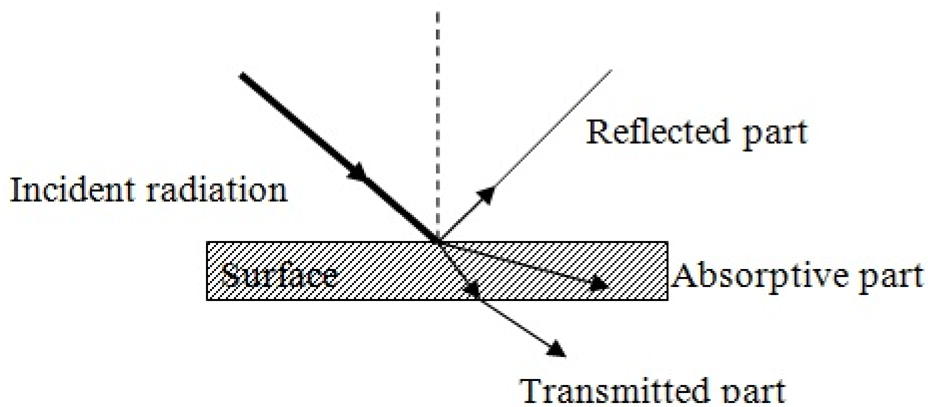


Fig. 7.2: Reflection, absorption and transmitted energy

Thus using energy conservation,

$$\rho + \alpha + \tau = 1 \tag{7.7}$$

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1 \qquad (7.7)$$

where, ρ , α and τ are total reflectivity, total absorptivity, and total transmissivity. The subscript λ indicates the monochromatic property.

In general the monochromatic and total surface properties are dependent on the system composition, its roughness, and on its temperature.

Monochromatic properties are dependent on the wavelength of the incident radiation, and the total properties are dependent on the spectral distribution of the incident energy.

Most gases have high transmissivity, i.e. $\tau \approx 1$ and $\rho = \alpha = 0$ (like air at atmospheric pressure). However, some other gases (water vapour, CO_2 etc.) may be highly absorptive to thermal radiation, at least at certain wavelength.

Most solids encountered in engineering practice are opaque to thermal radiation $(\tau \approx 0)$. Thus for thermally opaque solid surfaces,

$$\rho + \alpha = 1 \qquad (7.6)$$

Another important property of the surface of a substance is its ability to emit radiation. Emission and radiation have different concept. Reflection may occur only when the surface receives radiation whereas emission always occurs if the temperature of the surface is above the absolute zero. Emissivity of the surface is a measure of how good it is an emitter.

7.2 Blackbody radiation

In order to evaluate the radiation characteristics and properties of a real surface it is useful to define an ideal surface such as the perfect blackbody. The perfect blackbody is defined as one which absorbs all incident radiation regardless of the spectral distribution or directional characteristic of the incident radiation.

$$\alpha = \alpha_\lambda = 1$$

$$\rho = \rho_\lambda = 0$$

A blackbody is black because it does not reflect any radiation. The only radiation leaving a blackbody surface is original emission since a blackbody absorbs all incident radiation. The emissive power of a blackbody is represented by , and depends on the surface temperature only.

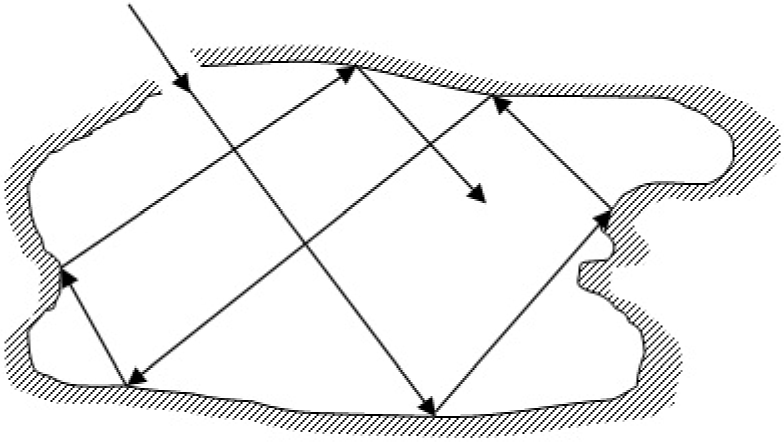


Fig. 7.3: Example of a near perfect blackbody

It is possible to produce a near perfect blackbody as shown in fig.7.3.

Figure 7.2 shows a cavity with a small opening. The body is at isothermal state, where a ray of incident radiation enters through the opening will undergo a number of internal reflections. A portion of the radiation absorbed at each internal reflection and a very little of the incident beam ever find the way out through the small hole. Thus, the radiation found to be evacuating from the hole will appear to that coming from a nearly perfect blackbody.

7.2.1 Planck's law

A surface emits radiation of different wavelengths at a given temperature (theoretically zero to infinite wavelengths). At a fixed wavelength, the surface radiates more energy as the temperature increases. Monochromatic emissive power of a blackbody is given by eq.7.10.

$$E_{b,\lambda} = \frac{2\pi h c^2 \lambda^{-5}}{\exp\left(\frac{hc}{\lambda k T}\right) - 1} \tag{7.7}$$

where, h = 6.6256 X 10^{-34} JS; Planck's constant
c = 3 X 10^{8} m/s; speed of light
T = absolute temperature of the blackbody
λ = wavelenght of the monochromatic radiation emitted
k = Boltzmann constant

Equation 7.10 is known as Planck's law. Figure 7.4 shows the representative plot for Planck's distribution.

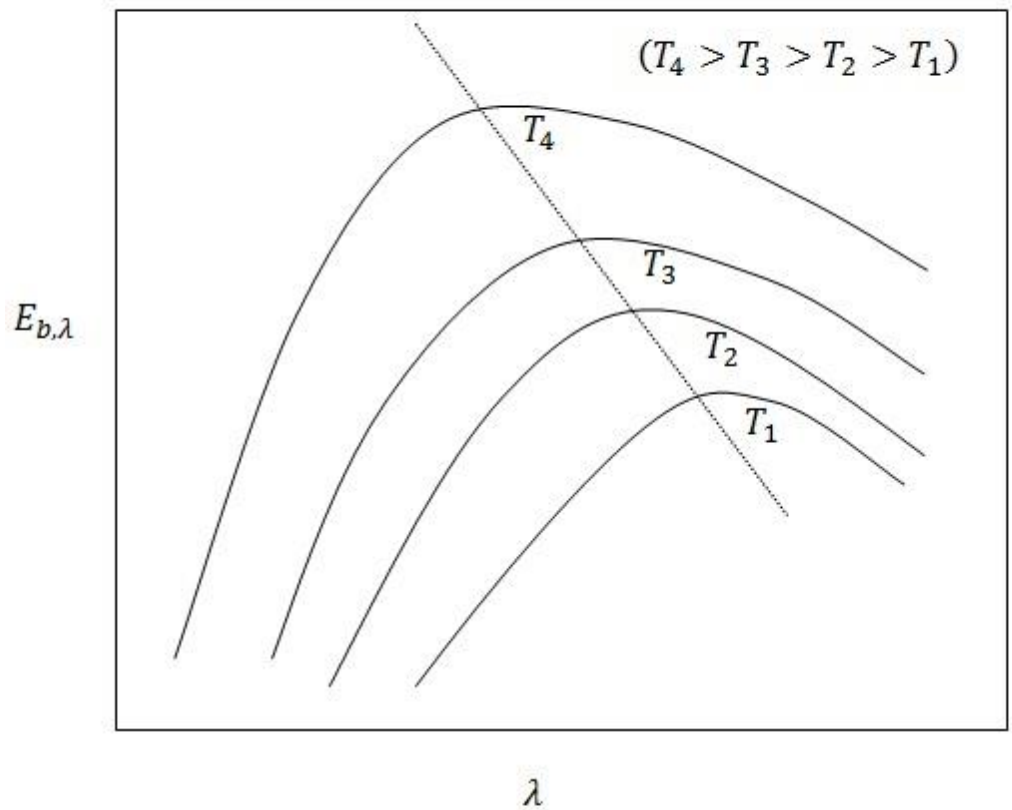


Fig. 7.4: Representative plot for Planck's distribution

7.2.2 Wien's law

Figure 7.4 shows that as the temperature increases the peaks of the curve also increases and it shift towards the shorter wavelength. It can be easily found out that the wavelength corresponding to the peak of the plot (λ_{max}) is inversely proportional to the temperature of the blackbody (Wein's law) as shown in eq. 7.11.

$$\lambda_{max} T = 2898 \tag{7.11}$$

Now with the Wien's law or Wien's displacement law, it can be understood if we heat a body, initially the emitted radiation does not have any colour. As the temperature rises the λ of the radiation reach the visible spectrum and we can able to see the red colour being height λ (for red colour). Further increase in temperature shows the white colour indicating all the colours in the light.

7.2.3 The Stefan-Boltzmann law for blackbody

Josef Stefan based on experimental facts suggested that the total emissive power of a blackbody is proportional to the fourth power of the absolute temperature. Later, Ludwig Boltzmann derived the same using classical thermodynamics. Thus the eq. 7.12 is known as Stefan-Boltzmann law,

$$E_b = \int_0^{\infty} E_{b\lambda}(\lambda, T)\, d\lambda$$

$$E_b = \sigma T^4 \qquad (7.12)$$

where, E_b is the emissive power of a blackbody, T is absolute temperature, and σ (= 5.67 X 10^{-8} $W/m^2/K^4$) is the Stefan-Boltzmann constant.

The Stefan-Boltzmann law for the emissive power gives the total energy emitted by a blackbody defined by eq.7.3.

7.2.4 Special characteristic of blackbody radiation

It has been shown that the irradiation field in an isothermal cavity is equal to E_b. Moreover, the irradiation was same for all planes of any orientation within the cavity. It may then be shown that the intensity of the blackbody radiation, I_b, is uniform. Thus, blackbody radiation is defined as,

$$E_b = \pi I_b \qquad (7.13)$$

where, $I_b = \int_0^{\infty} I_{b\lambda}\, d\lambda$ is the total intensity of the radiation and is called the spectral radiation intensity of the blackbody.

7.2.5 Kirchhoff's law

Consider an enclosure as shown in fig.7.2 and a body is placed inside the enclosure. The radiant heat flux (q) is incident onto the body and allowed to come into temperature equilibrium. The rate of energy absorbed at equilibrium by the body must be equal to the energy emitted.

$$EA = \alpha q A$$

$$E = \alpha q \qquad (7.14)$$

where, E is the emissive power of the body, α is absorptivity of the of the body at equilibrium temperature, and A is the area of the body.

Now consider the body is replaced by a blackbody i.e. $E \rightarrow E_b$ and $\alpha = 1$, the equation 7.14 becomes

$$E_b = q \qquad (7.15)$$

Dividing eq. 7.14 by eq.7.15,

$$\frac{E}{E_b} = \alpha \tag{7.16}$$

At this point we may define emissivity, which is a measure of how good the body is an emitter as compared to blackbody. Thus the emissivity can be written as the ratio of the emissive power to that of a blackbody,

$$\frac{E}{E_b} = \epsilon \tag{7.17}$$

On comparing eq.7.16 and eq.7.17, we get

$$\epsilon = \alpha \tag{7.18}$$

Equation 7.18 is the Kirchhoff's law, which states that the emissivity of a body which is in thermal equilibrium with its surrounding is equal to its absorptivity of the body. It should be noted that the source temperature is equal to the temperature of the irradiated surface. However, in practical purposes it is assumed that emissivity and absorptivity of a system are equal even if it is not in thermal equilibrium with the surrounding. The reason being the absorptivity of most real surfaces is relatively insensitive to temperature and wavelength. This particular assumption leads to the concept of grey body. The emissivity is considered to be independent of the wavelength of radiation for grey body.

7.3 Grey body

If grey body is defined as a substance whose monochromatic emissivity and absorptivity are independent of wavelength. A comparative study of grey body and blackbody is shown in the table 7.2.

Table-7.2: Comparison of grey and blackbody

Blackbody	**Grey body**
Ideal body	Ideal body
Emissivity (∈) is independent of wavelength	Emissivity (∈) is independent of wavelength

Absorptivity (α) is independent of wavelength	Absorptivity (α) is independent of wavelength
$\varepsilon = 1$	$\varepsilon < 1$
$\alpha = 1$	$\alpha < 1$

Illustration 7.1

The surface of a blackbody is at 500 K temperature. Obtain the total emissive power, the wavelength of the maximum monochromatic emissive power.

Solution 7.1

Using eq. 7.12, the total emissive power can be calculated,

$E_b = \sigma T^4$

where, σ (= 5.67 X 10^{-8} $W/m^2/K^4$) is the Stefan-Boltzmann constant. Thus at 500 K,

$E_b = (5.67 \text{ X } 10^{-8})(5000^4)\ W/m^2$

$E_b = 354.75\ W/m^2$

The wavelength of the maximum monochromatic emissive power can be obtained from the Wien's law (eq. 7.11),

$\lambda_{max} T = 2898$

$$\lambda_{max} = \frac{2898}{500} = 5.796\ \mu m$$

7.4 Radiative heat exchanger between surfaces

Till now we have discussed fundamental aspects of various definitions and laws. Now we will study the heat exchange between two or more surfaces which is of practical importance. The two surfaces which are not in direct contact, exchanges the heat due to radiation phenomena. The factors those determine the rate of heat exchange between two bodies are the temperature of the individual surfaces, their emissivities, as well as how well one surface can see the other surface. The last factor is known as view factor, shape factor, angle factor or configuration factor.

7.4.1 View factor

In this section we would like to find the energy exchange between two black surfaces having area A_1 and A_2, respectively, and they are at different temperature and have arbitrary shape and orientation with respect to each other. In order to find the radiative heat exchange between the bodies we have to first define the view factor as

F_{12} = fraction of the energy leavings surface 1 which reaches surface 2
F_{21} = fraction of the energy leaving surface 2 which reaches surface 1 or in general,
F_{mn} = fraction of the energy leaving surface m which raches surface n

Thus the energy leaving surface 1 and arriving at surface 2 is $E_{b1}A_1F_{12}$ and the energy leaving surface 2 and arriving at surface 1 is $E_{b2}A_2F_{21}$. All the incident radiation will be absorbed by the blackbody and the net energy exchange will be,

$$Q = E_{b1}A_1F_{12} - E_{b2}A_2F_{21}$$

At thermal equilibrium between the surfaces $Q_{12} = 0$ and $E_{b1} = E_{b2}$, thus

$$0 = E_{b1} (A_1F_{12} - A_2F_{21})$$

$$A_1F_{12} - A_2F_{21} \qquad (7.19)$$

Equation 7.19 is known as reciprocating relation, and it can be applied in general way for any blackbody surfaces.

$$A_iF_{ij} - A_jF_{ji} \qquad (7.20)$$

Though the relation is valid for blackbody it may be applied to any surface as long as diffuse radiation is involved.

7.4.1.1 Relation between view factors

In this section we will develop some useful relation of view factor considering fig. 7.5

In this section we will develop some useful relation of view factor considering fig. 7.5

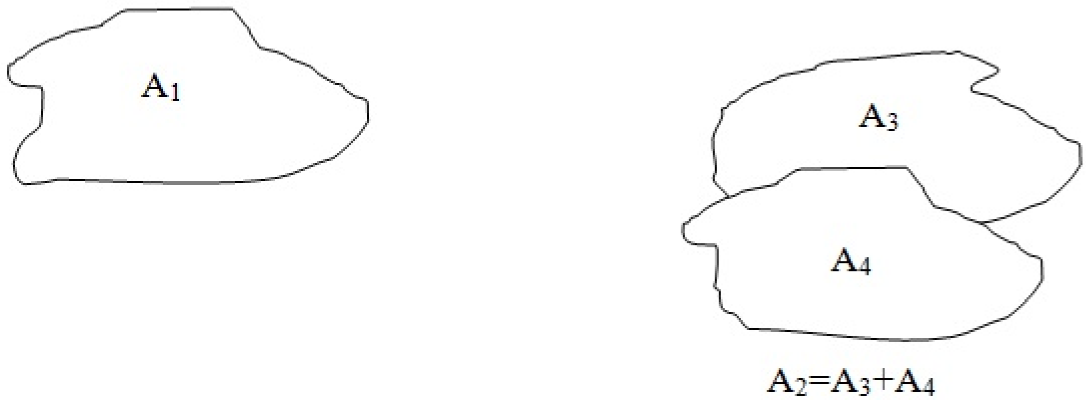


Fig. 7.5: Exchange of energy between area A1 and A2 (A is area of blackbody)

View factor for radiation from A_1 to the combined area A_2,

$$F_{12} = F_{13} + F_{14} \quad (7.21)$$

and using the reciprocating relations for surface 1 and 4,

$$A_1F_{14} = A_4F_{41} \quad (7.22)$$

Using eq. 7.21 and 7.22,

$$F_{41} = \frac{A_1}{A_4} F_{14}$$

$$F_{41} = \frac{A_1}{A_4} (F_{12} - F_{13})$$

Thus the unknown view factor F_{14} can be estimated if the view factors F_{12} and F_{13}, as well as their areas are (A_1, A_2) known.

Now, consider a flat plate (for eg.) which is emitting the radiation, it can be understood that the radiation of the flat plat cannot fall on its own surface (partly or fully). Such kind or surfaces are termed as “not able to see itself”. In such situations,

$$F_{11} = F_{22} = F_{33} = F_{44} = 0$$

However, if the surface can see itself like concave curved surfaces, which may thus see themselves and then the shape factor will not be zero in those cases.

Another property of the shape factor is that when the surface is enclosed, then the following relation holds,

$$\sum_{j=1}^{n} F_{ij} = 1 \tag{7.23}$$

where, F_{ij} is the fraction of the total energy leaving surface i which arrives at surface j.

In case of N-walled enclosure, some of the view factors may be evaluated from the knowledge of the rest and the total N^2 view factors may be represented in square matrix form shown below,

$$\begin{bmatrix} F_{11} & F_{12} & \cdots F_{1N} \\ F_{21} & F_{22} & \ldots F_{2N} \\ \cdots & \cdots & \cdots\cdots \\ F_{N1} & F_{N2} & \cdots F_{NN} \end{bmatrix}$$

It is also assumed that the radiosity and irradiation are uniform over each surface. As we have already discussed that the radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted (as opaque body), or

$$J = \epsilon E_b + \rho G \tag{7.24}$$

where, ϵ is the emissivity and E_b is the blackbody emissive power. Because the transmissivity is zero due to opaque surface and absorptivity of the body (grey) will be equal to its emissivity by Kirchhoff's law.

$$\rho = 1 - \alpha = 1 - \epsilon$$

Thus, eq.7.24 becomes

$$J = \epsilon E_b + (1 - \epsilon)G \tag{7.25}$$

The net energy leaving the surface is the difference between the radiosity and the irradiance (fig.7.6a),

$$\frac{\dot{Q}}{A} = J - G = \epsilon E_b + (1-\epsilon)G - G$$

$$\frac{\dot{Q}}{A} = \frac{\epsilon A}{1-\epsilon}(E_b - J)$$

$$\dot{Q} = \frac{E_b - J}{(1-\epsilon)/\epsilon A} \qquad (7.26)$$

The eq.7.26 can be analogous to the electrical circuit as shown in fig.7.6(b). The numerator of the eq.7.26 is equivalent to the potential difference, denominator is equivalent to the surface resistance to radiative heat, and left part is equivalent to the current in the circuit.

In the above discussion we have considered only one surface. Now we will analyse the exchange of radiant energy by two surfaces, A_1 and A_2, as shown in the fig.7.7a.

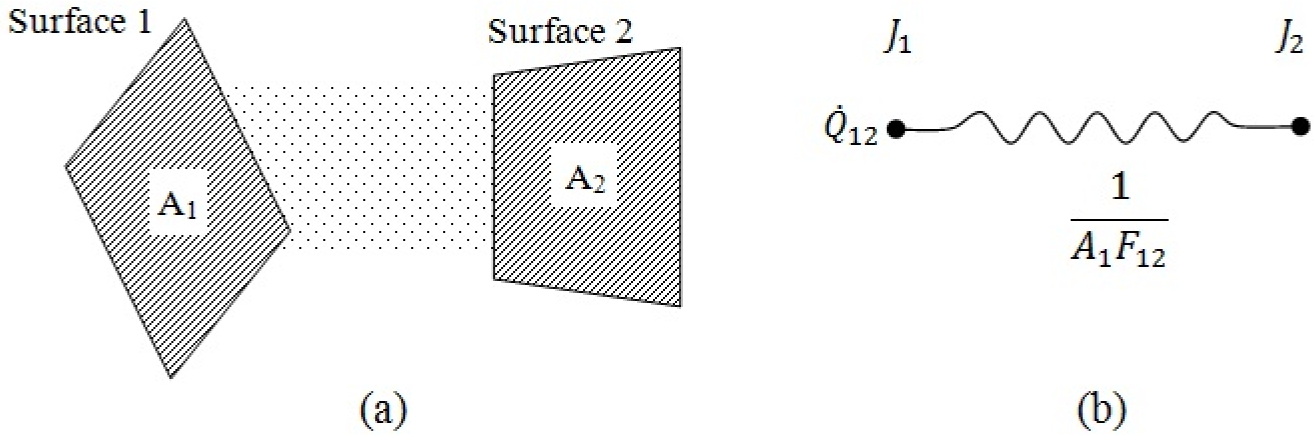


Fig. 7.7: (a) Energy exchange between two surfaces, (b) equivalent circuit diagram

7.8 Radiation combined with conduction and convection

In industrial processes, in general, the heat transfer at higher temperature has significant portion of radiation along with conduction and convection. For example, a heated surface is shown in the fig. 7.10 with all the three mechanism of heat transfer.

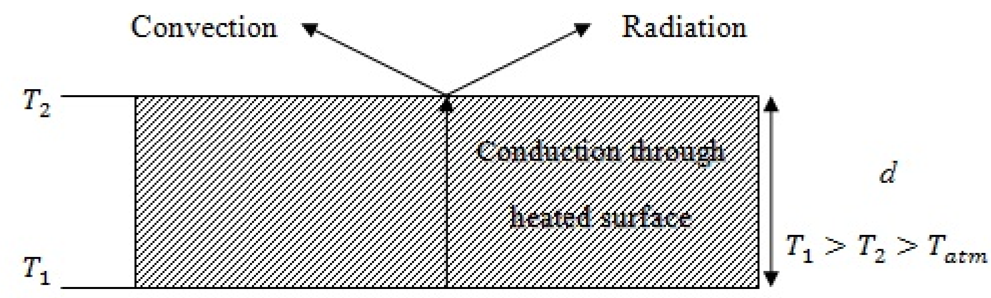


Fig. 7.11: Radiation combined with conduction and convection

At steady state

Heat flux by conduction = heat flux by convention + heat flux by radiation

$$\frac{K}{d}(T_1 - T_2) = h(T_2 - T_{atm}) + \epsilon\sigma(T_2^4 - T_{atm}^4)$$

where, *h* is the heat transfer coefficient at the surface in contact (outer surface) with atmosphere due to natural and forced convection combined together, ϵ is the emissivity of the outer surface, and T_{atm} is the atmospheric temperature.