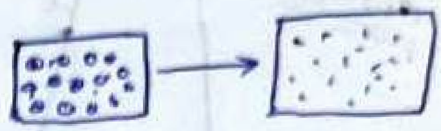
Entropy

- Measure of Randomness & disorder of any system

E → Energy, Equilibrium, Entropy ✓



Randomness, liquid < gas.

→ Entropy tells the amount of thermal energy which unsuitable for useful work

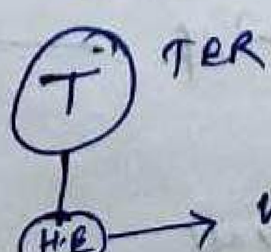
● Two Reversible adiabatic lines cannot intersect each others

Area ABC = Wnet

$\boxed{W_{net} = Q_{net}}$

In cycle ABC, the heat is being added from a single reservoir at const Temp (T)

Let this Qadd is taken from single temp. (TER)



$W_{net} = Q_{net}$

$Q_{rej} = 0$

↳ violation of kelvin plank statement

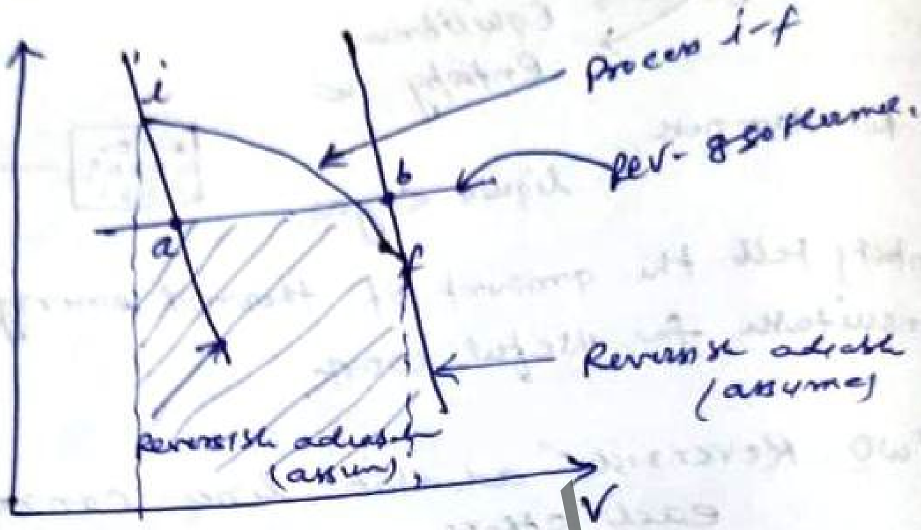
Here above is proved.

Clausius Theorem

$$\boxed{\oint_{Rev} \frac{dQ}{T} = 0}$$

Proof:

concept



i-f process

Area under i-f in pv-diagram $= W_{if}$

i-abf

i-a = Reversible adiab proc

ab = Rev. ISOthermal process

bf = Rev. adiab procer

$$W_{iabf} = W_{if}$$

For i-f process

$$Q_{i-f} = \Delta U_{if} + W_{if}$$

$$= U_f - U_i + W_{if} \quad \text{--- (1)}$$

for iabf process

$$Q_{iabf} = \Delta U_{if} + W_{iabf}$$

$$= U_f - U_i + W_{iabf} \quad \text{--- (2)}$$

from (I) & (II)

$$\boxed{Q_{if} = Q_{iabf}} \longrightarrow (3)$$

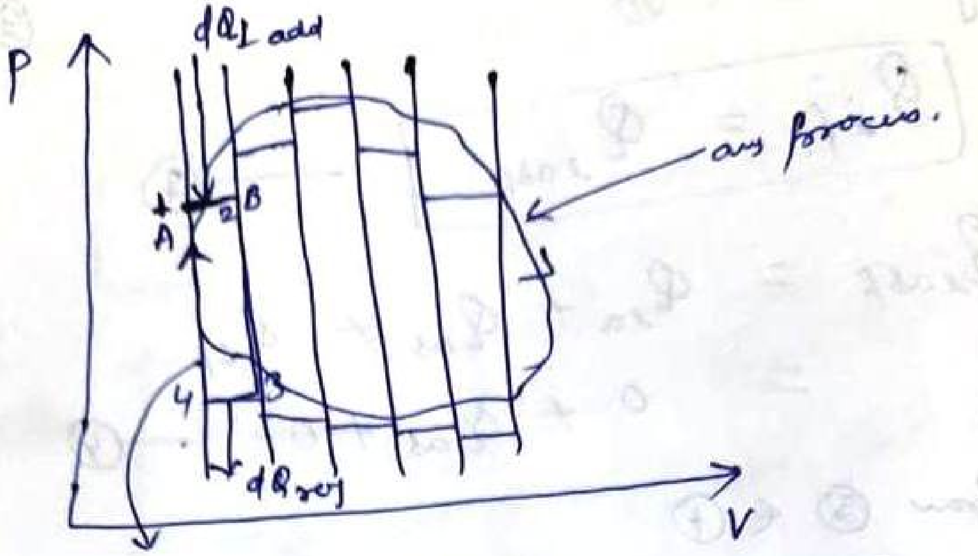
$$Q_{iabf} = Q_{ia} + Q_{ab} + Q_{bf}$$
$$= 0 + Q_{ab} + 0 \longrightarrow (4)$$

from (3) & (4)

$$\boxed{Q_{if} = Q_{ab}}$$

So, Any Rev. thermodynamic path can be replaced by an zig-zag path consisting of two rev. adiabatic & one Rev. isothermal process.

Such that the heat transfered in the original process will be equal to the heat transfered in the rev. isothermal process.



→ Elementary carnot cycle.

$$\eta_{th} = 1 - \frac{Q_{rej}}{Q_{add}} = 1 - \frac{dQ_2}{dQ_1}$$

$$= 1 - \frac{T_{rej}}{T_{add}} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow 1 - \frac{dQ_2}{dQ_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \frac{dQ_2}{dQ_1} = \frac{T_2}{T_1} \Rightarrow \frac{dQ_2}{T_2} = \frac{dQ_1}{T_1}$$

acc. to sign convention

$$\frac{dQ_2}{T_2} = -\frac{dQ_1}{T_1} \Rightarrow$$

$$\Rightarrow \frac{dQ_1}{T_1} + \frac{dQ_2}{T_2} = 0 \quad \text{--- (I)}$$

$$\frac{dQ_3}{T_3} + \frac{dQ_4}{T_4} = 0 \quad \text{--- (II)}$$

— — — —

— — —

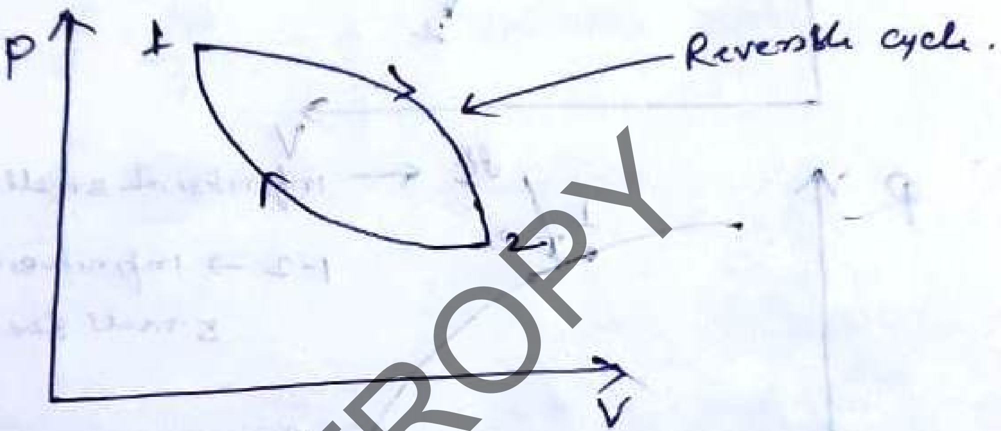
$$\frac{dQ_{n-1}}{T_{n-1}} + \frac{dQ_n}{T_n} = 0 \quad \text{--- (n)}$$

$$\frac{dQ_1}{T_1} + \frac{dQ_2}{T_2} + \frac{dQ_3}{T_3} + \cdots \frac{dQ_n}{T_n} = 0$$

$$\Rightarrow \boxed{\oint_{Rev} \frac{dQ}{T} = 0}$$

Hence, clausius theorem is proved.

Property of Entropy



$$\oint_{R_1R_2} \frac{dQ}{T} = 0$$

$$\int_1^2 \frac{dQ}{T} + \int_2^1 \frac{dQ}{T} = 0$$

$$\int_1^2 \frac{dQ}{T} = -\int_2^1 \frac{dQ}{T}$$

$$\left(\int_1^2 \frac{dQ}{T}\right)_{R_2} = \left(\int_1^2 \frac{dQ}{T}\right)_{R_2}$$

Path

So, $\frac{dQ}{T} \Rightarrow$ independent of path

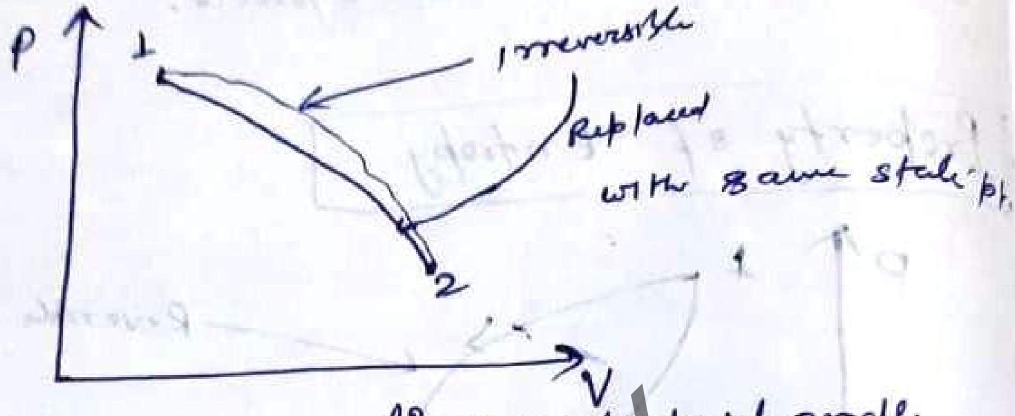
So it is a thermodynamic props.

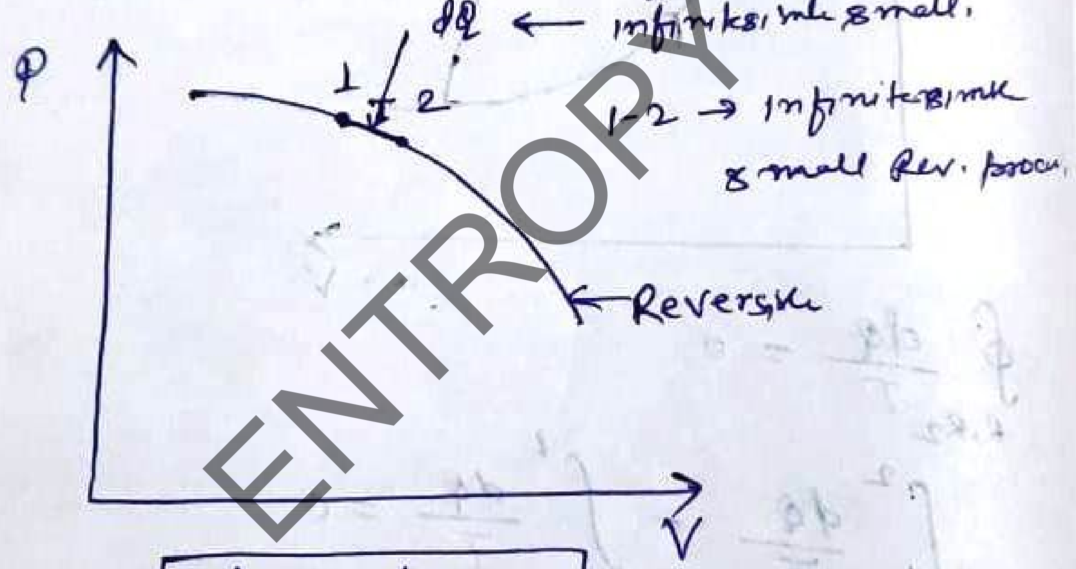
ds – exact diff.

Change in Entropy

$$\Delta S = \int_1^2 \frac{dQ}{T}$$

for $\Delta S \rightarrow$ for any Irreversible path





$$\boxed{ds = \frac{dQ_{Rev}}{T}}$$

Rev. process.

Unit ⇒ $S = J/K$ ← entropy ← extensive props.

- Specific entropy (s)

$s = J/kg \cdot K \simeq \frac{kJ}{kg \cdot K}$

↓ Intensive props.

$$\boxed{\Delta S = \int_1^2 \frac{dQ}{T}}$$

→ for any Process.

Clausius Inequality

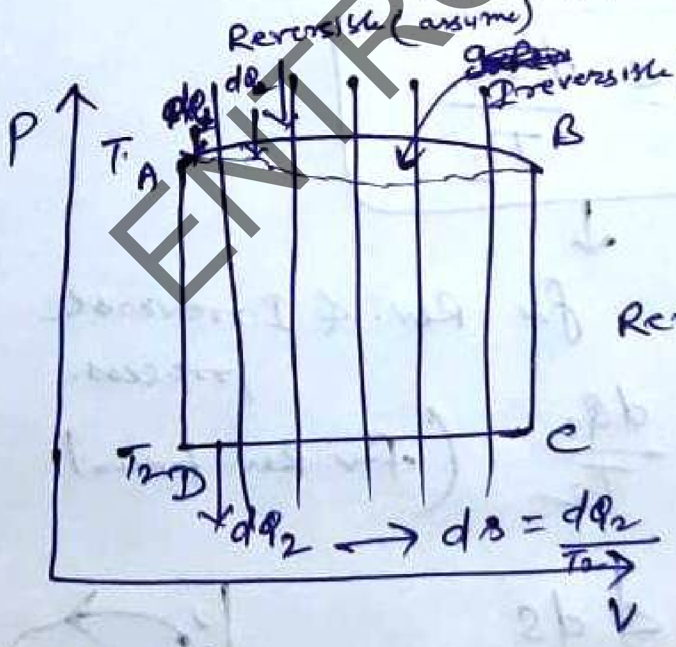
(i) $\oint \frac{dQ}{T} < 0$ Irreversible cycle.

(ii) $\oint \frac{dQ}{T} = 0$ Reversible cycles

(iii) $\oint \frac{dQ}{T} > 0$ Impossible

→ It violets the II-nd law of T.D

Proof



Reversible isothermal process.

Irreversible cycle,

$$\eta_{th} = 1 - \frac{dQ_2}{dQ}$$

Reversible cycle

$$\eta_{th} = \left(1 - \frac{dQ_2}{dQ}\right)_{Rev.}$$

from carnot theorem,

$$(\eta)_{irr} \le (\eta)_{rev.}$$

$$1-\frac{dQ_2}{dQ} \leq \left(1-\frac{dQ_2}{dQ}\right)_{rev.}$$

$$\therefore \frac{dQ_2}{dQ} \geq \left(\frac{dQ_2}{dQ}\right)_{rev.}$$

the thermal efficiency.

of, efficiency of H.E

$$1-\frac{dQ_2}{dQ} = 1-\frac{T_2}{T}$$

$$\boxed{\left(\frac{dQ_2}{dQ}\right)_{rev} = \frac{T_2}{T}}$$

$$\frac{dQ_2}{dQ} \leq \frac{T_2}{T}$$

$$\boxed{\frac{dQ}{T} \leq \frac{dQ_2}{T_2}}$$

for Rev. & Irreversible process.

$$dS = \frac{dQ_2}{T_2} \quad \text{(for Rev. process)}$$

$$\frac{dQ}{T} \leq dS$$

$$\oint \frac{dQ}{T} \leq \oint dS$$

$\oint$ thermody prop. $=0$

$$\boxed{\oint \frac{dQ}{T} \leq 0}$$