

SYLLABUS - UNIT 1

- **Logic Families**: CMOS Logic, CMOS Dynamic Electrical behaviour, Bipolar Logic: Diode Logic, Transistor Logic Inverter, TTL Logic, NMOS, CMOS/TTL Interface, ECL

- **Minimization Techniques and Logic gates**:

[Minimization Techniques]: [Boolean postulates and Laws] - [De Morgan Theorem] - Principle of Duality - Boolean expressions - [Minimization of Boolean Expressions] - [Minterms, Maxterms] [Sum of Products (SOP)], [Product of Sums (POS)], Karnaugh map minimization, Don't care conditions - Quine, Mc Cluskey method of minimization, [Number System: Representation of -ve numbers] & [1's Complement], [10's Complement], [Arithmetic Using 2's Complement]

- **CONVERSIONS**:

- Decimal to Binary - divide the num successively by 2 and record the remainder until 0 as quotient. Reverse the remainders.

for fractional part: multiply successively the fractional part by 2. Integral part each time 'd be digits of binary.

ex - 4.47	$\begin{array}{r} 2 \overline{) 4.0} \\ 2 \overline{) 2.0} \\ \hline 1 \end{array}$	$0.47 \times 2 = 0 + 0.94 \quad 0$
		$0.94 \times 2 = 1 + 0.88 \quad 1$
$\Rightarrow 100.011$		$0.88 \times 2 = 1 + 0.76 \quad 1$

- Binary to Decimal:

ex - 100.011

$$1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$$

4

+

0.25 + 0.125

4.375

- Decimal to Octal: same repeated division method as in
Decimal to Binary.

ex - $(266)_{10} = (412)_8$

8	266
8	33 2
8	4 1
	0 4

- Octal to Decimal:

ex - $(372)_8 = 3 \times 8^2 + 7 \times 8 + 2 \times 8^0 = (250)_{10}$

- Octal to Binary: 3 bit equivalent of each digit is written.

ex - $(472)_8 = (100111010)_2$

4 7 2

↓ ↓ ↓

100 111 010

← binary of individual digits
(using 3 bits of each octal digit)

2	4
2	2 0
2	1 0
	0 1

2	7
2	3 1
2	1 1
	1 1

2	2
2	1 0
2	0 1
	1 1

Base 2 to Base 8
 Couple 3 bits & convert to make each digit
 directly writing 3 bit equivalent

Base 2 to Base 16
 write 4 bit equivalent.
 directly coupling 4 bits & converting to write each digit for 4 binary.

We can directly convert bases with power, just the same way.

Base 4 \rightarrow Base 16 Base 3 \rightarrow Base 9

- Binary to Octal: bits of binary grouped into 3 bits from left to right
 ex- $(011010110)_2 = (326)_8$

$0 \rightarrow 2+1$ $0 \rightarrow 2+0$ $2 \rightarrow 2+0$

* Left Right ka chakkar mil hi ba jaha Odd ka sakte hi krlo jisse 3-3 ki grouping hojaye.

- Decimal to Hex: repeated division by 16.

remainders greater than 9 are represented by letters A to F.

A B C D E F
10 11 12 13 14 15

ex- $(423)_{10} = (1A7)_{16}$

16	423	
16	26	7
16	1	10
	0	1

- Hex to Decimal:

ex- $(356)_{16} = (3 \times 16^2 + 5 \times 16^1 + 6 \times 16^0) = (854)_{10}$

$(2AF)_{16} = (2 \times 16^2 + 10 \times 16^1 + 15 \times 16^0) = (687)_{10}$

- Binary to Hex: bits grouped into 4.

ex- $(11010111010)_2 = (2BA)_{16}$

↓ ↓ ↓
2 11 10
 B A

ex- $(10101110.010111)_2 = (AE.5C)_{16}$

↓ ↓ ↓ ↓
A E 5 C

- Hex to binary: write 4 bit binary equivalent of each digit

ex- $(9F2)_{16} = (100111110010)_2$

$$\begin{array}{r|l} 2 & 9 \\ \hline 2 & 4 \\ 2 & 2 \\ \hline & 1 \end{array} \begin{array}{l} 1 \\ 0 \\ 0 \end{array} \uparrow$$

1001

$$\begin{array}{r|l} 2 & 15 \\ \hline 2 & 7 \\ 2 & 3 \\ 2 & 1 \end{array} \begin{array}{l} 1 \\ 1 \\ 1 \\ 1 \end{array} \uparrow$$

1111

$$\begin{array}{r|l} 2 & 2 \\ \hline 2 & 1 \\ \hline & 0 \end{array} \begin{array}{l} 0 \\ 1 \end{array} \uparrow$$

0010

- Base 3 to Base 7

↘ Base 10 ↗

- Base 4 to Base 8

↘ Base 10 ↗
↘ Base 2 ↗

ex- $(3123)_4 \rightarrow (333)_8$

↓ ↓ ↓ ↓
11 10 10 11
↓ ↓
 $\frac{110}{3}$ $\frac{100}{3}$

- COMPLEMENT :

- 1's complement

- 2's complement

* Complements are to represent -ve number

for 1's complement, "reverse each bit"

ex- 100101 $\xrightarrow{\text{1's complement}}$ 011010

2's complement \rightarrow 1's complement + 1

ex- 100101 $\xrightarrow{\text{2's complement}}$ 011011

reverse each

bit after 1st 1 bit

starting from L to R

ex- 00010100

011010

+ 1

011011

$\xrightarrow{\text{2's complement}}$

11101100

11101011

reversing after this 1

+ 1

11101100

- Subtraction using 2's Complement

$$[A - B = A + \text{2's complement of } B]$$

Towards the end

\rightarrow If carry's there, \Rightarrow Ans is +ve for result, ignore the carry.

\rightarrow If carry not there \Rightarrow Ans is -ve for result, obtain the 2's complement of the Ans, that'd be the final result.

ex- A = 11001 B = 10110 find A-B.

11001

01010

100011

\rightarrow carry

hence Ans \rightarrow 00011

In every base, We can define 2 complements,

Base $r \rightarrow r$'s complement [Radix complement]

$\hookrightarrow (r-1)$'s complement [Diminished Radix complement]

$(r-1)$'s complement $\rightarrow (r^n - 1) - N$

(r) 's complement $\rightarrow r^n - N$

n is no. of digits in given question
 N is no. given in the question.

Subtraction doesn't exist in computer.

ex- $7 - 3$ can be written as $7 - 3 + 10 = 14$

ex- 124 [9's complement = $(10^3 - 1) - 124 = 875$] [10's $\rightarrow 10^3 - 124 = 876$]

- BOOLEAN ALGEBRA :

basic operations \rightarrow OR AND NOT

\hookrightarrow Closure property (answers should be in set)

- Boolean postulates & Theorem:

- $x + 0 = x$
- $x \cdot 1 = x$
- $x + 1 = 1$
- $x \cdot 0 = 0$
- $x + \bar{x} = 1$
- $x \cdot \bar{x} = 0$
- $x + xy = x(1 + y) = x$
- $A + \bar{A}B = (A + \bar{A})(A + B) = A + B$
- $A \cdot (B + C) = A \cdot B + A \cdot C$
- $A + (B \cdot C) = (A + B) \cdot (A + C)$

- De Morgan's Law:

$$(\overline{A+B}) = \bar{A} \cdot \bar{B}$$

$$(\overline{AB}) = \bar{A} + \bar{B}$$

- DUAL of an expression:

$$A + \bar{A} = 1 \quad \Leftrightarrow \quad A \cdot \bar{A} = 0$$

$$+ \longrightarrow \cdot$$

$$\cdot \longrightarrow +$$

$$1 \longrightarrow 0$$

$$0 \longrightarrow 1$$

ex- $A + \bar{A}B = A + B$

$$A \cdot (\bar{A} + B) = A \cdot B$$

$$A \cdot B = A \cdot B$$

* n bit variable - 2^n combinations

A	B	A+B	A.B
0	0	0	0
0	1	1	0
1	1	1	1
1	0	1	0

Design:

1. Identify the number of inputs and outputs

2. Write O/P for each combination of I/P.

(Make the Truth table).

	A	B	S	SOP	POS
0→	0	0	0	$\bar{A}\bar{B}$	$A+B$
1→	0	1	1	$\bar{A}B$	$A+\bar{B}$
2→	1	1	0	AB	$\bar{A}+\bar{B}$
3→	1	0	1	$A\bar{B}$	$\bar{A}+B$

$$SOP = \sum (1, 2)$$

$$= \bar{A}B + A\bar{B}$$

$$POS = \prod (0, 3)$$

$$= (A+B) \cdot (\bar{A}+\bar{B})$$

sequencing
through Gray Code

0→ \bar{A}, \bar{B}
1→ \bar{A}, B
minterms

0→ A, B
1→ A, \bar{B}
maxterms

↓
[collect all 1s] [collect all 0s]

Binary	Gray Code
0	0
1	1
2	3
3	2

* Gray code is used so that ek bit se next bit me jaane se sirf 1 bit ka change ae. Bad circuit me isse time difference nahi.

Each successive member differs just by 1 bit...

- CANONICAL FORM - STANDARD FORM of an expression.

ex- $Y = AB + B\bar{C}$ (Standard form) [SOP case]

$$\begin{aligned}
 & AB \cdot 1 \quad B\bar{C} (A + \bar{A}) \\
 & AB(C + \bar{C}) \quad AB\bar{C} + \bar{A}B\bar{C} \\
 & \underbrace{ABC + AB\bar{C}}_{2^2+2+1=7} \quad \underbrace{AB\bar{C} + \bar{A}B\bar{C}}_{2^2+2+0=6} \quad \underbrace{\quad}_{2=2} \\
 & Y = \sum (2, 6, 7)
 \end{aligned}$$

$$Y = \sum (2, 6, 7)$$

(Canonical form)

ex- $(A+B)(\bar{B}+C)$ (POS case)

$$\begin{aligned}
 & A+B+C\bar{C} \quad A\bar{A} + \bar{B} + C \\
 & (A+B+C)(A+B+\bar{C}) \quad (A+\bar{B}+C)(\bar{A}+\bar{B}+C) \\
 & \begin{array}{cc} 000 & 001 \\ \hline 0 & 1 \end{array} \quad \begin{array}{cc} 010 & 110 \\ \hline 2 & 6 \end{array}
 \end{aligned}$$

$$Y = \prod (0, 1, 2, 6)$$

* We can convert SOP to POS and vice versa by writing canonical form of the expression first, then change \sum to \prod or \prod to \sum as per question, then we can write the Standard form using the converted canonical form.

ex- Convert $AB + \bar{B}C$ to POS

$$AB(C + \bar{C}) \quad (A + \bar{A})\bar{B}C$$

$$\underbrace{ABC}_{7} + \underbrace{AB\bar{C}}_{6} \quad \underbrace{A\bar{B}C}_{5} + \underbrace{\bar{A}\bar{B}C}_{1}$$

$$Y = \sum (1, 5, 6, 7) = \prod (0, 2, 3, 4)$$

$$= (A + B + C) \cdot (A + \bar{B} + C) \cdot (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + C)$$

- COMPLEMENT OF A FUNCTION :

ex- $X = AB + BC$

$$\begin{aligned} \bar{X} &= \overline{(AB + BC)} = (\overline{AB}) \cdot (\overline{BC}) \\ &= (\bar{A} + \bar{B}) \cdot (\bar{B} + \bar{C}) \\ &= \bar{A}\bar{B} + \bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C} \end{aligned}$$

ex- $X = \sum (1, 2, 3)$

$$\bar{X} = \sum (0, 4, 5, 6, 7)$$

- CONSENSUS THEOREM :

When 3 terms, one with x one with x' , third gets cancelled. Recognise the pattern.

$$\begin{aligned} \text{ex- } xy + x'z + yz &= xy + x'z \\ xy' + x'z + yz &= xy' + x'z \\ xy' + x'z' + yz' &= xy' + x'z' \end{aligned}$$

derivations of consensus:

ex - $xy + x'z + yz$
 $xy + x'z + (x+x')yz$
 $xy + xyz + x'z + x'yz$
 $xy(1+z) + x'z(1+y) = xy + x'z$

ex - $xy' + x'z + y'z$
 $xy' + x'z + (x+x')y'z$
 $xy' + x'z + xy'z + x'y'z$
 $xy'(1+z) + x'z(1+y')$
 $= xy' + x'z$

- KARNAUGH MAP:

- 2 Variable

	\bar{B}	B
\bar{A}	0	1
A	2	3

- 3 variable

$\bar{A}\bar{B}$	0	1
$\bar{A}B$	2	3
AB	6	7
$A\bar{B}$	4	5

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	0	1	3	2
A	4	5	7	6

A	B	C	
0	0	0	→ 0
0	0	1	→ 1
0	1	0	→ 2
0	1	1	→ 3
1	0	0	→ 4
1	0	1	→ 5
1	1	0	→ 6
1	1	1	→ 7

- 4 variables

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
AB	12	13	15	14
$A\bar{B}$	8	9	11	10

CD \ AB	0	4	12	8
1	5	13	9	
3	7	15	10	
2	6	14	11	

- 5 variables

BC \ DE	0	1	3	2
4	5	7	6	
12	13	15	14	
8	9	11	10	

A = 0

BC \ DE	16	17	19	18
20	21	23	22	
28	29	31	30	
24	25	27	26	

A = 1

- GROUPING :

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	1
$\bar{A}B$	1	1	0	0
$A\bar{B}$	0	0	1	1
AB	0	0	1	1

$\bar{A}\bar{C}D$ (points to cell 1)
 $\bar{B}C\bar{D}$ (points to cell 1)
 AC (points to cell 1)
 $Y = \sum (1, 2, 4, 5, 10, 11, 14, 15)$
 $Y = AC + \bar{B}C\bar{D} + \bar{A}B\bar{C} + \bar{A}\bar{C}D$

- Prime Implicants :

check each group if member if the group contains atleast 1 member ie not the part of any other group.

- 5 variable KMap.

		$\bar{D}\bar{E}$ $\bar{D}E$ DE $D\bar{E}$			
$\bar{B}\bar{C}$	$\bar{D}\bar{E}$	0	0	0	0
	$\bar{D}E$	1	0	0	1
	DE	0	0	0	1
	$D\bar{E}$	0	0	0	0
$\bar{B}C$	$\bar{D}\bar{E}$	0	0	0	0
	$\bar{D}E$	1	0	0	1
	DE	0	0	0	1
	$D\bar{E}$	0	0	0	0

$A=0$ $\bar{A}\bar{C}\bar{E} + A\bar{C}\bar{E}$
 $= \bar{C}\bar{E}$

- DON'T CARE CONDITION:

need ke according kuchh bhī bhare skete h
 ex- $Y = \sum (0, 2, 5) + d(6, 7)$

		$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1	0	0	1	1
A	0	1	x	x	x

$Y = \bar{A}\bar{C} + AC$

		\bar{B}	B
\bar{A}	0	1	1
A	1	0	0

$\bar{A}B + A\bar{B} = A \oplus B$

Diagonal elements can be implemented
 using XOR or XNOR.