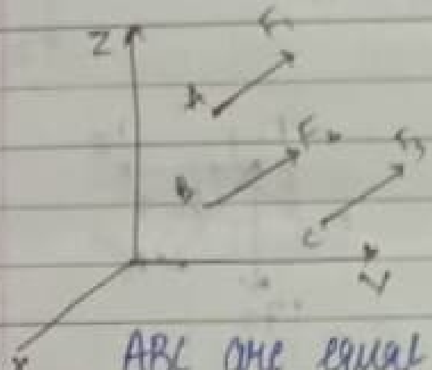
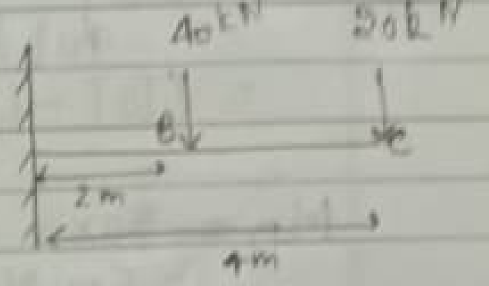
Unit - 1

Fundamentals of Engineering Mechanics

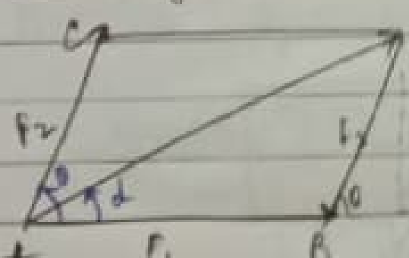


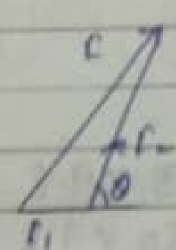
ABC are equal



40 & 20 kN are equivalent

- **Parallelogram law of forces**





(Triangle law of forces)

$$F = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$
$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\boxed{\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}}$$

* **Several ∥ forces acting**

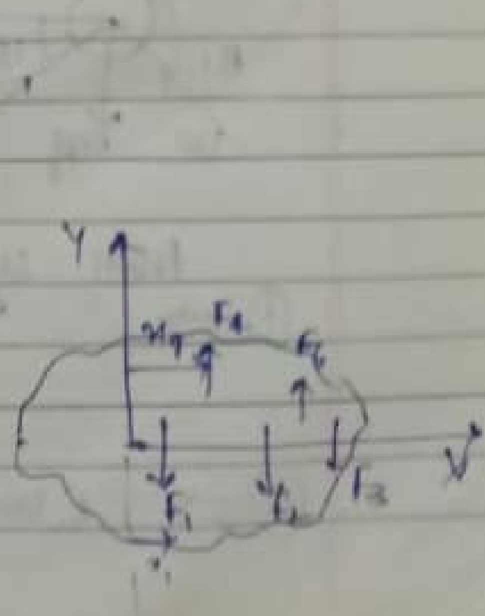
For eq^m

$\Sigma F_i = 0$

$F_1 + F_2 + F_3 + \dots = 0$

$\Sigma F_i x_i = 0$

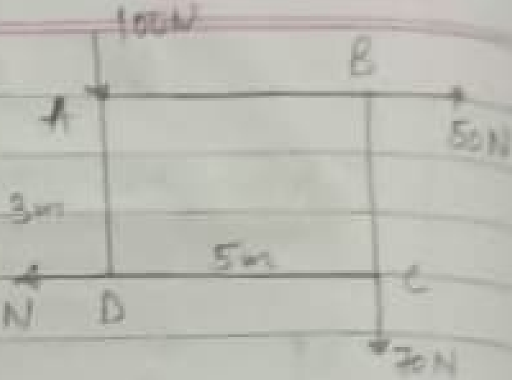
$F_1 x_1 + F_2 x_2 \dots F_n x_n = 0$



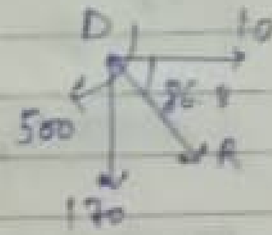
Example

$F_A = -100\hat{j}$ $\quad F_B = 50\hat{i}$

$F_C = -70\hat{j}$ $\quad F_D = -40\hat{i}$



$R = (-100\hat{j} - 70\hat{j} + 50\hat{i}$ 40N
$-40\hat{i})$ N
$= (10\hat{i} - 170\hat{j})$ N

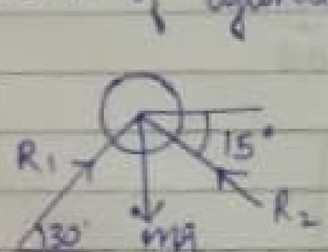


$M_D = 50 \times 3 + 70 \times 5$
$= 500$ N m

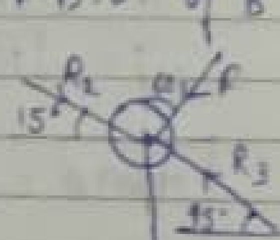
Example

$m_1 = 75 \times 9.8 = 735$ N
$m_2 = 50 \times 9.8 = 490$ N

F.B.D. of cylinder A



F.B.D. of B



From cylinder A FBD

(1) $\Rightarrow$ $R_1 \cos 30° = R_2 \cos 15°$
$R_1 = 1.1154 R_2$ — (1)

(2) $\Rightarrow$ $R_1 \sin 30 + R_2 \sin 15° = 735$
$R_2 = 900.2$ N

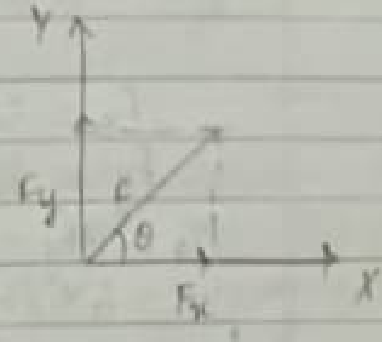
From FBD of B

$R_1 \cos 15 = F \sin 60 + R_2 \cos 45°$

$R_2 \sin 15 + F \cos 60° + m_B g = R_2 \sin 45$

$F = \underline{107.2\,N}$

* Rectangular components

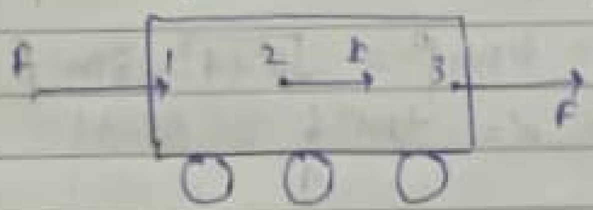


$F_x = F\cos\theta$

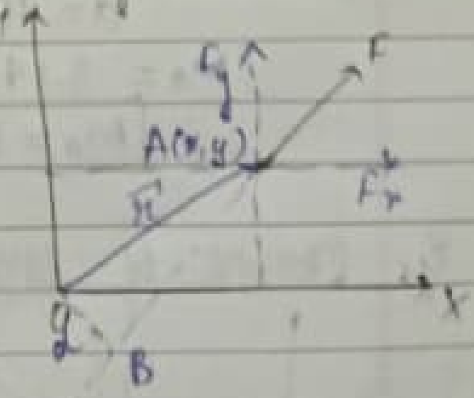
$F_y = F\sin\theta$

$F = F_x \hat{i} + F_y \hat{j}$

* Principle of transmissibility for force



* Moment of a force about origin



$\vec{r} = x\hat{i} + y\hat{j}$

$M_O = \vec{r} \times \vec{F}$ (cross product)

$= (x\hat{i} + y\hat{j}) \times (F_x\hat{i} + F_y\hat{j})$

$= (x F_y - y F_x)\hat{k}$

Also scalar method

$M_O = OB \times F = d \times F$

(+ve CCW, -ve CW)

Ex - A force of magnitude 100 N is passing through a point A (6, 4) m on a body as shown in figure. Force 100 N is inclined at angle 30° with the vertical. Determine the moment of force about 'O' of co-ordinate system.

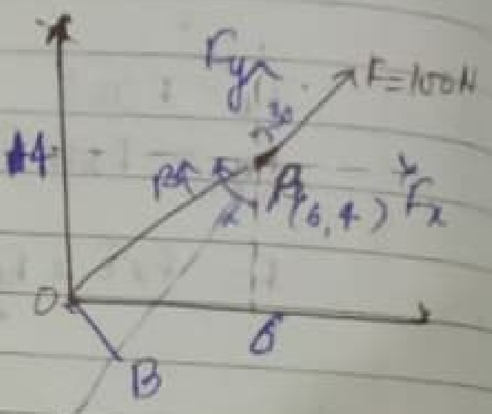
$F_x = 50\ N$

$F_y = 86.6\ N$

$M_0 = x F_y - y F_x$ → by dir.

$= 6 \times 86.6 + 4 \times 50$

$\boxed{M_0 = 314.6 \text{ CCW (N-m)}}$



2nd approach

Scalar Method

$OB = 4 \sin\beta = \sqrt{6^2+4^2}\ \sin\beta$

$\angle OAC = \alpha = \tan^{-1}\frac{6}{4} = 56.31$

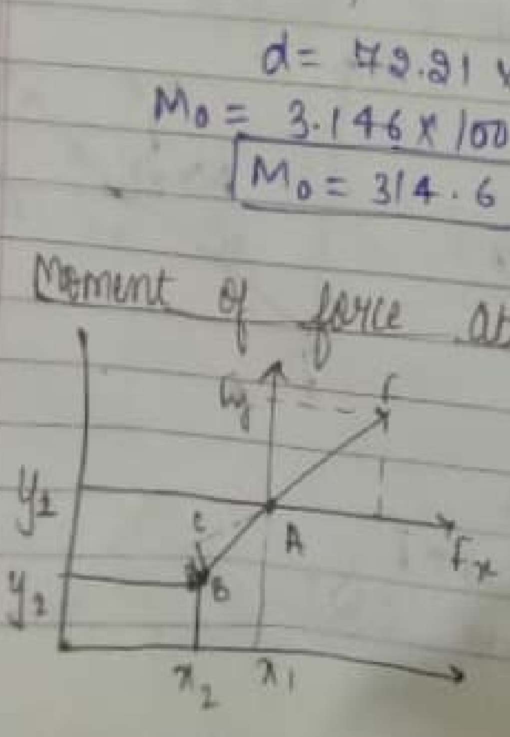
$\angle OAB = \beta = 56.31 - 30 = 26.31$

$d = 7.21 \times \sin 26.31 = 3.146$

$M_0 = 3.146 \times 100$

$\boxed{M_0 = 314.6\ Nm}$

* Moment of force about a point in space -



$M_B = \vec{r}_{BA} \times F$

$\vec{r}_{BA} = \vec{r}_A - \vec{r}_B$

$= (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j}$

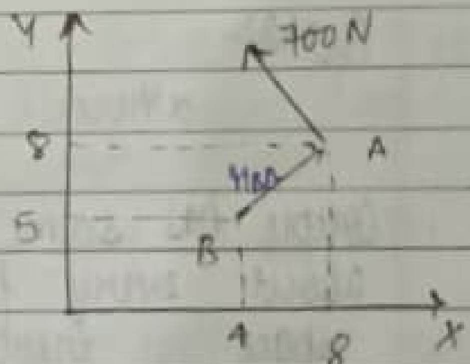
$M_B = (x_1 - x_2) F_y - (y_1 - y_2) F_x \quad \hat{k}$

Scalar Method

$M_B = BC \times F$

$= d \times F$

Ex- A force F, 700 N is applied at a point A on a body as shown. Force is inclined with x-axis at angle 30° as shown. Coordinates of point A are (8,8). What is the moment of this force about B(4,5) in the space



$F_x = -F\cos 30°$

$= -606.2N$

$F_y = F\sin 30°$

$= 350 N$

$r_{BA} = (8-4)\hat{i} + (8-5)\hat{j}$

$M_B = (x_A - x_B) F_y - (y_A - y_B) F_x$

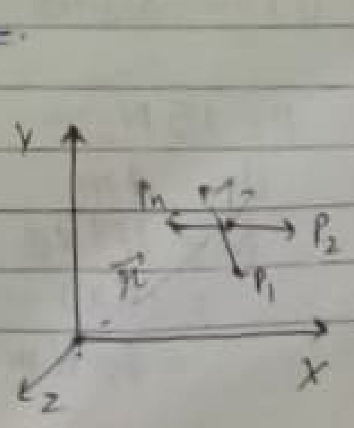
$= 4 \times 350 - 3(606.2)$

$= 3218.6 \hat{k}$ (CW N.m

* Varignon's Theorem

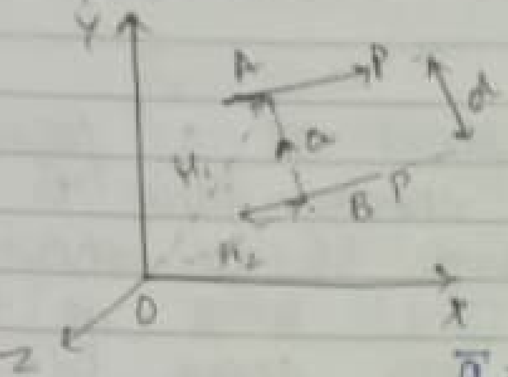
$M_0 = \vec{r} \times P_1 + r \times P_2 + \dots + r \times P_n$

Distributed property of cross product



$$M_0 = \vec{r} \times (P_1 + P_2 + \quad + P_n)$$
$$= \vec{r} \times \vec{P}_R \quad \text{(where } P_R \text{ is resultant forces)}$$

* <u>Couple and its Moments</u>



$$M = \vec{r_1} \times P + \vec{r_2}(-P$$
$$= (\vec{r_1} - \vec{r_2}) \times P$$

$$(\vec{r_1} - \vec{r_2}) = \vec{a}$$
$$= \vec{a} \times P$$

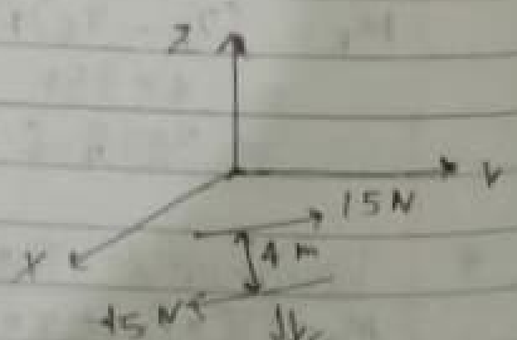
$\vec{a}$ = is displacement vector b/w

<u>Scalar</u>.
$$M = |P| \times d$$
where d is $\perp$r distance

Couple has same moment about every point in space, therefore couple moment is free vector

couple vector

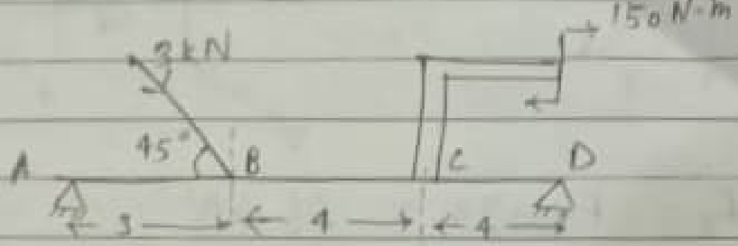
<u>Ex</u> Find couple moment
i) about origin
ii) about (2, 3, -2)



i) $P = 15\,N$
$d = 4\,m$
$M = |P| \times d$
$= 15 \times 4$
$= 60\,N{\cdot}m$ (Right-hand screw rule)
$M = -60\,N{\cdot}m$

ii) Since C is free vector. So, moment about (2, 3, -2) will be same.

Ex.

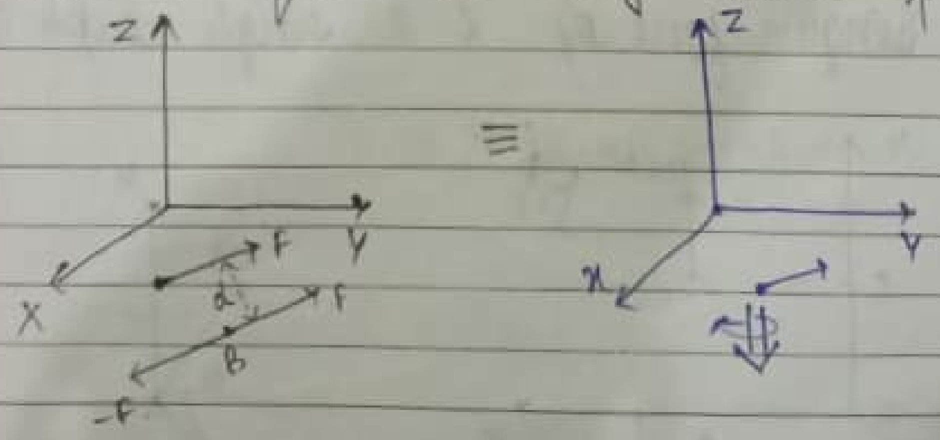


$$M_A = 3\sin 45 \times 3 \times 1000 + 150\ N\text{-}m$$
$$= 6513\ N\text{-}m \quad (CW)$$
$$M_C = 3\sin 45 \times 8 \times 1000 - 150\ N\text{-}m$$
$$= 16818\ N\text{-}m \quad (CCW)$$

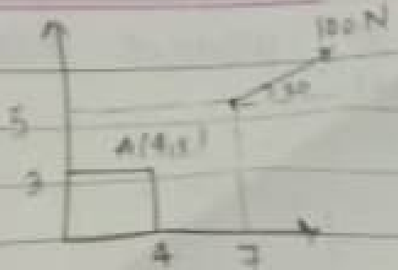
* Resultant Force System

i) Sum of concurrent forces is a single force
ii) Force may be moved along line of action.
iii) Effect of a couple is only a couple.

* Replacement of a force by equivalent couple and a force at any other point :



Ex



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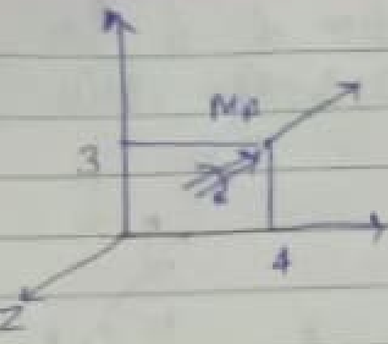
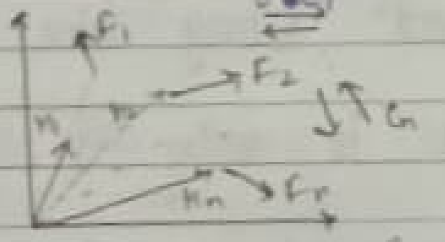


$$F_x = F\cos 30 = 86.6\,N$$
$$F_y = F\sin 30 = 50\,N$$

$$F_A = 86.6\,\hat{i} + 50\,\hat{j}\ N$$

$$M_A = (7-4)F_y - (5-3)F_x$$
$$= 150 - 173.2$$
$$= -23.2\ Nm$$

* Resultant of a force system

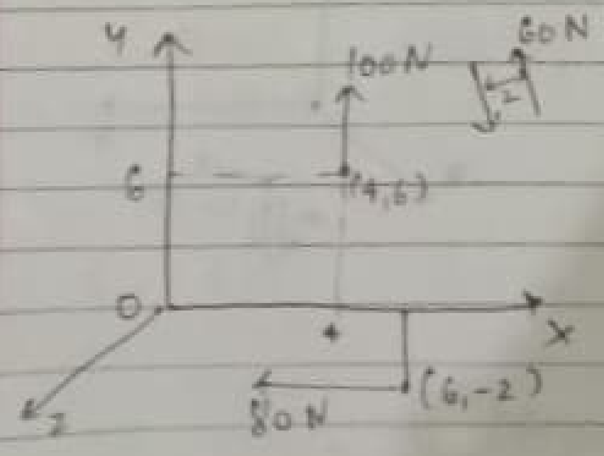


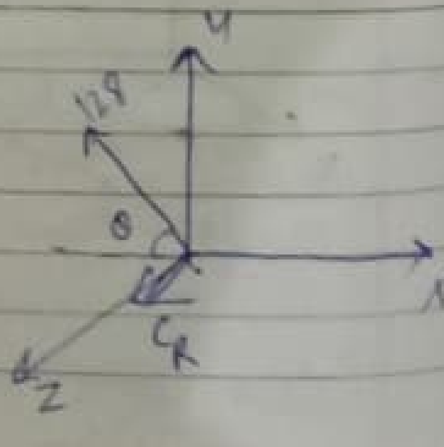
$$F_R = F_1 + F_2 + \dots + F_N$$
$$= \sum_{i=1}^{n} F_x\hat{i} + \sum_{i=1}^{n} F_y\hat{j}$$

$$C_R = r_1 \times F_1 + r_2 \times F_2 + \dots + r_n \times F_n + C_1 + C_2 + \dots + C_n$$

Single force F_R & a single couple C_R

Ex





$F_R = \sqrt{(80)^2 + (100)^2} = 128\,N$

Resultant Couple at O

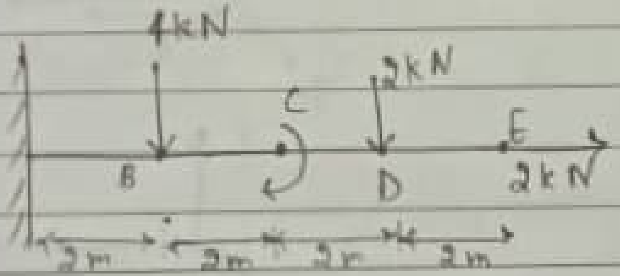
$M_O = -80 \times 2 + 100 \times 4 + 60 \times 2$

$= 360\,Nm$ CCW along z

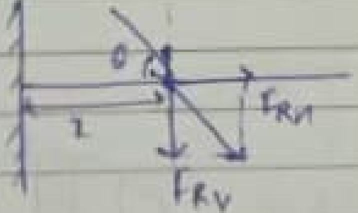
$\theta = \tan^{-1} \frac{100}{80} = 51.34°$

- <u>Wrench Resultant</u>

Ex -



Find the resultant of single force & its line of action



$F_{RV} = 4 + 2$

$= 6\,kN \downarrow$

$F_{RH} = 2\,kN \longrightarrow$

$F_{RV} \times x + F_{RH} \times 0 = 4 \times 2 + 2 \times 6 + 5\,kNm$

$= 25\,kNm$

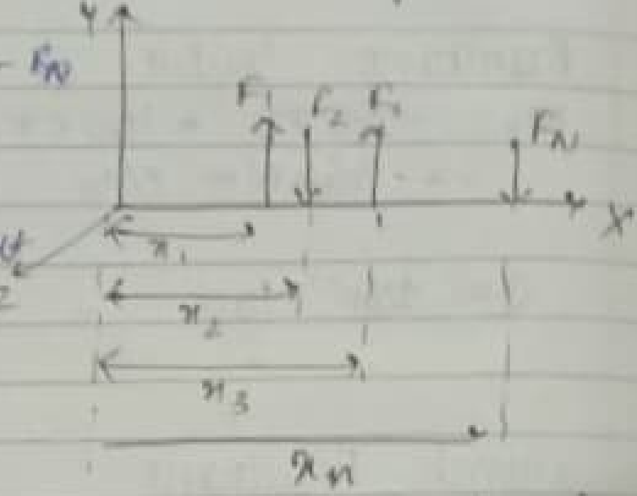
$x = \frac{25}{6} = 4.16\,m$

$F_R = \sqrt{6^2 + 2^2} = 6.32$

$\theta = \tan^{-1} \frac{F_{RV}}{F_{RH}}$

* Resultant of Parallel Force System –

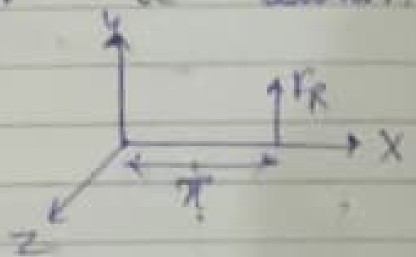
$F_R = F_1 - F_2 + F_3 + \dots - F_N$



Line of action
Taking moment about O

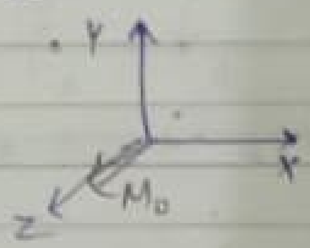
$M_0 = F_1 x_1 - F_2 x_2 + F_3 x_3 \dots F_n x_n$

Say resultant act at a distance $\bar{x}$ from O



$F_R \bar{x} = M_0$

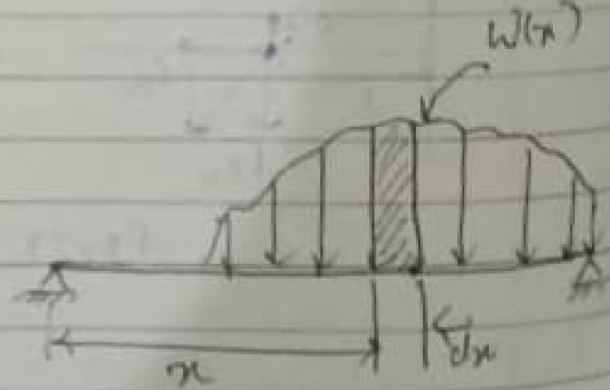
$\bar{x} = \dfrac{M_0}{F_R}$



* Distributed Force System

$dF_x = w(x)\, dx$

where $w(x)$ is intensity of loading



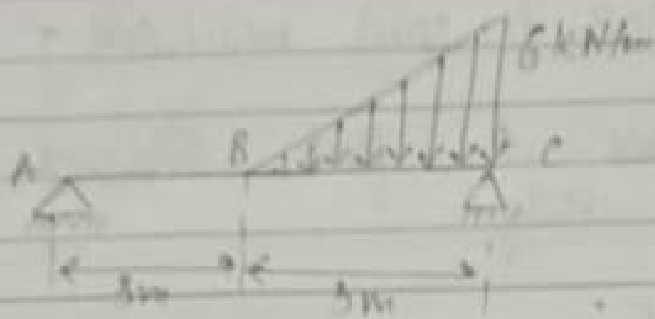
Resultant load on the beam

$F_R = -\int w(x)\, dx$

Position,

$\bar{x} = \dfrac{-\int x\, w(x)\, dx}{-\int w(x)\, dx}$

Ex -



say $w(x) = ax + b$

$w(x) = 0$ at $x = 3$

$w(x) = 6$ at $x = 6$

$\Rightarrow$ $0 = 3a + b$

$6 = 6a + b$

$a = 2$, $b = -6$

$w(x) = 2x - 6$

Resultant force $= F_R = \int_3^6 w(x)\,dx$

$= 9\,kN$

Moment of distributed load about A

$\bar{x}\,F_R = \int_3^6 x\,w(x)\,dx$

$= \int_3^6 x(2x-6)\,dx$

$= 45$

$\bar{x} = \underline{5\,m}$

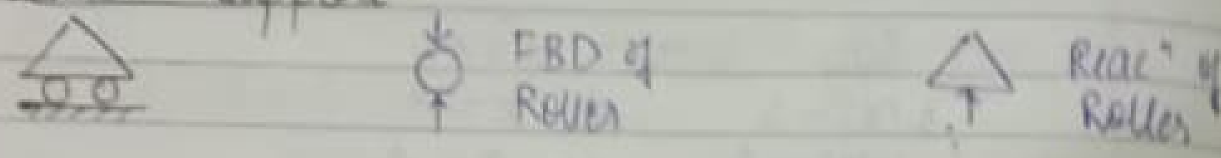
* Free Body Diagram (F.B.D) -

A sketch of the object showing all the actions & reactions at proper places is called F.B.D of the object

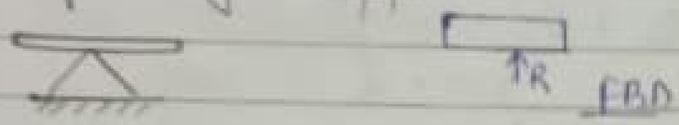
→ Common Supports & their reactions –

a) Belts Rope support: A force directed away from the body

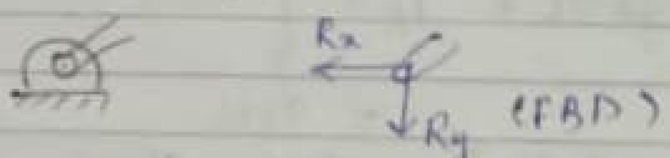
b) Roller support:



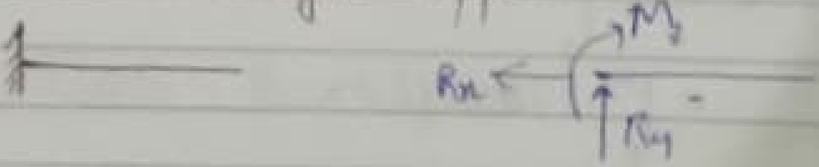
c) Knife Edge support:



d) Pin or Hinged Joint:



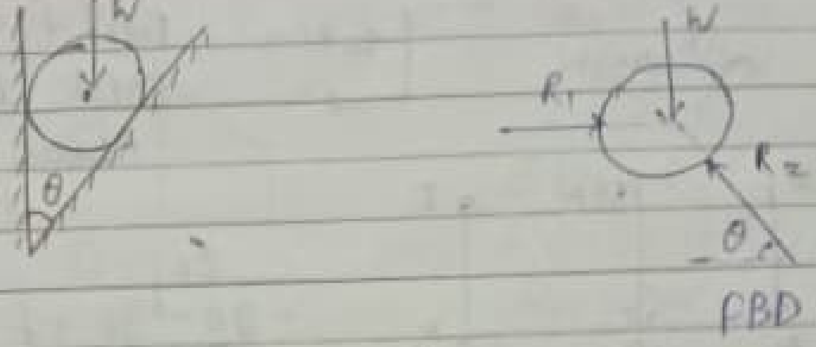
e) Fixed or Rigid support:



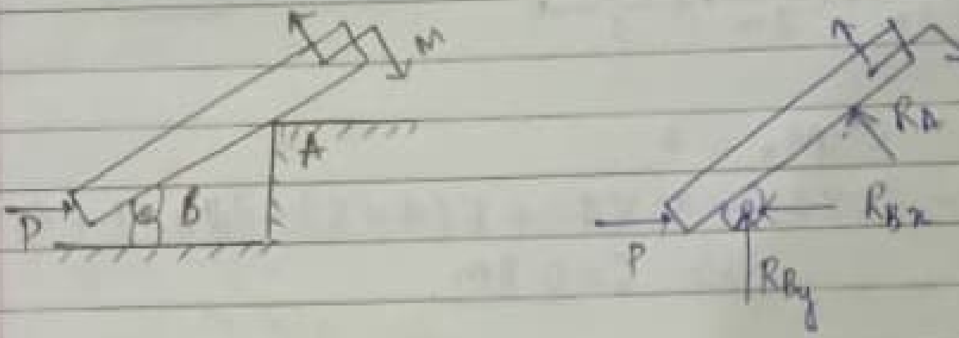
f) Smooth Surface:

g) Friction Surface:

Ex -



Ex.



* Equilibrium of Rigid Bodies -

When the resultant forces or couple become zero the body is said ~~in~~ to be in eq^m

For eq^m $F_R = 0$, $C_R = 0$

For 3D

$\Sigma F_x = 0$ — (1) $\Sigma F_y = 0$ — (2) $\Sigma F_z = 0$ — (3)

$\Sigma M_x = 0$ — (4) $\Sigma M_y = 0$ — (5) $\Sigma M_z = 0$ — (6)

For 2D coplanar

$\Sigma F_x = 0$ — (1)
$\Sigma F_y = 0$ — (2)
$\Sigma M_z = 0$ — (3)

} Eq^n of Equilibrium

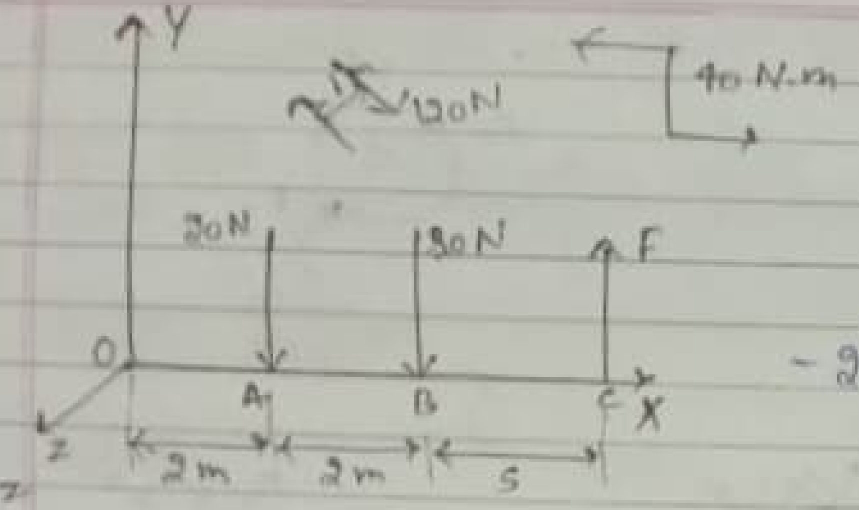
For concurrent forces

$\Sigma F_x = 0$ — (1)
$\Sigma F_y = 0$ — (2)

For ‖ force system

$\Sigma F_y = 0$ — (1)
$\Sigma M_z = 0$ — (2)

Ex



Find F & S
check equilibrium
about B

$\Sigma F_y = 0$

$-20 - 30 + F = 0$

$F = \underline{50N}$

$\Sigma M_z = 0$

$-20\times 2 - 30\times 4 + F(4+S) - 120\times 1 + 40 = 0$

$\Rightarrow S = \underline{0.8m}$

Check about B :- $F_R = 0$

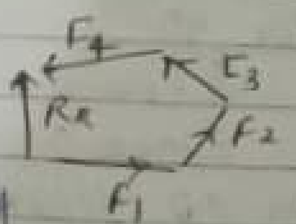
$C_{RB} = 20\times 2 + 50\times 0.8 - 120 + 40$

$\boxed{C_{RB} = 0}$ Equilibrium

* Law Of Polygon

Non-concurrent coplanar forces

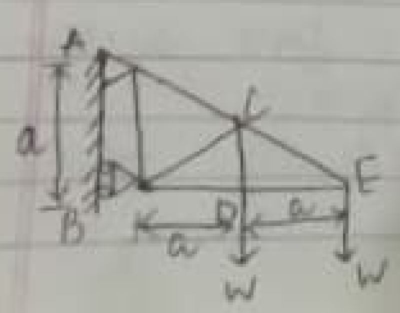
$F_1 + F_2 + \dots + F_N = 0$ vectorially

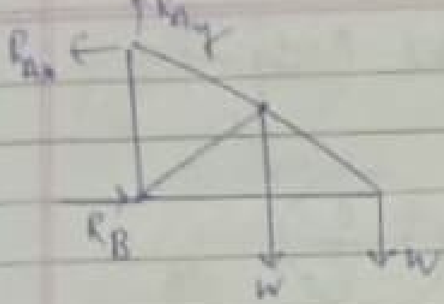


$(r_1\times F_1 + r_2\times F_2 + \dots + r_n\times F_n)$

$+ C_1 + C_2 \dots C_N = 0$

Ex





$R_{Ay} = W + W$

$= 2W \uparrow$

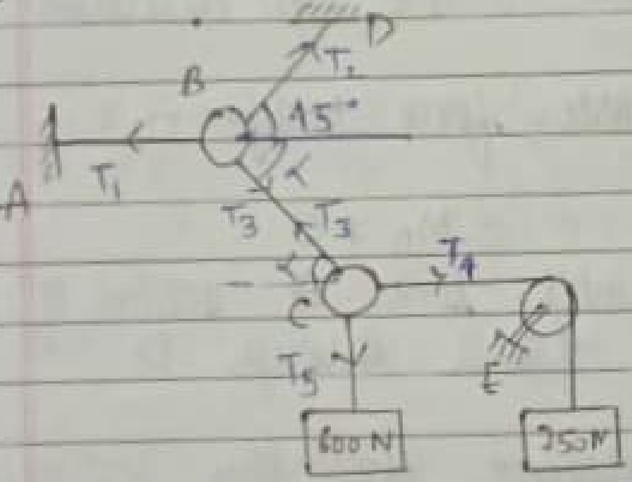
$\Sigma M_B = 0$

$W \times a + W \times 2a = R_{Ax} \times a$

$R_{Ax} = 3W$

$R_B = 3W$

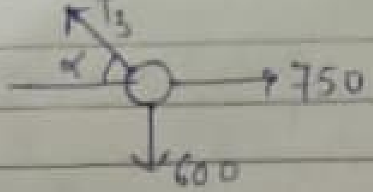
Ex -



$T_4 = 750N$

$T_5 = 600N$

FBD of C



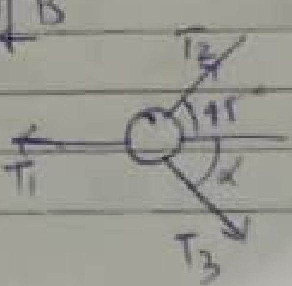
$T_3 \cos\alpha = T_4 = 750$

$T_3 \sin\alpha = T_5 = 600$

$\alpha = 38.66$

$T_3 = 961.5\ N$

FBD of B

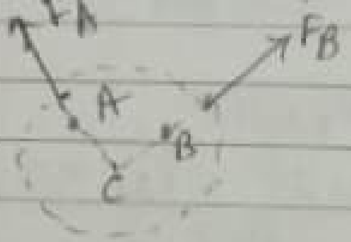


$T_1 = T_2 \cos 45 + T_3 \cos\alpha$

$T_2 \sin 45 = T_3 \sin\alpha$

$T_2 = 849.6\ N$

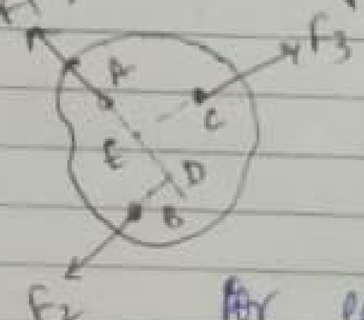
* Equilibrium of a Two force Body-



$\Sigma M_C = 0$
But $\Sigma F_x \neq 0$
$\Sigma F_y \neq 0$

Line of action should be same
| Same magnitude & opp. direction |

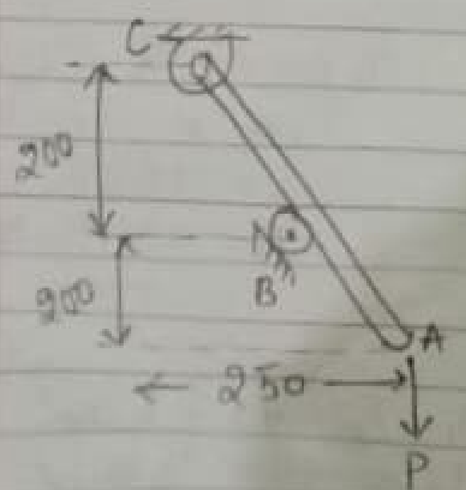
* Equilibrium of three force body-

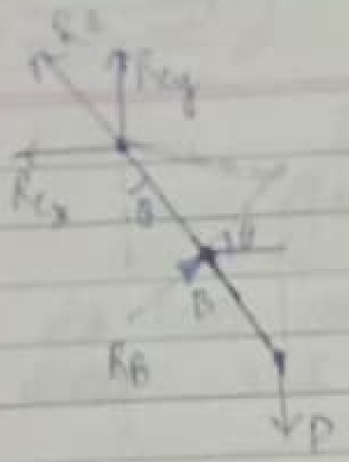


$\Sigma M_D \neq 0$
Until line of action of F_3 passes through D

For equilibrium the line of action of three forces must be concurrent (Only for non // forces)

Ex -





FBD

$$CB = \sqrt{(200)^2 + (125)^2}$$
$$= 235.8 \text{ mm}$$

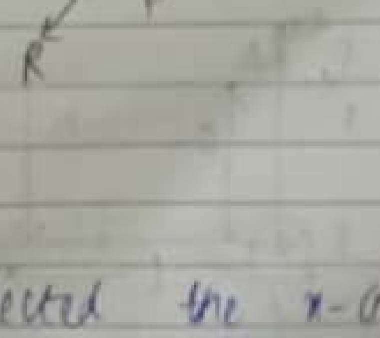
$$\Sigma M_c = 0$$
$$\Rightarrow R_B \times CB - 200 \times 250 = 0$$
$$R_B = 212 N$$

$$\tan\theta = \frac{250}{400} \Rightarrow \theta = 32^\circ$$

$$\Sigma F_x = 0 \Rightarrow R_B \cos\theta = R_{cx}$$
$$\Sigma F_y = 0 \Rightarrow R_B \sin\theta - 200 + R_{cy} = 0$$
$$R_{cy} = 87.6 N \qquad R_{cx} = 179.8 N$$

$$\underline{R_c = 200 N}$$

* Lami's Theorem



$$\Sigma F_x = 0 \quad \text{--- (1)}$$
$$\Sigma F_y = 0 \quad \text{--- (2)}$$

(2) $\Rightarrow$

$$Q \sin\gamma = R \sin\beta$$
$$\frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

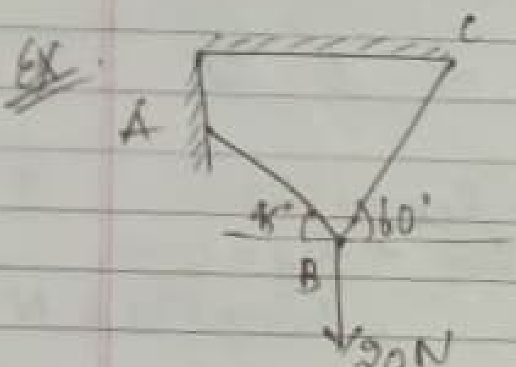
Similarly had we selected the x-axis coinciding with Q, the result would have been

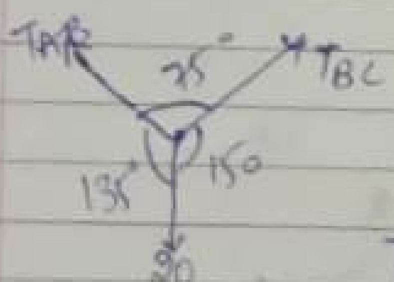
$$\frac{P}{\sin\alpha} = \frac{R}{\sin\gamma}$$

$$\Rightarrow \frac{P}{\sin\alpha} = \frac{R}{\sin\gamma} = \frac{Q}{\sin\beta}$$

The ratio of a force and sin of angle between other two forces is constant

Ex.

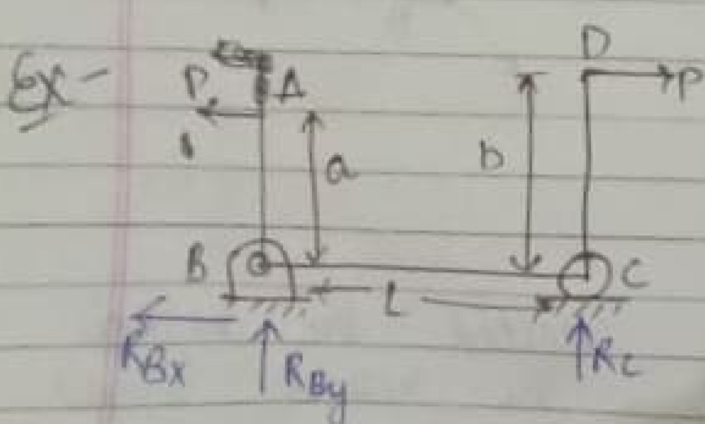




$$\frac{T_{AB}}{\sin 150} = \frac{T_{BC}}{\sin 135} = \frac{20}{\sin 75}$$

$T_{AB} = 1035\,N$ $\quad$ $T_{BC} = 1464\,N$

Ex -



$\Sigma M_B = 0 \Rightarrow P \times b - P(a) - R_C \times L = 0$

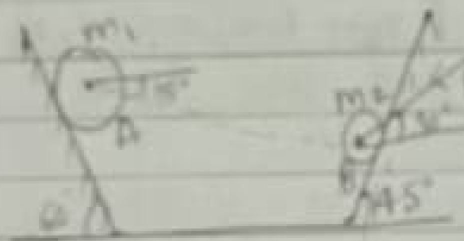
$$R_C = \frac{P(b-a)}{L}$$

$\Sigma F_y = 0$

$$R_{By} + R_C = 0$$
$$R_{By} = -\frac{P(b-a)}{L}$$

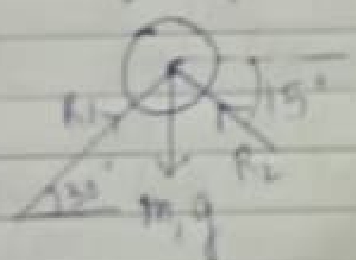
$\Sigma F_x = 0 \qquad R_{Bx} = 0$

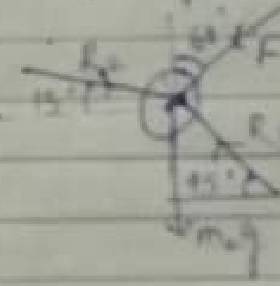
Ex



$$m_1 = 75 \times 9.8 = 735\,N$$
$$m_2 = 50 \times 9.8 = 490\,N$$

FBD of cylinder A | FBD of cylinder B





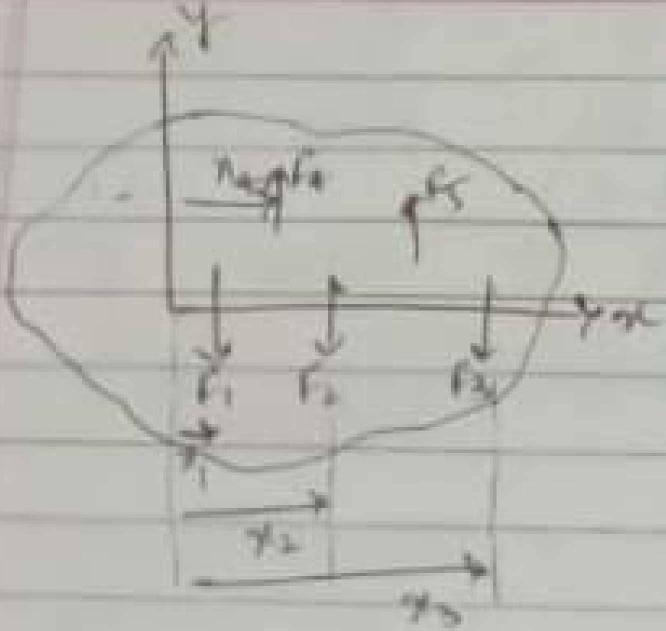
From cylinder A FBD

(1) ⇒ $R_1 \cos 30° = R_2 \cos 15°$
$$R_1 = 1.1154\,R_2 \quad \text{——} (1)$$

(2) ⇒ $R_1 \sin 30 + R_2 \sin 15 = 735 \quad \text{—}(2)$
$$\Rightarrow R_2 = 400.2\,N$$

From FBD of B,
$$R_2 \cos 15 = F \sin 60 + R_3 \cos 45$$
$$R_2 \sin 15 + F \cos 60 + m_2 g = R_3 \sin 45°$$
$$F = 107.2\,N$$

*



For equilibrium

$$\Sigma F_i = 0$$

$$\Rightarrow F_1 + F_2 + \cdots F_3 \cdots = 0$$

$$\Sigma F_i x_i = 0$$

$$F_1 x_2 + F_2 x_2 + \cdots F_n x_n = 0$$