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Subject → E. Mathematics (II)
(Assignment -03)

Branch → Computer Science & Engineering.

Assignment - 03

1. Find - - - - - - - - - - - - - - - - - - $\frac{b}{(S-a)^2+b^2}$

Ans $L[e^{(a+ib)t}] = \int_0^\infty e^{-st} e^{(a+ib)t} t^a \, dt$

$$\underbrace{\int_0^\infty e^{-t[s-(a+ib)]} t^{1-1} \, dt}_{\text{Gama fun}^n}$$

Using property :-

$$\int_0^\infty e^{-zt} t^{h-1} \, dt = \frac{\Gamma h}{z^h} \quad (h>0)$$

$$= \frac{\Gamma 1}{[s-(a+ib)]} \Rightarrow \Gamma n = (n-1)!$$

$$= \frac{1}{s-(a+ib)} \quad \text{Ans}$$

$$L[\cos bt] = \frac{s}{(s^2+b^2)}$$

we know that $L[f(t)e^{at}] = F(s-a)$

$$L[e^{at}\cos bt] = \frac{(S-a)}{(S-a)^2+b^2}$$

Hence proved

again

$$L[\sin bt] = \frac{b}{s^2+b^2}$$

using property $L[f(t)\, e^{at}] = F(s-a)$

$$L[e^{at} \sin bt] = \frac{b}{(s-a)^2+b^2}$$ Hence proved

② Find the Laplace of the following functions:-

ⓐ $t^3 \cos t$

$$L(\cos t) = \frac{s}{(s^2+1)}$$

using property $L[t^n \cos t] = (-1)^n \frac{d^n}{ds^n} \left\{ \frac{s}{(s^2+1)} \right\}$

$$L\{t^3 \cos t\} = -\frac{d^3}{ds^3} \cdot \frac{s}{(s^2+1)}$$

$$= \frac{d^2}{ds^2} \left\{ \frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2} \right\}$$

$$= \frac{d}{ds} \left\{ -\frac{d}{ds} \frac{1-s^2}{(s^2+1)^2} \right\}$$

$$= \frac{d}{ds} \left\{ \frac{(s^2+1)^2(2s) - (s^2-1)\, 2(s^2+1)\, 2s}{(s^2+1)^{4-1}} \right\}$$

$$= \frac{d}{ds} \left\{ \frac{2s(s^2+1) - 4s(s^2-1)}{(s^2+1)^3} \right\}$$

$$= 2\frac{d}{ds} \left[\frac{(3s-s^3)}{(s^2+1)^3} \right]$$

$$= 2 \left[\frac{(s^2+1)^3 (3-3s^2) - (3s-s^3)\, 3(s^2+1)^2(2s)}{(s^2+1)^{6-2}} \right]$$

$$= 2\left[\frac{(s^2+1)(3-3s^2) - 6s(3s-s^3)}{(s^2+1)^4}\right]$$

$$= \frac{2\cdot 3}{(s^2+1)^4}\left[(s^2+1)(1-s^2) - 2s(3s-s^3)\right]$$

$$= \frac{6}{(s^2+1)^4}\left[1-s^4-6s^2+2s^4\right]$$

$$= \frac{6}{(s^2+1)^4}\left[s^4-6s^2+1\right]$$ Ans

(b) $e^{3t}\sin t$

$$L(\sin t) = \frac{1}{(s^2+1)}$$

using property $[L\, f(t)\, e^{at}] = F(s-a)$

$$L[e^{3t}\sin t] = \frac{1}{(s-3)^2+1}$$ Ans

(3) $$f(t) = \begin{cases} 0 & ;\ 0 \le t \le \pi \\ \sin t & ;\ t \ge \pi \end{cases}$$

Now convert the impulse function (unit function)

$$f(t) = \sin t\, 4(t-\pi)$$

taking Laplace transform both sides

$$L[f(t)] = L[\sin t\, 4(t-\pi)]$$

using property

$$L[f(t)\, 4(t-\pi)] = e^{-as}\, L[f(t+a)]$$

$$L[f(t)] = e^{-\pi s}\, L[\sin(t+\pi)]$$

$$= -e^{-\pi s}\, L[\sin t]$$

$$L[f(t)] = \frac{-e^{-\pi s}}{(s^2+1)} \quad \text{Ans}$$

④ $\frac{(e^{-2t}\sin 3t)}{t}$ is

$$L[e^{-2t}\sin 3t] = \frac{3}{(s+2)^2+9}$$

$$L\left[e^{-2t}\frac{\sin 3t}{t}\right] = 3\int_s^{\infty} \frac{1}{(s+2)^2+s^2}\,ds$$

$$L\left[e^{-2t}\frac{\sin 3t}{t}\right] = \frac{3}{3}\tan^{-1}\left(\frac{s+2}{3}\right)\Big|_s^{\infty}$$

$$= \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s+2}{3}\right)$$

$$L\left[\frac{e^{-2t}\sin 3t}{t}\right] = \frac{\pi}{2} - \tan^{-1}\left(\frac{s+2}{3}\right)$$

$$= \cot^{-1}\left(\frac{s+2}{3}\right) \quad \text{Ans}$$

⑤ $f(t) = \begin{cases} t/a & ; 0 \le t \le a \\ (2a-t)/a & ; a \le t \le 2a \end{cases}$ & $f(t+2a) = f(a)$

Given $f(t+2a) = f(a)$ periodic function

$$L[f(t)] = \frac{1}{(1-e^{-2as})}\left[\underbrace{\int_0^a t/a\, e^{-st}\,dt}_{I_1} + \underbrace{\int_a^{2a} e^{-st}\,\frac{(2a-t)}{a}\,dt}_{I_2}\right]$$

$$L[f(t)] = \frac{1}{a(1-e^{-2as})}[I_1 + I_2]$$

$$I_1 = \int_0^a t\,e^{-st}\,dt$$

$$I_1 = -\frac{te^{-st}}{s} + \frac{1}{s}\int e^{-st}\,dt$$

$$(I_1) = \frac{-t\,e^{-st}}{s} - \frac{1}{s^2} e^{-st} \Big|_0^a$$

$$= \frac{t}{s} e^{-st} + \frac{1}{s^2} e^{-st} \Big|_a^0$$

$$= \frac{1}{s^2} - \left(\frac{a e^{-sa}}{s} + \frac{1}{s^2} e^{-as} \right)$$

$$(I_1) = \left(\frac{1}{s^2} - \frac{a}{s} e^{-as} - \frac{1}{s^2} e^{-as} \right)$$

$$(I_2) = \int_a^{2a} e^{-st} (2a - t)\, dt$$

$$(I_2) = -\frac{(2a-t)}{s} e^{-st} - \int \frac{(-1)}{(-s)} e^{-st}\, dt$$

$$(I_2) = \frac{-(2a-t)\, e^{-st}}{s} + \frac{1}{s^2} e^{-st} \Big|_a^{2a}$$

$$(I_2) = \frac{1}{s^2} e^{-2as} - \left[-\frac{(2a-a)\, e^{-as}}{s} + \frac{1}{s^2} e^{-as} \right]$$

$$(I_2) = \left(\frac{1}{s^2} e^{-2as} + \frac{a}{s} e^{-as} - \frac{1}{s^2} e^{-as} \right)$$

$$L\, f(t) = \frac{1}{a(1+e^{-2as})} \left[I_1 + I_2 \right]$$

$$= \frac{1}{a(1-e^{-2as})} \left[\frac{1}{s^2} - \cancel{\frac{a}{s} e^{-as}} - \frac{1}{s^2} e^{-as} + \frac{1}{s^2} e^{-2as} + \cancel{\frac{a}{s} e^{-as}} - \frac{1}{s^2} e^{-as} \right]$$

$$= \frac{1}{a(1-e^{-2as})} \left[\frac{1}{s^2} \left(1 - 2e^{-as} + e^{-2as}\right) \right]$$

$$= \frac{(1-e^{-as})^{\cancel{2}}}{a s^2 (1+e^{-as})(1-\cancel{e^{-as}})}$$

$$= \frac{(1-e^{-as})}{as^2(1+e^{-as})} \quad \text{Ans}$$

⑥ $f(t) = t^2 u_3(t)$:- - - - - - - - - - function.

Soln

$$f(t) = \begin{cases} 0 & ; t < 3 \\ t^2 & ; t \geq 3 \end{cases}$$

$$f(t) = (t-3+3)^2 \, u(t-3)$$

$$f(t) = [(t-3)^2 + 9 + 6(t-3)] \, u(t-3)$$

$$f(t) = [(t-3)^2 u(t-3) + 6(t-3)u(t-3) + 9u(t-3)]$$

(taking Laplace transform both sides)

$$L[f(t)] = L[(t-3)^2 u(t-3)] + 6L[(t-3)u(t-3)] + 9L(u(t-3))$$

$$L[f(t)] = e^{-3s} L(t^2) + 6e^{-3s} L(t) + 9e^{-3s} L(1)$$

$$L[f(t)] = e^{-3s}\left[\frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s}\right] \quad \text{Ans}$$

③ Find the inverse Laplace of following functions:-

ⓐ $\dfrac{s-2}{s(3+s)}$

Ans

$$\frac{(s-2)}{s(s+3)} = \frac{-2}{3s} + \frac{5}{3(s+3)}$$

taking inverse Laplace transform both sides:-

$$L^{-1}\left(\frac{(s-2)}{s(s+3)}\right) = -\frac{2}{3}t + \frac{5}{3}e^{-3t} \quad \text{Ans}$$

ⓑ $\dfrac{a}{s^2(s^2+a^2)}$

Ans

$$\frac{1}{a}\left[\frac{(s^2+a^2)-s^2}{s^2(s^2+a^2)}\right] = \frac{1}{a}\left[\frac{1}{s^2} - \frac{1}{s^2+a^2}\right]$$

taking inverse Laplace transform:-

$$\Rightarrow \frac{1}{a}\left[L^{-1}\left(\frac{1}{s^2}\right) - L^{-1}\left(\frac{1}{s^2+a^2}\right)\right]$$

$$\Rightarrow \frac{1}{a}\left[t - \frac{1}{a}\sin at\right]$$

$$\Rightarrow \frac{1}{a^2}(at - \sin at)$$ Ans

(4) Find the inverse Laplace transforms of the following functions using convolution

(a) $$\frac{1}{(s^2+a^2)^2}$$

Sol

$$\underset{F_1(s)}{\underbrace{\frac{1}{(s^2+a^2)}}} \cdot \underset{F_2(s)}{\underbrace{\frac{1}{(s^2+a^2)}}}$$

$$f_1(t) = L^{-1}[F_1(s)] = \frac{1}{a}\sin at$$

$$f_2(t) = L^{-1}[F_2(s)] = \frac{1}{a}\sin at$$

Apply convolution theorem:-

$$\boxed{L^{-1}[F_1(s)\,F_2(s)] = \int_0^t f_1(x) f_2(t-x)\,dx}$$

$$\Rightarrow \int_0^t \frac{1}{a}\sin ax \,\frac{1}{a}\sin(at - ax)\,dx$$

$$\Rightarrow \frac{1}{2a^2}\int_0^t [\cos(ax - at + ax) - \cos(\cancel{ax} + at - \cancel{ax})]\,dx$$

$$\Rightarrow \frac{1}{2a^2}\left[\int_0^t \cos(2ax - at)\,dt - \int_0^t \cos at\,dx\right]$$

$$\Rightarrow \frac{1}{2a^2}\left[\frac{1}{2a}\sin(2ax - at)\Big|_0^t - \cos at\,(t - 0)\right]$$

$$\Rightarrow \frac{1}{2a^2}\left[\frac{\sin at}{2a} - t\cos at + \frac{1}{2a}\sin at\right]$$

$$\Rightarrow \frac{1}{2a^3}\left[\sin at - at\cos at\right]$$

Ans

(6) $\frac{1}{(s-2)(s+3)}$

Soln

$\frac{1}{(s-2)}$ $\quad$ $\frac{1}{(s+3)}$

$\downarrow$ $\quad$ $\downarrow$

$F_1(s)$ $\quad$ $F_2(s)$

$f_1(t) = L^{-1}(F_1(s)) = L^{-1}\left(\frac{1}{s-2}\right)$

$f_1(t) = e^{2t}$

$f_2(t) = e^{-3t}$

$$\boxed{L^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(x) f_2(t-x)\,dx}$$

$$L^{-1}[F_1(s)F_2(s)] = \int_0^t e^{2x} e^{-3(t-x)}\,dx$$

$$= e^{-3t}\int_0^t e^{2x+3x}\,dx$$

$$= \frac{e^{-3t}}{5} e^{5x}\Big|_0^t$$

$$= \frac{e^{-3t}}{5}(e^{5t}-1)$$ Ans

(5) Using convolution, solve the initial value problem:—

$y'' + 9y = \sin 3t$, $\quad y(0) = 0$

$y'(0) = 0$

$y'' + 9y = \sin 3t$

(taking Laplace both sides)

$$s^2 y' - s y(0) - y'(0) + 9y = \frac{3}{(s^2+9)}$$

$$(s^2+9)\bar{y} = \frac{3}{(s^2+9)}$$

$$\bar{y} = \underset{\substack{\downarrow \\ f_1(s)}}{\frac{3}{(s^2+9)}} \cdot \underset{\substack{\downarrow \\ f_2(s)}}{\frac{1}{(s^2+9)}}$$

(Apply the inverse laplace both sides)

$$y = L^{-1}\left\{ \underset{\substack{\downarrow \\ f_1(s)}}{\frac{3}{s^2+9}} \quad \underset{\substack{\downarrow \\ f_2(s)}}{\frac{1}{s^2+9}} \right\}$$

$$f_1(t) = L^{-1}\left\{\frac{3}{s^2+9}\right\} = \frac{\cancel{3}}{\cancel{3}} \sin^{-1} 3t$$

$$f_1(t) = \sin 3t$$

$$f_2(t) = \frac{1}{3} \sin 3t$$

Apply the convolution theorem:-

$$L^{-1}\{f_1(s)\, f_2(s)\} = \int_0^t f_1(x)\, f_2(t-x)\, dx$$

$$= \int_0^t \sin 3x \times \frac{1}{3} \sin(3t-3x)\, dx$$

$$= \frac{1}{6}\int_0^t 2\sin 3x \sin(3t-3x)\, dx$$

$$= \frac{1}{6}\int_0^t \left[\cos(3x-3t+3x) - \cos(\cancel{3x}+3t-3x)\right] dx$$

$$= \frac{1}{6}\int_0^t \left[\cos(6x-3t) - \cos 3t\right] dx$$

$$= \frac{1}{6}\left\{\int_0^t \cos(6x-3t)\,dx - \int_0^t \cos 3t\, dx\right\}$$

$$= \frac{1}{6}\left(\frac{\sin(6x-3t)}{6}\Big|_0^t - \cos 3t \; t\right]$$

$$= \frac{1}{6}\left[\frac{2}{6}\sin 3t - t\cos 3t\right]$$

$$y = \frac{1}{36}\left[2\sin 3t - 6t\cos 3t\right]$$

$$y = \frac{1}{18}\left(\sin 3t - 3t\cos 3t\right) \quad \text{Ans}$$

(6) Solve the following initial value problems using Laplace transforms:-

(a) $4y'' - 8y' + 3y = \sin t$ $\qquad y(0) = 0$, $y'(0) = 2$

taking Laplace both sides

$$4[s^2\bar{y} - sy(0) - y'(0)] - 8[s\bar{y} - y(0)] + 3\bar{y} = \left(\frac{1}{s^2+1}\right)$$

$$4[s^2\bar{y} - sy(0) - y'(0)] - 8[s\bar{y} - y(0)] + 3\bar{y} = \frac{1}{s^2+1}$$

$$\bar{y}[s^2\bar{y} - 8y(0) - y'(0)] - 8[s\bar{y} - y(0)] + 3\bar{y} = \frac{1}{s^2+1}$$

$$\bar{y}[4s^2 - 8s + 3) - 4sy(0) - 4y'(0) + 8y(0) = \frac{1}{s^2+1}$$

$$(4s^2 - 8s + 3)\bar{y} - y(0)(8 - 4s) - 4y'(0) = \frac{1}{(1+s^2)}$$

$$\bar{y}(4s^2 - 8s + 9) - 8 = \frac{1}{(1+s^2)}$$

$$\bar{y} = \left[8 + \frac{1}{(1+s^2)}\right]\frac{1}{(2s-1)(2s-3)}$$

$$\bar{y} = \frac{(9 + 8s^2)}{(s^2+1)(2s-1)(2s-3)}$$

Degree of polynomial ($D_r > N_r$)
using Heaviside inverse formula:-

$$L^{-1}(\bar{y}) = L^{-1}\left[\frac{(9+8s^2)}{(s+i)(s-i)(2s-1)(2s-3)}\right]$$

Now roots D_r -

$-i, i, \frac{1}{2}$ & $\frac{3}{2}$ is :-

$$F(s) = (9+8s^2)$$
$$G(s) = (s^2+1)(4s^2-8s+3)$$
$$G(s) = (4s^4-8s^3+7s^2-8s+3)$$
$$G'(s) = (16s^3-24s^2+14s-8)$$

$F(-i) = 1$, $G'(-i) = 2i+16$

$F(i) = 1$, $G'(i) = -2i+16$

$F\left(\frac{1}{2}\right) = 11$, $G'\left(\frac{1}{2}\right) = -5$

$F\left(\frac{3}{2}\right) = 27$, $G'\left(\frac{3}{2}\right) = 13$

Now;

$$L^{-1}\left\{\frac{f(s)}{G(s)}\right\} = \frac{F(-i)}{G(-i)}e^{-it} + \frac{F(i)}{G'(i)}e^{it} + \frac{F\left(\frac{1}{2}\right)}{G'\left(\frac{1}{2}\right)}e^{t/2} + \frac{F(3/2)}{G'(3/2)}e^{3t/2}$$

$$= \frac{1}{2i+16}e^{-it} + \frac{1}{(-2i+16)}e^{it} + \frac{11}{5}e^{t/2} + \frac{27}{13}e^{3t/2}$$

$$= \frac{1}{2}\left[\frac{(8-i)e^{-it} + (8+i)e^{it}}{(64+1)}\right] - \frac{11}{5}e^{t/2} + \frac{27}{13}e^{3t/2}$$

$$= \frac{1}{130}\left[8\left(e^{-it}+e^{it}\right)+i\left(e^{it}-e^{-it}\right)\right]-\frac{11}{5}e^{t/2}+\frac{27}{13}e^{3t/2}$$

$$= \frac{1}{130}\left(16\cos t - 2\sin t\right) - \frac{11}{5}e^{t/2} + \frac{27}{13}e^{3t/2}$$

$$= \left(\frac{8}{65}\cos t - \frac{1}{65}\sin t - \frac{11}{5}e^{t/2} + \frac{27}{13}e^{3t/2}\right)$$ Ans

(b) $y'' + 2y' + 5y = \delta(t-2)$, $y(0) = 0$, $y'(0) = 0$

$y'' + 2y' + 5y$

taking Laplace transform both sides

$$\left[s^2\bar{y} - sy(0) - y'(0)\right] + 2\left[s\bar{y} - y(0)\right] + 5\bar{y} = \left(e^{-2s}\right)$$

$$(s^2+2s+5)\bar{y} + \underbrace{y(0)}_{0}(-s-2) - \underbrace{y'(0)}_{0} = e^{-2s}$$

$$\bar{y} = \frac{e^{-2s}}{(s+1)^2+2^2}$$

(taking inverse Laplace both sides)

$$y = L^{-1}\left\{\frac{e^{-2s}}{(s+1)^2+2^2}\right\}$$

we know that

$$L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\} = \frac{e^{-t}}{2}\sin 2t$$

$$L^{-1}\left\{\frac{e^{-2s}}{(s+1)^2+2^2}\right\} = \frac{e^{-(t-2)}}{2}\sin 2(t-2)\, u(t-2)$$

$$y = \frac{e^{-(t-2)}}{2}\sin(2t-4)\, u(t-2)$$ Ans

ⓒ $ty'' + 2ty' + 2y = 2$ $\qquad y(0) = 1$

$y'(0) = $ arbitary Con^n

(taking laplace transform both sides)

$$-\left[s^2\frac{d\bar{y}}{ds} + \bar{y}\,2s - y(0)\right] - 2\left[s\frac{d\bar{y}}{ds} + \bar{y}\right] + 2\bar{y} = \left(\frac{2}{s}\right)$$

$$-s^2\frac{d\bar{y}}{ds} - 2s\bar{y} - 2s\frac{d\bar{y}}{ds} = \left(\frac{2}{s} - 1\right)$$

$$\frac{d\bar{y}}{ds}(-s^2 - 2s) - 2s\bar{y} = \left(\frac{2}{s} - 1\right)$$

$$\frac{d\bar{y}}{ds}(s^2 + 2s) + 2s\bar{y} = \left(1 - \frac{2}{s}\right) = \frac{s-2}{s}$$

$$\frac{d\bar{y}}{ds} + \frac{2}{(s+2)}\bar{y} = \frac{(s-2)}{s\,(s)(s+2)}$$

$$\frac{d\bar{y}}{ds} + \frac{2}{(s+2)}\bar{y} = \frac{1}{s^2} - \frac{4}{s^2(s+2)}$$

Equating the equation:-

$$\left(\frac{d\bar{y}}{ds} + P\bar{y} = Q\right)$$

$$P = \frac{2}{(s+2)}, \quad Q = \left[\frac{1}{s^2} - \frac{4}{s^2(s+2)}\right]$$

$$IF = e^{\int \frac{2}{s+2}ds} = (s+2)^2$$

$$\bar{y}(s+2)^2 = \int (s+2)^2\left[\frac{1}{s^2} - \frac{4}{s^2(s+2)}\right]ds + c$$

$$\bar{y}(s+2)^2 = \left[\int \frac{(s+2)^2}{s^2}ds - \int \frac{4(s+2)}{s^2}ds\right] + c$$

$$\bar{y}(s+2)^2 = \left[\int\left(\frac{(s^2+4+4s)}{s^2}\right)ds - 4\int\left(\frac{1}{s}+\frac{2}{s^2}\right)ds\right] + c$$

$$\bar{y}(s+2)^2 = \left[\int\left(1+\frac{4}{s^2}+\frac{4}{s}\right)ds - 4\int\left(\frac{1}{s}+\frac{2}{s^2}\right)ds\right] + c$$

$$\bar{y}(s+2)^2 = \left[s - \frac{4}{s} + 4\ln s - 4\ln s - 4\left(-\frac{2}{s}\right)\right] + c$$

$$\bar{y}(s+2)^2 = \left[s - \frac{4}{s} + \frac{8}{s}\right] + c$$

$$\bar{y} = \frac{s}{(s+2)^2} - \frac{4}{s(s+2)^2} + \frac{8}{s(s+2)^2} + \frac{c}{(s+2)^2}$$

$$\bar{y} = \frac{s}{(s+2)^2} + \frac{4}{s(s+2)^2} + \frac{c}{(s+2)^2}$$

(taking inverse Laplace both sides)

$$L^{-1}(\bar{y}) = \underbrace{L^{-1}\left[\frac{s}{(s+2)^2}\right]}_{(I_1)} + 4\underbrace{\left[L^{-1}\left(\frac{1}{s(s+2)^2}\right)\right]}_{(I_2)} + cL^{-1}\left\{\frac{1}{(s-2)^2}\right\}$$

$$y = I_1 + 4I_2 + ce^{-2t}t$$

$$I_1 = L^{-1}\left[\frac{s}{(s+2)^2}\right]$$

we know that :- $L^{-1}\left(\frac{1}{(s+2)^2}\right) = \left(e^{-2t}t\right)$

$$L^{-1}\left(\frac{s}{(s+2)^2}\right) = \frac{d}{dt}\left(te^{-2t}\right) + f(0)\,\delta(t)$$

$$= t(-2e^{-2t}) + e^{-2t}$$

$$I_1 = L^{-1}\left(\frac{s}{(s+2)^2}\right) = -2te^{-2t} + e^{-2t}$$

$$I_2 = L^{-1}\left\{\frac{1}{s(s+2)^2}\right\}$$

We know that:-

$$L^{-1}\left\{\frac{1}{(s+2)^2}\right\} = e^{-2t}t$$

$$L^{-1}\left\{\frac{1}{s(s+2)^2}\right\} = \int_0^t e^{-2t}\, t\, dt$$

$$I_2 = \frac{te^{-2t}}{-2} + \frac{1}{2}\int e^{-2t}\, dt$$

$$= \frac{-te^{-2t}}{2} - \frac{1}{4}e^{-2t}\Big|_0^t$$

$$= \frac{te^{-2t}}{2} + \frac{1}{4}e^{-2t}\Big|_t^0$$

$$I_2 = \frac{1}{4} - \left(\frac{te^{-2t}}{2} + \frac{1}{4}e^{-2t}\right)$$

Now;

$$y = -2te^{-2t} + e^{-2t} + \frac{1}{4} - \frac{te^{-2t}}{2} - \frac{1}{4}e^{-2t} + ce^{-2t}t$$

$$y = \left(-\frac{5}{2}te^{-2t} + \frac{3}{4}e^{-2t} + \frac{1}{4}\right) + ce^{-2t}t.$$

$$y'(t) = \frac{-5}{2}\left[t(-2e^{-2t}) + e^{-2t}\right] + \frac{3}{4}(-2e^{-2t}) + c\left[e^{-2t} + t(-2e^{-2t})\right]$$

Put $(t=0)$ $\quad y'(0) = a$ (arbitary consn)

$$a = \frac{-5}{2} - \frac{3}{2} + c$$

$$c = a + 4$$

$$y(t) = \frac{-5}{2}te^{-2t} + \frac{3}{4}e^{-2t} + \frac{1}{4} + (a+4)te^{-2t}$$ Ans