

24/12/2020

Electrical Engineering

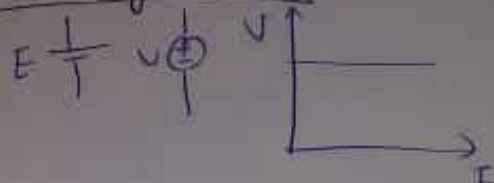
Unit 1

Circuit

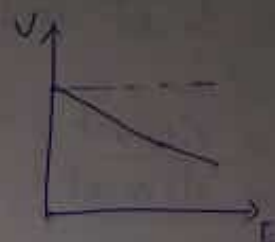
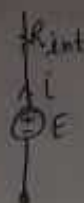


Source Voltage Source
Current Source

Ideal voltage Source +



Practical Voltage Source



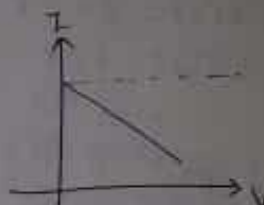
$I \uparrow V \downarrow$

24/12/2020

Current Source



$I_L < I$



Dependent Voltage / Current

Also called Controlled Source

- 1- Voltage Controlled Voltage Source (VCVS)
- 2- Current " " (CCVS)
- 3- Voltage " " (VCCS)
- 4- Current " " (CCCS)

Voltage source



Current source

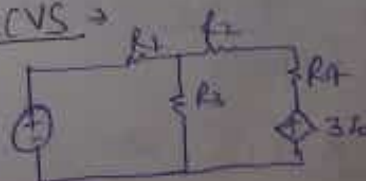


① VCVS →



②

CCVS →



Unilateral / bilateral element

Unilateral → p-n junction, BJT

Bilateral → R, L, C



Active & Passive elements:

If any element has internal source or own energy source, then ^{called} active element voltage source & if does not have energy source then called passive element.

Active element → Voltage source, Semi Conductor devices

Passive element → R, L, C

Linearity property →

- ① Homogeneity
- ② Superposition

① Homogeneity → $f(ax) = a f(x)$
(Scaling property)

Ex ① $f(x) = y = 7x$ ② $f(x) = 3x^2$
 $f(ax) = 7ax$
 $= a \cdot 7x$
 $f(ax) = a f(x)$ Homogeneity

$f(ax) = 3(ax)^2$
 $= a^2 \cdot 3x^2$
 $f(ax) \neq a f(x)$ not homogenous

$f(ax) = a f(x)$
Homogeneity

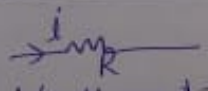
② Superposition →
(Additive property)

Ex ① $f(x) = 7x$
 $f(x_1 + x_2) = 7(x_1 + x_2)$
 $= 7x_1 + 7x_2$
 $f(x_1 + x_2) = f(x_1) + f(x_2)$
Additive property

② $f(x) = 3x^2$
 $f(x_1 + x_2) = 3(x_1 + x_2)^2$
 $= 3(x_1^2 + x_2^2 + 2x_1x_2)$
 $f(x_1 + x_2) \neq f(x_1) + f(x_2)$
not superposition

$f(x_1 + x_2) = f(x_1) + f(x_2)$
Additive property

Element R, L, C are linear element →

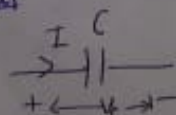
① R → $V = IR$ 

~~W H H A~~

③ C →

$q = CV$

$I = C \frac{dV}{dt}$

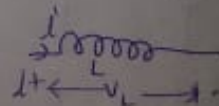
$I = C \frac{dV}{dt}$ 

$V = \frac{1}{C} \int I dt$

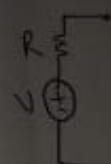
② L →

$V_L = L \frac{dI}{dt}$

$I = \frac{1}{L} \int V dt$



Source

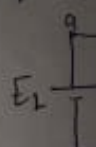


Series

Series

Parallel

Loop A



Node

Ex

Junct

Ex

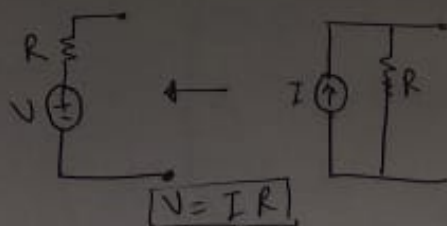
Branch

Loop

Mesh

nal energy
voltage
d passive

Source Transformation



Series & \parallel Resistance

Series $R_{eq} = R_1 + R_2$

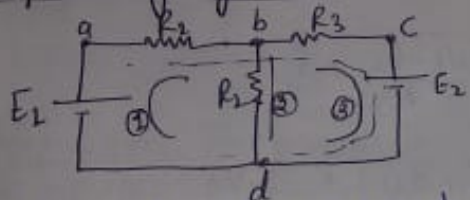
$L_{eq} = L_1 + L_2$

Parallel $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$

Loop Analysis



Node → A node network is equipotential pt. where two or more element are connected.

Ex → a, b, c, d

Junction → A junction network is equipotential pt. where three or more element are connected.

Ex → b, d.

Branch → A branch is that part of network which lies b/w junctions.

Ex → 3 branch (1)

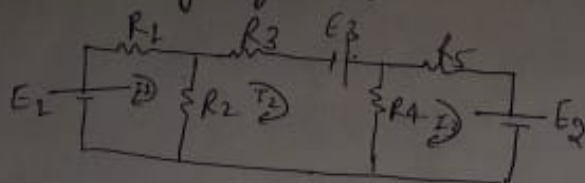
Loop → Any close path in the circuit is a loop.

Ex → 3 loop (2)

Mesh → Elementary loop are called mesh

Ex → 2 Mesh ()

Mesh Analysis



Solⁿ By applying K.V.L in loop (i)

$$-E_1 + I_1 R_1 + (I_1 - I_2) R_2 = 0 \quad \text{--- (i)}$$

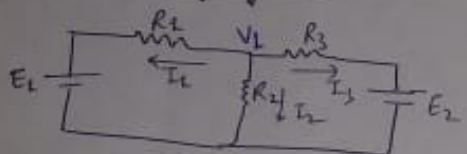
By applying K.V.L in loop (ii)

$$R_3 I_2 - E_3 + (I_1 - I_2) R_4 - (I_1 - I_2) R_2 = 0 \quad \text{--- (ii)}$$

By applying K.V.L in loop (iii)

$$R_5 I_3 + E_2 - (I_2 - I_3) R_4 = 0 \quad \text{--- (iii)}$$

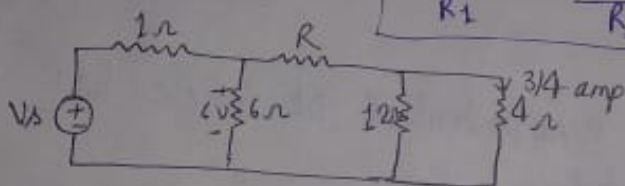
Nodal Analysis



$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_1 - E_1}{R_1} + \frac{V_1 - 0}{R_2} + \frac{V_1 - E_2}{R_3} = 0$$

Ans



Find (i) R & Vs

(ii) power out of source (Vs)

Solⁿ (i) $V_{4\Omega} = \frac{3}{4} \times 4 = 3V$

$$V_{12\Omega} = 3V$$

$$I_{12\Omega} = \frac{1}{4} \text{ amp}$$

$$I_{\text{total}} (12\Omega \text{ \& } 4\Omega) = 1 \text{ amp}$$

$$I_{6\Omega} = \frac{1}{2} = 1 \text{ amp}$$

$$\text{So } I_{\text{total}} = I_{6\Omega}$$

So resistance should be equal

$$6 = R + \frac{12 \times 4}{12 + 4}$$

$$6 = R + 3$$

$$\boxed{R = 3\Omega}$$

$$I_{\text{total}} (\text{whole}) = 1 + 1 = 2 \text{ amp}$$

$$V_{1\Omega} = 1 \times 1 = 1 \text{ Volt}$$

$$V_s = 1 + 6$$

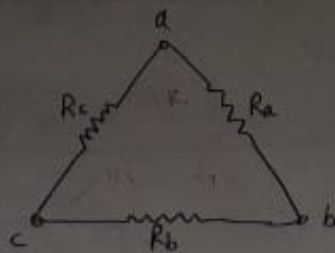
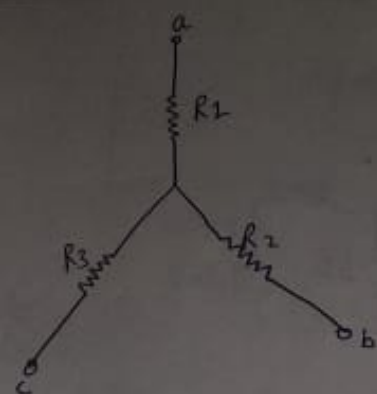
$$\boxed{V_s = 7V}$$

(ii) $P = VI$

$$P = 8 \times 2$$

$$\boxed{P = 16 \text{ Watt}}$$

Star-Delta Transformation -



$$\begin{aligned} R_a &= R_1 + R_2 + \frac{R_1 R_2}{R_3} \\ R_b &= R_3 + R_2 + \frac{R_2 R_3}{R_1} \\ R_c &= R_1 + R_3 + \frac{R_1 R_3}{R_2} \end{aligned}$$

$$\begin{aligned} R_1 &= \frac{R_a R_c}{R_a + R_b + R_c} \\ R_2 &= \frac{R_a R_b}{R_a + R_b + R_c} \\ R_3 &= \frac{R_b R_c}{R_a + R_b + R_c} \end{aligned}$$

Proof for a & b

$$R_1 + R_2 = R_a \parallel (R_b + R_c)$$

$$R_1 + R_2 = \frac{R_a (R_b + R_c)}{R_a + R_b + R_c} \quad \text{--- (i)}$$

$$R_2 + R_3 = \frac{R_b (R_c + R_a)}{R_a + R_b + R_c} \quad \text{--- (ii)}$$

$$R_3 + R_1 = \frac{R_c (R_a + R_b)}{R_a + R_b + R_c} \quad \text{--- (iii)}$$

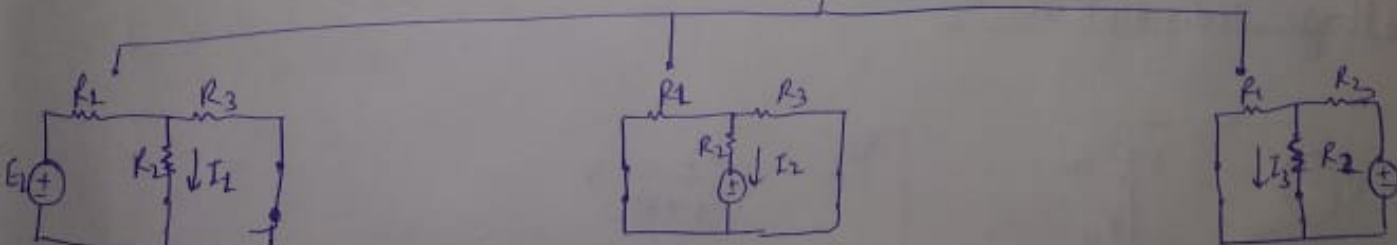
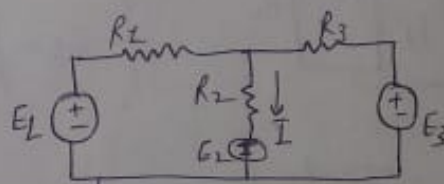
eqⁿ (i) + (ii) + (iii) then
subtract from (i) & (ii) & (iii)

Network Theorems +

- + Superposition Theorems
- + Thevenin's Theorems
- + Norton's Theorems
- + Maximum power transfer Theorem

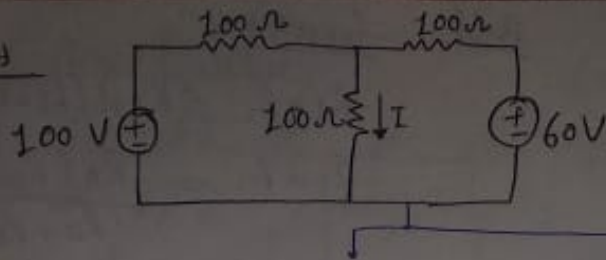
(i) Superposition Theorems +

- * Active linear bilateral network
- * Two or more sources



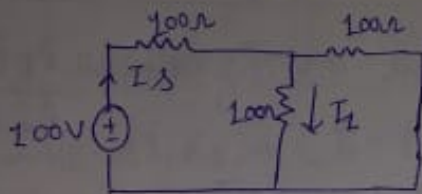
$$I = I_1 + I_2 + I_3$$

Ques →



Find I using superposition theorem

Solⁿ →



$$I_s = \frac{100}{150}$$

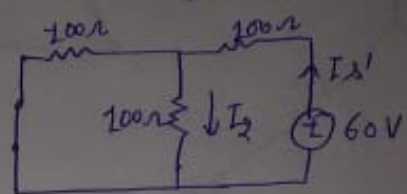
$$I_s = \frac{2}{3}$$

$$I_1 = \frac{I_s}{2} = \frac{1}{3} \text{ amp}$$

$$I = I_1 + I_2$$

$$I = \frac{1}{3} + \frac{3}{15}$$

$$I = \frac{8}{15} \text{ amp}$$



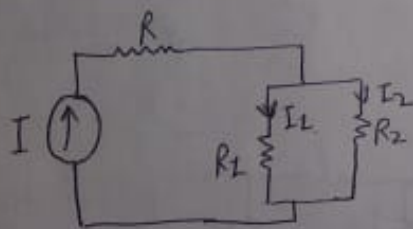
$$I_s' = \frac{60}{150}$$

$$I_s' = \frac{2}{5}$$

$$I_2 = \frac{I_s'}{2} = \frac{3}{15} \text{ amp}$$

✶ for verification solve by other method
✶ find equal current.

Current Division Rule +



$$I_2 = \frac{R_1}{R_1 + R_2} I$$

$$I_1 = \frac{R_2}{R_1 + R_2} I$$

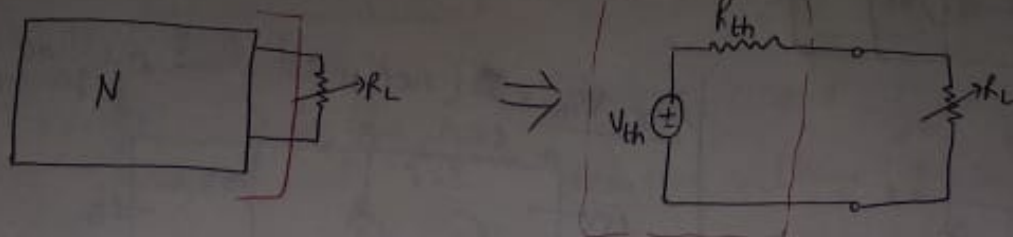
Voltage Division Rule +



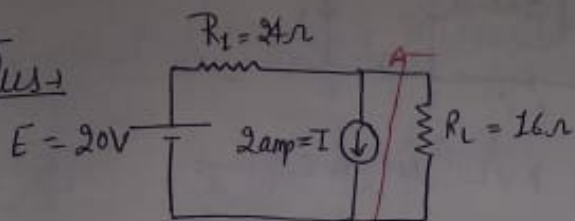
$$V_1 = \frac{R_1}{R_1 + R_2} V$$

$$V_2 = \frac{R_2}{R_1 + R_2} V$$

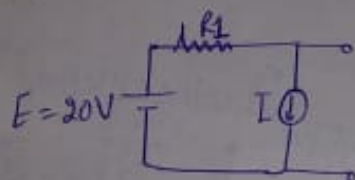
Thevenin's Theorems :



Ex
Ques →

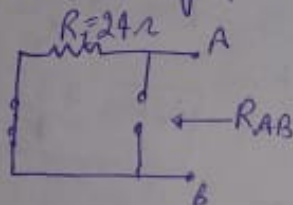


Step 1 Remove the load Resistance & $V_s \rightarrow$ ~~short~~ ^{open} circuit
Current source \rightarrow O.C.



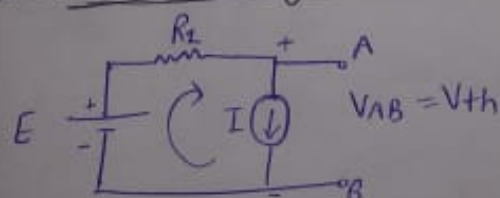
Step 2 For finding the value of R_{th}

Replace battery & Current with short circuit



$$R_{AB} = R_{th} = R_1 = 24\Omega$$

Step 3 For finding the value of V_{th}



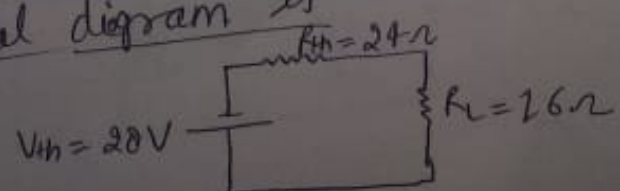
By using K.V.L

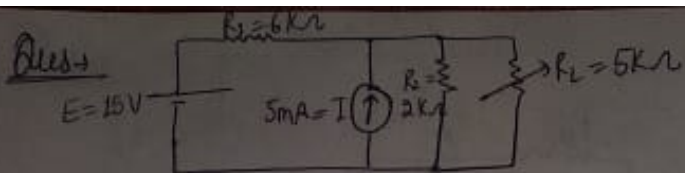
$$-E + IR_2 + V_{AB} = 0$$

$$V_{AB} = E - IR_2$$

$$V_{AB} = -28V$$

So final diagram is





Find the Current through load Resistance.

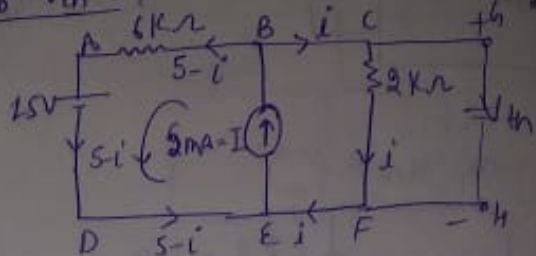
Solⁿ → for R_{th} →



$$R_{AB} = R_{th} = \frac{6 \times 2}{6 + 2} = \frac{3}{2}$$

$$R_{th} = \frac{3}{2} k\Omega$$

for V_{th} → (not used this method) generally



apply KVL in A C D F ÷

$$15 - 2i + 6(5 - i) = 0$$

$$45 - 8i = 0$$

$$i = \frac{45}{8} mA$$

And apply K.V.L in A D G H

$$6(5 - i) + 15 - V_{th} = 0$$

$$6\left(5 - \frac{45}{8}\right) + 15 = V_{th}$$

$$V_{th} = \frac{6 \times 5 - 5}{8} + 15$$

$$V_{th} = \frac{45}{4} = 11.25 \text{ Volt}$$

"Another Method"

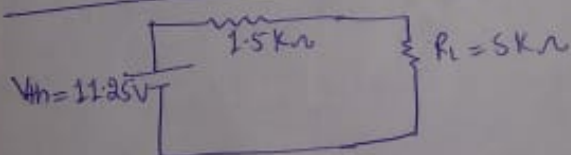


nodal analysis

$$\frac{V_A - E}{R_1} - I + \frac{V_A - 0}{R_2} = 0$$

$$V_A = \frac{45}{4} = 11.25 \text{ Volt}$$

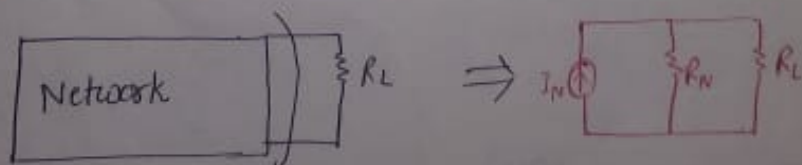
so equivalent circuit



$$i(R_L) = \frac{11.25}{6.5}$$

$$i(R_L) = 1.730 \text{ amp} \quad \text{Ans}$$

Norton's Theorem ÷

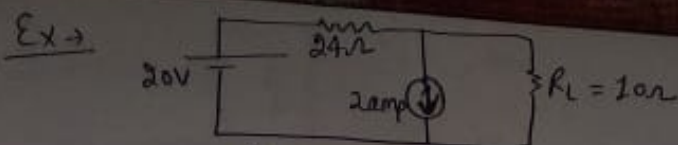


① R_N is calculated same as R_{th}

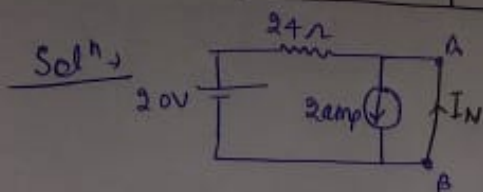
① remove R_L

② $V_I \rightarrow$ Short Circuit

③ $I_I \rightarrow$ Open Circuit



$R_L \rightarrow S.C$
 $I_N \rightarrow I_S$

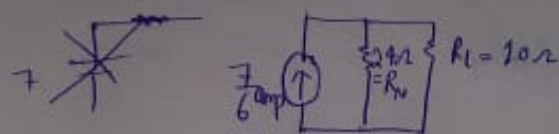


deactive current $\downarrow \frac{20}{24} = \frac{5}{6} = i_1$

deactive voltage $\uparrow 2Amp = i_2$

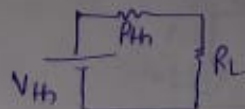
net $\uparrow (2 - \frac{5}{6}) = \frac{7}{6} amp$

equivalent Norton's Circuit



$$I_N = \frac{V_{th}}{R_{th}}$$

Thevenin's Circuit

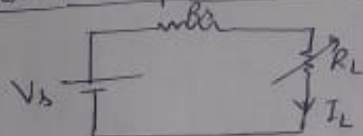


$$V_{th} = I_N \times R_N$$

Interchange by source transformation

The type of circuit converted into other type of circuit is called dual Circuit.

Maximum power transfer theorem :



$$I_L = \frac{V_S}{R_S + R_L}$$

$$P_L (\max) = I_L^2 R_L$$

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L} \left(\frac{V_S}{R_S + R_L} \right)^2 R_L = 0$$

$$\frac{(R_S + R_L)(R_S + R_L - 2R_L)}{(R_S + R_L)^4} = 0$$

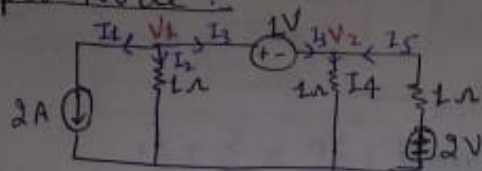
$$R_S - R_L = 0$$

$$\boxed{R_S = R_L} \text{ condition for max power}$$

∴ maximum power

$$\boxed{P_L = \frac{V_S^2}{4R_L}}$$

Super Node ÷

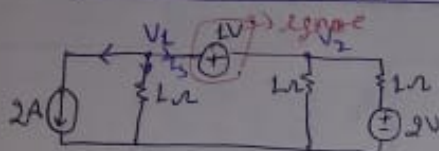


$$I_1 + I_2 + I_3 = 0 \quad \text{--- (i)}$$

$$I_3 = I_4 + I_5$$

by using eqⁿ ①

$$I_1 + I_2 + I_4 + I_5 = 0$$



$$I_3 = \frac{8}{3} \text{ amp}$$

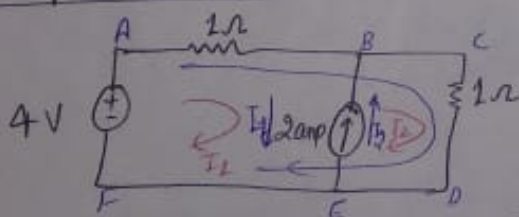
$$(1V)P = VI = \frac{8}{3} \text{ amp}$$

$$2 + \frac{V_1 - 0}{1} + \frac{V_2 - 0}{1} + \frac{V_2 - 2}{1} = 0 \quad \text{--- (ii)}$$

$$V_2 + 1 = V_1 \quad \text{--- (i)}$$

$$V_2 = -1/3 \quad V_1 = 2/3$$

Super Mesh ÷



in A CDF Apply KVL

$$-4 + I_1 \times 2 + I_2 \times 1 = 0$$

$$I_1 + I_2 = 4 \quad \text{--- (i)}$$

$$I_2 - I_1 = 2 \quad \text{--- (ii)}$$

$$I_2 = 3 \text{ amp}$$

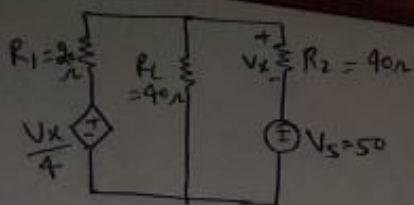
$$I_1 = 1 \text{ amp}$$

Ans

Thevenin's/Norton's (Dependent Source) ÷

At last page

Ques-1



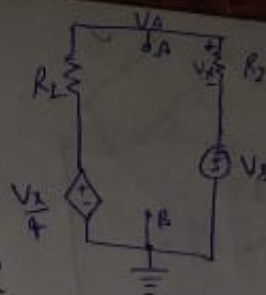
Putting the value of V_x in eqn ①

$$\frac{4V_A - V_A + 50}{\frac{4}{20}} + \frac{2(V_A - 50)}{2 \times 40} = 0$$

$$\boxed{V_A = 10}$$

$$\text{Vth } \boxed{V_{AB} = V_A = 10}$$

for V_{th}



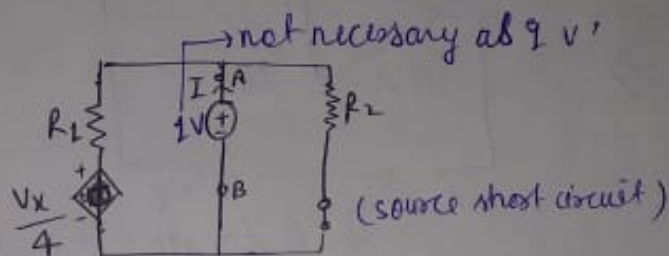
at 'A'

$$\frac{V_A - \frac{V_x}{4}}{R_1} + \frac{V_A - V_s}{R_2} = 0 \quad \text{--- (i)}$$

$$\nrightarrow V_x = V_A - V_s$$

$$V_x = V_A - 50$$

For R_{th}



apply K.V.L at A

$$1 - \frac{V_x}{4} - I + \frac{1-0}{40} = 0$$

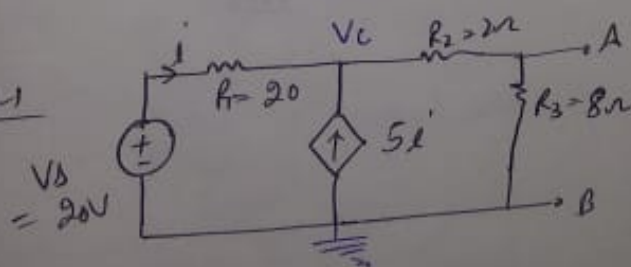
$$1 - \frac{1}{4} - I + \frac{1-0}{40} = 0$$

$$\boxed{I = \frac{1}{16}}$$

$$R_{AB} = \frac{V}{I} = \frac{1}{I}$$

$$\boxed{R_{AB} = 16 \Omega}$$

Ques-1



soln

$$\frac{V_c - V_s}{R_1} - 5i + \frac{V_c - 0}{R_2 + R_3} = 0$$

$$\frac{V_c - 20}{20} - 5i + \frac{V_c}{8+8} = 0$$

$$\text{for } i \rightarrow i = \frac{V_s - V_c}{R_1} = \frac{20 - V_c}{20} \quad \text{--- (ii)}$$

using ① + ②

$$\frac{V_c - 20}{20} - \left(\frac{20 - V_c}{4} \right) + \frac{V_c}{10} = 0$$

$$\boxed{V_c = 15V}$$

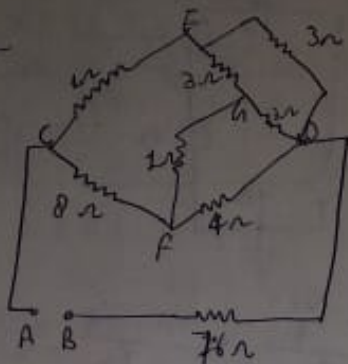
$$I_{R_3} = \left(\frac{V_c}{R_2 + R_3} \right) = 1.5$$

$$V_{AB} = I_{R_3} \times R_3$$

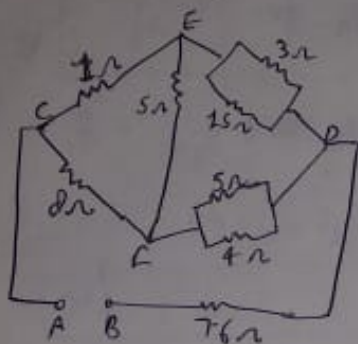
$$V_{AB} = 1.5 \times 8$$

$$\boxed{V_{th} = V_{AB} = 12V}$$

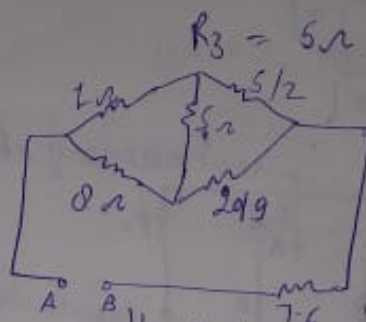
Ans -



↓ reduced to



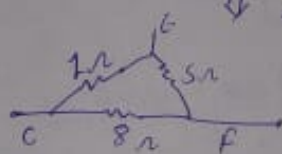
⇒



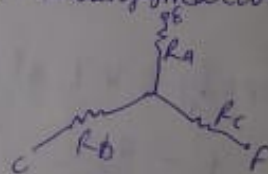
$$R_1 = 3 + 3 + \frac{3 \times 3}{1} = 15\Omega$$

$$R_2 = 3 + 1 + \frac{3 \times 1}{3} = \frac{15}{3} = 5\Omega$$

$$R_3 = 5\Omega$$

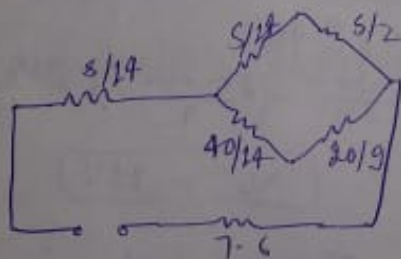


Δ to star transformation



$$R_a = \frac{5}{14}, R_b = \frac{8}{14}, R_c = \frac{40}{14}$$

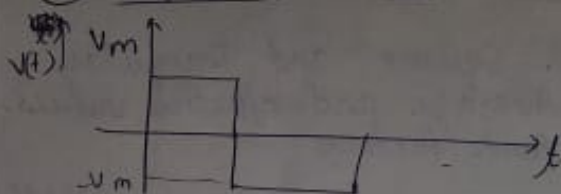
Equivalent circuit will be



$$R_{AB} = 10\Omega$$

Ans

② Square Wave form:



$$V_{avg} = \frac{1}{T/2} \int_0^{T/2} V_m dt$$

$$= \frac{V_m}{T/2} (t)_0^{T/2}$$

$$= \frac{V_m}{T/2} (T/2 - 0)$$

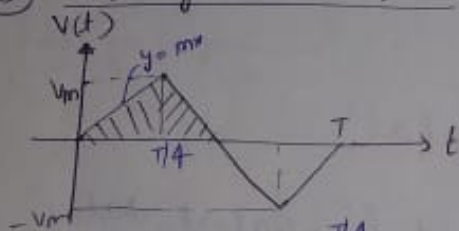
$$\boxed{V_{avg} = V_m}$$

$$V_{rms} = \sqrt{\frac{1}{T/2} \int_0^{T/2} V_m^2 dt}$$

$$= \sqrt{\frac{V_m^2}{T/2} \times T/2}$$

$$\boxed{V_{rms} = V_m}$$

③ Triangular Wave form →



$$V_{avg} = \frac{1}{T/4} \int_0^{T/4} \frac{V_m}{T/4} t dt$$

$y = mx$
 $y = \frac{V_m}{T/4} t$

$$V_{avg} = \frac{V_m}{(T/4)^2} \int_0^{T/4} t dt$$

$$V_{avg} = \frac{V_m}{(T/4)^2} \left(\frac{t^2}{2} \right)_0^{T/4}$$

$$\boxed{V_{avg} = \frac{V_m}{2}}$$

$$V_{rms} = \sqrt{\frac{1}{T/4} \int_0^{T/4} \left(\frac{V_m}{T/4} t \right)^2 dt}$$

$$= \sqrt{\frac{V_m^2}{(T/4)^3} \int_0^{T/4} t^2 dt}$$

$$= \sqrt{\frac{V_m^2}{(T/4)^3} \times \left[\frac{t^3}{3} \right]_0^{T/4}}$$

$$= \sqrt{\frac{V_m^2}{(T/4)^3} \times \left(\frac{T/4)^3}{3} \right)}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{3}}}$$

Form fa

i) For S

ii) For

iii) For

Peak fa

1) Sine

ii) Squa

(iii) Tran

Phasor

Form factors

$$\text{Form factor} = \frac{\text{RMS Value}}{\text{Avg. Value}}$$

i) For Sine wave = $\frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$

ii) For Square wave = 1

iii) For Triangular wave = $\frac{V_m}{\sqrt{3}} / \frac{V_m}{3} = \frac{3}{\sqrt{3}} = 1.732$

Peak factor / Crest factor :-

$$\text{Peak factor} = \frac{\text{Maximum Value}}{\text{RMS Value}}$$

i) Sine wave = $V_m/V_m/\sqrt{2} = \sqrt{2}$

ii) Square wave = 1

iii) Triangular wave = $\frac{V_m}{V_m/\sqrt{3}} = \sqrt{3}$

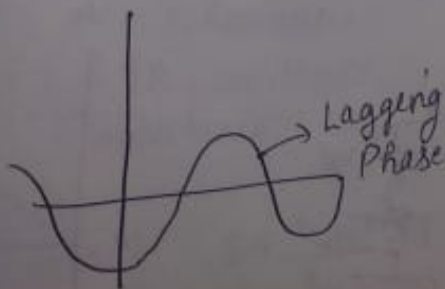
Phasors



$$V(t) = V_m \sin(\omega t + \phi)$$

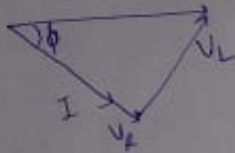
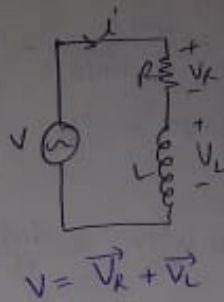
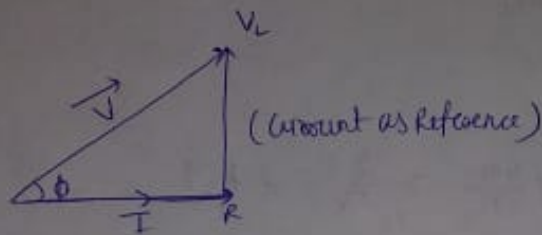
↑
Phase

$$\omega = 2\pi f = \frac{2\pi}{T}$$

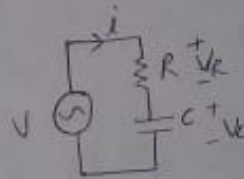
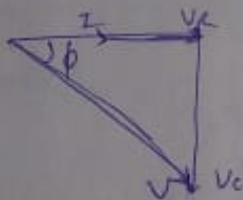


Analysis of series, 11^{el} and series 11^{el} RLC \rightarrow

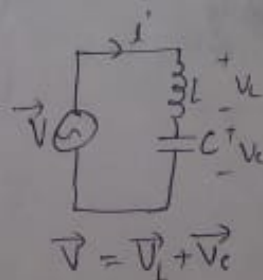
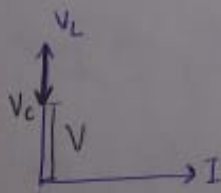
① R-L Circuit \rightarrow



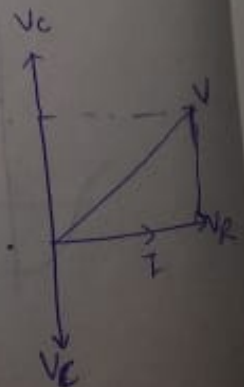
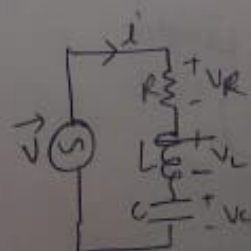
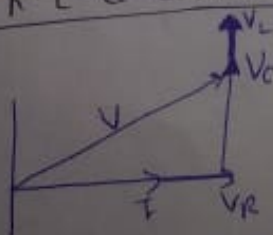
② R-C Circuit \rightarrow



③ L-C-Circuit \rightarrow

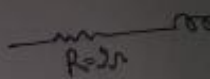


④ R-L-C Circuit \rightarrow

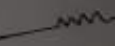


Analysis of

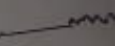
① R-L \rightarrow



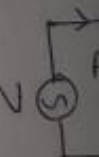
② R-C \rightarrow



③ RLC \rightarrow

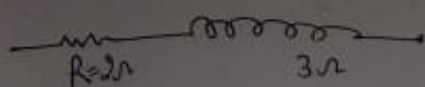


Parallel



Analysis of Series Circuit →

① R-L →



$$Z = R + jX_L$$

→ same as complex number "i"
 → Inductive Reactance
 $\omega L = 2\pi fL$

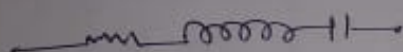
② R-C →



$$Z = R - jX_C$$

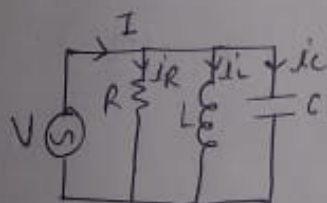
→ Capacitive reactance = $\frac{1}{\omega C} = \frac{1}{2\pi fC}$

③ R-L-C →



$$Z = R + (X_L - X_C)j$$

Parallel Circuit →



$$\vec{I} = \vec{I}_R + \vec{I}_L + \vec{I}_C$$

$$Z = R // jX_L // jX_C$$

or

$$Z = R + jX$$

→ Reactance

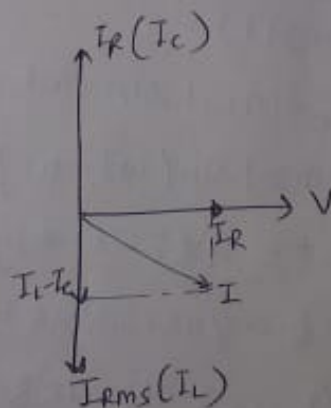
$$Y = G + jB$$

→ susceptance
→ conductance

and admittance

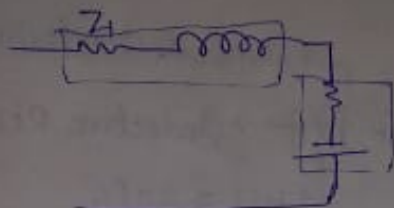
$$G = \frac{1}{R}, B = \frac{1}{X_C}$$

$$Y = Y_1 + Y_2 + Y_3$$



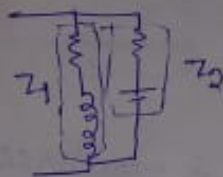
$$Z = R + jX \quad , \quad Y = G + jB$$

series →



$$Z = Z_1 + Z_2$$

Parallel →



$$Z = Z_1 || Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\frac{1}{Z_1} \rightarrow Y_1 \quad \frac{1}{Z_2} \rightarrow Y_2$$

$$Y = Y_1 + Y_2 \rightarrow Z = \frac{1}{Y}$$

Power and Power Factors →

$$P = VI \quad \text{--- DC circuit}$$

$$V(t) = V_m \sin \omega t$$

$$i(t) = I_m \sin(\omega t - \phi)$$

$$P_i = V_i \times i_i$$

$$P(t) = V(t) \times i(t)$$

$$p = V_m I_m \sin \omega t \sin(\omega t - \phi) \quad \left(\because V = \frac{V_m}{\sqrt{2}} \quad , \quad I = \frac{I_m}{\sqrt{2}} \right)$$

$$p = VI [2 \sin \omega t \sin(\omega t - \phi)]$$

$$p = VI [\cos \phi - \cos(2\omega t - \phi)]$$

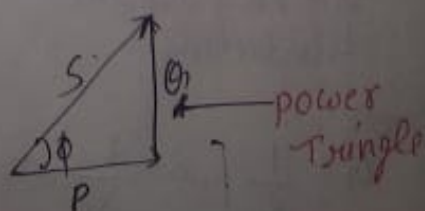
$$p = VI [\cos \phi - \{\cos 2\omega t \cos \phi + \sin 2\omega t \sin \phi\}]$$

$$p = VI \cos \phi (1 - \cos 2\omega t) - \underbrace{VI \sin \phi \sin 2\omega t}_{\theta_r \text{ (Reactive power)}}$$

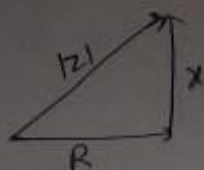
P (active power)

$$S = \sqrt{P^2 + \theta_r^2}$$

$$S = VI = \text{Apparent power}$$



$$Z = R + jX$$



Impedance Triangle

$$|Z| = \sqrt{R^2 + X^2}$$

$$p = P(1 - \cos 2\omega t) - \theta \sin 2\omega t$$

$$P_{avg} = \frac{\int_0^T V(t) i(t) dt}{T}$$

$$= \frac{\int_0^{2\pi} P(1 - \cos 2\omega t) - \theta \sin 2\omega t d\omega t}{2\pi}$$

$$\left(\because \int_0^{2\pi} \cos 2\omega t = \int_0^{2\pi} \sin 2\omega t = 0 \right)$$

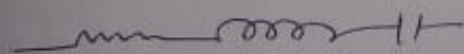
$$= \frac{P \int_0^{2\pi} d\omega t}{2\pi}$$

$$= \frac{P \times 2\pi}{2\pi}$$

$$P_{avg} = P \leftarrow \text{active power}$$

$$S = VI^* = P + jQ$$

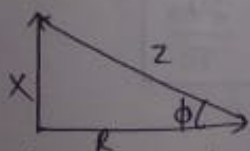
+ve for lagging power factor



$$Z = R + j(X_L - X_C)$$

$$\vec{V} = \vec{V}_R + \vec{V}_L + \vec{V}_C$$

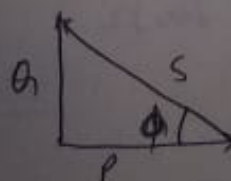
Power factor :



(Impedance triangle)

$$P.F. = \cos \phi = \frac{R}{Z}$$

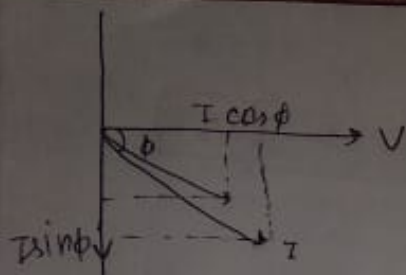
$$P.F. = \cos \phi = \frac{R}{\sqrt{R^2 + X^2}}$$



(Power triangle)

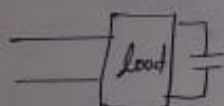
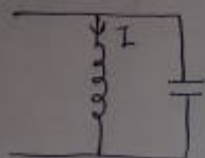
$$P.F. = \cos \phi = \frac{P}{S} = \frac{VI \cos \phi}{VI}$$

$$P.F. = \cos \phi = \frac{P}{\sqrt{P^2 + Q^2}}$$

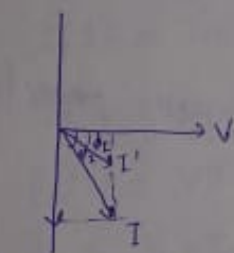
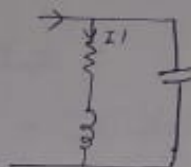


$$P = VI \cos \phi$$

$$\left\{ \begin{array}{l} \phi = 1, \cos \phi = 1 \\ \text{T.e. } \phi \text{ lags} \end{array} \right\}$$



$$P = VI \cos \phi$$



$$(I' < I)$$

$$\cos \phi_1 = \frac{P_1}{S_1}$$

$$\cos \phi_2 = \frac{P_2}{S_2}$$

$$S_2 = P_2 + jQ_2$$

Ans $V(t) = 200 \cos(314t - 30^\circ)$ Find $V_{avg} = ?$ & $V_{rms} = ?$

Solⁿ $V_{avg} = \frac{1}{\pi} \int_0^\pi 200 \cos(314t - 30^\circ) d\omega t$

$$= \frac{200}{\pi} \sin(314t - 30^\circ) \Big|_0^\pi$$

$$= \frac{200}{\pi} \times 2$$

$$\boxed{V_{avg} = \frac{400}{\pi}}$$

$$\text{or } \boxed{V_{avg} = \frac{2V_m}{\pi}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 100\sqrt{2}$$

$$(2) \quad V(t) = 100 \cos(314t - 30^\circ) - 200 \sin(314t - 60^\circ)$$

$$\Rightarrow 100 \cos(314t - 30^\circ) = 100 (\cos 314t \cos 30^\circ + \sin 314t \sin 30^\circ)$$

$$\Rightarrow 200 \sin = 50 \times 1.732 \cos 314t + 50 \sin 314t \quad (i)$$

$$200 \sin(314t - 60^\circ) = 100 \sin 314t - 100 \times 1.732 \cos 314t \quad (ii)$$

eqn (i) + (ii)

$$V(t) = 100 \cos(314t - 30^\circ) - 200 \sin(314t - 60^\circ) = 150 \sin 314t - 86.6 \cos 314t$$

$$V_{rms} = \sqrt{(150)^2 + (86.6)^2}$$

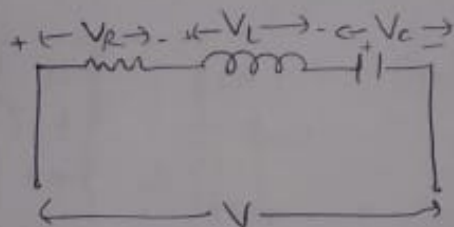
$$V_{rms} = 173.20 \text{ Am} \quad \text{or}$$

$$V(t) = 100 \cos(314t - 30^\circ) - 200 \cos(314t - 150^\circ)$$

$$V^2 = V_1^2 + V_2^2 + 2V_1V_2 \cos 60^\circ$$

$$V = 173.20$$

Ans:



$$V_R = 6 \text{ V}$$

$$V_L = 12 \text{ V}$$

$$V_C = 4 \text{ V}$$

Soln:

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{36 + 64}$$

$$V = 10 \text{ Volts}$$

Ans:

$$V = 100 \sin(\omega t - 30^\circ)$$

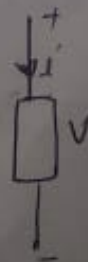
$$I = 20 \sin(\omega t - 60^\circ)$$

Soln:

$$P = VI \cos \phi$$

$$P = \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \cos 30^\circ$$

$$P = 966 \text{ Watt}$$



Ques $v(t) = 100 \sin(\omega t - 30^\circ)$
 $i(t) = 20 \sin(\omega t - 60^\circ)$

Solⁿ
 $P = VI \cos \phi$
 $P = \frac{100 \times 20 \times 3}{\sqrt{2} \sqrt{2} \sqrt{5}}$
 $P = \frac{1200}{2} = 600 \text{ Watt}$

$Q = VI \sin \phi$

$Q = 100 \times 20 \times \frac{4}{5}$

$Q = 800 \text{ Volt amp (Var)}$

VAR \rightarrow volt ampere reactive
 or

$S = V I^* = \frac{100}{\sqrt{2}} \angle -30^\circ \times 10\sqrt{2} \angle +83^\circ$

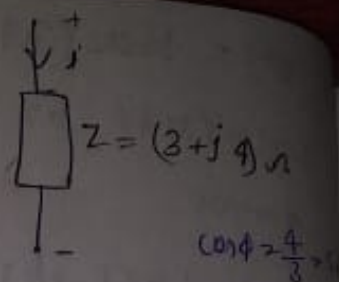
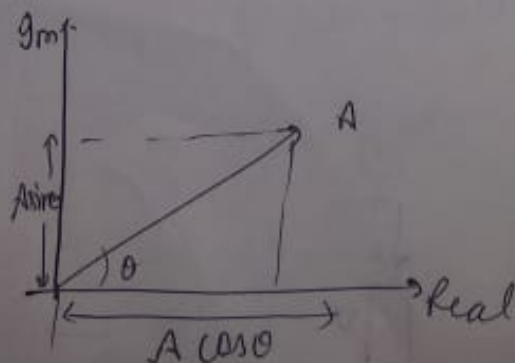
$S = 1000 \angle 53^\circ$

$S = \boxed{1000 \cos 53^\circ + j 1000 \sin 53^\circ}$

$S = \boxed{600 + 800j}$

$V I^*$

$A \cos \theta = A \cos \theta + j A \sin \theta$



$I = 10\sqrt{2} \angle +83^\circ$

$I^* = 10\sqrt{2} \angle 83^\circ$

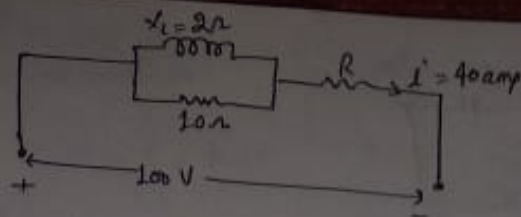
for I $I = \frac{V_{rms}}{Z}$

$I = \frac{100 \angle -30^\circ}{\sqrt{2} \angle 53^\circ}$

$I = 10\sqrt{2} \angle -30^\circ - 53^\circ$

$I = \boxed{10\sqrt{2} \angle 83^\circ}$

Ans 1



① Find $R = ?$

② Power factor of circuit

solⁿ

① $Z = R + 10 \parallel 20j$

$$Z = R + \frac{20j}{10 + 20j}$$

$$Z = R + \frac{20j(10 - 20j)}{104}$$

$$Z = \left(R + \frac{40}{104}\right) + \frac{200}{104}j$$

$$Z = \sqrt{\left(R + \frac{40}{104}\right)^2 + \left(\frac{20}{104}\right)^2}$$

$$\frac{100}{40} = \sqrt{\left(R + \frac{40}{104}\right)^2 + \left(\frac{20}{104}\right)^2}$$

$$R = 1.21 \Omega$$

②

$$Z = \left(1.21 + \frac{40}{104}\right) + \frac{j200}{104}$$

$$\tan \phi = \frac{200}{104}$$

$$\frac{1.21 + \frac{40}{104}}{104}$$

$$\tan \phi = \frac{200}{1.21 \times 104 + 40}$$

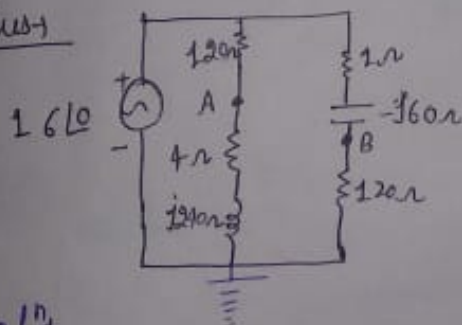
$$\cos \phi = 0.638$$

or

$$\cos \phi = \frac{R}{Z} = \frac{R + \frac{40}{104}}{\frac{100}{40}}$$

$$\cos \phi = 0.638$$

Ans 2



Find $V_{AB} = ?$

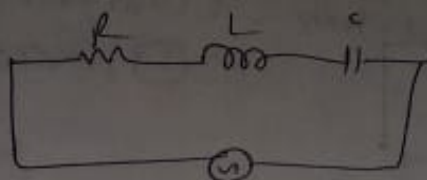
solⁿ

$$V_A = \frac{4 + j240}{120 + 4 + j240} \times 16 = 14.21 \angle 26.56^\circ$$

$$V_B = \frac{120}{120 + 1 - j \times 60} \times 16 = 14.21 \angle 26.56^\circ$$

$$V_A - V_B = 0 \text{ Ans}$$

Resonance



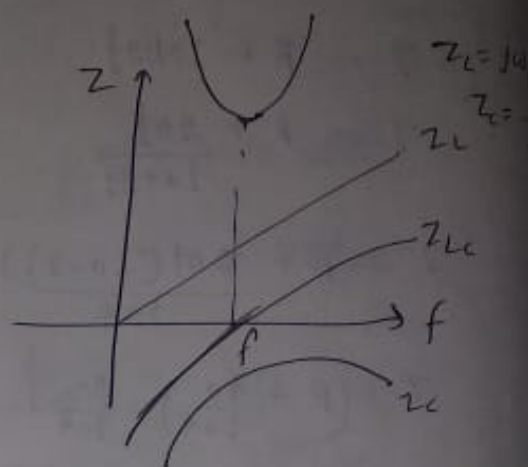
$$Z = R + j \left(\omega L - \frac{1}{\omega C} \right)$$

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$I_0 = \frac{V}{R}$$



Selectivity

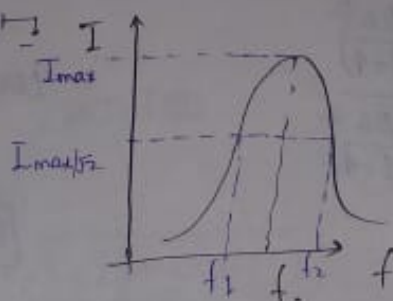
$$P_0 = I_{\max}^2 R$$

$$P_1 = \left(\frac{I_{\max}}{\sqrt{2}} \right)^2 R$$

$$P_1 = \frac{P_0}{2}$$

similarly

$$P_2 = \frac{P_0}{2}$$

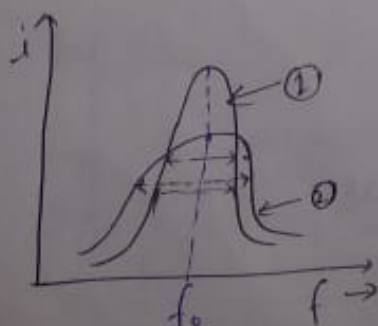


f_1 & f_2 are called half power frequency

$$\text{Band width} = f_2 - f_1 = \Delta f$$

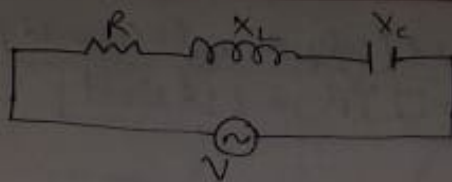
$$\text{Selectivity} = \frac{\text{Resonance freq}^n}{\text{Band width}} = \frac{f_0}{f_2 - f_1}$$

Ques



Band width ① > Band width ②
so ① graph has more selectivity than ②

Quality factor +



suppose circuit has resonance

$$V = I_{\max} R$$

$$|V_L| = |V_C| = I_{\max} X_L = I_{\max} X_C$$

$$\text{Voltage magnification} = \frac{V_L}{V} = \frac{I_{\max} X_L}{I_{\max} R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$= \frac{V_C}{V} = \frac{X_C}{R} = \frac{1}{\omega R C} \Rightarrow \frac{\omega_0}{\Delta \omega}$$

Quality factor (Q) = Voltage magnification

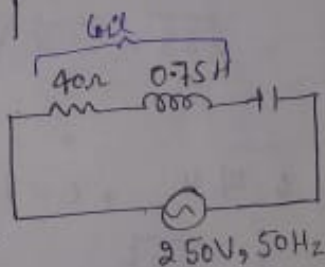
$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{\frac{1}{\omega_0 R C}} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Ans: $f_0 = 55 \text{ Hz}$
(Resonant freq)



Find ① the line current = ?

② Power factor?

③ Power Consumed?

④ Voltage across the coil?

Solⁿ → ① $f_0 = \frac{1}{2\pi\sqrt{LC}}$ $C = \frac{1}{L} \left(\frac{1}{2\pi f_0} \right)^2 = 11.115 \text{ mF}$

$$I = \frac{V}{|Z|} = \frac{250}{63.63} = 3.93 \text{ Amp}$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + (-43.48)^2} = 63.63$$

$$\text{① power factor} = \frac{R}{Z} = \frac{40}{63.63} = 0.629 = 0.63$$

iii) power consumed (only R will consume power)

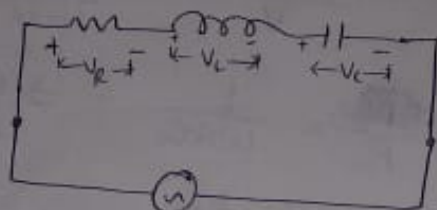
$$P = (I)^2 R = 6.18 \text{ Watt}$$

iv) Voltage across coil = $I \sqrt{X_L^2 + R^2}$

$$V_{\text{coil}} = 3.93 \times \sqrt{X_L^2 + (40)^2}$$

$$V_{\text{coil}} = 939.2 \text{ Volts}$$

Ques



$$\begin{aligned} V_R &= 5 \text{ Volt} \\ V_L &= 8 \text{ Volt} \\ V_C &= 10 \text{ Volt} \end{aligned}$$

Find f_0 (Resonance freq)
 ii) $V_{\text{net}} = ?$
 $f_0 = \frac{1}{2\pi\sqrt{LC}}$

solⁿ: ① $X_L = \frac{V_L}{I} \Rightarrow L = \frac{V_L}{2\pi f I}$

$$X_C = \frac{V_C}{I} \Rightarrow C = \frac{I}{2\pi f V_C}$$

$$\Rightarrow LC = \left(\frac{1}{2\pi f}\right)^2 \frac{V_L}{V_C}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1 \times 2\pi f \sqrt{\frac{V_C}{V_L}}}{2\pi} = 50 \sqrt{\frac{10}{8}} = \frac{50\sqrt{5}}{2}$$

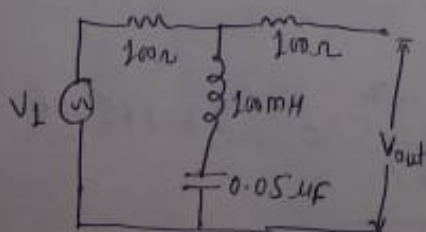
$$= 55.9 \text{ Hz}$$

ii) $V_{\text{net}} = \sqrt{(5)^2 + (10-8)^2} = \sqrt{29} = 5.38 \text{ Volt Ans}$

Ques: $R = 10 \Omega$, $L = 0.01 \text{ H}$, $C = 10 \mu\text{F}$ Series Circuit Find Quality factor

solⁿ: $Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{10} \sqrt{\frac{0.01}{1000 \times 10^{-6}}} = \frac{1}{10} \sqrt{10^3} = 3.16 \text{ Ans}$

Ques



Find the frequency at which $V_{\text{out}} = 0$

solⁿ: V_{out} is zero if $X_L = X_C \Rightarrow V_L = V_C$ so resonance freq

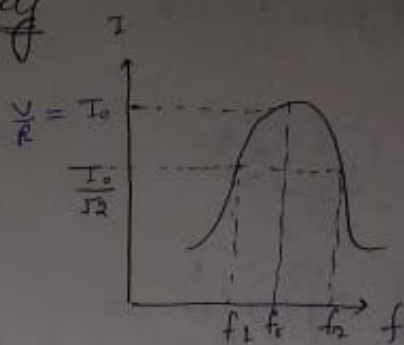
$$f = \frac{1}{2\pi\sqrt{LC}} = 2250 \text{ Hz Ans}$$

Corner frequency / Edge Frequency

At f_1 the current -

$$\frac{I_0}{\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}}$$

$$\frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}}$$



Compare

$$2R^2 = R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2$$

$$\omega_1 L - \frac{1}{\omega_1 C} = \pm R$$

Consider -ve sign ($\frac{1}{\omega_1 C} > \omega_1 L$)

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\omega_1^2 LC - 1 = -R\omega_1 C$$

$$\omega_1^2 LC + R\omega_1 C - 1 = 0 \Rightarrow \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

$$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4 \times 1 \left(-\frac{1}{LC}\right)}}{2 \times 1} \Rightarrow \frac{-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \omega_1$$

$$\text{so } \omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

-ve not possible because frequency is -ve.

At f_2 the current

$$\frac{I_0}{\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2}}$$

$$\frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2}}$$

$$2R^2 = R^2 + \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2 \Rightarrow \pm R = \omega_2 L - \frac{1}{\omega_2 C}$$

At f_2 $X_L > X_C$ so taking +ve sign

$$\omega_2 L - \frac{1}{\omega_2 C} = +R$$

$$\omega_g^2 - \frac{R}{L}\omega_g - \frac{1}{LC} = 0$$

$$\omega_g = \frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}$$

$$\omega_g = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

-ve cannot consider due to -ve freqn so

$$\boxed{\omega_g = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

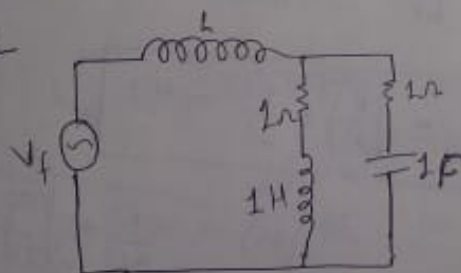
and

$$\boxed{\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Band width $\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$

$$\boxed{\omega_1 \omega_2 = \frac{1}{LC} = \omega_g^2}$$

Ques →

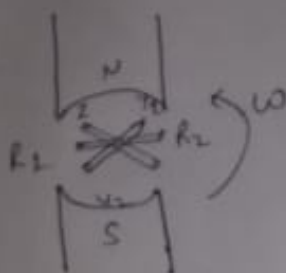
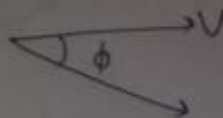
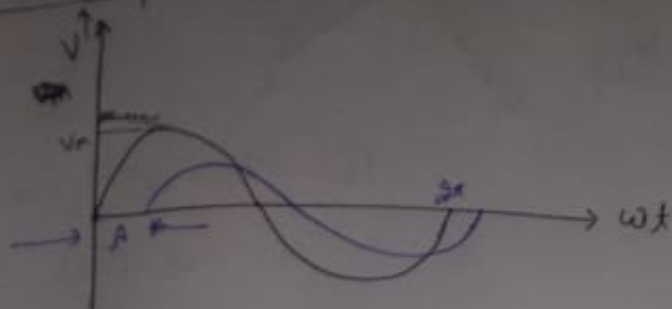


find condition for Resonance?

Soln →

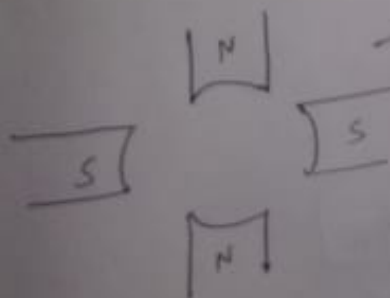
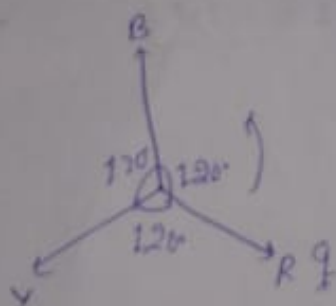
Unit - 3

Three phase A.C. Circuits



$$\left[\frac{360^\circ}{n} \right]$$

RYB RBY
 YBR BYR
 BRY YRB



→ No of poles are 4
 $P = 4$

$$\theta_{mech} = \frac{\theta_{elec}}{P/2}$$

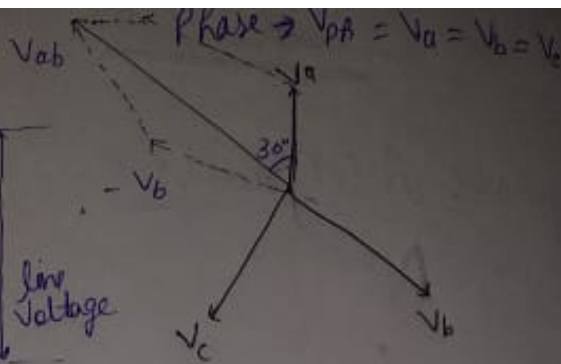
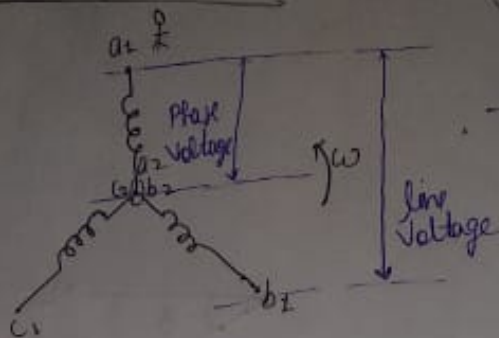
($\because P$ is no of poles here)

$$\begin{aligned}
 V_R &= V_m \sin \omega t \\
 \text{or} \\
 V_Y &= V_m \sin(\omega t + 120^\circ) \\
 V_B &= V_m \sin(\omega t - 120^\circ) \\
 \text{or} \\
 V_B &= V_m \sin(\omega t + 240^\circ)
 \end{aligned}$$



Star / Delta Connection :-

Star →



Line → $V_L = V_{ab} = V_{bc} = V_{ca}$
 $V_a + (-V_b)$

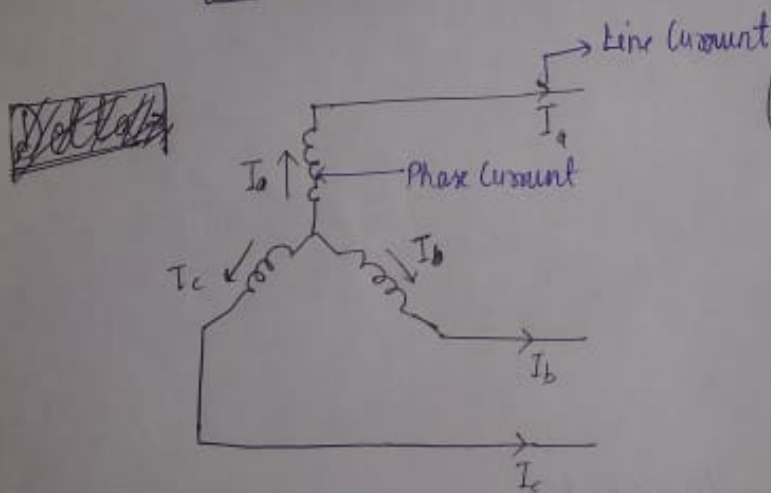
$$V_{ab} = \sqrt{V_a^2 + V_b^2 + 2V_a V_b \cos 60^\circ}$$

$$V_{ab} = \sqrt{V_a^2 + V_b^2 + 2V_a V_b \times \frac{1}{2}}$$

$\therefore V_a = V_b$

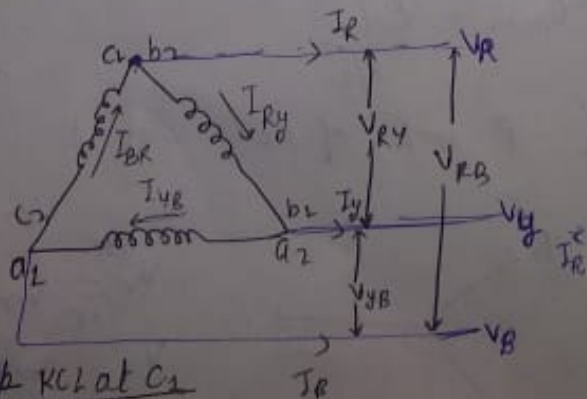
so $V_{ab} = \sqrt{3} V_a$

$$V_L = \sqrt{3} V_{ph}$$

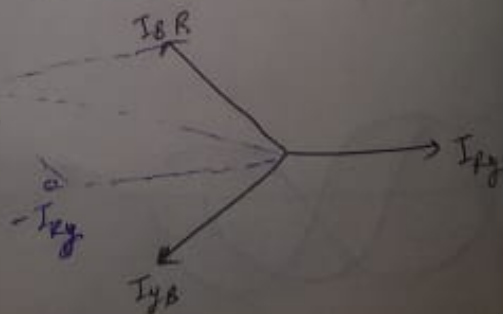


$$I_L = I_{ph}$$

Δ-Connection →



$$V_L = V_{ph}$$



KCL at C_1
 $-I_{BR} + I_{BY} + I_R = 0$

$$I_R = I_{BR} - I_{BY}$$

$$= V_c$$

$$\vec{V}_R = V_{ph} \angle 0^\circ = V_{ph} (1 + j0)$$

$$\vec{V}_Y = V_{ph} \angle 120^\circ = V_{ph} (\cos 120^\circ + j \sin 120^\circ) = V_{ph} \left(-\frac{1}{2} + j \frac{\sqrt{3}}{2}\right)$$

$$\vec{V}_B = V_{ph} \angle 240^\circ = V_{ph} \left(-\frac{1}{2} - j \frac{\sqrt{3}}{2}\right)$$

Parallel supply -

$$\Rightarrow \boxed{\vec{V}_R + \vec{V}_Y + \vec{V}_B = 0}$$

$$= V_{bc} = V_{ca}$$

$$+ (-V_b)$$