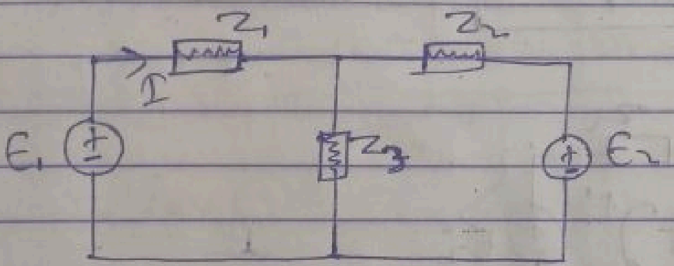
① <u>Super position Theorem</u> :- In any linear, bilateral network with two or more than two independent sources, the net response will be phasor sum of individual responses. Individual responses are responses of individual sources keep other sources deactivated and replaced by their internal impedances or resistance.

But in this theorem dependent sources are left as they are.

<u>proof</u> :- let consider a circuit,

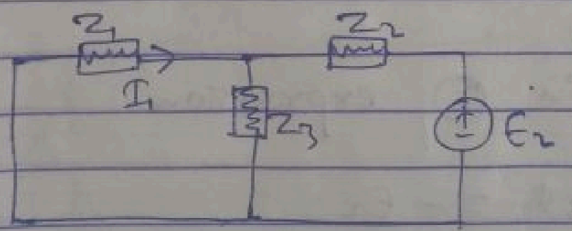
(i)



let in this case current through Z_1 is I.

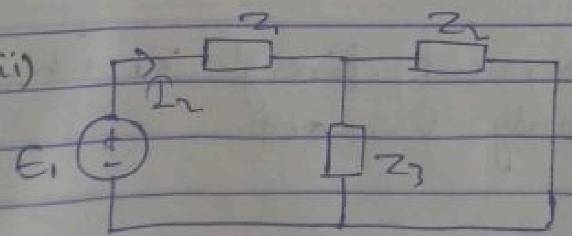
And Now, we find current through Z_1 when E_1 and E_2 are acting individually,

(ii)



let current through Z_1 is I_1,

(iii)

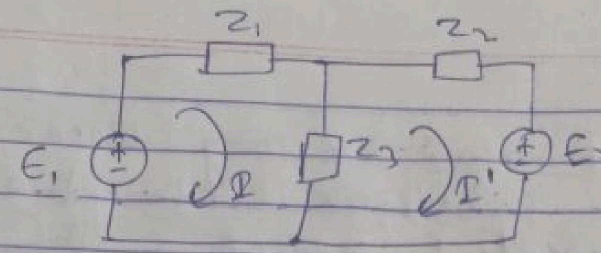


let current through Z_1 is I_2.

So, According to superposition Theorem,

$I = I_1 + I_2$

from (i),



KVL to loop ①, $-E_1 + IZ_1 + (I - I')Z_3 = 0$

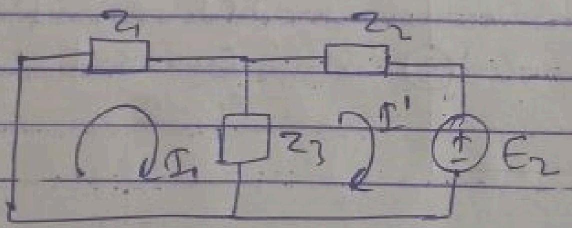
$$I(Z_1+Z_3) - I'Z_3 = E_1 \quad -①$$

$I'Z_2 + E_2 + (I' - I)Z_3 = 0 \Rightarrow I'(Z_2+Z_3) - IZ_3 = -E_2 \; -②$

By solving both eqn., we get,

$$\boxed{I = \frac{E_1Z_2 + E_1Z_3 - E_2Z_3}{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}} \rightarrow \text{Current through } Z_1$$

From (ii),



$Z_1I_1 + Z_3(I_1 - I') = 0 \Rightarrow I' = \dfrac{I_1(Z_1+Z_3)}{Z_3}$

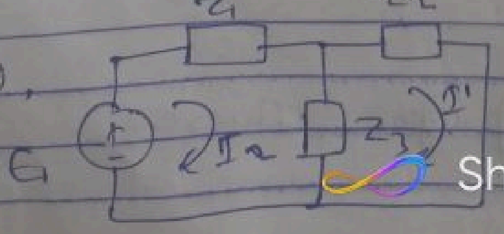
$Z_2I' + E_2 + Z_3(I' - I_1) = 0 \Rightarrow (Z_2+Z_3)I' - Z_3I_1 = -E_2$

put value of 'I'' in ② expression.

$$(Z_2+Z_3)\frac{I_1(Z_1+Z_3)}{Z_3} - Z_3I_1 = -E_2$$

$$\boxed{I_1 = \frac{-E_2Z_3}{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}} \rightarrow \text{current through } Z_1 \text{ when only } E_2 \text{ act.}$$

from (iii),



$Z_1I_2 + Z_3(I_2 - I') - E_1 = 0$

$I_2(Z_1+Z_3) - I'Z_3 = E_1 \quad -(a)$

$$Z_2 I' + Z_3(I' - I_2) = 0 \Rightarrow I' = \frac{I_2 Z_3}{Z_2 + Z_3}$$

put this value of I' in (a) expression.

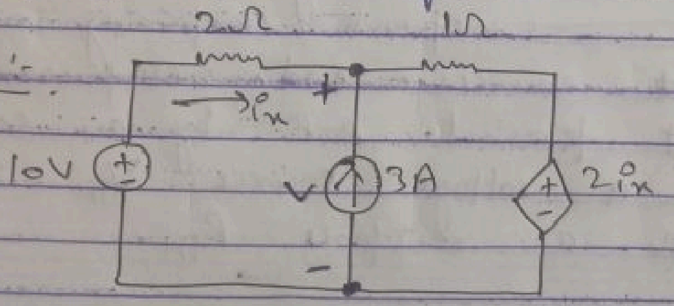
$$I_2(Z_1 + Z_3) - \frac{Z_3^2 I_2}{Z_2 + Z_3} = E_1$$

$$\boxed{I_2 = \frac{E_1(Z_2 + Z_3)}{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}}$$

$$I = I_1 + I_2$$

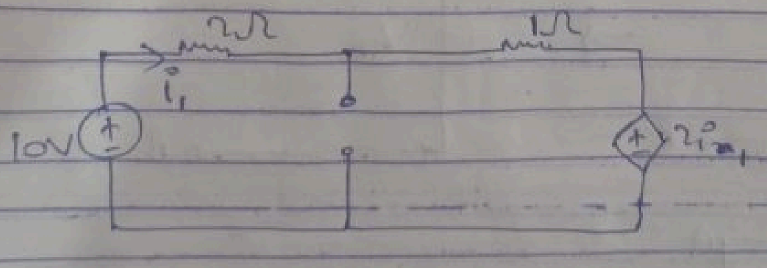
We can clearly see that, $I = I_1 + I_2$.
Superposition theorem proved.

Example:-



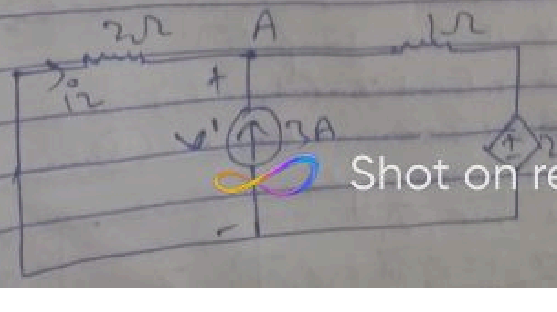
$i_x = ?$

⇒ First open the current source.



$$2i_1 + i_1 + 2i_1 - 10 = 0$$
$$\boxed{i_1 = 2A}$$

Now short the voltage source.



Apply KCL at node A.

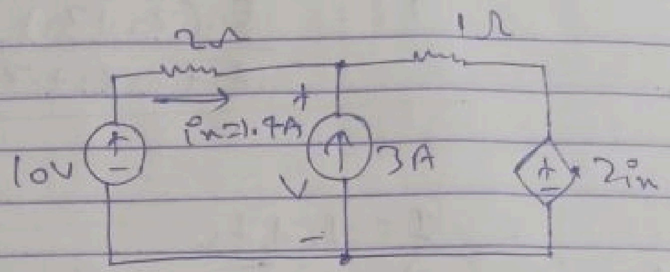
$$i_2 + 3 = \frac{V' - 2i_2}{1} \quad -(1)$$

$$V' = -2i_2 \quad -(2)$$

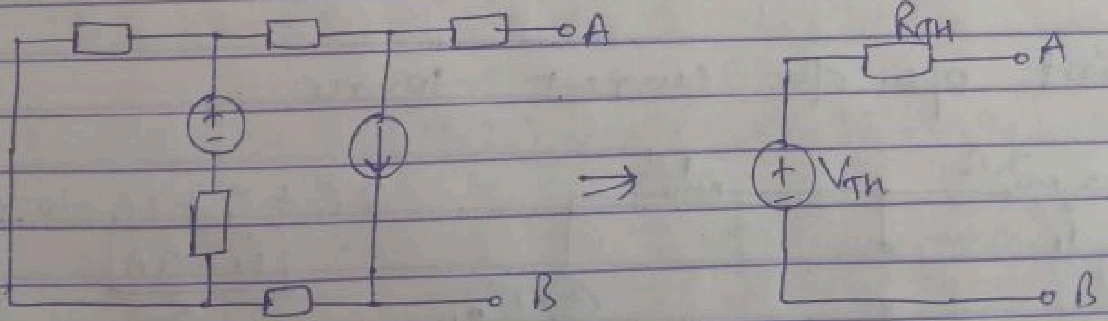
$$i_2 + 3 = -4i_2 \Rightarrow i_2 = -0.6A$$

Hence total $i_x = i_1 + i_2 = 1.4A$

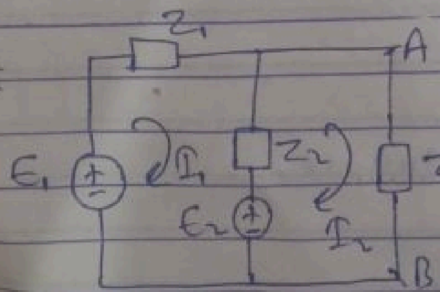
$$\underline{i_x = 1.4A}$$



(2) Thevenin's Theorem :- For any linear electrical network containing only voltage sources, current sources and resistances can be replaced at terminals A-B by an equivalent combination of a voltage source V_{TH} in a series connection with a resistance R_{TH}.



Proof :-



KVL in loop (1):

$$I_1 Z_1 + (I_1 - I_2) Z_2 + E_2 - E_1 = 0$$

$$I_1 (Z_1 + Z_2) - I_2 Z_2 = E_1 - E_2 \quad -(1)$$

KVL in loop (2):

$$I_2 Z_3 - E_2 + Z_2 (I_2 - I_1) = 0$$

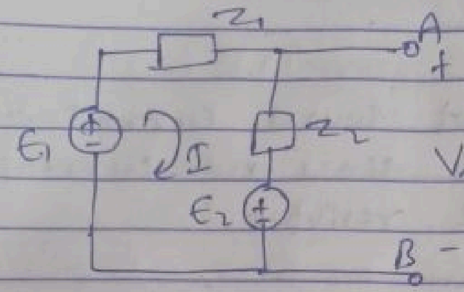
$$I_2 (Z_2 + Z_3) - I_1 Z_2 = E_2 \quad -(2)$$

After solving (1) & (2), we get

$$\boxed{I_2 = \frac{E_1 Z_2 + E_2 Z_1}{\ldots}} \rightarrow \text{current through } Z_3$$

To find thevenin equivalent circuit first we have to find V_{TH} that is the voltage across A-B terminal.

Remove the load.



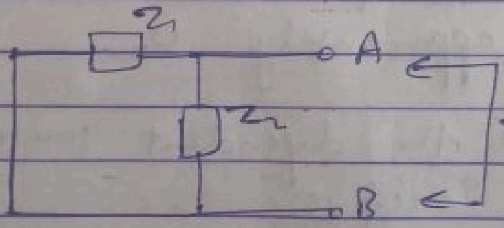
$$IZ_1 + IZ_2 + E_2 - E_1 = 0$$

$$\boxed{I = \frac{E_1 - E_2}{Z_1 + Z_2}}$$

$$IZ_2 + E_2 - V_{TH} = 0$$

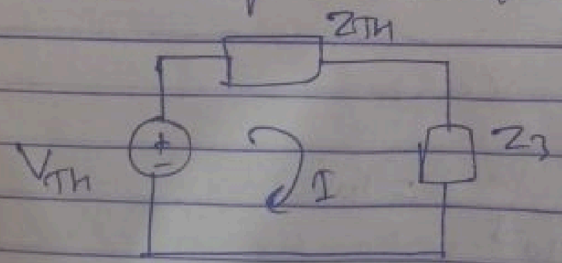
$$V_{TH} = E_2 + \frac{Z_2(E_1 - E_2)}{Z_1 + Z_2} \Rightarrow \boxed{V_{TH} = \frac{E_1Z_2 + E_2Z_1}{Z_1 + Z_2}}$$

Now to find Z_{TH} we have to replace all the sources by their respective internal impedances and find equivalent resistance across A-B terminal.



$$\boxed{Z_{TH} = \frac{Z_1Z_2}{Z_1 + Z_2}}$$

Hence equivalent thevenin circuit is.



Current through Z_3 is.

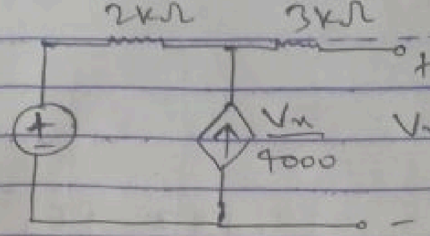
$$I(Z_3 + Z_{TH}) = V_{TH}$$

$$I = \frac{V_{TH}}{Z_{TH} + Z_3}$$

$$I = \frac{\frac{E_1Z_2 + E_2Z_1}{Z_1 + Z_2}}{\frac{Z_1Z_2}{Z_1 + Z_2} + Z_3} \Rightarrow \boxed{I = \frac{E_1Z_2 + E_2Z_1}{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}}$$

Hence, $I_L = I$ Thevenin's theorem proved.

Example:



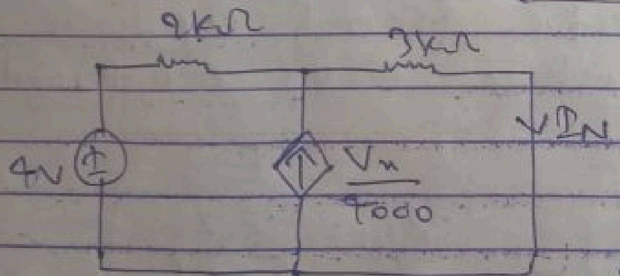
Determine Thevenin equivalent?

⇒ To find V_x, the dependent source current must pass through 2kΩ resistor, since no current can flow through the 3kΩ resistor.

KVL in outer loop $2000 \times \frac{V_x}{4000} + 4 - V_x + 3 \times 0 = 0$

$$\frac{V_x}{2} = 4 \Rightarrow \underline{V_x = 8V = V_{th}}$$

We know that, $\boxed{R_{th} = \frac{V_{TH}}{I_N}}$



In this case V_x is apparently 0.

So, the dependent source becomes inactive.

Hence,

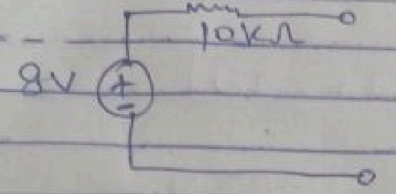
$$I_N = \frac{4}{5 \times 10^3} = 0.8mA$$

So, $R_{th} = \frac{V_{TH}}{I_N} = \frac{8}{0.8 \times 10^{-3}} = 10k\Omega$

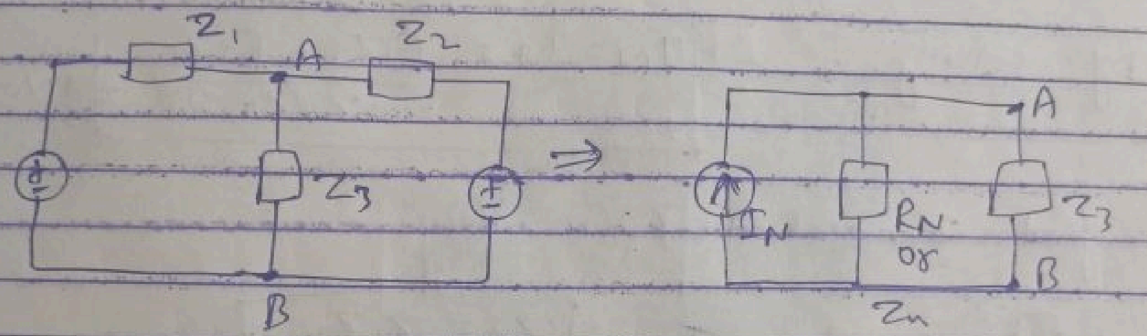
$R_{TH} = 10k\Omega$, $V_{TH} = 8V$

Shot on realme 8

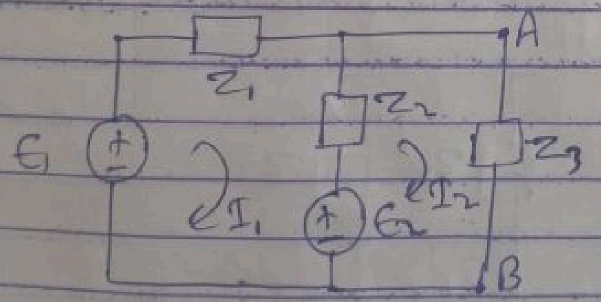
So, thevenin's equivalent circuit is,



③ <u>Norton's Theorem :-</u> Any linear circuit containing several energy sources and resistances can be replaced by a single constant current source in ~~pallel~~ parallel with a single resistor.



<u>Proof</u>:- Consider a circuit,



KVL in loop ①.

$I_1 Z_1 + Z_2(I_1 - I_2) + E_2 - E_1 = 0$

$I_1(Z_1 + Z_2) - I_2 Z_2 = E_1 - E_2$ — ①

KVL in loop ②.

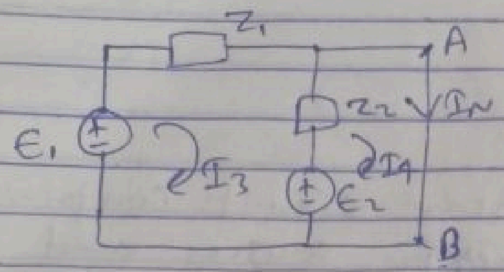
$Z_3 I_2 - E_2 + Z_2(I_2 - I_1) = 0$

$I_2(Z_2 + Z_3) - I_1 Z_2 = E_2$ — ②

By solving both equations we get,

$$\boxed{I_2 = \frac{E_1 Z_2 + E_2 Z_1}{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}}$$ → current through Z_3

To find norton's current remove the load Z_3 and short the load,



Here $\boxed{I_4 = I_N}$

KVL in loop ①,

$$I_3Z_1 + Z_2(I_3 - I_4) + E_2 - E_1 = 0$$

$$I_3(Z_1 + Z_2) - I_4Z_2 = E_1 - E_2 \quad -①$$

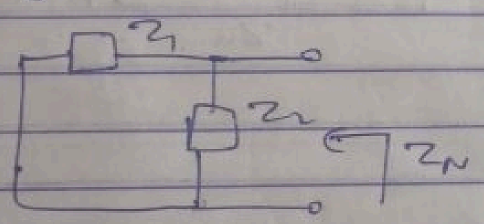
KVL in loop ②,

$$-E_2 + Z_2(I_4 - I_3) = 0 \Rightarrow Z_2I_4 - Z_2I_3 = E_2 \quad -②$$

By solving,

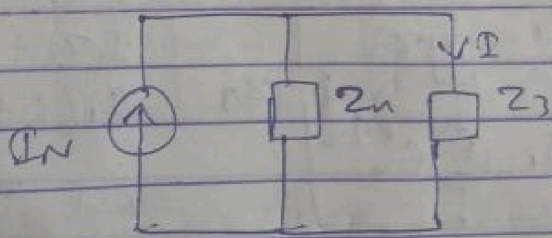
$$\boxed{I_N = I_4 = \frac{E_1Z_2 + E_2Z_1}{Z_1Z_2}} \rightarrow \text{Norton's Current}$$

To find Z_N or Z_{TH},



$$\boxed{Z_n = \frac{Z_1Z_2}{Z_1 + Z_2}}$$

Norton's equivalent,

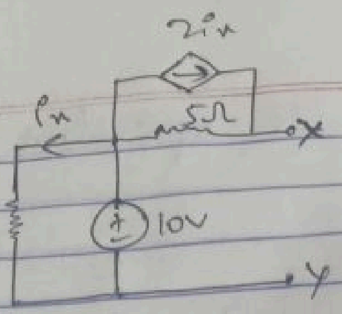


Current through Z_3 is,

$$I = \frac{Z_n}{Z_3 + Z_n} \times I_N$$

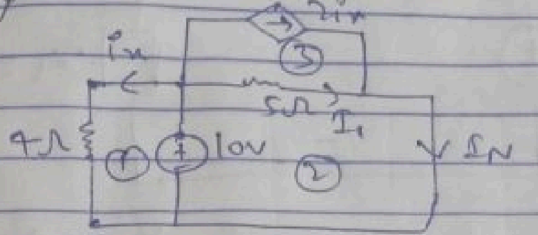
$$I = \frac{\frac{Z_1Z_2}{Z_1+Z_2}}{Z_3 + \frac{Z_1Z_2}{Z_1+Z_2}} \times \frac{E_1Z_2 + E_2Z_1}{Z_1Z_2} \Rightarrow I = \frac{E_1Z_2 + E_2Z_1}{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}$$

Hence, $\boxed{I = I_n}$ [illegible] proved.

Example:- 

Develop the Norton's equivalent circuit b/w the terminals x & y.

→ first calculate I_N.



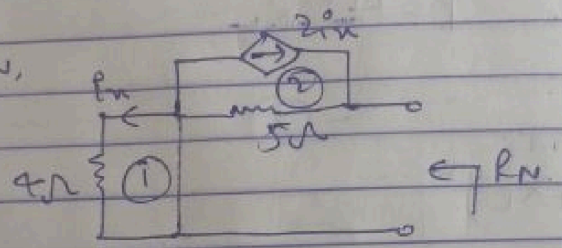
Here, $I_N = I_1 + 2i_x$ — ①

In ① loop, $i_x = \frac{10}{4} = 2.5A$

In ② loop, $5I_1 = 10 \Rightarrow I_1 = 2A$

Then, $I_N = 2 + 5 = 7A \Rightarrow \boxed{I_N = 7A}$

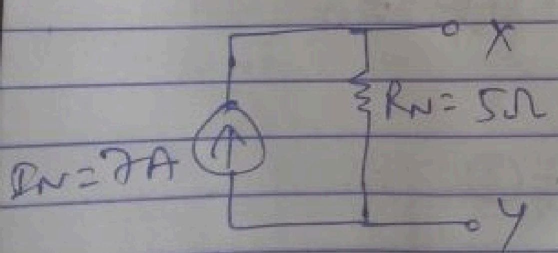
To find R_N,



In loop ①,
$i_x = 0$
then $2i_x = 0$

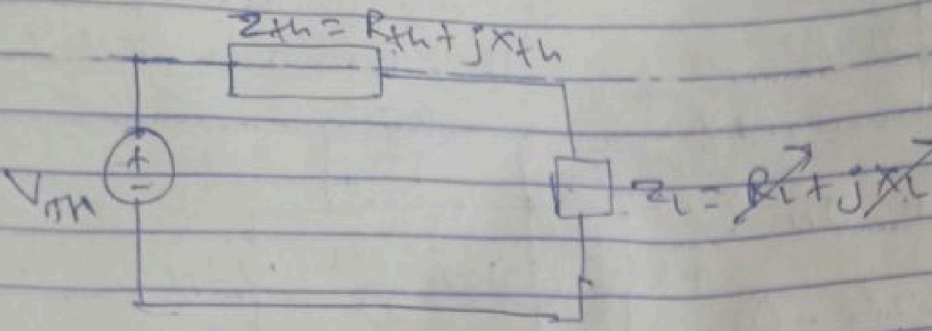
Hence, $R_N = 5\Omega$

So, Norton's equivalent circuit is,



④ **Maximum Power Transfer Theorem:** This theorem states that, to obtain maximum external power from a source with a finite internal impedance, the impedance of the load must equal the conjugate impedance of the source as viewed from its o/p terminals.

Proof :- Consider a thevenin's equivalent circuit.



Current flowing through circuit is.

$$|I| = \frac{V_{th}}{Z_{equ.}} = \frac{V_{th}}{\left((R_{th}+R_L)^2 + (X_{th}+X_L)^2\right)^{1/2}}$$

We know that, $P = |I|^2 R_L$

$$P_L = \frac{V_{th}^2 R_L}{(R_{th}+R_L)^2 + (X_{th}+X_L)^2} \quad -①$$

To get max. power transfer we should differentiate P_L w.r.t. X_L and R_L,

$$\frac{dP_L}{dX_L} = \frac{-V_{th}^2 R_L}{\left((R_{th}+R_L)^2 + (X_{th}+X_L)^2\right)^2} \times 2(X_{th}+X_L) = 0$$

we get, $\boxed{X_{th} = -X_L}$ $-②$

Now put ② equ. in ①,

$$P_L = \frac{V_{th}^2 R_L}{(R_{th}+R_L)^2} \Rightarrow \cancel{\frac{dP_L}{dR_L} = \text{ex}}$$

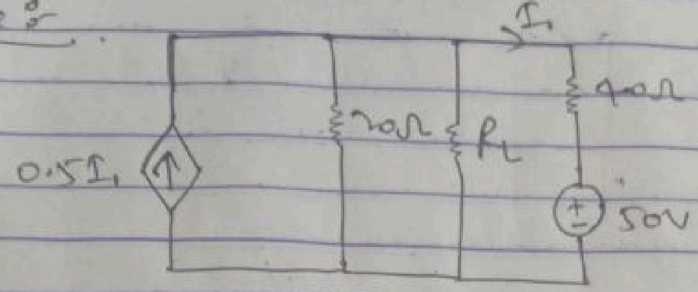
$$\frac{dP_L}{dR_L} = V_{th}^2 \left[\frac{1 \times (R_{th}+R_L)^2 - R_L \times 2(R_{th}+R_L)}{(R_{th}+R_L)^4}\right] = 0$$

we get, $\boxed{R_{th} = R_L}$

$Z_L = R_L + jX_L = R_{th} - jX_{th}$

$\boxed{Z_L = \overline{Z}_{th}}$ proved

Example :-

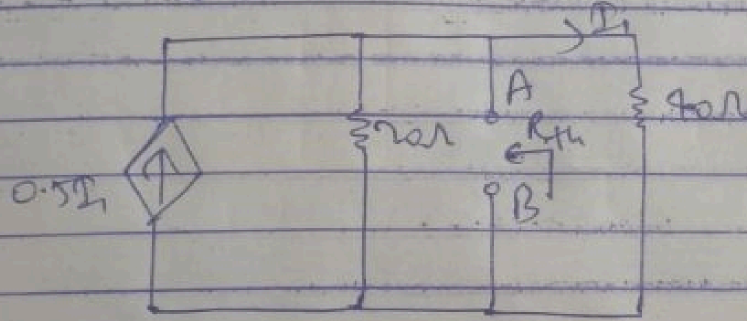


The max. power is delivered to R_L if R_L is.?

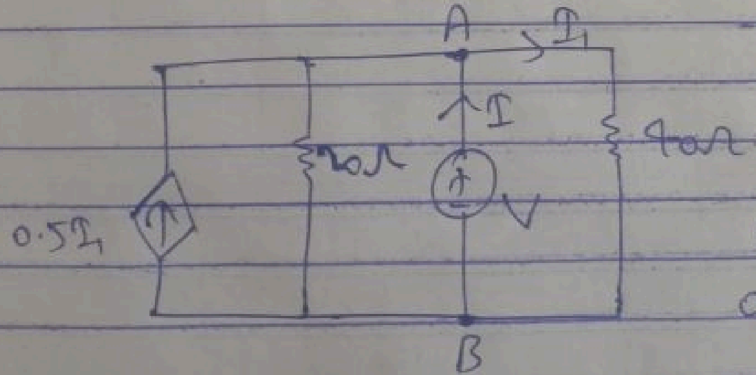
$R_L = ?$

→ for P_{max}, $\boxed{R_L = R_{Th}}$

To find R_{th} across R_L short the voltage source and Remove the R_L,



Now to solve above circuit consider a voltage source at open terminals,



Here, $\boxed{R_{th} = \frac{V}{I}}$

Apply KCL at A,

$0.5I_1 + I = \frac{V}{20} + I_1$

Here, $\boxed{I_1 = \frac{V}{40}}$

$\frac{V}{20} = I - 0.5I_1$

$\frac{V}{20} = I - 0.5 \times \frac{V}{40}$

$\frac{5V}{80} = I \Rightarrow \frac{V}{I} = R_{th} = R_L = 16\Omega$