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NUMBER SYSTEM

Natural Numbers

The counting numbers are commonly called natural numbers.
For example Natural Number, N = {1, 2, 3 …}

- All natural numbers are positive.
- The smallest natural number is 1.
- Zero (0) is not a natural number.

Whole Numbers

All the natural numbers including Zero are called Whole Numbers. It is also known as non-negative integers.
For example Whole Numbers, W = {0, 1, 2, 3 …}.

Integers

Whole numbers as well as negative numbers form the set of integers. It can be classified into two types,
(i) Positive integers → {1, 2, 3…}
(ii) Negative integers → {– 1, – 2, – 3…}
(iii) Zero is neither a positive nor a negative integer.

Even Numbers

All the counting numbers which are divisible by 2 are called even numbers.
For example 2, 4, 6, 8, 10…2n

- Unit place of an Even number is 0, 2, 4, 6, or 8.

Odd Numbers

The numbers which are not divisible by 2 are called odd numbers.
For Example 1, 3, 5, 7, 9, 11 … (2n-1).

- Unit place of an odd number is 1, 3, 5, 7,or 9.

Odd * Odd = Odd
Even * Even = Even
Odd * Even = Even

Prime Numbers

The numbers which are having exactly 2 distinct factors namely itself and 1 are called Prime Numbers.
For Example: 2, 3, 5, 7, 11, 13 …
The number 5 is a prime number and the factors of 5 are 1 and 5. These are two distinct factors and so 5 is a Prime number.

- The smallest odd prime number is 3.
- 2 is the only even prime number.
- All prime numbers greater than 3 can be represented by 6n+1 or 6n-1, where n is an integer.

How to check if a number is Prime or not?
If A is a given number,
(i) Find a number S, such that $S > \sqrt{A}$.
(ii) Consider all the prime numbers less than equal to S.
(iii) If none of these divides A, then A is prime.
Example: Find A= 137 is prime or not?

$$12 > \sqrt{137}$$

Prime numbers up to 12 are: 2, 3, 5, 7, and 11.
None of these divide 137 exactly, so 137 is a Prime number.

Composite Numbers

Composite numbers are non-prime natural numbers. They must have at least one factor except 1 and itself.

- 1 is neither prime nor composite.

Co Primes

Two natural numbers are said to be Co primes if the HCF of the two numbers is 1.
For example (7, 9) and (18, 19)

- Two composite numbers having no common factor except 1 are always co prime.
- Two consecutive numbers are always co prime.

DIVISIBILITY TESTS

Divisibility by 2
When the unit digit of a number is even, the number is divisible by 2.
For example
16, 98, 1000 etc., are divisible by 2.

Divisibility by 3
When the sum of the digits of a number is a multiple of 3, then the number is divisible by 3.
For example
4518 = 4 + 5 + 1 + 8 = 18 which is a multiple of 3, so 1233 must be divisible by 3.

Divisibility by 4
When the last two-digits of a number are a multiple of 4, then the number is divisible by 4.
For example
4596 is divisible by 4 as the last two digits 96 of the number are divisible by 4.

Divisibility by 5
Numbers having 0 or 5 at the unit place are divisible by 5.
For example
55, 2350, 22850 etc., are divisible by 5 as they have 0 or 5 at the unit place.

Divisibility by 6
When a number is divisible by both 3 and 2, then the number is divisible by 6 also.
For example
12, 1440 etc., are divisible by 6 as they are divisible by both 3 and 2.

Divisibility by 7
A number is divisible by 7 when the difference between twice the digit at the units place and the number formed by the other digits is either zero or a multiple of 7.
For example
679 is divisible by 7 because 67 – (2 × 9) = 67 - 18 = 49. As 49 is a multiple of 7, the number 679 is divisible by 7.

Divisibility by 8
When the number made by last three digits of a number is a multiple of 8, then the number is divisible by 8.
For example
2208, as 208 (the last three digits of 2208) is divisible by 8, the number 2208 is also divisible by 8.

Divisibility by 9
When the sum of all the digits of a number is a multiple of 9, then the number is also divisible by 9.
For example
936819 → 9+3 + 6 + 8 + 1 + 9= 36 which is divisible by 9. Therefore, 936819 is divisible by 9.

Divisibility by 10
When the digit at the unit place of a number is zero, then the number is divisible by 10.
For example
20, 40, 150, 123450, 478970 etc., are divisible by 10 as they all end with zero.

Divisibility by 11
When the difference between the sums of odd position digits of the number and the even position digits of the number are multiples of 11 or zero, the number is divisible by 11.
For example
661749 → Sum of digits at odd places (A) = 6 + 1 + 4 = 11
Sum of digits at even places (B) = 6 + 7 + 9 = 22 →A – B = 22 – 11 = 11.
661749 is divisible by 11.

HCF or GCD

HCF of a given set of numbers is the greatest common number that divides all the numbers of the set. Hence it is called HCF of the given set.
To find the HCF of the given numbers

- Factorize each of the given set of numbers into prime factors and their powers thereof
- Take the common prime factors that contain the minimum power available and multiply. The product is known as the HCF of the given set of numbers.

Least Common Multiple

LCM of any given set of numbers is the smallest such number which is divisible by each number of the given set.
To find the LCM of the given numbers,

- Factorize the number into prime factors and their powers thereof.

- Select all the prime factors, with their respective maximum power, and multiply them.

For Example, Consider 12, 24
$12 = 2 \times 2 \times 3 = 2^2 \text{ X } 3^1$
$24 = 2 \times 2 \times 2 \times 3 = 2^3 \text{ X } 3^1$
HCF of 12, 24 $\rightarrow 2^2 \text{ X } 3^1 = 12$
LCM of 12, 24 $\rightarrow 2^3 \text{ X } 3^1 = 24$

FACTORS or DIVISORS

In order to find the factors of a number Nidentify the prime factors and their respective powers thereof and rewrite the number where a, band c are the prime factors and x, y and z are their respective powers as
$N=a^x * b^y * c^z$
Number of factors = (x+1)(y+1)(z+1)

Remainder theorem

The basic remainder theorem formula is:
Dividend = Divisor* Quotient + Remainder
If remainder = 0, then the number is divisible by the divisor and divisor is a factor of the number.
For example when 8 divides 40, the remainder is 0 and it can be said that 8 is a factor of 40.

Cyclicity of Remainders:

Cyclicity is the property of remainders, due to which the remainders start repeating after a certain point.
Euler's theorem
Euler's theorem states that for any co prime numbers P and Q,
$R(\frac{P^{\varphi(n)}}{Q}) = 1$. Where φ (n) is Euler's totient.
It is applicable only for co-prime numbers.
Euler's totient
$\varphi(n) = n \times (1 - 1/P_1) \times (1 - 1/P_2) \times (1 - 1/P_3) \times \ldots$
Fermat's theorem
Remainder of $\frac{a^{p-1}}{} = 1$, which is Fermat's little theorem,where p is a prime number and a and p are co primes.

CLASSWORK PROBLEMS

1. Find the rational form of the recurring rational 0.2333333333
(a)11/99
(b) 1/3
(c) 7/30
(d) 13/30

2. What two numbers have a product of 48 and, when the larger number is divided by the smaller, a quotient of 3?
(a) 4 and 12
(b) 6 and 12
(c) 6 and 12
(d) 5 and 10

3. When a two digit number is reversed it gets decreased by 72, what is the number?
(a) 81
(b) 72
(c) 91
(d) 64

4. Find the number of 7's between 100 and 700.
(a) 120
(b) 101
(c) 121
(d) 100

5. If all the 6 are replaced by 9, then the algebraic sum of all the numbers from 1 to 100 (both inclusive) varies by ___.
(a) 500
(b) 360
(c) 450
(d) 330

6. Rahul scored 78, 56 and 89 marks in Science, Social and Mathematics. Later it was found that his marks were reversed and entered. How much marks should be added to get correct total?
(a) 18
(b) 24
(c) 27
(d) 15

7. A group of friends goes for dinner and gets bill of Rs 2400. Two of them says that they have forgotten their purse so remaining make an extra contribution of Rs 100 to pay up the bill. Tell the no. of person in that group.
(a) 8 persons
(b) 7 persons

(c) 12 persons
(d) None of these

8. If 14p0p0p4 which is a 8 digit number and is divisible by 12 then the number of possible values of p is:
(a) 2
(b) 3
(c) 4
(d) 5

9. If 21pq33pq is a 8-digit number which is divisible by 12 then how many 2 digit numbers as pq are possible?
(a) 8
(b) 18
(c) 12
(d) 28

10. Let N = 80pq2pq (7digit number). If N is exactly divisible by 120 then the sum of the digits in N is equal to:
(a) 18
(b) 22
(c) 24
(d) 12

11. If 53p26p3 is a 7 digit number divisible by 9 and if 757qp is divisible by 8 then the minimum value of p + q is:
(a) 4
(b) 12
(c) 16
(d) 8

12. Find the LCM of 12, 15, 72 and 75
(a)1800
(b) 1500
(c) 1440
(d) 1040

13. 6 bells of a Church toll at different intervals of 5seconds, 8 seconds, 10 seconds, 6 seconds, 12 seconds and 15 seconds respectively. If they toll together at 12 noon, how many times will they toll together till 1 pm?
(a) 15 times
(b) 20 times
(c) 31 times
(d) None of these

14. Find the HCF 2/4, 10/8, 4/12, 6/15
(a) 1/60
(b) 1/2
(c) 2/56
(d) 4/60

15. If HCF and LCM of two numbers 15 and 1440 respectively, one of the numbers is 75 find the other number.
(a) 240
(b) 288
(c) 250
(d) 285

16. If 24586 and 22584 both leaves a same remainder when divided by a divisor which of the following can be the maximum possible value of the divisor?
(a) 143
(b) 77
(c) 91
(d) 2

17. Find the number of factors of 35.
(a) 4
(b) 6
(c) 8
(d)10

18. Find the number of factors of 120
(a) 4
(b) 8
(c) 12
(d) 16

19. Find the number of factors of 330.
(a) 2
(b)8
(c) 16
(d) 32

20. Find the number of factors of 1000.
(a) 12
(b) 16
(c) 10
(d) 18

21. Find the number of factors of 1560.
(a) 6
(b) 16
(c) 12
(d) 32

22. How many factors of 340 are even?
(a) 12
(b) 16
(c) 8
(d) 6

23. How many factors of 408 are even?
(a) 12
(b) 16
(c) 8
(d) 10

24.How many factors of 1024 are even?
(a) 12
(b) 10
(c) 8
(d) 6

25. In how many ways can 120 be written as a product of two numbers?
(a) 12
(b) 8
(c) 16
(d) 10

26. In how many ways can 200 be written as a product of two numbers?
(a) 4
(b) 6
(c) 8
(d) 10

27. In how many ways can 450 be written as a product of two numbers?
(a) 12
(b) 16
(c) 18
(d) 6

28. In how many ways can 840 be written as the product of 2 numbers?
(a) 6
(b) 16
(c) 18
(d) 32

29. In how many ways 12 can be written as a product of two co-prime factors?
(a) 2
(b) 6
(c) 8
(d) 6

30. In how many ways 320 can be written as a product of two co-prime factors?
(a) 2
(b) 6
(c) 1
(d) 5

31. In how many ways 540 can be written as a product of two co-prime factors?
(a) 1
(b) 2
(c) 3
(d) 4

32. In how many ways 1024 can be written as a product of two co-prime factors?
(a) 2
(b) 1
(c) 3
(d) 4

33. Find the number of zeroes in 154!.
(a) 35
(b) 31
(c) 34
(d) 37

34. Find the number of trailing zeroes in 56!
(a) 13
(b) 11
(c) 12
(d) 6

35. Find the remainder when 234! is divided by 560.
(a) 2
(b) 0
(c) 1
(d) 13

36. Find the unit digit of 2354^{1048}
(a) 4
(b) 6
(c) 8
(d) None of these

37. Find the unit digit of 248^{1587}.
(a) 2
(b) 4
(c) 8

(d) 6

38. Find the first non zero digit in 100!
(a) 6
(b) 8
(c) 4
(d) 3

39. Find the remainder when 171054 is divided by 15.
(a) 4
(b) 11
(c) 12
(d) 13

40. Find the remainder when 2145123 is divided by 6.
(a) 0
(b) 3
(c) 4
(d) 5

ASSIGNMENT PROBLEMS

1. What minimum number must be added to 454 so that the number is divisible by both 5 and 7?
(a) 1
(b) 5
(c) 34
(d) 35

2. What minimum number must be added to or subtracted from 261 so that the number is exactly divisible by 2, 6 and 8?
(a) 1
(b) 2
(c) 3
(d) 21

3. What is the smallest number which gives a remainder of 2 when divided by 3, 6 and 5?
(a) 5
(b) 8
(c) 32
(d) None

4.Which is the smallest number greater than 1000 that gives a remainder of 5 when divided by both 6 and 8?
(a) 1008
(b) 1001
(c) 1013
(d) 1012

5. What is the largest number smaller than 300 which when divided by both 4 and 7 gives a remainder of 3?
(a) 3
(b) 31
(c) 283
(d) 308

6. Find the largest 4 digit number that will give a remainder of 6 when divided by 8 and 7?
(a) 9974
(b) 9984
(c) 9969
(d) none of these

7. In a leap race ram takes a leap of 6 feet and shyam takes a leap of 8 feet. Find the distance of the point (from starting point) where both will land at the same spot 3rd time. (Assume their average speed is same)
(a) 24 feet
(b) 72 feet
(c) 36 feet
(d) 144 feet

8.What is the maximum possible length of a thread required to exactly measure 352 cm, 220 cm and 308 cm?
(a) 22
(b) 44
(c) 11
(d) 88

9. Find the largest number which will divide 458, 515 and 648 leaving same remainder in all cases.
(a) 19
(b) 57
(c) 38
(d) 133

10. LCM of three numbers are in the ratio 7:3:12 is 2016. What is the HCF of these numbers?
(a) 4
(b) 8
(c) 12
(d) 24

11. Find the largest 4 digit number which when divided by 42 and 24 leaves 33 and 15 as remainders respectively.
(a) 9921
(b) 9903
(c) 9912
(d) 9901

12. Alarms of three clocks ring at the interval of 4 minutes, 8 minutes and 6 minutes respectively. All of them rang together at 8.00 a.m. Now, find the number of times they will ring together before 9.45 a.m.
(a) 5
(b) 4
(c) 6
(d) 8

13. In a circular race, A completes 1 round in 15 minutes and B completes 1 round in 25 minutes. After making how many rounds will A meet B again at the starting point, if they start running together from the starting point?
(a) 3
(b) 5
(c) 6
(d) 2.5

14. Find the number of composite factors of 42.
(a) 1
(b) 4
(c) 5

15.Find the number of prime factors of 300.
(a) 2
(b) 3
(c) 4
(d) 5

16. Find the number of factors of 72.
(a) 9
(b) 10
(c) 11
(d) 12

17. Find the last digit of $456^{87}*307^{42}$
(a) 1
(b) 4
(c) 5
(d) 7

18. . What will be the last digit of 48^{67}?
(a) 2
(b) 4
(c) 8
(d) 6

19. A number 1245674567 is divided by 11. What will be the remainder?
(a) 8
(b) 9
(c) 10
(d) 5

20. What is the remainder when $(259^{19}+34^{17})$ is divided by 10?
(a) 3
(b) 5
(c) 7
(d) 9