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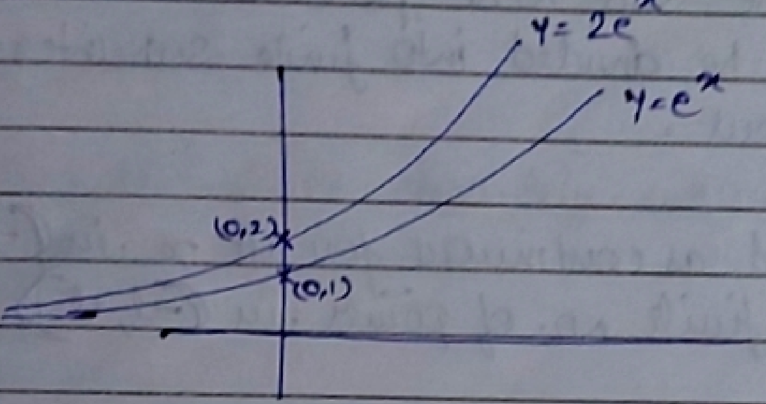
Maths - 3 (RMA - 201)

Unit 1: Transform Methods → Fourier integral, condition of convergence, fourier sin & cosine integral, fourier transform, existence

Fourier integral → Fourier sine integral
→ fourier cosine integral



$y_1 = c_1 e^x$ → a family of curves

Example of :-

Initial value problem → Solve the eq^n when $y = -1$ when $x = 0$

& boundary values, interval is mentioned. func^n value is provided at 2 points.

Unit 2: Functions of complex variable (Differentiation

Limit, Continuity, Differentiability, Analyticity.

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→ Fourier Integral

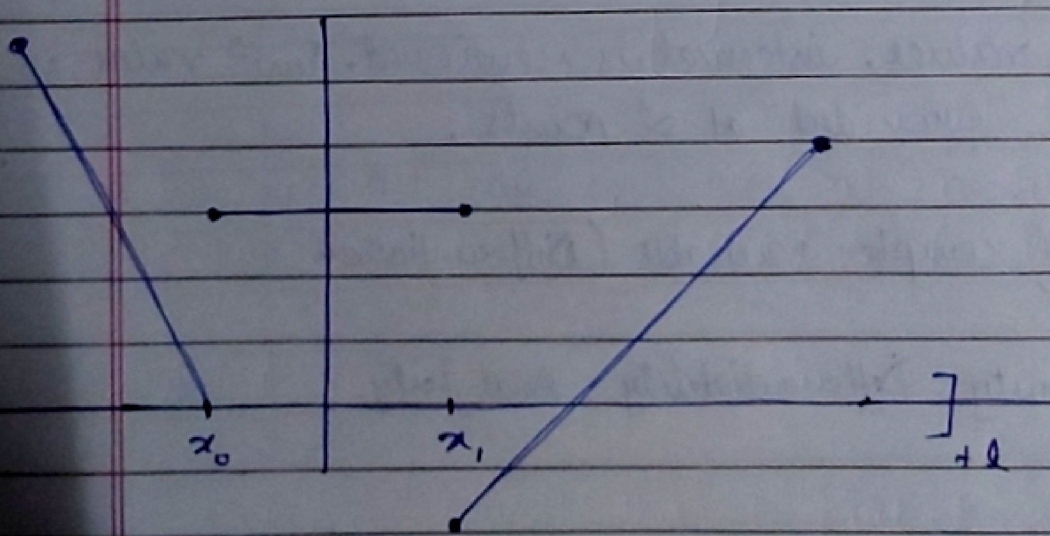
Conditions for applicability

① funⁿ should be piecewise continuous on arbitrary interval $[-l, l]$

A funⁿ can be said to be piecewise continuous if interval can be divided into finite sub-intervals on it is continuous.

→ $f(x)$ is defined as continuous for all x in $[-l, l]$ ex may be at a finite no. of points in $[-l, +l]$

→ At any point $x_0 \in (-l, l)$ where $f(x)$ is not conti both the one-sided limits $\lim_{x \to x_0^-} f(x)$ and $\lim_{x \to x_0^+} f(x)$ exist an are finite.



- If $f(x)$ is piecewise continuous on any interval $(0,l)$ and periodic then it can be represented by fourier series over the entire real no. line.

- If $f(x)$ is non periodic, then it cannot be represented by fourier series but can be represented in an Integral form if the func^n has the following properties :-

① $f(x)$ should be piecewise continuous on each interval $[-l, l]$

② $f(x)$ should be absolutely integrable on the x axis (integral should be convergent on $-\infty$ ∞)

$$\int_{-\infty}^{\infty} |f(x)|\,dx \quad \text{converges}$$

(proper integral is always converg

③ At each x on the real line, ~~f(x)~~ $f(x)$ has left & right hand derivative.

FOURIER INTEGRAL (DERIVATION)

Consider the fourier series representation of $f(x)$ on the interv $[-l, l]$.

$$\left[f(x)\right] = \left[\frac{a_0}{2} + \left(\sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}\right) + \left(\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}\right)\right]$$

Where

$$a_0 = \frac{1}{l} \int_{-l}^{+l} f(t)\, dt$$

$$a_n = \frac{1}{l} \int_{-l}^{l} f(t) \cos\left(\frac{n\pi t}{l}\right) dt$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(t) \sin\left(\frac{n\pi t}{l}\right) dt$$

Assume a quantity

$$\omega_n = \frac{n\pi}{l} \quad \text{and} \quad \Delta\omega = \omega_n - \omega_{n-1}$$

$$= \frac{\pi}{l}$$

Replacing above nota^n into fourier series formula

$$f(x) = \frac{\Delta\omega}{2\pi} \int_{-l}^{+l} f(t)\, dt + \frac{1}{\pi} \sum_{n=1}^{\infty} \left[\left\{ \int_{-l}^{l} f(t) \cos(\omega_n t)\, dt \right\} \cos \right.$$

$$\left. + \left\{ \int_{-l}^{l} f(t) . \sin \omega_n t\, dt \right\} \sin \omega_n x \right] \Delta\omega \quad \text{— (1)}$$

* to represent our eq^n on real no. line, ∴ we have to extent our limits $[-l, l]$ to $[-\infty, \infty]$

As $l \to \infty$, → $\Delta\omega \to 0$

due to this, 1st part of eq^n (1) becomes 0.

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*Concept → Riemann's sum of a definite integral

$$\lim_{\Delta\omega \to 0} \sum_{n=1}^{\infty} \int f(n) . \Delta\omega$$

↓

$$\int_0^{\infty} f(\omega)\, d\omega$$

↓

using this in eq^n (1)

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left[\left\{ \int_{-\infty}^{\infty} f(t) \cos \omega_n t\, dt \right\} \cos \omega_n x + \left\{ \int_{-\infty}^{\infty} f(t) \sin \omega_n t\, dt \right\} \sin \omega_n \right] d\omega$$

Since ω_n is not discreet now,

$\omega_n \rightarrow \omega$

$$\Rightarrow f(x) = \frac{1}{\pi} \int_0^{\infty} \left[\left\{ \int_{-\infty}^{\infty} f(t) \cos \omega t\, dt \right\} \cos \omega x + \left\{ \int_{-\infty}^{\infty} f(t) \sin \omega t\, dt \right\} \sin \, d\omega \right]$$

$$\Rightarrow f(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ A(\omega) \cos \omega x + B(\omega) \sin \omega x \right\} d\omega$$

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Convergence Of fourier Integral

fourier integral of any func^n f(x) converges to value of the func^n f(x) at the point of continuity &

→ Fourier Sin & cosine Integral

1. fourier Sine integral

Let f(x) is odd func^n

$$A\omega = \int_{-\infty}^{\infty} \underbrace{f(t)}_{\text{odd}} \underbrace{\cos\omega t}_{\text{even}} \, dt$$

product of odd × even = odd

$$\boxed{\therefore A\omega = 0}$$

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Putting value into fourier integral

$$f(x) = \frac{2}{\pi}\int_0^\infty \sin \omega x \left\{ \int_0^\infty f(t) \cdot \sin \omega t \, dt \right\} d\omega$$

$$\Rightarrow \boxed{\frac{1}{\pi}\int_0^\infty B(\omega) \sin(\omega x) \, d\omega}$$

Fourier sine integral

2. Fourier cosine Integral

Let f(x) is even func^n

$$B\omega \to 0$$

$$\therefore f(x) = \frac{2}{\pi}\int_0^\infty \cos \omega x \left\{ \underbrace{\int_0^\infty f(t) \cos \omega t \, dt}_{A\omega} \right\} d\omega$$

$$\Rightarrow \boxed{\frac{1}{\pi}\int_0^\infty A(\omega) \cos(\omega x) \, d\omega}$$

Fourier cosine integral