

Chapter-1 Relativistic Mechanics

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① Outline of Relativity.

Einstein demonstrated that many parameters which were earlier regarded as absolute (space, time interval, mass, velocities of moving objects) are no more absolute but are affected by the velocity of source and observer.

② State of Rest and state of Motion.

Rest \rightarrow position do not change w.r.t time.

Motion \rightarrow position changes w.r.t time

Eg car which is running on the road w.r.t to an observer standing on the ground.

③ Event

Something which happens in space at some instant is known as event.

Eg (a) Shot of a bullet at some instant.

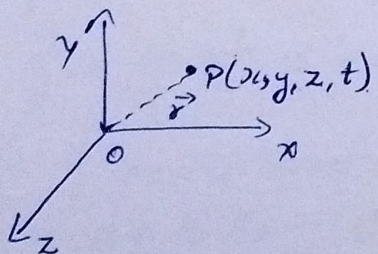
(b) Swinging pendulum while it passes through the mean position.

④ Frames of Reference

\rightarrow It is a geometrical framework.

\rightarrow This is used to describe the position of object in space.

\rightarrow An event occurring in space is described by space coordinates (x, y, z, t) .



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

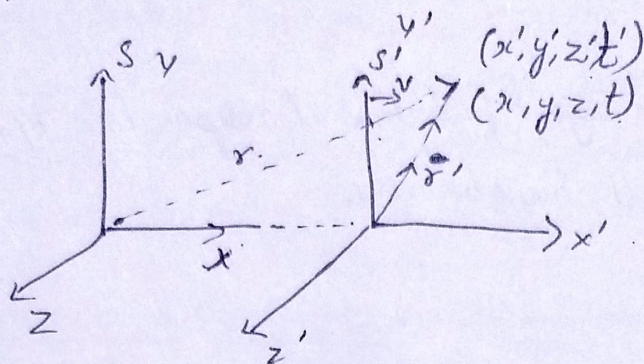
$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

\downarrow
space time continuum

* It is difficult to imagine a frame of reference completely at rest. Eg: ground.
Reason \rightarrow Earth is moving in the orbit around the sun as well as around its own axis. In both situations motion is accelerated, but for practical purpose we assume frame attached to earth to be at rest.

Galilean Transformation

Equations which transform the coordinates of an ~~off~~ object from one ~~point~~ inertial frame to another inertial frame is known as Galilean Transformations.



Assuming two frames, both S & S' is moving with a constant velocity v.

Position vector of particle P is at any instant connected by eq =

$$r' = r - vt$$

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

time was taken identical by Galileo
 $t' = t$

Consequences of Galilean Transformation

i) Length of an object is invariant under Galilean transformations.

The length of moving object at any instant for S'-frame of observer (assuming rod to be in same frame)

$$L' = \sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2}$$

using Galilean transformations :-

$$x_2' = x_2 - vt$$

$$x_1' = x_1 - vt$$

$$y_2' = y_2$$

$$y_1' = y_1$$

$$z_2' = z_2$$

$$z_1' = z_1$$

$$\begin{aligned}
 &= \sqrt{(x_2 - vt - x_0 + vt)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\
 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\
 &= L
 \end{aligned}$$

$$L' = L.$$

\Rightarrow Length of an object (which is also separation b/w two points in space) is same for all the observers.

Addition of velocity

On differentiating $x' = x - vt$

$$\boxed{V' = V - v} \rightarrow \text{velocity of } S' \text{ frame.}$$

where V & V' are the velocities of ~~the~~ the moving objects as observed by the observer of S & S' frames. velocities of moving object will be diff for diff. observer.

Acceleration

$$\frac{dV'}{dt} = \frac{dV}{dt} - 0$$

$$a' = a$$

acceleration of moving objects will be identical for S & S' frame.

$\Rightarrow F' = F$ force is also identical.

Galilean Hypothesis of invariance (Principle of Relativity)

① The fundamental laws of physics are identical in all the frames of reference which moves with a constant velocity with respect to one another.

Eg. Assume a windowless spaceship moving with a uniform speed relative to fixed stars, then all the exp performed inside it and all phenomena occurring in the spaceship will appear to be the same to an

observers inside it as if the spaceship were completely at rest.

Michelson - Morley Exp

The necessity of a material medium for transmission of all kind of wave compelled scientists to imagine an invisible medium to be present throughout the universe. It was accepted that the transmission of light waves require some material medium which was named "Ether". This hypothesis was also known as Ether Hypothesis.

Characteristics of this imaginary invisible substance:-

- ① It is highly elastic
- ② It is transparent.
- ③ It exhibits negligible density.
- ④ Material bodies (celestial bodies) can move through it without disturbing it.

Objective of the experiment

- ① To justify the presence of ether throughout the universe
- ② To see whether the speed of light gets modified or not with the velocity of source or observer, or whether Galilean transformations are valid for speed of light or not.
- ③ To measure and to detect the velocity of earth relative to ether.
- ④ To justify the existence of "Absolute frame of reference".

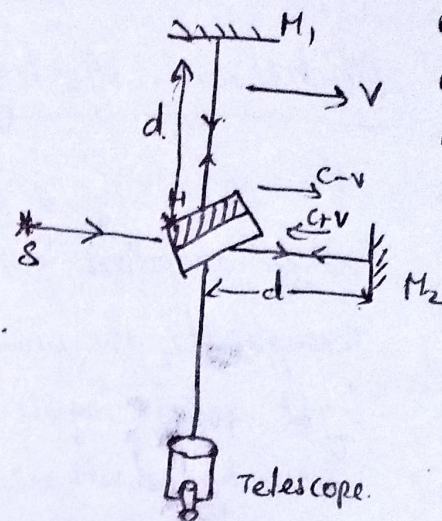
Instrument →

Michelson interferometer (based on interference phenomena of light) which was most sophisticated, highly precision instrument capable of measuring even a fractional shift in a fringe-pattern.

Instrument is fixed on heavy ~~st~~ sand stone and floated in a pond of mercury, so that it could be rotated in any direction.

Experiment :-

- i) Monochromatic light source
- ii) half silvered glass plate H.
- iii) two mirrors M_1, M_2 . \perp to each other.
- iv) Telescope T to observe the interference pattern.



Earth is assumed to be at rest.

time taken by light signal to reach M_1 and back = t_1 ,
 " " " " " " " M_2 " " = t_2

$$\Delta t = t_2 - t_1 = 0$$

\Rightarrow No fringe shift will be observable in telescope T.

But in reality: Earth is not at rest

Assuming Galilean transformations valid for c and also taking into consideration the earth's velocity, instrument will be moving with earth's vel = v .

Light signal travelling along $M_2 = c-v$ relative to ether. = onward journey
 $= c+v$ " " " = backward journey.

Time taken by the light signal to reach M_2 and back =

$$t_2 = \frac{d}{c-v} + \frac{d}{c+v} = \frac{d(c+v) + d(c-v)}{c^2 - v^2}$$

$$= \frac{2dc}{c^2 - v^2} = \frac{2d}{c} \left[1 - \frac{v^2}{c^2} \right]^{-1}$$

$$\boxed{t_2 = \frac{2d}{c} \left[1 + \frac{v^2}{c^2} + \dots \right]}$$

$$t_1 = \frac{2d}{\sqrt{c^2 - v^2}}$$