

UNIT 1

NUMBER SYSTEM

DIVISIBILITY OF A NUMBER

<i>Divisibility Tests</i>	<i>Example</i>
A number is divisible by 2, if the last digit is 0, 2, 4, 6 or 8.	168 is divisible by 2 since the last digit is 8.
A number is divisible by 3, if the sum of the digits is divisible by 3.	168 is divisible by 3 since the sum of the digits is 15 (1+6+8=15), and 15 is divisible by 3.
A number is divisible by 4, if the number formed by the last two digits is divisible by 4.	316 is divisible by 4 since 16 is divisible by 4.
A number is divisible by 5, if the last digit is either 0 or 5.	195 is divisible by 5 since the last digit is 5.
A number is divisible by 6, if it is divisible by 2 AND it is divisible by 3.	168 is divisible by 6 since it is divisible by 2 AND it is divisible by 3.
A number is divisible by 8, if the number formed by the last three digits is divisible by 8.	7,120 is divisible by 8 since 120 is divisible by 8.
A number is divisible by 9, if the sum of the digits is divisible by 9.	549 is divisible by 9 since the sum of the digits is 18 (5+4+9=18), and 18 is divisible by 9.
A number is divisible by 10, if the last digit is 0.	1,470 is divisible by 10 since the last digit is 0.

Divisibility Rule for 7

Subtract 2 times the last digit from remaining truncated number. Repeat the step as necessary. If the result is divisible by 7, the original number is also divisible by 7.

For example : **945**

$94 - (2 \times 5) = 84$. Since 84 is divisible by 7, the original no. 945 is also divisible

Divisibility Rule for 11

For a test of divisibility by 11 start from the right and add every second digit. Now subtract from that total the sum of the remaining digits. The resulting number is divisibly by 11 if and only if the number you started with is divisible by 11.

For example consider **678234**.

$$(4 + 2 + 7) - (3 + 8 + 6) = 13 - 17 = -4$$

which is not divisible by 11 so **678234 is not divisible by 11**.

Now, try **908193**

$$(3 + 1 + 0) - (9 + 8 + 9) = -22 \text{ which is divisible by 11. So, } \mathbf{908193 \text{ is divisible by 11.}}$$

Divisibility Rule for 13

Add 4 times the last digit to the remaining truncated number. Repeat the step as necessary. If the result is divisible by 13, the original number is also divisible by 13.

For example: **3146**

$$314 + (4 \times 6) = 338 \therefore 33 + (4 \times 8) = 65. \text{ Since 65 is divisible by 13, the original no. 3146 is also divisible}$$

Divisibility Rule for 17

Subtract 5 times the last digit from remaining truncated number. Repeat the step as necessary. If the result is divisible by 17, the original number is also divisible by 17

For example : **2278**

$227 - (5 \times 8) = 187$. Since 187 is divisible by 17, the original number 2278 is also divisible.

Divisibility Rule for 19

Add 2 times the last digit to the remaining truncated number. Repeat the step as necessary. If the result is divisible by 19, the original number is also divisible by 19

For example : **11343**

$1134 + (2 \times 3) = 1140$. (Ignore the 0): $11 + (2 \times 4) = 19$. Since 19 is divisible by 19, original no. 11343 is also divisible

LCM and HCF

Important Terms:

- 1) **Factors:** Factor is a number which exactly divides other number.
- 2) **Multiple:** A number is said to be multiple of another number, when it is exactly divisible by other number.
- 3) **Common multiple:** A common multiple of two or more numbers is a number which is exactly divisible by each of them.
- 4) **Highest Common Factor (HCF) or Greatest Common Factor (GCF) :** HCF of two or more numbers is the greatest number which divides each number exactly.
- 5) **Lowest Common Multiple (LCM):** The least number exactly divisible by each one of the given numbers is called least common multiple.

Tips and Tricks:

1) H.C.F. and L.C.M. of Fractions

$$\text{a) H.C.F.} = \frac{\text{H.C.F. of Numerator}}{\text{L.C.M. of Denominator}}$$

$$\text{b) L.C.M.} = \frac{\text{L.C.M. of Numerator}}{\text{H.C.F. of Denominator}}$$

2) Product of two numbers = Product of their H.C.F. and L.C.M.

This condition is only true for two given numbers. If H.C.F. and L.C.M. of three or more numbers are given, then this rule is not applicable.

Method to Find H.C.F. of Given Numbers

Prime Factorization Method

Steps to follow :

- 1) Express the given numbers as product of their prime factors.
- 2) Check for common prime factors and find least index of each common prime factor
- 3) The product of all common prime factors with the respective least indices is H.C.F of given numbers.

Example : H.C.F. of 12, 36, 48

Prime Factors of 12, 36, 48

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$36 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$$

2 & 3 are common factors. 2^2 & 3 have least indices.

H.C.F. of 12, 36, 48 = Product of common prime factors with least indices.

$$\text{H.C.F. of 12, 36, 48} = 2^2 \times 3 = 12$$

$$\text{H.C.F. of 12, 36, 48} = 12$$

Division Method

Steps to follow:

- 1) Draw a table as shown and arrange the given numbers horizontally.
- 2) Divide the numbers with their common factors.
- 3) Divide till the given numbers have no common factors.
- 4) Finally multiply the common factors on left hand side of the table to find the H.C.F.

Example: H.C.F. of 12, 36, 48

2	12	36	48
2	6	18	24
3	3	9	12
	1	3	4

H.C.F or G.C.F = $2 \times 2 \times 3 = 12$

H.C.F of 12, 36, 48 = 12

FACTORS OF A NUMBER

Given an integer N, there is a simple way to find the total number of its factors. The main tool for the feat is the **prime number decomposition theorem**.

These are certain basic formulas pertaining to factors of a number N, such that,

$$N = p^a \times q^b \times r^c$$

Where, p, q and r are the prime factors of the number N. a, b and c are non-negative powers/ exponents.

1. Number of factors of $N = (a+1)(b+1)(c+1)$
2. Number of odd factors of $N =$ product of only odd numbers power increased by 1.
3. Number of even factors of $N =$ Total factors – odd factors
4. Number of prime factors of $N =$ addition of powers $= a+b+c$.
5. Product of factors of $N = N^{\text{No. of factors}/2}$

6. Sum of factors of $N = (p^0 + p^1 + \dots + p^a)(q^0 + q^1 + \dots + q^b)(r^0 + r^1 + \dots + r^c)$

Example- Consider the number 120. Find the following for n:

1. Sum of factors.
2. Number of factors.
3. Product of factors.
4. Odd factors.
5. Even factors.
6. Prime factors.

Solution- The prime factorization of 120 is $2^3 \times 3^1 \times 5^1$. By applying the formulae, 1.

Sum of factors = $[(2^0+2^1+2^2+2^3)(3^0+3^1)(5^0+5^1)] = 1560$

2. **Number of factors** = $(3+1)(1+1)(1+1) = 16$

3. **Product of factors** = $120(16/2) =$

12084. **Odd factors** = $(1+1)*(1+1) = 4$

5. **Even factors** = $16-4 = 12$

6. **Prime Factors** = $3+1+1 = 5$

FACTORIALS

The factorial function (symbol “!”) means to multiply a series of descending natural numbers.

An older notation for the factorial is \underline{n}

$$N! = N(N-1)(N-2) \dots 1.$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

Note- $0! = 1$ and $1! = 1$.

Trailing zeros or ending zeros in $N!$

For example, $5! = 120$. So, it has only one zero in end.

Rule for finding trailing zeros- Divide the given number by the powers of 5 till it is divisible by powers of 5. It means numerator is greater or equal to denominator.

$$N/5 + N/5^2 + N/5^3 + \dots \quad N \geq 5^n$$

Here we take only quotient of it.

Example- Find the trailing zeros in $102!$

$102/5 + 102/25 = 20 + 4 = 24$ (Here $100/125$ is not possible, so divide by 5's powers till it is less or equal to number) So, $102!$ has 24 zeros.

Highest power of a number in a factorial or in a product

Highest power of p (prime number) in $N!$ is $[N/p] + [N/p^2] + [N/p^3] + \dots + [N/p^n]$ till

$N \geq p^n$. Take only quotient of these divisions.

Example 1- Highest power of 2 in $50!$? $50/2 + 50/4 + 50/8 + 50/16 + 50/32 = 25 + 12 + 6 + 3 + 1 = 47$

Example 2- Highest power of 6 in $20!$?

6 is a composite number. To find the highest power of composite number write it into prime factorization, i.e., $6 = 2 \times 3$. Now, find the highest power of 2 and 3 in $20!$.

Highest power of 2 is $= 20/2 + 20/4 + 20/8 + 20/16 = 10 + 5 + 2 + 1 = 18$ Highest

power of 3 is $= 20/3 + 20/9 = 6 + 2 = 8$

Highest power of 6 is the least value which of individual highest powers. Here values are 18 and 8. So, the highest power of 6 is 8.

Highest power of p^a in $N!$ is $[N/p] + [N/p^2] + [N/p^3] + \dots + [N/p^n] / a$

(a – natural Number & p – prime)

Example - Highest power of 72 in

$$50! / 72 = 8 \times 9 = 2^3 \times 3^2$$

Highest power of 2^3 $= [50/2] + [50/4] + [50/8] + [50/16] + [50/32] / 3 = [25 + 12 + 6 + 3 + 1] / 3 = 15$

Highest power of 3^2 $= [50/3] + [50/9] + [50/27] / 2 = [16 + 5 + 1] / 2 = 11$

So, the highest power of 72 is 11.

REMAINDER

Remainder Theorem:- Dividend = Divisor \times Quotient +

Remainder When dividend is of the form $a^n + b^n$ or $a^n - b^n$:

Theorem 1: $a^n + b^n$ is divisible by $a + b$ when n is **ODD**.

Theorem 2: $a^n - b^n$ is divisible by $a + b$ when n is **EVEN**.

Theorem 3: $a^n - b^n$ is ALWAYS divisible by $a - b$.

When $f(x) = a + bx + cx^2 + dx^3 + \dots$ is divided by $x - a$

The remainder when $f(x) = a + bx + cx^2 + dx^3 + \dots$ is divided by $x - a$ is $f(a)$.

So, If $f(a) = 0$, $(x - a)$ is a factor of $f(x)$.

Example:- What is the remainder when the product $1998 \times 1999 \times 2000$ is divided by 7?
 Find the individual remainders of 1998, 1999, and 2000 are divided by 7 are 3, 4, and 5 respectively. Hence, the final remainder is the remainder when the product $3 \times 4 \times 5 = 60$ is divided by 7. So, the final remainder is 4.

Fermat's theorem-

This theorem is stated in the following form: if p is a prime and a is an integer co-prime to p , then $a^{(p-1)} - 1$ will be evenly divisible by p . In other words, $[a^{(p-1)}]/p$ gives remainder 1. **Example:-**
 Find the remainder when 72^{40} divide by 41?
 Answer: So here we see that 41 is a prime number, so we will target Fermat's little theorem instead of Euler's theorem.
 Again 72 and 41 are co-prime. so we can apply our little theorem in this problem easily.
 \rightarrow remainder $[72^{40}/41] = 1$.

Wilson's Theorem-

This theorem state that for a prime number p , $(p-1)!$ Divide by p , then the remainder is $p-1$.
Example:- Find the remainder when $16!$ is divided by 17. $16! = (16! + 1) - 1 = (16! + 1) + 16 - 17$
 Every term except 16 is divisible by 17 in the above expression. Hence the remainder = the remainder obtained when 16 is divided by 17 = Rem (16).

UNIT DIGIT

Unit digit of product- Multiply last digits of each number.
Example:- $121 \times 76 \times 528 \times 172 = 1 \times 6 \times 8 \times 2 = 96 = 6$ is unit digit here.
Unit digit of powers- Either use cyclicity of number or use simple method.

2	3	4	5	6	7	8	9
$2^1=2$	$3^1=3$	$4^1=4$	$5^1=5$	$6^1=6$	$7^1=7$	$8^1=8$	$9^1=9$
$2^2=4$	$3^2=9$	$4^2=6$	$5^2=5$	$6^2=6$	$7^2=9$	$8^2=4$	$9^2=1$
$2^3=8$	$3^3=7$	$4^3=4$	$5^3=5$	$6^3=6$	$7^3=3$	$8^3=2$	$9^3=9$
$2^4=6$	$3^4=1$	$4^4=6$	$5^4=5$	$6^4=6$	$7^4=1$	$8^4=6$	$9^4=1$
$2^5=2$	$3^5=3$	$4^5=4$	$5^5=5$	$6^5=6$	$7^5=7$	$8^5=8$	$9^5=9$
$2^6=4$	$3^6=9$	$4^6=6$	$5^6=5$	$6^6=6$	$7^6=9$	$8^6=4$	$9^6=1$
$2^7=8$	$3^7=7$	$4^7=4$	$5^7=5$	$6^7=6$	$7^7=3$	$8^7=8$	$9^7=9$

Example:- Find the unit digit in 2^{49} ?
 We know in case of 2, it repeats itself after a cycle of 4 . We will divide 49 by 4
 remainder is 1
 We write it as $2^{49} = 2^1 = 2$. That means the unit digit in the 2^{49} is 2.

Rule for numbers ending in digits 0 or 1 or 5 or 6 :-
 Unit digits of that numbers are same as there last digits ending in 0 or 1 or 5 or 6 whatever the power is.
Eg.- $(235)^{27} =$ unit digit 5 $(126)^{344} =$ unit digit 6

Rule for numbers ending in digits 2,3,4,7,8 and 9 :-

Divide the power by 4 find the remainder. Make that remainder to the power of last digit of the number will give us the unit digit.

Note- if remainder is 0 (power completely divisible by 4) take remainder as 4 not 0.

Example.1-

$$(327)^{22} \quad 22/4 \\ = \text{Rem}(2)$$

Last digit is 7. Make remainder 2 to power of 7 = $7^2 = 49$ So, 9 is a unit digit.

Example.2- $(28)^{36}$

$36/4 = \text{Rem}(0)$. Here take remainder as 4. Last digit is 8. Then, $8^4 = 64 \times 64 = 4 \times 4 = 16$. So, unit digit is 6.

ARITHMETIC & GEOMETRIC PROGRESSION

An Arithmetic Progression (A.P.) is a sequence in which the difference between any two consecutive terms is constant.

Let a = first term, d = common difference

Then, **n th term $a_n = a + (n-1)d$**

The sum of n terms of an A.P. whose first term is a and common difference is d , is given by

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

The sum of n terms of an A.P. whose first term is a and last term is l is given by the formula:

$$S_n = \frac{n}{2} [a + l]$$

AM (Arithmetic mean): If a, b, c are in AP then the arithmetic mean is given by **$b = \frac{a+c}{2}$** Inserting AM:

To insert k means between a and b the formula for common difference is given by **$d = \frac{(b-a)}{k+1}$**

For Example: Insert 4 AM's between 4 and 34 $d =$

$$(34-4)/(4+1) = 30/5 = 6$$

\therefore The 4 AM are $4+6=10, 10+6=16, 16+6=22, 22+6=28$

Geometric Progression: Geometric sequences are powers r^k of a fixed number r , such as 2^k and 3^k . The general form of a geometric sequence is

The n -th term of a geometric sequence with initial value a and common ratio r is given by

$$a_n = ar^{n-1}$$

Such a geometric sequence also follows the recursive relation

$$a_n = r a_{n-1} \text{ for every integer } n \geq 1.$$

Sum of G.P. = $a(1-r^n)/(1-r)$

GM (Geometric mean): If a, b, c are in GP Then the GM is given by **$b = \sqrt{ac}$**

Note: 1. $AM > GM > HM$ 2. $GM^2 = AM \times HM$

Inserting GM: To insert k means between a and b the formula for common ratio is given by

$$r = (b/a)^{1/(k+1)}$$

For example: Insert 4 GM's between 2 and
 $486r = (486/2)^{(1/(4+1))} = (243)^{(1/5)} = 3$
 \therefore The 4 GM are $2 \times 3 = 6$, $6 \times 3 = 18$, $18 \times 3 = 54$, $54 \times 3 = 162$.

General Questions on Number System

- Q1.** For the product $n*(n + 1)*(2n + 1)$, where n is a natural number. Which one of the following is not necessarily true?
 (a) It is even. (b) Divisible by 3 (c) Divisible by 6 (d) Never divisible by 12
- Q2.** If two digit integers M and N are positive and have same digits, but in reverse order, which of the following cannot be the sum of M and N ?
 (a) 181 (b) 165 (c) 121 (d) 99
- Q3.** What is the value of $(x-a)(x-b)(x-c) \dots (x-z)$?
 (a) 1 (b) 3 (c) 2 (d) 0
- Q4.** If you write first 252 natural numbers in a straight line, how many times do you write the digit 4? (a) 55 (b) 53 (c) 50 (d) 48
- Q5.** There are three consecutive natural numbers such that the square of the second minus twelve times the first is three less than twice the third. What is the largest of the three numbers?
 (a) 14 (b) 13 (c) 15 (d) 18
- Q6.** Which one of the following is the minimum value of the sum of two integers whose product is 36? (a) 37 (b) 20 (c) 15 (d) 12
- Q7.** Four digits of the number 29138576 are omitted so that the result is as large as possible. The largest omitted digit is?
 (a) 5 (b) 6 (c) 7 (d) 8
- Q8.** A boy writes all the numbers from 100 to 999. The number of zeroes that he uses is 'a', the number of 5's that he uses is 'b' and the number of 8's he uses is 'c'. What is the value of $b + c - a$?
 (a) 280 (b) 380 (c) 180 (d) 80
- Q9.** The product of 4 consecutive even numbers is always divisible by?
 (a) 600 (b) 768 (c) 864 (d) 364
- Q10.** A set has exactly five consecutive positive integers starting with 1. What is the percentage decrease in the average of the numbers when the greatest one of the numbers is removed from the set?
 (a) 8.54 (b) 12.56 (c) 15.25 (d) 16.66

Questions on Rules of Divisibility

- Q21.** What least value should be assigned to * so that the number 451*603 is exactly divisible by 9? (a) 2 (b) 5 (c) 8 (d) 7
- Q22.** What least value should be assigned to * so that the number 63576*2 is divisible by 8? (a) 2 (b) 1 (c) 4 (d) 3
- Q23.** If 256X561 is divisible by 11, then what can be the value of 'X'?
 (a) 3 (b) 0 (c) 6 (d) 8
- Q24.** If ABC0 is a 4 digit number divisible by 4, then how many such 4 digit number exist? (a) 360 (b) 400 (c) 450 (d) 500
- Q25.** If a number 968A96B is to be divisible by 72, the respective values of A and B can be?
 (a) 7 and 8 (b) 7 and 0 (c) 5 and 8 (d) 0 and 8
- Q26.** The number $(6n^2 + 6n)$ for any natural number n is always divisible by which maximum number? (a) 6 (b) 24 (c) 12 (d) 18

Q27. It is given that $(2^{32} + 1)$ is exactly divisible by a certain number. Which of the following is also definitely divisible by the same number?

- (a) $(2^{16} + 1)$ (b) $(2^8 + 1)$ (c) $(2^{16} - 1)$ (d) $(2^{96} + 1)$

Lowest Common Multiple (LCM) & Highest Common Factor (HCF)

Q45. The LCM of 5, 8, 12, 20 will not be a multiple of?

- (a) 3 (b) 9 (c) 8 (d) 5

Q46. Find L.C.M. of 1.05 and 2.1?

- (a) 1.3 (b) 1.25 (c) 2.1 (d) 4.30

Q47. How many numbers between 200 and 600 are divisible by 4, 5 and 6?

- (a) 5 (b) 6 (c) 7 (d) 8

Q48. For how many values of k the L.C.M of 6^6 , 8^8 and k is 12^{12} (k is a natural number)?

- (a) 1 (b) 24 (c) 25 (d) Infinite

Q49. Three bells toll at intervals of 9, 12 and 15 minutes respectively. All three begins to toll at 8 a.m. At what time will they first toll together again?

- (a) 11 a.m. (b) 8:30 a.m. (c) 10 a.m. (d) 10:30 a.m.

Q50. A person has to completely put each of the three liquids i.e. 403 liters of petrol, 465 litres of diesel and 496 liters of Mobil oil in bottles of equal size without mixing any of the three types of liquids such that each bottle is completely filled. What is the least possible number of bottles required?

- (a) 44 (b) 34 (c) 31 (d) None of these

Q51. Five bells begin to toll together at intervals of 9 s, 6 s, 4 s, 10 s and 8 s, respectively. How many times will they toll together in the span of one hour (excluding the toll at the start)?

- (a) 5 (b) 8 (c) 10 (d) None of these

Q52. The least perfect square number which is divisible by 3, 4, 5, 6 and 8, is?

- (a) 900 (b) 1200 (c) 2500 (d) 3600

Q53. Monica, Veronica and Rachat begin to jog around a circular stadium. They complete their revolutions in 42s, 56s and 63s, respectively. After how many seconds will they be together at the starting point?

- (a) 366 (b) 252 (c) 504 (d) Cannot be determined

Q60. In a meet, persons from five different places have assembled in Bangalore High School. From the five places the persons come to represent are 42, 60, 210, 90 and 84. What is the minimum number of rooms that would be required to accommodate so that each room has the same number of occupants and occupants are all from the same places?

- (a) 44 (b) 62 (c) 81 (d) 96

Q61. The product of two numbers is 12960 and their HCF is 36. How many pairs of such numbers can be formed?

- (a) 3 (b) 4 (c) 5 (d) 2

Q62. Calculate H.C.F. of $\frac{2}{3}$, $\frac{16}{81}$ and $\frac{8}{9}$?

- (a) $\frac{2}{9}$ (b) $\frac{8}{3}$ (c) $\frac{2}{81}$ (d) $\frac{3}{16}$

Q63. H.C.F. of two numbers is 13. If these two numbers are in the ratio of 15: 11, then find the numbers? (a)

- 230, 140 (b) 215, 130 (c) 195, 143 (d) 155, 115

Q64. The L.C.M. of two numbers is 2310 and their H.C.F. is 30. If one of these numbers is 210, the second number is?

- (a) 330 (b) 1470 (c) 2100 (d) 16170

Factors & Factorials

Q86. Find the following for the number 84?

I. Number of odd factors. II. Number of even factors.

- (a) 4, 8 (b) 5, 5 (c) 8, 12 (d) 7, 9

Q87. How many factors of 1200 are odd integers?

- (a) 6 (b) 8 (c) 12 (d) 22

- Q88.** Find the total no of prime factors in $4^{11} \times 7^5 \times 11$?
 (a) 17 (b) 27 (c) 28 (d) 30
- Q89.** Find the sum of factors of 18?
 (a) 6 (b) 13 (c) 39 (d) 35
- Q90.** Find the number of factors of 6!?
 (a) 25 (b) 30 (c) 35 (d) 32
- Q91.** Find the number of trailing zeroes in the expansion of 23!?
 (a) 5 (b) 4 (c) 20 (d) 21
- Q92.** Find the number of trailing zeroes in the expansion of 1000!?
 (a) 250 (b) 300 (c) 249 (d) 245
- Q93.** Find the number of zeros in $2 \times 3 \times 4 \times 5 \times \dots \times 125$?
 (a) 30 (b) 35 (c) 38 (d) 31
- Q94.** Find the highest power of 24 in 150!?
 (a) 48 (b) 72 (c) 58 (d) 45
- Q95.** Find the highest power of 30 in 40!?
 (a) 12 (b) 10 (c) 8 (d) 9
- Q96.** pqr is a three digit natural number such that $pqr = p! + q! + r!$. What is the value of $(q+r) \times p$?
 (a) 1296 (b) 3125 (c) 19683 (d) 9

Remainders

- Q97.** A number when divided by 54 leaves a remainder of 31. Find the remainder when the same number is divided by 27?
 (a) 4 (b) 23 (c) 15 (d) (a) or (b)
- Q98.** Find the remainder when 2^{93} is divided by 7?
 (a) 1 (b) 2 (c) 4 (d) 6
- Q99.** Find the remainder when 24^5 is divided by 5?
 (a) 0 (b) 1 (c) 4 (d) None of these
- Q100.** The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is?
 (a) 4 (b) 15 (c) 0 (d) 18
- Q101.** What is the remainder when 4^{96} is divided by 6?
 (a) 0 (b) 2 (c) 3 (d) 4
- Q102.** $(7^{4n} - 6^{4n})$, where n is an integer > 0 , is divisible by?
 (a) 13 (b) 5 (c) 17 (d) All of these
- Q103.** Find the remainder when n is divided by 12 where $N = 1821 \times 1823 \times 1827$?
 (a) 9 (b) 12 (c) 15 (d) 18
- Q104.** A number when divided by 5, leaves 3 as remainder. What will be the remainder when the square of this number is divided by 5?
 (a) 0 (b) 1 (c) 2 (d) 4
- Q105.** In a division sum, the remainder is 6 and the divisor is 5 times the quotient and is obtained by adding 2 to the thrice of the remainder. The dividend is?
 (a) 40 (b) 42 (c) 80 (d) 86

UNIT DIGIT

- Q122.** If the unit's digit in the product of $(47ax729 \times 345 \times 343)$ is 5, then how many values that a can take?
 (a) 9 (b) 3 (c) 7 (d) 5
- Q123.** The rightmost non - zero digit of the number 30^{2720} is?

- (a) 1 (b) 3 (c) 7 (d) 9
Q124. What is the unit digit in 2^9 ?
 (a) 1 (b) 3 (c) 2 (d) 4
Q125. What is the unit's digit of the number $(6^{256} - 4^{256})$?
 (a) 0 (b) 1 (c) 4 (d) 7
Q126. Find the unit digit in the product $(243 \times 397 \times 2497 \times 3913)$?
 (a) 4 (b) 3 (c) 7 (d) 1
Q127. What are the respective digits in the unit's place in the expansions of 7^7 and 17^7 ?
 (a) 2, 6 (b) 3, 3 (c) 1, 4 (d) 9, 9
Q128. Find the unit's digit in $(264^{102} + 264^{103})$?
 (a) 0 (b) 2 (c) 4 (d) 6
Q129. Which digits should come in place of @ and # if the number $62684@ \#$ is divisible by both 8 and 5?
 (a) 4, 0 (b) 0, 4 (c) 4, 4 (d) 1, 1
130. What will be the last digit of the multiplication $3^{153} \times 7^{162}$?
 (a) 5 (b) 9 (c) 7 (d) 6
Q131. The digit in the unit place of the number 7295×3158 is?
 (a) 7 (b) 2 (c) 6 (d) 4
Q132. Find the unit digit of $(23)^{251}$?
 (a) 0 (b) 2 (c) 3 (d) 1
Q133. The unit digit of $(137^{13})^{47}$ is?
 (a) 1 (b) 3 (c) 5 (d) 7
Q134. The unit digit of $35^{87} + 93^{46}$ is?
 (a) 2 (b) 4 (c) 6 (d) 8
Q135. The unit digit of $44^{91} \times 73^{37}$ is?
 (a) 2 (b) 4 (c) 6 (d) 8
Q136. The unit digit of $12^{34} - 5^9$ is?
 (a) -1 (b) 1 (c) 9 (d) None of these
Q137. Find the unit digit of given product $(2^{34} \times 14^{832} \times 17^{21})$?
 (a) 6 (b) 8 (c) 2 (d) 7

Arithmetic Progression & Geometric Progression

- Q138.** Find the number of terms in the series 8, 12, 16, ... 72?
 (a) 10 (b) 12 (c) 17 (d) 16
Q139. The sum of third and ninth term of an A.P is 8. Find the sum of the first 11 terms of the progression?
 (a) 44 (b) 22 (c) 19 (d) None of the above
Q140. Find $4 + 7 + 10 + 13 + 16 + \dots$ up to 20 terms?
 (a) 600 (b) 650 (c) 540 (d) 454
Q141. Find 5th term in the series 5, 15, 45, ?
 (a) 405 (b) 345 (c) 450 (d) 340
Q142. Given $A = 2^{65}$ and $B = (2^{64} + 2^{63} + 2^{62} + \dots + 2^0)$. Which one is correct option?
 (a) $B = 2^{64} + A$ (b) $A = B$ (c) $B = A + 1$ (d) $A = B + 1$
Q143. If $\log 2$, $\log (2^x - 1)$ and $\log (2^x + 3)$ are in A.P, then x is equal to...?
 (a) 5252 (b) $\log_2 5$ (c) $\log_3 2$ (d) 32
Q144. Which term of the A.P. 3, 8, 13 is 78?
 (a) 16th (b) 17th (c) 20th (d) 25th
Q145. Is (-150) a term of the series 11, 8, 5, 2, ...?
 (a) Yes (b) No (c) Can't be determined (d) Data Insufficient

Q146. Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 3.

- (a) 162 (b) 175 (c) 178 (d) 180

Q147. Which term of the A.P. 3, 15, 27, 39 ... will be 132 more than its 54th term?

- (a) 82^{nd} (b) 75^{th} (c) 60^{th} (d) 65^{th}

Q148. Write down the 8th term in the Geometric Progression 1, 3, 9, ...

- (a) 2187 (b) 2185 (c) 2287 (d) 2021

Q149. Find the number of terms in the geometric progression 6, 12, 24, ...1536

- (a) 10 (b) 9 (c) 15 (d) 13

Q150. The sum of n terms of an A.P. is $3n^2 + n$, find the nth term.

- (a) $6n - 4$ (b) $4n - 4$ (c) $6n - 2$ (d) $4n - 2$

Q151. Find the sum of the following series: $3 + 7 + 11 + 15 + \dots$ to 30 terms.

- (a) 1830 (b) 1840 (c) 1800 (d) 1940

Q152. Find the position of 62 in the following series 2, 5, 8,?

- (a) 26 (b) 21 (c) 23 (d) 20

Q153. If you save 1 paise today, 2 paise next day and 3 paise the succeeding day and so on, what will be your savings in 365 days?

- (a) 666.75 (b) 665.35 (c) 668.85 (d) 667.95

AVERAGE

AVERAGE

The result obtained by adding several quantities together and then dividing this total by the number of quantities is called Average.

Average = Sum of quantities / Number of Quantities

An average is the mean value of a set of numbers or values. It is given by:-

Average = $(x_1 + x_2 + x_3 + \dots + x_n) / n$

Example: If the ages of 4 students are 20 years, 22 years, 18 years and 24 years, then what is the average age of the students?

Solution: Average Age = $(20 + 22 + 18 + 24) / 4$

Important Points to Remember

1. If all the numbers are increased by 'a' then their average is also increased by 'a'.
2. If all the numbers are decreased by 'a' then their average is also decreased by 'a'.
3. If all the numbers are multiplied by 'a' then their average is also multiplied by 'a'.
4. If all the numbers are divided by 'a' then their average is also divided by 'a'.

Age and Average

1. If the average age of n persons decreases by x years. Then, the total age of n persons decreases by $(n \times x)$ yr
2. If the average age of n persons increases by x years. Then, the total age of n persons increases by $(n \times x)$ yr

Example: The average age of 6 persons is increased by 2 years when one of them, whose age is 26 years is replaced by a new man. What is the age of the new person?

Solution: Total age increased = $6 \times 2 = 12$

year Age of new persons = $(26 + 12) = 38$

year

The increase in the total age of 6 persons is due to the replacement of a person aged 26 year with a person who is 12 years older to him.

Average of Some Important Series of Numbers

The average of odd numbers from 1 to n,
= **$(\text{Last odd number} + 1) / 2$**

(n = Last odd number)

The average of even numbers from 2 to n,
= **$(\text{Last even number} + 2) / 2$**

(n = Last even number)

Important Points

1. Average of first 'n' natural numbers = $(n+1)/2$
2. The average of first 'n' consecutive even numbers = $(n+1)$
3. The average of first 'n' consecutive odd numbers = n
4. The average of consecutive numbers = $(\text{First Number} + \text{Last Number})/2$
5. The average of 1 to 'n' odd numbers = $(\text{Last Odd Number} + 1)/2$
6. The average of 1 to 'n' even numbers = $(\text{Last Even Number} + 2)/2$
7. The average of square of natural numbers till n = $[(n+1)(2n+1)]/6$
8. The average of cubes of natural numbers till n = $[n(n+1)^2]/4$
9. Correct Sum = Wrong Sum - Wrong Value + Right Value
10. The average of squares of 1st n consecutive even no's = $[2(n+1)(2n+1)]/3$
11. The average of squares of consecutive even no's from 1 to n = $[(n+1)(n+2)]/3$
12. The average of squares of consecutive odd no's from 1 to n = $[n(n+2)]/3$
13. If the average of n₁ observation is a₁ and n₂ observation is a₂. Then, the average of all the observations is:-

$$A = \frac{n_1 a_1 + n_2 a_2 + n_3 a_3 + \dots}{n_1 + n_2 + n_3 + \dots}$$
14. If the average of 'm' observations is 'a' and average of 'n' observations taken out of 'm' is 'b'. Then, Average of rest of the observations = $(ma - nb)/(m - n)$

Average Speed

1. Average Speed = Total Distance / Total Time

Let the distance between two points A and B is d and speed in travelling from point A to B is x km/hr and from point B to A is y km/hr.

Then, average speed = $(2xy) / (x+y)$

Example: If a person travels two equal distances at 10 km/hr. and 30 km/hr. What is the average speed for the entire journey?

Solution: Average Speed = $2xy / (x+y)$

$$= (2 \times 30 \times 10) / (30 + 10)$$

$$= 600 / 40 = 15 \text{ km/hr.}$$

2. If a person covers three equal distances at a speed of A km/hr, B km/hr and C km/hr. Then, the average speed for the whole journey will be = **$\frac{3ABC}{AB+BC+CA}$**

3. If a person covers 'P' part of his total distance with a speed of 'x', 'Q' part of his total distance with a speed of 'y', 'R' part of his total distance with a speed of 'z'. Then,

$$\text{Average Speed} = \frac{xyz}{Pyz + Qxz + Rxy}$$

Type 3 - With/Without Replacement

Q12. When a student weighing 45 kg left a class, the average weight of the remaining 59 students increased by 200 grams. What is the average weight of the remaining 59 students?

- A. 50 B. 57 C. 65 D. 80

Q13. There were 35 students in a hostel. Due to the admission of 7 new students the expenses of the mess were increased by Rs.42 per day while the average expenditure per head diminished by Re.1. What was the original expenditure of the mess?

- A. 240 B. 440 C. 420 D. 540

Q14. The average age of 40 students of a class is 18 years. When 20 new students are admitted to the same class the average age of the class is increased by 6 months. The average age of the newly admitted students is?

- A. 19 Years 6 months B. 19 years C. 18 Years D. 20 years 2 months

Type 4 - Mistaken Average

Q15. The average of 8 observations was 25.5. It was noticed later that two of those observations were wrongly taken. One observation was 14 more than the original value and the other observation was wrongly taken as 31 instead of 13. What will be the correct average of those 8 observations?

- A. 22.5 B. 21.5 C. 25 D. 24.5

Q16. The Arithmetic mean of 100 numbers was computed as 89.05. It was later found that two numbers 92 and 83 have been misread as 192 and 33 respectively. What is the correct Arithmetic Mean of the numbers?

- A. 88.66 B. 88.55 C. 77.02 D. 90.54

Q17. In an examination, the average marks of all the students calculated to be 58 marks. It was later found that marks of 60 students were wrongly written as 70 instead of 50. If the corrected average is 55, find the total number of students who took the exam?

- A. 500 B. 450 C. 400 D. 420

Type 5 – Problems on Cricket

Q18. A cricketer has completed 10 innings and his average is 21.5 runs. How many runs must he make in his next innings so as to raise his average to 24?

- A. 50 B. 24 C. 49 D. 52

Q19. A cricketer had a certain average of runs for his 64th innings. In his 65th innings, he is bowled out for no score on his part. This brings down his average by 2 runs. His new average of run is?

- A. 135 Runs B. 128 Runs C. 150 Runs D. 132 Runs

Q20. The batting average of a cricket player for 64 innings is 62 runs. His highest score exceeds his lowest score by 180 runs. Excluding these two innings, the average of the remaining innings becomes 60 runs. His highest score is?

- A. 212 Runs B. 220 Runs C. 214 Runs D. 241 Runs

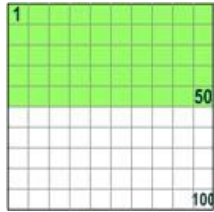
UNIT2

PERCENTAGE

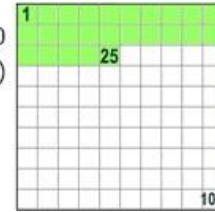
PERCENT

When we say "Percent" we mean "per 100"

One percent (1%) means 1 per 100.



50% means 50 per 100
(50% of this box is green)



25% means 25 per 100
(25% of this box is green)

Remember: $x\%$ of $y = y\%$ of $x = xy/100$

Example: Find 8% of 50.

8% of 50 is the same as 50% of 8

And 50% of 8 is 4

So, 8% of 50 is 4



Decimals, Fractions & Percentages are just different ways of showing the same value:

A Half can be written as:



Common Fractions with Decimal and Percent Equivalents

Here is a table of commonly used values shown in Percent, Decimal and Fraction form:


Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	0.333...	33.333...%
$\frac{2}{3}$	0.666...	66.666...%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%
$\frac{4}{5}$	0.8	80%
$\frac{1}{6}$	0.1666...	16.666...%
$\frac{5}{6}$	0.8333...	83.333...%
$\frac{1}{8}$	0.125	12.50%
$\frac{3}{8}$	0.375	37.50%
$\frac{5}{8}$	0.625	62.50%
$\frac{7}{8}$	0.875	87.50%
$\frac{1}{9}$	0.111...	11.111...%
$\frac{2}{9}$	0.222...	22.222...%
$\frac{4}{9}$	0.444...	44.444...%
$\frac{5}{9}$	0.555...	55.555...%
$\frac{7}{9}$	0.777...	77.777...%
$\frac{8}{9}$	0.888...	88.888...%
$\frac{1}{10}$	0.1	10%
$\frac{1}{12}$	0.08333...	8.333...%
$\frac{1}{16}$	0.0625	6.25%
$\frac{1}{32}$	0.03125	3.13%

LET'S PRACTICE THE CONVERSIONS Now -

A. FROM PERCENT TO DECIMAL:

To convert from percent to decimal : divide by 100, and remove the "%" sign.

The easiest way to divide by 100 is to **move the decimal point 2 places to the left:**

From Percent		To Decimal	
75%		0.75	move the decimal point 2 places to the left , and remove the "%" sign.

B. FROM DECIMAL TO PERCENT:

To convert from decimal to percent : multiply by 100, and add a "%" sign.

The easiest way to multiply by 100 is to **move the decimal point 2 places to the right:**

From Decimal		To Percent	
0.125		12.5%	move the decimal point 2 places to the right , and add the "%" sign.

Or you can simply multiply 0.125 with 100 and add the % sign to get 12.5%.

C. FROM FRACTION TO DECIMAL:

The easiest way to convert a fraction to a decimal is to divide the top number by the bottom number (divide the numerator by the denominator in mathematical language)

Example: Convert $\frac{2}{5}$ to a decimal.

Divide 2 by 5: $2 \div 5 = 0.4$

Answer: $\frac{2}{5} = 0.4$

D. FROM DECIMAL TO FRACTION:

To convert a decimal to a fraction , remove the decimal by adding the denominator with appropriate number of zeroes and then simplify the fraction.

Example: To convert 0.75 to a fraction

Remove the decimal $\Rightarrow 0.75 = 75/100$

Simplify the fraction $\Rightarrow 75/100 = 3/4$

Answer: $\frac{3}{4} = 0.75$

E. FROM FRACTION TO PERCENTAGE:

The easiest way to [convert a fraction to a percentage](#) is to multiply the fraction by 100 and reduce it to decimal form and add the "%" sign.

Example: Convert $\frac{3}{8}$ to a percentage

Multiply $\frac{3}{8}$ by 100: 37.5

Add the "%" sign: 37.5%

Answer: $\frac{3}{8} = 37.5\%$

F. FROM PERCENTAGE TO FRACTION:

To [convert a percentage to a fraction](#), first convert to a decimal (divide by 100), then use the steps for converting decimal to fractions (like above).

ATTENTION PLEASE!!!

REMEMBER THAT THE BASE TAKEN IS ALWAYS THE ORIGINAL QUANTITY!!!

Q13. The price of a shirt is increased by 15% and then reduced by 15%. The final price of the shirt is?

- A. 1.25% increases B. 1.25% decreases C. 2.25% increases D. 2.25% decreases

Q14. A's salary increased by 12% over last year and has become Rs. 6720. What will be his next year salary if it increases by 20% over last year's salary?

- A. Rs. 8000 B. Rs. 8064 C. Rs. 7500 D. Rs. 7200

Type 3 – Expenditure and Consumption

Q15. Price of sugar rises by 20%. By how much percent should the consumption of sugar be reduced so that the expenditure does not change?

- A. 20 B. 10 C. $16\frac{2}{3}$ D. 15

Q16. The price of an article is cut by 30%. To restore it to the former value the new price must be increased by?

- A. 30% B. $300/13\%$ C. $300\frac{1}{13}\%$ D. $300/7\%$

Q17. A reduction of 20% in the price of sugar enables a housewife to purchase 6 kg more for Rs. 240. What is original price per kg of sugar?

- A. Rs.10/kg B. Rs.8/kg C. Rs.6/kg D. Rs.5/kg

Q18. A 10% hike in the price of rice forces a person to purchase 2 kg less for rupees 110. Find the price per kg of rice?

- A. Rs.5/kg B. Rs.5.5/kg C. Rs.6/kg D. None of these

Type 4 – Venn Diagram and Miscellaneous

Q19. 30% of the men are more than 25 years old and 80% of the men are less than or equal to 50 years old. 20% of all men play football. If 20% of the men above the age of 50 play football, what percentage of the football players are less than or equal to 50 years?

- A. 15% B. 20% C. 80% D. 70%

Q20. A bag contains 600 coins of 25p denomination and 1200 coins of 50p denomination, If 12% of 25p coins and 24% of 50p coins are removed, the percentage of money removed from the bag is nearly?

- A. 21.6 B. 22.5 C. 20.6 D. 12.6

Q21. In an election contested by two parties, Party D secured 12% of the total votes more than Party R. If party R got 132,000 votes and there are no invalid votes, by how many votes did it lose the election?

- A. 300000 B. 168000 C. 36000 D. 24000

Q22. In a game show, the percentage of participants qualified to the number of participants participated from team A is 60%. In team B, the number of participants participated is 40% more than the participants participated from team A and the number of participants qualified from team B is 40% more than the participants qualified from team A. What is the percentage of participants qualified to the number of participants participated from team B?

- A. 20% B. 40% C. 60% D. 80%

Q23. A student has to secure 40% marks to pass. He gets 178 marks and fails by 22 marks. What are the maximum marks?

- A. 500 B. 450 C. 560 D. 600

Q24. Forty percent of the employees of a company are men, and 75 percent of the men earn more than Rs.25,000 per year. If 45 percent of the company's employees earn more than Rs.25,000 per year, what fraction of the women employed by the company earn Rs.25,000 per year or less?

- A. $\frac{2}{11}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{3}{4}$

Q25. In a library, 20% of the books are in Hindi. 50% of the remaining in English and 30% of the remaining are in French. The remaining 6,300 books are in regional languages. What is the total number of books in library?

- A. 19,500 B. 20,500 C. 21,500 D. 22,500

PROFIT and LOSS

Basic Terminology

Cost Price: C.P. is the price at which one buys anything.

Selling Price: S.P. is the price at which one sells anything.

Profit/Loss: This is the difference between the selling price and the cost price. If the difference is positive it is called the profit and if negative it is called as loss.

Profit/Loss %: This is the profit/loss as a percentage of the C.P.

Margin: Normally is in % terms only. This is the profit as a percentage of S.P.

Marked Price: This is the price of the product as displayed on the label.

Discount: This is the reduction given on the marked price before selling it to a customer. If the trader wants to make a loss he can offer a discount on the cost price as well

Mark-up: This is the increment on the cost price before being sold to a customer.

It is also known as list price or Tag price which is written on the item. The markup price written is always greater than the actual C.P of the item and the percentage rise in the mark-up price is on the C.P of the item.

Percentage increase in the Mark-up price = $(MP - CP) / CP \times 100$

IMPORTANT FORMULAE

1. $\text{Gain} = (\text{S.P.}) - (\text{C.P.})$

2. $\text{Loss} = (\text{C.P.}) - (\text{S.P.})$

1. Loss or gain is always reckoned on C.P.

2. Loss Percentage: (Loss %)

$$\text{Loss \%} = \frac{\text{Loss} \times 100}{\text{C.P.}}$$

3. Selling Price: (S.P.)

$$\text{SP} = \left[\frac{(100 + \text{Gain \%})}{100} \right] \times \text{C.P.}$$

4. Selling Price: (S.P.)

$$\text{SP} = \frac{(100 - \text{Loss \%})}{100} \times \text{C.P.}$$

5. Cost Price: (C.P.)

$$\text{C.P.} = \frac{100}{(100 + \text{Gain \%})} \times \text{S.P.}$$

6. Cost Price: (C.P.)

$$\text{C.P.} = \frac{\text{S.P.} \times 100}{(100 - \text{Loss \%})}$$

7. If an article is sold at a gain of say 35%, then S.P. = 135% of C.P.

8. If an article is sold at a loss of say, 35% then S.P. = 65% of C.P.

9. When there are two successive profits of a% and b%, then the resultant profit percent = ____

$$\left(a + b + \frac{a \times b}{100}\right) \%$$

10. When there is a profit of a% and a loss of b% in a transaction, then the resultant profit percent

$$= \left(a - b - \frac{a \times b}{100}\right) \%$$

11. Successive Discounts

In case of successive discounts of a% and b%, the effective discount = $\left(a + b - \frac{a \times b}{100}\right) \%$

12. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain \%} = \left[\frac{\text{Error}}{\text{True Value} - \text{Error}} \times 100 \right] \%$$

Type 1 – Profit & Loss Percentage

Q1. If the cost price is 96% of selling price then what is the profit %?

- A. 3.13 B. 2.45 C. 2.34 D. 4.17

Q2. Monika purchased a pressure cooker at 9/10th of its selling price and sold it at 8% more than its S.P. Find her gain percent?

- A. 20% B. 10% C. 15% D. 30%

Q3. A vendor bought bananas at 6 for Rs.10 and sold them at 4 for Rs.6. What is the gain/ loss percent?

- A. 12% profit B. 20% loss C. 10% loss D. 15% profit

Q4. A vendor bought toffees at 6 for a rupee. How many for a rupee must he sell to gain 20%?

- A. 10 B. 5 C. 15 D. 22

Q5. A shopkeeper buys scientific calculators in bulk for Rs. 15 each. He sells them for Rs. 40 each. Calculate the profit on each calculator as percentage of the cost price.

- A. 166.67% B. 150% C. 66.67% D. 123%

Q6. If the cost price of a book is Rs. 150 and selling price is 137.50, then calculate the percentage loss on the book?

- A. 12.33% B. 8.33% C. 10% D. 15%

Q7. What is the loss percent if a man loses Rs.10 on selling an article for Rs.100?

- A. 120/13 B. 111/12 C. 100/11 D. 120/11

Q8. If selling price is doubled, the profit triples. Find the profit percent?

- A. 300% B. 200% C. 150% D. 100%

Type 2 – Cost Price in Terms of Selling Price

Q9. The cost price of 21 articles is equal to selling price of 18 articles. Find gain or loss %?

- A. 50/3% gain B. 60/3% gain C. 70/3% loss D. 80/3% loss

Q10. A man sells 320 mangoes at the cost price of 400 mangoes. His gain percent is?

- A. 25% B. 30% C. 35% D. 15%

Q11. If the cost of 30 articles is equal to the selling of 20 articles, find the profit percent?

- A. 40 B. 50 C. 45 D. 55

Type 3 – Error in Weight and Dishonest Dealer

Q12. A dishonest dealer professes to sell his goods at cost price but uses a weight of 900 grams for a kg weight. Find his gain percent.

- A. 11.11 B. 33.33 C. 12 D. Cannot be determined

Q13. A shopkeeper claims that he is selling sugar at Rs 23/kg which cost him Rs 25/kg but he is giving

800gm instead of 1000gm. What is his percentage profit or loss?

- A. 15% profit B. 15% loss C. no profit no loss D. Cannot be determined

Q14. Lalit marks up his goods by 40% and gives a discount of 10%. Apart from this, he uses a faulty balance also, which reads 1000 gm for 800 gm. What is his net profit percentage?

- A. 57.5% loss B. 57.5% profit C. 60% profit D. Cannot be determined

Q15. A shopkeeper sells rice to a customer, using false weights and gains $100/8\%$ on his cost. What weight has he substituted for a kilogram?

- A. 750 gms B. 800 gms C. 880 gms D. 888.89 gms

Type 4 – When SP is Same for Two Items

Q16. A man sells 2 flats for Rs 675958 each. On one he gains 16% while on the other his losses 16%. How much does his gain/loss in the whole transaction?

- A. 3.56% loss B. 3.56% gain C. 2.56% gain D. 2.56% loss

Q17. If a shopkeeper sells two items at the same price. If he sells one of them at a profit of 10% and the other at a loss of 10%, find his profit/loss percentage?

- A. 1% profit B. 1% loss C. No profit no loss D. None of these

Type 5 – Single and Successive Discounts

Q18. A shopkeeper marks the price of the article at Rs.80. Find the cost if after allowing a discount of 10%, he still gains 20% on the cost price?

- A. 60 B. 40 C. 29 D. 39

Q19. An article was sold for Rs. Y after giving a discount of x%. Then, its list price is _____?

- A. $100y/(100-x)$ B. $(100-x)/y$ C. $(100-x)/90y$ D. $x/(100-y)$

Q20. Find the single discount equivalent to successive discounts of 40% and 20%.

- A. 52% B. 45% C. 46% D. 48%

Q21. An article is listed at Rs. 65. A customer bought this article for Rs. 56.16 and got two successive discounts of which the first one is 10%. What was the other rate of discount of this scheme that was allowed by the shopkeeper?

- A. 3% B. 4% C. 6% D. 2%

Q22. Tarun got 30% concession on the labelled price of an article and sold it for Rs. 8750 with 25% profit on the price he bought. What was the labelled price?

- A. 10000 B. 12000 C. 13000 D. 14000

Type 6 – Goods Passing Through Successive Hands

Q23. Peter bought an item at 20% discount on its original price. He sold it with 40% increase on the price he

bought it. The new sale price is by what percentage more than the original price?

- A. 12% B. 13% C. 15% D. 17%

Q24. A man bought an article and sold it at a gain of 5 %. If he had bought it at 5% less and sold it for Re 1 less, he would have made a profit of 10%. The C.P. of the article was?

- A. Rs. 100 B. Rs. 150 C. Rs. 200 D. Rs. 250

Q25. A trader sold an article at a loss of 5% but when he increased the selling price by Rs.65 he gained 3.33% on the cost price. If he sells the same article at Rs. 936, what is the profit percentage?

- A. 15% B. 16.66 % C. 20 % D. Data Insufficient

Q26. A person incurs a loss of 5% by selling a watch for Rs. 1140. At what price should the watch be sold to earn 5% profit?

- A. Rs.1200 B. Rs.1230 C. Rs.1260 D. Rs.1290

Q27. The marked price of an article is increased by 25% and the selling price is increased by 16.66%, then the amount of profit doubles. If the original marked price be Rs. 400 which is greater than the corresponding cost price by 33.33%, what is the increased selling price?

- A. 240 B. 360 C. 420 D. 600

Q28. Bhajan Singh purchased 120 reams of paper at Rs 80 per ream. He spent Rs 280 on transportation, paid octroi at the rate of 40 paise per ream and paid Rs 72 to the coolie. If he wants to have a gain of 8 %, what must be the selling price per ream?

- A. 90 B. 89 C. 87.48 D. 86

Q29. If the manufacturer gains 10 %, the wholesale dealer 15 % and the retailer 25 %, then find the cost of production of a table if the retail price was Rs 1265

- A. Rs. 750 B. Rs. 800 C. Rs. 850 D. Rs. 900

Interest

SIMPLE INTEREST

If the interest on a sum borrowed for certain period is calculated uniformly, it is called **simple interest (SI)**. Simple interest is a quick method of calculating the interest charge on a loan.

Principal: The amount borrowed or invested.

Loan period or duration: Is the time that the principal amount is either borrowed or invested. It is usually given in years, but in some cases, it may be quoted in months or even days.

Interest: Is the extra money paid by the borrower to the owner (lender) as a form of compensation for the use of the money borrowed.

The statement "**rate of interest 10% per annum**" means that the interest for one year on a sum of **Rs.100** is **Rs.10**. If not stated explicitly, rate of interest is assumed to be for one year.

Formula

$$\text{SIMPLE INTEREST} = \frac{\text{PRINCIPAL} \times \text{RATE OF INTEREST} \times \text{TIME}}{100}$$

Example: Calculate the simple interest on Rs. 1000 at the rate of 5% per annum for a time period of 2 years.

Solution: Principal=1000

Rate of interest=5%

p.a. Time= 2 years

$$\text{SIMPLE INTEREST} = \frac{P \times R \times T}{100} = \frac{1000 \times 5 \times 2}{100} = \text{Rs.100}$$

COMPOUND INTEREST

Compound Interest is the interest calculated on a sum of money which includes principal and interest calculated for the previous year.

Example: Calculate the interest if compounded annually for an amount of Rs. 100 for a time period of 3 years at the rate of 10 % per annum.

Solution: Here, Principal

=Rs. 100 Time Period=3

years

Rate of interest =10% per annum

compounding is regular addition of interest

100	<u>interest for 1st year</u>	110	<u>interest for 2nd year</u>	121	<u>interest for 3rd year</u>	133.31
	at 10% p.a. is 10		at 10% p.a. is 11		at 10% p.a. is 12.1	

Amount 110 is the principal for the 2nd year, amount 121 is the principal for the 3rd year, and amount 133.1 is the principal for the 4th year.

Under compound interest, Amount is found by the formula given below:

$$A = P \left(1 + \frac{R}{100} \right)^n$$

Let Principal = P, Rate = R% per annum, Time = n years.

1. When interest is compound Annually:

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^n$$

2. When interest is compounded Half-yearly:

$$\text{Amount} = P \left[1 + \frac{(R/2)}{100} \right]^{2n}$$

3. When interest is compounded quarterly:

$$\text{Amount} = P \left[1 + \frac{(R/4)}{100} \right]^{4n}$$

4. Present worth of Rs. x due n years hence is given by:

$$\text{Present Value} = \frac{x}{\left(1 + \frac{R}{100}\right)^n}$$

5. Compound interest, **C.I.** = (Amount, **A**) – (Principal, **P**)

Type 1 – Simple Interest

Q1. A sum of money at simple interest amounts to Rs. 815 in 3 years and to Rs. 945 in 5 years. The sum is? A. 650 B. 690 C. 620 D. 700

Q2. How much time will it take for an amount of Rs. 450 to yield Rs. 81 as interest at 4.5% per annum of simple interest?

A. 3.5 years B. 4 years C. 4.5 years D. 5 years

Q3. A sum of Rs. 12,500 amounts to Rs. 15,500 in 4 years at the rate of simple interest. What is the rate of

interest? A. 3% B. 4% C. 5% D. 6%

Q4. What will be the ratio of simple interest earned by certain amount at the same rate of interest for 6 years and that for 9 years?

A. 1: 3 B. 1: 4 C. 2: 3 D. Data inadequate

Q5. A person borrows Rs. 5000 for 2 years at 4% p.a. simple interest. He immediately lends it to another person at $6\frac{1}{4}\%$ per annum for 2 years. Find his gain in the transaction per year?

A. Rs. 112.50 B. Rs. 125 C. Rs. 150 D. Rs. 167.50

Q6. A father left a will of Rs.35 lakhs between his two daughters aged 8.5 and 16 such that they may get equal amounts when each of them reach the age of 21 years. The original amount of Rs.35 lakhs has been instructed to be invested at 10% p.a. simple interest. How much did the elder daughter get at the time of the will?

A. 17.5 lakhs B. 21 lakhs C. 15 lakhs D. 20 lakhs

Q7. At what rate percent per annum will a sum of money double in 8 years?

A. 12.5% B. 13.5% C. 11.5% D. 14.5%

Q8. A sum of Rs. 725 is lent in the beginning of a year at a certain rate of interest. After 8 months, a sum of Rs. 362.50 more is lent but at the rate twice the former. At the end of the year, Rs. 33.50 is earned as interest from both the loans. What was the original rate of interest?

A. 3.46% B. 5% C. 4.5% D. 6%

Type 2 – Compound Interest

Q9. The compound interest on Rs. 30,000 at 7% per annum is Rs. 4347. The period (in years) is? A. 2 B. 2.5 C. 3

D. 4

Q10. The Compound interest on Rs. 20,480 at $6\frac{1}{4}\%$ per annum for 2 years 73 days is?

A. Rs. 2929 B. Rs. 2219 C. Rs. 3021 D. Rs. 3049

Q11. A man invests Rs.5000 for 3 years at 5% p.a. compound interest reckoned yearly. Income tax at the rate of 20% on the interest earned is deducted at the end of each year. Find the amount at the end of the third year?

A. Rs. 5624.32 B. Rs. 5423 C. Rs. 5634 D. Rs. 5976

Q12. The population of a town was 3600 three years back. It is 4800 right now. What will be the population three years down the line, if the rate of growth of population has been constant over the years and has been compounding annually?

A. Rs. 600 B. Rs. 6400 C. Rs. 6500 D. Rs. 6600

Q13. A tree increases annually by $\frac{1}{5}$ th of its height. If its height today is 50 cm, what will be the height after 2 years?

A. 64 cm B. 72 cm C. 66 cm D. 84 cm

Q14. The compound interest on Rs. 30,000 at 7% per annum is Rs. 4347. The period (in years) is? A. 1 B. 2 C. 3 D.

3.5

Q15. A sum amounts to Rs. 882 in 2 years at 5% compound interest. The sum is?

- A. Rs. 800 B. Rs. 822 C. Rs. 840 D. Rs. 816

Q16. What annual payment will discharge a debt of Rs. 1025 due in 2 years at the rate of 5% compound interest? A. Rs. 560 B. Rs. 560.75 C. Rs. 551.25 D. Rs. 550

Q17. The present worth of Rs. 242 due in 2 years at 10% per annum compound interest is?

- A. Rs. 180 B. Rs. 240 C. Rs. 220 D. Rs. 200

Q18. If in a certain number of years Rs. 10000 amounts to Rs. 160000 at compound interest, in half that time Rs. 10000 will amount to?

- A. Rs. 50000 B. Rs. 40000 C. Rs. 80000 D. Rs. 60000

Q19. The compound interest on Rs. 30,000 at 7% per annum is Rs. 4347. The period (in years) is?

- A. 1 B. 2 C. 3 D.

3.5

UNIT 3

NUMBER SERIES

Series completion

In this type of questions, some numbers and/or alphabetical letters are given. They all form a series and the series changes in certain order.

The series may also have one or more numbers/letters missing.

The candidates are required to observe that specific order in which the series changes and then complete the series.

Similarly, the candidates have to decide about the missing letter or number that would suit for the blank space if they continue to change in some order. Some common types are explained in the following slides.

Types of Series:

Number

Series

Alpha

series

Letter

series

Number and letter Analogy

Tricks to solve series completion

Step 1: Observe are there any familiar numbers in the given series like primes numbers, perfect squares, cubes and so on which are easy to identify.

Step 2: Calculate the differences between the numbers. Observe the pattern in the differences.

If the differences are growing rapidly it might be a square series, cube series or multiplicative series. If the numbers are growing slowly, then it is an addition or subtraction series.

If the differences are not having any pattern then,

1. It might be a double or triple series. Here every alternate number or every 3rd number forms series
2. It might be a sum or average series. Here sum of two consecutive numbers gives 3rd number or average of first two numbers give next number.

Step 3: Sometimes number will be multiplied and will be added another number.

Types of number series:

I. Prime number Series:

Example: 2, 3, 5, 7, 11, 13,

Solution: The given series is prime number series. The next prime number is 17.

Example: 2, 5, 11, 17, 23, 41.

Solution: The prime numbers are written alternately.

II. Difference Series:

Example: 2, 5, 8, 11, 14, 17... 23.

Answer: The difference between the numbers is 3. ($17+3 = 20$)

Example: 45, 38, 31, 24, 17... 3.

Answer: The difference between the numbers is 7. ($17-7=10$).

III. Multiplication Series:

Example: 2, 6, 18, 54, 162... 1458.

Answer: The numbers are multiplied by 3 to get next number. ($162 \times 3 = 486$).

IV. n^2 Series:

Example: 1, 4, 9, 16, 25,, 49

Answer: The series is $1^2, 2^2, 3^2, 4^2, 5^2, \dots$. The next number is $6^2=36$;

Example : 0, 4, 16, 36, 64,144.

Answer: The series is $0^2, 2^2, 4^2, 6^2$, etc. The next number is $10^2=100$.

V. n^2-1 Series :

Example : 0, 3, 8, 15, 24, 35, 48,,

Answer : The series is $1^2-1, 2^2-1, 3^2-1$ etc. The next number is $8^2-1=63$.

Another logic : Difference between numbers is 3, 5, 7, 9, 11, 13 etc. The next number is ($48+15=63$).

VI. n^2+1 Series :

Example: 2, 5, 10, 17, 26, 37,, 65.

Answer: The series is $1^2+1, 2^2+1, 3^2+1$ etc. The next number is $7^2+1=50$.

Example: 3, 12, 48, 192,, 3072.

Answer : The numbers are multiplied by 4 to get the next number. ($192 \times 4 = 768$).

VII. Division Series:

Example : 720, 120, 24,, 2, 1

Answer: $720/6=120, 120/5=24, 24/4=6, 6/3=2, 2/2=1$. **

Example : 32, 48, 72, 108,, 243.

Answer: . Number $\times 3/2 =$ next number. $32 \times 3/2=48, 48 \times 3/2=72, 72 \times 3/2=108, 108 \times 3/2=162$.

VIII. n^2+n Series (or) n^2-n Series :

Example : 2, 6, 12, 20,, 42.

Answer : The series is $1^2+1, 2^2+2, 3^2+3, 4^2+4$ etc. The next number = $5^2+5=30$.

Another Logic : The series is $1 \times 2, 2 \times 3, 3 \times 4, 4 \times 5$. The next number is $5 \times 6=30$.

Another Logic : The series is $2^2-2, 3^2-3, 4^2-4, 5^2-5$. The next number is $6^2-6=30$.

IX. n^3 Series :

Example : 1, 8, 27, 64, 125, 216,

Answer : The series is $1^3, 2^3, 3^3$, etc. The missing number is $7^3=343$.

X. n^3+1 Series :

Example : 2, 9, 28, 65, 126, 217, 344,

Answer : The series is $1^3+1, 2^3+1, 3^3+1$, etc. The missing number is $8^3+1=513$.

XI. n^3-1 Series :

Example : 0, 7, 26, 63, 124,, 342.

Answer: The series is $1^3-1, 2^3-1, 3^3-1$ etc. The missing number is $6^3-1=215$.

XII. n^3+n Series :

Example : 2, 10, 30, 68, 130,, 350.

Answer : The series is $1^3+1, 2^3+2, 3^3+3$ etc. The missing number is $6^3+6=222$.

XIII. n^3-n Series :

Example : 0, 6, 24, 60, 120, 210,,

Answer : The series is $1^3-1, 2^3-2, 3^3-3$, etc. The missing number is $7^3-7=336$.

Another Logic : The series is $0 \times 1 \times 2, 1 \times 2 \times 3, 2 \times 3 \times 4$, etc. The missing number is $6 \times 7 \times 8=336$.

XIV. n^3+n^2 Series :

Example : 2, 12, 36, 80, 150,,

Answer: The series is $1^3+1^2, 2^3+2^2, 3^3+3^2$ etc. The missing number is $6^3+6^2=252$

XV. n^3-n^2 Series

Example: 0, 4, 18, 48, 100,,

Answer : The series is $1^3-1^2, 2^3-2^2, 3^3-3^2$ etc. The missing number is $6^3-6^2=180$

XVI. $xy, x+y$ Series:

Example: 48, 12, 76, 13, 54, 9, 32,,

Answer : $4+8=12, 7+6=13, 5+4=9, 3+2=5$.

XVII. Factorial Series:

Example: 1, 1, 2, 6, 24, 120,,

Answer : $0!=1, 1!=1, 2!=2, 3!=6, 4!=24, 5!=120, 6!=720$

PRACTICE EXERCISE

Q1. In following question, a number series is given with one term missing. Choose the correct alternative that will same pattern and fill in the blank spaces.: 1, 4, 9, 16, 25, x

- A. 35** **B. 36** **C. 48** **D. 49**

Q2. In following question, a number series is given with one term missing. Choose the correct alternative that will same pattern and fill in the blank spaces.: 1, 6, 13, 22, 33,

- A. 44** **B. 45** **C. 46** **D. 47**

Q3. In following question, a number series is given with one term missing. Choose the correct alternative that will same pattern and fill in the blank spaces.: 19, 2, 38, 3, 114, 4,

- A. 228** **B. 256** **C. 352** **D. 456**

Q4. In following question, a number series is given with one term missing. Choose the correct alternative that will same pattern and fill in the blank spaces.: 4, 5, 9, 18, 34, (.....)

- A. 43** **B. 49** **C. 50** **D. 59**

Q5. In following question, a number series is given with one term missing. Choose the correct alternative that will same pattern and fill in the blank spaces.: 2, 1, 2, 4, 4, 5, 6, 7, 8, 8, 10, 11,

- A. 9** **B. 10** **C. 11** **D. 12**

Q6. In following question, a number series is given with one term missing. Choose the correct alternative that will same pattern and fill in the blank spaces.: 11, 10, (.....), 100, 1001, 1000, 10001

- A. 101** **B. 110** **C. 111** **D. None of these**

Q7. In following question, a number series is given with one term missing. Choose the correct alternative that will same pattern and fill in the blank spaces.: 123456147, 12345614, 2345614, 234561,

- A. 3456** **B. 2345** **C. 23456** **D. 34561**

Q8. In following question, a number series is given with one term missing. Choose the correct alternative that will same pattern and fill in the blank spaces.: In the Series 3, 9, 15, ... what will be the 21st term ?

- A. 117** **B. 121** **C. 123** **D. 129**

Q9. In following question, a number series is given with one term missing. Choose the correct alternative that will same pattern and fill in the blank spaces.: Which term of the series 5, 8, 11, 14, ... is 320 ?

- A. 104th** **B. 105th** **C. 106th** **D. 64th**

Q10. In following questions, one term in number series is incorrect. : Find out the incorrect number 24, 27, 31, 33, 36

- A. 24** **B. 27** **C. 31** **D. 33**

**CODING DECODING
&
LETTER SERIES**

Coding Decoding

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 1 2 3 4 5 6 7 8 9 10 11 12 13

To remember them use the Code- **EJOTY (5, 10, 15, 20, 25)**

A-Z , B-Yare opposite to each other. The sum of two opposite letters is 27. A=1 , Z=26 so $A+Z=1+26=27$.

Number coding

In this, either the numerals are assigned to the alphabets of the given code or the alphabets are assigned to the numerals. The candidate has to observe the direction of solving the problem.

Mixed coding

In this, three or more complete messages are given. The procedure to solve is any two messages bearing the common word are picked up. Proceeding similarly, all possible combinations of two messages are analyzed.

Mixed number coding

It is the same as mixed coding but instead of alphabetical codes numerical codes are given.

Decoding

Conversion of the coded numbers or alphabets to the original text. The procedure to decode is the same as coding. That is, find the pattern that is followed in the given series.

SYMBOLS CODING

In this type of coding, symbols like!, @, # and so on will be used for coding the numbers or alphabets.

PRACTICE EXERCISE

Q1. If COURSE is coded as FRXUVH, how is RACE coded as?

- A.ABHF B.UDFH C.DUHF D.WQYF

Q2. In a certain code, MONKEY is written as XDJMNL. How is TIGER written in that code?

- A.QDFHS B.FHSQD C.DQSFH D.STFDQ

Q3. If BOMBAY is written as MYMYMY, how will TAMIL NADU be written in that code?

- A.YMNYMNYMN B.ABHABHABH C.ABCDABCDA
D.MNUMNUMNU

Q4. In a certain code, TOGETHER is written as RQEGRJCT. In the same code, what will PAROLE be written as?

- A.PQJGNC B.CNGJPQ C.NCPQJG D.NCJQPG

Q5. If in a certain language, COUNSEL is coded as BITIRAK, how is GUIDANCE written in that code?

- A.OHYFZJBB B.OFHBZYB C.BJZBHFYO D.FOHYZJBB

Q6. If in a certain code, TWENTY is written as 863985 and ELEVEN is written as 323039, how is TWELVE written in that code?

- A.203863 B.368302 C.863203 D.320368

Q7. In a certain code, if LOGIC is coded as 1512201824, how is PEARL coded as?

- A.112226915 B.113331596 C.112226571
D.113336734

Q8. If APPLE is written as 24991320, how is LOVELY coded as?

- A.13101310130 B.13101320130 C.13101350140 D.13101340120

Q9. If ENGLAND is written as 1234526 and FRANCE is written as 785291, how is GREECE coded?

- A.117186 B.381171 C.131871 D.112235

Q10. If tee see pee means drink fruit juice, see kee lee means juice is sweet, lee reemee means he is intelligent, then which word means sweet?

- A.See B.Pee C.Tee D.Kee

Q11. If white is called blue, blue is called red, red is called yellow, yellow is called green, green is called black, black is called violet and violet is called orange, what would be the color of human blood?

A.Blue

B.Yellow

C.Black

D.Violet

Q12. If the animals which can walk are called swimmers, animals who crawl are called flying, those living in water are called snakes and those which fly in the sky are called hunters, then what will a lizard be called?

A.Flying

B.Swimmer

C.Snakes

D.Hunters

Q13. In a certain code language, 'col tip mot' means 'singing is appreciable', 'mot baj min' means 'dancing is good' and 'tip nopbaj' means 'singing and dancing', then, which of the following means 'good' in that code language ?

A.Mot

B.Bai

C.Min

D.Nop

Q14. In a certain code language, '851' means 'good sweet fruit', '783' means 'good red rose' and '341' means 'rose and fruit'. Which of the following digits stands for 'sweet' in that language ?

A.2

B.3

C.4

D.5

Q15. In a certain code, 2 is coded as P, 3 as N, 9 as Q, 5 as R, 4 as A and as B. How is 599423 coded in that code?

A.QRQPAN

B.RQQAPN

C.AQPQRN

D.QRANPA

Q16. In a certain code language, '123' means 'hot filtered coffee'

, '356' means 'very hot day' and '589' means 'day and night'. Which digit stands for 'very'?

A.3

B.6

C.9

D.7

Q17. In a certain code, '256' means 'you are good'; '637' means 'we are bad' and '358' means 'good and bad'. Which of the following represents 'and' in that code?

A.5

B.6

C.7

D.8

Q18. If in a certain language NZTUJGZ is coded as MYSTIFY, how is OFNFTJT coded in that language?

A.REGULAR

B.MORNING

C.MINDFUL

D.NEMESIS

Q19. In a certain code, SQHOOKD is written as TRIPPLE. How CHRONRD is written in that code ?

A.GLITTER

B.TROUSER

C.JANUARY

D.DISPOSE

Q20. If HUMJTK is coded as FRIEND, how is EDRIRL written in that code ?

A.SUNDAY

B.MONDAY

C.BEAUTY

D.CANDLE

Q21. In a certain code language TUTDNES is written as STUDENT. How will SUORECS be written in that code language?

A.BATTERY

B.FASHION

C.SOURCES

D.LIMITED

Q22. ZA5, Y4B, XC6, W3D,

A.E7V

B.V2E

C.VE5

D.VE7

Q23. In a certain code 'TOME' is written as '@ \$ * ?' and ARE is written as '• £ ?' How can 'REMOTE' be written in that code?

A. ?* \$@? £

B. *\$@? £?

C. £?* \$@?

D. *\$? £@?

24. In a certain code 'PALM' is coded as '!@?\$', and 'ARM' is written as '@*\$', how can 'ALARM' be written in that code?

A. @!@?\$

B. @\$?!@

C. ?@@!\$

D. NONE OF THESE

UNIT 4

RATIO & PROPORTION and PARTNERSHIP

RATIO

Ratio is a comparison of two quantities by division. Ratio represents the relation that one quantity bears to the other. If **a** and **b** are two quantities of the same kind, then **a/b** is known as the ratio of **a** and **b**.

Denoted as **a: b**, where the first term of the ratio is called as **antecedent**, while the second term is called as **consequent**.

A "**ratio**" is just a comparison between two different things. The ratio between 30 kg and 50 kg is 3:5.

Example: In the park mentioned above, the ratio of ducks to geese is 16 to 9. How many of the 300 birds are geese?

Solution: The ratio tells that, out of every $16 + 9 = 25$ birds, 9 are geese. That is, $\frac{9}{25}$ of the birds are geese. Then there are $(\frac{9}{25}) (300) = 108$ geese.

Example: In a school the ratio of number of boys and girls is 9:6. If there are present 180 boys. Find the total number of students in the school?

Solution: Let the number of boys and girls be $9x$ and $6x$. Then $9x = 180$, $x = 20$

Therefore, the total number of students $= 15x$, Thus, $15(20) = 300$

Different Types of Ratios

1. Duplicate Ratio:

$a^2 : b^2$ is called duplicate ratio of $a : b$

2. Triplicate Ratio:

$a^3 : b^3$ is called triplicate ratio of $a : b$

3. Compound Ratio:

$ab : cd$ is the compound ratio of $a : c$ and $b : d$. It is the ratio of the products of the antecedents to that of the consequents of the two or more given ratios.

PROPORTION

The equality of two ratios is called as proportion. a, b, c , and d are said to be in proportion if,

$$a : b = c : d \quad \text{or} \quad a : b :: c : d$$

In a proportion, the first and fourth terms are known as extremes, while second and third terms are known as means.

PRODUCT OF EXTREMES = PRODUCT OF MEANS

$$a * d = b * c$$

Continued Proportion

Four quantities: a, b, c and d are said to be in continued proportion, if **$a:b=b:c=c:d$** .

Three quantities: a, b and c are said to be in continued proportion, if **$a:b=b:c$** or **$ac=b^2$**

b is said to be the **mean proportional** between **a** and **c** and **c** is said to be a **Third proportional** to **a** and **b**.

Example: If 40, x, x, 40 are in proportion, then find the value of x.

Solution: **Product of means = product of extremes**

$$x * x = 40 * 40$$

$$\Rightarrow x^2 = 1600 \quad \Rightarrow \quad x = 40$$

FOURTH Proportion – If four quantities a, b, c and x are such that $a : b :: c : x$, then $ax=bc$ and x is called fourthproportion of a, b and c.

Example: A can do a piece of work in 12 days, B is 60% more efficient than A. Find the number of days that B takes to do the same piece of work.

Solution: Ratio of efficiencies of A and B=100

$$: 160 = 5 : 8$$

Since, **efficiency is inversely proportional to the number of days.**

Ratio of days taken to complete the

job=8:5 No. of days taken by B=5/8

$$*12=15/2$$

Variation

If two quantities are related in such a way that as quantity 'x' changes, it also brings a change in the second quantity 'y', then the two quantities are in variation. There are two types of variations:-

1.Direct Variation: The quantity 'x' is in direct variation to 'y', if an increase in 'x' causes an increase in 'y' and decrease in 'x' causes 'y' to decrease proportionally. Therefore, **$x=ky$** , where 'k' is constant of proportionality.

2. Inverse Variation: The quantity 'x' is in inverse variation to 'y', if an increase in 'x' causes an decrease in 'y' and decrease in 'x' causes 'y' to increase proportionally. Therefore, **$x=k/y$** , where 'k' is constant of proportionality.

3. Joint Variation: If there are more than 2 quantities x,y and z; and x varies with both y and z, then x is in jointvariation to y and z. It can be expressed as $k y z$, where k is constant of proportionality.

Example: Men doing a work in some number of days working certain hours a day.

Partnership

Persons two or more than two persons when start and run the new business jointly of their own choice, the persons who start the business are called **partners** and the agreement between them is called **partnership**.

Working and Inactive partners:

A partner who manages the business is called **working/active partner** and the one who simply invests the money is called **inactive partner**.

Ratio of division of gains:

1. The amount investment of all the partners are for the same time period, the gain or loss amount is distributed among the partners in the ratio of their invested amount.

2. When investments are for different time periods

Example: A invests Rs. **R1 for T1 months** and B invests Rs. **R2 for T2 months**, then **(A's share of profit) : (B's share of profit) = A*T1 : B*T2**

Partnership is of two types:

1. Simple Partnership
2. Compound Partnership

13. Simple Partnership: When investments of all the partners are for the same period of time, the profit or loss is distributed among the partners in the ratio of their original investments.

Suppose A and B invest ` p and ` q respectively for a year in a business, then at the end of the year.

Share of A's profit (loss) : Share of B's profit (loss) = p : q

14. Compound Partnership: When investments of all the partners are for different period of time, then equivalent capitals are calculated for a unit of time and the profit or loss is divided in the ratio of the product of time and investment.

Suppose A and B invest ` p and ` q for x months and y months respectively,

then Share of A's profit (loss) : Share of B's profit (loss) = px : qy

Example: A and B started a business investing Rs. 90,000 and Rs 20,000 respectively. In what ratio should the profit earned after 2 years be divided between A and B respectively?

- A. 9:2 B. 3:2 C. 18:20 D. 18:4

Solution: Exp: A: B = 90000 : 20000 = 90 : 20 = 18 : 4 = 9 : 2

Example: Ajay, Bhavan and Chetan started a business together. Thrice the investment of Ajay, twice the investment of Bhavan and the investment of Chetan are equal. Find the ratio of their respective profits at the end of the year?

- A. 1:2:1 B. 2:3:6 C. 3:2:1 D. 1:2:3

Solution: Let the investments of Ajay, Bhavan and Chetan be Rs. a, Rs. b and Rs. c respectively.

$$3a = 2b = c, a = c/3, b = c/2.$$

Ratio of profits of Ajay, Bhavan and Chetan at the end of one year = Ratio of their respective investments

=2:3:6.

Type 1 – Percentage & Ratio

Q1. The salaries of A, B, C are in the ratio 2:3:5. If the increments of 15%, 10% and 20% are allowed respectively in their salaries, then what will be new ratio of their salaries?
A. 3:3:10 B. 10:11:20 C. 23:33:60 D. Can't be determined

Q2. In a class of 125, 20% students can dance. $\frac{2}{5}$ of the total students can sing and $\frac{2}{5}$ of the remaining students are good at sports. What is the respective ratio of the students who can dance to students who are good at sports?
A. 5:4 B. 3:2 C. 4:5 D. 3:7

Q3. X: Y: Z is in the ratio of 3: 2: 5. Then how much money will Z get out of Rs 500?
A. Rs. 200 B. Rs. 250 C. Rs. 300 D. Rs. 350

Q4. Rate of income tax is increased from 4% to 5%. However, the total tax liability of a person remains the same as was in the last year. If his income for the last year was Rs.10000, find his present income.
A. 9000 B. 8000 C. 5000 D. 6000

Q5. Mohan distributed his assets to his wife, three sons, two daughters and five grandchildren in such a way that each grandchild got one-eighth of each son and one-tenth of each daughter. His wife got 40% of the total share of his sons and daughter together. If each daughter receives asset of Rs.1.25 lakhs, what is the salary of his wife?
A. 2.5 Lakhs B. 2.7 Lakhs C. 2.2 Lakhs D. 3.2 Lakhs

Type 2 - Coin Based Problem

Q6. A sum of Rs. 36.90 is made up of 180 coins which are either 10 p coins or 25 p coins. The number of 10 p coins is?
A. 48 B. 54 C. 56 D. 60

Q7. A bag contains Rs 410 in the form of Rs 5, Rs 2 and Rs 1 coins. The numbers of coins are in the ratio 4:6: 9. So, find the number of 2 Rs coins.
A. 40 B. 50 C. 60 D. 70

Q8. A bag contains 50 P, 25 P and 10 P coins in the ratio 5: 9: 4, amounting to Rs. 206. Find the number of coins of each type respectively.
A. 360, 160, 200 B. 160, 360, 200 C. 200, 360, 160 D. 200, 160, 300

Q9. A bag contains some coins in the denominations 50, 20 and 10 paisa coins in the ratio 4:2:1. If their total value is Rs 12.50, then the number of 10 paisa coins is?
A. 10 B. 5 C. 20 D. 15

Q10. In a bag, there are coins of 25 p, 10 p and 5 p in the ratio of 1 : 2 : 3. If there is Rs. 30 in all, how many 5 p

coins are there?

- A. 50 B. 100 C. 150 D. 200

Type 3 - Income and Expenditure

Q11. Share of Rs.4200 among Rahul, Vijay and Mahinder in the ratio of 2:4:6.Find the amount received by Mahinder?

- A. 3100 B.2500 C.2100 D.4200

Q12. The ratio of the incomes of four persons A, B, C and D is 5:3:9:4.The sum of the incomes of A and C is 84,000.Find the difference of the incomes of B and D?

- A. 5000 B.7000 C.6000 D.8000

Q13. The ratio of income of A and B is 3:4. The Ratio of expenditure of both is 2: 3 and each saves RS 200. Findthe income of A and B.

- A. Rs 500,600 B. Rs 600,800 C.Rs 600,900 D.Rs 800, 1000

Q14. The salary of two friend's Ramu and Raju are in the ratio of 4:5.If the salary of each one increases by Rs.6000, then the new ratio becomes 48:55.What is Raju's present salary?

- A. 11,500 B.10,500 C.9000 D.8,500

Type 4 - Ratios of Ratios

Q15. In a school, the ratio to the number of boys and girls is 4:9, after inclusion of 32 new girls, the ratio becomes 4:17.How many boys were present at the starting in this school?

- A. 20 B.16 C.25 D.18

Q16. In an examination, the number of those who passed and the number of those who failed were in the ratio 25:4.If five more had appeared and the number of failures was 2 less than earlier, the ratio of passers to failureswould have been 22:3.The number of students who appeared at the examination, is?

- A. 154 B.145 C.160 D.150

Q17. The students in the three classes are in the ratio 2:3:5.If 20 students are increased in each class the ratiochanges to 4:5:7. What was the total number of students in the three classes before the increase?

- A. 125 B.130 C.100 D.150

Q18. At a start of seminar, the ratio of the number of male participants to the number of female participants was 3:1.During the tea break 16 participants left and 6 more participants registered. The ratio of the male to the female participants now becomes 2:1.What was the total number of participants at the start of the seminar?

- A. 50 B.60 C.30 D.40

Q19. The numerator and denominator of a fraction are in the ratio 2:3.If 6 is subtracted from the numerator thevalue of the fraction becomes $\frac{2}{3}$ of the original fraction. The numerator of the original fraction is?

- A. 6 B.18 C.5 D.5

Q20. The ratio of the first and the second class train fares between two stations is 3:1 and that of the number of passengers travelling between the two stations by first and second class is 1:50. If on a particular day, Rs.1325 are collected from passengers travelling between the two stations, then the amount collected from the second class passenger is?

- A. 1250 B.1350 C.1520 D.1400

Type 5 - Simple & Compound Partnership

Q21. A, B, C subscribes together Rs.50, 000 for business. A subscribes Rs.4000 more than B and B Rs.5000 more than C. Out of a total profit Rs.35000, A receives?

- A. 14, 700 B.15, 500 C.16, 500 D.17, 400

Q22. A and B joined a partnership business by investing Rs.30, 000 and Rs.50, 000 respectively. If they earn a profit of Rs.4, 000, find A's share in profit.

- A. 2500 B.1500 C.2000 D.500

Q23. A starts a business with Rs.7, 000 and after 5 months, B joined as a partner. After a year, the profit is divided in ratio 2:3. The capital of B is?

- A. 18,000 B.7,000 C.10,000 D.16,000

Q24. A and B starts a business jointly. A invests Rs.16, 000 for 8 months and B remains in the business for 4 months. Out of total, B claims $\frac{2}{7}$ of the profit. How much money was contributed by B?

- A. 12,500, B.12, 000 C.12,800 D.13,000

Q25. A and B are partners and invested Rs.50,000 and Rs.60,000 respectively. After 8 months B leaves and C joins with a capital of Rs.90,000. If the profit for 1 year is Rs.36,000, find A's share of profit.

- A. 15000 B. 12000 C.9000 D.14000

Q26. A, B and C started a business with investment in ratio 5:6:8 respectively. After 1 year, C withdrew 50% of his capital and A increase his capital by 60% of his investment. After 2 years, in what ratio should the earned profit be distributed among A, B and C respectively?

- A. 12:12:13 B.13:12:12 C.12:13:13 D.13:12:13

Q27. A began with Rs.45000 and was joined afterwards by B with Rs.54000. After how many months did B join, if the profits at the end of the year were divided in the ratio 2:1?

- A. 7 months B.9 months C. 5 months D. 7.5 months

Type 6 - Partnership with Ratio

Q28. A, B and C shared profits in ratio of 5:7:8. They partnered for 14 months, 8 months and 7 months respectively. Find the ratio of their investments.

- A. 64:49:20 B.49:64:20 C.20:49:64 D.20:64:49

Q29. A and B invests in the business in ratio 3:2. Assume that 5% of total profit goes to charity. If A's share is Rs.855, what is the total profit?

- A. 1000 B. 4275 C.2525 D.1500

Q30. In a business, A and C invested amounts in the ratio 2:1, whereas the ratio between amount invested by

A and B was 3:2. If Rs. 1, 57,300 was their profit, how much amount did B receive?

- A. 48,400 B. 46,400 C. 72,600 D. 36,300

Q31. A and B are partners. A contributes $\frac{1}{4}$ of the capital for 15 months and B received $\frac{2}{3}$ of the profit. For how many months B's money was used?

- A. 15 months B. 18 months C. 10 months D. 8 months

Q32. A, B and C started a business with capitals in the ratio 5:6:8. At the end of 1 year, they shared profits in the ratio 5:3:12 find the ratio of time for which they had contributed their capitals?

- A. 2:1:3 B. 1:2:3 C. 2:3:1 D. 2:3:3

Type 7 - Partnership and Shares

Q33. A and B started a business with Rs. 4000 and Rs. 3000 respectively. After 6 months, C joined them by investing Rs. 4,000. At the end of 2 years, profit was Rs. 5,000, then find B's share of profit?

- A. 2000 B. 1500 C. 2500 D. 1000

Q34. A started a business with capital of Rs. 1,00,000. 1 year later, B joined him with capital of Rs. 2,00,000. At the end of 3 years, from the start of the business, profit was Rs. 84,000. B's share in profit exceeded A's share in profit by?

- A. 12,000 B. 24,000 C. 48,000 D. 60,000

Q35. P, Q and R started a business by investing Rs. 120,000, Rs. 135,000 and Rs. 150,000 respectively. Find the share of Q, out of annual profit of Rs. 56,700?

- A. 16,800 B. 21,000 C. 18,900 D. 27,000

ALLIGATIONS & MIXTURES

ALLIGATIONS

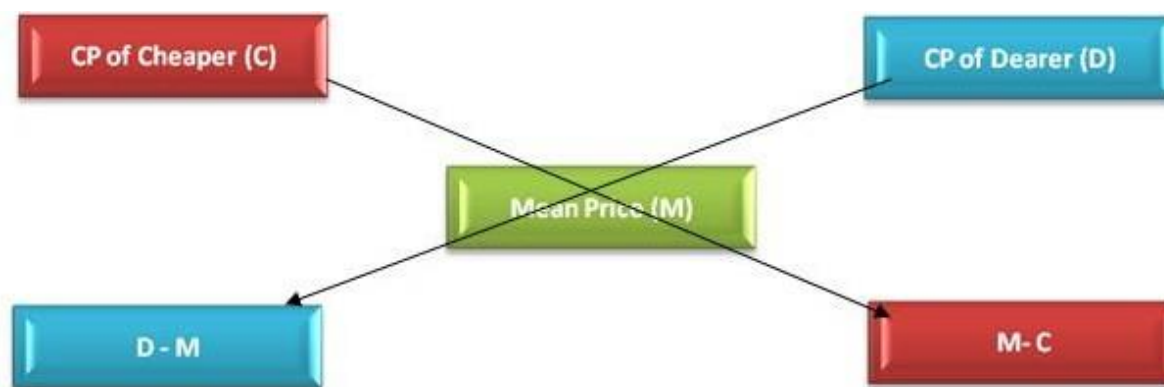
The technique of alligation is applicable in all the cases where two extreme values are given and one average value is given. It is a very useful technique which can be applied in chapters like Percentage, Simple interest, Ratio & proportion, Average etc.

This technique enables us to calculate the ratio in which extreme values/ prices/ interests/ ratios and averages should be mixed so that a given average value/price/interest/ratio and average can be obtained.

Alligation is the rule that enables us to find the proportion in which the two or more ingredients at the given price must be mixed to produce a mixture at a given price. Thus,

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{(\text{C.P. of dearer}) - (\text{Mean Price})}{\text{Mean Price} - \text{CP of cheaper}}$$

Find it complicated to remember the Formula?? Don't worry, keep in mind the below short cut by following the direction of the arrows:



Attention please!!

1. Mean price is always less than dearer price and is always more than cheaper price.
2. The price of the first kind should always be on the left hand side.
3. Keep in mind the simple point that the order of the ratio follows the order of what is written at the top.

MIXTURES

Mixture or alloys contains two or more ingredients of certain quantity mixed together to get a desired quantity. The quantity can be expressed as a ratio or percentage. For ex: 1 liter of a mixture contains 250ml water and 750ml milk. That means, $\frac{1}{4}$ of mixture is water and $\frac{3}{4}$ of mixture is milk. In other words, 25% of mixture is water and 75% of mixture is milk.

Concept 1: Finding the Quantity of an Ingredient in the Mixture

Illustration 1:

A mixture contains alcohol and water in the ratio 4 : 3. If 7 litres of water is added to the mixture, the ratio of alcohol and water becomes 3 : 4. Find the quantity of alcohol in the mixture.

Solution:

Let the alcohol : water be $4x : 3x$.

Adding 7 litres of water, the fraction becomes $4x/(3x + 7) = 3/4$. On solving, we get $x = 3$ and alcohol = $4x = 12$.

Concept 2: Quantity of Ingredient to be Added to Increase the Content of Ingredient in the Mixture to $y\%$

Illustration 2:

A mixture of water and milk contains 80% milk. In 50 litres of such a mixture, how many litres of water is required to increase the percentage of water to 50%?

Solution:

Total mixture = 50 litres

Milk = 80% of 50 = 40 litres

Water = 20% of 50 = 10

litres Let 'x' litres of water is added.

Now, milk = 40

litres Water = $10 + x$

Total = $50 + x$

Now, 50% of total = Water

$\frac{1}{2} \times (50 + x) = 10 +$

x = 30 litres

Concept 3: Quantity of Ingredient to be Added to Change the Ratio of Ingredients in a Mixture

Illustration 3:

729 ml of a mixture contains milk and water in the ratio 7 : 2. How much more water is to be added to get a new mixture containing milk and water in the ratio of 7 : 3?

Solution:

Milk and water in the original liquid = $7/9 \times 729 = 567$ and water = $2/9 \times 729 = 162$.

Let water to be added = x .

Then, $567/(162 + x) = 7/3$

Hence, we get $1701 = 1134 + 7x$; or $7x = 567$; or $x = 81$

Concept 4: Replacement of a Part of a Solution

If a vessel contains A liters of milk and if B litres of milk is withdrawn and replaced by water, and again if B litres of mixture is withdrawn and replaced by water and this operation is repeated n times in all, then

$$\frac{\text{(Quantity of milk left after } n^{\text{th}} \text{ operation)}}{\text{(Initial quantity of milk)}} = \left(\frac{A - B}{A} \right)^n$$

Thus, quantity of milk/alcohol left after nth operation = $A \left(1 - \frac{B}{A} \right)^n$ Or in other words,

Final Amount of ingredient that is not replaced =

$$\text{Initial Amount} \times \left(\frac{\text{Vol. after removal}}{\text{Vol. after replacing}} \right)^n$$

Type 1- Alligation

- Q1.** In what ratio must rice at Rs. 43/kg be mixed with rice at Rs 56/kg, so that mixture be worth Rs. 51/kg?
 A. 3:7 B. 5:8 C. 7:3 D. 7:5
- Q2.** In what ratio must rice at Rs. 20/kg be mixed with rice at Rs 12/kg, so that mixture be sold at Rs. 18/kg, with profit of 20%?
 A. 3:5 B. 5:3 C. 7:5 D. 7:3
- Q3.** In what ratio must rice at Rs. 42/kg be mixed with rice at Rs 24/kg, so that by selling the mixture at 40/kg, shopkeeper gain 25%?
 A. 3:4 B. 5:4 C. 4:5 D. 4:3
- Q4.** A shopkeeper has 50 kg rice, some part of rice he sold at 8 % profit & remaining at 18% profit. He gain 14% on the whole transaction. Find the quantity of rice sold at 8 % profit?
 A. 20 kg B. 21 kg C. 22 kg D. 23 kg
- Q5.** A merchant has 25 kg rice, some part of rice he sold at 10 % profit & remaining at 5% loss. He gain 7% on the whole transaction. Find the quantity of rice sold at 10 % profit?
 A. 20 kg B. 30 kg C. 25 kg D. 35 kg
- Q6.** A shopkeeper has 1000 kg sugar, some part he sold at 14 % profit & remaining at 6% loss. He lost 4% on the whole transaction. Find the quantity of rice he sold at 6 % loss?
 A. 700 kg B. 900 kg C. 800 kg D. 600 kg

Type 2- Mixtures

- Q7.** When 16 liter water be mixed with 108 Rs/liter pure milk. The price of mixture becomes 90 Rs/liter. Find the quantity of pure milk in the mixture?
 A. 83 liters B. 80 liters C. 82 liters D. 81 liters
- Q8.** When 25 liter water be mixed with Rs. 12/liter pure milk so that the cost of mixture becomes Rs. 2 /liter. Find the quantity of pure milk in the mixture?
 A. 3 liters B. 4 liters C. 5 liters D. 6 liters
- Q9.** How much water must be added to a bucket containing 40 liter of milk at 3.5 Rs/liter so that the cost of mixture becomes 2 Rs/liter?
 A. 30 liters B. 40 liters C. 50 liters D. 60 liters

Type 3 –Removal of Some Quantity of the Mixture

- Q10.** From 100 liter milk 10 liter milk is taken out instead of milk 10 liter water is added & this process repeated 2 more times than find quantity of pure milk left after 3 such processes (in liter)?
 A. 70 B. 80 C. 72.9 D. 80.9
- Q11.** From 100 liter milk 10 liter milk is taken out. Instead of milk, 10 liter water is added, again 9 liter milk is taken out instead of this 9 liter water is added, again 8 liter water is taken out instead 8 liter water is added. Find the quantity of pure milk left after such processes (in liter)?
 A. 74 B. 80 C. 75.34 D. 76

Q12. A container has 80 litres mixture of milk & water, if we pour out 70 % milk & 30 % water then an average 55 % container is empty, find quantity of milk and water in container?

- A. 30 lt, 50 lt B. 50 lt, 40 lt C. Rs. 50 lt, 30 lt D. 20 lt, 30 lt

Q13. A can contains a mixture of two liquids A and B in the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially?

- A. 10 B. 20 C. 21 D. 25

Q14. A jar contains a mixture of two liquids A and B in the ratio 4 : 1. When 10 litres of the mixture is taken out and 10 litres of liquid B is poured into the jar, the ratio becomes 2 : 3. How many litres of liquid A was contained in the jar?

- A. 14 litres B. 18 litres C. 20 litres D. 16 litres

Type 4 – Mixing of Mixtures

Q15. Two equal glasses having milk & water in ratio 3:2 & 4:1. Both glasses get mixed in third glass, then ratio of milk & water in third glass is?

- A. 3:7 B. 7:3 C. 7:2 D. 2:7

Q16. Three equal glasses are having milk & water in ratio 9:2, 7:4 & 6:5. These glasses are mixed in fourth glass, then ratio of milk & water in fourth glass is?

- A. 2:1 B. 1:2 C. 3:1 D. 1:3

Q17. Two equal glasses having milk & water in ratio 4:3 & 3:2 respectively. If content of both glasses are mixed in third glass, then ratio of milk & water in third glass is?

- A. 41:29 B. 29:41 C. 40:15 D. 15:40

Q18. Milk and water in two vessels are in ratio 4:3 & 2:3. In what ratio the liquid in both the vessels should be mixed to obtain the new mixture in vessel C, containing half milk & half water?

- A. 7:5 B. 5:3 C. 5:7 D. 3:5

Q19. Zinc and copper in two parts A & B are in ratio 1:2 & 2:3. In what ratio zinc & copper from both the parts can be mixed to obtain the new mixture in part C, in the ratio of 5:8?

- A. 10:3 B. 3:10 C. 5:10 D. 10:5

Q20. A vessel contains a mixture of 2 liquids A & B in the ratio 3:2, when 20 litres of mixture is taken out & 20 litres of liquid of type B is added, then ratio becomes 1:4. Find quantity of liquid A & B in the container (in litres)?

- A. 18, 12 B. 20, 12 C. 12, 20 D. 12, 18

Q21. One type of liquid contains 25% of milk, the other contains 30% of milk. A container is filled with 6 parts of the first liquid and 4 parts of the second liquid. The percentage of milk in the mixture is?

- A. 27% B. 31% C. 29% D. 33%

Q22. There are 2 bottles containing a mixture of wine, water and alcohol. The first bottle contains wine, water and alcohol in the ratio 3 : 5 : 2. The second bottle contains water and wine in the ratio 5 : 4. 1 litre of the first and 2 litres of the second are mixed together. What fraction of the mixture is alcohol?

- A. 1/15 litres B. 6/13 litres C. 2/15 litres D. 6/19 litres

Type 5- Applications

- Q23.** In what ratio milk and water be mixed so that the mixture be sold at CP, The milkman gain 20%?
A. 1:3 B. 2:3 C. 3:4 D. 5:1
- Q24.** In what ratio milk and water be mixed so that the mixture be sold at CP, The milkman gain 25%?
A. 4:1 B. 1:4 C. 1:5 D. 5:1
- Q25.** In what ratio must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price?
A. 1:6 B. 6:1 C. 2:3 D. 4:3
- Q26.** A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is?
A. 4 % B. $6\frac{1}{4}\%$ C. 20 % D. 25 %
- Q27.** A man purchased, 150 pen at the rate 12 Rs/pen, out of them he sold 50 pen @ 10 % profit, remaining pens should be sold at what profit, if he earns a total profit of 15 %?
A. 4 % B. 17.5 % C. 20 % D. 25 %
- Q28.** A man purchased, 200 pen at the rate Rs. 15/pen, out of them he sold 75 pen @ 5 % loss, remaining pens should be sold at what percent to gain 10% on the whole transaction?
A. 16 % B. 17 % C. 19 % D. 20 %
- Q29.** In a class there are 65 students & 39 Rs is distributed among them in such a way that each boy gets 80 paise and each girl gets 30 paise. Find the number of boys and girls?
A. 39, 26 B. 26, 36 C. 26, 39 D. 25, 35
- Q30.** In a class there are 75 students & 48 Rs is distributed among them in such a way that each boy gets 1 Rs and each girl gets 40 paise. Find the number of boys and girls?
A. 30, 20 B. 20, 30 C. 45, 30 D. 30, 45

Unit 5

PERMUTATION AND COMBINATION

Principal Of Multiplication:

AND suggests the use of Multiplication and shows that more than one operation has to be performed at a time. It also gives the idea that there should be one starting point and one end point.

Multiplication

If an event can occur in m different ways, and following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is $m * n$

Principal Of Addition:

OR suggests the use of Addition and shows that exactly one operation has to be performed at a time out of the given set of all the possible operations.

PERMUTATION

A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

Linear Arrangement

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is $P(n,r)$ kind, and where repetition is not allowed, is

$$= \frac{n!}{p! q! r! \dots}$$

(Where, $p+q+r \dots \leq n$)

Number of permutations of n objects, when all of them are identical = $n!/n!$

Circular Arrangement

Number of ways to arrange n distinct objects on n places around a circle = $(n-1)!$

Number of arrangements of n beads forming a necklace = $(n-1)!/2$

(In case of the necklace or garland, anticlockwise and clockwise arrangements are same)

Number of selection of k consecutive things out of n things in a circle

$$= n, \quad \text{when } k < n$$

$$= 1, \quad \text{when } k = n$$

Polygon Arrangement

Number of ways to arrange n distinct objects along the sides of a r sided regular polygon with every side having n/r objects = $n!/r$

If the polygon is not regular, then the number of arrangements will be

$${}^n P_r = \frac{n!}{(n-r)!} \quad (0 < r < n)$$

Number of permutations of n distinct objects among r different places, where repetition is allowed, is n^r

Number of permutations of n objects in which p objects are alike of one kind, q are alike of second, r are alike of third and so on and remaining are of different

If n people are to be arranged around a rectangular table, such that there are equal number of people on each side of the table, then total number of arrangements will be $n!/2$

Derangement

Number of arrangements of n distinct things in a row, such that none of them occupies its original place is

$$= n! \left[\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n / n! \right]$$

$$\text{Darr.}(2) = 1, \text{Darr.}(3) = 2,$$

COMBINATION

A combination is a selection, in no definite order, of a number of objects taken some or all at a time.

Number of combinations of n distinct objects taken r at a time, where repetition is not allowed, is $C(n, r)$

$$\text{Darr.}(4) = 9, \text{Darr.}(5) = 44$$

Miscellaneous

$${}^nC_r = \frac{n!}{r! (n-r)!} \quad (0 \leq r \leq n)$$

Number of ways 4 different letters can be posted in 7 different letter boxes = 4^7

Number of ways n identical things can be arranged among r different places = r^n

e.g. Number of ways 4 identical rings can be worn in 5 fingers of a hand = 5^4

Number of ways n different things can be arranged among r different places

$$= (n+r-1)/(r-1)!$$

e.g. Number of ways 4 different rings can be worn in 5 fingers of a hand = $5 \cdot 6 \cdot 7 \cdot 8$

Sum of all ' r ' digit numbers formed by using each of the given ' n ' non-zero distinct digits exactly once (no repetition) = (Sum of all the digits) (1111... r times) ${}^nP_{r-1}$

Sum of all ' r ' digit numbers formed by using each of the given ' n ' non-zero distinct digits (with repetition) = (Sum of all the digits) (1111... r times) n^{r-1}

Number of combinations of n distinct objects among r different places, where repetition is allowed, is ${}^{n+r-1}C_r$

Number of combinations or distributions of n identical objects among r different places is ${}^{n+r-1}C_{r-1}$

Also the whole number solutions of Equation, $(x + y + z + \dots (r \text{ variables}) = n)$ $= {}^{n+r-1}C_{r-1}$

Number of combinations or distributions of n identical objects among r different places such that each place gets at least 1 is ${}^{n-1}C_{r-1}$

Also the natural number solutions of Equation, $(x + y + z + \dots (r \text{ variables}) = n)$ $= {}^{n-1}C_{r-1}$

Number of selections out of n distinct objects

$= (\text{Select None}) + (\text{Select One}) + (\text{Select Two})$

$$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$$

Number of ways in which a selection can be made by taking some or all out of $p + q + r + \dots$ things where p are alike of one kind, q alike of second, r alike of third and so on is $(p+1)(q+1)(r+1)\dots - 1$

Number of zero or more selections out of n same objects $= 1 + 1 + 1 + \dots + 1 = n + 1$

Number of one or more selections out of n same objects $= 1 + 1 + 1 + \dots + 1 = n$

Number of lines in a plane formed by n points (where no three points are collinear) $= {}^nC_2$

Number of diagonals in a regular polygon $= {}^nC_2 - n$

Number of triangles formed in a plane using n points (where no three points are collinear) $= {}^nC_3$

Formulae related to Combination

a) ${}^nC_0 = 1 = {}^nC_n$

b) ${}^nC_1 = n = {}^nC_{n-1}$

c) ${}^nC_{n-r} = {}^nC_r$

d) ${}^nC_a = {}^nC_b \Rightarrow a + b = n$

e) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

f) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1} + {}^nC_n = 2^n$

g) ${}^nC_0 + {}^nC_2 + {}^nC_4 + \dots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots = 2^{n-1}$

GROUPING & DISTRIBUTION

Number of ways in which n distinct objects can be distributed equally among r people

$$= n! / p! q! r! \dots \quad (n = p + q + r \dots)$$

Number of ways in which n distinct objects can be distributed equally among r groups

$$= n! / [(n/r)!]^r \text{ (if groups are distinct)}$$

$$= n! / r! [(n/r)!]^r \text{ (if groups are not distinct)}$$

Practice Questions

1. How many 3 digit number can be formed with the digits 5, 6, 2, 3, 7 and 9 which are divisible by 5 and none of its digit is repeated?

- a) 12 b) 16 c) 20 d) 24

2. In how many different ways can the letter of the word ELEPHANT be arranged so that vowels always occur together?

- a) 2060 b) 2160 c) 2260 d) 2360

3. There are 4 bananas, 7 apples and 6 mangoes in a fruit basket. In how many ways can a person make a selection of fruits from the basket.

- a) 269 b) 280 c) 279 d) 256

4. There are 15 points in a plane out of which 6 are collinear. Find the number of lines that can be formed from 15 points.

- a) 105 b) 90 c) 91 d) 95

5. In how many ways 4 Indians, 5 Africans and 7 Japanese be seated in a row so that all person of same nationality sits together

- a) $4! 5! 7! 3!$ b) $4! 5! 7! 5!$ c) $4! 6! 7! 3!$ d) can't be determined

6. In how many ways 5 Americans and 5 Indians be seated along a circular table, so that they are seated in alternative positions

- a) $5! 5!$ b) $6! 4!$ c) $4! 5!$ d) $4! 4!$

7. 4 matches are to be played in a chess tournament. In how many ways can result be decided?

- a) 27 b) 9 c) 81 d) 243

Q(8 –9) There are 6 players in a cricket which is to be sent to Australian tour. The total number of members is 12.

If 2 particular member is always included

- a) 210 b) 270 c) 310 d) 420

If 3 particular player is always excluded

- a) 76 b) 82 c) 84 d) 88

10. In a group of 6 boys and 5 girls, 5 students have to be selected. In how many ways it can be

done so that at least 2 boys are included

- a) 1524 b) 1526 c) 1540 d) 1560

11. How many words of 4 letters with or without meaning be made from the letters of the word 'NUMBER', when repetition of letters is not allowed?

- A) 480 B) 360 C) 240 D) 360

12. In how many ways the letters of the word 'ALLIGATION' be arranged taking all the letters?

- A) 120280 B) 453600 C) 360340 D) 3628800

13. In how many ways all the letters of the word 'MINIMUM' be arranged such that all vowels are together?

- A) 60 B) 30 C) 90 D) 70

14. In how many ways a group of 4 men and 3 women be made out of a total of 8 men and 5 women?

- A) 720 B) 140 C) 120 D) 360

15. How many 3 digit numbers are divisible by 4?

- A) 256 B) 225 C) 198 D) 252

16. How many 3 digits numbers have exactly one digit 2 in the number?

- A) 225 B) 240 C) 120 D) 160

17. There are 8 men and 7 women. In how many ways a group of 5 people can be made such that the particular woman is always to be included?

- A) 860 B) 1262 C) 1001 D) 1768

18. There are 6 men and 7 women. In how many ways a committee of 4 members can be made such that a particular man is always to be excluded?

- A) 280 B) 420 C) 220 D) 495

19. How many 4 digit words can be made from the digits 7, 8, 5, 0, and 4 without repetition?

- A) 70 b) 96 c) 84 d) 48

20. In how many ways 8 students can be given 3 prizes such that no student receives more than 1 prize?

- A) 348 B) 284 C) 224 D) 336

21. A box contains 27 marbles some are blue and others are green. If a marble is drawn at random from the box, the probability that it is blue is $\frac{1}{3}$. Then how many number of green marbles in the box?

- A. 10 b) 15 c) 14 d) 18

22. In how many ways can 3 prizes be given away to 12 students when each student is eligible for all the prizes?

A.1234 B.1728 C.5314
D.1331

23. Total no of ways in which 30 sweets can be distributed among 6 persons?

A. $35C5$ B. $36C5$ C. $36C6$ D. $35!/5!$

24. A bag contains 4 red balls and 5 black balls. In how many ways can I make a selection so as to take at least 1 red ball and 1 black ball?

A.564 B.345 C.465 D.240

25. In how many ways can 7 beads be strung into a necklace?

A.2520 B.5040 C.720 D.360

26. Find the no of 3 digit numbers such that at least one of the digits is 6 (with repetitions)?

A.252 B.345 C.648 D.560

27. In how many ways can 7 girls and 4 boys stand in a row so that no 2 boys are together?

A.8467200 B.9062700 C.7407000 D.8407200

28. In how many ways the letters of the word PERMUTATION be arranged?

A. $10!/2!$ B. $10!$ C. $11!$ D. $11!/2!$

29. How many numbers can be formed with the digits 1, 7, 2, 5 without repetition?

A.89 B.56 C.64 D.72

30. There are 3 boxes and 6 balls. In how many ways these balls can be distributed if all the balls and all the boxes are different?

A.243 B.512 C.729 D.416

31. In how many ways can 4 books be selected out of 10 books on different subjects?

A.210 B.320 C.716 D.5040

32. In how many ways can 5 boys and 4 girls can be seated in a row so that they are in alternate position.

a) 2780 b) 2880 c) 2800 d) 2980

33. In how many ways 5 African and five Indian can be seated along a circular table, so that they occupy alternate position.

a) $5!5!$ b) $4!5!$ c) $5!4!$ d) $4!4!$

34. There is a meeting of 20 delegates to be held in a hotel. In how many ways these delegates can be seated along a round table, if three particular delegates always seat together.

a) $17!3!$ b) $18!3!$ c) $17!4!$ d) can't be determined

35. In how many 8 prizes can be given to 3 boys, if all boys are equally eligible of

getting the prize.

- a) 512 b) 343 c) 256 d) 526

36. There are 15 points in a plane out of which 6 are collinear. Find the number of lines that can be formed from 15 points.

- a) 105 b) 90 c) 91 d) 95

37. In party there is a total of 120 handshakes. If all the persons shake hand with every other person. Then find the number of persons present in the party.

- a) 15 b) 16 c) 17 d) 18

38. There are 8 boys and 12 girls in a class. 5 students have to be chosen for an educational trip. Find the number of ways in which this can be done if 2 particular girls are always included

- a) 812 b) 816 c) 818 d) 820

39. In how many different ways the letters of the word INSIDE be arranged in such a way that all vowels always come together

- a) 64 b) 72 c) 84 d) 96

40. How many 3 digit numbers can be formed by 0, 2, 5, 3, 7 which is divisible by 5 and none of the digits is repeated.

- a) 24 b) 36 c) 48 d) 60

PROBABILITY

Probability or chance is a common term used in day-to-day life. For example, we generally say, 'it may rain today'. This statement has a certain uncertainty.

Probability is a quantitative measure of the chance of occurrence of a particular event.

If all the possible outcomes of an experiment are known but the exact output cannot be predicted in advance, that experiment is called a random experiment.

Examples

Tossing of a fair coin

When we toss a coin, the outcome will be either Head (H) or Tail (T)

Throwing an unbiased die

Die is a small cube used in games. It has six faces and each of the six faces shows a different number of dots from 1 to 6. Plural of die is dice.

When a die is thrown or rolled, the outcome is the number that appears on its upper face and it is a random integer from one to six, each value being equally likely.

Drawing a card from a pack of shuffled cards

A pack or deck of playing cards has 52 cards which are divided into four categories as given below

Spades (♠) Clubs (♣)

Hearts (♥) Diamonds (♦)

Each of the above mentioned categories has 13 cards, 9 cards numbered from 2 to 10, an Ace, a King, a Queen and a Jack

Hearts and Diamonds are red faced cards whereas Spades and Clubs are black faced cards.

Kings, Queens and Jacks are called face cards

Taking a ball randomly from a bag containing balls of different colours

Sample Space

Sample Space is the set of all possible outcomes of an experiment. It is denoted by S .

Examples

When a coin is tossed, $S = \{H, T\}$ where H = Head and T = Tail

When a dice is thrown, $S = \{1, 2, 3, 4, 5, 6\}$

When two coins are tossed, $S = \{HH, HT, TH, TT\}$ where H = Head and T = Tail

Events are said to be equally likely if there is no preference for a particular event over the other.

Examples

When a coin is tossed, Head (H) or Tail is equally likely to occur.

When a dice is thrown, all the six faces (1, 2, 3, 4, 5, 6) are equally likely to occur.

Two or more than two events are said to be mutually exclusive if the occurrence of one of the events excludes the occurrence of the other

This can be better illustrated with the following examples

When a coin is tossed, we get either Head or Tail. Head and Tail cannot come simultaneously. Hence occurrence of Head and Tail are mutually exclusive events.

When a die is rolled, we get 1 or 2 or 3 or 4 or 5 or 6. All these faces cannot come simultaneously. Hence occurrences of particular faces when rolling a die are mutually exclusive events.

Note : If A and B are mutually exclusive events, $A \cap B = \emptyset$ where \emptyset represents empty set.

Consider a die is thrown and A be the event of getting 2 or 4 or 6 and B be the event of getting 4 or 5 or 6. Then

$A = \{2, 4, 6\}$ and $B = \{4, 5, 6\}$

Here $A \cap B \neq \emptyset$. Hence A and B are not mutually exclusive events.

Events can be said to be independent if the occurrence or non-occurrence of one event does not influence the occurrence or non-occurrence of the other.

Example : When a coin is tossed twice, the event of getting Tail(T) in the first toss and the event of getting Tail(T) in the second toss are independent events. This is because the occurrence of getting Tail(T) in any toss does not influence the occurrence of getting Tail(T) in the other toss.

Exhaustive Event is the total number of all possible outcomes of an experiment.

Examples

When a coin is tossed, we get either Head or Tail. Hence there are 2 exhaustive events.

When two coins are tossed, the possible outcomes are (H, H), (H, T), (T, H), (T, T). Hence there are 4 ($=2^2$) exhaustive events.

When a dice is thrown, we get 1 or 2 or

3 or 4 or 5 or 6. Hence there are 6 exhaustive events.

Let A and B are two events with sample space S. Then

$A \cup B$ is the event that either A or B or Both occur. (i.e., at least one of A or B occurs)

$A \cap B$ is the event that both A and B occur

Let E be an event and S be the sample space. Then probability of the event E can be defined as

$$P(E) = n(E)/n(S)$$

where $P(E)$ = Probability of the event E, $n(E)$ = number of ways in which the event can occur and $n(S)$ = Total number of outcomes possible

$$P(S) = 1$$

$$0 \leq P(E) \leq 1$$

$$P(\varphi) = 0$$

Addition theorem

Let A and B be two events associated with a random experiment. Then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$ because for mutually exclusive events, $P(A \cap B) = 0$

If A and B are two independent events, then $P(A \cap B) = P(A).P(B)$

Let A be any event and A^c be its complementary event (i.e., A^c is the event that A does not occur). Then $P(A^c) = 1 - P(A)$

Let E be an event associated with a random experiment. Let x outcomes are favourable to E and y outcomes are not favourable to E, then

Odds in favour of E are x:y, i.e., x/y and Odds against E are y:x, i.e., y/x

$$P(E) = x/x+y \quad P(E^c) = y/x+y$$

Practice questions

1. A bag contains 5 red balls and 7 blue balls. Two balls are drawn at random without replacement, and then find the probability of that one is red and other is blue.
a) $33/65$ b) $35/66$ c) $37/66$ d) $41/65$
2. A bag contains 3 red balls and 8 black balls and another bag contains 5 red balls and 7 black balls, one ball is drawn at random from either of the bag, find the probability that the ball is red.
a) $93/264$ b) $95/264$ c) $91/264$ d) $97/264$
3. 12 persons are seated at a circular table. Find the probability that 3 particular persons always seated together.
a) $9/55$ b) $7/55$ c) $4/55$ d) $3/55$
4. P and Q are two friends standing in a circular arrangement with 10 more people. Find the probability that exactly 3 persons are seated between P and Q.
a) $5/11$ b) $4/11$ c) $2/11$ d) $3/11$
5. A basket contains 5 black and 8 yellow balls. Four balls are drawn at random and not replaced. What is the probability that they are of different colours alternatively.
a) $56/429$ b) $57/429$ c) $61/429$ d) $68/429$
- Direction (Q6 – Q8):
A bag contains 6 red balls and 8 green balls. Two balls are drawn at random one after one with replacement.
6. What is the probability that Both the balls are green
a) $13/49$ b) $15/49$ c) $16/49$ d) $17/49$
7. First one is green and second one is red
a) $16/49$ b) $14/49$ c) $11/49$ d) $12/49$
8. Both the balls are red
a) $14/49$ b) $9/49$ c) $11/49$ d) $12/49$
9. Find the probability that in a leap year, the numbers of Mondays are 53?
a) $1/7$ b) $2/7$ c) $3/7$ d) $4/7$
10. A urn contains 4 red balls, 5 green balls and 6 white balls, if one ball is drawn at random, find the probability that it is neither red nor white.
a) $1/3$ b) $\frac{1}{4}$ c) $1/5$ d) $2/3$
11. A six-digit is to be formed from the given numbers 1, 2, 3, 4, 5 and 6. Find the probability that the number is divisible by 4.
a) $3/17$ b) $4/15$ c) $4/19$ d) $4/17$
12. A bag contains 6 red balls and 7 white balls. Another bag contains 5 red balls and 3 white balls. One ball is selected from each. Find the probability that one ball is red and one is white?
a) $53/104$ b) $47/104$ c) $63/104$ d) $51/104$
13. A lottery is organized by the college ABC through which they will provide scholarship of rupees one lakhs to only one student. There are 100 fourth year students, 150 third year students, 200 second year students and 250 first year students. What is the probability that a second year student is chosen.
a) $1/7$ b) $2/7$ c) $3/7$ d) $4/7$
14. A card is drawn from a pack of 52 cards. The card is drawn at random; find the probability that it is neither club nor queen?

- a) $\frac{4}{13}$ b) $\frac{5}{13}$ c) $\frac{7}{13}$ d) $\frac{9}{13}$

15. A box contains 50 balls, numbered from 1 to 50. If three balls are drawn at random with replacement. What is the probability that sum of the numbers are odd?

- a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{7}$ d) $\frac{1}{5}$

16. From a pack of cards, if three cards are drawn at random one after the other with replacement, find the probability that one is ace, one is jack and one is queen?

- a) $\frac{16}{7725}$ b) $\frac{16}{5525}$ c) $\frac{18}{5524}$ d) $\frac{64}{5515}$

17. A and B are two persons sitting in a circular arrangement with 8 other persons. Find the probability that both A and B sit together.

- a) $\frac{1}{9}$ b) $\frac{2}{7}$ c) $\frac{2}{9}$ d) $\frac{2}{5}$

18. Find the probability that in a random arrangement of the letters of words in the word 'PROBABILITY' the two I's come together.

- a) $\frac{2}{11}$ b) $\frac{1}{11}$ c) $\frac{3}{11}$ d) $\frac{4}{11}$

19. In a race of 12 cars, the probability that car A will win is $\frac{1}{5}$ and of car B is $\frac{1}{6}$ and that of car C is $\frac{1}{3}$. Find the probability that only one of them won the race.

- a) $\frac{2}{7}$ b) $\frac{7}{10}$ c) $\frac{9}{10}$ d) $\frac{3}{7}$

20. A bag contains 3 red balls and 8 black balls and another bag contains 5 red balls and 7 black balls, one ball is drawn at random from either of the bags, find the probability that the ball is red.

- a) $\frac{93}{264}$ b) $\frac{95}{264}$ c) $\frac{91}{264}$ d) $\frac{97}{264}$

21. In a bag there are 4 white, 4 red and 2 green balls. Two balls are drawn at random. What is the probability that at least one ball is of red colour?

- A. $\frac{4}{3}$ B. $\frac{7}{3}$ C. $\frac{1}{3}$ D. $\frac{2}{3}$

22. Sahil has two bags (A & B) that contain green and blue balls. In the Bag 'A' there are 6 green and 8 blue balls and in the Bag 'B' there are 6 green and 6 blue balls. One ball is drawn out from any of these two bags. What is the probability that the ball drawn is blue?

- A. $\frac{15}{28}$ B. $\frac{13}{28}$ C. $\frac{17}{28}$ D. $\frac{23}{28}$

23. In an examination, there are three sections namely Reasoning, Maths and English. Reasoning part contains 4 questions. There are 5 questions in maths section and 6 questions in English section. If three questions are selected randomly from the list of questions then what is the probability that all of them are from maths?

- A. $\frac{7}{91}$ B. $\frac{8}{91}$ C. $\frac{2}{91}$ D. $\frac{4}{91}$

24. A basket contains 5 red 4 blue 3 green marbles. If three marbles are picked up at random, What is the probability that either all are green or all are red?

- A. $\frac{1}{20}$ B. $\frac{7}{20}$ C. $\frac{3}{20}$ D. $\frac{9}{20}$

25. A basket contains 5 red 4 blue 3 green marbles. If three marbles are picked up at random, What is the probability that at least one is blue?

- A. $\frac{41}{55}$ B. $\frac{53}{55}$ C. $\frac{47}{55}$ D. $\frac{49}{55}$

26. A basket contains 5 red 4 blue 3 green marbles. If two marbles are picked up at random, What is the probability that both are red?

- A. $\frac{4}{33}$ B. $\frac{5}{33}$ C. $\frac{7}{33}$ D. $\frac{8}{33}$

27. A bag contains 5 red caps, 4 blue caps, 3 yellow caps and 2 green caps. If three caps are picked at random, what is the probability that two are red and one is green?

- A. $\frac{22}{55}$ B. $\frac{15}{81}$ C. $\frac{10}{91}$ D. $\frac{5}{91}$

28. A bag contains 5 red caps, 4 blue caps, 3 yellow caps and 2 green caps. If four caps are picked at random, what is the probability that two are red, one is blue and one is green?

- A. $\frac{22}{1001}$ B. $\frac{80}{1001}$ C. $\frac{21}{1001}$ D. $\frac{55}{1001}$

29. A bag contains 2 red caps, 4 blue caps, 3 yellow caps and 5 green caps. If three caps are picked at random, what is the probability that none is green?

- A. $\frac{2}{13}$ B. $\frac{3}{13}$ C. $\frac{1}{13}$ D. $\frac{5}{13}$

30. A bag contains 5 red and 7 white balls. Four balls are drawn out one by one and not replaced. What is the probability that they are alternatively of different colours?

- a) $\frac{7}{99}$ b) $\frac{11}{99}$ c) $\frac{14}{99}$ d) $\frac{19}{99}$

31. P and Q are sitting in a ring with 11 other persons. If the arrangement of 11 persons is at random, then the probability that there are exactly 4 persons between them?

- a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{5}$ d) $\frac{1}{6}$

32. 10 persons are seated around a round table. What is the probability that 4 particular persons are always seated together?

- a) $\frac{1}{21}$ b) $\frac{4}{21}$ c) $\frac{8}{21}$ d) $\frac{11}{21}$

33. A box contains 4 red, 5 black and 6 green balls. 3 balls are drawn at random. What is the probability that all the balls are of same colour?

- a) $\frac{33}{455}$ b) $\frac{34}{455}$ c) $\frac{44}{455}$ d) $\frac{47}{455}$

34. An apartment has 8 floors. An elevator starts with 4 passengers and stops at 8 floors of the apartment. What is the probability that all passengers travel to different floors?

- a) $\frac{109}{256}$ b) $\frac{135}{256}$ c) $\frac{105}{256}$ d) $\frac{95}{256}$

35. A speaks truth in 60% cases and B in 80% cases. In what percent of cases they likely to contradict each other narrating the same incident?

- a) $\frac{9}{25}$ b) $\frac{7}{25}$ c) $\frac{11}{25}$ d) $\frac{13}{25}$

36. A box contains 30 electric bulbs, out of which 8 are defective. Four bulbs are chosen at random from this box. Find the probability that at least one of them is defective?

- a) $\frac{432}{783}$ b) $\frac{574}{783}$ c) $\frac{209}{784}$ d) $\frac{334}{784}$

37. Two persons A and B appear in an interview. The probability of A's selection is $\frac{1}{5}$ and the probability of B's selection is $\frac{2}{7}$. What is the probability that only one of them is selected?

- a) $\frac{11}{35}$ b) $\frac{12}{35}$ c) $\frac{13}{35}$ d) $\frac{17}{35}$

38. A 4-digit number is formed by the digits 0, 1, 2, 5 and 8 without repetition. Find the probability that the number is divisible by 5.

- a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

39. A bag contains 6 red balls and 8 green balls. 2 balls are drawn at random one by one with replacement. Find the probability that both the balls are green.

- a) $\frac{16}{49}$ b) $\frac{25}{49}$ c) $\frac{12}{49}$ d) $\frac{21}{49}$

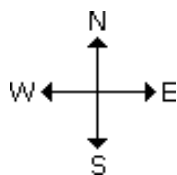
40. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts. Find the Probability of the lost card being a heart?

- A. $\frac{12}{50}$ B. $\frac{8}{50}$ C. $\frac{11}{50}$ D. $\frac{9}{50}$

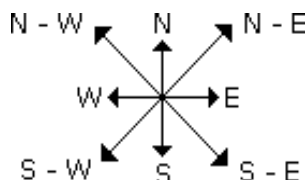
UNIT 6

DIRECTION SENSE TEST

1. There are four main directions - East, West, North and South as shown below:



2. There are four cardinal directions - North-East (N-E), North-West (N-W), South-East (S-E), and South-West (S-W) as shown below:



3. At the time of sunrise if a man stands facing the east, his shadow will be towards west.
4. At the time of sunset the shadow of an object is always in the east.
5. If a man stands facing the North, at the time of sunrise his shadow will be towards his left and at the time of sunset it will be towards his right.
6. At 12:00 noon, the rays of the sun are vertically downward hence there will be no shadow.
7. Left Right Movement:-
- ✚ A person facing north, on taking left will face towards west and on taking the right turn towards east. ✚ A person facing west, on taking left will face towards south and on taking right turn towards north.
 - ✚ A person facing east, on taking left will face towards north and on taking the right turn towards south. ✚ A person facing south, on taking left will face towards east and on taking the right turn towards west. ✚
- Whenever a person moves to his left side, he will move towards anti- clockwise direction.
- ✚ Whenever a person moves to his right side, he will move towards clockwise direction.
8. When a question says moved towards left or right side, we assume that the movement is at an angle of 90degrees.

Q11. Raju moved to his North- West side for 2 km. From there he turned 90 degrees clockwise & moved 2 km. From there he turned 90 degrees clockwise & travelled 2km, then he would be in which direction from the original position?

- A. South East Region B. North East Region C. South West Region D. Western Region

Q12. Ravi started walking from his house east direction on Bus stop which is 3km away. Then he set off in the bus straight towards his right to the school 4 km away. What is the crow flight distance from his house to the school?

- A. 1 km B. 5 km C. 7 km D. 12 km

Q13. Debu walks towards East then towards North and turning 45° right walks for a while and lastly turns towards left. In which direction is he walking now?

- A. North B. East C. South-East D. North-West

Q14. Suman is 40 metres South-West of Ashok. Prakash is 40 meters South-East of Ashok. Prakash is in which direction of Suman?

- A. South B. West C. East D. North-East

Q15. Mohan started from point 'A' and proceeded 7 km straight towards East, then he turned left and proceeded straight for a distance of 10 km. He then turned left again and proceeded straight for a distance of 6 km, and then turned left again and proceeded straight for another 10 km. In which direction is Mohan from his starting point?

- A. East B. West C. North D. South

Q16. One evening before sunset Rekha and Hema were talking to each other face to face. If Hema's shadow was exactly to the right of Hema, which direction was Rekha facing?

- A. North B. South C. West D. Data Inadequate

Q17. K is 40 m South-West of L. If M is 40 m South-East of L, then M is in which direction of K?

- A. East B. West C. North-East D. South

Q18. A is east of B and west of C. H is south-west of C, B is south-east of X. Which is the farthest west?

- A. A B. B C. C D. X

Q19. Rahul put his timepiece on the table in such a way that at 6 p.m. hour-hand points to North. In which direction the minute-hand will point at 9.15 p.m.?

- A. South-East B. South C. North D. West

Q20. P started from his house towards west. After walking a distance of 25 m, he turned to the right and walked 10 m. He then again turned to the right and walked 15 m. After this he is to turn right at 135 degree and to cover 30 m. In which direction should he go?

- A. West B. South C. South-West D. South-East

Q21. A boy rode his bicycle northward, then turned left and rode 1 km and again turned left and rode 2 km. He found himself 1 km west of his starting point. How far did he ride northward initially?

- A. 1 Km B. 2 Km C. 3 Km D. 5 Km

Q22. Starting from the point X, Jai walked 15 m towards west. He turned left and walked 20 m. He then turned left and walked 15 m. After this he turned to his right and walked 12 m. How far and in which directions is now Jai from X?

- A. 32 m, South B. 47 m, East C. 42 m, North D. 27 m, South

Q23. Two cars start from the opposite places of a main road, 150 km apart. First car runs for 25 km and takes a right turn and then runs 15 km. It then turns left and then runs for another 25 km and then takes the direction back to reach the main road. In the mean time, due to minor break down the other car has run only 35 km along the main road. What would be the distance between two cars at this point?

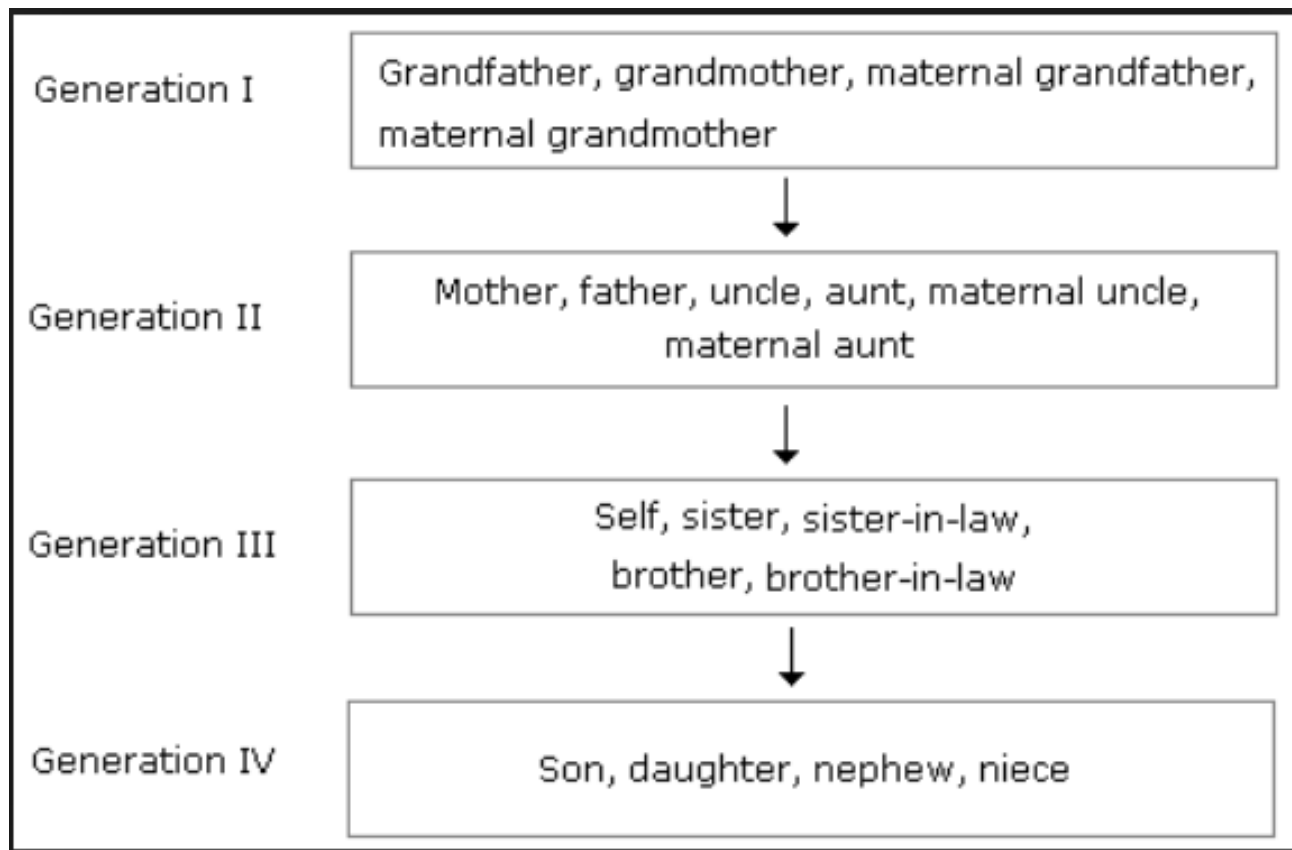
- A. 65 Km B. 75 Km C. 80 Km D. 85 Km

Q24. Rajat walked 20 m towards north. Then he turned right and walks 30 m. Then he turns right and walks 35 m. Then he turns left and walks 15 m. Finally he turns left and walks 15 m. In which direction and how many metres is he from the starting position?

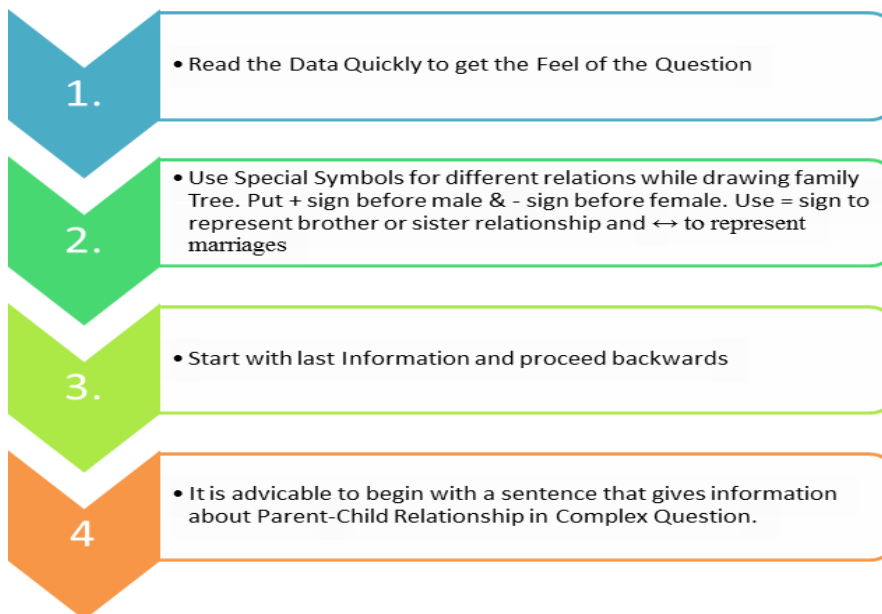
- A. 15 m West B. 30 m East C. 30 m West D. 45 m East

BLOOD RELATION TEST

COMMON RELATIONS-



How To Solve Questions:



PRACTICE EXERCISE

Q1. Pointing to a photograph, a man said, "I have no brother or sister but that man's father is my father'sson." Whose photograph was it?

- A. His own B. His son C. His Father D. His Grandfather

Q2. Pointing to a man, a woman said, "His mother is the only daughter of my mother." How is the woman related to the man?

- A. Mother B. Daughter C. Sister D. Brother

Q3. Pointing to the photograph, Vipul said, "She is the daughter of my grandfather's only son." How is Vipul related to the girl in the photograph?

- A. Father B. Sister C. Brother D. Son

Q4. Pointing to a girl in photograph. Amar said, "Her mother's brother is the only son of my mother's father." How the girl's mother related to Amar?

- A. Mother B. Sister C. Aunt D. Father

Q5. Pointing to a gentleman, Deepak said, "His only brother is the father of my daughter's father." How is gentleman related to Deepak?

- A. Brother B. Sister C. Father D. Uncle

Q6. If Kamal says, "Ravi's mother is the only daughter of my mother", how is Kamal related to Ravi?

- A. Brother B. Sister C. Maternal Uncle D. Aunt

Q7. A's father is B's son-in-law. C, A's sister, is the daughter of P. How is P related to B?

- A. Brother B. Sister C. Mother D. Can't be determined

Q8. Divyansh said to Nimish, "The boy playing with the football is the younger of the two brothers of the daughter of my father's wife." How is the boy playing football related to Divyansh?

- A. Cousin B. Brother C. Son D. Brother-in-law

Q9. B is the brother of A, S is the sister of B, E is the brother of D, D is the daughter of A, F is the father of S. Then, the uncle of E is?

- A. A B. F C. B D. D

Q10. R is the brother of G. Q is the sister of R. O is the brother of N. N is the daughter of G. L is the father of Q, who is the uncle of O?

- A. R B. L C. G D. Q

Q11. Pointing to Sagar in a photograph, Manjula said, "His brother's father is the only son of my grandfather. How is Manjula related to Sagar?

- A. Aunt B. Sister C. Mother D. None of these

Q12. Sia introduced Raghav as the son of the only daughter of the father of her uncle. How is Raghav related to Sia?

- A. Brother B. Cousin C. Nephew D. Can't be determined

Q13. Introducing a woman, Nisha said, 'She is the daughter-in-law of the grandmother of my father's only son.' How is the woman related to Nisha?

- A. Grandmother B. Sister-in-law C. Sister D. Mother

Q14. A man said to a lady, "Your mother's husband's sister is my aunt". How is that lady related to that man?

- A. Daughter B. Sister C. Grand-daughter D. Mother

Q15. Anupam said to a lady sitting in a car, "The only daughter of the brother of my wife is the sister-in-law of the brother of your sister." How the husband of the lady is related to Anupam?

- A. Maternal Uncle B. Uncle C. Father D. Son-In-Law

Q16. Pointing to Varman, Madhav said, "I am the only son of one of the sons of his father." How is Varman related to Madhav?

- A. Nephew B. Uncle C. Father or Uncle D. Father

Q17. Pointing to Gopi, Nalni Says, "I am the daughter of the only son of his grandfather." How Nalni is related to Gopi?

- A. Niece B. Daughter C. Sister D. Indeterminable

Q18. Introducing a woman, Shashank said, "She is the mother of the only daughter of my son." How that woman is related to Shashank?

- A. Daughter B. Sister-in-law C. Wife D. Daughter-in-law

Q19. A man introduced the boy coming with him as "He is son of the father of my wife's daughter". What relation did the boy bear to the man?

- A. Son-in-law B. Son C. Brother D. Father

Q20. If B says that his mother is the only daughter of A's mother, how is A related to B?

- A. Son B. Father C. Brother D. Uncle

Q21. Veena who is the sister-in-law of Ashok, is the daughter-in-law of Kalyani. Dheeraj is the father of Sudeep who is the only brother of Ashok. How Kalyani is related to Ashok?

- A. Mother-in-law B. Aunt C. Wife D. Mother

Q22. If $A + B$ means A is the mother of B; $A - B$ means A is the brother of B; $A \% B$ means A is the father of B and $A \times B$ means A is the sister of B, which of the following shows that P is the maternal uncle of Q?

- A. $Q - N + M \times P$ B. $P + S \times N - Q$ C. $P - M + N \times Q$ D. $Q - S \% P$

Q23. If $A + B$ means A is the brother of B; $A \times B$ means A is the son of B; and $A \% B$ means B is the daughter of A then which of the following means M is the maternal uncle of N?

- A. $M + O \times N$ B. $M \% O \times N + P$ C. $M + O \% N$ D. None of these

Q24. If $A + B$ means A is the father of B; $A - B$ means A is the brother of B; $A \% B$ means A is the wife of B and $A \times B$ means A is the mother of B, which of the following shows that M is the maternal grandmother of T?

- A. $M \times N \% S + T$ B. $M \times N - S \% T$ C. $M \times S - N \% T$ D. $M \times N \times S \% T$

Q25. If D is the brother of B, how B is related to C? To answer this question which of the statements is/are necessary?

1. The son of D is the grandson of C.

2. B is the sister of D.

- A. Only 1 B. Only 2 C. Either 1 or 2 D. 1 and 2 both are required

Q26. Pointing to Sahil, Neeru says, "I am the daughter of the only son of his grandfather." How Neeru is related to Sahil?

- A. Daughter B. Mother C. Sister D. Cousin