Shivam Gulia 200106065

Unit - 4

solution 1

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

let B be inverse matrix

$$AB = I$$

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

solution - 2

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

let B be inverse of matrix A

$$AB = I$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 - 2R_1$$

$$R_3 \to R_3 - 3R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -1 \\ 0 & -1 & -3 \end{bmatrix} B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$C_2 \to C_2 - 2C_1 \quad , \quad C_3 \to C_3 - 3C_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & -3 \end{bmatrix} B = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 5 & 6 \\ -3 & 6 & 10 \end{bmatrix}$$

$$R_2 \to R_2 - R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & -3 \end{bmatrix} B = \begin{bmatrix} 1 & -2 & -3 \\ 1 & -1 & -4 \\ -3 & 6 & 10 \end{bmatrix}$$

$$R_3 \to R_3 + R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} B = \begin{bmatrix} 1 & -2 & -3 \\ 1 & -1 & -4 \\ -2 & 5 & 6 \end{bmatrix}$$

$$R_2 \to R_2 + 2R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} B = \begin{bmatrix} 1 & -2 & -3 \\ -3 & 9 & 8 \\ -2 & 5 & 6 \end{bmatrix}$$

$$R_3 \to (-1) R_3$$

$$B = \begin{bmatrix} 1 & -2 & -3 \\ -3 & 9 & 8 \\ 2 & -5 & -6 \end{bmatrix}$$

solution 3

unitary Matrix $A^{\theta}A = I$

$A^{\theta} = (\bar{A})^{T}$
conjugate's Impose

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

$$\bar{A} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}$$

$$A^{\theta} = (\bar{A})^{T} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

$$A.A^{\theta} = \frac{1}{3}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

hence proved

$A^{\theta}.A = I$ unitary Matrix

solution 4

$$A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$$

$$A^{\theta} = (\bar{A})^{T} = \begin{bmatrix} \alpha - i\gamma & -\beta - i\delta \\ \beta - i\delta & \alpha + i\gamma \end{bmatrix}^{T}$$

$$= \begin{bmatrix} \alpha - i\gamma & \beta - i\delta \\ -\beta - i\delta & \alpha + i\gamma \end{bmatrix}$$

$$A.A^{\theta} = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}\begin{bmatrix} \alpha - i\gamma & \beta - i\delta \\ -\beta - i\delta & \alpha + i\gamma \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \gamma^2 + \beta^2 + \delta^2 & (\alpha + i\gamma)(\beta - i\delta) + (\alpha + i\gamma)(-\beta + i\delta) \\ (\beta + i\delta)(\alpha - i\gamma) + (\alpha - i\gamma)(-\beta - i\delta) & \alpha^2 + \beta^2 + \gamma^2 + \delta^2 \end{bmatrix}$$

since A is unitary matrix

$$A A^{\theta} = I$$

$$\alpha^{4} + y^{2} + \beta^{2} + \delta^{2} = 1$$

Solution 5-

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Cayley Hamilton Theory

↓

matrix itself satisfies its characteristic eqn

Characteristic eqn of matrix eqn A

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[(2-\lambda)^2 - 1] + 1[(\lambda-2)+1] + 1[1-(2-\lambda)] = 0$$

$$(2-\lambda)[4+\lambda^2-4\lambda+1] + (\lambda-1] + (\lambda-1] = 0$$

$$(2\lambda^2 - 8\lambda + 6) - \lambda^3 + 4\lambda^2 - 3\lambda + 2\lambda - 2 = 0$$

$$-\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$$

$$\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$$

$$A^3 - 6A^2 + 9A - 4 = 0 \quad -(1)$$

Acc to Cayley-Hamilton

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Putting values in eqn (1)

$$\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 22-36+18-4 & -22+30-9 & 21-30+9 \\ -21+30+9 & 22+36+18-4 & -21+30-9 \\ 21-30+9 & -21+30-9 & 22-36+18-4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

hence proved

solution - 6

$$(A/b) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 0 & 0 & \lambda-3 & \mu \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

(i) no solution $\to b \neq 0$

$R(A/b) \neq R(A)$

$\mu - 10 \neq 0$

$\lambda - 3 = 0$

$\lambda = 3$ & $\mu \neq 10$

(ii) unique solution $\to b \neq 0$

$R(A/b) = R(A) = n$

$\lambda - 3 \neq 0$

since $n = 3$

$\mu - 10 \neq 0$

$\lambda \neq 3$ & $\mu \neq 10$

(iii) Infinite solⁿ $\longrightarrow b \neq 0$

$R(A/b) = R(A) < n$

$\lambda - 3 = 0$

$\mu - 10 = 0$

$\lambda = 3 \text{ \& } \mu = 10$

solution 6-(a)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank (A) = 2

solution 7

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 2 & 10 & 1 \end{bmatrix}$$

$C_3 \to C_3 - 2C_1$

$C_4 \to C_4 - C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 2 & 10 & 1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$R_4 \to R_4 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 14 & -1 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 14 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_3 \to C_3 + 2C_2$
$C_4 \to C_4 - C_2$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 14 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

<u>Rank $(A) = 3$</u>

$R_3 \to R_3/14$ $\qquad C_4 \to C_4 + \frac{1}{14} C_3$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/14 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

<u>Solution</u> – (3)

$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$

$A = I_n A I_n$

$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 \to R_2 - R_1$

$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_3 \to R_3 + R_2$

$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3/2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3/2$ $\qquad R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & -1 & -1/2 \\ -1 & 1 & -1/2 \\ 0 & 0 & 1/2 \end{bmatrix}}_{P} A \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{Q}$$

Solution - 9

Echelon ↓ Row only

$$A = \begin{bmatrix} 1 & -2 & 1 & -1 \\ 1 & 1 & -2 & 3 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 4R_1$
$R_4 \to R_4 - 5R_1$

$$\begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 3 & -3 & 4 \\ 0 & 9 & -9 & 12 \\ 0 & 3 & -3 & 4 \end{bmatrix}$$

$R_4 \to R_4 - R_2$
$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 3 & -3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank $(A) = 2$

Solution - 10

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$$ Rank $(A) = 2$

$R_3 \to R_3 - 2R_1 - R_2$

$$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b-2 & 0 & 0 \end{bmatrix}$$

R_3 must be 0 $\qquad b - 2 = 0$

$b = 2$

solution - 11

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -2 \\ 2 & 4 & 7 \end{bmatrix} \quad b = \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = 1(7+8) - 1(21+4) + 1(12-2)$$

$$= 15 - 25 + 0$$

$$= 0$$

since, $|A| = 0$ solution is in consistant

$b \neq 0$

$$(A|b) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 3 & 1 & -2 & -2 \\ 2 & 4 & 7 & 7 \end{array}\right]$$

$R_3 \to R_3 - 2R_1$, $R_2 \to R_2 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 2 & 5 & 13 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 0 & 0 & 6 \end{array}\right]$$

$R(A|b) \neq R(A)$

$\to$ No Sol^n

solution - 12

$3x + 3y + 2z = 1$

$x + 2y = 4$

$10y + 3z = -2$

$2x - 3y - z = 5$

$$(A|b) = \left[\begin{array}{ccc|c} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 3 & 3 & 2 & 1 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{array}\right]$$

solution -13

$2x + 3y + 5z = 9$
$7x + 3y - 2z = 8$
$2x + 3y + \lambda z = \mu$

(i) unique solⁿ

$R(A|b) = R(A) = n$

(ii) no solⁿ

$R(A|b) \neq R(A)$

$$(A|b) = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$= \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

Case I Unique solⁿ

$R(A|b) = R(A) = n$ $\quad$ $n = 3$ $\quad$ unknows

$\lambda - 5 \neq 0$
$\lambda \neq 5$

Case II no solⁿ

$\lambda = 5$ $\qquad$ $\mu - 9 \neq 0$
$\mu \neq 9$

solution-14

$2x + y + 2z = 0$
$x + y + 3z = 0$
$4x + 3y + \lambda z = 0$

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & \lambda \end{bmatrix}$$

for non-zero solⁿ $|A| \neq 0$

$2(\lambda - 9) - 1(\lambda - 12) + 2(3 - 4) = 0$

$\lambda - 18 + 12 - 2 = 0$

$\lambda = 8$ $\qquad \Rightarrow \lambda \neq 8$

solutions-15

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 1 & 6 \\ -1 & 4 & 7 \end{bmatrix}$$

using characteristic eqn

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 4 & 1 \\ 2 & 1-\lambda & 6 \\ -1 & 4 & 7-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)\left[(1-\lambda)(7-\lambda) - 24\right] - 4\left[2(7-\lambda)+6\right] + 1(8+1-\lambda) = 0$$

$$(3-\lambda)\left[7+\lambda^2-8\lambda-24\right] - 4\left[14-2\lambda+6\right] + 9-\lambda = 0$$

$$(3-\lambda)\left[\lambda^2-8\lambda-17\right] - 4\left[20-2\lambda\right] + 9-\lambda = 0$$

$$3\lambda^2 - 24\lambda - 51 - \lambda^3 + 8\lambda^2 + 17\lambda - 8(10-\lambda) + 9 - \lambda = 0$$

$$-\lambda^3 + 11\lambda^2 + \lambda - 122 = 0$$

$$\lambda^3 - 11\lambda^2 - \lambda + 122 = 0$$

By cayley-Hamilton

$$A^3 - 11A^2 - A + 122 = 0$$

$$A^2 - 11A - I + 122A^{-1} = 0$$

$$A^{-1} = \frac{1}{122}\left(-11A - A^2 + I\right)$$

$$A^2 = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 1 & 6 \\ -1 & 4 & 7 \end{bmatrix}\begin{bmatrix} 3 & 4 & 1 \\ 2 & 1 & 6 \\ -1 & 4 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 20 & 34 \\ 2 & 33 & 50 \\ -2 & 28 & 72 \end{bmatrix}$$

$$A^{-1} = \frac{1}{122}\left[\begin{bmatrix} 33 & 44 & 11 \\ 22 & 11 & 66 \\ -11 & 44 & 77 \end{bmatrix} - \begin{bmatrix} 16 & 20 & 34 \\ 2 & 33 & 50 \\ -2 & 28 & 72 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{122}\begin{bmatrix} 18 & 24 & -23 \\ 20 & -21 & 16 \\ -9 & 16 & 6 \end{bmatrix}$$

solution - 16

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

characteristic eq^n

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{vmatrix} = (2-\lambda)\left[(1-\lambda)(2-\lambda)\right] - 1[0] + 1(\lambda-1) = 0$$

$$(2-\lambda)\left[\lambda^2 - 3\lambda + 2\right] + \lambda - 1 = 0$$

$$-\lambda^3 + 5\lambda^2 - 7\lambda + 3 = 0$$

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

By Caley - Hamilton theorem

$$A^3 - 5A^2 + 7A - 3 = 0$$

A/Q

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

$$A^5\overset{0}{\cancel{(A^3 - 5A^2 + 7A - 3)}} + A\overset{0}{\cancel{(A^3 - 5A^2 + 7A - 3)}} + A^2 + A + I$$

$$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix}$$

$$\Rightarrow A^2 + A + I$$

$$\Rightarrow \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

solution-17

$|A - \lambda I| = 0$

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \qquad |A - \lambda I| = \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$$

$$(8-\lambda)\left[(7-\lambda)(3-\lambda) - 16\right] + 6\left[-6(3-\lambda) + 8\right] + 2\left[24 - 2(7-\lambda)\right] = 0$$

$$(8-\lambda)\left[\lambda^2 - 10\lambda + 5\right] + 6(6\lambda - 10) + 2(10 + 2\lambda) = 0$$

$$8\lambda^2 - 80\lambda + \cancel{40} - \lambda^3 + 10\lambda^2 - 5\lambda + 40\lambda - \cancel{40} = 0$$

$$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$$

$$\lambda(\lambda^2 - 15\lambda - 3\lambda + 45) = 0$$

$$\lambda\left[\lambda(\lambda - 15) - 3(\lambda - 15)\right] = 0$$

$$\lambda(\lambda - 15)(\lambda - 3) = 0$$

$\lambda = 0, 3, 15$ Eigen values

Case I $\lambda = 0$

Corresponding Eigen vector $(A - 0I)X_1 = 0$

$$\begin{bmatrix} -8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}$$

$$R_1 \leftrightarrow R_3 \quad \begin{bmatrix} 2 & -4 & 3 \\ -6 & 7 & 4 \\ 8 & -6 & 2 \end{bmatrix}$$

$$\begin{array}{l} R_3 \to R_3 - 4R_1 \\ R_2 \to R_2 + 3R_1 \end{array} \quad \begin{bmatrix} 2 & -4 & 3 \\ 0 & -5 & 13 \\ 0 & 10 & -10 \end{bmatrix}$$

$\underline{\underline{\lambda = 3}}$

corresponding $(A - \lambda I)x = 0$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_1 \to \frac{R_3 - R_1}{2}$ $\begin{bmatrix} 1 & -2 & 0 \\ -6 & 4 & -4 \\ 5 & -6 & 2 \end{bmatrix}$

$R_3 \to R_3 - 5R_1$
$R_2 \to R_2 + 6R_1$ $\begin{bmatrix} 1 & -2 & 0 \\ 0 & -8 & -4 \\ 0 & 4 & 2 \end{bmatrix}$

$R_2 \to R_2/-2$ $\begin{bmatrix} 1 & -2 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 2 \end{bmatrix}$

$R_3 \to R_3 - 2R_2$ $\begin{bmatrix} 1 & -2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 - 2x_2 = 0$

$2x_2 + x_3 = 0$

$x_2 = K$

$x_3 = -2x_2$

$= -2K$

$x_1 = 2x_2 = 2K$

$\underline{\underline{\text{Eigen vector}}} \Rightarrow \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

$\underline{\underline{\text{Case } \lambda = 15}}$

$(A - \lambda I)x = 0$

$$\begin{bmatrix} -16 & 2 & -2 \\ 1 & -13 & 1 \\ -1 & 1 & -15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \to R_3 - R_1$ $\begin{bmatrix} 1 & 1 & 15 \\ 1 & -13 & 1 \\ -16 & 2 & -2 \end{bmatrix} \Rightarrow$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 + 16R_1$ $\begin{bmatrix} 1 & 1 & 15 \\ 0 & -14 & -14 \\ 0 & 18 & 238 \end{bmatrix}$

solution - 18

$$A = \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{vmatrix}$$

Characteristic eqn

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$-(\lambda+2)[\lambda(\lambda-1)-12] - 2[-2\lambda-6] - 3[-4+(1-\lambda)] = 0$$

$$-(\lambda+2)[\lambda^2-\lambda-12] + 4\lambda + 12 + 9 + 3\lambda = 0$$

$$(\lambda+2)(\lambda^2-\lambda-12) - 7\lambda - 21 = 0$$

$$\lambda^3 + \lambda^2 - 21\lambda - 45 I = 0$$

eigen value → $\lambda = 5, -3, 1$

Case I $\lambda = 5$ corresponding $(A - \lambda I)X = 0$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$ $\begin{bmatrix} -1 & -2 & -5 \\ 2 & -4 & -6 \\ -7 & 2 & -3 \end{bmatrix} \Rightarrow$ $R_2 \to R_2 + 2R_1$, $R_3 \to R_3 - 7R_1$ $\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix}$

$R_3 \to R_3 + 2R_2$ $\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix}$

$R_2 \to -\frac{1}{8} R_2$, $R_1 \to (-1)R_1$ $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 + 2x_2 + 5x_3 = 0$

$x_2 + 2x_3 = 0$

$x_3 = k$

$x_2 = -2k$

$x_1 = -2x_2 - 5x_3$

$= +4k - 5k$

$= -k$

eigen vector $\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$ for $\lambda = 5$

for $\lambda = -3$ corresponding $(A - \lambda I)X = 0$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_3 \to R_3 + R_1$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 - 3x_3 = 0$$

let $x_3 = k_1$, $x_2 = k_2$

$$x_1 = 3k_1 - 2k_2$$

$$\begin{bmatrix} 3k_1 - 2k_2 \\ k_2 \\ k_1 \end{bmatrix} = \begin{bmatrix} 3k_1 \\ k_2 \\ k_1 \end{bmatrix} + \begin{bmatrix} -2k_2 \\ k_2 \\ k_1 \end{bmatrix}$$

$$= k_1 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

solution-19

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$R_3 \to R_3 - R_1$

$R_1 \to -R_1$

$$\begin{bmatrix} 1 & -2 & 2 \\ 0 & 4 & -2 \\ 0 & -3 & 2 \end{bmatrix}$$

$C_2 \to C_2 + 2C_1$

$C_3 \to C_3 - 2C_1$

$$(-1)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & -3 & 2 \end{bmatrix}$$

$R_2 \to R_2 + R_3$

$$(-1)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 2 \end{bmatrix}$$

$R_3 \to R_3 + 3R_2$

$$-1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

solution - 20

$$A = \begin{bmatrix} 2 & 3-4i \\ 3+4i & 5 \end{bmatrix}$$

for Hermitite matrix $A = A^{\theta}$

$$A^{\theta} = (\bar{A})^{T}$$

$$= \begin{bmatrix} 2 & 3+4i \\ 3-4i & 5 \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 2 & 3-4i \\ 3+4i & 5 \end{bmatrix} = A$$

hence, A is hermitite matrix.