Chapter - 2

Wave Particle Duality and Heisenberg Uncertainty Principle

Contents

Dual Nature of Matter & radiation

Introduction:

The radiation exhibit a wide spectrum of phenomena: namely reflection, refraction, Interference, diffraction, polarization etc. All of these confirm the wave character of the radiation. On the other hand there are a variety of other phenomena too exhibited by the radiation e.g. energy spectrum of a black body, Photo-electric effect, Compton Effect etc. which could not be understood on the basis of Wave character i.e. wave nature of radiation fails to explain these. This was definitely the down fall of the classical theory or wave theory of the radiation which put a question mark on it. This setback to the wave theory certainly forced scientists to develop some other formulation which could also answer to these phenomena that could not get explanation on the basis of wave theory. This was the time when a need was realized to reinvestigate the character of the radiation and of the matter as well. However the above phenomena could be explained on assuming the radiation exhibiting particle-like and applying Planck's Quantization concept of the energy. We have realized that it is not the radiation only which sometimes appear to behave like wave and sometimes exhibit particle like characteristics; the matter also presents itself both ways. This is known as wave particle duality of radiation.

2.1 de-Broglie Hypothesis: Matter Wave Concept or Wave particle duality

Louis de Broglie proposed in 1924 that like radiation matter also consist dual characteri.e. wave and particle nature. According to de Broglie '**A wave is associated with a moving material body or a moving material body present itself wave-like similar to the radiation.** This hypothetical wave was named as 'matter wave' or **de-Broglie wave** and the wave length of the wave associated was named as **de Broglie wavelength**. This duality concept of matter and radiation was the part of his Ph.D. thesis of Louis de Broglie. For this remarkable discovery Louis de Broglie received Nobel Prize just only after Four years i.e. in 1929.

In the beginning it was not having any experimental evidence. But after few years many experiments confirmed it.

We know that From Einstein's Mass Energy relation

$$E=mc^2 \qquad \ldots\ldots\ldots\ldots..(1)$$

From Plank's Quantum theory $E=h\nu$ ………..…(2)

And $\nu=c/\lambda$

So $E= hc/\lambda$ ……………..(3)

From Equation (1) and (3)………………………

$$hc/\lambda=mc^2$$

$$\lambda = \frac{h}{mc}$$

This is the velocity of Photon for matter particle c can be replaced by v

$$\lambda = \frac{h}{m\upsilon} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

2.2 Different forms of de Broglie Wave-length:

The de Broglie wave length can be expressed in different forms depending upon the situation in Non-relativistic and relativistic manner.

Non-relativistic Case:

(a) In Terms of Kinetic Energy : For a classical particle of mass m traveling with a small velocity the kinetic energy is given by :

$$K = \frac{p^2}{2m}$$

=> $$p = \sqrt{2mK}$$

and de Broglie wavelength - $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}}$

i.e. $$\lambda = \frac{h}{\sqrt{2mK}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

(b) In Terms of Accelerating Potential:

If a charged particle of mass m is accelerated to a potential difference V, its kinetic energy is given by :

K = q V where $q \rightarrow$ charge

Then de Broglie wavelength; $\lambda = \dfrac{h}{\sqrt{2mqV}}$

For an electron the de Broglie wavelength then become

$$\lambda = \frac{6.625\times10^{-34}}{2\times9.1\times10^{-31}\times1.6\times10^{-19}.V}$$

on simplifying we get –

$$\lambda = \frac{12.3}{\sqrt{V}} A^0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Relativistic Case:

(a) In terms of kinetic Energy : For a particle of rest mass m_0, the relativistic total energy is by:

$$E^2 = p^2 c^2 + m_0^2 c^4 \qquad \ldots\ldots\ldots\ldots\ldots(7)$$

$$P = \frac{1}{c}\sqrt{E^2 - m_o^2 c^4} \qquad \ldots\ldots\ldots\ldots\ldots(8)$$

The total energy of a particle as per Einstein's relation is given by :

$$E = (m - m_o)\, c^2 + m_o\, c^2$$

$$E = K + m_o c^2 \qquad \ldots\ldots\ldots\ldots\ldots(9)$$

using this in above relation :

$$P = \frac{1}{c}\sqrt{(K+m_oc^2)^2 - m_o^2c^4}$$

$$= \frac{1}{c}\sqrt{K(K+2m_oc^2)}$$

The de Broglie wavelength therefore will be:

$$\boxed{\lambda = \frac{hc}{\sqrt{K(K+2m_oc^2)}}} \quad \ldots\ldots\ldots\ldots\ldots(10)$$

(b) In Terms of Accelerating Potential: If kinetic energy in the above relation is replaced by qV, then de Broglie wave length can be expressed as:

$$\lambda = \frac{hc}{\sqrt{qV(2m_oc^2 + qV)}} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

$$\boxed{\lambda = \frac{h}{\sqrt{2m_oqV\left(1+\frac{qV}{2m_oc^2}\right)}}} \quad \ldots\ldots\ldots\ldots\ldots(12)$$

2.3 de-Broglie Wave-Velocity:

From Planck's quantization Hypothesis, the energy of a quantum of radiation

$$E = h\nu = \frac{hc}{\lambda} \quad \ldots\ldots\ldots\ldots(1)$$

If this quantum behaves as particle of mass m then, from Einstein's Mass-Energy relation, the energy associated with a particle:

$$E = mc^2 \quad \ldots\ldots\ldots\ldots(2)$$

Since a wave is associated with a moving material body, therefore it is quite reasonable to think that this 'wave' should travel with the velocity of the moving material particle. Let us now estimate this velocity:

de Broglie wave velocity $w = \nu \ \lambda$ ....(3)

$$= \frac{E}{h}.\frac{h}{p} = \frac{E}{p} = \frac{mc^2}{m\upsilon} = \frac{c^2}{\upsilon}$$

As we know from relativistic mechanics, the velocity of moving material particle will always be less than c, therefore:

$$w >> c$$

This means de Broglie wave velocity is very-very greater than c. The above result seems absurd and therefore is unacceptable. This controversy suggested Erwin Schrödinger to think in some other way and to modify the hypothesis proposed by Louis de Broglie. The solution to this controversy is described in the forthcoming section:

2.4 Schrödinger's Explanation:

The above controversial result was carefully solved by Schrödinger. He again suggested a mathematical solution and out that there may be associated a number of waves rather than a single wave. He thought to resolve the problem and looked the situation like **'production of beats'** in sound where a number of waves superpose each other to produce a resultant wave (wave group) due to which we receive a sounds after some fixed intervals. For mathematical simplicity, the formation of such a wave-group was imagined by him considering the two waves slightly differing in angular frequency and propagation vector.

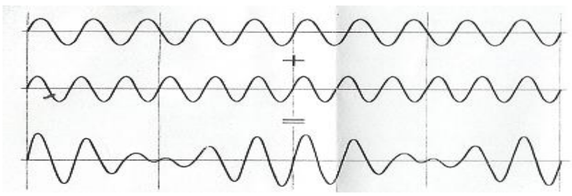

Fig: Production of Beats in Sound

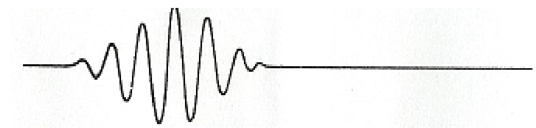

Fig: de-Broglie Wave group

Let the equations of the wave forming a wave-group are:

$$y_1 = A\cos(\omega t - kx) \quad \text{……….(1)}$$

the equation of second wave :

$$y_2 = A\cos(\omega + d\omega)\,t - (k + dk)\,x] \quad \text{……….(2)}$$

using superposition principle, the equation of resultant wave :

$$y = y_1 + y_2 \quad \text{……….(3)}$$

$$= A\cos(\omega t - kx) + A\cos(\omega + d\omega)\,t - (k + dk)\,x]$$

$$= 2A\cos\frac{1}{2}[(2\,\omega + d\omega)t - (2\,k + dk)\,x]\cos\frac{1}{2}(\,d\omega t - dk.\,x)$$

Assuming $d\omega$ and dk very small compared to ω and k, can write:

$$2\omega + d\omega = 2\omega \quad , \qquad 2k + d\,k = 2\,k$$

On using these approximations in above, we get –

$$y = 2\,A\cos(\omega t - kx)\cos\left[\frac{d\omega}{2}.t - \frac{dk}{2}.x\right] \quad \text{…………(4)}$$

The resultant wave given above is comprised of the **original wave having angular frequency $\boldsymbol{\omega}$** and propagation vector k superimposed upon it **a modulation** with angular frequency $\frac{d\omega}{2}$ and propagation vector $\frac{dk}{2}$. **The effect of this modulation is to produce a successive wave- groups.**

The ratio **of angular frequency $\boldsymbol{\omega}$ to the propagation vector $\boldsymbol{k}$** of the original wave is known as **Phase velocity$\boldsymbol{v_p}$ or wave velocity** and the ratio of **angular frequency $\boldsymbol{\frac{d\omega}{2}}$ to the propagation vector $\boldsymbol{\frac{dk}{2}}$** is known as **group velocity $\boldsymbol{v_g}$**:

$$\boldsymbol{v_p = \frac{\omega}{k}} \qquad\qquad \boldsymbol{v_g = \frac{d\omega}{dk}}$$

2.5 Wave Velocity (Phase Velocity) and Group Velocity (υ_p & υ_g):

The ratio of angular frequency ω and propagation vector k of the original wave is known as **wave velocity or phase velocity** and is denoted by υ_p. Since ($\omega t - kx$) is phase of the motion in the equation of wave, it means this is the velocity of that wave which come into existence when the particles with the constant phase travels: i.e.

$$\omega t - kx = \text{constant} \quad \text{…………… (1)}$$

or $$\frac{d}{dt}(\omega t - kx) = 0 \quad \text{………….. (2)}$$

or ω - k . $\frac{dx}{dt} = 0$

$$\upsilon_p = \frac{dx}{dt} = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda \qquad \text{............(3)}$$

If a wave group contains a number of frequency components in very small frequency interval then the above relation can be expressed in terms of frequency as:

$$\upsilon_g = \frac{d\omega}{dk} = \frac{2\pi d\nu}{2\pi d\left(\frac{1}{\lambda}\right)} = -\lambda^2 \frac{d\nu}{d\lambda} \qquad \text{..................(4)}$$

2.6 Relation between Phase Velocity and Group Velocity:

(a) In Dispersive Medium: The phase velocity is given by –

$$\upsilon_p = \frac{\omega}{k} \quad \text{or} \quad \omega = k\upsilon_p$$

and group velocity $\upsilon_g = \frac{d\omega}{d\kappa} = \frac{d(k\upsilon_p)}{dk}$ ....(5)

$$= \upsilon_p + k\frac{d\upsilon_p}{dk}$$

Replacing k by $\frac{2\pi}{\lambda}$ in the above expression we get –

$$\upsilon_g = \upsilon_p + \frac{2\pi}{\lambda}\frac{d\upsilon_p}{d\left(\frac{2\pi}{\lambda}\right)}$$

or

$$\upsilon_g = \upsilon_p + \frac{1}{\lambda}\frac{d\upsilon_p}{d\left(\frac{1}{\lambda}\right)}$$

On simplifying we get –

$$\upsilon_g = \upsilon_p - \lambda\frac{d\upsilon_p}{d\lambda} \qquad \text{..................(6)}$$

From the above one can conclude that the group velocity is always less to the phase velocity in a non-dispersive medium.

(b) In Non- dispersive Medium: (Free Space)

For a non-dispersive medium phase velocity is independent of λ i.e. :

$$\frac{d\upsilon_p}{d\lambda} = 0$$

Therefore $\boxed{\upsilon_g = \upsilon_p}$

Which means that the group velocity υ_g is always same as phase velocity in free space.

Velocity of Wave-group (Wave Packet):

As suggested by Schrödinger that a wave-group may be associated with a moving material particle. So let us calculate the velocity of this wave-group or wave-packet.

From Einstein's Relation the energy of moving material particle is –

$$E = m\,c^2 = \frac{m_o c^2}{\sqrt{1-\upsilon^2/c^2}} \qquad \text{.....................(7)}$$

and the momentum of the same particle –

$$P = m\,\upsilon = \frac{m_o \upsilon}{\sqrt{1-\upsilon^2/c^2}} \qquad \text{....................(8)}$$

Frequency of the energy quantum(assuming that it exhibit the particle character)–

$$\nu = \frac{E}{h} = \frac{m_o c^2}{h\sqrt{1-\upsilon^2/c^2}} \qquad \text{.....................(9)}$$

Where m_0 is the rest mass of the particle.

and $$\omega = 2\pi\nu = \frac{2\pi m_0 c^2}{h\sqrt{1-\upsilon^2/c^2}} \qquad \text{....................(10)}$$

on differentiating above w.r.t. v we get –

$$\frac{d\omega}{d\upsilon} = \frac{2\pi m_0 c^2}{h\left(1-\frac{\upsilon^2}{c^2}\right)^{3/2}}\left[-\frac{1}{2}\times -\frac{2\upsilon}{c^2}\right]$$

or $$\frac{d\omega}{d\upsilon} = \frac{2\pi m_o c^2}{h\left[1-\frac{\upsilon^2}{c^2}\right]^{3/2}}$$

the wave length of de Broglie wave associated with the moving material particle :

$$\lambda = \frac{h}{p} = \frac{h\left(1-\frac{\upsilon^2}{c^2}\right)^{1/2}}{m_o \upsilon}$$

and propagation vector $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi m_o \upsilon}{h\left(1-\frac{\upsilon^2}{c^2}\right)^{1/2}}$

On differentiating above w.r.t. υ we get -

$$\frac{dk}{d\upsilon} = \frac{2\pi m_o}{h}\,\frac{\left[1-\frac{\upsilon^2}{c^2}\right]^{1/2} - \upsilon.\frac{1}{2}\left[1-\frac{\upsilon^2}{c^2}\right]^{-1/2}\times\left[-\frac{2\upsilon}{c^2}\right]}{\left[1-\frac{\upsilon^2}{c^2}\right]}$$

or $$\frac{dk}{d\upsilon} = \frac{2m_o}{h}\left[\left\{1-\frac{\upsilon^2}{c^2}\right\}^{-1/2} + \frac{\upsilon^2}{c^2}\left\{1-\frac{\upsilon^2}{c^2}\right\}\right]$$

on simplifying we get :

$$\frac{dk}{d\upsilon} = \frac{2\pi m_o}{h\left[1-\frac{\upsilon^2}{c^2}\right]^{3/2}}$$

The group velocity (velocity of wave-packet) υ_g is given by :

$$\upsilon_g = \frac{d\omega}{dk} = \frac{d\omega/d\upsilon}{dk/d\upsilon} \quad \ldots\ldots\ldots\ldots\ldots(11)$$

On Substituting $\frac{d\omega}{d\upsilon}$ and $\frac{dk}{d\upsilon}$ in the above equation and simplifying we get :

$$\boxed{\upsilon_g = \upsilon}$$

i.e. the velocity of wave-group (wave-packet) υ_g associated with a material body travel with the velocity of the material particle itself. Of course this result is acceptable. The above conclusion can also be stated as-

"A moving material body presents itself as a wave-group or wave-packet associated with a moving material particle travel with the velocity of moving material body".

Therefore the de Broglie hypothesis was slightly modified as –

"**There is associated a wave group or a wave packet with a moving material body."**

2. 7 Relation between υ_p and υ_g for a Non-relativistic Free Particle:

Let υ_p be the phase velocity and υ the group velocity ($\upsilon_g = \upsilon$) for a non- relativistic free particle of mass m.

The de Broglie wavelength $\lambda = \frac{h}{m\upsilon}$

The phase velocity $\upsilon_p = \upsilon$

Since, total energy of free particle E = kinetic energy = $\frac{1}{2}m\upsilon^2$

Also E = hν

$$=> \qquad \nu = \frac{E}{h} = \frac{1}{2}\frac{m\upsilon^2}{h}$$

Therefore $\upsilon_p = \nu\lambda = \frac{1}{2}\frac{m\upsilon^2}{h}.\frac{h}{m\upsilon} = \frac{\upsilon}{2}$

ie. $\qquad \upsilon_p = \frac{\upsilon}{2} = \frac{\upsilon_g}{2}$

Therefore for a non- realistic free particle, the phase velocity is half of the group velocity.

2.8 Experimental Confirmation of Matter Waves:

Our next target is to show experimentally, that the accelerated electron is able to show diffraction. For this we require a grating(slit system) whose size of the slit must match with the wave length of the electron which is assumed to be accelerated to let 100V.

when we calculate the de Broglie wavelength for an electron accelerated to 100 V, it comes out nearly 1.23Å which well matches with the inter planer separation of a single crystal. This suggested that the fast moving electrons, (if behave wave-like) should undergo diffraction exactly similar as observed by WL Bragg for X-rays.

The following below experiments are sufficient enough to justify that fast moving material particles exhibit a wave-character:

- **Electron Microscope**
- **Davisson-Germer Experiment**
- **GP Thomson Experiment**

All the above experiments utilize accelerated electron beam which ultimately undergo diffraction and hence confirm the wave nature.

2.9 Davisson- Germer Experiment:

The main objective of Davisson- Germer experiment was to demonstrate the existence of matter-waves experimentally. This experiment was performed in 1927 with the following aims:

- To provide experimental confirmation of Matter-Waves
- Fast moving electrons can undergo diffraction which is essential wave nature confirmation.

The following below are given the schematic diagram and the main components of the experimental set-up and results obtained:

Experimental Set-up: The main parts of experimental set-up are:

- **Electron Gun -** The purpose of electron gun is to produce, Collimate and accelerate the electron beam.
- **Nickel Crystal –** Inter planer separation of this crystal matches roughly with the wave-length of electron waves and therefore Ni-crystal can act as slit system (grating).
- **Detector –** This is the device which collect the scattered radiation and convert it into the current which is the measure of intensity.

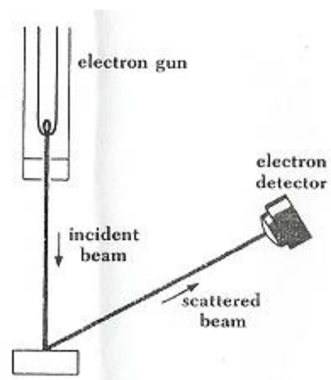

Fig: Experimental Arrangement of Davisson Germer Expt.

Experimental Observation: The experiment was performed at various varying voltages and the polar plots were recorded at each voltage which is given below.

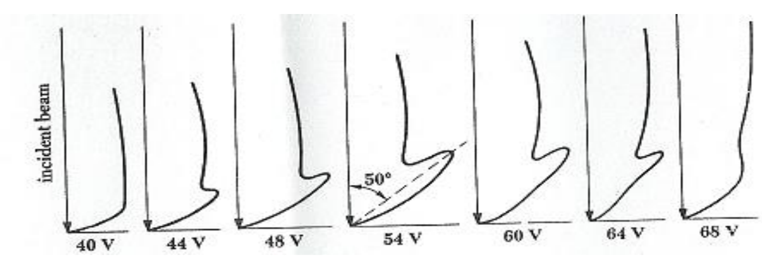

Fig : Results of Davisson - Germer Experiment

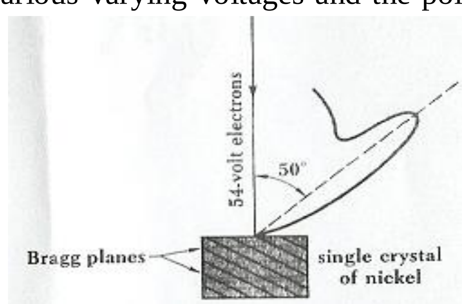

Fig : Diffraction of Electrons by Bragg's Planes in Ni crystal

Explanation:

As per classical physics, the scattering of electrons should occur in all the directions but reality was not this. The results obtained were quite surprising.

Well defined bump started to appear with a more pronounced peak appeared at 54 volts at an angle of 50^0. If fast moving electron beam behave wave-like, it must undergo diffraction and then Bragg's Law can be applied. Let us now calculate the wavelength of de Broglie wave (electron-wave) considering the wave behaviors of electron beam and applying Bragg's equation:

$$2d \sin\theta = n\lambda \qquad \ldots\ldots\ldots\ldots(1)$$

For nickel crystal d = 0.91 A and $\theta = 50^0$ (but this becomes 65^0 w.r.t. Bragg's Planes). Assuming first order Bragg's diffraction:

$$\lambda = 2 \times 0.91 \times \sin 65^0$$
$$= 1.65 \text{ Å}$$

Let us calculate wavelength of electron waves using de-Broglie formula:

$$\lambda = \frac{h}{m\upsilon} = \frac{h}{\sqrt{2mE_k}} = \frac{6.64\times10^{-34}}{\sqrt{2\times9.1\times10^{-3}\times54\times1.6\times10^{-19}}}$$

where E_k and υ are the kinetic energy and velocity of electron beams. On simplifying:

$$\lambda = 1.66 \text{ Å}$$

The observed wavelength as calculated using de-Broglie formula comes very close to the wavelength obtained using Bragg's law. This confirms the wave character of fast moving electrons**. Hence the material particle (matter) also exhibit wave like nature under certain circumstances similar to the radiation.**

2.10 Principle of Complementarity:

This state's "**The two natures exhibited by matter and radiation although appear entirely different, but both are equally important to understand the atomic system."** These two natures are as essential, as the two faces of the same coin. The existence of coin is meaningless is in absence of either of the two. But it is also true that when one of the behaviors dominates other is found absent. In other words we can say that a compact wave group must reveal particle like characteristics and under this situation wave behavior will not be visible.

2.11 Representation of Particle of Microscopic (Sub-atomic) Region:

We have seen in the previous discussion that a microscopic particle presents itself as a wave packet that travel with the speed of a material particle. Such a wave – packet has been given below in the figure traveling along positive x-axis.

The momentum and energy of such a particle may be given as:

$$P = \hbar k$$
$$E = \hbar\omega$$

where k = $\frac{2\pi}{\lambda}$ is known as propagation vector and ω = 2 $\pi\nu$ is the angular frequency.

2.12 Interesting Characteristics of a Wave-Packet:

The most important and interesting property of wave packet is that 'if Δx is the spatial extent and Δk be its wave number range, then it is always observed that:

$$\Delta k \;\; \Delta x \;\; \geq 1$$

This means it is impossible to reduce both Δx & Δk. The smaller the Δx, the larger will be Δk and vice-versa. This general feather of wave-packet has very deep implications in quantum mechanics. The above stated relation is known as reciprocity relation.

Heisenberg Uncertainty Principle:

In classical mechanics, the simultaneous measurement of particle's momentum and position or any canonically conjugate physical quantities with a very high degree of precision is always possible as both are independent of each other. Now when we arrive at microscopic region (sub atomic level) ,the simultaneous measurement of particle's momentum and position with a high degree of accuracy is always impossible. Increasing the accuracy in measurement of one quantity leads another quantity highly imprecise (less accurate). The reason behind it is the wave character exhibited by the microscopic material particle.

Position Momentum Uncertainty Relation: According to Heisenberg, if Δx be the error (uncertainty) in measurement of position of a particle and Δp be the error in measurement of its momentum; the product of these two errors will always be greater or equal to $\frac{\hbar}{2}$

i.e. $$\Delta x \ \Delta p \geq \frac{\hbar}{2} \qquad \text{...............(1)}$$

This means, **"It is impossible to specify both the position and momentum of a particle simultaneously with a high degree of precision."**

The above relation can be generalized in three dimensions as given below:

$$\Delta x . \Delta p_x \geq \frac{\hbar}{2}$$

$$\Delta y . \Delta p_y \geq \frac{\hbar}{2}$$

$$\Delta z . \Delta p_z \geq \frac{\hbar}{2}$$

The above restriction applies only on complementary pairs, $\Delta x, \Delta p_x ; \Delta y, \Delta p_y and \Delta z, \Delta p_z$

Energy-time Uncertainty Relation: The **Uncertainty principle** can be expressed in other way also. One may be interested in measurement of the energy emitted by some atomic process during the time interval $\boldsymbol{\Delta t}$. If the energy is in the form of electromagnetic radiations, the limited time available imposes a restriction to the accuracy with which the frequency υ and therefore the energy E of the wave can be determined. This relation can be derived using the position-momentum uncertainty relation. The energy and momentum of a free particle are related as given:

$$E = \frac{p^2}{2m} \qquad \text{...................(2)}$$

therefore $$\Delta E = \frac{2 p \Delta p}{2m} = \frac{p}{m} \Delta p = \upsilon \Delta p \qquad \text{.....................(3)}$$

where υ is the velocity of a particle.

If the spatial width of a wave-packet representing a particle be Δx then the time required for this wave-packet to a pass given point and therefore error (uncertainty) in time Δt is :

$$\Delta t = \frac{\Delta x}{\upsilon} \qquad \text{....................(4)}$$

Multiplying the above (3) and (4) we get :

$$\Delta E . \Delta t = \upsilon \Delta p . \frac{\Delta x}{\upsilon} = \Delta p \Delta x \geq \frac{h}{2}$$

This relation connects the uncertainty ΔE in the determination of the energy of a system with the time-interval Δt available for the energy determination and state that **the product of the uncertainty** ΔE **in the**

energy measurement and the uncertainty Δt in the time in which the measurement was done cannot be minimized below to the Planck's constant. In the context of wave-mechanics this means that the state of finite duration cannot have precisely defined values of energy. It should be kept in mind that the error described above has nothing to do with the experimental errors that occurs is actual measurements. These are inherent and the dual character of matter and radiation is responsible for these. The uncertainty principle is valid for both: the material particles as well as for radiations (photons):

2.14 Heisenberg Gamma- ray Microscope:

This is not an experiment in the reality. It is a thought experiment in which Heisenberg thought to reduce the error in measurement of position of a material particle. In this experiment an attempt has been made to locate an electron (material particle) by observing it through a γ - ray microscope. Naturally this required a source of radiation emitting $\gamma - radiations$. The electron will only be visible, when at least one photon must strike the electron and the scattered photon be available in the field of view of the microscope.

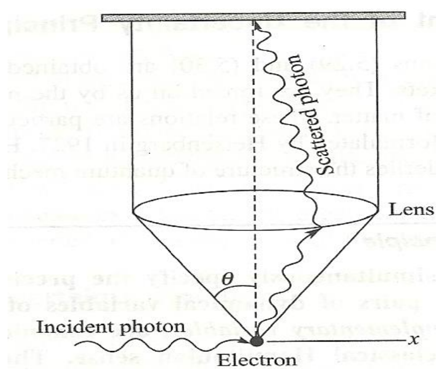

Fig: Gama ray- microscope

From the knowledge of diffraction phenomena, we know that one cannot measure the position of the particle very accurately. The accuracy in the position can never be minimized below to the resolving limit of microscopic i.e.

Uncertainty in the position of the electron is given by $\Delta x = \dfrac{\lambda}{2\sin\alpha}$ …………(1)

Where α is the half angle subtended by the lens at the electron position. From the above formula it is also evident that error Δx can be minimized by using a source having extremely low wavelength. That is why Heisenberg thought to use γ -ray source.

Suppose if Δx be minimized by using a $\gamma - ray$ source, then it will create a problem in the measurement of the momentum of electron. From de Broglie hypothesis, $\left(\lambda = \dfrac{h}{p}\right)$ the momentum of incident photon will become too large. Hence this will create a high disturbance to the momentum of the electron and therefore accuracy in the momentum measurement will fall down leading to large Δp. Converse will also be true.

From the figure, the x-component of the momentum of the scattered photon ranges from - $\frac{h}{\lambda}$ sin α to + $\frac{h}{\lambda}\sin\alpha$.

Therefore uncertainty in the measurement of the momentum of electron

$$\Delta p_x = 2\frac{h}{\lambda}\sin\alpha \qquad \text{……………..(2)}$$

Now from the above relation it is clear that Δp_x, can be minimized by increasing λ or by decreasing α But in doing so Δx will increase.

On multiplying eqn. (1) and (2) –

$$\boxed{\Delta x.\Delta p_x \approx h}$$

Therefore ideally also one cannot reduce the product of errors in Δx and Δp_x below to the Planck's constant.

2.15 Single Slit diffraction:

Let a beam of material particle like electron (or photons) incident upon a slit of width a with a well defined momentum *p*. Then the y- component of the position of the electron passing through the slit is determined to an accuracy of.

$$\Delta y = a \qquad \text{…………..(1)}$$

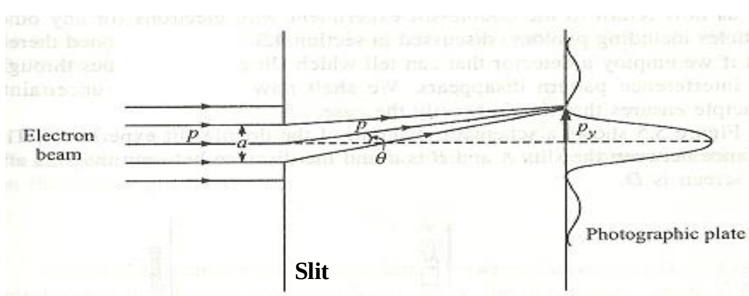

Fig: Single Slit Diffraction

The accuracy in the measurement of position can be increased by minimizing the slit width a.

From de-Broglie hypothesis $p = \frac{h}{\lambda}$. In the above figure we can see that as the photon emerges from the slit, its momentum is not well defined. It makes some angle with the horizontal. Therefore the uncertainty is y – component of the momentum is at least as large as $p\sin\theta$

Therefore $\quad \Delta p_y \geq p.\sin\theta \qquad$ ………………(2)

Angular spread of the central maxima is given by –

$$\sin\ \theta = \pm\frac{\lambda}{a} \qquad \text{…………………(3)}$$

Hence $\quad \Delta p_y \geq \frac{h}{x}.\frac{x}{a} \geq \frac{h}{a} \geq \frac{h}{\Delta y}$

$$\boxed{\Delta p_y.\Delta_y \geq h}$$

This is consistent with uncertainty principle.

2.16 Derivation of Uncertainty Principle:

The Heisenberg uncertainty relation can be obtained using the wave-packet concept proposed by the Schrödinger.

The resultant equation of a wave-packet is given by

$$y = 2\,A\cos(\omega t - kx)\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) \quad \ldots\ldots\ldots\ldots(1)$$

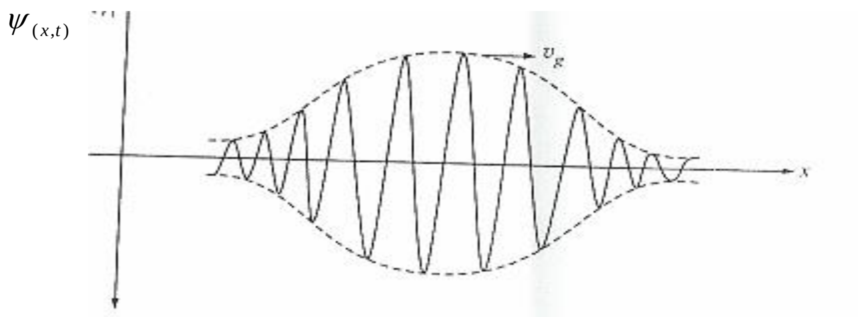

Fig : A Wave Packet traveling along x – direction

From the above we can note that the particle must be present somewhere within the wave-packet (loop) but at the nodes it will be certainly be absent. Applying this condition at the node (y = 0).

$$\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) = 0 \quad \ldots\ldots\ldots(2)$$

==> $$\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x = (2n+1)\frac{\pi}{2}$$

Where n = 0, 1, 2, 3……….

If x_n and x_{n+1} are the positions of two successive nodes:

$$\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_n\right] = (2n+1)\frac{\pi}{2} \quad \ldots\ldots\ldots\ldots(3)$$

and $$\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_{n+1}\right] = (2n+3)\frac{\pi}{2} \quad \ldots\ldots\ldots..(4)$$

On subtracting we get :

$$\frac{\Delta k}{2}(x_{n+1} - x_n) = \pi$$

or $$\frac{\Delta k}{2}.\Delta x = \pi$$

therefore $$\Delta x = \frac{2\pi}{\Delta k} \quad \ldots\ldots\ldots\ldots(5)$$

since $$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$$

therefore $$\Delta k = \frac{2\pi}{h}\Delta p \quad \ldots\ldots\ldots\ldots(6)$$

$$\Delta p = \frac{h\Delta k}{2\pi} \qquad \text{……………(7)}$$

on multiplying (6.50) and (6.51) we get :

$$\Delta x.\Delta p. = \frac{2\pi}{\Delta K}.\frac{h.\Delta K}{2\pi} = h$$

$$\boxed{\Delta x.\Delta p = h}$$

2.17 Consequences of Heisenberg Uncertainty Principle:

A large number of well known facts can be justified on the basis of **Heisenberg uncertainty principle**. Since the order of magnitude of the Planck's constant is too small, the uncertainty principle plays a significant role only at microscopic levels. Its effect can be ignored at macroscopic level.

(i) Radius of Hydrogen Atom (Bohr atom) and Ground State Energy: The uncertainty relation can be used to estimate the ground state energy and radius of Hydrogen atom. Hydrogen atom has pride of being the simplest atom containing one proton and one revolving electron.

The total energy of the atom can therefore be obtained by summing up the potential and kinetic energy:

$$E = \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0 r} \qquad \text{…………..(1)}$$

where p, r and m are momentum, radius and mass of the revolving electron.

Assuming r $\approx \Delta x$ and $p \approx \Delta p$;

Therefore $\Delta x \;\; \Delta p \approx h$

$$r.\, p \approx h$$

$$p \approx \frac{h}{r} \qquad \text{…………..(2)}$$

Using this in the above equation we get –

$$E = \frac{h^2}{2mr^2} - \frac{e^2}{4\pi\varepsilon_0 r} \qquad \text{…………..(3)}$$

The system will be in the lowest energy state when:

$$\frac{dE}{dr} = 0$$

or

$$\frac{h^2}{2mr^2} - \frac{e^2}{4\pi\varepsilon_0 r} = 0 \qquad \text{…………..(4)}$$

On simplifying we get:

$$r = r_0 = \frac{(4\pi\varepsilon_0)h^2}{me^2}$$

On substituting values of ε_0, h, m & e we find.

$$r_0 = 0.53\,\text{Å}$$

This is popularly known as **radius of the Bohr's first orbit of H-atom.**

Using (4) in eqn. (3) we get:

$$E = -\frac{me^4}{2h^2(4\pi\varepsilon_0)^2} = \frac{h^2}{2mr_0^2}$$

On substituting the values of h, m, r_0 in the above we get:

$$E = -13.6eV$$

This is the lowest Energy state of Bohr-atom and is named as **ground state energy** of hydrogen atom.

(ii) Electron cannot exist in the Nucleus: This is another well known fact beyond any doubt. Nuclear size is of order of 10^{-14} m. Assuming the electron to be within the nucleus, the uncertainty in its position cannot go beyond this value i.e.

$$\Delta p \approx 10^{-14} m$$

using uncertainty relation :

$$\Delta x \;\; \Delta p \approx \hbar$$

The uncertainty in the momentum of the electron (if it is inside the nucleus) will be –

$$\Delta p \approx \frac{h}{\Delta x} = \frac{1.054\times 10^{-34}}{10^{-14}}$$

$$= 1.1\times 10^{-20} kgms^{-1}$$

An electron having such a large momentum must have kinetic energy many times greater to the rest mass energy $m_o c^2$.

Using relativistic formula for finding the kinetic energy –

$$K^2 = p^2 c^2 + m_o^2 c^4$$

Neglecting $m_o^2 c^4$ for simplicity –

$$K = pc = \frac{1.1\times 10^{-20} \times 3.0\times 10^{8}}{1.6\times 10^{-19}}$$

$$= \frac{3.3}{1.6}\times 10^{-12+9}$$

$$= \frac{3.3}{1.6}\times 10^{7} eV$$

$$= 20.6 \text{ MeV}$$

But β - ray experiments show that electron's energy is only in between 2-3 MeV which contradict the above assumption. Therefore Electron cannot reside in the nucleus.

(iii). Zero Point Energy (Lowest) of Harmonic Oscillator: Harmonic oscillations occur at microscopic regions too. There are countless example at microscopic levels in which the motion is found simple harmonic e.g. vibrations of atoms in a crystal lattice, vibrating diatomic molecule etc.

Classically the total energy of simple Harmonic oscillation is expressed as –

$$E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 \qquad \text{……………..(1)}$$

where ω is the frequency of vibration. Classically the lowest energy of the oscillator will be zero when it will be in the equilibrium (mean) position i.e. in non- oscillatory condition. But the uncertainty principle does not allow this.

Let us assume that the oscillator is restricted to move in a region and it can go upto maximum displacement a. Then uncertainty in the position will be -

$$x \approx \Delta x \approx a \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

Using real statement of uncertainty principle:

$$\Delta x.\Delta p \geq \frac{\hbar}{2}$$

we get $\quad p = \Delta p = \dfrac{\hbar}{2a} \qquad \ldots\ldots\ldots\ldots\ldots(3)$

The total energy therefore be –

$$E = \frac{\hbar}{8ma^2} + \frac{1}{2}m\omega^2 a^2 \qquad \ldots\ldots\ldots\ldots\ldots(4)$$

on minimizing –

$$\frac{dE}{da} = o$$

which gives - $\quad a = \left[\dfrac{\hbar}{2m\omega}\right]^{1/2}$

on using this in eqn. (4) we get –

$$\boxed{E_{\min} = \frac{1}{2}\hbar\omega_0} \qquad \omega_0 \rightarrow \text{frequency in ground state}$$

This is known as **zero point energy. Therefore, contrary to the classical prediction, the quantum mechanical oscillator exhibit a minimum non –zero value of energy as given above. This result is entirely different with the well known classical result.**

Significance of Zero Point energy:

The uncertainty principle says that, "**No physical system can be completely at rest, even at absolute zero."** This is entirely different from classical point of view. One important consequence of non-zero value of lowest energy is – "the Helium does not solidify even at very low temperature while many substances solidify.

(iv) Finite Width of Spectral Lines: The spectral line is observed when an atom in the excited state (high energy state) comes to the ground state. This process is known as **'transition'** which is associated with the emission of some energy (photon). The transition-time is of the order of 10^{-8} sec.

Energy-time uncertainty can be used to estimate-how much accurate can be frequency of a line be measured.

Assuming $\quad \Delta t \approx 10^{-8}$ sec

$$\Delta E.\Delta t \approx h$$

$$\Delta E \approx \frac{h}{\Delta t} = \frac{h}{10^{-8}}$$

or

$$h\Delta\nu = \frac{h}{2\pi \times 10^{-8}}$$

$$\Delta\nu = 1.6 \times 10^{7}\,Hz$$

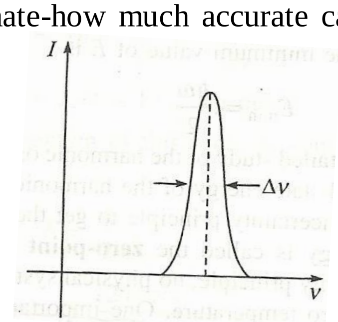

Fig: Width of Spectral line

The quantity $\Delta\nu$ can never be minimized below to this value and is known as **'natural width'**. Hence one cannot observe the spectral line infinitely sharp. The above uncertainty will definitely be there which is inherent. It is a quantum mechanical phenomenon.

(v) Energy of Particle in a Box: We will now discuss the case of a particle whose freedom of motion is restricted in a given region. i.e. a particle trapped to move in an infinite potential well of width l. This particle cannot cross the walls of the box. Under this situation the maximum uncertainty in the position of this particle will be:

$$(\Delta x)_{max} = l$$

using uncertainty principle :

$$\Delta x \ \ \Delta p \approx h$$

$$\Delta p \approx \frac{h}{l}$$

The kinetic energy of the particle:

$$E_k = \frac{p^2}{2m} = \frac{h^2}{2ml^2}$$

i.e.

$$E_k = \frac{h^2}{2ml^2}$$

This is the **Lowest energy** of a particle which is not free to move and is confined to move only in a given region.

2.18 Bohr's Atomic Model and Uncertainty Principle: Bohr's theory of Hydrogen atom has been regarded as one of the most popular theory explaining numerous puzzles related with the atom. Let us inspect this from the point of view of the **Heisenberg Uncertainty Principle.**

This model of is a hybrid of two rival theories: the classical theory and the quantum theory.

From energy- time uncertainty principle –

$$\Delta E.\Delta t \geq \frac{h}{2}$$

According to Bohr's first postulate -

The electrons are only allowed to revolve in certain discrete orbits which have a well defined energy. If this is true then-

$$\boxed{\Delta E = 0}$$

From the above relation this gives $\Delta t = \infty$ i.e. the energy states of the atom must have infinite life-time which is contradictory as it is of order of 10^{-8} seconds only. Therefore Bohr's model is not consistent from the point of view of the Heisenberg uncertainty principle.

Solved Examples

1. A ball of mass 10 g. has velocity 100 cm/s. Calculate the wavelength associated with it. Why this wave nature does not appear in our daily observations? Given h = 6.62 × 10^{-34} joule sec.

Solution –

The de-Broglie wavelength λ is given by -

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34} J.s}{10 \times 10^{-3} \times 1.0 \times kg.m/s)} = 6.62 \times 10^{-22} \text{ m.}$$

The wavelength is much smaller than the dimensions of the ball; and hence wave-like properties of matter cannot be observed in this case.

2. (a) Find the de-Broglie wavelength of an electron which is accelerated to V volts.

(b) Calculate the wavelength associated with an electron subjected to a potential difference of 50 volts.

Solution –

The de-Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \qquad \left[\text{since E} = \frac{1}{2} m\upsilon^2, \text{ so that m}\upsilon = \sqrt{2mE}\right]$$

Given E = e V electron.volt = 1.6 × 10^{-19} V joule

mass of the electron m = 9.1 × 10^{-31} kg.

and Planck's constant h = 6.62 × 10^{-34} joule.sec

$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}} \text{ m}$$

$$= \frac{12.28 \times 10^{-10}}{\sqrt{V}} \text{ m} = \frac{12.28}{\sqrt{V}} \text{ A}^0$$

(b) The de-Broglie wavelength λ is given by

$$\lambda = \frac{12.28}{\sqrt{V}} \text{Å}$$

Here V = 50 Volts $\qquad \lambda = \frac{12.28}{\sqrt{50}} A^0 = 1.73 A^0$

3. Calculate the de-Broglie wavelength associated with a proton moving with a velocity equal 1/20th of the velocity of light.

Solution: velocity of proton $\upsilon = \frac{3 \times 10^8 m/s}{20} = 1.5 \times 10^7$ m/s

Given, mass of the proton, m = 1.67 × 10^{-27} kg.

Planck's constant h = 6.6 × 10^{-34}Js

$\therefore$ de Broglie wavelength $\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.5 \times 10^7}$

= 2.634 × 10^{-14} m.

4. Find the velocity and kinetic energy of the neutron, having de-Broglie wavelength 1A^0. Given mass of the neutron = 1.67 × 10^{-27} kg.

Solution -

The de-Broglie wavelength λ associated with a neutron is given by:

$$\lambda = \frac{h}{mv}$$

$$v = \frac{h}{m\lambda}$$

Here h = 6.63 × 10^{-34} J-s, m = 1.67 × 10^{-27} kg.

and $\lambda = 1A^0 = 10^{-10} m.$

$$\therefore \quad v = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1 \times 10^{-10}}$$

$= 3.97 \times 10^3$ ms^{-1}

(ii) Kinetic energy $E = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.67 \times 10^{-27} \times (3.97 \times 10^3)^2$

$= 13.6 \times 10^{-19}$ J $= \frac{13.16 \times 10^{-19}}{1.6 \times 10^{-19}} eV$

= 8.2 eV

5. **Calculate the de-Broglie wavelength of neutron with kinetic energy of 1eV. Given h = 6.63 × 10^{-34} joule-sec, m = 1.67 × 10^{-27} kg.**

Solution –

The de-Broglie wavelength is given by $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$ where E is kinetic energy.

Here h = 6.63 × 10^{-34} joule sec., m = 1.67 × 10^{-27} kg.

E = 1eV = 1 × 1.6 × 10^{-19} joule = 1.6 × 10^{-19} joule.

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2} \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19}} = \frac{6.63 \times 10^{-34}}{2.31 \times 10^{-23}}$$

= 2.87 × 10^{-11} m = 0.287 × 10^{-10} m = 0.287 A^0

6. **Can a photon and an electron of the same momentum have the same wavelength? Compare their wavelengths if the two have the same energy.**

Solution –

Yes, the de Broglie wavelengths associated with a photon and an electron having same momentum are the same.

Reason : de Broglie wavelength associated with an electron is

$$\lambda_e = \frac{h}{p} \qquad \text{…………(1)}$$

Momentum of a photon, $p = \frac{h}{\lambda_{ph}} \Rightarrow \lambda_{ph} = \frac{h}{p}$ …………(2)

From eqns. (1) and (2) it is clear that for same momentum p, $\lambda_e = \lambda_{ph}$

Let a photon and an electron possess same energy E.

Energy of a photon $E = h\nu = \dfrac{hc}{\lambda_{ph}}$

$\therefore$ de-Broglie wavelength.

$$\lambda_{ph} = \frac{hc}{E} \qquad \text{……….(3)}$$

For an electron of energy E,

$$\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \qquad \text{……….(4)}$$

$\therefore$ Dividing eqn. (3) by eqn. (4)

$$\frac{\lambda_{ph}}{\lambda_e} = \frac{hc/E}{h\sqrt{2mE}} = c\sqrt{\frac{2m}{E}} = \sqrt{\frac{2mc^2}{E}}$$

7. **Calculate the voltage that must be applied to an electron microscope to produce electrons of wavelength 0.50 A^0? Given** $h = 6.62 \times 10^{-34}$ joule-sec., $m = 9 \times 10^{-31}$ kg. $e = 1.6 \times 10^{-19}$ C

Solution –

The de-Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

But E = eV, where V is voltage in volts,

so that $\lambda = \dfrac{h}{\sqrt{2mE}}$

Here $h = 6.62 \times 10^{-34}$ joule-sec., $m = 9 \times 10^{-31}$ kg.
$e = 1.6 \times 10^{-19}$ coulomb, $\lambda = 0.5A^0 = 0.5 \times 10^{-10}$ m, V = ?

$$\therefore \quad 0.5 \times 10^{-10} = \frac{6.62\times10^{-34}}{\sqrt{2\times9\times10^{-31}\times1.6\times10^{-19}V}}$$

or

$$V = \frac{(6.62\times10^{-34})^2}{(0.5\times10^{-10})^2\times2\times9\times10^{-31}\times1.6\times10^{-19}}$$

$= 608.7$ volts.

8. **Calculate the wavelength of thermal neutrons at 27^0C; assuming energy of a particle at absolute temperature T is of the order of kT, where k is Boltzmann's constant.**

Solution –

The de-Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

But $E = kT$ $\quad\therefore\quad$ $\lambda = \dfrac{h}{\sqrt{2mE}}$

$h = 6.62 \times 10^{-34}$ joule-sec
$m = 1.67 \times 10^{-27}$ kg
$k = 1.38 \times 10^{-23}$ joule per K
$T = 27^0C = (273 + 27)\ K = 300\ K$

$\therefore \qquad \lambda = \dfrac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.38\times10^{-23}\times300}}$

$= 1.78 \times 10^{-10}\,\text{m} = 1.78\ \text{A}^0$

9. Calculate the de-Broglie wavelength of an α – particle accelerated through a potential difference of 200 volts. Given mass of proton = 1.67 × 10 $^{-27}$ kg,. Planck's constant = 6.62 × 10 $^{-34}$ joule second.

Solution –

The de-Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

Here $\qquad E = qV$ joules

where V is in volts and q is the charge on α - particle.

But $\qquad q = 2e \qquad \therefore E = 2eV$

so that $\qquad \lambda = \dfrac{h}{\sqrt{2m\times2eV}} = \dfrac{h}{\sqrt{4meV}}$

Hence m = mass of α - particle = 4 × mass of proton

= 4 × 1.67 × 10 $^{-27}$ kg.

e = 1.6 × 10 $^{-19}$ coulomb, h = 6.62 × 10 $^{-34}$ joule-sec, V = 200 volts

$\therefore \qquad \lambda = \dfrac{6.62\times10^{-34}}{\sqrt{4\times4\times1.67\times10^{-27}\times1.6\times10^{-19}\times200}} = \dfrac{6.62\times10^{-34}}{9.25\times10^{-22}}$

$= 7.2 \times 10^{-13}\,\text{m} = 7.2 \times 10^{-3}\ \text{A}^0.$

10. A particle of rest mass m_0 has a kinetic energy K. Calculate the value of its de Broglie wavelength. What will be its value if $K << m_0 c^2$?

Solution –

The relativistic relation between momentum and energy is

$$E^2 = p^2c^2 + m_0^2c^4$$

Total Energy E = kinetic energy + rest $= K + m_0c^2$

$\therefore \qquad (K + m_0c^2)^2 = p^2c^2 + m_0^2c^4$

$\Rightarrow \qquad K^2 + m_0^2c^4 + 2Km_0c^2 = p^2c^2 + m_0^2c^4$

$\Rightarrow \qquad p^2c^2 = K^2 + 2K\,m_0c^2 = K\,(K + 2m_0c^2)$

$\therefore \qquad p^2 = \dfrac{K(K+2m_0c^2)}{c^2}$

Momentum, $\qquad p = \dfrac{\sqrt{K(K+2m_0c^2)}}{c}$

$\therefore$ de Broglie wavelength, $\qquad \lambda = \dfrac{h}{p} = \dfrac{hc}{\sqrt{K(K+2m_0c^2)}}$

$$= \frac{hc}{\sqrt{\left[2m_0c^2K\left[1+\dfrac{K}{2m_0c^2}\right]\right]}}$$

$$= \frac{h}{\sqrt{2m_0K\left[1+\frac{K}{2m_0c^2}\right]}}$$

Special Case : If K < < m_0 c^2, then $\frac{K}{2m_0c^2} << 1$ so above equation takes the form

$$\lambda = \frac{h}{\sqrt{2m_0K}} \Rightarrow \lambda = \frac{h}{\sqrt{2m_0K}}$$

This is usual non-relativistic case.

11. Show that the de-Brgolie wavelength for a material particle of rest mass m_0 and charge q, accelerated from rest through a potential difference of V volts relativistically is given by

$$\lambda = \frac{h}{\sqrt{2m_0qV\left(1+\frac{qV}{2m_0c^2}\right)}}$$

Solution:

In this case kinetic energy K = qE but K $\neq \frac{1}{2}mv^2$,m since velocity is relativistic. So, we can not find momentum, directly form E_k. Instead, we use the relativistic formula.

we have

$$E^2 = p^2c^2 + m_0^2c^4,\ E = K + m_0c^2$$

so that $p^2c^2 = E^2 - m_0^2c^4 = (K + m_0c^2)^2 - m_0^2c = K^2 + 2Km_0c^2$

$\therefore$ $p^2 = 2m_0K\left(1+\frac{K}{2m_0c^2}\right)$ or p = $\sqrt{2m_0K\left(1+\frac{K}{2m_0c^2}\right)}$

$\therefore$ dc Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0K\left(1+\frac{K}{2m_0c^2}\right)}}$$

Now substituting, K = qV

$$\lambda = \frac{h}{\sqrt{2m_0qV\left(1+\frac{qV}{2m_0c^2}\right)}}$$

12. A beam of neutrons at 27^0C is allowed to fall on a crystal. A first order reflection is observed at a glancing angle 30^0, calculate the inter-planer spacing of the crystal. Given Planck's constant, h = 6.62×10^{-34} joule-sec.

Mass of neutron, m = 1.67×10^{-27} kg.

Boltzmann's constant, k = 1.38×10^{-23} joule per K.

Solution.

According to Bragg's law

$$2d \sin\theta = n\lambda$$

Given $\theta = 30^0, n = 1$

$\therefore$ 2d sin 30^0 = λ or $\lambda = d$

The energy of the neutron E = k T.

where k is Boltzmann's constant and T is absolute temperature.

Here T = 27^0C = (273 + 27) K = 300 K

k = 1.38×10^{-23} joule per K

$\because$ E = $1.38 \times 10^{-23} \times 300 = 4.14 \times 10^{-21}$ joule

We know p = $\sqrt{(2mE)}$

$\therefore$ p = $\sqrt{(2 \times 1.67 \times 10^{-27} \times 4.14 \times 10^{-21})}$

so that d = $\lambda = \frac{h}{p} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 4.14 \times 10^{-21}}}$

= $1.78 \times 10^{-10} m$ = 1.78 A^0.

13. Electrons are accelerated through 344 volts and are reflected from a crystal. The first reflection maximum occurs when glancing angle is 60^0. Determine the spacing of the crystal. Given h = 6.63×10^{-34} joule sec., e = 1.6×10^{-19} coulomb, m = 9.1×10^{-31} kg.

Solution –

The wavelength of an electron of mass m associated with an electron of kinetic energy E is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

If V is the accelerating potential, then

E = eV joule

so that $\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 344}} = 6.63 \times 10^{-11} m$

According to Bragg's law

2d sin $\theta = n\lambda$

Here $\theta = 60^0, n = 1$ and $\lambda = 6.63 \times 10^{-11} m$

$\therefore$ 2d sin 60^0 = $1 \times 6.63 \times 10^{-11}$

or 2d. $\sqrt{\frac{3}{2}} = 6.63 \times 10^{11}$

or d = $\frac{1}{\sqrt{3}} \times 6.63 \times 10^{-11} = 0.26 \times 10^{10} m$

= 0.26 A^0

14. **How does the concept of Bohr's orbit violate the uncertainty relation ? Explain.**

Solution :

According to uncertainty relation between energy and time, $\Delta E \Delta t \approx \hbar$.

According to Bohr's theory an electron revolves in a quantized orbit, and therefore possesses well defined energy, with no uncertainty i.e., $\Delta E = 0$

$\therefore$ $\Delta t = \frac{\hbar}{\Delta E} = \frac{\hbar}{0} =$ infinite

This physically means that all energy states of atom must have infinite life time; but experimental observations show that the excited states of atom has a life time of the order of 10^{-8} sec. Thus the concept of Bohr's orbit violates the uncertainty principle.

15. Calculate the zero point energy of a linear harmonic oscillator of frequency 50 Hz.

Solution –

Zero point energy of linear harmonic oscillator

$$E = \frac{1}{2}\hbar\omega_0 = \frac{1}{2}\left(\frac{h}{2\pi}\right)(2\pi r) = \frac{1}{2}h\nu_0$$

Here, $h = 6.63 \times 10^{-34}$ J – s

Frequency, $v = 50$ Hz

$$\therefore \quad E = \frac{1}{2}\times 6.63\times 10^{-34}\times 50 = 1.66\times 10^{-32}\,J$$

16. Calculate the lowest possible uncertainty in the position of an electron having velocity 3×10^{7} m/s.

Solution –

Let $(\Delta x)_{min}$ be the minimum uncertainty in the position of the electron and $(\Delta p)_{max}$ the maximum uncertainty in the momentum of the electron. Then, we must have $(\Delta x)_{min}\ (\Delta p)_{max} \approx \hbar$.

But $(\Delta p)_{max} = p$ (momentum of the electron)

$$= mv = \frac{m_0 v}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$ (m_0 being rest mass of the electron)

Substituting this in above equation, we get :

$$(\Delta x)_{min} = \frac{\hbar}{m_0 v/\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

cr

$$(\Delta x)_{min} = \frac{\hbar\sqrt{\left(1-\frac{v^2}{c^2}\right)}}{m_0 v}$$

We have $\hbar = 1.05 \times 10^{-34}$ joule/ sec., $v = 3 \times 10^{7}$ m /s

$m_0 = 9 \times 10^{-31}$ kg. $c = 3 \times 10^{8}$ m/s.

$$\therefore \quad (\Delta x)_{min} = \frac{1.05\times 10^{-34}\sqrt{\left\{1-\frac{(3\times 10^{7})^2}{(3\times 10^{8})^2}\right\}}}{9\times 10^{-31}\times 3\times 10^{7}}$$

$$= \frac{1.05\times 10^{-31}\times 0.99}{9\times 10^{-31}\times 3\times 10^{7}} = 3.8\times 10^{-9}\,m = 38A^0$$

17. Calculate the uncertainty in velocity of electron which is confined in a box of length $10A^0$. Given $m = 9 \times 10^{-31}$ kg., $\hbar = 1.05\times 10^{-34}$ joule second.

Solution –

According to uncertainty relation, $\Delta x \Delta p = \hbar$, so that if Δx is maximum, Δp must be minimum, i.e., ($\Delta x)_{max} (\Delta p)_{min} = \hbar$

Given ($\Delta x)_{max}$ = maximum uncertainty in position = 10 A^0 = 10^{-9} m, $\hbar = 1.05 \times 10^{-34}$ Js.

So that, we have $(\Delta p)_{min} = \dfrac{\hbar}{(\Delta x)_{max}} = \dfrac{1.05 \times 10^{-34}}{10^{-9}} kg - m/s$

$= 1.05 \times 10^{-25}$ kg – m/s

But $(\Delta p)_{min} = m(\Delta v)_{min}$ so that, we have m $(\Delta v)_{min} = 1.05 \times 10^{-25}$

or $$(\Delta v)_{min} = \frac{1.05 \times 10^{-25}}{m} = \frac{1.05 \times 10^{-25}}{9 \times 10^{-31}}$$

$= 1.17 \times 10^{5}$ ms^{-1}

s

18. Find the uncertainty in the momentum of a particle when its position is determined within 0.01 cm. Find also the uncertainty in the velocity of an electron and an $\alpha -$ *particle* respectively when they are located within 5 × 10^{-8} cm.

Solution –

According to Heisenberg' uncertainty principle

$$\Delta x \Delta p \approx \hbar$$

$$\therefore \quad \Delta p = \frac{\hbar}{\Delta x}$$

Here $\hbar = 1.05 \times 10^{-34}$ joule –second. Δx = 0.01 × 10^{-2} meter.

$$\therefore \quad \Delta p = \frac{1.05 \times 10^{-34}}{0.01 \times 10^{-2}} = 1.05 \times 10^{-30} \text{ kg m / s}$$

.

In Δv is the uncertainty is the velocity of particle of mass m, we have

$$\Delta p = m \Delta v$$

$\therefore$ Uncertainty principle may be written as $\Delta x . m \Delta v = \hbar$

$$\therefore \quad \Delta v = \frac{\hbar}{m \Delta x}$$

Uncertainty in the velocity of electron :

Hence $\hbar = 1.05 \times 10^{-34}$ joule second, m = 9 × 10^{-31} kg., and

Uncertainty in the position of electron Δx = 5 ×$10^{-10} m$

$$\therefore \quad \Delta v = \frac{1.05 \times 10^{-34}}{9 \times 10^{-31} \times 5 \times 10^{-10}} = 2.33 \times 10^{5} \text{ m/s.}$$

Uncertainty in the velocity of α - particle:

Here mass of $\alpha -$particle = 4 × mass of proton

= 4 × 1.67 × 10^{-27} = 6.68 × 10^{-27} = 6.68 × 10^{-27} kg.

$$\Delta x = 5 \times 10^{-10} m$$

$$\therefore \quad \Delta v = \frac{1.05 \times 10^{-34}}{6.68 \times 10^{-27} \times 5 \times 10^{-10}} = 31.4 m/s$$

19. The speed of an electron is measured to be 5.00 × 10^{3} m/s to an accuracy of 0.003%. Find the uncertainty in determining position of this electron. Given h = 6.63 × 10^{-34} J-s. Mass of electron = 9.11 × 10^{-31} kg.

Solution –

The uncertainty in velocity = $\frac{0.003}{100} \times 5.00 \times 10^3$ m/sec. = 0.15 m /s

$\because$ The uncertainty in momentum $\Delta p = m\Delta v$

= $9.11 \times 10^{-31} \times 0.15 = 1.366 \times 10^{-31}$ kg. m/s

$$\hbar = \frac{h}{2\pi} = \frac{6.63 \times 10^{-34}}{2 \times 3.14} = 1.055 \times 10^{-34} J - s$$

But $\Delta p \Delta x = \hbar$

$$\therefore \quad \Delta x = \frac{\hbar}{\Delta p} = \frac{1.055 \times 10^{-34}}{1.366 \times 10^{-31}} m$$

= 7.72×10^{-4} m.

20. A hydrogen atom is 0.53 A^0 in radius. Use uncertainty principle to estimate the minimum energy an electron can have in this atom. [Rest mass of electron = 9.1×10^{-31} kg]

Solution –

Given radius of hydrogen atom, r = 0.53 A^0 = 0.53×10^{-10} m.

Uncertainty in the position of electron, $\Delta x = 2\pi r$

If Δp is uncertainty in momentum, then

$$\Delta x \ \Delta p \approx \hbar \quad \text{or} \quad \Delta p \approx \frac{\hbar}{\Delta x}$$

This is minimum momentum

$$\therefore \quad \Delta p_{min} = \frac{\hbar}{\Delta x} = \frac{\hbar}{2\pi r}$$

$$= \frac{1.05 \times 10^{-34}}{2 \times 3.14 \times 0.53 \times 10^{-10}} kg - m/s$$

= 3.15×10^{-25} kg m / s

$\therefore$ Minimum kinetic energy, $K_{min} = \frac{\Delta p^2_{min}}{2m}$

$$= \frac{(3.15 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} = 5.46 \times 10^{-20} \text{ joule.}$$

Total energy of electron in hydrogen atom is sum of kinetic and potential energy ; which is equal to negative of kinetic energy.

Thus minimum energy of electron in hydrogen atom = -5.46×10^{-20} joule

The uncertainty relation gives only the order of magnitude of energy, which is

$E_{min} = -10^{-19}$ joule.

21. The average time that an atom retains excess excitation energy before re-emitting it in the form of electromagnetic radiations is 10^{-8} second. Calculate the limit of accuracy with which the excitation energy of the emitted radiation can be determined.

Solution-

According to uncertainty principle $\Delta E.\Delta t \approx \hbar$, where ΔE is the uncertainty in energy and Δt that in time.

Given $\Delta t = 10^{-8}$ second.

$\therefore \quad \Delta E = \frac{\hbar}{\Delta t} = \frac{1.05\times10^{-34}\ joule-\sec ond}{10^{-8}} = 1.05\times10^{-26}$ joule-second

$= \frac{1.05\times10^{-26}}{1.6\times10^{-19}} eV = 6.56 \times 10^{-8}$ eV

$\therefore$ Uncertainty in energy $= 6.56 \times 10^{-8}$ eV.

22. The average life time of hydrogen in excited state is 2.5 × 10^{-14} second, calculate the uncertainty in the measurement of energy in this state.

Solution-

According to the uncertainty principle

$$\Delta E.\Delta t \approx \hbar ,$$

where ΔE is the uncertainty in energy and Δt that in time.

Given $\Delta t = 2.5\times10^{-14} \sec ond$

$\therefore \quad \Delta E = \frac{\hbar}{\Delta t} = \frac{1.05\times10^{-34}\ joule-\sec ond}{2.5\times10^{-14}\sec ond}$

$= 4 \times 10^{-21}$ joule-second

$= \frac{4\times10^{-21}}{1.6\times10^{-19}} eV = 2.5 \times 10^{-2} = 0.025$ eV

$\therefore$ Uncertainty in energy = 0.025 eV.

23. An electron moving with velocity 30m/s. This velocity has an accuracy of99.99%. Determine the uncertainty in locating its position.

Solution:

According to Heisenberg' uncertainty principle:

$$\Delta x \Delta p \approx \hbar$$

$\therefore$ Since the velocity of electron has an accuracy of99.99% ,the Uncertainty in it will be 0.01%.

The momentum p of the electron = $9.0 \times 10^{-31} \times 30 = 2.7\times10^{-29}$ kgm/s

Therefore the uncertainty in the momentum will be:

$$\Delta p = \frac{0.01}{100}\times 2.7\times10^{-29} = 2.7\times10^{-33} \text{ kgm/s}$$

Therefore the uncertainty in the position of electron:

$$\Delta x = \frac{h}{2\pi\Delta p} = \frac{6.63\times10^{-34}}{2\pi\times2.7\times10^{-33}}$$

$$= 3.8\times10^{-2}\text{m.}$$

24. **A microscope using photons is used to locate an electron in an atom within a distance of 0.1A^0. Find the uncertainty in the momentum of the electron.**

Solution:

According to Heisenberg' uncertainty principle:

$$\Delta x \Delta p \approx \hbar$$

$\therefore$

Therefore the uncertainty in the momentum will be:

, $$\Delta p = \frac{h}{2\pi\Delta x} = \frac{6.63\times10^{-34}}{2\times3.14\times0.1\times10^{-10}}$$

$$= 1.05\times10^{-24} kgm/s$$

25. If the uncertainty in the location of a particle is equal to de Broglie wavelength, what will be the uncertainty in the velocity?

Solution:

From the uncertainty principle we have:

$$\Delta x\Delta p \approx \hbar$$

So, $$\Delta p = \frac{h}{2\pi\Delta x}$$

But, from the problem; $\Delta x = \lambda = \frac{h}{p}$

On substituting this above $\Delta p = m\Delta v = \frac{h}{2\pi\frac{h}{mv}}$

Therefore the uncertainty in the velocity; $\Delta v = v$

26 **.Estimate the minimum uncertainty in measurement of the frequency of a photon whose life time is about 10^{-8}sec.**

Solution:

According to the Heisenberg uncertainty principle

$$\Delta E.\Delta t \approx \hbar ,$$

where ΔE is the uncertainty in energy and Δt that in time.

Given $\Delta t = 10^{-8}\ \sec ond$

$\therefore$ $$\Delta E = h\Delta\nu = \frac{\hbar}{\Delta t} = \frac{1.05\times10^{-34}\ joule-\sec ond}{10-8}$$

$$= 10^{8}\ \text{Hz} \quad \text{Approx.}$$

27. The width of a spectral line having wavelength 5000A^{0} is found to be 10^{-4}A^{0} . Determine the minimum time required for the atomic system to retain the corresponding excitation energy.

Solution:

According to the Heisenberg uncertainty principle

$$\Delta E.\Delta t \approx h ,$$

So, hEE $$\Delta t = \frac{h}{\Delta E}$$

Since, $$E = h\nu = \frac{hc}{\lambda}$$

$$\Delta E = h\nu = \frac{hc}{\lambda^2}d\lambda$$

Therefore,

$$\Delta t = \frac{h}{\Delta E} = \frac{h\lambda^2}{2\pi hcd\lambda} = \frac{\lambda^2}{2\pi cd\lambda}$$

On substituting the values of parameters above:

$$\Delta t = \frac{(5.0\times10^{-7})^2}{2\pi\times3.0\times10^8\times10^{-14}} = 1.32\times10^{-8}\text{ sec}$$

28. The average life time of an excited state is 10 $^{-12}$second; calculate the uncertainty in the measurement of energy of $\gamma - ray\ photon$ in this state.
Solution-

Using Heisenberg's uncertainty principle:

$$\Delta E.\Delta t \approx \hbar\ ,$$

where ΔE is the uncertainty in energy and Δt that in time.

Given $\Delta t = 10^{-12}\text{ sec}ond$

$$\therefore \qquad \Delta E = \frac{\hbar}{\Delta t} = \frac{1.05\times10^{-34}\ joule - \sec ond}{10^{-12}\sec ond}$$

= 1.05 × 10 $^{-12}$ joule

29. If the center of hydrogen atom be located with a precision of 0.01A^0, find the corresponding uncertainty in the velocity.

Solution:

From the uncertainty principle we have:

$$\Delta x\Delta p \approx \hbar$$

So, $$\Delta p = m\Delta v = \frac{h}{2\pi\Delta x}$$

Therefore, $$\Delta v = \frac{h}{2\pi m\Delta x}$$

Given, $\Delta x = 0.01\ A\ \ and\ mass\ of\ H - atom\ m = 1.67\times10^{-27}kg$

Therefore, $$\Delta v = \frac{6.63\times10^{-34}}{2\times3.14\times1.67\times10^{-27}\times10^{-10}}$$

$$= 6.3\times10^4 m/s$$

Therefore the uncertainty in the velocity of electron in H-atom is $6.3\times10^4 m/s$.

30. How many photons of green light with wavelength 4800A^0 constitute 2.0 joules of energy?

Solution:

Given $\lambda = 4800A^0$ and Energy of photons = 2.0 Joules

Energy of photons is given by $E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3.0\times10^{8}}{4800\times10^{-10}}$

$= 4.13 \times 10^{-19} joules$

Given; $n \times Energy\ of\ photons = 2.0$

Therefore, $n = \frac{2.0}{4.13\times10^{-19}} = 4.84 \times 10^{18}$

31. The de-Broglie wavelength of an electron is 0.1A^0 Find the potential difference by which the electron should be accelerated .

Solution:

Given $\lambda = 0.1 \times 10^{-10} m$

de-Broglie wavelength of an electron is given by:

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Or,
$$2mqV = \frac{h^2}{\lambda^2}$$

Therefore, $$V = \frac{h^2}{2mq\lambda^2} = \frac{(6.63\times10^{-34})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times(10^{-11})^2}$$

$$= 15.05\text{kV}$$

32. If the electron beam in the TV picture tube is accelerated to 10kV,find the de-Broglie wavelength associated with the electron.

Solution:

de-Broglie wavelength associated with electron is given by:

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 10000}}$$

$= 0.128 \times 10^{-10}$

= 0.128A^0

Summary

The topics covered in this chapter are summarized below:

1. Louis de-Broglie proposed that a material particle in motion present wave-like properties and the wavelength associated with it is given by
$$\lambda = \frac{h}{p} = \frac{h}{m\upsilon}$$
2. de-Broglie wavelength associated with moving material particles with kinetic energy K is
$$\lambda = \frac{h}{\sqrt{2mK}}$$
3. For charged particles accelerated through a potential difference V; $\lambda = \dfrac{h}{\sqrt{2mqV}}$
4. A material particle is not equivalent to a single wave but it is equivalent to a group of waves or wave packet; which travels with group velocity. The group velocity of wave packet is equal to the particle velocity.
5. Relation between have velocity an particle velocity is $\upsilon_p = \dfrac{c^2}{\upsilon}$.
6. Relation between phase velocity (v_p) and group velocity υ_g of wave-packet is $\upsilon_p \upsilon_g = c^2$
7. General relation between phase velocity (u) an group velocity υ_g is $\upsilon_g = \upsilon_p - \lambda \dfrac{\upsilon_p}{d\lambda}$.
8. Heisenberg's Uncertainty Relation: It states that the product of uncertainties in finding the position and momentum of particle simultaneously is of the order of $\hbar\left(=\dfrac{h}{2\pi}\right)$ i.e.,
$$\Delta x \Delta p \approx \frac{\hbar}{2}$$
$$\Delta E \Delta t \approx \frac{\hbar}{2}$$
9. The wave nature of material particles was first proposed by Louis de-Broglie.
10. All material particles, charged or uncharged, show wave nature.
11. The first experimental evidence for wave nature of matter was given by Davisson and Germer. It is diffraction experiment with slow electrons.
12. A material particle is not equivalent to a single wave, but it is equivalent to a wave packet (or group of waves). The velocity of wave packet is called group velocity (υ_g). It is equal to particle velocity.
$\upsilon_p \upsilon_g = c^2$ (υ_p = phase velocity, υ_g = group velocity)
13. The Schrödinger equation is used to locate the particle within the wave packet.
14. The uncertainty principle holds for all systems/ particles (microscopic and macroscopic) and is independent of experimental technique.
15. According to uncertainty principle the position and momentum of a particle cannot be exactly determined simultaneously with high degree of accuracy.
16. According to uncertainty principle electrons cannot exist in the atomic nucleus.
17. A free particle can have any energy; while a particle confined in a box can have only discrete values of energy.

Exercises

1. What is de-Broglie concept of stationary orbits? Are the de-Broglie stationary orbits different from Bohr's stationary orbits?
2. Show that de-Broglie wavelength is a function of wavelength even in free space.
3. State and explain and prove Heisenberg's uncertainty principle.
4. Derive Heisenberg's uncertainty principle. Use this principle to prove that electrons cannot exist in the nucleus.
5. Calculate radius of Bohr's first orbit by the help of uncertainty principle.
6. What is de-Broglie hypothesis? How has it been verified experimentally?
7. What are de-Broglie waves? Show that the de-Broglie wavelength of a particle of momentum p is (h/p).
8. What are matter waves? Obtain an expression for the wavelength associated with moving material particles.
9. What are matter waves? Show that the wavelength λ associated with a particle of mass m and kinetic energy E is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

10. Describe Davisson and Germer experiment to demonstrate the wave nature of particles.
11. Derive the formula for the de-Broglie wavelength of a particle in terms of kinetic energy and its rest energy m_0c^2.
12. Discuss the basic results which led to the formulation of the wave concept of matter. What are de-Broglie waves? Discuss their wavelength and explain how they afford a correct description of the motion of a particle.
13. Explain de-Broglie waves? How do they help in interpretation of the Bohr's quantization rule?
14. Can a photon and an electron of the same momentum have the same wavelength? Compare their wavelengths if the two have the same energy.
15. Prove that the velocity of matter waves is found to be greater than the velocity of the light.
16. Define phase velocity and group velocity .Show that for no-relativistic free particle the phase velocity is half of the group velocity.
17. Show that the phase velocity of de-Broglie waves is greater than the velocity of light, but the group velocity is equal to the velocity of the particle with which the waves are associated.
18. Discuss the significance of Heisenberg's uncertainty principle and prove the relation $\Delta x.\Delta p=h$.
19. State and explain Heisenberg uncertainty principle. Give two examples to illustrate the principle.

OBJECTIVE QUESTIONS

1. **The de-Broglie hypothesis is associated with**:
(a) wave nature of electron only (b) wave nature of α - particles only
(c) wave nature of radiations (d) wave nature of all material particles.

2. **Neglecting variation of mass with velocity, the wavelength associated with electron having a kinetic energy, E is proportional to :**
(a) $E^{1/2}$ (b) E (c) $E^{-1/2}$ (d) E^{-2}

3. **Matter waves :**
(a) are longitudinal (b) are electromagnetic
(c) always travel with speed of light (d) show diffraction

4. **If a charged particle of mass m is accelerated through a p.d. of V volts, the de-Broglie wavelength is proportional to :**
(a) V (b) $V^{-1/2}$ (c) V^2 (d) $V^{1/2}$

5. **According to Schrödinger a particle is equivalent to a :**
(a) a single wave (b) a wave packet
(c) light wave (d) can not behave as wave.

6. Of the following moving with the same velocity, the one which has largest wavelength is :
(a) an electron (b) a photon (c) a neutron (d) an α - particle

7. Of the following having the same kinetic energy, which has the longest wavelength :
(a) an electron (b) proton (c) neutron (d) α - particle

8. The expression $|\psi(r,t)|^2$ stands for :
(a) position (b) position probability density
(c) normalisation (d) time probability density

9. The phase velocity (v_p) and the group velocity (v_g) of a de-Broglie wave in free space (speed of light = c) are related as :
(a) $\frac{\upsilon_p}{\upsilon_g} = \sqrt{2}$ (b) $\upsilon_p \upsilon_g = c^2$ (c) $\frac{\upsilon_p}{\upsilon_g} = 0.5$ (d) $\upsilon_p \upsilon_g = \sqrt{2}\, c^2$

10. The de-Broglie wavelength of material particles which are in thermal equilibrium at temperature T is :
(a) $\frac{h}{\sqrt{2mkT}}$ (b) $\frac{\hbar}{\sqrt{2mkT}}$ (c) $\frac{h}{\sqrt{mkT}}$ (d) $\frac{h}{\sqrt{2kT}}$

11. Which one of the following pairs of phenomena illustrates the particle aspect of wave particle duality?
(a) Compton effect and Bragg's law (b) Photoelectric effect and Compton effect
(c) Compton effect and Pauli's principle (d) Bragg's law and photoelectric effect

12. An electron of mass m and charge e initially at rest gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t, ignoring relativistic effects is :
(a) $-\frac{h}{eEt^2}$ (b) $-\frac{eht}{E}$ (c) $-\frac{mh}{eEt^2}$ (d) $-\frac{h}{eE}$

13. The uncertainty relation holds for :
(a) microscopic particles only (b) macroscopic particles only
(c) microscopic and macroscopic particles both (d) neither microscopic nor macroscopic particles

14. According to uncertainty relation the minimum uncertainty in the velocity of electron orbiting around the nucleus of radius r is :

(a) $\frac{\hbar}{2\pi mr}$ (b) $\frac{\hbar}{2mr}$ (c) 2hmr (d) 0

15. The duration of radar pulse is 10^{-6} s. The uncertainty in its energy would be :
(a) 0 (b) 1.05×10^{-35} J (c) 1.05×10^{-28} J (d) 1.05×10^{-21} J

16. Let uncertainty relation be $\Delta x \Delta p \approx h$. If such of nucleus is 10^{-15} m, the uncertainty in the momentum of proton remaining within the nucleus is of the order of :
(a) 6.62×10^{-19} kg – m/s (b) 6.62×10^{-49} kg .ms^{-1}
(c) 10^{-23} kg m / s (d) 10^{-27} kg-m/s

17. The uncertainty relation cannot hold for the following pairs :
(a) position and momentum (b) energy and time
(c) linear momentum and angle (d) angular momentum and angle

18. The wavelength λ associated with a particle of mass m moving with velocity v is given by:
(a) $\lambda = \frac{h}{mv}$ (b) $\lambda = \frac{m}{hv}$
(c) $\lambda = \frac{mv}{h}$ (d) $\lambda = \frac{hv}{m}$

==19. The de-Broglie hypothesis is associated with:
(a) wave nature of electrons only
(b) wave nature of α-particle only
(c) wave nature of radiations
(d) wave nature of all materials particles.

20. The equation of motion of matter wave was derived by:
(a) Heisenberg (b) Bohr
(c) de-Broglie (d) Schrödinger.

21. If the momentum of a particle is increased to four times, then the de-Broglie wavelength will become:
(a) two times (b) four times
(c) half times (d) one fourth times

22. de-Broglie wavelength of an electron which has been accelerated from rest through a potential difference of 100 V is:
(a) 12.27Å (b) 1.227Å
(c) 15Å (d) 1.5Å

23. The wavelength of the matter waves is independent of:
(a) mass (b) velocity
(c) charge (d) momentum

24. The rest mass of a photon is:
(a) zero (b) 1.67×10^{-31}kg
(c) 1.9×10^{-27}kg (d) infinity

25. A photon and an electron have the same wavelength then:
(a) photon has greater momentum (b) electron has greater momentum
(c) both have the same momentum (d) equal to the phase velocity

26. Uncertainty principle states that the error in measurement is due to:
(a) dual nature of particles (b)due to small size of particles
(c) due to large size of particles (d) due to error in measuring instrument

27. The product of uncertainties between position and momentum is given by:
(a) ΔxΔp= 1 (b) ΔxΔp = h
(c) ΔxΔp= nh (d) ΔxΔp = mv

28. If the uncertainty in the location of a particle is equal to de-Broglie wavelength, the uncertainty in its

velocity will be:

(a) equal to its velocity
(b) half of its velocity
(c) twice its velocity
(d) four times its velocity

(For this assume $\Delta x.\Delta p \sim h$)

29. Which of the following can act as both a particle and as a wave ?

(a) photon
(b) electron
(c) neutron
(d) all of these

30. Which of the following phenomena cannot be expressed by wave nature of light?

(a) Interference
(b) Diffraction
(c) Polarization
(d) photoelectric effect

31. An α- particle and a photon have the same kinetic energy. The ratio of their wavelength is ($m\alpha = 4m_p$)

(a) 1:2
(b) 2:1
(c) 1:4
(d) 4:1

Answers

1. (d) 2. (c) 3. (d) 4. (b) 5. (b) 6. (a) 7. (a) 8. (b) 9. (b) 10. (a) 11. (b) 12. (a) 13. (c) 14. (a) 15. (c) 16. (a) 17. (c) 18. (a) 19. (d) 20. (d) 21. (d) 22. (b) 23. (c) 24. (a) 25. (c) 26. (a) 27. (b) 28. (a) 29. (d) 30. (d) 31. (c)

Fill in the Blanks

1. **The de-Broglie hypothesis is associated with** dual nature of all material particles and radiation.
2. **Matter waves** which are supposed to be associated with moving material particle show diffraction.
3. **According to Schrodinger a particle is equivalent to a wave group.**
4. The expression $|\psi(r,t)|^2$ stands for time probability density of finding a particle.
5. The phase velocity (v_p) and the group velocity (v_g) of a de-Broglie wave in free space (speed of light = c) are related as $\upsilon_p \upsilon_g = c^2$.
6. The equation of motion of matter wave was derived by Erwin Schrödinger.
7. The wavelength of the matter waves is independent of charge.
8. The rest mass of a photon is zero, still it is regarded as material particle.
9. A photon and an electron have the same wavelength then both have the same momentum.
10. Uncertainty principle states that the error in measurement is due to dual nature of particles .