Matrices

Definition ⇒ A system of mn numbers arranged in a ractangular array formation along m rows and n columns and bounded by the brackets is called an $m \times n$ matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

① Row Matrices ⇒ A matrix having a single row is called row matrix

$$\begin{bmatrix} 1 & 3 & 5 & 7 \end{bmatrix}$$

② Column Matrices ⇒ A matrix having a single column is called column matrix.

$$\begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix}$$

{Note ⇒ Row and column matrices are sometimes called row vector and column vector}.

③ Square Marix ⇒ A matrix having n rows & n columns is called a square matrix of order n.

A ⇒ square matrix

* The determinant having the same element as the square matrix A is called the determinant of the matrix and is denoted by $|A|$

④ The diagonal of this matrix containing the elements 1, 3, 5 is called the leading diagonal or principal diagonal.

$\Rightarrow A' = A$

$$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

(8) Skew - Symmetric Matrix $\Rightarrow$ If $a_{ij} = -a_{ji}$ for all i & j so that all the leading diagonal elements are zero. then the matrix is called skew-symmetric.

$$\begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ -g & -f & 0 \end{bmatrix}$$

(i) $A' = -A$
(ii) diagonal $= 0$

(9) Triangular Matrices $\Rightarrow$ (i) A square matrix all of whose elements below the leading diagonal are zero, is called an upper triangular matrix.

$$\begin{bmatrix} a & h & g \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix} \Rightarrow \text{Upper triangular}$$

(ii) A square matrix all of whose elements above the leading diagonal are zero, is called an ~~upper~~ lower triangular matrix.

$$\begin{bmatrix} a & 0 & 0 \\ h & b & 0 \\ g & f & c \end{bmatrix} \Rightarrow \text{Lower triangular.}$$

(10) Transpose of a Matrix $\Rightarrow$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 5 \\ 6 & 7 & 8 \end{bmatrix}, \quad A' = \begin{bmatrix} 2 & 1 & 6 \\ 3 & 0 & 7 \\ 4 & 5 & 8 \end{bmatrix}$$

(11) Orthogonal Matrix $\Rightarrow$ A square matrix A is said to be orthogonal

if $\boxed{AA' = A'A = I}$

COMPLEX MATRICES

(12) Conjugate of a Matrix $\Rightarrow$

$$A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & i & 3+2i \end{bmatrix}$$

(13) Conjugate Transpose of Matrix $(A^{\theta}) \Rightarrow$

$$A^{\theta} = (\bar{A})'$$
$$= \begin{bmatrix} 1-i & 7+2i \\ 2+3i & i \\ 4 & 3+2i \end{bmatrix}$$

(14) Unitary Matrix $\Rightarrow$ $\boxed{A^{\theta}A = AA^{\theta} = I}$

(15) Hermition Matrix $\Rightarrow$ $\boxed{A = A^{\theta}}$

(16) Skew Hermition Matrix $\Rightarrow$ $\boxed{A^{\theta} = -A}$

(17) Idempotent Matrix $\Rightarrow$ $\boxed{A^2 = A}$

(18) Periodic Matrix $\Rightarrow$ $\boxed{A^{k+1} = A}$

k is (+)ive integer
k is called the period of A.

(15) Nilpotent Matrix $\Rightarrow$ $A^k = 0$ where k is (+)ive integer. k is the index of Nilpotent matrix.

(20) ~~Inv~~

- Inversion of a Matrix $\Rightarrow$

If A be any matrix, then a matrix B if it exist such that

$$AB = BA = I ,$$

is called inverse of A.

OR

The inverse of a matrix A is denoted by A^{-1}

$$\boxed{A^{-1} = \frac{adj.A}{|A|}}$$

(4) The inverse of a matrix is unique

(4) $(AB)^{-1} = B^{-1}A^{-1}$

Qs: Find the inverse of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

$A_1 \quad A_2 \quad A_3$

$$adj\,A = \Bigg[\begin{vmatrix} 3 & -3 \\ -4 & -4 \end{vmatrix}, - \begin{vmatrix} 1 & -3 \\ -2 & -4 \end{vmatrix}, \begin{vmatrix} 1 & 3 \\ -2 & -4 \end{vmatrix}, - \begin{vmatrix} 1 & 3 \\ -4 & -4 \end{vmatrix},$$

$$+ \begin{vmatrix} 1 & 3 \\ -2 & -4 \end{vmatrix}, - \begin{vmatrix} 1 & 1 \\ -2 & -4 \end{vmatrix}, + \begin{vmatrix} 1 & 3 \\ 3 & -3 \end{vmatrix}, - \begin{vmatrix} 1 & 3 \\ 1 & -3 \end{vmatrix}$$

$$+ \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix},$$

$= \cancel{0} -24 + 10 + 2 - 8 + 2 + 2 + 6 + 6 + 2$

$$= \begin{vmatrix} -24 & 10 & 2 \\ -8 & 2 & 2 \\ 6 & 6 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} -24 & -8 & -12 \\ 10 & 2 & 6 \\ 2 & 2 & 2 \end{vmatrix}$$

$$|A| = 1(-12-12) - 1(-10) + 3(2)$$

$$= -24 + 10 + 6$$

$$= -8$$

$$= -\frac{1}{8} \begin{bmatrix} -24 & -8 & -12 \\ 10 & 2 & 6 \\ 2 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

Ans.

Qs: ~~A=~~ If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ find A^{-1}

Elementary Tranformation →

Any one of the following operations on a matrix is called an elementary tranformation.

1) Interchanging any two rows (or columns) This tranformation is indicated by R_{ij} if the i^{th} row and j^{th} row are interchanged.

2) Multiplication of the elements of any row R_i (or column) by a non zero scalar quantity k is denoted by (kR_i)

3) Addition of constant multiplication of the elements of any row R_j to the corresponding elements of any other row R_i is denoted by $(R_i + kR_j)$

If a matrix B is obtained from a matrix A by one or more E operations, then B is said to be equivalent to A.

Qus: Find the inverse of the following matrix employing elementary tranformation.

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \qquad \boxed{A \sim I}$$

Solⁿ

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\sim \begin{bmatrix} 1 & -1 & 4/3 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_1 \to R_1/3$$

$$\sim \begin{bmatrix} 1 & -1 & 4/3 \\ 0 & -1 & 4/3 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ -2/3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_2 \to (-2)R_1 + R_2$$

$R_{ij}(\lambda) \rightarrow R_i + \lambda R_j \Rightarrow R_i$
$C_{ij}(\lambda) \rightarrow C_i \rightarrow C_i + \lambda C_j$

$$\sim \begin{bmatrix} 1 & -1 & 4/3 \\ 0 & 1 & -4/3 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ 2/3 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 \rightarrow (-1) R_2$$

$$\sim \begin{bmatrix} 1 & -1 & 4/3 \\ 0 & 1 & -4/3 \\ 0 & 0 & -1/3 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ 2/3 & -1 & 0 \\ 2/3 & -1 & 1 \end{bmatrix} \quad R_3 \rightarrow R_2 + R_3$$

$$\sim \begin{bmatrix} 1 & -1 & 4/3 \\ 0 & 1 & -4/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ 2/3 & -1 & 0 \\ -2 & 3 & -3 \end{bmatrix} \quad R_3 \rightarrow (-3) R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2/3 & -1 & 0 \\ -2 & 3 & -3 \end{bmatrix} \quad R_1 \rightarrow R_1 + R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \quad R_2 \rightarrow \frac{4}{3} R_3 + R_2$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

Ans.

Qs: Find the inverse of the matrix A by applying elementary transformation.

$$\begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}$$

Ques. Sol^n

$$\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$