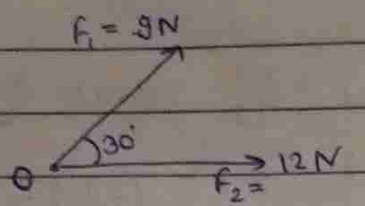
Date __/__/____ Page No.: Assignment-1

Exercise-1//

1- Determine magnitude of resultant of two forces of magnitude 12 N and 9 N acting at a point if angle b/w two forces is 30°

sol"



Let resultant force = R

$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{9^2 + 12^2 + 2\times9\times12\times\cos30^\circ}$

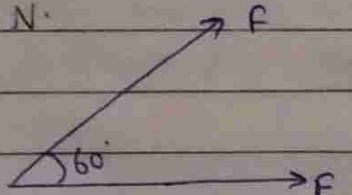
$= \sqrt{81 + 144 + 108\sqrt{3}}$

$= \sqrt{412.06}$

$\therefore$ Resultant force (R) = 20.29 N Ans

2- Find magnitude of two equal force acting at a point with an angle of 60° b/w them, if the resultant is equal to $30\sqrt{3}$ N.

sol"



Let forces are F.

$R = 30\sqrt{3}$ (given)

$\therefore R^2 = F^2 + F^2 + 2F^2\cos60^\circ$

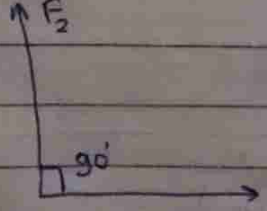
$(30\sqrt{3})^2 = 3F^2$

$F^2 = 900$ N $\quad\therefore F = 30$ N

$\therefore$ magnitude of force is 30 N. Ans

3- The resultant of two forces when they act at right angles is 10N. whereas when they act an angle of 60° the resultant is $\sqrt{148}$. Determine the magnitude of two forces.

sol: Case-1



$R = 10N$ (given)

$10^2 = F_1^2 + F_2^2 + 2F_1F_2\cos90^\circ$

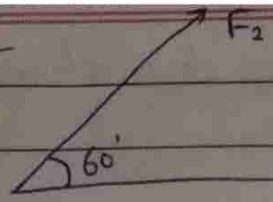
$2F^2 = 100$

$\therefore F_1^2 + F_2^2 = 100$ ——— (i)

$(F_1+F_2)^2 - 2F_1F_2 = 100$

$F_1 + F_2 = 14$ N ——— (iii)

Case-2



$R = \sqrt{148}$ N (given)

$(\sqrt{148})^2 = F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cos 60°$

$148 = F_1^2 + F_2^2 + F_1 F_2$

$F_1 \cdot F_2 = 48$ ——— (ii)

from eq^n (ii) & (iii)

$\because (F_1 - F_2)^2 = (F_1 + F_2)^2 - 4F_1 \cdot F_2$

$= (14)^2 - 4 \times 48$

$(F_1 - F_2)^2 = 4$

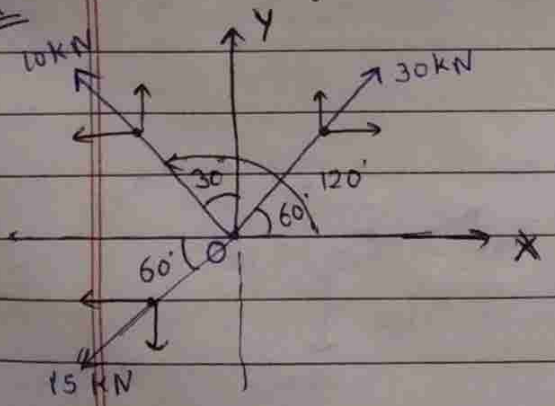
$\therefore F_1 - F_2 = 2$ N ——— (iv)

after solving (iii) & (iv) -

we find $F_1 = 8$ N & $F_2 = 6$ N

4- Three forces of magnitude 30 kN, 10 kN & 15 kN are acting at point O. The angles made by 30 kN force, 10 kN force & 15 kN force with x-axis are 60°, 120° & 240° respectively. Determine magnitude & dir^n of force.

Sol^n



Resultant force in X-dir^n -

$\Sigma F_x = 30\cos 60° - 10\cos 60° - 15\cos 60°$

$= (15 - 5 - 7.5)$ kN

$= 2.5$ kN

Resultant force in Y-dir^n -

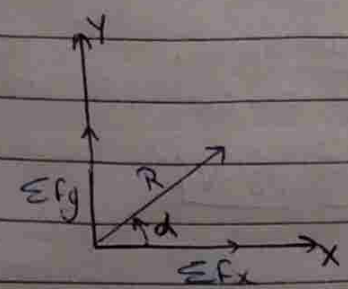
$\Sigma F_y = 30\sin 60° + 10\sin 60° - 15\sin 60°$

$= 21.65$ kN

$\therefore$ Resultant force $(R) = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$= \sqrt{(2.5)^2 + (21.65)^2}$

$R = 21.79$ kN / Ans.



$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x}$

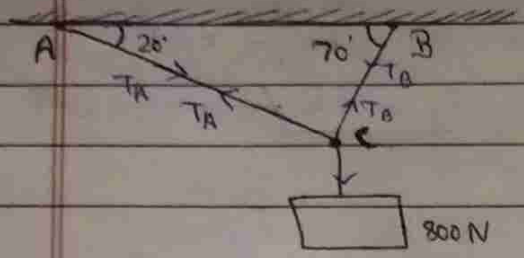
$\alpha = \tan^{-1}\left(\frac{21.65}{2.5}\right)$

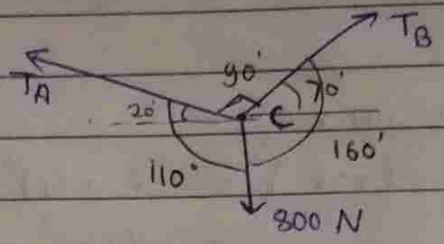
$\alpha = 83°41'$ / Ans.

$\therefore$ magnitude of force is 21.79 kN

& dir^n of force is 83°41' from the X-axis.

5- A weight of 800 N is supported by two chains, Determine the tension in each chain.





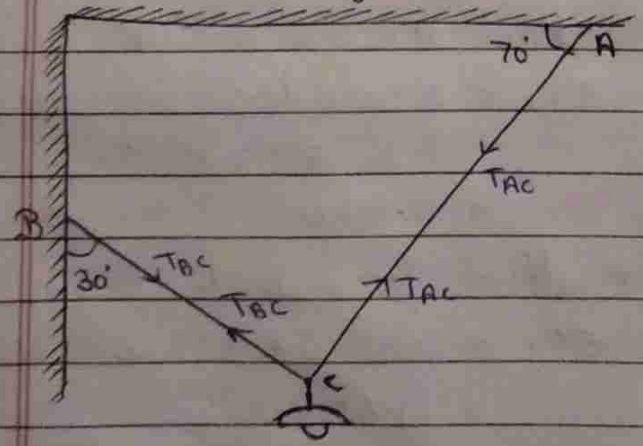
Sol:- apply Lami's theorem -

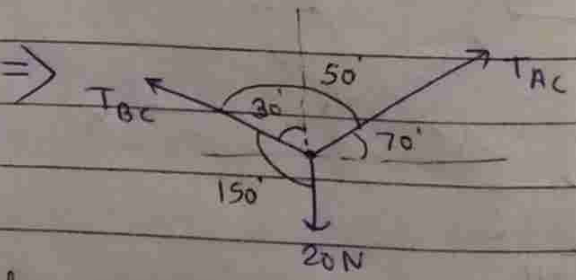
$$\frac{T_A}{\sin 160^\circ} = \frac{T_B}{\sin 110^\circ} = \frac{800}{\sin 90^\circ}$$

$T_A = 800 \sin 160^\circ$, $T_B = 800 \cdot \sin 110^\circ$

$T_A = 273.61\ N$ $T_B = 751.75\ N$ Ans

6- Using Lami's theorem or otherwise determine the forces in the string AC and BC, an electric lamp of 20 N hing from C.





apply Lammi's theorem -

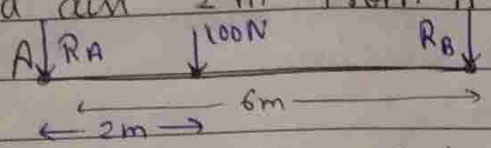
$$\frac{T_{AC}}{\sin 150^\circ} = \frac{T_{BC}}{\sin 160^\circ} = \frac{20}{\sin 50^\circ}$$

$T_{AC} = \dfrac{20 \times \sin 150^\circ}{\sin 50^\circ}$ $T_{BC} = \dfrac{20 \times \sin 160^\circ}{\sin 50^\circ}$

$T_{AC} = 13.05\ N$ $T_{BC} = 8.92\ N$

Ans

7- A beam AB of span 6 m carries a point load of 100 N at a dist" 2 m from A. Determine the beam reaction.



Sol"

moment about A -

$R_B \times 6 = 100 \times 2$

$R_B = 200/6$

$R_B = 33.33\,N$ /

Net moment about B -

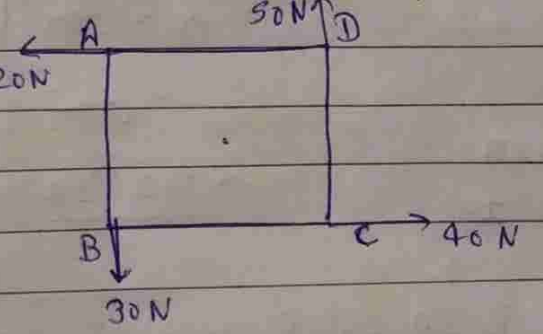
$R_A \times 6 = 100 \times 4$

$R_A = 400/6$

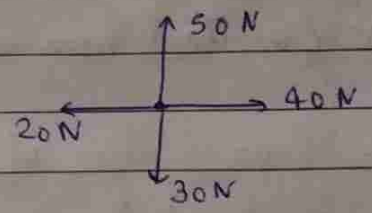
$R_A = 66.67\,N$ /

8- Four forces of magnitude 20 N, 30 N, 40 N and 50 N are acting respe- along the four sides of square taken in order. Determine the magnitude, dir" and position of resultant force.

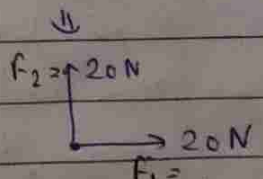
Sol"



$\Rightarrow$ Principle of transmissibility of force -



Position of resultant force -



∴ magnitude of resultant force = $\sqrt{F_1^2 + F_2^2}$

$= \sqrt{20^2 + 20^2}$

$R = 20\sqrt{2}\,N$ / Ans

directions

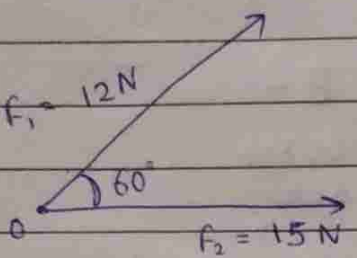
$\alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$= \tan^{-1}(1)$

$\alpha = 45°$ / Ans

9- Two forces magnitude 15 N & 12 N are acting at a point. If the angle b/w two force is 60°, determine the resultant of the forces in magnitude and direction.

Sol"



Let resultant force (R)

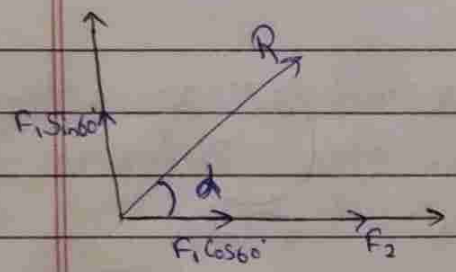
$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 60^\circ}$$

$$= \sqrt{(12)^2 + (15)^2 + 2\times 12 \times 15 \times \frac{1}{2}}$$

$$= \sqrt{144 + 125 + 180}$$

$$= \sqrt{449} = 23.43 \text{ N}$$

for direction -



$$\Sigma F_x = 12\cos 60^\circ + 15 = 21 \text{ N}$$

$$\Sigma F_y = 6\sqrt{3}$$

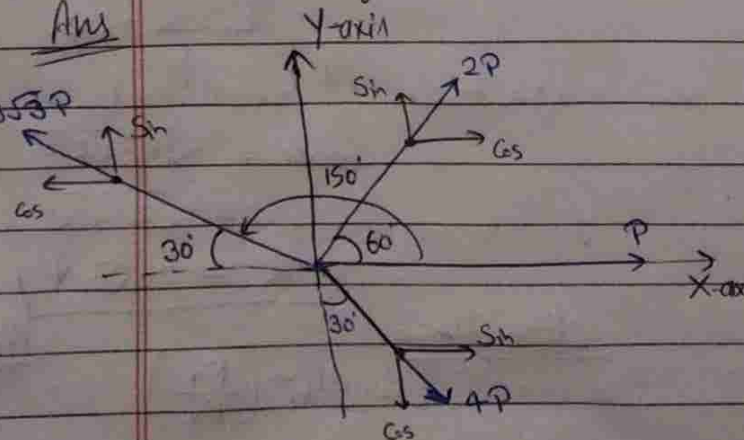
$$\Rightarrow \alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{6\sqrt{3}}{21}\right)$$

$$\alpha = 26^\circ 28'$$

∴ magnitude of resultant force 23.43 N
and dir" of resultant from +ive X-axis = 26° 28'

10- Four forces of magnitude P, 2P, $3\sqrt{3}$P and 4P are acting at a point O. The angle made by these forces with X-axis are 0°, 60°, 150° & 300° respectively. Find magnitude and dir" of resultant force.

Ans



Resultant force in X-dir"

$$\Sigma F_x = P + 2P\cos 60^\circ - 3\sqrt{3}P\cos 30^\circ + 4P\sin 30^\circ$$

$$= -\frac{1}{2}P$$

Resultant force in Y-dir"

$$\Sigma F_y = 3\sqrt{3}\sin 30^\circ + 2P\sin 60^\circ - 4P\cos 30^\circ$$

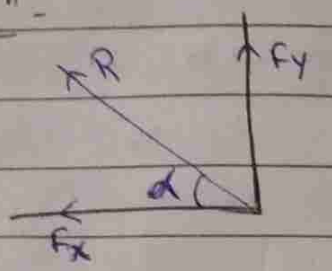
$$= \frac{\sqrt{3}}{2}P$$

∴ resultant force (R) = $\sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$= \sqrt{\left(-\frac{1}{2}P\right)^2 + \left(\frac{\sqrt{3}}{2}P\right)^2}$$

$$= P$$

dir^n -



$$\alpha = \tan^{-1}\left(\frac{F_Y}{F_x}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}/2\,P}{-1/2\,P}\right) = \tan^{-1}(-\sqrt{3})$$

$$= -60^\circ \text{ in C.W. dir}^n$$

so, magnitude of resultant force is P
and direction from +ive X-axis is 120° in C.C.W. dir^n.

Exercise - 2

1- Three collinear horizontal forces of magnitude 300 N, 100 N a 250 N are acting on rigid body. Determine resultant of forc analytically & graphically when -

(a) all forces are acting in the same dir^n.

(b) the force 100 N acts in the opposite dir^n.

sol^n - (a) (By analytically) -

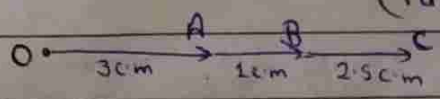
so, $R = F_A + F_B + F_C$

$= 300 + 100 + 250$

$R = 650$ N

(By graphically)

suitable scale
(100 N = 0.1 cm)

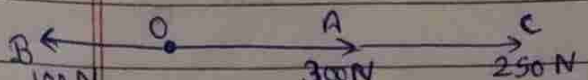


$R = 6.5$ cm

$= 6.5 \times \frac{100}{0.1} = 650$

so, R = 650 N Ans//

(b) (By analytically)

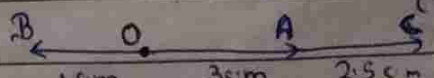


so, $R = F_A + F_C - F_B$

$(R = 450\ N)$

(By graphically)

suitable scale -
(100 N = 0.1 cm)



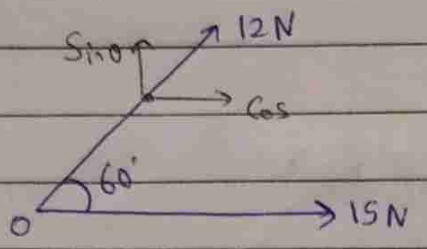
$R = 5.5$ cm $- 1$ cm

$= 4.5$ cm $\Rightarrow 4.5 \times \frac{100}{0.1}$

$(R = 450\ N)$ Ans

2- Two forces of magnitude 15 N and 12 N are acting at a poin The angle b/w forces is 60°. Find resultant in magnitude.

Sol"



$$\Sigma F_x = 15 + 12\cos 60^\circ$$
$$= 21\ N$$

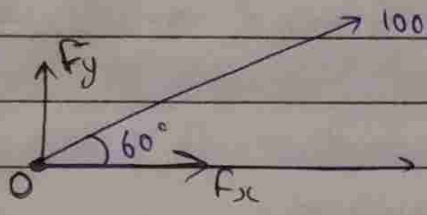
$$\Sigma F_y = 12\sin 60^\circ$$
$$= 6\sqrt{3}\ N$$

$$\therefore R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(21)^2 + (6\sqrt{3})^2} = \sqrt{549}\ N$$

$$\therefore (R = 23.43\ N)$$ Ans

3- A force of 1000 N is acting at a point, making an angle of 6 with the horizontal. Determine the components of force alon horizontal & vertical dir".

Sol"



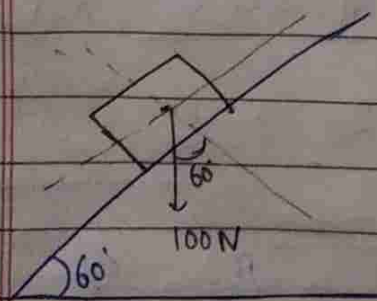
along X-dir"-

$$F_x = 1000 \times \cos 60^\circ$$
$$= 500\ N$$ Ans

along Y-dir"-

$$F_y = 1000 \times \sin 60^\circ$$
$$= 500\sqrt{3} = 866.02\ N$$ Ans

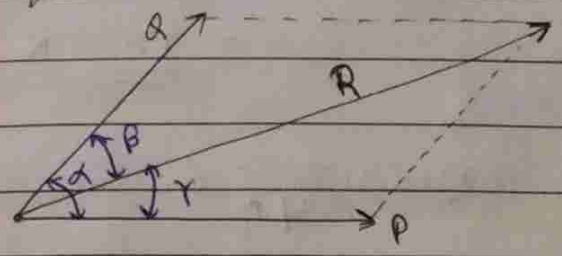
4- A small block of weight 100 N placed on inclined plane which makes 60° with horizontal, Find the components of weight - (i) Lular to inclined plane (ii) ∥ to inclined plane.



(i) component of weight in ⊥ular to ∠ plane
$$= 100\cos 60^\circ = 50\ N$$ Ans

(ii) ∥el to inclined plane
$$= 100\sin 60^\circ$$
$$= 50\sqrt{3} = 86.6\ N$$ Ans

5- Two forces P and Q are acting at a point O. The force P = 264.9 N and force Q = 195.2 N, if the resultant forces is equal to 400 N then find β, γ and α.



Sol^n- Resultant force (R) = $\sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

$(400)^2 = (264.9)^2 + (195.2)^2 + 2\times(264.9)(195.2)\cos\alpha$

$\cos\alpha = \dfrac{(400)^2 - (264.9)^2 - (195.2)^2}{2\times 264.9 \times 195.2}$

$= \dfrac{51.724}{103.41} = 0.5 = \dfrac{1}{2}$

$\therefore \alpha = 60^\circ$

along X-dir^n-

$\Sigma F_x = P + Q\cos\alpha$

$= 264.9 + 195.2\times\cos 60^\circ$

$= 362.5$ N

along Y-dir^n

$\Sigma F_y = Q\sin\alpha$

$= 195.2\times\sin 60^\circ$

$= 169.04$

$\therefore \gamma = \tan^{-1}\left(\dfrac{\Sigma F_y}{\Sigma F_x}\right)$

$= \tan^{-1}\left(\dfrac{169.04}{362.5}\right)$

$\gamma = 24^\circ 38' \approx 25^\circ$

$\therefore \alpha = 60^\circ$

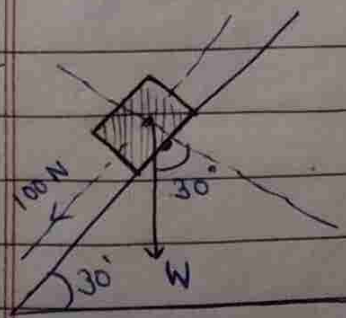
$\gamma = 25^\circ$

$\beta = \alpha - \gamma$

$\beta = 35^\circ$

6- A small block of unknown weight placed on inclined plane which makes angle 30° with horizontal, component of weight ⊥ to the inclined plane is 100 N, find weight of block-

Sol^n



W_x = 100 N

∴ Horizontal component of weight-

$\Rightarrow W\sin\theta = 100$

$\therefore W = \dfrac{100}{\sin 30^\circ}$

$W = 2000$ N Ans

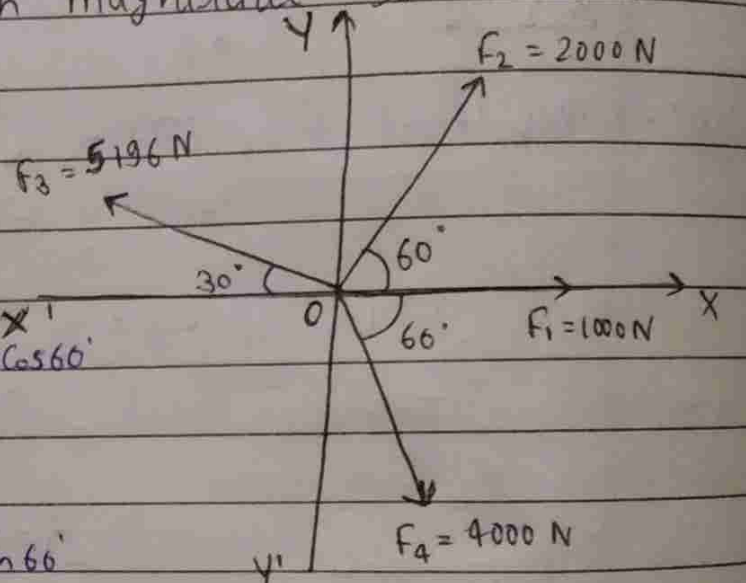
7- In q^n 6, find component of weight ⊥ular to inclined plane.

perpendicular component of weight = $W\cos 30^\circ$

$= 2000 \times \sqrt{3}/2$

$= 17.32\ N$ Ans.

8- Determine the resultant in magnitude & direction analytically and graphically.



Sol^n

*** Analytically:

along X-axis -

$\Sigma F_x = 1000 + 2000\cos 60^\circ - 5196\cos 30^\circ + 4000\cos 60^\circ$

$= -499.86\ N$

along Y-axis -

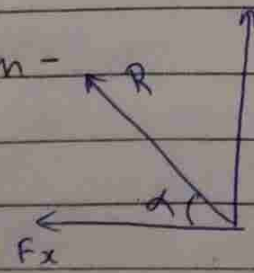
$\Sigma F_y = 2000\sin 60^\circ + 5196\sin 30^\circ - 4000\sin 66^\circ$

$= 865.94\ N$

so, Resultant $(R) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-499.86)^2 + (865.94)^2}$

$R = 998.85\ N \approx 1000\ N$ // Ans.

direction -



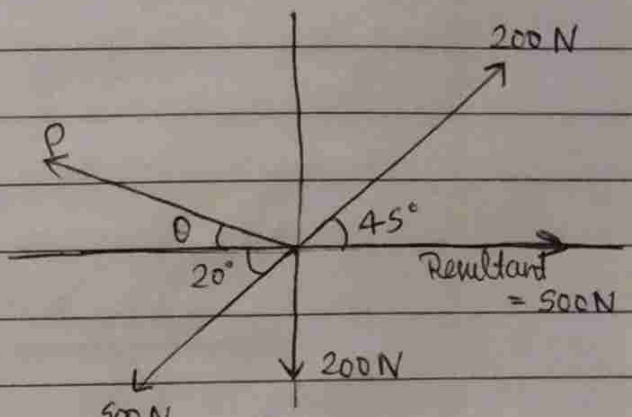
$\alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$= \tan^{-1}(-1.732)$

$\alpha = -60^\circ$ (C.W dir^n)

so, resultant force is 1000 N & direction is 120° ~~from~~ with OX in C.C.W. dir^n.

9- The four coplanar forces are acting at a pt. One of the force is unknown with magnitude P. The resultant is having a magnitude 500 N & acting along x-axis. Determine unknown force P & its dir^n with X-axis.



sol^n ∵ resultant force along X-axis

so, forces along Y-axis cancel each-other.

$200 \sin 45^\circ + P \sin\theta = 500 \sin 20 + 200$

so, $P \sin\theta = 500 \sin 20^\circ + 200 - 200 \sin 45^\circ$

$$\boxed{P \sin\theta = 229.58\ N} \quad \text{——(i)}$$

∵ resultant force along X-axis = 500N

$$200 \cos 45^\circ - P \cos\theta - 500 \cos 20^\circ = 500$$

$$P \cos\theta = 200 \cos 45^\circ - 500 \cos 20^\circ - 500$$

$$\boxed{P \cos\theta = -828.42\ N} \quad \text{——(ii)}$$

from eq^n (i) & (ii)

$$P^2 (\sin^2\theta + \cos^2\theta) = (229.58)^2 + (-828.42)^2$$

so, $P = \sqrt{(229.58)^2 + (-828.42)^2}$

so, $P = 859.64\ N$ Ans

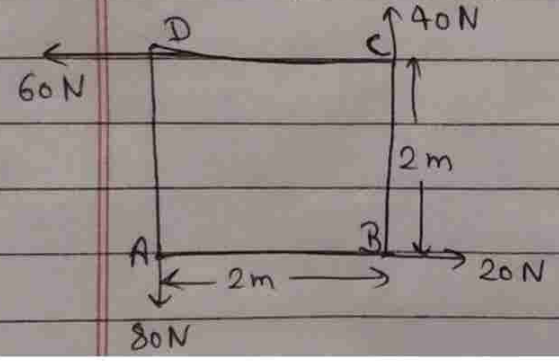
from eq^n (i)

$$\sin\theta = \frac{229.5}{859.64}$$

so, $\theta = 15^\circ 48'$

Exercise-3

1- Four forces of magnitudes 20 N, 40 N, 60 N & 80 N are acting respectively along four sides of square ABCD. Determine resultant moment about point A. (Each side of square = 2m)



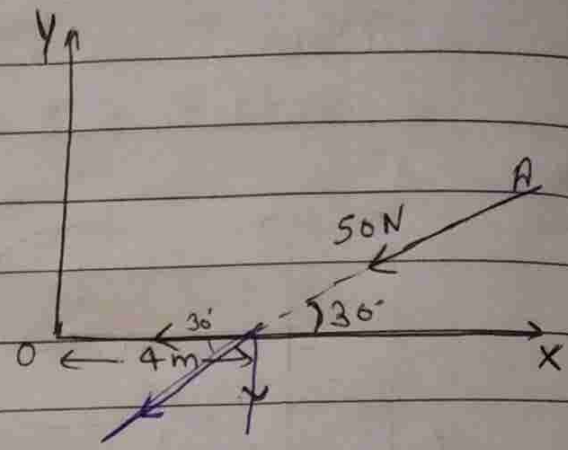
Resultant moment about point A

$\Rightarrow (80 \times 2)_{C.C.W} + (20 \times 2)_{C.C.W}$

$= 200$ Nm C.C.W direction //

2- A force of 50 N is acting at point A. Determine the moment of this force about O.

Sol^n



$F = 50\ N$

$F_y = 50 \sin 30°$

$= 25\ N$

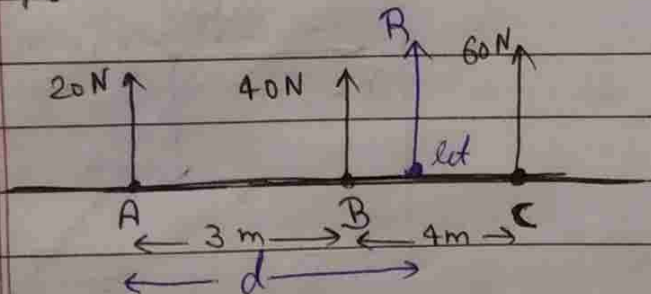
$d = 4\ m$

$\therefore M_o = F_y \times d$

$= 25 \times 4 = 100\ Nm$ clockwise Ans

3- Three like || forces 20N, 40N & 60N are acting at pt's A, B & C respectively on straight line ABC. The distances are AB = 3m & BC = 4m. Find resultant & also dist^n of resultant from point A on line ABC.

Sol^n



Resultant Force

$= 20N + 40N + 60N$

$= 120\ N$ Ans

let dist^n of resultant from point A = d.

$\Sigma M_A = 40 \times 3 + 60 \times 7$

$= 540\ N\text{-}m$

$M_R = 120 \times d$

apply Varignon's theorem -

$M_R = \Sigma M_A$

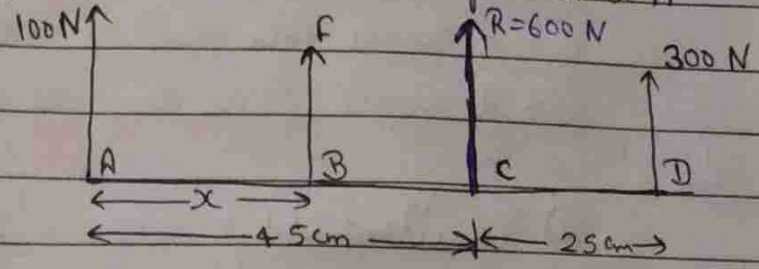
$120d = 540\ N\text{-}m$

$\therefore d = 540/120 = 4.5\ m$

$\therefore$ Resultant force = 120N
& dist^n from point A = 4.5 m } Ans

4- Three like ∥ forces 100 N, F and 300 N ~~are acting~~. If the Resultant R = 600 N and is acting at a dist^n of 45 cm from A, then find magnitude of force F and dist^n of F and A.

sol^n-



Resultant (R) = ΣF_y

$600 = 100 + F + 300$

so, $F = 200\ N$ // Ans //

Total moment about pt. A -

$\Sigma M_A = F \times x + 300 \times 70$

$= (200x + 21000)\ N\text{-}m$

Total moment of Resultant (R) about A -

$M_R = (600 \times 45)\ N\text{-}m$

apply Varignon's theorem -

$M_R = \Sigma M_A$

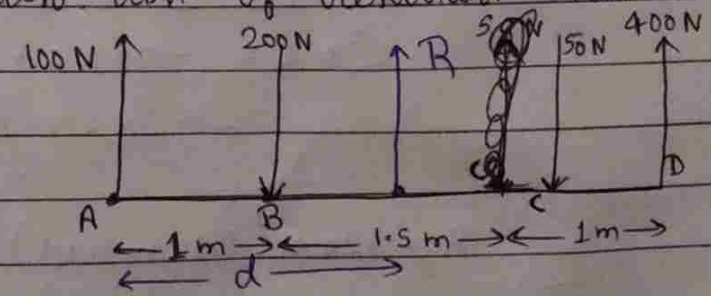
$600 \times 45 = 200x + 21000$

$200x = 6000 \Rightarrow x = 30\ cm$ // Ans //

so, magnitude of force (F) = 200 N & x = 30 cm,

5- Four ∥ forces of magnitudes 100 N, 200 N, 50 N and 400 N. Determine magnitude of resultant and also dist^n of resultant from point A -

sol^n



Resultant (R) = ΣF_y

$R = 100 - 200 - 50 + 400$

so, $R = 250\ N$ //

Total moment about pt. A -

$\Sigma M_A = -200 \times 1 - 50 \times 2.5 + 400 \times 3.5$

$= 1075\ N\text{-}m$

Total moment of Resultant (R) about A -

$M_R = R \times d$

$= 250 \times d$

apply Varignon's theorem -

$M_R = \Sigma M_A$

$250 \times d = 1075$

so, $d = 4.3\ m$

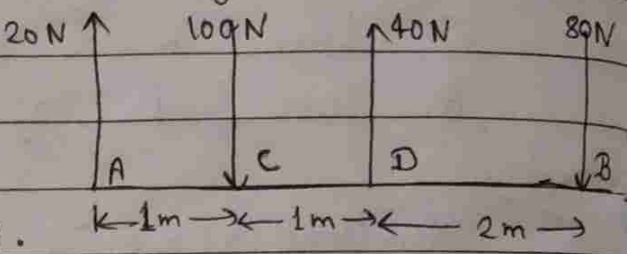
so, Resultant (R) = 250 N & dist^n from pt A = 4.3 m

6- A system of 11 forces are acting on a rigid bar. Reduce this system to -

(i) a single force

(ii) a single force and a couple at A.

(iii) a single force and a couple at B.



Sol^n

(i) Resultant (R) = ΣF_y = 20 - 100 + 40 - 80

= 120 N (downward)

Total moment about pt. A - moment of resultant (R) about A -

ΣM_A = -100×1 + 40×2 - 80×4 ⇒ -120 × d

= -340 N-m

apply varignon's theorem -

-120 × d = -340

so, d = 2.83 from A.

so, Resultant (R) = 120 N (downward)

& dist^n of R_M from A = 2.83 m

(ii) R = 120 N (downward)

Total moment about pt. A -

ΣM_A = -100×1 + 40×2 - 80×4

= -340 N-m

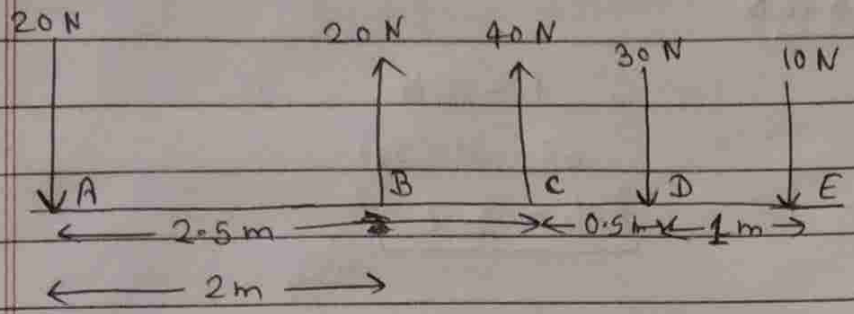
(iii) R = 120 N (downward)

Total moment about pt. B -

ΣM_B = -40×2 + 100×3 - 20×4

= 140 N-m

7- Five forces are acting on a body. Determine the resultant



Resultant Force (R) = $-20+20+40-30-10$

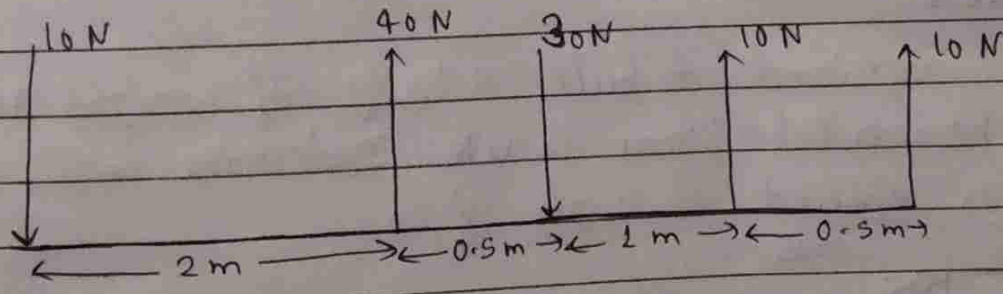
$R = 0$

Resultant couple about A -

$M_A = 20 \times 2 + 40 \times 2.5 - 30 \times 3 - 10 \times 4$

$= 10$ N-m

8- Determine the resultant of 11 forces -



Resultant Force (R) = $-10+40-30+10+10$

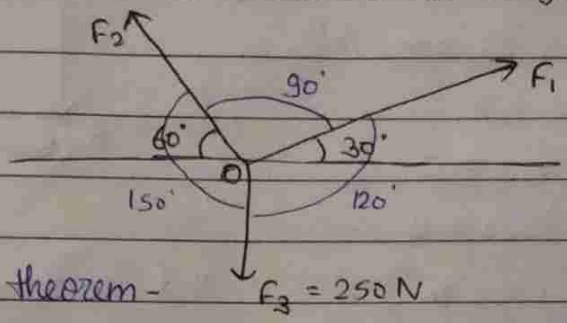
$= 20$ N (upward)

Date ___/___/_____ Page No.: _____

Exercise-4

1- Three forces F_1, F_2 & F_3 are acting on a body, body in equilib
If magnitude of force F_3 is 250 N, find magnitude of force F_1 and F_2.

Sol^n



apply Lammi's theorem -

$$\frac{F_1}{\sin 150^\circ} = \frac{F_2}{\sin 120^\circ} = \frac{250}{\sin 90^\circ}$$

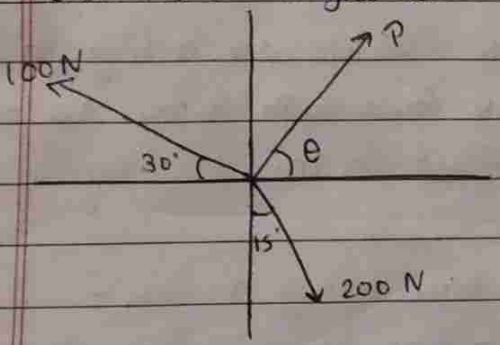
$$F_1 = 250 \cdot \sin 150^\circ, \qquad F_2 = 250 \sin 120^\circ$$

$$F_1 = 125 \text{ N} \qquad F_2 = 216.5 \text{ N}$$

Ans

2- Three forces of magnitude P, 100 N and 200 N are acting at point O, determine magnitude and dir^n of force P.

Sol^n



since, forces are in equilibrium

so, $\Sigma F_x = 0$ & $\Sigma F_y = 0$

$\Sigma F_x \Rightarrow P\cos\theta + 100\cos 30^\circ + 200 \sin 15^\circ = 0$

$$P\cos\theta = 100\cos 30^\circ - 200\sin 15^\circ$$

$$P\cos\theta = 34.83 \text{ N} \quad \text{---(i)}$$

$\Sigma F_y \Rightarrow P\sin\theta + 100\sin 30^\circ - 200\cos 15^\circ = 0$

$$P\sin\theta = 200\cos 15^\circ - 100\sin 30^\circ$$

$$P\sin\theta = 143.18 \text{ N} \quad \text{---(ii)}$$

after squaring and add eq^n (i) & (ii)

$$P^2(\sin^2\theta + \cos^2\theta) = (34.83)^2 + (143.18)^2$$

$$P = \sqrt{(34.83)^2 + (143.18)^2}$$

$$\text{so, } P = \sqrt{21713.64}$$

$$P = 147.35 \text{ N} \quad \text{Ans}$$

direction

$$\frac{P\sin\theta}{P\cos\theta} = \frac{143.18}{34.83} \Rightarrow \theta = \tan^{-1}(4.110)$$

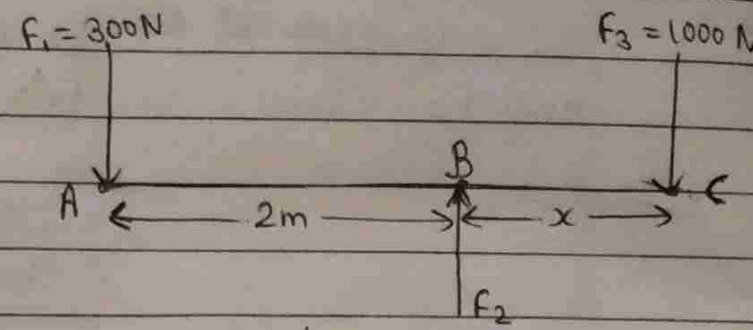
$$= 76^\circ 32'$$

Ans

$P = 147.35 N$

$\theta = 76^\circ 32'$

3- Body is in equilibrium, then determine the magnitude of force F_2 and dist^n of F_3 from force F_2.



since, Body is in equilibrium.

so, Net Force = 0

$F_1 + F_2 + F_3 = 0$

$-300 + F_2 - 1000 = 0$

so, $F_2 = 1300\ N$ /

Net moment about pt B = 0

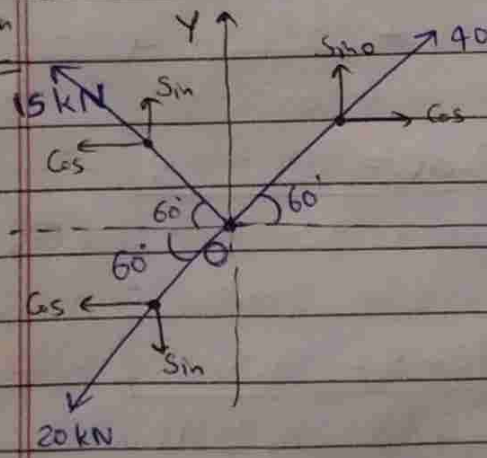
b/c body is in equilibrium.

$300 \times 2 = 1000 \times x$

so, $x = 0.6\ m$ /

4- Three forces of magnitude 40kN, 15kN & 20kN are acting at point O. The angles made by 40kN, 15kN & 20kN forces with x-axis are 60°, 120° & 240°. Determine magnitude & dir^n resultant force.

Sol^n



net force along X-axis -

$\Sigma F_x = 40\cos 60° - 15\cos 60° - 20\cos 60°$

$= 2.5\ N$

net force along Y-axis -

$\Sigma F_y = 40\sin 60° + 15\sin 60° - 20\sin 60°$

$= 30.31\ N$

magnitude of resultant force -

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(2.5)^2 + (30.31)^2}$

$R = 30.41\ kN$ Ans

dir^n of resultant force -

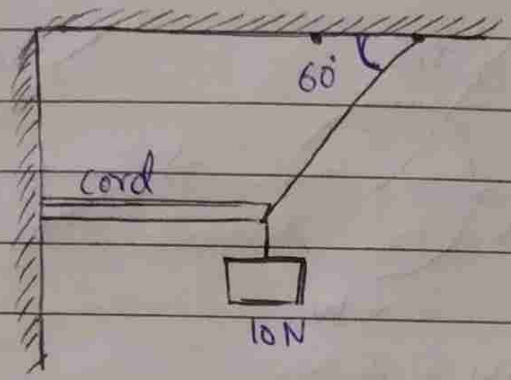
$\alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

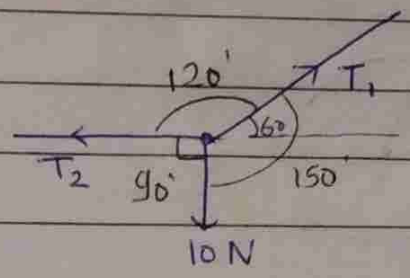
$= \tan^{-1}\left(\frac{30.31}{2.5}\right)$ so, $\alpha = 85.25°$ with X-axis

Ans

5- A lamp of weight 10 N suspended from ceiling by chain, It is pulled aside by horizontal cord, untill chain makes angle 60° with ceiling. Find tensions in chain & chord by applying Lami's theorem.

Sol:





using lammi's theorem-

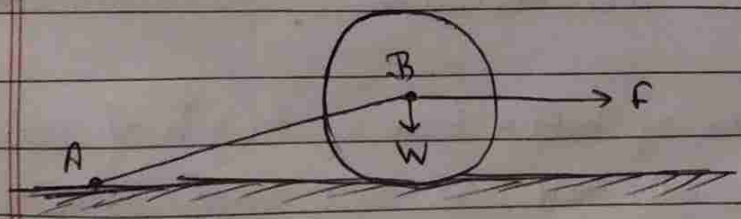
$$\frac{T_1}{\sin 90^\circ} = \frac{10}{\sin 120^\circ} = \frac{T_2}{\sin 150^\circ}$$

$$T_1 = \frac{10}{\sin 120^\circ} \qquad T_2 = \frac{10}{\sin 120^\circ} \cdot \sin 150^\circ$$

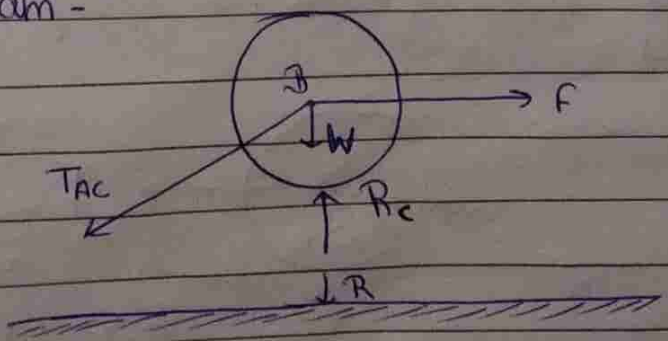
$$T_1 = 11.54\ N \qquad T_2 = 5.77\ N$$

so, tension in chain is 11.54 N & tension in chord is 5.77 N.

6- Draw the F.B.D digram of following figure -

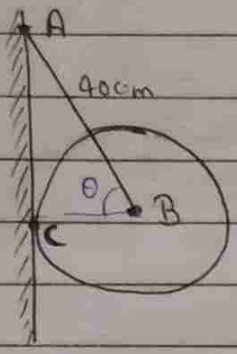


F.B.D. digram -

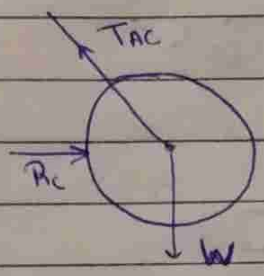


7- A circular roller of weight 1000 N & radius 20 cm hangs by tie rod AB = 40 cm & rests against a vertical wall at C, determine tension in tie rod & reaction Rc at point C.

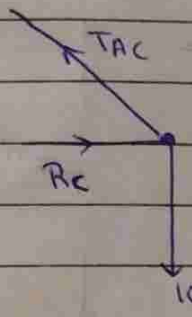
sol^n



F.B.D. digram -



⇒



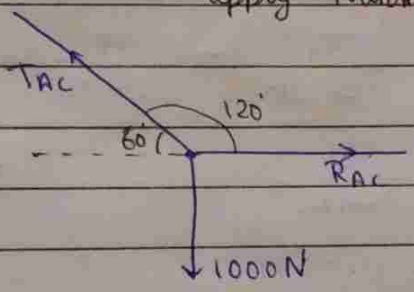
AB = 40 cm
BC = 20 cm

$\cos\theta = \frac{BC}{AC}$

$= 1/2$

$(\theta = 60^\circ)$

apply transmissibility of force -



apply lammi's theorem -

$$\frac{T_{AC}}{\sin 90^\circ} = \frac{R_{AC}}{\sin 150} = \frac{1000}{\sin 120^\circ}$$

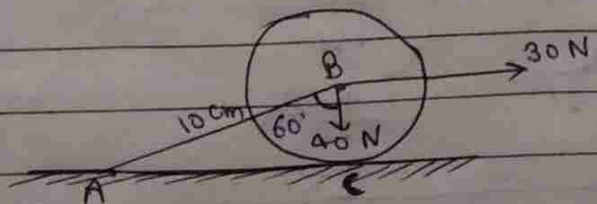
$\therefore T_{AC} = \frac{1000}{\sin 120^\circ}$ $\quad R_{AC} = 1000 \frac{\sin 150^\circ}{\sin 120^\circ}$

$= 1154.70\ N$ $\quad = 577.35\ N$

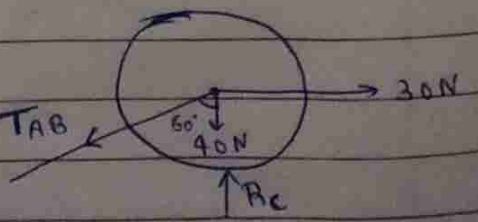
∴ tension in tie rod is 1154.70 N
and reaction force is 577.35 N

8- In problem 6, if radius of ball = 5 cm & length of string AB = 10 cm, weigh of ball (W) = 40 N, & horizontal force (F) = 30 N then find tension in string & vertical reaction Rc at point C.

sol^n



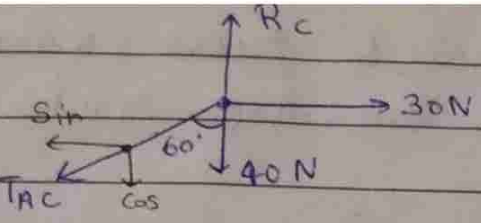
F.B.D digram -



BC = 5 cm
AB = 10 cm

$\cos\theta = \frac{BC}{AB} = 1/2$

$(\theta = 60^\circ)$



since, ball in equilibrium

$\Sigma F_x = 0$

$30 - T_{AC} \sin 60^\circ = 0$

$T_{AC} = 30 / \sin 60^\circ$

$= 34.64\ N$ ///

$\Sigma F_y = 0$

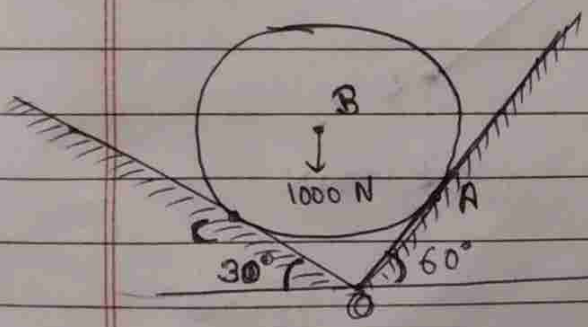
$R_C - 40 - T_{AC} \cos 60^\circ = 0$

$R_C = 40 + 34.64 \cdot \cos 60^\circ$

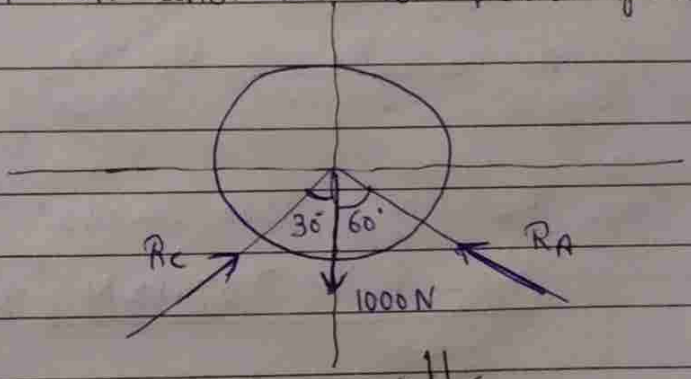
$R_C = 57.32\ N$

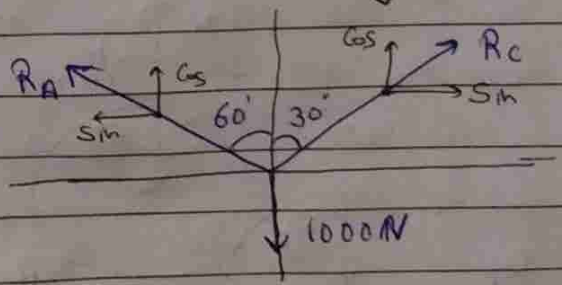
so, tension in string is 34.64 N & vertical reaction $R_C = 57.32\ N$.

9- Determine the reaction R_A and R_C at point of contact:



Sol^n





Equilibrium eq^n -

$\Sigma F_x = 0$

$R_C \sin 30^\circ - R_A \sin 60^\circ = 0$

$R_C = \dfrac{R_A \sin 60^\circ}{\sin 30^\circ}$ ——— (1)

$R_C = \dfrac{500 \times \sin 60^\circ}{\sin 30^\circ}$

$R_C = 866.02\ N$ / Ans.

$\Sigma F_y = 0$

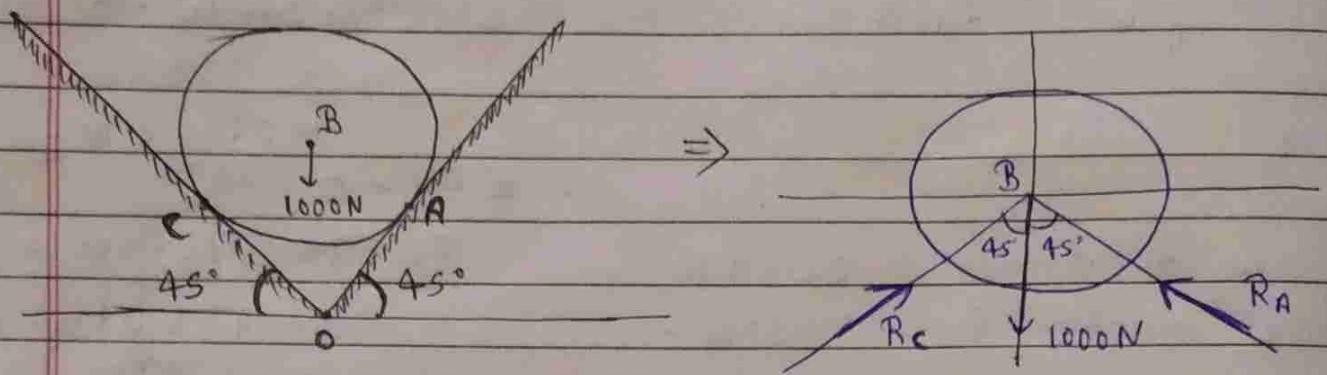
$R_C \cos 30^\circ + R_A \cos 60^\circ - 1000 = 0$

$R_A \left(\dfrac{\sin 60^\circ \cdot \cos 30^\circ}{\sin 30^\circ} + \cos 60^\circ \right) = 1000$

$R_A = \dfrac{1000}{2} = 500\ N$

$R_A = 500\ N$ / Ans

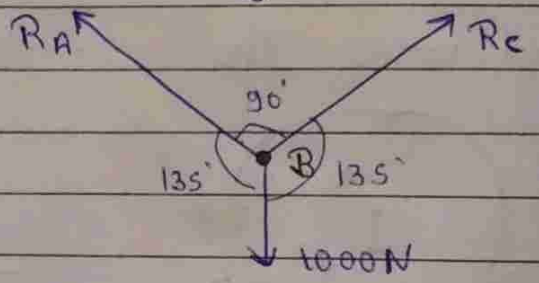
10- In the above problem, sides of groove makes an angle of 45° with horizontal, then find reaction R_A & R_C.



Sol^n

apply Lammi's theorem-

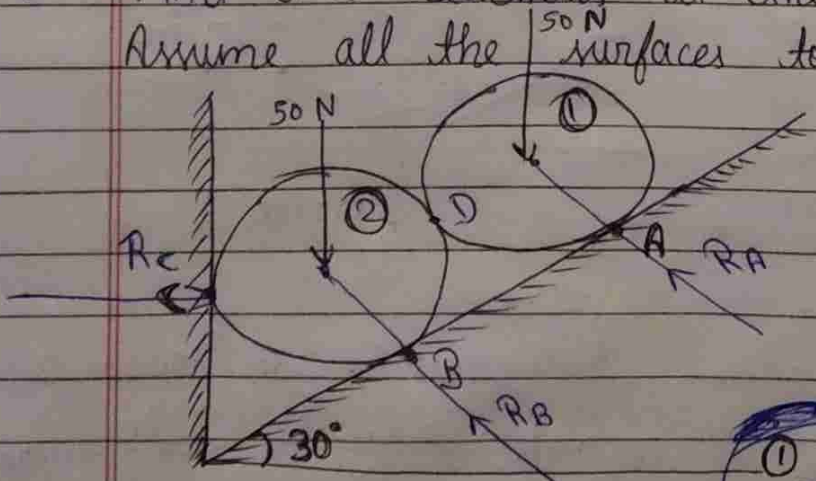
$$\frac{R_A}{\sin 135^\circ} = \frac{R_C}{\sin 135^\circ} = \frac{1000}{\sin 90^\circ}$$



so, $R_A = R_C = 1000 \cdot \sin 135^\circ$

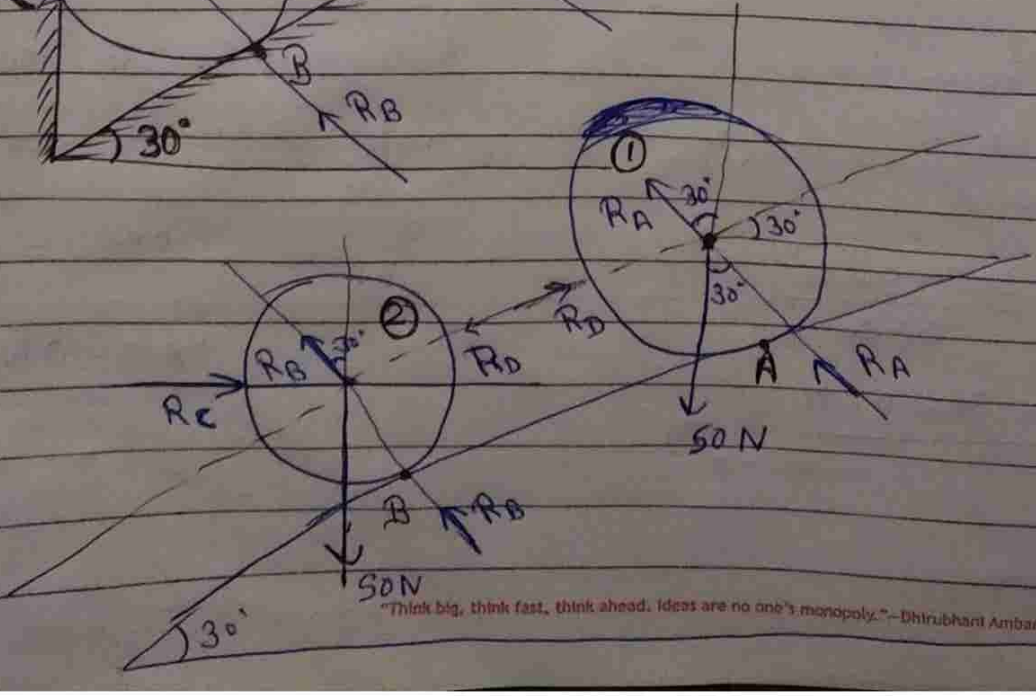
$R_A = R_C = 707.10\,N$ // Ans.

11- Find the reactions at the points of supports A, B and C. Assume all the surfaces to be smooth.

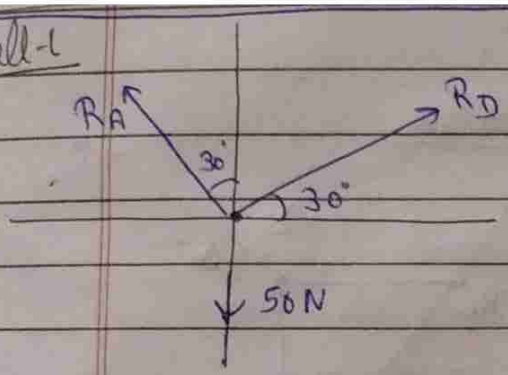


(Let ball contact at point D)

Sol^n



Ball-1



$$\frac{R_A}{\sin 120^\circ} = \frac{R_D}{\sin 150^\circ} = \frac{50}{\sin 90^\circ}$$

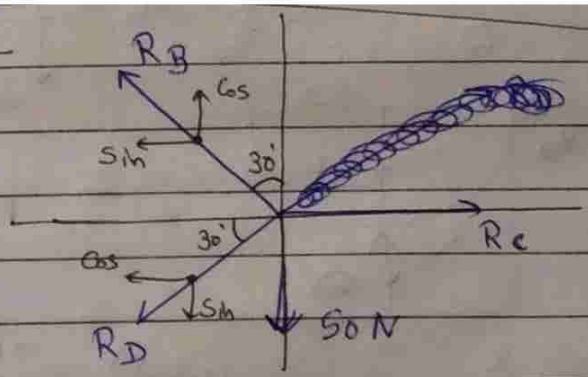
$$\therefore R_A = 50 \sin 120^\circ = 43.3\ N$$

$$R_D = 50 \times \frac{\sin 150^\circ}{\sin 90^\circ} = 25\ N$$

$$\boxed{R_D = 25\ N}$$

$$\therefore R_A = 43.3\ N$$
$$R_B = 72.16\ N$$
$$R_C = 57.73\ N$$

Ball-2



$$\Sigma F_x = 0$$

$$R_C - R_B \sin 30^\circ - R_D \cos 30^\circ = 0$$

$$R_C = 25 \cos 30^\circ + R_B \sin 30^\circ$$

$$\Sigma F_y = 0$$

$$R_B \cos 30^\circ - R_D \sin 30^\circ - 50 = 0$$

$$R_B = \frac{50 + 25 \sin 30^\circ}{\cos 30^\circ}$$

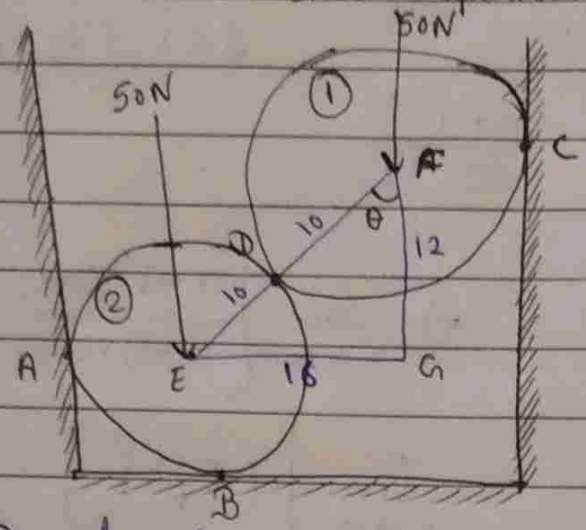
$$\therefore R_B = 72.16\ N$$

after solving

$$R_C = 57.73\ N$$

12- Two spheres each of weight 50 N & radius 10 cm rest in a horizontal channel of width 36 cm. Find the reactions on the points of contact at A, B and C.

Sol:



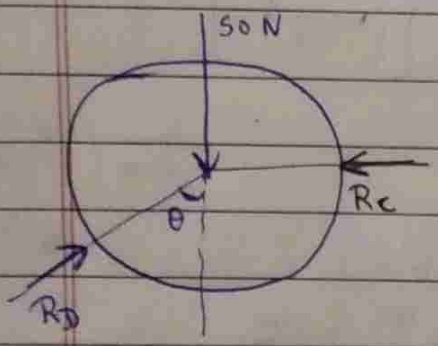
weight of each radius = 50N

In $\triangle EFG$-

$\sin\theta = \frac{EG}{EF} = \frac{16}{20} = 4/5$

$\cos\theta = \frac{FG}{EF} = \frac{12}{20} = 3/5$

F.B.D of block-1



$\Sigma F_y = 0$

$R_D \cos\theta = 50$

$R_D = \frac{50}{\cos\theta} = 50 \times \frac{5}{3} = \frac{250}{3}$ N

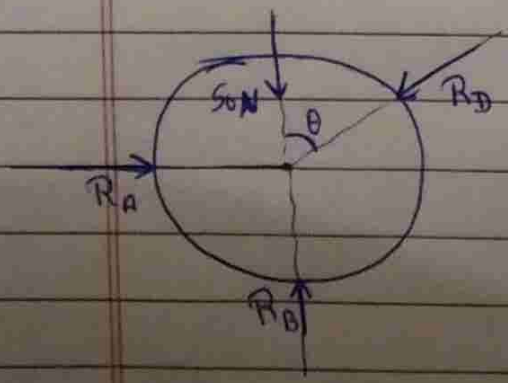
$\Sigma F_x = 0$

$R_C - R_D \sin\theta = 0$

$R_C = R_D \cdot \sin\theta = \frac{250}{3} \times \frac{4}{5}$

$R_C = \frac{200}{3} \approx 10$, $\boxed{R_C = 66.67\ N}$

F.B.D of block-2



$\Sigma F_y = 0$,

$R_B - R_D \cos\theta - 50 = 0$

$R_B = 50 + R_D \cos\theta = 50 + \frac{250}{3} \times \frac{3}{5}$

$\boxed{R_B = 100\ N}$

$\Sigma F_x = 0$

$R_A - R_D \sin\theta = 0$

$R_A = R_D \sin\theta$

$= \frac{250}{3} \times \frac{4}{5}$ $\approx$ 10, $\underline{R_A = 66.67\ N}$

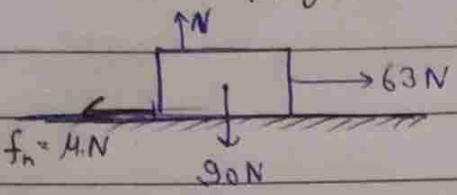
$\approx$, $R_A = R_C = 66.67\ N$

& $R_B = 100\ N$

Chapter-8

1- A body of weight 90 N placed on rough horizontal plane. Determine co-efficient of friction if a horizontal force of 63 N is apply.

sol"



since, $N = 90$

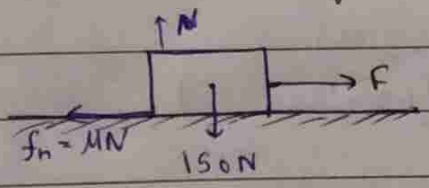
$\mu N = 63$

so, $\mu = 63/90 = 0.7$

so, $\mu = 0.7$

2- A body of weight 150 N placed on rough horizontal plane, if coeff" of friction b/w body and plane is 0.4. Determine the horizontal force.

sol"



since, $N = 150$

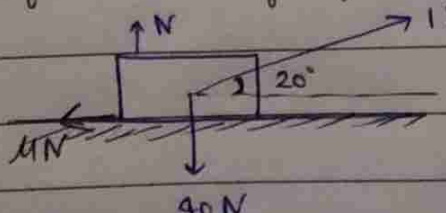
$\mu_s N = F$

$0.4 \times 150 = F$

so, $F = 60$ N

3- The force required to pull a body of weight 40N on a rough horizontal plane is 15 N. Determine coeff of friction if force is applied at angle 20°

sol"



$\Sigma F_x = 0$

$15 \cos 20^\circ = \mu N$ ——— (i)

$\Sigma F_y = 0$

$N + 15 \sin 20^\circ = 40$

$N = 40 - 15 \sin 20^\circ$

$N = 34.86$ Newt

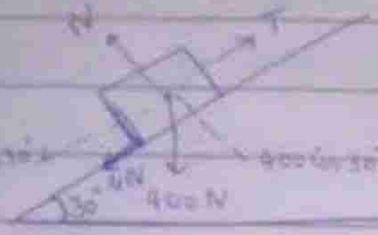
Put value of N in eq (i)

$$\mu = \frac{15 \cos 20^\circ}{34.86}$$

so, $\mu = 0.404$

8- A body of weigh 400 N is pulled up an inclined plane having inclined 30°. If coeff" of friction "b/w body & plane is 0.3 and force is applied // to inclined plane, find force required. Find also work done if body moves 10 m along the plane.

Sol"



$\Sigma F_y = 0$, $N = 400 \cos 30^\circ$

$\Sigma F_x = 0$

$T - 400 \sin 30^\circ - 0.3(400 \cos 30^\circ) = 0$

$T = 400 \sin 30^\circ + 0.3(400 \cos 30^\circ)$

$T = 303.92\ N$ / Ans.

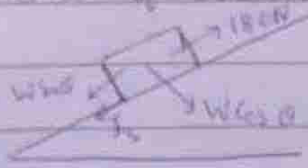
Work done $= T \cdot d$

$= (303.92) \times 10 = 3039.2\ J$ Ans

9- An effort of 180 N is required just to move a body up an inclined plane of angle 15°. If the angle of inclination is made 20°, the effort required is found to be 210 N. Find weight of body and co-effient of friction.

Sol"

Case-1



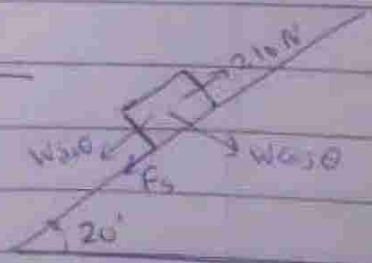
$N = W \cos\theta$

$F = F_s + W \sin\theta$

$180 = \mu W \cos 15^\circ + W \sin 15^\circ$

$180 = \mu W(0.9659) + W(0.2588)$ ——— (i)

Case-2



$N = W \cos 20$

$F = F_s + W \sin 20^\circ$

$210 = \mu W(0.9396) + W(0.342)$ ——— (ii)

(ii) — (i)

$$\frac{W(\mu \times 0.9396 + 0.3420)}{W(\mu \times 0.9659 + 0.2588)} = \frac{210}{180} = 1.16$$

$0.9396\,\mu + 0.3420 = 1.1268\,\mu + 0.3002$

$0.0417 = 0.1872\,\mu$

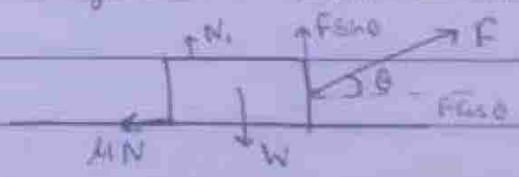
$\mu = 0.22$ / Ans.

$180 = W(0.2067) + W(0.2588)$

$\therefore W = 386.68\ N$ / Ans

6- Find the least force required to pull a body of weight W placed on rough plane, when force is applied at angle θ with the horizontal.

Sol:



$\Sigma F_x = 0$, $F\cos\theta - \mu N_1 = 0$ ——— (i)

$\Sigma F_y = 0$,

$N_1 + F\sin\theta = W$

$\therefore N_1 = W - F\sin\theta$ ——— (ii)

$F\cos\theta = \mu(W - F\sin\theta)$

$\therefore F = \frac{\mu W}{\cos\theta + \mu\sin\theta}$ ——— (iii)

Lets find θ for which $\frac{dF}{d\theta} = 0$,

$\frac{dF}{d\theta} = \frac{(\cos\theta + \mu\sin\theta)0 - \mu W(-\sin\theta + \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)^2} = 0$

$\therefore -\sin\theta + \mu\cos\theta = 0$

$\mu = \tan\theta$

putting $\tan\theta = \mu$ in eq (iii)

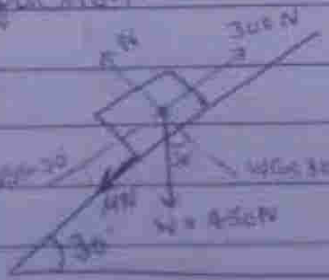
$F = \frac{W\tan\theta}{\cos\theta + \tan\theta\sin\theta}$

$= \frac{(\tan\theta)W}{\cos^2\theta + \sin^2\theta}\cos\theta$

$\therefore (F = W\sin\theta)$ Ans

7- A body of weight 450 N is pulled on inclined plane by force 300 N. The inclination of plane is 30°. the force is applied ∥ to plane. Determine coeff of friction.

Sol:



$\Sigma F_y = 0$

$N = 450\cos 30°$

$\Sigma F_x = 0$

$300 - 450\sin 30 - \mu(450\cos 30) = 0$

$\therefore \mu = \frac{300 - 450\sin 30}{450\cos 30}$

$= \frac{75}{450\cos 30}$

$\therefore \mu = 0.192$

4- A body of weight 60 N is placed on rough plane. To just move the body, a push of 18 N inclined at 20° to horizontal plane is required. Find coeff° of friction -

Sol°

$\Sigma F_y = 0$

$N - 18\sin 20° - 60 = 0$

$N = 60 + 18\sin 20°$

$\Sigma F_x = 0$

$18\cos 20° = \mu N$

$18\cos 20° = \mu(60 + 18\sin 20°)$

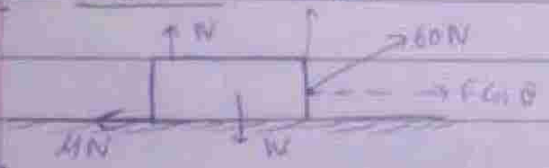
$\therefore \mu = \frac{18\cos 20°}{60 + 18\sin 20°}$

$\therefore \mu = 0.255$

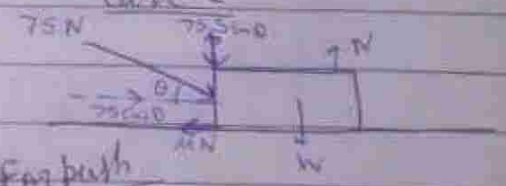
5- A pull of 60 N inclined at 25° to horizontal plane, is required just to move a body placed on rough horizontal plane. But the push required to move body is 75 N. If push is inclined at 25°, find weight of the body & coeff° of friction.

Sol°

Case-1



Case-2



For pull

$\Sigma F_y = 0$

$N = W - F\sin\theta$

$= W - 60\sin 25° = W - 25.356$

$\Sigma F_x = 0$

$F_s = F\cos\theta = 54.378 = \mu N$

$\mu(W - 25.356) = 54.386$ ——— (i)

For push

$\Sigma F_y = 0$

$N = W + F\sin\theta = W + 31.695$

$\Sigma F_x = 0$

$F_s = F\cos\theta = 67.97$

$\mu(W + 31.695) = 67.97$ ——— (ii)

Substracting (i) from (ii),

$\mu W + 31.695\mu = 67.97$

$\mu N - 25.356\mu = 54.378$

$57.051\mu = 13.592$

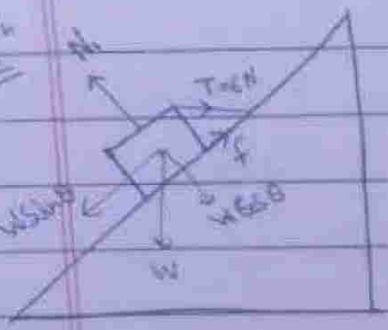
$\mu = 0.238$ Ans

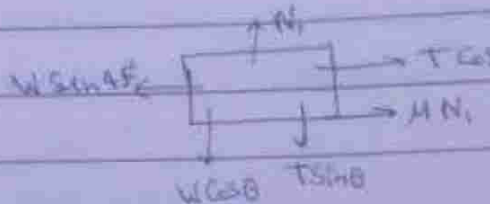
$W = \frac{54.378}{0.238} + 25.356$

$\therefore W = 253.83\ N$

10- Block A weight 20 N is rectangular prism resting on rough inclined plane. The block is tied up by horizontal string which has a tension 6N. Find
(i) the friction force on the block.
(ii) the normal reaction of the inclined plane and
(iii) the co-efficient of friction b/w surface of contact.

Sol^n





The body is in equilibrium -

$\Sigma F_y = 0$, ✗

$N_1 = 6\sin 45° + W\cos 45°$

$= 4.242 + \frac{W}{1.414} = 18.382\ N$

$\Sigma F_x = 0$ ✗

$W\sin 45° = \mu N_1 + 6\cos 45°$

$\frac{W}{1.414} = \mu N_1 + 4.242$

$14.142 = \mu(18.382) + 4.242$

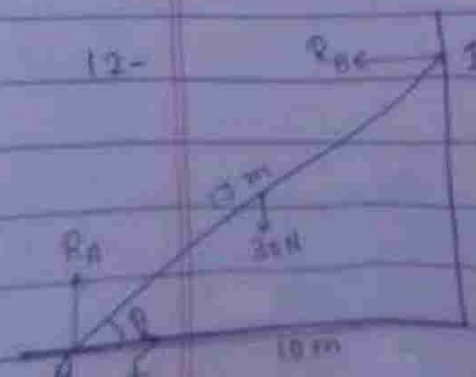
$\mu = 0.538$

✗, (i) Frictional force $= \mu N_1 = (0.538)(18.382) = 9.888\ N$

(ii) Normal rx^n of inclined plane - $N_1 = 18.382\ N$

(iii) Coeff^n of friction $= \mu = 0.538$

12-



since ladder is about ✗ to slip ✗

$\Sigma F_x = \Sigma F_y = 0$

$\Sigma F_y = 0$, ✗, $R_B = 30\ N$

$\Sigma F_x = 0$, ✗, $R_A = F$

and $\Sigma M_A = 0$, ✗, $-30\left(\frac{10}{2}\right) + R_B(10) = 0$

$30(5) = R_B(8.3)$

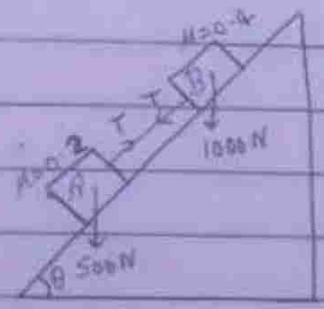
$(R_B = 18.06\ N)$

$\therefore 18.06 = \mu(30)$

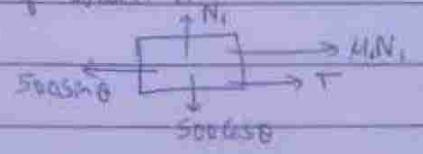
$(\mu = 0.602)$

11- A cord connected two bodies of weight 500 N and 1000 N. The two bodies placed on an inclined plane and chord is ll to inclined. The coeff" of friction of weight 500 N is 0.2 & that of 1000 N is 0.4. Determine the inclination of plane & tension in cord, when motion is down the inclined plane. The body weight 500 N is below the body weighting 1000 N.

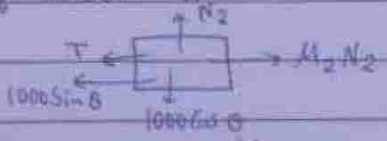
Sol"



F.B.D of block 'A' -



F.B.D of block 'B' -



solving for Block 'A' -

$\Sigma F_y = 0$; $N_1 = 500\cos\theta$ ——— (i)

$\Sigma F_x = 0$, $(0.2)N_1 = 500\sin\theta$

$T = 500\sin\theta - (0.2) \times 500\cos\theta$ ——— (ii)

solving for block 'B' -

$\Sigma F_y = 0$; $N_2 = 1000\cos\theta$

$\Sigma F_x = 0$, $0.4 N_2 = T + 1000\sin\theta$

$T = -1000\sin\theta + 0.4 \times 1000\cos\theta$ ——— (ii)

eq" (i) & (ii)

$500\sin\theta - 100\cos\theta = 400\cos\theta - 1000\sin\theta$

$1500\sin\theta = 500\cos\theta$

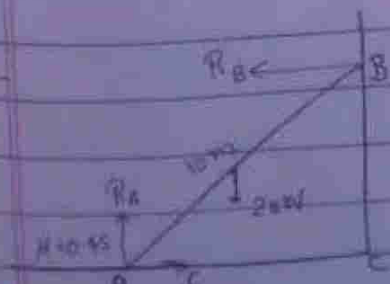
$\theta = \tan^{-1}(1/3) = 18.41°$ / Ans.

$T = 500\sin\theta - 100\cos\theta$

$= 500(0.315) - 100(0.9488)$

$T = 63\ N$ / Ans.

13-



$\Sigma F_y = 0$, so, $R_A = 20\ N$

$\Sigma F_x = 0$ so, $f = R_B$

$\Sigma M_A = 0$, so, $-20(3) + R_B(8) = 0$

$(R_B = 7.5N)$ $(R_A = 20N)$

so $(f = 7.5\ N)$

and max" value of friction that can act

is $\mu N = (0.45)(20) = 9\ N$, so, body is in equilibrium.