Assignment-I

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Q.1 Integrate $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ to obtain the equation of the curve satisfying this eq^n and passing through the origin.

Sol^n

$$(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$$

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$$

clearly above eq^n is linear diff. eq^n of form,

$$\frac{dy}{dx} + Py = Q$$

Hence, $I.F = e^{\int P dx} = e^{\int \frac{2x}{1+x^2}dx}$

$$1+x^2 = t$$
$$2x\,dx = dt$$

$$I.F = e^{\int \frac{dt}{t}}$$
$$= e^{\log t} = t$$
$$I.F = 1+x^2$$

$$y \times I.F = \int I.F \times Q\,dx$$

$$y(1+x^2) = \int \cancel{(1+x^2)} \times \frac{4x^2}{\cancel{(1+x^2)}}\,dx$$

$$\boxed{y(1+x^2) = \frac{4x^3}{3} + C}$$

As above curve passes through origin

$$0(1+0^2) = \frac{4\times(0)^3}{3} + C$$

$$C = 0$$

eq^n of curve => $\boxed{y(1+x^2) = \frac{4x^3}{3}}$

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Q.2) Solve: $x(dy/dx) + y\log y = xye^x$

Sol^n
=

$$x\left(\frac{dy}{dx}\right) + y\log y = xye^x$$

$$\frac{1}{y}\frac{dy}{dx} + \frac{1}{x}\log y = e^x$$

let, $\log y = t$

$$\frac{1}{y}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + \frac{t}{x} = e^x$$

Clearly above eq^n is linear diff. eq^n of form

$$\frac{dy}{dx} + Py = Q$$

$$I.F. = e^{\int P dx}$$

$$= e^{\int \frac{dx}{x}} = e^{\log x}$$

$$= x$$

$I.F = x$

$$x.t = \int x.e^x dx$$

$$\boxed{x\log y = e^x(x-1)+c}$$

Q.3) Solve $(D^3 - 5D^2 + 7D - 3)y = e^{2x}\cosh x$

$$(D^3 - 5D^2 + 7D - 3)y = e^{2x}\left(\frac{e^x + e^{-x}}{2}\right)$$

$$(D^3 - 5D^2 + 7D - 3)y = \frac{e^{3x}}{2} + \frac{e^{-x}}{2}$$

A.E $\quad m^3 - 5m^2 + 7m - 3 = 0$

$$(m-1)(m^2 - 4m + 3) = 0$$

$$(m-1)(m-1)(m-3) = 0$$

~~(m-1) (m-1~~ $\quad m = 1, 1, 3$

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$C.F = (c_1 + x c_2) e^x + c_3 e^{3x}$

$P.I = \frac{R}{f(D)} = \frac{e^{3x} + e^x}{2 f(D)}$

$= \frac{1}{2}\left(\frac{x e^{3x}}{f'(D)} + \frac{x e^x}{f'(D)}\right)$

$= \frac{1}{2}\left(\frac{x e^{3x}}{3D^2 - 10D + 7} + \frac{x e^x}{3D^2 - 10D + 7}\right)$

$= \frac{1}{2}\left(\frac{x e^{3x}}{27 - 30 + 7} + \frac{x^2 e^x}{6D - 10}\right)$

$= \frac{1}{2}\left(\frac{x e^{3x}}{4} - \frac{x^2 e^x}{4}\right)$

$= \frac{1}{2} x e^x (e^{2x} - x)$

$y = C.F + P.I$

$y = (c_1 + x c_2) e^x + c_3 e^{3x} + \frac{1}{8} x e^x (e^{2x} - x)$

Q.4 Solve $(D^2 - 1) y = x e^x + \cos^2 x$

Sol^n $(D^2 - 1) y = x e^x + \cos^2 x$

$(D^2 - 1) y = x e^x + \frac{1}{2} + \frac{\cos 2x}{2}$

A.E $\Rightarrow$ $m^2 - 1 = 0$

$(m - 1)(m + 1) = 0$

$m = 1, -1$

$C.F = c_1 e^x + c_2 e^{-x}$

$P.I = \frac{R}{f(D)} = \frac{x e^x + \frac{1}{2} e^{0x} + \frac{1}{2}\cos 2x}{D^2 - 1}$

$P.I = \frac{x e^x}{D^2 - 1} + \frac{1}{2}\frac{e^{0x}}{D^2 - 1} + \frac{1}{2}\frac{\cos 2x}{D^2 - 1}$

$$= \frac{xe^x}{D^2-1} - \frac{2D}{(D^2-1)^2}e^x + \frac{1}{2}\cdot\frac{1}{(-1)} + \frac{1}{2}\frac{\cos 2x}{(-4-1)}$$

$$= x \times \frac{xe^x}{2D} - 2D\left(\frac{xe^x}{2(D^2-1)\times 2D}\right) - \frac{1}{2} - \frac{1}{10}\cos 2x$$

$$= \frac{x^2e^x}{2} - \frac{xe^x}{2(D^2-1)} - \frac{1}{2} - \frac{1}{10}\cos 2x$$

$$= \frac{x^2e^x}{2} - \frac{x^2e^x}{4D} - \frac{1}{2} - \frac{1}{10}\cos 2x$$

$$= \frac{x^2e^x}{2} - \frac{x^2e^x}{4} - \frac{1}{2} - \frac{1}{10}\cos 2x$$

$$P.I. = \frac{x^2e^x}{4} - \frac{1}{2} - \frac{1}{10}\cos 2x$$

Solⁿ

$$\boxed{y = C_1e^x + C_2e^{-x} + \frac{x^2e^x}{4} - \frac{1}{2} - \frac{1}{10}\cos 2x}$$

Q.5 Solⁿ $\left(\frac{d^2y}{dx^2}\right) - 9y = x + e^{2x} - \sin 2x$ using Method of undetermined coeff. for finding particular integral.

Solⁿ

$$\frac{d^2y}{dx^2} - 9y = x + e^{2x} - \sin 2x \quad \text{—(1)}$$

$$R = x + e^{2x} - \sin 2x$$

Hence $y = C_0 + C_1x + C_3e^{2x} + C_4\sin 2x + C_5\cos 2x$

$$y_1 = C_1 + 2C_3e^{2x} + 2C_4\cos 2x - 2C_5\sin 2x$$

$$y_2 = 4C_3e^{2x} - 4C_4\sin 2x - 4C_5\cos 2x$$

from (1)

$$4C_3e^{2x} - 4C_4\sin 2x - 4C_5\cos 2x - 9(C_0 + C_1x + C_3e^{2x} + C_4\sin 2x + C_5\cos 2x) = x + e^{2x} - \sin 2x$$

$= -9C_0 - 9C_1 x - 5C_3 e^{2x} - 13C_4 \sin 2x - 13C_5 \cos 2x$
$= x + e^{2x} - \sin 2x$

$-9C_0 = 0$ $\quad$ $-9C_1 = 1$
$C_0 = 1$ $\quad$ $C_1 = -1/9$ $\quad$ $\Rightarrow -5C_3 = 1$
$C_4 = \frac{-1}{13}$ $\quad$ $\Rightarrow C_5 = 0$ $\quad$ $C_3 = -1/5$

$$y = \frac{-x}{9} - \frac{e^{2x}}{5} + \frac{1}{13}\sin 2x$$

Hence $P.I = y = \frac{-x}{9} - \frac{e^{2x}}{5} + \frac{1}{13}\sin 2x$

A.E $(m^2 - 9) = 0$

$m = -3, 3$

$C.F = c_1 e^{3x} + c_2 e^{-3x}$

Sol^n

$$\boxed{y = c_1 e^{3x} + c_2 e^{-3x} - \frac{x}{9} - \frac{e^{2x}}{5} + \frac{1}{13}\sin 2x}$$

Q) Solve $y''' + y' = 2x^2 + 4\sin x$

$y''' + y' = 2x^2 + 4\sin x$

$\frac{d^3y}{dx^3} + \frac{dy}{dx} = 2x^2 + 4\sin x$

$\Rightarrow D^3y + Dy = 2x^2 + 4\sin x$

$D(D^2+1)y = 2x^2 + 4\sin x$

A.E. $\Rightarrow$ $m(m^2+1) = 0$

$m = 0, \pm i$

$C.F = C_1 e^{0x} + e^{0x}(C_2 \cos x + C_3 \sin x)$

$C.F = C_1 + C_2 \cos x + C_3 \sin x$

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$$P.I. = \frac{2x^2+4\sin x}{f(D)} = \frac{2x^2+4\sin x}{D^3+D}$$

$$= \frac{2x^2}{D(1+D^2)} + \frac{4\sin x}{D^3+D}$$

$$= \frac{2}{D}(1+D^2)^{-1}x^2 + \frac{4.x\sin x}{3D^2+1}$$

$$= \frac{2}{D}(1-D^2+D^4 \dots)x^2 + \frac{4x\sin x}{-2}$$

$$= 2\left(\frac{1}{D} - D + D^3 + \dots\right)x^2 - 2x\sin x$$

$$= 2\left(\frac{x^3}{3} - 2x\right) - 2x\sin x$$

$$P.I. = \frac{2x^3}{3} - 4x - 4x\sin x$$

Soln

$$\boxed{y = C_1 + C_2\cos x + C_3\sin x + \frac{2x^3}{3} - 4x - 4x\sin x}$$

Q. 4) Reduce the equation

$$2x^2y\frac{d^2y}{dx^2} + 4y^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$$

to homogeneous form by making the substitution $y = z^2$ and hence solve it.

Soln

$$2x^2y\frac{d^2y}{dx^2} + 4y^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$$

$$y = z^2$$

$$\frac{dy}{dx} = 2z\frac{dz}{dx}$$

$$\frac{d^2y}{dx^2} = 2z\frac{d^2z}{dx^2} + 2\left(\frac{dz}{dx}\right)^2$$

$$2x^2z^2\left(2z\frac{d^2z}{dx^2} + 2\left(\frac{dz}{dx}\right)^2\right) + 4z^4 = 4x^2z^2\left(\frac{dz}{dx}\right)^2 + 2xz^2 \times 2z\frac{dz}{dx}$$

$$4x^2z^3\frac{d^2z}{dx^2} + 4x^2z^2\left(\frac{dz}{dx}\right)^2 + 4z^4 = 4x^2z^2\left(\frac{dz}{dx}\right)^2 + 4xz^3\frac{dz}{dx}$$

$$x^2\frac{d^2z}{dx^2} + z = x\frac{dz}{dx}$$

$$\Rightarrow x^2\frac{d^2z}{dx^2} - x\frac{dz}{dx} + z = 0$$

Clearly above eqn is Cauchy's homogeneous eqn.

$$D(D-1)z - Dz + z = 0$$

$$(D^2 - 2D + 1)z = 0$$

$$(D-1)^2 z = 0$$

A.E $\Rightarrow (m-1)^2 = 0$

$$m = 1, 1$$

$$C.F = (C_1 + C_2t)e^t$$

$$z = (C_1 + C_2t)e^t$$

$$\sqrt{y} = (C_1 + \log x\, C_2)x \quad \text{Ans}$$

Q) Solve $[(5+2x)^2D^2 - 6(5+2x)D + 8]y = x^2+1$

Soln Clearly above eqn is Legendre's eqn

putting $5+2x = e^t$

$$x = \frac{e^t-5}{2}$$

$$D(D-1)y - 6Dy + 8y = \frac{2}{4}(e^t-5)^2 + 1$$

$$(D^2 - 7D + 8)y = \frac{1}{2}(e^{2t} + 25 - 10e^t) + 1$$

$$(D^2 - 7D + 8)y = \frac{e^{2t}}{2} + \frac{27}{2} - 5e^t$$

A.E $= m^2 - 7m + 8 = 0$

$$m = \frac{7 \pm \sqrt{49-32}}{2}$$

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$$m = \frac{7 \pm \sqrt{17}}{2}$$

$$m = \frac{7}{2} + \frac{\sqrt{17}}{2} \quad , \quad m = \frac{7}{2} - \frac{\sqrt{17}}{2}$$

$$C.F = e^{\frac{7t}{2}}\left(c_1 \cosh \frac{\sqrt{17}}{2} t + c_2 \sinh \frac{\sqrt{17}}{2} t \right)$$

$$P.I = \frac{\frac{e^{2t}}{2} + \frac{27}{2} e^{0t} - 5e^{t}}{D^2 - 7D + 8}$$

$$= \frac{e^{2t}}{2(4-14+8)} + \frac{27}{2 \times 8} - \frac{5e^{t}}{2}$$

$$P.I = \frac{27}{16} - \frac{5}{2} e^{t} - \frac{e^{2t}}{4}$$

Sol

$$y = e^{\frac{7t}{2}}\left(c_1 \cosh \frac{\sqrt{17}}{2} t + c_2 \sinh \frac{\sqrt{17}}{2} t \right) + \frac{27}{16} - \frac{5e^{t}}{2} - \frac{e^{2t}}{4}$$

$$y = (5+2x)^{7/2}\left(c_1 \cosh \frac{\sqrt{17}}{2} \log(5+2x) + c_2 \sinh \frac{\sqrt{17}}{2} \log(5+2x) \right) + \frac{27}{16} - \frac{5}{2}(5+2x) - \frac{(5+2x)^2}{4}$$

Q Solve

$$tDx + 2(x-y) = t, \quad tDy + x + 5y = t^2, \text{ where } D = \frac{d}{dt}$$

Solⁿ

$$tDx + 2(x-y) = t$$

$$tDy + x + 5y = t^2$$

$$(tD+2)x - 2y = t \longrightarrow (i)$$

$$(tD+5)y + x = t^2 \longrightarrow (ii)$$

from (i)

$$(tD+2)\left(t^2 - (tD+5)y\right) - 2y = t$$

$$tDt^2 - tD(tD+5)y + 2t^2 - 2(tD+5)y - 2y = t$$

$$4t^2 - t^2D^2y - 5tDy - 2tDy - 10y - 2y = t$$

$$4t^2 - t = t^2D^2y + 7tDy + 12y$$

Putting $t = e^u$

$D(D-1)y + 7Dy + 12y = 4e^{2u} - e^u$

$(D^2 + 6D + 12)y = 4e^{2u} - e^u$

A.E $\Rightarrow$ $m^2 + 6m + 12 = 0$

$$m = \frac{-6 \pm \sqrt{36-48}}{2}$$

$$m = \frac{-6 \pm 2\sqrt{3}i}{2}$$

$m = -3 \pm \sqrt{3}i$

C.F $= e^{-3u}(C_1 \cos\sqrt{3}u + C_2 \sin\sqrt{3}u)$

$$P.I = \frac{4e^{2u} - e^u}{D^2 + 6D + 12} = \frac{4e^{2u}}{4+12+12} - \frac{e^u}{1+6+12}$$

$$P.I = \frac{e^{2u}}{7} - \frac{e^u}{19}$$

$$y = e^{-3u}(C_1 \cos\sqrt{3}u + C_2 \sin\sqrt{3}u) + \frac{e^{2u}}{7} - \frac{e^u}{19}$$

$$y = \frac{1}{t^3}(C_1 \cos\sqrt{3}\log t + C_2 \sin\sqrt{3}\log t) + \frac{t^2}{7} - \frac{t}{19}$$

dt

(ii) $x = t^2 - (tD + 5)y$

$x = t^2 - tDy - 5y$

$$x = t^2 - tD\left(\frac{1}{t^3}(C_1 \cos\sqrt{3}\log t + C_2 \sin\sqrt{3}\log t) + \frac{t^2}{7} - \frac{t}{19}\right) - 5y$$

$$x = t^2 - t\left(\frac{-3}{t^4}(C_1 \cos\sqrt{3}\log t + C_2 \sin\sqrt{3}\log t) + \frac{1}{t^3}\left(-C_1 \sin\sqrt{3}\log t \cdot \frac{\sqrt{3}}{t} + C_2 \cos\sqrt{3}\log t \cdot \frac{\sqrt{3}}{t}\right)\right.$$

$$x = \frac{-10t^2}{7} - \frac{4t}{19} - \frac{1}{t^3}\cos\sqrt{3}\log t\,(\sqrt{3}C_2 - 8C_1) - \frac{1}{t^3}\sin\sqrt{3}\log t\,(2C_2 - \sqrt{3}C_1)$$

Q) Solve $\frac{dx}{dt} - y = e^t$, $\frac{dy}{dt} + x = \sin t$; $x(0) = 1$, $y(0) = 1$

Sol^n =

$$\frac{dx}{dt} - y = e^t \qquad \frac{dy}{dt} + x = \sin t$$

$$Dx - y = e^t \quad \text{--- (I)}$$

$$Dy + x = \sin t \quad \text{--- (II)}$$

$$D(\sin t - Dy) - y = e^t$$

$$\cos t - D^2y - y = e^t$$

$$\cos t - e^t = (D^2+1)y$$

$$(D^2+1)y = \cos t - e^t$$

A.E $\Rightarrow (m^2+1) = 0$

$$m = \pm i$$

$$C.F = e^{0t}(c_1 \cos t + c_2 \sin t)$$

$$C.F = c_1 \cos t + c_2 \sin t$$

$$P.I. = \frac{\cos t - e^t}{D^2+1} = \frac{t \cdot \cos t}{2D \cdot D} - \frac{e^t}{D^2+1}$$

$$= \frac{t \times -\sin t}{2 \times -1} - \frac{e^t}{2}$$

$$PI = \frac{t \sin t - e^t}{2}$$

Sol^n =

$$\boxed{y = c_1 \cos t + c_2 \sin t + \frac{t \sin t}{2} - \frac{e^t}{2}}$$

$\Rightarrow$ from (II) $\quad x = \sin t - Dy$

$$x = \sin t - D\left(c_1 \cos t + c_2 \sin t + \frac{t \sin t}{2} - \frac{e^t}{2}\right)$$

$x = \sin t + c_1 \sin t + c_2 \cos t - \frac{\sin t}{2} - \frac{t\cos t}{2} + \frac{e^t}{2}$

$x = c_1 \sin t + c_2 \cos t + \frac{\sin t - t\cos t + e^t}{2}$

at $t=0,\ x=1$

$1 = c_2 + \frac{1}{2} \Rightarrow \boxed{c_2 = \frac{1}{2}}$

at $t=0,\ y=1$

$1 = c_1 - \frac{1}{2} \Rightarrow c_1 = \frac{3}{2}$

$y = \frac{3}{2}\cos t + \frac{1}{2}\sin t + \frac{t\sin t - e^t}{2}$

$x = \frac{4\sin t + \cos t - t\cos t + e^t}{2}$

Q8) $x^2y'' + xy' - y = 0$ given that $x + \frac{1}{x}$ is one integral.

Sol^n

$x^2y'' + xy' - y = 0$

$y'' + \frac{y'}{x} - \frac{y}{x^2} = 0$

Given $u = x + \frac{1}{x}$

$y = uv$

$\Rightarrow \frac{d^2v}{dx^2} + \left(\frac{2}{u}\frac{du}{dx} + P\right)\frac{dv}{dx} = \frac{R}{u}$

$\frac{d^2v}{dx^2} + \left(\frac{2x}{x^2+1}\left(1 - \frac{1}{x^2}\right) + \frac{1}{x}\right)\frac{dv}{dx} = 0$

$\frac{d^2v}{dx^2} + \left(\frac{2(x^2-1)}{x(x^2+1)} + \frac{1}{x}\right)\frac{dv}{dx} = 0$

$\frac{d^2v}{dx^2} + \left(\frac{3x^2-1}{x(x^2+1)}\right)\frac{dv}{dx} = 0$

$$\frac{dz}{dx} + \left(\frac{3x^2-1}{x(x^2+1)}\right) z = 0$$

$$\frac{dz}{z} = -\left(\frac{3x^2+1-2}{x^3+x}\right) dx$$

$$\frac{dz}{z} = -\left(\frac{3x^2+1}{x^3+x}\right) dx + \frac{2}{x(x^2+1)} dx$$

$$\frac{dz}{z} = -\left(\frac{3x^2+1}{x^3+x}\right) dx + \frac{2\left((x^2+1)-x^2\right)}{x(x^2+1)} dx$$

$$\int \frac{dz}{z} = -\int\left(\frac{3x^2+1}{x^3+x}\right)dx + \int \left(\frac{2}{x} - \frac{2x}{x^2+1}\right) dx$$

$$\log z = -\log|x^3+x| + 2\log x - \log|x^2+1| + \log c$$

$$\log|zx(x^2+1)^2| = \log x^2 c$$

$$z = \frac{xc}{(x^2+1)^2}$$

$$\frac{dv}{dx} = \frac{xc}{(x^2+1)^2}$$

$$x^2+1 = t \Rightarrow 2x\,dx = dt$$

$$\int dv = \frac{c}{2}\int \frac{dt}{t^2}$$

$$v = \frac{-c}{2t} + c_1$$

$$v = \frac{-c}{2(x^2+1)} + c_1$$

$$y = \left(x+\frac{1}{x}\right)\left(c_1 - \frac{c}{2(x^2+1)}\right)$$

$$\boxed{y = \frac{c_1(x^2+1)}{x} - \frac{c}{2x}}$$

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Q) $x^2y'' + xy' - 9y = 0$

$\Rightarrow y'' + \frac{y'}{x} - \frac{9}{x^2}y = 0$

Given $u = x^3$

$y = uv$

$y = x^3 v$

Sol^n,

$$\frac{d^2v}{dx^2} + \left(\frac{2}{u}\frac{du}{dx} + P\right)\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} + \left(\frac{2}{x^3} \times 3x^2 + \frac{1}{x}\right)\frac{dv}{dx} = 0$$

$$\frac{d^2v}{dx^2} + \frac{7}{x}\frac{dv}{dx} = 0$$

$$\frac{dz}{dx} + \frac{7}{x} z = 0$$

$$\frac{dz}{dx} = -\frac{7z}{x}$$

$$\int \frac{dz}{z} = -7\int \frac{dx}{x}$$

$$\log z = -7 \log x + \log c$$

$$\log z x^7 = \log c$$

$$z x^7 = c$$

$$z = \frac{c}{x^7}$$

$\Rightarrow \frac{dv}{dx} = \frac{c}{x^7}$

$v = \int \frac{c}{x^3} dx$

$v = \frac{-c}{6x^6} + c_1$

Sol^n

$y = uv$

$y = x^3\left(\frac{-c}{6x^6} + c_1\right)$

$$\boxed{y = c_1 x^3 - \frac{c}{6x^3}}$$

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Q $x^2 y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$

$\Rightarrow \quad y_2 - \frac{2}{x}(1+x)y_1 + \frac{2}{x^2}(1+x)y = x$

$P = \frac{-2}{x}(1+x) \quad , \quad Q = \frac{2}{x^2}(1+x)$

$P + Qx = \frac{-2}{x}(1+x) + \frac{2}{x}(1+x) = 0$

Hence $\quad u = x$

$y = xv$

$\frac{d^2v}{dx^2} + \left(\frac{2}{u}\frac{du}{dx} + P\right)\frac{dv}{dx} = \frac{R}{u}$

$\frac{d^2v}{dx^2} + \left(\frac{2}{x} - \frac{2}{x} - 2\right)\frac{dv}{dx} = 1$

$\frac{dz}{dx} - 2\frac{dv}{dx} = 1$

$\frac{dz}{dx} - 2z = 1$

$I.F = e^{-2\int dx} = e^{-2x}$

$z \cdot e^{-2x} = \int e^{-2x} \cdot 1 dx$

$z \cdot e^{-2x} = \frac{e^{-2x}}{-2} + c$

$z = -\frac{1}{2} + ce^{2x}$

$v = \int z dx = \int\left(-\frac{1}{2} + ce^{2x}\right) dx$

$v = -\frac{x}{2} + \frac{ce^{2x}}{2} + c_1$

$$\boxed{y = \frac{-x^2 + cxe^{2x} + 2c_1 x}{2}}$$

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Q) $(1-x^2)y'' + xy' - y = x(1-x^2)^{3/2}$

$$y'' + \frac{xy'}{1-x^2} - \frac{y}{1-x^2} = x(1-x^2)^{1/2}$$

$$P + Qx = \frac{x}{1-x^2} - \frac{x}{1-x^2} = 0$$

Hence $\quad u = x$

$$y = xv$$

$$\frac{d^2v}{dx^2} + \left(\frac{2}{u}\frac{du}{dx} + P\right)\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} + \left(\frac{2}{x} + \frac{x}{(1-x^2)}\right)\frac{dv}{dx} = (1-x^2)^{1/2}$$

$$\Rightarrow \frac{dz}{dx} + \left(\frac{2}{x} - \frac{2x}{2(x^2-1)}\right)z = (1-x^2)^{1/2}$$

$$\text{I.F.} = e^{\int\left(\frac{2}{x} - \frac{2x}{2(x^2-1)}\right)dx}$$

$$= e^{2\int\frac{dx}{x}} \cdot e^{-\frac{1}{2}\int\frac{2x}{x^2-1}dx}$$

$$x^2-1 = t$$

$$2x\,dx = dt$$

$$= e^{2\log x} \cdot e^{-\frac{1}{2}\int\frac{dt}{t}}$$

$$= x^2 \cdot e^{\log(\sqrt{t})^{-1}}$$

$$\text{I.F} = x^2(\sqrt{x^2-1})^{-1}$$

$$z \cdot x^2(\sqrt{x^2-1})^{-1} = \int x^2\,dx$$

$$z \cdot x^2(\sqrt{x^2-1})^{-1} = \frac{x^3}{3} + C$$

$$z \cdot \frac{x^2}{\sqrt{x^2-1}} = \frac{x^3}{3} + C$$

$$z = \frac{x\sqrt{x^2-1}}{3} + \frac{C\sqrt{x^2-1}}{x^2}$$

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$$v = \int z \, dx$$

$$v = \int \frac{x\sqrt{x^2-1}}{3} dx + c\int \frac{\sqrt{x^2-1}}{x^2} dx$$

$x^2 - 1 = t$ $\qquad\qquad$ $x^2 - 1 = \omega^2$

$2x\,dx = dt$ $\qquad\qquad$ $2x\,dx = 2\omega\,d\omega$

$$v = \frac{1}{6}\int \sqrt{t}\,dt + c\int \frac{\omega}{(1+\omega^2)} \frac{\omega}{\sqrt{1+\omega^2}} d\omega$$

$$v = \frac{1}{6} \times \frac{2t^{3/2}}{3} + c\int \frac{1+\omega^2-1}{(1+\omega^2)^{3/2}} d\omega$$

$$v = \frac{t^{3/2}}{9} + c\int \left(\frac{1}{\sqrt{1+\omega^2}} - \frac{1}{(1+\omega^2)^{3/2}}\right) d\omega$$

$$v = \frac{(x^2-1)^{3/2}}{9} + c \cdot \log|\sqrt{\omega^2+1} + \omega| - c\int \frac{-\omega\, d\omega}{(\omega^2+1)^{3/2}}$$

$$\frac{1}{\sqrt{1+\omega^2}} = a$$

$$\frac{-2\omega\, d\omega}{2(1+\omega^2)^{3/2}} = da$$

$$\frac{d\omega}{(1+\omega)^{3/2}} = -\frac{da}{\omega} = \frac{-da}{\sqrt{\frac{1}{a^2} - 1}}$$

$$\Rightarrow v = \frac{(x^2-1)^{3/2}}{9} + c\log|\sqrt{x^2-1} + x| - c\int \frac{-a\,da}{\sqrt{1-a^2}}$$

$$v = \frac{(x^2-1)^{3/2}}{9} + c\log|x + \sqrt{x^2-1}| - c\sqrt{1-a^2}$$

$$v = \frac{(x^2-1)^{3/2}}{4} + c\log|x + \sqrt{x^2-1}| - c\sqrt{1 - \frac{1}{x^2}}$$

$$\boxed{y = \frac{x(x^2-1)^{3/2}}{9} + (x\log|x + \sqrt{x^2-1}|) - c\sqrt{x^2-1}}$$

(8) $\frac{d}{dx}\left(\cos^2 x \frac{dy}{dx}\right) + y\cos^2 x = 0$

$$\cos^2 x \frac{d^2y}{dx^2} - 2\cos x \sin x \frac{dy}{dx} + y\cos^2 x = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} + y = 0$$

$\Rightarrow \quad u = e^{-\frac{1}{2}\int P\,dx}$

$$u = e^{-\frac{1}{2}\int -2\tan x\,dx}$$

$$u = e^{\log \sec x}$$

$$\boxed{u = \sec x}$$

$\Rightarrow \quad I = Q - \frac{P^2}{4} - \frac{1}{2}\frac{dP}{dx}$

$$I = 1 - \frac{4\tan^2 x}{4} - \frac{1}{2} \times -2\sec^2 x$$

$$I = 1 - \tan^2 x + \sec^2 x$$

$$I = 2$$

$\Rightarrow \quad S = \frac{R}{u} = \frac{0}{u} = 0$

Normal form $\Rightarrow \quad \frac{d^2v}{dx^2} + Iv = S$

$$\frac{d^2v}{dx^2} + 2v = 0$$

A.E $= (m^2+2) = 0$

$$(m-\sqrt{2}i)(m+\sqrt{2}i) = 0$$

$$m = \sqrt{2}i, \quad m = -\sqrt{2}i$$

$$C.F. = e^{0x}(C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x)$$

$$v = C_1\cos\sqrt{2}x + C_2\sin\sqrt{2}x$$

Solⁿ $\quad y = uv \Rightarrow \boxed{y = \sec x(C_1\cos\sqrt{2}x + C_2\sin\sqrt{2}x}$

(8) $\quad xy_2 - y_1 + 4x^3y = x^5$

$\Rightarrow \quad y_2 - \frac{y_1}{x} + 4x^2y = x^4$

By changing the, independent variable

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

$$\left(\frac{dz}{dx}\right)^2 = 4x^2$$

$$\frac{dz}{dx} = 2x$$

$$\int dz = \int 2x\,dx$$

$$\boxed{z = x^2}$$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx^2}\right)^2} = \frac{2 - \frac{1}{x} \times 2x}{(2x)^2} = 0$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = \frac{4x^2}{4x^2} = 1$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{x^4}{4x^2} = \frac{x^2}{4} = \frac{z}{4}$$

$$\Rightarrow \quad \frac{d^2y}{dz^2} + 0.\frac{dy}{dz} + y = \frac{z}{4}$$

$$D^2y + y = \frac{z}{4}$$

A.E $\Rightarrow$ $m^2 + 1 = 0$ $\qquad$ $C.F = e^{0x}(c_1\cos x + c_2\sin x)$

$$(m-i)(m+i) = 0$$

$$m = \pm i$$

$$P.I. = \frac{z}{4(D^2+1)} = \frac{1}{4}(1+D^2)^{-1}z$$

$$= \frac{1}{4}(1 - D^2 + \text{------})z = \frac{z}{4}$$

$$P.I. = \frac{z}{4}$$

Solⁿ, $y = c_1\cos z + c_2\sin z + \frac{z}{4}$

$$\boxed{y = c_1\cos x^2 + c_2\sin x^2 + \frac{x^2}{4}}$$

(9) $(1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + 4y = 0$

$$\frac{d^2y}{dx^2} + \frac{2x}{1+x^2}\frac{dy}{dx} + \frac{4}{(1+x^2)^2}y = 0$$

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after changing of independent variable.

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

$$\left(\frac{dz}{dx}\right)^2 = \frac{4}{(1+x^2)^2}$$

$$\int dz = \int \frac{2}{1+x^2}\,dx$$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} = \frac{\frac{-4x}{(1+x^2)^2} + \frac{4x}{(1+x^2)^2}}{\frac{4}{(1+x^2)^2}} = 0$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = \frac{\frac{4}{(1+x^2)^2}}{\frac{4}{(1+x^2)^2}} = 1$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = 0 \Rightarrow \frac{d^2y}{dz^2} + y = 0$$

A.E $\Rightarrow (D^2+1) = 0$

$D = \pm i$

C.F $= C_1 \cos z + C_2 \sin z$

Soln $y = C_1 \cos z + C_2 \sin z$

$$\boxed{y = C_1 \cos(2\tan^{-1}x) + C_2 \sin(2\tan^{-1}x)}$$ Ans

(3) $(1-x)y_2 + xy_1 - y = 2(x-1)^2 e^{-x}$

Let $y = C_1 e^x + C_2 x$

$y_1 = C_1 e^x + C_2$

$y_2 = C_1 e^x$

L.H.S $= (1-x)y_2 + xy_1 - y$

$= (1-x)C_1 e^x + x(C_1 e^x + C_2) - C_1 e^x - C_2 x$

$= c_1e^x - xe^xc_1 + xe^xc_1 + xc_2 - c_1e^x - c_2x$

$= 0$

Hence $y = c_1e^x + c_2x$ is solⁿ of given diff eqⁿ clearly x & e^x are also solⁿ of give differential eqⁿ.

Solⁿ $(1-x)y_2 + xy_1 - y = 2(x-1)^2e^{-x}$

$$y_2 + \frac{x}{1-x}y_1 - \frac{y}{1-x} = -2(x-1)e^{-x}$$

$\Rightarrow$ $P + xQ = \frac{x}{1-x} - \frac{x}{1-x} = 0$

Hence $u = x$

$y = vu$

$y = xv$

eqⁿ
$$\frac{d^2v}{dx^2} + \left(\frac{2}{x} + \frac{x}{1-x}\right)\frac{dv}{dx} = \frac{-2(x-1)e^{-x}}{x}$$

$$\frac{dz}{dx} + \left(\frac{2}{x} - \frac{x}{x-1}\right)z = \frac{-2(x-1)e^{-x}}{x}$$

$$I.F = e^{\int\left(\frac{2}{x} - \frac{x-1+1}{x-1}\right)dx}$$

$$I.F = e^{\int\left(\frac{2}{x} - 1 - \frac{1}{x-1}\right)dx}$$

$$= e^{2\log x} \cdot e^{-x} \cdot e^{-\log|x-1|}$$

$$I.F = \frac{x^2}{x-1}e^{-x}$$

$$\Rightarrow z \cdot \frac{x^2}{x-1}e^{-x} = \int \frac{x^2}{x-1}e^{-x} \times \frac{-2(x-1)}{x}e^{-x}dx$$

$$z \cdot \frac{x^2}{x-1}e^{-x} = -2\int xe^{-2x}dx$$

$$z \cdot \frac{x^2}{x-1}e^{-x} = -2\left(x\frac{e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2}dx\right)$$

$$z\frac{x^2}{x-1}e^{-x} = -2\left(\frac{xe^{-2x}}{-2} - \frac{e^{-2x}}{4}\right) + c$$

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$$z \cdot \frac{x^2}{x-1} e^{-x} = \left(x+\frac{1}{2}\right) e^{-2x} + c$$

$$z = \left(x+\frac{1}{2}\right) \frac{(x-1)}{x^2} e^{-x} + c \cdot \frac{x-1}{x^2} e^{x}$$

$$z = \frac{(2x+1)(x-1)}{2x^2} e^{-x} + c\left(\frac{1}{x} - \frac{1}{x^2}\right) e^{x}$$

$$z = (e^{-x}) - \frac{e^{-x}}{2}\left(\frac{1}{x} + \frac{1}{x^2}\right) + c\left(\frac{1}{x} - \frac{1}{x^2}\right) e^{x}$$

$$v = \int e^{-x} dx + c\int e^{x}\left(\frac{1}{x} - \frac{1}{x^2}\right) dx - \frac{1}{2}\int e^{-x}\left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$v = -e^{-x} + \frac{ce^{x}}{x} + \frac{1}{2}\int e^{t}\left(\frac{-1}{t} + \frac{1}{t^2}\right) dt$$

$$v = \frac{ce^{x}}{x} - e^{-x} + \frac{1}{2} e^{t}\left(\frac{-1}{t}\right) + c_1$$

$$\Rightarrow \boxed{v = \frac{ce^{x}}{x} - e^{-x} + \frac{e^{-x}}{2x} + c_1}$$

$$y = xv \Rightarrow \boxed{y = ce^{x} + \frac{e^{-x}}{2} - xe^{-x} + c_1 x}$$

Q) $(x^2+1) y_2 - 2xy + 2y = 6(x^2+1)^2$

$$\Rightarrow y_2 - \frac{2x}{1+x^2} y_1 + \frac{2}{1+x^2} y = 6(x^2+1)$$

$$P + Qx = \frac{-2x}{1+x^2} + \frac{2x}{1+x^2} = 0$$

Hence $u = x$

$y = xv$

eq^n is $\dfrac{d^2v}{dx^2} + \left(\dfrac{2}{u}\dfrac{du}{dx} + P\right)\dfrac{dv}{dx} = \dfrac{R}{u}$

$$\Rightarrow \frac{d^2v}{dx^2} + \left(\frac{2}{x} - \frac{2x}{1+x^2}\right)\frac{dv}{dx} = \frac{6(x^2+1)}{x}$$

$$I.F = e^{\int\left(\frac{2}{x} - \frac{2x}{1+x^2}\right) dx}$$

$$= e^{2\log x} \cdot e^{-\log|1+x^2|}$$

$$I.F = \frac{x^2}{1+x^2}$$

$$z \cdot \frac{x^2}{1+x^2} = \int 6x \, dx$$

$$z \cdot \frac{x^2}{1+x^2} = \frac{6x^2}{2} + c$$

$$z = 3(1+x^2) + c\left(1+\frac{1}{x^2}\right)$$

$$v = \int z\,dx = \int \left(3+3x^2+c+\frac{c}{x^2}\right) dx$$

$$v = 3x + x^3 + cx - \frac{c}{x}$$

Sol^n is $\boxed{y = 3x^2 + x^4 + cx^2 - c}$

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