Unit-I RELATIVISTIC MECHANICS

*Event ÷ Occurance of something at one point at an instant in space. Ex ÷ shot of a bullet, swing of a pendulam when it comes in the mean position.

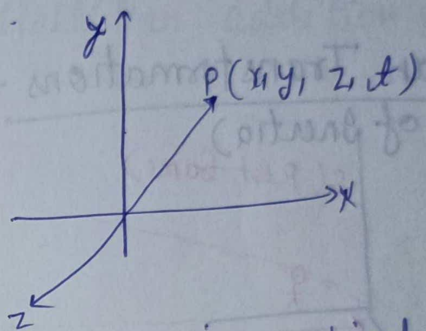
* Observer ÷ who observes the occurence of event in the space. (Rest or moving together)

* frame of Reference ÷ Geometrical frame work (normal Cartesian System) required to describe the occurance of event in the space.

$(x, y, z, t) \rightarrow$ space - lines

↓

special Co ordinate



*Position Vector ÷ The position vector of the moving object (at any instant) →

$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

Velocity of the moving object ÷

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}}$$

Acceleration ÷

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j} + \frac{dV_z}{dt}\hat{k}}$$

→ Classification of frame of Reference ÷

Inertial frame

* The frame which follows the law of inertia (Newton's I^{st} law)
* In Inertial frame

$$a = \frac{d^2r}{dt^2} = 0 \text{ , or}$$

* $\frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} = \frac{d^2z}{dt^2} = 0$

Non Inertial frame

* Do not follow law of inertia
* In non Inertial frame

$$a = \frac{d^2r}{dt^2} \neq 0 \text{ or}$$

$$\frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} = \frac{d^2z}{dt^2} \neq 0$$

* Since these have zero acc^n so it is called as Non accelerated frame.

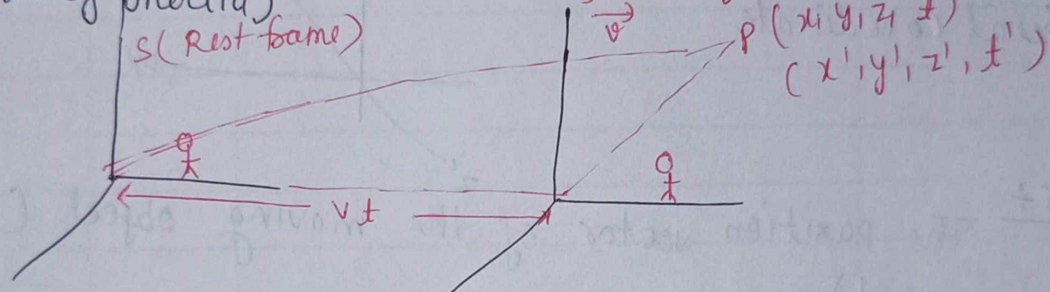
* Theory of relativity which is applicable to inertial frame of Reference known as special theory of relativity.

* Particle will experience some force due to accelerated frame of reference. ①

* Particle will experience force, because of acc of FOR known as Fictitious force or pshudo force.

* The relative which is applicable to non inertial frame is known as General theory of relativity.

Gallilean Transformations :-
(Law of Inertia)



at $t = t' = 0$ origin of both frames coincides

$$r = r' + vt$$

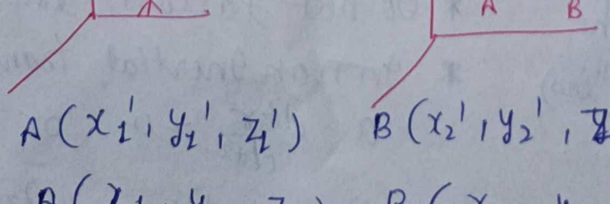
Component form

$$\boxed{\begin{matrix} & x = x' + vt \\ y' = y \quad , & z' = z \\ & t' = t \quad \boxed{x' = x - vt} \end{matrix}}$$

Gallilean transformation

Component of G.I. :-

Length of object is obsalute (invorient)



S' Frame $A(x_1', y_1', z_1')$ $B(x_2', y_2', z_2')$

S Frame $A(x_1, y_1, z_1)$ $B(x_2, y_2, z_2)$

S' frame observer

$$L = \sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2}$$

Let us transform (x_1', y_1', z_1') using G.T

$$L = \sqrt{(x_2 - vt - x_1 + vt)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$\boxed{L = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

w.r.t S frame

$\boxed{L' = L}$ → Length is in-vairent under Gallilian Transformation

velocity of moving object ÷

$$r' = r - vt \quad -(i)$$

$$\frac{dr'}{dt} = \frac{dr}{dt} - v,$$

$\boxed{V' = V - v}$ → F.O.R. $\boxed{V = V' + v}$ Gallilian addition of velocities

moving object

Acceleration ÷

$$\frac{d^2 r'}{dt^2} = \frac{d^2 r}{dt^2} - 0$$

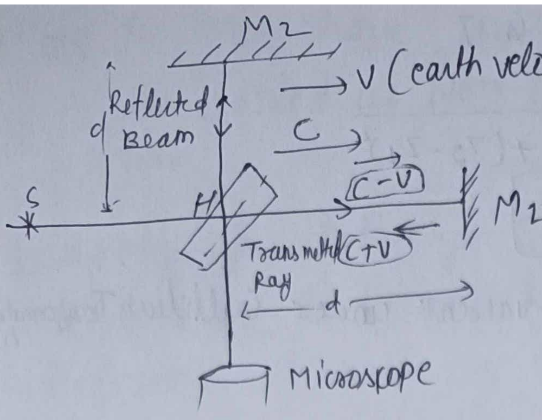
$$\boxed{a' = a} \Rightarrow \boxed{F' = F}$$

All the law of physics are identical for all the observer for inertial frame → Gallilian hypothesis of invarience

Michelson Morly experiment ÷

Objectives ÷ - Whether speed of light "c" get modified in accordance with G.T.

- Aether - Matarial mediam which was supposed to be present throughout the universe
  - * Perfectly elastic
  - * highly transparent
  - * Negligible density
- existance of aether was assumed as absolute frame of Reference Relative to which the motion of bodies can be detected
- To justify the aether hypothesis.



$$\Delta t = \frac{2d}{c} - \frac{2d}{c}$$

$$\boxed{t_2 - t_1 = 0}$$

When velocity of earth at aether frame is zero.

$\because \Delta t = 0 \Rightarrow \Delta x = 0$
no frinze pattern is visible

When earth is moving
$\Delta t = t_2 - t_1 \neq 0$
Some path diff^n will definitely occure b/w transmitted & reflected ray

$$\boxed{\text{Path diff}^n = \frac{dv^2}{c^2}}$$

frinze pattern should appear & visible

for prove that to shift frinze (interfrence) this turn into 90°
but no frinze shift was experimintly observed.
so $\boxed{V = 0}$

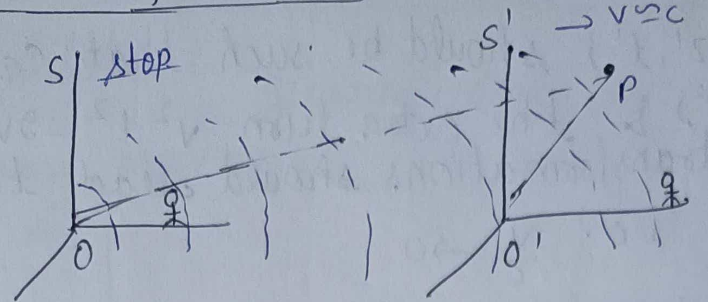
therefore
→ Motion of earth could not be detected relative to aether.
→ Earth is absolutly rest in aether
→ This is known as negative or null result.
→ The speed of light is universal constant identical for all the observes of inertial frame.

Basic Postulates of Relativity :-

i) All the laws of physics are identical for all the observer of the initial ~~figure~~ frame that move with a constant velocity relative to one - another.

ii) The velocity of light (c) is a universal constant and is identical for all the observers of the initial frame. This is also known as constancy of velocity of light.

Lorentz transformation :



Initially at $t = t' = 0$
origions coinciding

When S & S' are coinciding, a light signal emits from O
P co-ordenate from :

Rest frame observer (S) :- (x, y, z, t)

S' observer :- (x', y', z', t')

time taken by signal to reach P for S frame observer

$$\boxed{t = \frac{OP}{c} = \frac{\sqrt{x^2+y^2+z^2}}{c}}$$

$$\boxed{x^2+y^2+z^2 = c^2t^2} \quad \text{---(i)}$$

For S' frame observer

$$t' = \frac{O'P}{c} = \frac{\sqrt{x'^2+y'^2+z'^2}}{c}$$

$$\boxed{x'^2+y'^2+z'^2 = c^2t'^2} \quad \text{---(ii)}$$

$\because$ c is identical for all observer
The new transformation will be such that eq^n (ii) transform into (i)
let's apply G.T. again in eq^n (2)

$x' = x - vt$ $\qquad (x-vt)^2 + y^2 + z^2 = c^2t^2$

$y' = y$ $\qquad x^2+y^2+z^2 + \underline{v^2t^2 - 2vtx} = c^2t^2$ ---(iii)

$z' = z$

$t' = t$

Compairing eq^n (1) & (3)
Extra term $\rightarrow v^2t^2 - 2vxt$

$\Rightarrow$ G.T. fail or need some modification.

$\Longrightarrow$

∵ New transformation b/w (x, y, z, t) and (x', y', z', t') should be such that eq^n(iii) transforms into eq^n(i) & the extra term $v^2t^2 - 2vxt$ should cancel. Also new transformations should lead to G.T. for smaller velocities i.e. $\frac{v}{c} \to 0$.

Let the new or modified eq^n's can be written as
$x' = \alpha(x - vt)$, $t' = \alpha'(t + fx)$
where α, α' & f are Constant.
substituting these in eq^n (ii)

$$\alpha^2 (x - vt)^2 + y^2 + z^2 = c^2\alpha'^2 (t + fx)^2$$
$$\alpha^2 (x^2 + v^2t^2 - 2xvt) + y^2 + z^2 = c^2\alpha'^2 (t^2 + f^2x^2 + 2fxt)$$

~~on s.w. compa~~ on equating coffient of x^2, x & constant
we get,
$$x^2[\alpha^2 - f^2\alpha'^2c^2] - 2xt[\alpha^2 v + fc^2\alpha'^2] + y^2 + z^2 = c^2t^2\left[\alpha'^2 - \frac{\alpha^2v^2}{c^2}\right]$$

on compairing with eq^n (ii) -
$$\alpha^2 - f^2\alpha'^2c^2 = 1$$
$$\alpha^2 v + fc^2\alpha'^2 = 0$$
$$\alpha'^2 - \frac{\alpha^2v^2}{c^2} = 1$$

on solving these eq^n
$$\boxed{\alpha = \alpha' = \frac{1}{\sqrt{1 - v^2/c^2}}} \; ; \; \boxed{f = \frac{-v}{c^2}}$$

(event O)
on subsituting these in eq^n above

Direct Lorentz Transformation ⇐ $$\boxed{x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} , \; y' = y , \; z' = z , \; t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}}$$

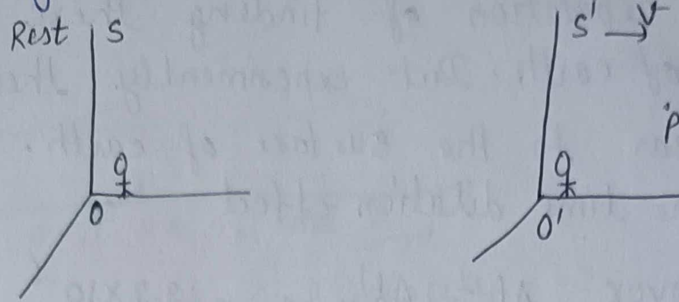
For small velocity $\frac{v}{c^2} \to 0$
Gallilian transformation ⇐ $$\boxed{x' = x - vt \; ; \; y' = y \; ; \; z' = z \; ; \; t' = t}$$

additional
event at {O'}
$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} , \; y = y' \; \; z = z' , \; t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Consequences of Lorentz Transformations :-

① Time dilation :-

Slowing down of clock relative to stationary observers.



t_1' Event 1

t_2' Event 2 (mean return)

at S'(in)

For S' observer → $(\Delta t')_{rest} = t_2' - t_1'$ — (i) (Proper time interval)

for S observer →

'experiment is moving' wrt 'S' observer

$t_1 \quad t_2$

observed time interval (Improper) $(\Delta t)_{motion} = t_2 - t_1$ — (ii)

on applying laranzt transformation in eqⁿ (ii) (due change position)

$$(\Delta t)_{motion} = \frac{t_2' + \frac{Vx'}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} - \frac{t_1' + \frac{Vx'}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \Rightarrow \frac{t_2' - t_1'}{\sqrt{1-(V/c)^2}}$$

$$\boxed{(\Delta t)_{motion} = \frac{(\Delta t')_{rest}}{\sqrt{1-(V/c)^2}}}$$

$$\boxed{(\Delta t)_{motion} > (\Delta t')_{rest}}$$

↳ time dilation

Time dilation is a Real effect :-

* Example from high Energy Physics (Elementary Particles)

μ-mesons are elementry particles which are produced in the upper atmosphare at high altitudes by the action of cosmic ray showers on π-mesons. These are highly unstable and their life time in own frame of Reference is 2.2×10^{-6} sec. So the distance travelled by μ-mesons in this life time —

⇒

Travelled distance in own frame d = life time $\times$ velocity

$= 2.2\times10^{-6}\times 0.998c$

$\boxed{d = 660 \text{ meter}}$

Therefore there is no expetation of finding these particles near to the surface of earth. But experimently these particles have been detected near to the surface of earth. This can be justified considering the time dilation effect.

For laborartry frame observer $\Delta t = \dfrac{\Delta t'}{\sqrt{1-(V/c)^2}} = \dfrac{2.2\times10^{-6}}{\sqrt{1-\left(\frac{0.098c}{c}\right)^2}}$

$\boxed{\Delta t = 3.17\times10^{-5} \sec}$

New distance (observed by laborartry frame observer)

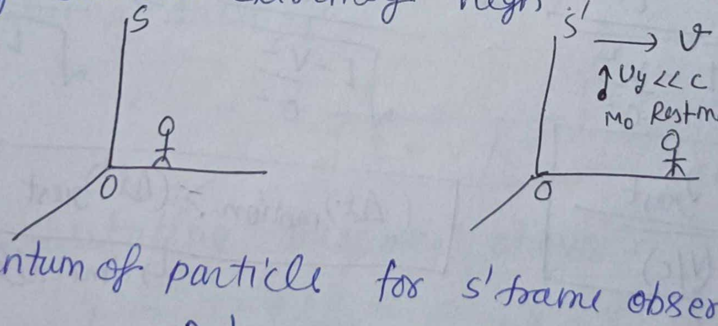
$d' = 3.17\times10^{-5}\times 3\times10^{8} \text{ or } (0.99c)$

$\boxed{d' = 9500 m}$ (approx)

$\Rightarrow$ The presence of μ-meson near to surface of earth is justified. This mean Time dilation is real effect.

Mass of particle :-

Mass undergoes variation with velocity when velocity becomes extremly high.



dy' in dt'

momentum of particle for s' frame observer

$P_y' =$ mass $\times$ velocity $= m_0 U_y = m_0 \dfrac{dy'}{dt'}$ — (i)

momentum of same particle for s frame observer

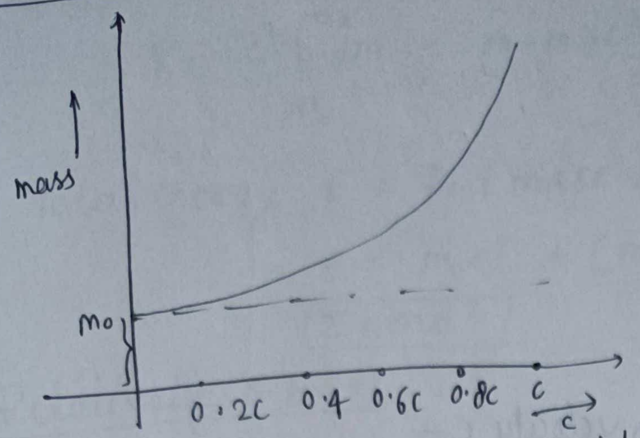
$dy' = dy \qquad dt = \dfrac{dt'}{\sqrt{1-(V/c)^2}}$

using in eq^n (1)

$P_y = \dfrac{m_0\, dy}{dt\sqrt{1-\left(\frac{V}{c}\right)^2}} = \boxed{\dfrac{m_0 U_y}{\sqrt{1-\left(\frac{V}{c}\right)^2}}} \Rightarrow \boxed{P_y = m U_y}$

$\boxed{m = \dfrac{m_0}{\sqrt{1-\left(\frac{V}{c}\right)^2}}}$

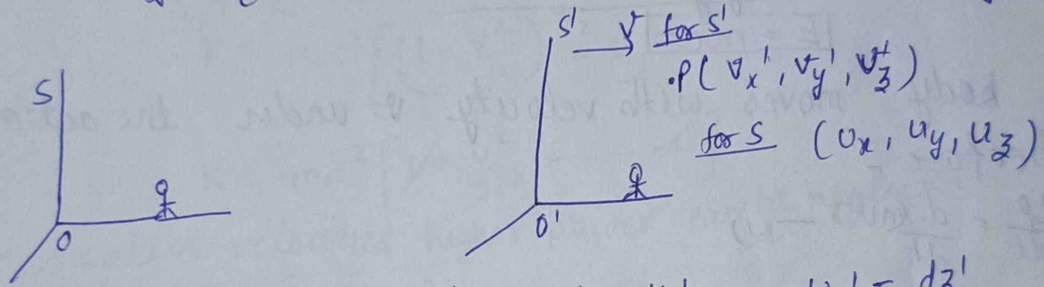
Graphical Representation ÷



Relativistic addition of velocity ÷

Larentz transformation are

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \quad ; \quad y' = y \quad ; \quad z' = z \quad ; \quad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$



velocity component $U_x' = \frac{dx'}{dt'}$, $U_y' = \frac{dy'}{dt'}$; $U_z' = \frac{dz'}{dt'}$

on differenciating larenzt transformation eqn –

$$dx' = \frac{dx - vdt}{\sqrt{1-\frac{v^2}{c^2}}} \quad , \quad dy' = dy \quad , dz' = dz \quad , \quad dt' = \frac{dt - \frac{vdx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$U_x' = \frac{dx'}{dt'} = \frac{dx - vdt}{\cancel{\sqrt{1-\frac{v^2}{c^2}}}} \times \frac{\cancel{\sqrt{1-\frac{v^2}{c^2}}}}{dt - \frac{vdx}{c^2}}$$

$$U_x' = \frac{dx - vdt}{dt - \frac{v}{c^2}dx} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}}$$

Therefore

$$\boxed{U_x' = \frac{V_x - V}{1 - \frac{V U_x}{c^2}}} \quad \text{--- (i)}$$

similarly $U_y' = \frac{dy'}{dt'} = \frac{dy\sqrt{1-\frac{v^2}{c^2}}}{dt - \frac{v}{c^2}dx} \Rightarrow \frac{\frac{dy}{dt}\sqrt{1-\frac{v^2}{c^2}}}{1 - \frac{v}{c^2}\frac{dx}{dt}}$

$$\boxed{U_y' = \frac{U_y\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{vU_x}{c^2}}} \quad \text{— (ii)}$$

$$\boxed{U_z' = \frac{dz'}{dt'} = \frac{U_z\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{vU_x}{c^2}}} \quad \text{— (iii)}$$

Let the moving object moving with velocity c -

$$U_x' = \frac{c-v}{1-\frac{vc}{c^2}} = c \Rightarrow$$ Confirm second basic postulate

Einestenis Mass Energy Relation :-

$$\boxed{E = mc^2}$$

Consider a body moves with velocity v under the action of force 'F', therfore -

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} \quad \text{— (i)}$$

In relativistic mechanics, both mass & velocity are variable

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \text{— (ii)}$$

work done on the body $dw = dK = F ds$

$$dK = m\frac{dv}{dt}ds + \frac{v\,dm}{dt}ds$$

$$dK = mv\,dv + v^2 dm \quad \text{— (iii)}$$

mass can be repreoent as -

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow m^2\left[1-\frac{v^2}{c^2}\right] = m_0^2$$

$$\Rightarrow m^2[c^2 - v^2] = m_0^2c^2$$

$$m^2c^2 - m^2v^2 = m_0^2c^2 \quad \text{— (iv)}$$

on diff^n eq^n (iv)

$$\cancel{2}mc^2dm - [m^{\cancel{2}}\cancel{2}v\,dv + v^2\cancel{2}m\,dm] = 0$$

$$mv\,dv + v^2dm = c^2dm$$

on subsituting this in eq^n (iii)

$$dK = c^2dm \quad \text{— (v)}$$

Let K be the K.E. of the body (during movement $m_0 \to m$) (22)

$$K = c^2 \int_{m_0}^{m} dm = (m - m_0)c^2$$

Total energy E = Rest mass Energy + K.E.

$$E = m_0c^2 + (m - m_0)c^2$$

$$\boxed{E = mc^2}$$

Relativistic K.E →

$$K = (m - m_0)c^2 \quad (i), \quad E = mc^2$$

classical value of K.E $= \frac{1}{2}m_0v^2$

using eqn ①

$$K = \left(\frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0\right)c^2$$

$$K = m_0c^2\left[\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1\right]$$

$$K = m_0c^2\left[\cancel{1} + \frac{v^2}{2c^2} + \cdots - \cancel{1}\right]$$

For smaller velocities higher power can be neglected

$$\boxed{K.E = \frac{1}{2}m_0v^2}$$

Relativistic Momentum & Energy :-

$$\boxed{E^2 = p^2c^2 + m_0^2c^4}$$

$$E^2 - p^2c^2 = m_0^2c^4$$

L.H.S $\quad (mc^2)^2 - p^2c^2 \Rightarrow m^2c^4 - m^2v^2c^2$

$$= \frac{m_0^2c^4}{1 - \frac{v^2}{c^2}} - \frac{m_0^2c^2v^2}{1 - \frac{v^2}{c^2}}$$

$$= \frac{m_0^2c^2}{1 - \frac{v^2}{c^2}}\left[c^2 - v^2\right]$$

$$= m_0^2c^4$$