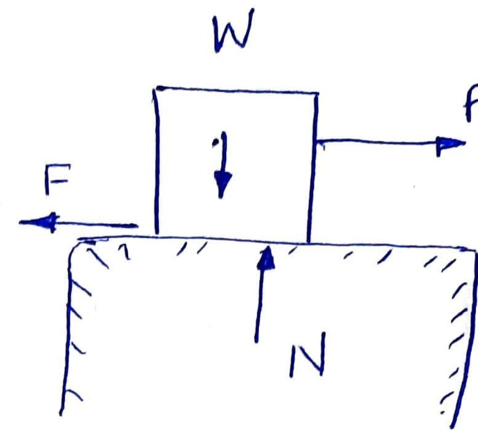
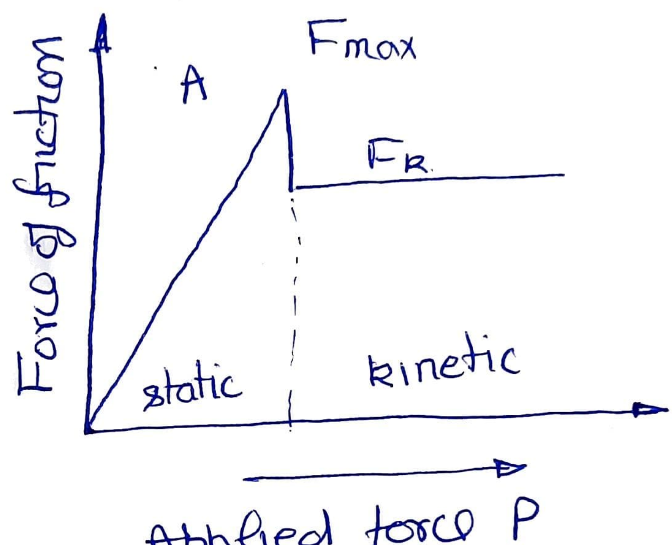
Friction

For smooth surface friction force is zero.



A, condition of impending motion.



F ∝ P upto limit Fmax

As the motion starts, there is slight reduction in friction force Fmax to FR

FR → kinetic friction.

Laws of Dry Friction or Coulomb Friction

Coulomb Friction

$$F_s \propto N$$
$$F_k \propto N$$

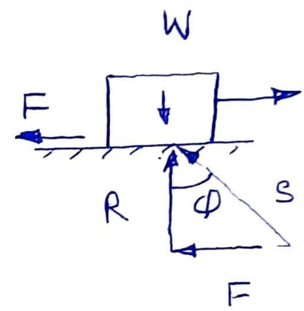
$$F_s = \mu_s N \quad \& \quad F_k = \mu_k N$$

$\mu_s \rightarrow$ coefficient of static friction

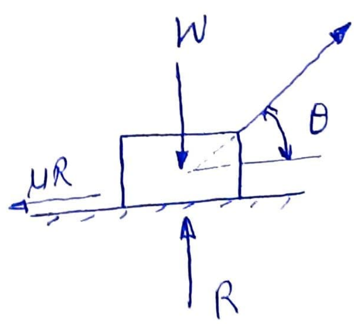
$\mu_k \rightarrow$ Coefficient of kinetic friction.

μ_k is 20-25% less than μ_s

Angle of Friction



$$\tan\phi = \frac{F}{R}$$
$$= \frac{\mu R}{R}$$
$$= \mu$$



$$F = P\cos\theta$$
$$\mu R = P\cos\theta$$
$$R + P\sin\theta = W$$
$$R = W - P\sin\theta$$

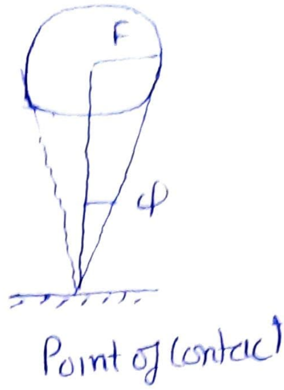
If W, P, & θ known

R can be find out.

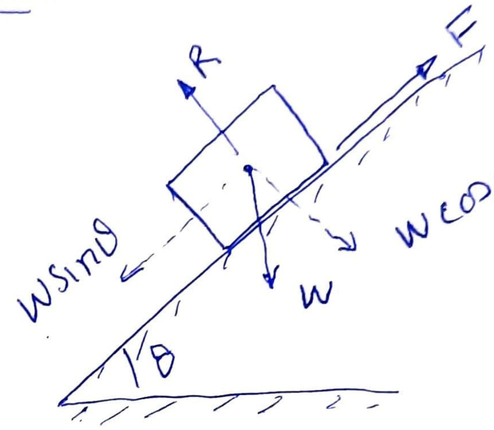
μ " " " "

Cone of Friction

ϕ is Angle of friction.



Angle of Repose



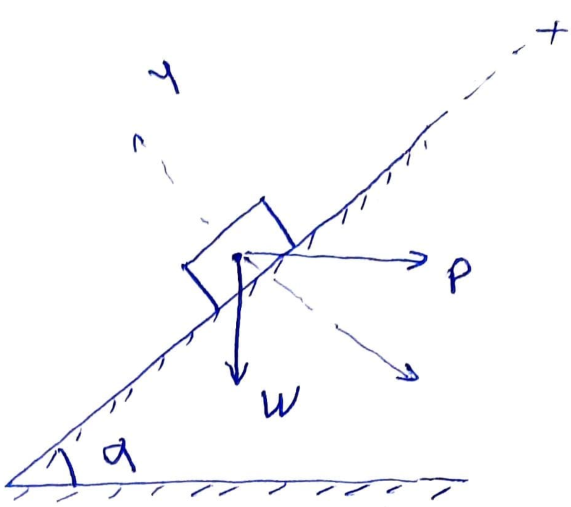
$$W\sin\theta = F$$
$$W\cos\theta = R.$$

$$\tan\theta = \frac{F}{R} = \frac{\mu R}{R} = \mu$$
$$= \tan\phi$$
$$\theta = \phi.$$

Example

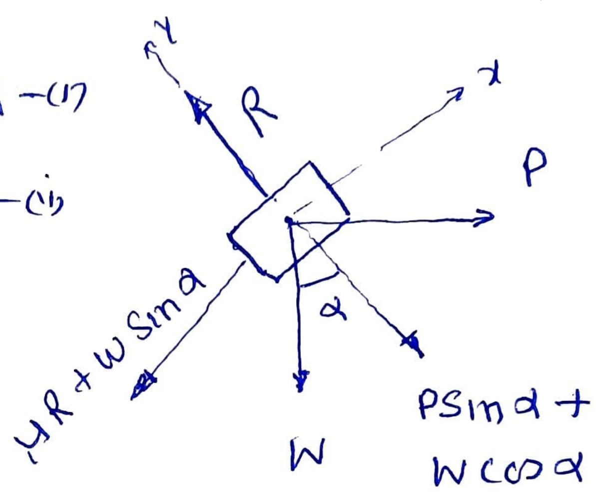
(a)

Resolving the forces along the plane & ⊥ to plane



$W\sin\alpha + \mu R = P\cos\alpha$ — (i)

$W\cos\alpha + P\sin\alpha = R$ — (ii)



FBD.

Substituting value of R in (i)

$$W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) = P\cos\alpha.$$

$$P(\cos\alpha - \mu\sin\alpha) = W(\sin\alpha + \mu\cos\alpha)$$

$$P = W\left(\frac{\sin\alpha + \tan\phi\cos\alpha}{\cos\alpha - \tan\phi\sin\alpha}\right)$$

$$\boxed{P = W\tan(\alpha+\phi)}$$

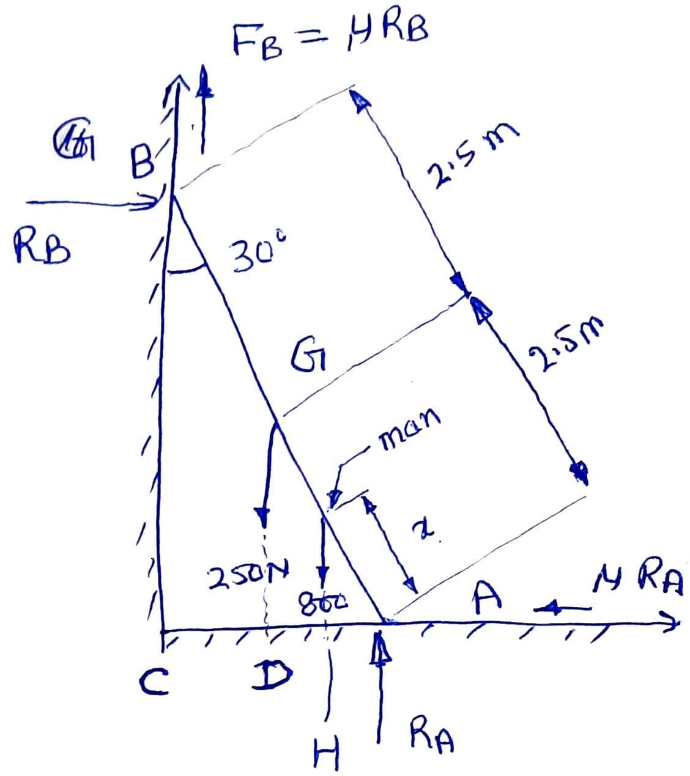
(b) Going down

$$\boxed{P = W\tan(\alpha-\phi)}$$

(i) Block on inclined plane
(ii) Ladder
(iii) Wedges.

Example

Let x be distance climbed by man when slipping start.



(2) ⇒

$\Sigma F_y = 0$

$R_A + \mu R_B = 250 + 800$ — (1)

$\Sigma F_x = 0$

$R_B - \mu R_A = 0$

$R_B = 0.2 R_A$ — (2)

(1) and (2) gives $R_A = 1009.6N$ $R_B = 201.9N$

$AD = 2.5 \cos 60^\circ = 1.25$

$AH = x \cos 60^\circ = x/2$

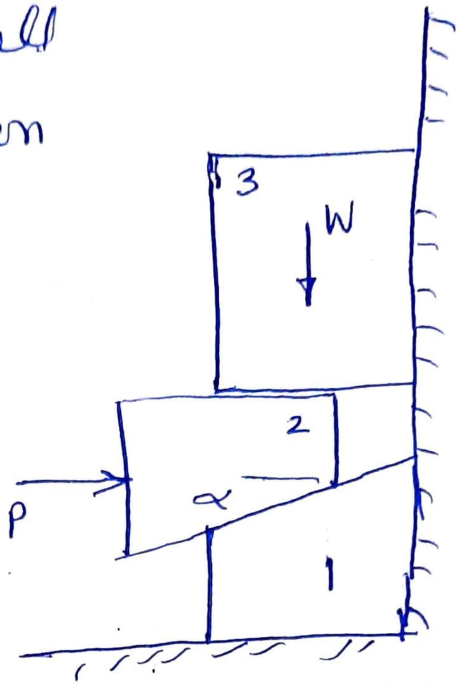
$\Sigma M_z = 0$ taking moment about A

$800 \times AH + 250 \times AD = R_B . BC + F_B \times AC$

$BC = AB \cos 30^\circ = 4.33$ $AC = 2.5$

$\boxed{x = 1.657}$

Wedge :- Wedge is useful m/c for small adjustment in position of body.



FBD (1)

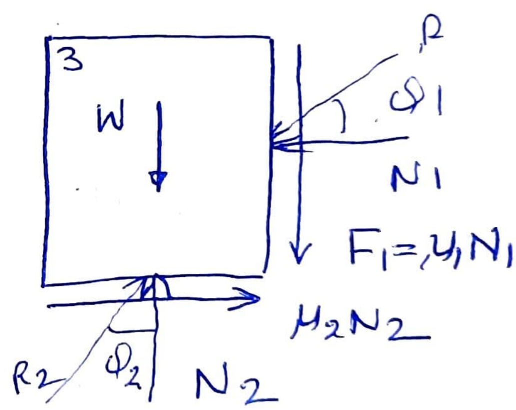
From FBD (1)

$\Sigma F_y = 0$

$W = N_2 - \mu_1 N_1$ — (1)

$\Sigma F_x = 0$

$\mu_2 N_2 = N_1$ — (2)



From (1) & (2)

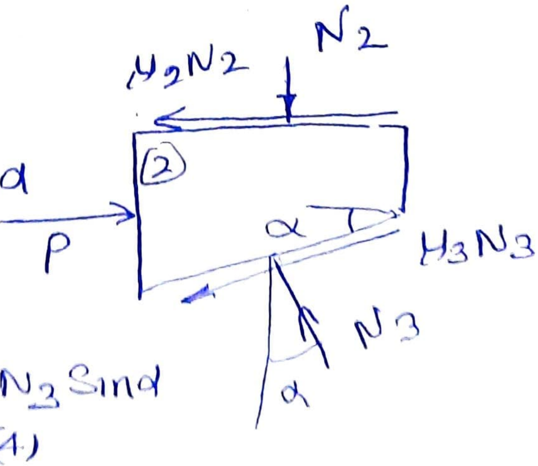
$$N_2 = \frac{W}{1 - \mu_1 \mu_2}$$

$$N_1 = \frac{\mu_2 W}{1 - \mu_1 \mu_2}$$

From FBD (2)

$\Sigma F_y = 0$

$N_2 = N_3 \cos\alpha - \mu_3 N_3 \sin\alpha$ — (3)



$P = \mu_2 N_2 + \mu_3 N_3 \cos\alpha + N_3 \sin\alpha$ — (4)

FBD (2)

α is known

N_3 & P can be calculated.

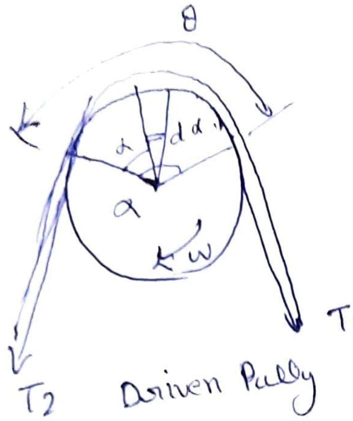
6.10 Belt & Rope Drive

Power transmission depends upon the frictional resistance between belt & surface drum.

For rough surface → tension in belt vary throughout.

Frictional resistance → increases exponential maner.

belt is in contact over angle θ. (Angle of lab)



Driven Pully

$T_1 > T_2$

For Equilibrium $\Sigma F_x = 0$

$(T+dT)\cos\frac{d\alpha}{2} - T\cos\frac{d\alpha}{2} - dF = 0$

$d\alpha \to 0 \quad \cos d\alpha/2 \to 1.$

$\Rightarrow dF = dT$

$\mu dN = dT \quad — \quad (1)$

$\Sigma F_y = 0$

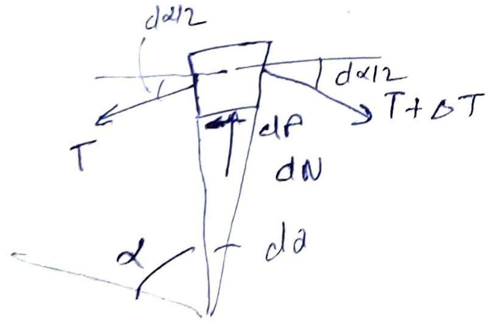
$(T+dT)\sin\frac{d\alpha}{2} + T\sin\frac{d\alpha}{2} - dN = 0$

$\sin d\alpha/2 \simeq d\alpha/2$

$\Rightarrow dN = 2T\frac{d\alpha}{2} + dT\frac{d\alpha}{2}$

$x = 0$

$= T d\alpha. \quad — \quad (2)$



(1) & (2)

$\frac{dT}{\mu} = T\, d\alpha.$

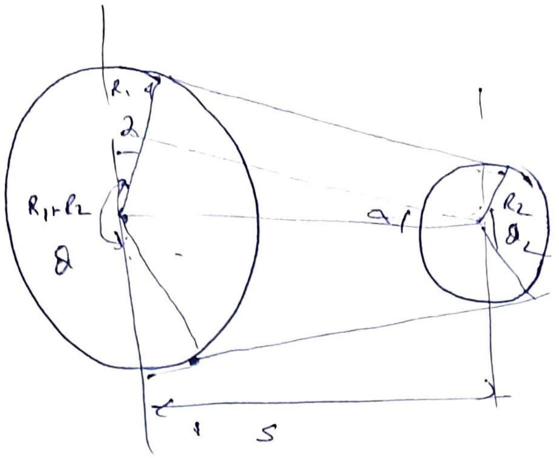
$\Rightarrow \int_{T_2}^{T_1} \frac{dT}{T} = \int_0^\theta \mu\, d\alpha$

$\log_e T \Big|_{T_2}^{T} = \mu\theta$

$\Rightarrow T_1/T_2 = e^{\mu\theta}.$

θ must be in radians

Types of Belt Drive: (i) open belt & (ii) crossed belt



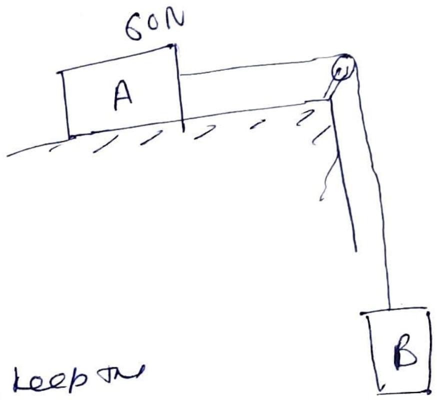
Torque

$T = (T_1 - T_2) R$

$$\theta_1 = \pi + 2\alpha = \pi + 2\sin\frac{R_1 - R_2}{S}$$

$$\theta_2 = \pi - 2\alpha = \pi - 2\sin\frac{R_1 - R_2}{S}$$

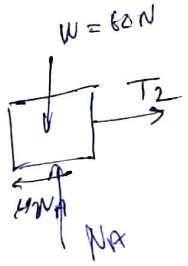
Ex A system of two blocks connected by a string which passes over pulley as shown in fig. $\mu = 0.3$ for block & plane & bet & pulley. Determine min weight B to keep the system in eqm.



Sol

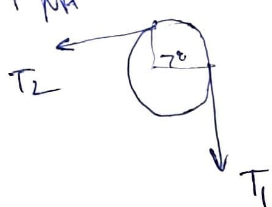
$T_2 = \mu N_A = 0.3 N_A$

$N_A = 60 \Rightarrow T_2 = 18N$



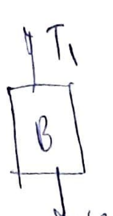
$\frac{T_1}{T_2} = e^{\mu\theta}$ $(\theta = 90°)$

$= e^{0.3 \times 90 \times \pi/180} = 1.6$

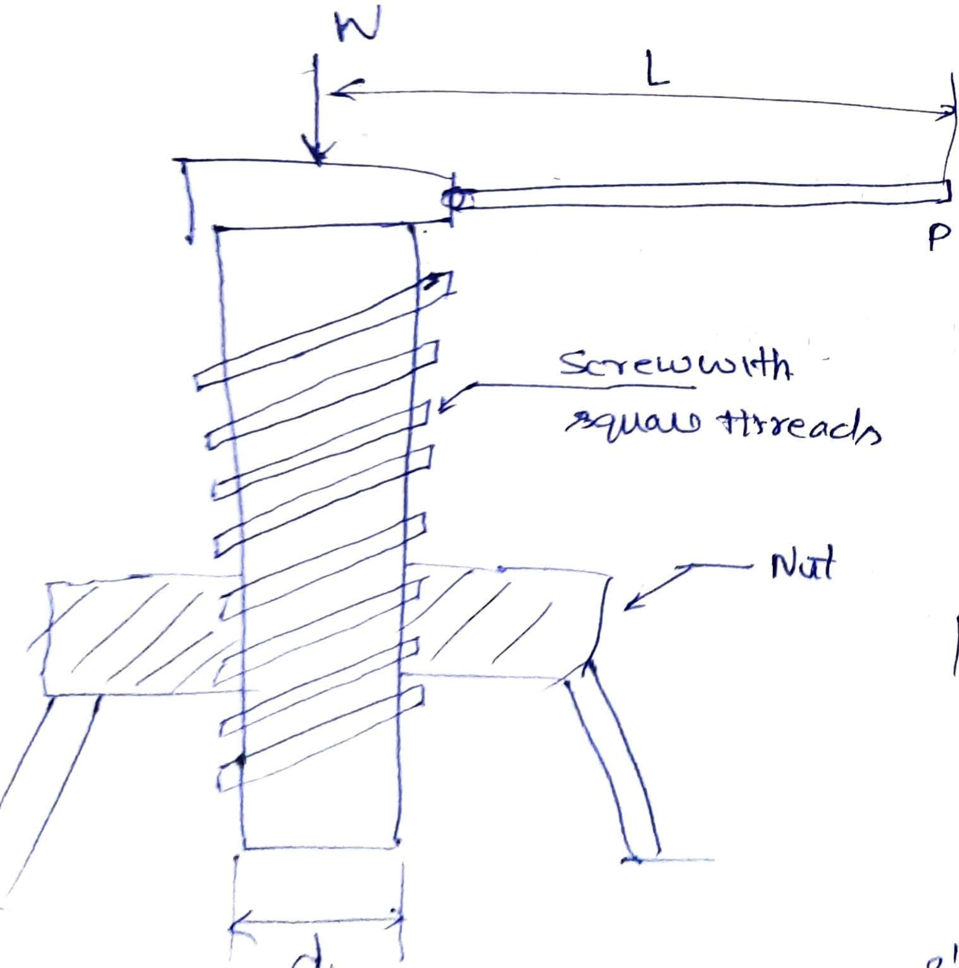


$T_1 = 1.6 T_2 = 28.8$

$W_B = T_2 = 28.8$ Ans



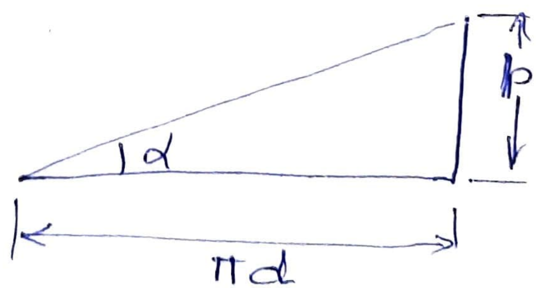
Screw - Jack



d = mean diameter of thre srew

α = Angle of srew or helix angle

ϕ = Angle of friction.



$$\tan\alpha = \frac{p}{\pi d}$$

$$P' = W \tan(\alpha + \phi) \quad \text{--- (1)} \uparrow$$

$$P = W \tan(\alpha - \phi) \quad \downarrow$$

$$P \times L = P' \times \frac{d}{2} = Q' \frac{d}{2} \quad \text{--- (2)}$$

$$P' = W \tan(\alpha + \phi)$$

$$P \times L = P' \times \frac{d}{2} = W \tan(\alpha + \phi) \times \frac{d}{2}$$

$$P_{actual} = \frac{d}{2L} W \tan(\alpha + \phi)$$

$$\text{or } \frac{d}{2L} W \tan(\alpha - \phi)$$

$$P_{ideal} = \frac{d}{2L} W \tan\alpha .$$

P.T.O

8.32

The efficiency of a screw-jack is 55%, when a load of 1500 N is lifted by an effort applied at the end of a handle of length 50 cm. Determine the effort applied if the pitch of the screw thread is 1 cm.

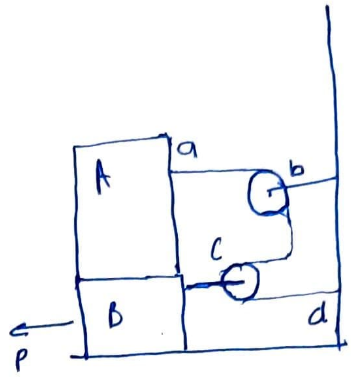
Ans $\eta = \frac{55}{100} = 0.55$

$$VR = \frac{2\pi \times 0.5}{0.01} = 314.16$$

$$MA = \frac{W}{P} = \frac{1500}{P}$$

$$\eta = \frac{MA}{VR} = \frac{1500/P}{314.16} \Rightarrow P = 8.68\ N.$$

6.3 [OCJ] Two blocks A and B of mass 60kg and 90 Kg respectively are placed on horizontal plane. Both are connected to string abcd as shown in fig. The Coefficient for friction for all contacting plane is 0.3. Determine the Value of the largest force P that can be applied without moving the blocks A & B.



Soln Block A:

$N_1 = W_A = 60 \times 9.81 = 588.6$

$T = F_1 = 0.3 N_1 = 176.6\ N$

Block B.

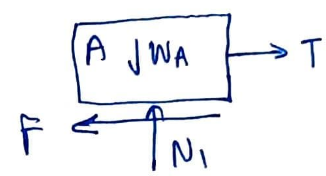
$N_1 + W_B = N_2$

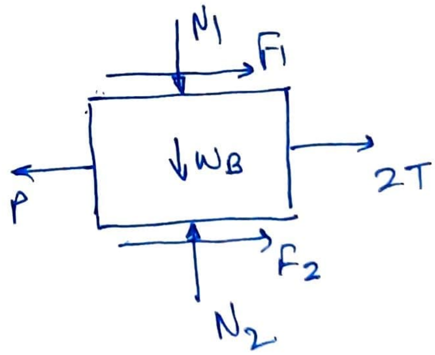
$588.6 + 90 \times 9.81 = N_2$

$N_2 = 1471.5\ N$

$F_2 = 0.3 N_2 = 441.45\ N$

$2T = 2 \times 176.6 = 353.2\ N$





$\Sigma F_x = 0$

$P = F_1 + F_2 + 2T$

$= 176.6 + 441.45 + 353.2$

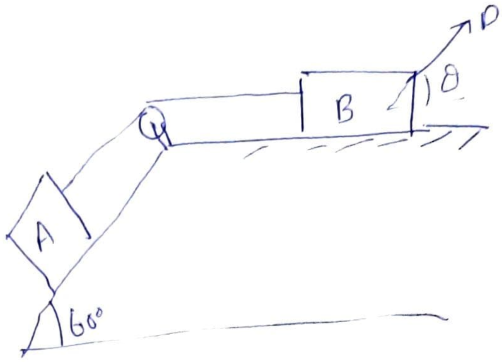
$= 91.25\ N$ ✗

$= 970$

P6.14

~~Wha~~ What is the least value of P required to cause the motion impend in the arrangement shown below. Assume the coefficient of friction on all contact surface as 0.2 weight of block A and B are 840 N and 560 N respectively

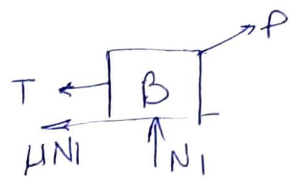
Soln Block B FBD



$T + \mu N_1 = P\cos\theta$ — ①

$N_1 = 560 - P\sin\theta$ ②

$\Rightarrow T + \mu(560 - P\sin\theta) = P\cos\theta$ — ③



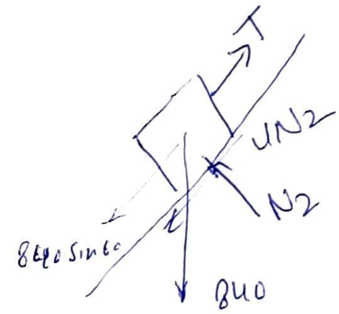
Block A

$N_2 = 840\cos 60°$

$= 420\ N \Rightarrow \mu N_2 = 84$

$840\sin 60° + \mu N_2 = T$ — ④

Putting value of T in ④ from ③



Eq ③ $\Rightarrow$

$T + 112 - 0.2P\sin\theta = P\cos\theta$

$\Rightarrow 840\sin 60° + 84 + 112 - 0.2P\sin\theta = P\cos\theta$

$\Rightarrow P[\cos\theta + 0.2\sin\theta] = 923.44$

Now $0.2 = \tan\varphi$

$\Rightarrow P\left[\cos\theta + \frac{\sin\varphi}{\cos\varphi}\sin\theta\right] = 923.44$

$P = \frac{923 \cdot \cos\varphi}{\cos(\theta - \varphi)}$

P will be min when $\cos(\theta - \varphi) = 1$

$\Rightarrow \cos(\theta - \varphi) = 1 = \cos 0°$

$\theta - \varphi = 0$

$\Rightarrow \theta = \varphi$

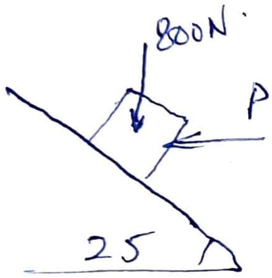
$\varphi = 11.31°$

$P_{least} = 923.44 \times 0.98 = 905.5\ N$

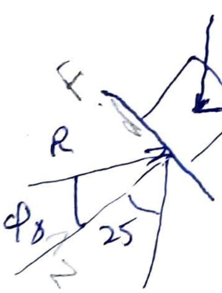
8.21 BEER

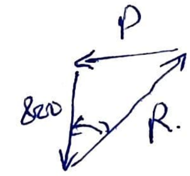
A support block is acted upon by two forces as shown. Knowing that the coefficient of friction between the block and inclined plane are $\mu_s = 0.35$ and $\mu_k = 0.25$ determine the force P required (a) to start the block moving up the incline. (b) to keep it moving up (c) to prevent it from sliding down.

Solution.



(a)

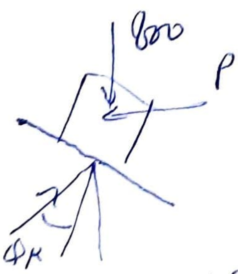


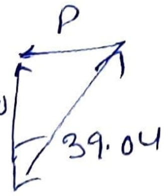


$\phi_s + 25$
$= 44.29$

$P = (800\ N) \tan 44.29$
$= 780 N$ ←

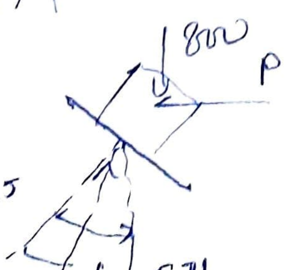
(b)

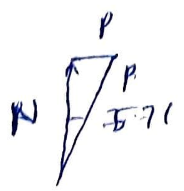




$P = 800 \tan 39.04$
$= 649 N.$

(c)





$P = 800 N \tan 5.75$
$= 80 N.$