《数据分析与 R 软件 (第二版)》 习题解答

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第一章　探索性数据分析

1. 程序:

```
w <- c(72,61,72,61,66,61,80,
      71,80,80,72,71,67,61,
      76,68,68,73,81,66,75,
      75,61,76,91,96,86)
# 求均值
w.mean <- mean(w); w.mean
# 求中位数
w.median <- median(w); w.median
# 求分位数
q.quantile = quantile(w); q.quantile
# 求下四分位数
Q1 = q.quantile[2]; Q1
# 求上四分位数
Q3 = q.quantile[4]; Q3
# 求三均值
M3 = Q1*(1/4) + q.quantile[3]*(1/2) + Q3*(1/4); M3
```

输出结果:

```
> # 求均值
> w.mean <- mean(w); w.mean
[1] 72.85185
> # 求中位数
> w.median <- median(w); w.median
[1] 72
> # 求分位数
> q.quantile = quantile(w); q.quantile
  0% 25% 50% 75% 100%
61.0 66.5 72.0 78.0 96.0
> # 求下四分位数
> Q1 = q.quantile[2]; Q1
 25%
66.5
> # 求上四分位数
> Q3 = q.quantile[4]; Q3
75%
 78
> # 求三均值
> M3 = Q1*(1/4) + q.quantile[3]*(1/2) + Q3*(1/4); M3
```

```
  25%
72.125
```

主要结论：

均值为 72.85185；中位数为 72；下四分位数为 66.5；上四分位数为 78.0；三均值为 72.125。

2. 程序：

```
w <- c(72,61,72,61,66,61,80,
      71,80,80,72,71,67,61,
      76,68,68,73,81,66,75,
      75,61,76,91,96,86)
m <- mean(w); m
# 求方差
v <- var(w); v
# 求标准差
s <- sd(w); s
# 求极差
R <- max(w) - min(w); R
# 求变异系数
cv <- (s/m); cv
# 求四分位极差
q.quantile = quantile(w)
Q1 = q.quantile[2]
Q3 = q.quantile[4]
R1 <- Q3 - Q1; R1
# 求上截断点
Qu <- Q3 + 1.5*R1; Qu
# 求下截断点
Qd <- Q1 - 1.5*R1; Qd
```

输出结果：

```
> m <- mean(w); m
[1] 72.85185
> # 求方差
> v <- var(w); v
[1] 83.59259
> # 求标准差
> s <- sd(w); s
[1] 9.142898
> # 求极差
> R <- max(w) - min(w); R
[1] 35
```

```
> # 求变异系数
> cv <- (s/m); cv
[1] 0.1254999
> # 求四分位极差
> q.quantile = quantile(w)
> Q1 = q.quantile[2]
> Q3 = q.quantile[4]
> R1 <- Q3 - Q1; R1
 75%
11.5
> # 求上截断点
> Qu <- Q3 + 1.5*R1; Qu
  75%
95.25
> # 求下截断点
> Qd <- Q1 - 1.5*R1; Qd
  25%
49.25
```

主要结论：

方差为 83.59259；标准差为 9.142898；极差为 35；变异系数为 0.1254999；四分位极差为 11.5；上截断点为 95.25；下截断点为 49.25。

3. 程序：

```
source("summarize.R")
w <- c(72,61,72,61,66,61,80,
      71,80,80,72,71,67,61,
      76,68,68,73,81,66,75,
      75,61,76,91,96,86)
summarize(w)
```

输出结果：

```
> summarize(w)
   N Mean Var std_dev Median CV CSS USS M3 R R1
1 27 72.85185 83.59259 9.142898 72 0.1254999 2173.407 145473 72.125 35 11.5
   Skewness Kurtosis
1 0.7044859 0.3881179
```

主要结论：

数量为 27；均值为 72.85185；方差为 83.59259；标准差为 9.142898；中位数为 72；变异系数为 0.1254999；样本校正平方和为 2173.407；样本未校正平方和为 145473；三均值为 72.125；极差为 35；四分位极差为 11.5；偏度系数为 0.7044859；峰度系数为 0.3881179。

4. 程序:

```
w <- scan("ex1_4-data.txt")
# 绘制密度直方图
hist(w, freq=FALSE, xlim = c(2,8), ylim = c(0,0.6))
# 绘制密度估计曲线
lines(density(w), type="l")
x <- 2:8
# 绘制正态分布概率密度曲线
lines(x, dnorm(x, mean(w), sd(w)), type="b")
```

输出结果:

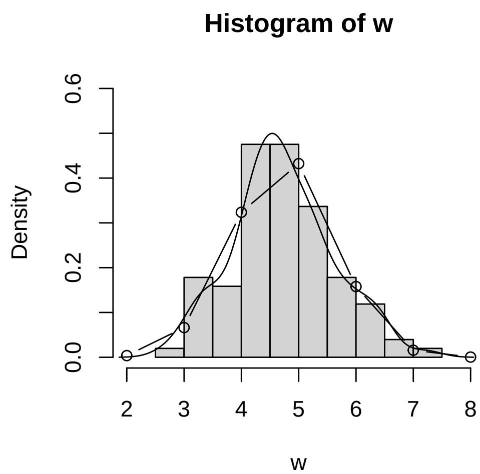


图 1: 题 4 图

主要结论:

由线条显示的曲线是密度估计曲线，而点和线交错显示的曲线是相应正态分布的概率密度曲线。密度估计曲线与正态分布的概率密度曲线有一定差别，但不是很大，可近似认为样本符合正态分布。

5. 程序:

```
w <- scan("ex1_5-data.txt")
# 绘制茎叶图
stem(w)
# 绘制箱线图
boxplot(w)
# 计算五数总括
fivenum(w)
```

输出结果:

```
> stem(w)
```

```
The decimal point is at the |

12 | 069
14 | 477901112223336899
16 | 0111233446677902233345555567789
18 | 001223338891123355579
20 | 0005689003588
22 | 089
24 | 346816
26 |
28 |
30 | 0
```

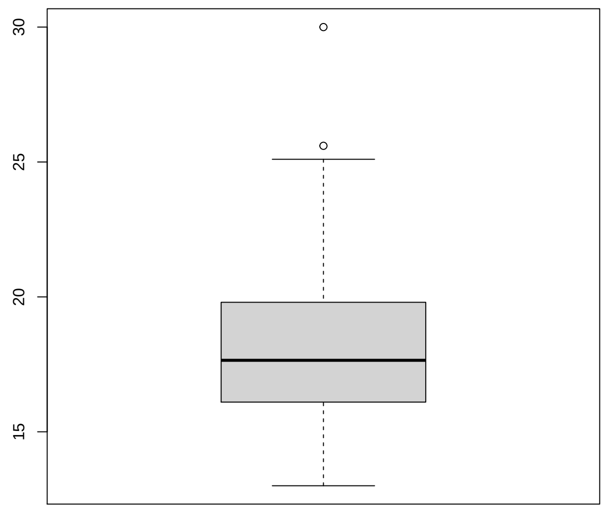


图 2: 题 5 箱线图

```
> fivenum(w)
[1] 13.00 16.10 17.65 19.80 30.00
```

主要结论：

(1) 从茎叶图可以看出，绝大部分数据集中在 $14 \sim 20$，在 $16 \sim 18$ 形成一个高峰；数据分布不对称，有异常数据 30.0；

(2) 从箱线图可以看出，数据存在异常值 30.0，集中在较大值一侧，分布呈右偏态；

(3) 五数总括中，最小值为 13.00，下四分位数为 16.10，中位数为 17.65，上四分位数为 19.80，最大值为 30.00。

6. 程序：

```
w <- scan("ex1_5-data.txt")
# 绘制经验分布函数曲线
plot(ecdf(w), verticals=TRUE, do.p=FALSE)
```

```
x <- 12:31
# 绘制正态分布函数曲线
lines(x, pnorm(x, mean(w), sd(w)))
```

输出结果：

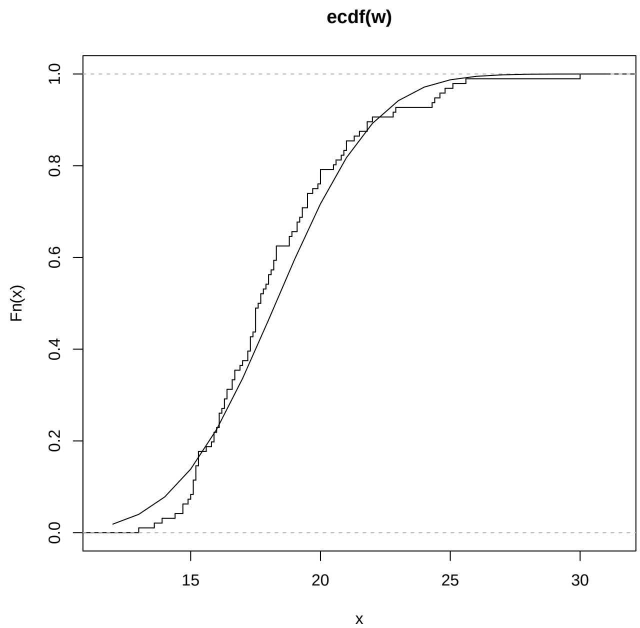


图 3: 题 6 图

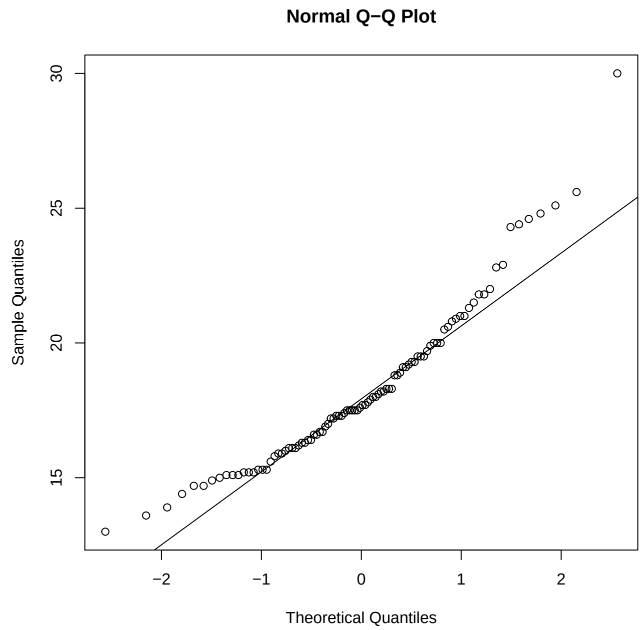
主要结论：

儿童体重的经验分布函数曲线用正态分布函数曲线拟合效果良好，可以认为数据来自正态总体。

7. 程序：

```
w <- scan("ex1_5-data.txt")
# 绘制正态QQ图
qqnorm(w)
qqline(w)
```

输出结果:



主要结论:

这些点中间部分近似在一条直线上，但首尾偏差较大，无法认为样本数据来自正态总体。

8. 程序:

```
w <- scan("ex1_5-data.txt")
n <- length(w);
k <- floor(1+3.3*log10(n));
R <- max(w) - min(w);
d <- round(R/k); d
A <- table(cut(w, br=c(13,15,17,19,21,23,30))); A
p <- pnorm(c(15,17,19,21,23,30), mean(w), sd(w));
p <- c(p[1],p[2]-p[1],p[3]-p[2],p[4]-p[3],p[5]-p[4],1-p[5]); p
chisq.test(A, p=p)
```

输出结果:

```
> d <- round(R/k); d
[1] 2
> A <- table(cut(w, br=c(13,15,17,19,21,23,30))); A

(13,15] (15,17] (17,19] (19,21] (21,23] (23,30]
     7 28 27 19 7 7
> p <- c(p[1],p[2]-p[1],p[3]-p[2],p[4]-p[3],p[5]-p[4],1-p[5]); p
[1] 0.13841648 0.19775300 0.25927817 0.22211154 0.12430578 0.05813504
> chisq.test(A, p=p)
```

```
        Chi-squared test for given probabilities

data:  A
X-squared = 10.185, df = 5, p-value = 0.07016
```

主要结论:

H_0 : 来自正态总体，H_1 : 不来自正态总体，p 值 $= 0.07016 > 0.05$，则接受原假设，认为数据来自正态总体。

9. 程序:

```
red <- seq(420, 640, by = 20)
num <- c(2,4,7,16,20,20,24,22,16,2,6,1)
w <- rep(red, num)
ks.test(w, "pnorm", mean(w), sd(w))
```

输出结果:

```
> ks.test(w, "pnorm", mean(w), sd(w))

        One-sample Kolmogorov-Smirnov test

data:  w
D = 0.10583, p-value = 0.0869
alternative hypothesis: two-sided

Warning message:
In ks.test(w, "pnorm", mean(w), sd(w)) :
  Kolmogorov - Smirnov检验里不应该有连结
```

主要结论:

H_0 : 数据来自正态总体，H_1 : 数据不是来自正态总体，p 值为 $0.0869 > 0.05$，则接受原假设，认为数据来自正态总体。

10. 程序:

```
w <- c(0.54,0.81,0.87,0.21,0.31,
       0.40,0.46,0.17,0.62,0.63,
       0.99,0.71,0.14,0.12,0.64,
       0.51,0.68,0.50,0.60,0.78)
# X^2拟合优度检验
A <- table(cut(w, br=c(0,0.2,0.4,0.6,0.8,1))); A
p <- punif(c(0.2,0.4,0.6,0.8,1), 0, 1);
p <- c(p[1],p[2]-p[1],p[3]-p[2],p[4]-p[3],1-p[4]); p
chisq.test(A, p=p)
```

```
# K-S检验
ks.test(w, "punif", 0, 1)
```

输出结果：

```
> # X^2拟合优度检验
> A <- table(cut(w, br=c(0,0.2,0.4,0.6,0.8,1))); A

  (0,0.2] (0.2,0.4] (0.4,0.6] (0.6,0.8] (0.8,1]
        3 3 5 6 3
> p <- c(p[1],p[2]-p[1],p[3]-p[2],p[4]-p[3],1-p[4]); p
[1] 0.2 0.2 0.2 0.2 0.2
> chisq.test(A, p=p)

      Chi-squared test for given probabilities

data: A
X-squared = 2, df = 4, p-value = 0.7358

Warning message:
In chisq.test(A, p = p) : Chi-squared近似算法有可能不准
> # K-S检验
> ks.test(w, "punif", 0, 1)

      One-sample Kolmogorov-Smirnov test

data: w
D = 0.16, p-value = 0.6285
alternative hypothesis: two-sided
```

主要结论：

(1) H_0 : 来自 $[0,1]$ 上的均匀分布，H_1 : 不来自 $[0,1]$ 上的均匀分布，χ^2 拟合优度检验的 p 值为 $0.7358 > 0.05$，则接受原假设，认为数据来自 $[0,1]$ 上的均匀分布。

(2) H_0 : 来自 $[0,1]$ 上的均匀分布，H_1 : 不来自 $[0,1]$ 上的均匀分布，K-S 检验的 p 值为 $0.6285 > 0.05$，则接受原假设，认为数据来自 $[0,1]$ 上的均匀分布。

11. 程序：

```
w <- c(36,32,16,15,35)
p <- c(0.2,0.2,0.2,0.2,0.2)
chisq.test(w, p=p)
```

输出结果：

```
> chisq.test(w, p=p)

	Chi-squared test for given probabilities

data:  w
X-squared = 16.224, df = 4, p-value = 0.002733
```

主要结论：

H_0 : 不合格率相同，H_1 : 不合格率不相同，p 值为 $0.002733 < 0.05$，则拒绝原假设，认为五个工作日的产品不合格率不相同。

12. 程序：

```
w <- c(36,32,16,15,35)
p <- c(0.3,0.25,0.1,0.1,0.25)
chisq.test(w, p=p)
```

输出结果：

```
> chisq.test(w, p=p)

	Chi-squared test for given probabilities

data:  w
X-squared = 1.2687, df = 4, p-value = 0.8667
```

主要结论：

H_0 : 这种说法正确，H_1 : 这种说法不正确，p 值为 $0.8667 > 0.05$，则接受原假设，认为这种说法正确。

第二章　非参数统计

1. 程序：

```
library(tseries)
w <- c(9.8,10.1,9.7,9.9,10,10,9.8,9.7,9.8,9.9)
binom.test(sum(w>10), sum(w<10)+sum(w>10), al='two.sided')
```

输出结果：

```
> binom.test(sum(w>10), sum(w<10)+sum(w>10), al='two.sided')

	Exact binomial test

data: sum(w > 10) and sum(w < 10) + sum(w > 10)
number of successes = 1, number of trials = 8, p-value = 0.07031
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.003159724 0.526509671
sample estimates:
probability of success
              0.125
```

主要结论：

H_0 : 中位数 $= 10$，不需要调整，H_1 : 中位数 $\neq 10$，需要调整，p 值为 $0.07031 > 0.05$，则接受原假设，认为不需要调整。

2. 程序：

```
library(tseries)
w <- c(432.00,82.46,46.19,43.14,
      21.59,12.61,23.61,22.20,
      11.31,105.53,83.57,21.77,
      11.95,42.17,6.30,39.03)
binom.test(sum(w>32.32), sum(w<32.32)+sum(w>32.32), al='greater')
```

输出结果：

```
> binom.test(sum(w>32.32), sum(w<32.32)+sum(w>32.32), al='greater')

	Exact binomial test

data: sum(w > 32.32) and sum(w < 32.32) + sum(w > 32.32)
number of successes = 8, number of trials = 16, p-value = 0.5982
alternative hypothesis: true probability of success is greater than 0.5
```

```
95 percent confidence interval:
 0.2786027 1.0000000
sample estimates:
probability of success
                   0.5
```

主要结论:

H_0 : 中位数并无增加，H_1 : 中位数有所增加，p 值为 $0.5982 > 0.05$，则接受原假设，认为中位数并无增加。

3. 程序:

```
library(tseries)
x <- c(2.300,2.396,2.100,2.442,1.81,2.007,1.766,1.224,1.411,1.438,0.892,1.167)
D <- numeric()
if(length(x)%%2==0){
	c=length(x)/2
	for(i in 1:c)
		D[i]=x[i]-x[i+c]}
if(length(x)%%2==1){
	c=(length(x)+1)/2
	for(i in 1:c-1)
		D[i]=x[i]-x[i+c]}
binom.test(sum(D>0), sum(D<0)+sum(D>0), al='less')
```

输出结果:

```
> binom.test(sum(D>0), sum(D<0)+sum(D>0), al='less')

	Exact binomial test

data: sum(D > 0) and sum(D < 0) + sum(D > 0)
number of successes = 6, number of trials = 6, p-value = 1
alternative hypothesis: true probability of success is less than 0.5
95 percent confidence interval:
 0 1
sample estimates:
probability of success
                     1
```

主要结论:

$H_0 : M \geq 0$，无上升趋势，$H_1 : M < 0$，有上升趋势，p 值为 $1 > 0.05$，则接受原假设，认为销售量没有单调增加趋势。

4. 程序:

```
library(tseries)
w <- c(3.6,3.9,4.1,3.6,3.8,3.7,3.4,4.0,3.8,4.1,3.9,4.0,3.8,4.2,4.1)
x <- numeric()
for(i in 1:length(w)){
   if(w[i]-median(w)>0)
      x[i]=1
      if(w[i]-median(w)<=0)
      x[i]=0}
x <- factor(x)
runs.test(x)
```

输出结果:

```
> runs.test(x)

      Runs Test

data: x
Standard Normal = 1.008, p-value = 0.3134
alternative hypothesis: two.sided
```

主要结论:

H_0：质量的变动是随机的，H_1：质量的变动不是随机的，p 值为 $0.3134 > 0.05$，则接受原假设，认为质量的变动是随机的。

5. 程序:

```
library(tseries)
w <- c(11,19,17,15,15,16,193)
x <- numeric()
for(i in 1:length(w)){
   if(w[i]-median(w)>0)
      x[i]=1
      if(w[i]-median(w)<=0)
      x[i]=0}
x <- factor(x)
runs.test(x)
```

输出结果:

```
> runs.test(x)

      Runs Test

```

```
data: x
Standard Normal = -0.3638, p-value = 0.716
alternative hypothesis: two.sided
```

主要结论：

H_0：一周内各日的死亡数是随机的，H_1：一周内各日的死亡数不是随机的，p 值为 $0.716 > 0.05$，则接受原假设，认为一周内各日的死亡数是随机的。

6. 程序：

```
x <- c(54.0,55.1,53.8,54.2,52.1,54.2,55.0,55.8,55.1,55.3)
wilcox.test(x, alternative="two.sided", mu=54, exact=FALSE, conf.level=0.95)
```

输出结果：

```
> wilcox.test(x, alternative="two.sided", mu=54, exact=FALSE, conf.level=0.95)

	Wilcoxon signed rank test with continuity correction

data: x
V = 34, p-value = 0.1906
alternative hypothesis: true location is not equal to 54
```

主要结论：

H_0：零件质量正常，等于 54 克，H_1：零件质量不正常，不等于 54 克，p 值为 $0.1906 > 0.05$，则接受原假设，认为零件质量正常。

7. 程序：

```
dcxfp <- c(0.6,1.2,2.0,2.4,3.1,4.1,5.0,5.9,7.4,13.9)
pzczdz <- c(9.8,10.2,10.6,13.0,14.0,14.8,15.6,15.6,21.6,24.0)
wilcox.test(pzczdz, dcxfp, alternative="l", paired=FALSE, exact=FALSE)
```

输出结果：

```
> wilcox.test(pzczdz, dcxfp, alternative="l", paired=FALSE, exact=FALSE)

	Wilcoxon rank sum test with continuity correction

data: pzczdz and dcxfp
W = 96, p-value = 0.9998
alternative hypothesis: true location shift is less than 0
```

主要结论：

H_0：皮质醇增多症的血浆总皮醇测定值高于单纯性肥胖，H_1：皮质醇增多症的血浆总皮醇测定值不高于单纯性肥胖，p 值为 $0.9998 > 0.05$，则接受原假设，认为皮质醇增多症的血浆总皮醇测定值高于单纯性肥胖。

8. 程序：

```
zf <- c(8.2,10.7,7.5,14.6,6.3,9.2,11.9,5.6,12.8,5.2,4.9,13.5)
jk <- c(4.7,6.3,5.2,6.8,5.6,4.2,6.0,7.4,8.1,6.5)
ansari.test(zf, jk, alternative="two.sided", exact=F, conf.level=0.95)
```

输出结果：

```
> ansari.test(zf, jk, alternative="two.sided", exact=F, conf.level=0.95)

	Ansari-Bradley test

data: zf and jk
AB = 60.5, p-value = 0.1269
alternative hypothesis: true ratio of scales is not equal to 1
```

主要结论：

H_0 : 尿酸浓度的变异相同，H_1 : 尿酸浓度的变异不同，p 值为 $0.1269 > 0.05$，则接受原假设，认为尿酸浓度的变异相同。

9. 程序：

```
x <- c(55,74,68,80,77,69,57,72,63,52,85,66,71,48,83,78,51,67)
y <- c(41,64,61,70,75,60,53,59,61,48,79,65,70,47,81,69,50,62)
d <- x-y
# 符号检验
binom.test(sum(d>5), sum(d>5)+sum(d<5), al="two.sided")
# Wilcoxon符号秩检验
wilcox.test(x-5, y, alternative="two.sided", paired=T, exact=F, correct=F)
```

输出结果：

```
> # 符号检验
> binom.test(sum(d>5), sum(d>5)+sum(d<5), al="two.sided")

	Exact binomial test

data: sum(d > 5) and sum(d > 5) + sum(d < 5)
number of successes = 8, number of trials = 17, p-value = 1
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.2298327 0.7218817
sample estimates:
probability of success
            0.4705882
```

```
> # Wilcoxon符号秩检验
> wilcox.test(x-5, y, alternative="two.sided", paired=T, exact=F, correct=F)

	Wilcoxon signed rank test

data: x - 5 and y
V = 89, p-value = 0.5516
alternative hypothesis: true location shift is not equal to 0
```

主要结论：

H_0 : 城市噪音已下降 5 分贝，H_1 : 城市噪音没有下降 5 分贝，

(1) 符号检验的 p 值为 $1 > 0.05$，则接受原假设，认为城市噪音已下降 5 分贝。

(2) Wilcoxon 符号秩检验的 p 值为 $0.5516 > 0.05$，则接受原假设，认为城市噪音已下降 5 分贝。

综上，接受原假设，认为城市噪音已下降 5 分贝。

10. 程序：

```
pjz <- c(83.46,49.25,70.21,50.06,43.83,44.03,
        43.59,43.61,42.57,55.71,52.85,
        44.61,45.28,46.37,42.19,42.43)
jj <- c(5118,3148,2574,2216,2186,1759,
       1829,2008,1514,3939,2865,3156,
       1816,2530,2476,2651)
cor.test(pjz, jj, alternative="two.sided", method="spearman")
```

输出结果：

```
> cor.test(pjz, jj, alternative="two.sided", method="spearman")

	Spearman's rank correlation rho

data:  pjz and jj
S = 288, p-value = 0.02156
alternative hypothesis: true rho is not equal to 0
sample estimates:
      rho
0.5764706
```

主要结论：

H_0 : 二者不相关，H_1 : 二者相关，p 值为 $0.02156 < 0.05$，则拒绝原假设，认为二者相关，相关系数为 0.5764706。

11. 程序：

```
pjz <- c(83.46,49.25,70.21,50.06,43.83,44.03,
       43.59,43.61,42.57,55.71,52.85,
       44.61,45.28,46.37,42.19,42.43)
jj <- c(5118,3148,2574,2216,2186,1759,
      1829,2008,1514,3939,2865,3156,
      1816,2530,2476,2651)
cor.test(pjz, jj, alternative="two.sided", method="kendall", exact=F)
```

输出结果：

```
> cor.test(pjz, jj, alternative="two.sided", method="kendall", exact=F)

      Kendall's rank correlation tau

data:  pjz and jj
z = 2.3412, p-value = 0.01922
alternative hypothesis: true tau is not equal to 0
sample estimates:
      tau
0.4333333
```

主要结论：

H_0 : 二者不相关，H_1 : 二者相关，p 值为 $0.01922 < 0.05$，则拒绝原假设，认为二者相关，相关系数为 0.4333333。

12. 程序：

```
x <- matrix(c(15,13,20,18), nr=2)
chisq.test(x, correct=F)
```

输出结果：

```
> chisq.test(x, correct=F)

      Pearson's Chi-squared test

data:  x
X-squared = 0.0057171, df = 1, p-value = 0.9397
```

主要结论：

H_0 : 体育达标水平与性别相互独立，即无关，H_1 : 体育达标水平与性别不独立，即有关，p 值为 $0.9397 > 0.05$，则接受原假设，认为体育达标水平与性别相互独立，即无关。

第三章　多元统计分析相关基础

1. 程序：

```
library(ICSNP)
boy <- read.table("data.exam3.4.1.txt", header=TRUE)
mu0 <- c(155,38,2205)
# 取F统计量
HotellingsT2(boy, mu=mu0, test="f")
# 取X^2统计量
HotellingsT2(boy, mu=mu0, test="chi")
```

输出结果：

```
> # 取F统计量
> HotellingsT2(boy, mu=mu0, test="f")

      Hotelling's one sample T2-test

data:  boy
T.2 = 1.3725, df1 = 3, df2 = 26, p-value = 0.2732
alternative hypothesis: true location is not equal to c(155,38,2205)

> # 取X^2统计量
> HotellingsT2(boy, mu=mu0, test="chi")

        Hotelling's one sample T2-test

data: boy
T.2 = 4.4343, df = 3, p-value = 0.2182
alternative hypothesis: true location is not equal to c(155,38,2205)
```

主要结论：

H_0 : 数据均值向量等于 $(155,38,2205)^{\mathrm{T}}$，$H_1$: 数据均值向量不等于 $(155,38,2205)^{\mathrm{T}}$，

(1) 用统计量 F 检验，p 值为 $0.2732>0.05$，则接受原假设，认为数据均值向量等于 $(155,38,2205)^{\mathrm{T}}$。

(2) 用统计量 χ^2 检验，p 值为 $0.2182>0.05$，则接受原假设，认为数据均值向量等于 $(155,38,2205)^{\mathrm{T}}$。

综上，接受原假设，认为数据均值向量等于 $(155,38,2205)^{\mathrm{T}}$。

2. 程序：

```
A <- matrix(c(0.702,0.541,0.184,0.253,
            0.541,0.712,0.228,0.258,
            0.184,0.228,0.392,0.346,
```

```
            0.253,0.258,0.346,0.806), nr=4)
mu0 <- c(22.75,32.75,51.50,61.50)
x0 <- c(22.82,32.79,51.45,61.38)
n <- 21
p <- 4
T2 <- (n-1)*n*t(x0-mu0)%*%solve(A)%*%(x0-mu0);
F <- (n-p)/((n-1)*p)*T2;
pvf <- 1 - pf(F,p,n-p); pvf
```

输出结果:

```
> pvf <- 1 - pf(F,p,n-p); pvf
         [,1]
[1,] 0.03538181
```

主要结论:

$H_0: \boldsymbol{\mu} = \boldsymbol{\mu}_0, H_1: \boldsymbol{\mu} \neq \boldsymbol{\mu}_0$，$p$ 值为 $0.03538181 < 0.05$，则拒绝原假设，认为 $\boldsymbol{\mu} \neq \boldsymbol{\mu}_0$。

3. 程序:

```
library(ICSNP)
# (1)
s_ve <- as.matrix(iris[1:100,1:4]);
s_ve_G <- as.factor(iris[1:100,5]);
HotellingsT2(s_ve~s_ve_G)
# (2)
ve_vi <- as.matrix(iris[51:150,1:4]);
ve_vi_G <- as.factor(iris[51:150,5]);
HotellingsT2(ve_vi~ve_vi_G)
# (3)
# setosa
setosa <- as.matrix(iris[1:50,1:4]);
# versicolor
versicolor <- as.matrix(iris[51:100,1:4]);
# vinginica
vinginica <- as.matrix(iris[101:150,1:4]);
n <- 50
p <- 4
avg_s <- as.matrix(colSums(setosa)/50);
avg_ve <- as.matrix(colSums(versicolor)/50);
avg_vi <- as.matrix(colSums(vinginica)/50);
avg <- (avg_s + avg_ve + avg_vi)/3;
A <- 50*(avg_s - avg) %*% t(avg_s - avg) + 50*(avg_ve - avg) %*% t(avg_ve - avg) +
   50*(avg_vi - avg) %*% t(avg_vi - avg);
```

```
E <- matrix(0,4,4);
for (i in seq(1,n)){
      E = E + (t(t(setosa[i,])) - avg_s) %*% t(t(t(setosa[i,])) - avg_s);
}
for (i in seq(1,n)){
      E = E + (t(t(versicolor[i,])) - avg_ve) %*% t(t(t(versicolor[i,])) - avg_ve)
         ;
}
for (i in seq(1,n)){
      E = E + (t(t(vinginica[i,])) - avg_vi) %*% t(t(t(vinginica[i,])) - avg_vi);
}
Lambda <- det(E)/(det(E+A));
pvf3 <- 1-pf((150-3-4+1)/4 * (1 - sqrt(Lambda))/sqrt(Lambda), 8, 2*(150-3-4+1));
   pvf3
```

输出结果：

```
> # (1)
> HotellingsT2(s_ve~s_ve_G)

      Hotelling's two sample T2-test

data:  s_ve by s_ve_G
T.2 = 625.46, df1 = 4, df2 = 95, p-value < 2.2e-16
alternative hypothesis: true location difference is not equal to c(0,0,0,0)

> # (2)
> HotellingsT2(ve_vi~ve_vi_G)

        Hotelling's two sample T2-test

data: ve_vi by ve_vi_G
T.2 = 86.148, df1 = 4, df2 = 95, p-value < 2.2e-16
alternative hypothesis: true location difference is not equal to c(0,0,0,0)

> # (3)
> pvf3
[1] 0
```

主要结论：

(1) $H_0: \mu_{setosa} = \mu_{versicolor}, H_1: \mu_{setosa} \neq \mu_{versicolor}$，$p$ 值 $< 2.2 \times 10^{-16} < 0.05$，则拒绝原假设，认为 setosa 与 versicolor 的均值向量不相等。

(2) $H_0: \mu_{versicolor} = \mu_{vinginica}, H_1: \mu_{versicolor} \neq \mu_{vinginica}$，$p$ 值 $< 2.2 \times 10^{-16} < 0.05$，则拒绝原假设，认为 versicolor 与 vinginica 的均值向量不相等。

(3) H_0：三种花的均值向量相等，H_1：三种花的均值向量不相等，p 值为 $0 < 0.05$，则拒绝原假设，认为三种花的均值向量不相等。

4. 程序：

```
x <- read.table("ex3_4-data.txt", header=TRUE, row.names=1)
# 均值向量
colMeans(x)
# 协方差矩阵
cov(x)
# 相关系数矩阵
cor(x)
```

输出结果：

```
> # 均值向量
> colMeans(x)
       X1 X2 X3 X4 X5 X6
 1487.42125 685.85438 10874.18750 449.13000 58.87375 62.83937
> # 协方差矩阵
> cov(x)
         X1 X2 X3 X4 X5 X6
X1 2093336.6 841893.88 18479476 703207.2 118473.934 139988.10
X2 841893.9 350404.23 7036375 277089.8 48305.919 54154.89
X3 18479475.7 7036374.81 212249748 6910248.0 1017813.524 1424766.40
X4 703207.2 277089.84 6910248 250159.3 38767.802 50785.80
X5 118473.9 48305.92 1017814 38767.8 7008.546 7660.30
X6 139988.1 54154.89 1424766 50785.8 7660.300 10541.30
> # 相关系数矩阵
> cor(x)
        X1 X2 X3 X4 X5 X6
X1 1.0000000 0.9829992 0.8766909 0.9717521 0.9781137 0.9423764
X2 0.9829992 1.0000000 0.8159071 0.9358961 0.9747690 0.8910577
X3 0.8766909 0.8159071 1.0000000 0.9483349 0.8345083 0.9525174
X4 0.9717521 0.9358961 0.9483349 1.0000000 0.9258677 0.9889784
X5 0.9781137 0.9747690 0.8345083 0.9258677 1.0000000 0.8912195
X6 0.9423764 0.8910577 0.9525174 0.9889784 0.8912195 1.0000000
```

主要结论：

(1) 数据的均值向量为

$$(1487.42125, 685.85438, 10874.18750, 449.13000, 58.87375, 62.83937)^{\mathrm{T}}$$

(2) 数据的协方差矩阵为

$$\begin{pmatrix} 2093336.6 & 841893.88 & 18479476 & 703207.2 & 118473.934 & 139988.10 \\ 841893.9 & 350404.23 & 7036375 & 277089.8 & 48305.919 & 54154.89 \\ 18479475.7 & 7036374.81 & 212249748 & 6910248.0 & 1017813.524 & 1424766.40 \\ 703207.2 & 277089.84 & 6910248 & 250159.3 & 38767.802 & 50785.80 \\ 118473.9 & 48305.92 & 1017814 & 38767.8 & 7008.546 & 7660.30 \\ 139988.1 & 54154.89 & 1424766 & 50785.8 & 7660.300 & 10541.30 \end{pmatrix}$$

(3) 数据的相关系数矩阵为

$$\begin{pmatrix} 1.0000000 & 0.9829992 & 0.8766909 & 0.9717521 & 0.9781137 & 0.9423764 \\ 0.9829992 & 1.0000000 & 0.8159071 & 0.9358961 & 0.9747690 & 0.8910577 \\ 0.8766909 & 0.8159071 & 1.0000000 & 0.9483349 & 0.8345083 & 0.9525174 \\ 0.9717521 & 0.9358961 & 0.9483349 & 1.0000000 & 0.9258677 & 0.9889784 \\ 0.9781137 & 0.9747690 & 0.8345083 & 0.9258677 & 1.0000000 & 0.8912195 \\ 0.9423764 & 0.8910577 & 0.9525174 & 0.9889784 & 0.8912195 & 1.0000000 \end{pmatrix}$$

5. 程序:

```
data <- read.table("ex3_4-data.txt", header=TRUE, row.names=1)
x <- data[2:5,]
# 轮廓图
source("outline.R")
outline(x)
# 蛛网图
source("spider.R")
spider(x)
# 调和曲线图
source("harmonic.curve.R")
harmonic.curve(x)
```

输出结果：

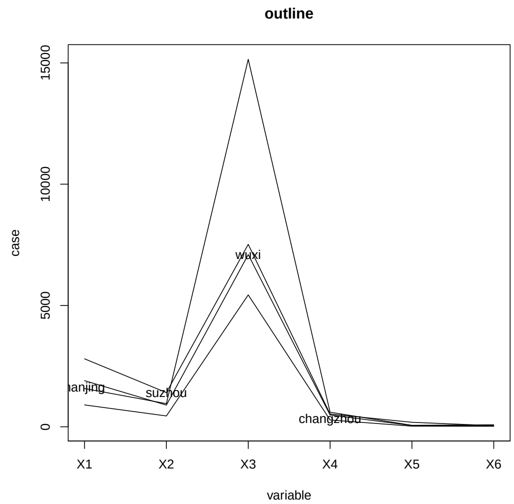


图 4: 题 5 轮廓图

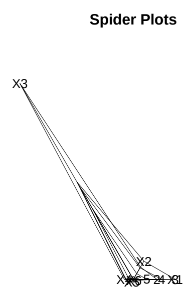


图 5: 题 5 蛛网图

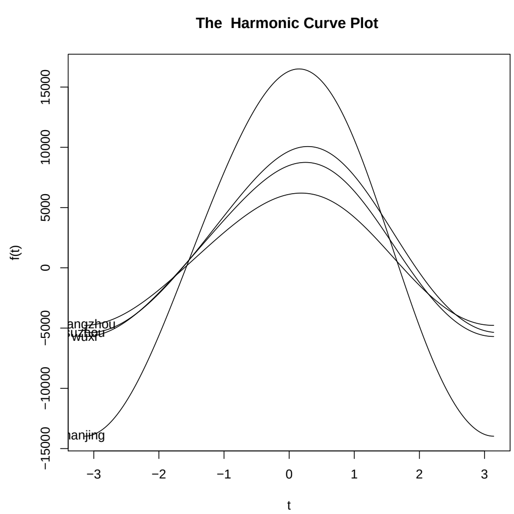


图 6: 题 5 调和曲线图

第四章　回归分析

1. 程序：

```
x <- read.table("ex4_1-data.txt", header=TRUE)
lm.1 <- lm(Y~X1+X2, data=x)
summary(lm.1)
R = sqrt(summary(lm(Y~X1+X2, data=x))$r.sq); R
```

输出结果：

```
> summary(lm.1)

Call:
lm(formula = Y ~ X1 + X2, data = x)

Residuals:
   Min 1Q Median 3Q Max
-3.9340 -0.8672 0.5711 0.7114 4.4507

Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) -61.47289 17.60192 -3.492 0.00580 **
X1 2.12039 0.18148 11.684 3.75e-07 ***
X2 0.39431 0.08631 4.569 0.00103 **
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.968 on 10 degrees of freedom
Multiple R-squared: 0.9417, Adjusted R-squared: 0.9301
F-statistic: 80.83 on 2 and 10 DF, p-value: 6.709e-07

> R
[1] 0.9704358
```

主要结论：

根据输出，可以得到

(1) Y 关于 X_1, X_2 的二元线性回归方程为

$$Y = -61.47289 + 2.12039X_1 + 0.39431X_2.$$

(2) 自变量 X_1 的系数 p 值为 $3.75 \times 10^{-7} < 0.05$，认为 X_1 在 0.05 的显著性水平下是显著的；
自变量 X_2 的系数 p 值为 $0.00103 < 0.05$，认为 X_2 在 0.05 的显著性水平下是显著的；

(3) 复相关系数为 0.9704358。

2. 程序:

```
x <- read.table("ex4_2-data.txt", header=TRUE)
# (1)
lm.1 <- lm(Y~X1+X2, data=x)
coefficients(lm.1)
# (2)
attach(x)
# 散点图
plot(Y~X1)
abline(lm(Y~X1))
plot(Y~X2)
abline(lm(Y~X2))
# 残差图
resid <- residuals(lm.1)
y.pre <- predict(lm.1)
plot(y.pre, resid)
# 残差QQ图
plot(lm.1,2)
```

输出结果:

```
> coefficients(lm.1)
(Intercept) X1 X2
 -4.9513762 1.5465083 -0.9539819
```

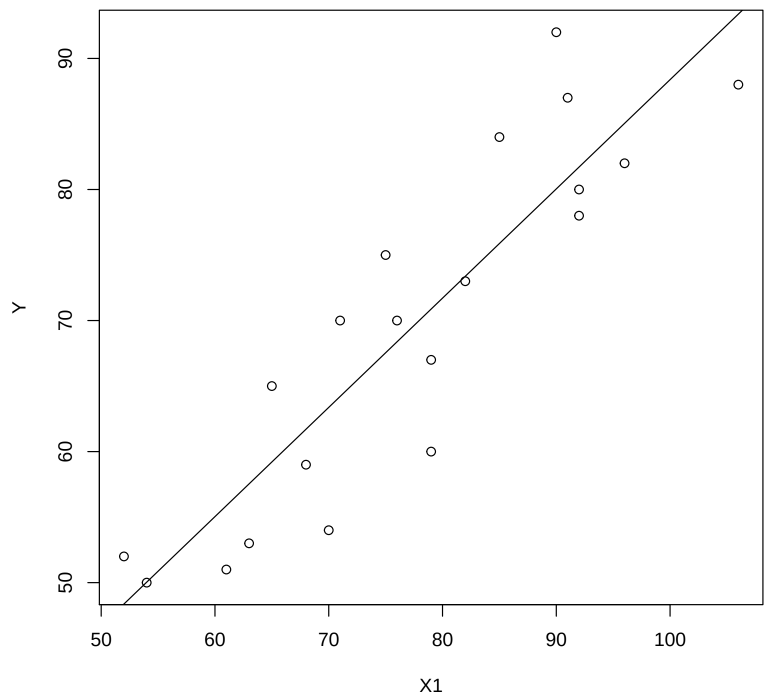


图 7: 题 2 中 Y 关于 X_1 的散点图

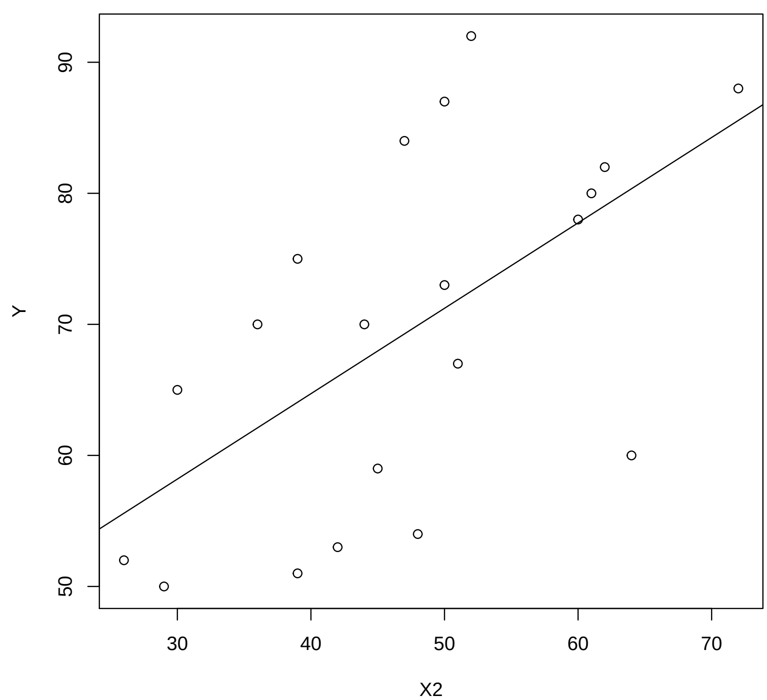


图 8: 题 2 中 Y 关于 X_2 的散点图

主要结论：
根据输出，可以得到

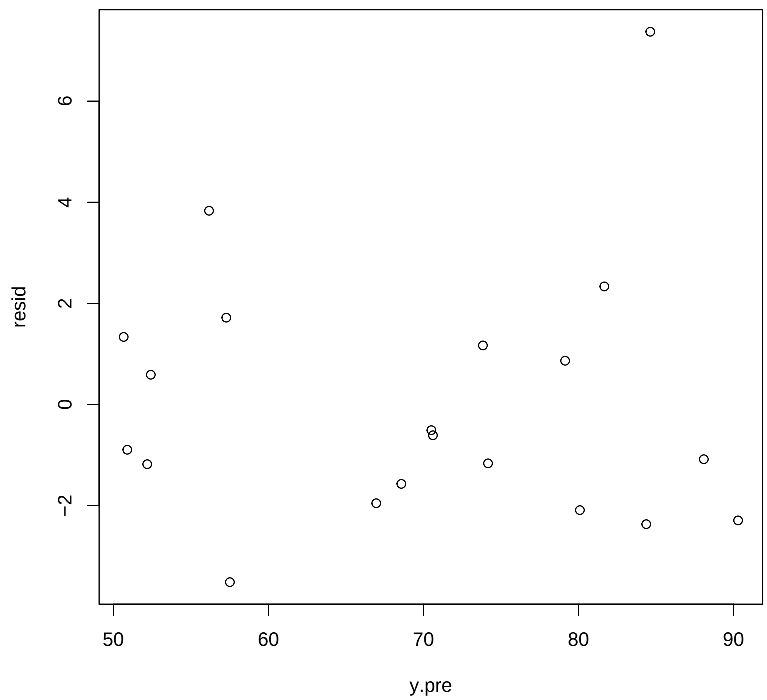


图 9: 题 2 中的残差图

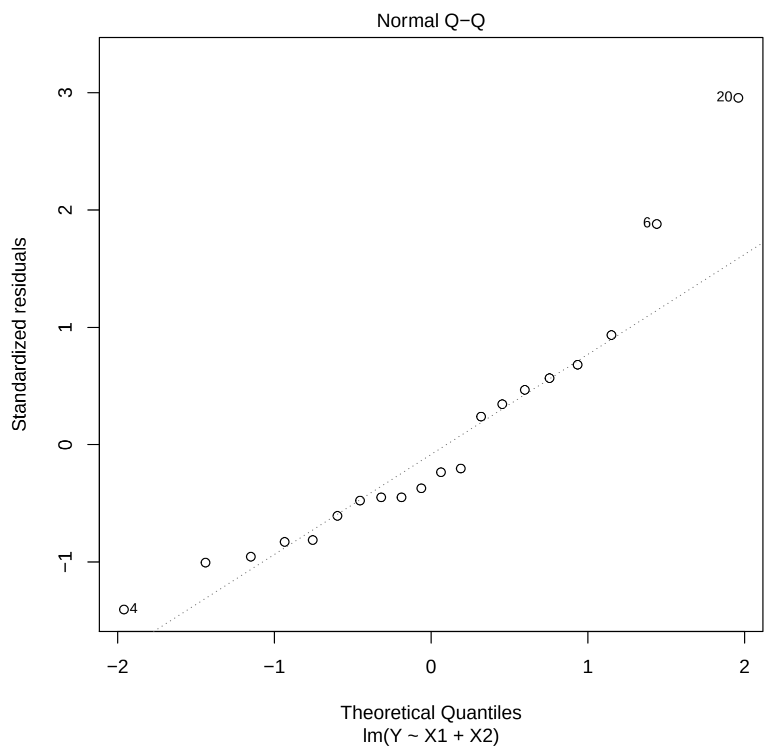


图 10: 题 2 中的残差 QQ 图

主要结论：
根据输出，可以得到

(1) Y 关于 X_1, X_2 的二元线性回归方程为

$$Y = -4.9513762 + 1.5465083X_1 - 0.9539819X_2.$$

(2) 通过两张散点图和残差图，可以认为残差随时间变换呈线性变化，因此回归函数中应包含时间的线性项；通过残差 QQ 图，可以认为残差所属总体为 $N(0,1)$，因此总体分布是正态分布。

3. 程序：

```
x <- read.table("ex4_3-data.txt", header=TRUE)
source("step.regression.R")
step.regression(x, x[[6]], c(2,3,4,5), 0.05, 0.05)
attach(x)
lm.1 <- lm(Y2~Y1+X3+X2, data=x)
summary(lm.1)
```

输出结果：

```
> step.regression(x, x[[6]], c(2,3,4,5), 0.05, 0.05)
  Varible Enter.Exclude F.value P.value
1 Y1 Enter 173.500426 0.000000000
2 X3 Enter 8.307257 0.005765805
3 X2 Enter 11.918944 0.001139657

> summary(lm.1)

Call:
lm(formula = Y2 ~ Y1 + X3 + X2, data = x)

Residuals:
    Min 1Q Median 3Q Max
-0.28869 -0.04378 0.00621 0.04249 0.56721

Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.3542790 0.1177948 11.497 1.20e-15 ***
Y1 0.0010070 0.0001892 5.322 2.43e-06 ***
X3 0.0048378 0.0010773 4.491 4.20e-05 ***
X2 0.0045750 0.0013252 3.452 0.00114 **
---
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1169 on 50 degrees of freedom
Multiple R-squared: 0.8399, Adjusted R-squared: 0.8303
```

```
F-statistic: 87.42 on 3 and 50 DF, p-value: < 2.2e-16
```

主要结论：

根据输出，所得最优回归方程为

$$Y_2 = 1.3542790 + 0.0010070Y_1 + 0.0048378X_3 + 0.0045750X_2.$$

4. 程序：

```
x <- read.table("ex4_4-data.txt", header=TRUE)
log.lm <- glm(Y~X1+X2+X3, family=binomial, data=x)
summary(log.lm)
```

输出结果：

```
> summary(log.lm)

Call:
glm(formula = Y ~ X1 + X2 + X3, family = binomial, data = x)

Deviance Residuals:
   Min 1Q Median 3Q Max
-1.4903 -0.8790 -0.7097 0.9873 1.7984

Coefficients:
          Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.03785 0.92265 -0.041 0.9673
X1 -1.70448 0.71784 -2.374 0.0176 *
X2 0.01118 0.01771 0.631 0.5280
X3 0.30887 0.70744 0.437 0.6624
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

   Null deviance: 61.29 on 44 degrees of freedom
Residual deviance: 54.70 on 41 degrees of freedom
AIC: 62.7

Number of Fisher Scoring iterations: 4
```

主要结论：

根据输出，所得关系为

$$P\{Y=1\} = \frac{\exp(-0.03785 - 1.70448X_1 + 0.01118X_2 + 0.30887X_3)}{1+\exp(-0.03785 - 1.70448X_1 + 0.01118X_2 + 0.30887X_3)}.$$

第五章　主成分分析

1. 解:

矩阵 $\mathbf{\Sigma}$ 的特征值为 $\lambda_1=5,\lambda_2=3,\lambda_3=1$，对应的正交单位化特征向量为

$$\boldsymbol{e}_1=(1,0,0)^{\mathrm{T}},\boldsymbol{e}_2=\left(0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)^{\mathrm{T}},\boldsymbol{e}_3=\left(0,\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)^{\mathrm{T}},$$

则

$$\begin{cases}Y_1=\boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{X}=X_1,\\ Y_2=\boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{X}=\frac{\sqrt{2}}{2}X_2+\frac{\sqrt{2}}{2}X_3,\\ Y_3=\boldsymbol{e}_3^{\mathrm{T}}\boldsymbol{X}=\frac{\sqrt{2}}{2}X_2-\frac{\sqrt{2}}{2}X_3.\end{cases}$$

各主成分的方差为

$$\mathrm{Var}(Y_1)=\lambda_1=5,\mathrm{Var}(Y_2)=\lambda_2=3,\mathrm{Var}(Y_3)=\lambda_3=1,$$

各主成分的累计贡献率为

$$\begin{aligned}\psi(Y_1)&=\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}=\frac{5}{9}=55.56\%<85\%,\\ \psi(Y_2)&=\frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+\lambda_3}=\frac{8}{9}=88.89\%>85\%,\\ \psi(Y_3)&=\frac{\lambda_1+\lambda_2+\lambda_3}{\lambda_1+\lambda_2+\lambda_3}=\frac{9}{9}=100.0\%>85\%.\end{aligned}$$

因为各主成分互不相关，所以各主成分间的相关系数均为 0，即

$$\rho_{(Y_i,Y_j)}=0,\quad i\neq j,\quad i,j=1,2,3.$$

综上，

(1) $\boldsymbol{X}$ 累计贡献率 $\geq 85\%$ 的主成分为 Y_1,Y_2；

(2) $\mathrm{Var}(Y_1)=5,\mathrm{Var}(Y_2)=3,\mathrm{Var}(Y_3)=1$；

(3) $\rho_{(Y_i,Y_j)}=0,\quad i\neq j,\quad i,j=1,2,3$。

2. 证:

令 $P=[p_1,p_2,p_3]$，

$$P^TX\text{是}X\text{的主成分}\iff p_i\text{是}X\text{的协方差矩阵}\Sigma\text{的特征向量}\iff \Sigma p_i=\lambda_i p_i\iff$$
$$(\Sigma+\sigma^2 I)p_i=(\lambda_i+\sigma^2)p_i\iff p_i+\sigma^2\text{是}Y\text{的协方差矩阵}\Sigma+\sigma^2 I\text{的特征向量}\iff P^TY\text{是}Y\text{的主成分}.$$

3. 证:

协方差矩阵 Σ 的特征值为

$$\begin{cases}\lambda_1=\sigma^2+\sigma_{12}-\sigma_{13}-\sigma_{14}\\ \lambda_2=\sigma^2-\sigma_{12}+\sigma_{13}-\sigma_{14}\\ \lambda_3=\sigma^2-\sigma_{12}-\sigma_{13}+\sigma_{14}\\ \lambda_4=\sigma^2+\sigma_{12}+\sigma_{13}+\sigma_{14}\end{cases}$$

且形式 $P^T X$ 中的 $P = [p_1, p_2, p_3, p_4]$ 如下

$$\begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \end{pmatrix}$$

很容易验证：$\Sigma p_i = \lambda_i p_i$，则证明完毕。

4. 程序：

```
x <- read.table("ex5_4-data.txt", header=TRUE)
std1.x <- scale(x)
rownames(std1.x) <- seq(1,42)
std.x <- as.data.frame(std1.x)
# 样本相关阵出发
prin1 <- princomp(std.x, cor=TRUE)
summary(prin1)
loadings(prin1)
# 样本协方差阵出发
prin2 <- princomp(std.x, cor=FALSE)
summary(prin2)
loadings(prin2)
```

输出结果：

```
# 样本相关阵出发
> summary(prin1)
Importance of components:
                     Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6
Standard deviation 1.5270812 1.1627564 1.0870278 0.8587576 0.8340510 0.7390645
Proportion of Variance 0.3331395 0.1931432 0.1688042 0.1053521 0.0993773 0.0780309
Cumulative Proportion 0.3331395 0.5262828 0.6950870 0.8004391 0.8998164 0.9778473
                      Comp.7
Standard deviation 0.39378827
Proportion of Variance 0.02215274
Cumulative Proportion 1.00000000
> loadings(prin1)

Loadings:
  Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7
X1 0.253 0.203 0.672 0.520 0.327 0.240
X2 -0.221 -0.529 0.190 -0.755 0.245
X3 -0.546 -0.107 0.253 0.483 0.224 -0.585
```

```
X4 -0.377 0.481 -0.351 -0.332 0.416 0.468
X5 -0.488 0.193 0.226 0.178 0.134 -0.717 0.331
X6 -0.330 -0.587 0.433 -0.227 0.354 0.416
X7 -0.314 0.258 0.564 -0.163 -0.605 0.163 -0.313

            Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7
SS loadings 1.000 1.000 1.000 1.000 1.000 1.000 1.000
Proportion Var 0.143 0.143 0.143 0.143 0.143 0.143 0.143
Cumulative Var 0.143 0.286 0.429 0.571 0.714 0.857 1.000

# 样本协方差阵出发
> summary(prin2)
Importance of components:
                     Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6
Standard deviation 1.5087921 1.1488307 1.0740090 0.8484728 0.8240620 0.7302131
Proportion of Variance 0.3331395 0.1931432 0.1688042 0.1053521 0.0993773 0.0780309
Cumulative Proportion 0.3331395 0.5262828 0.6950870 0.8004391 0.8998164 0.9778473
                      Comp.7
Standard deviation 0.38907207
Proportion of Variance 0.02215274
Cumulative Proportion 1.00000000
> loadings(prin2)

Loadings:
  Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7
X1 0.253 0.203 0.672 0.520 0.327 0.240
X2 -0.221 -0.529 0.190 -0.755 0.245
X3 -0.546 -0.107 0.253 0.483 0.224 -0.585
X4 -0.377 0.481 -0.351 -0.332 0.416 0.468
X5 -0.488 0.193 0.226 0.178 0.134 -0.717 0.331
X6 -0.330 -0.587 0.433 -0.227 0.354 0.416
X7 -0.314 0.258 0.564 -0.163 -0.605 0.163 -0.313

            Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7
SS loadings 1.000 1.000 1.000 1.000 1.000 1.000 1.000
Proportion Var 0.143 0.143 0.143 0.143 0.143 0.143 0.143
Cumulative Var 0.143 0.286 0.429 0.571 0.714 0.857 1.000
```

主要结论：

根据输出，可以得到

(1) 样本相关阵与样本协方差阵所得结果的特征根有细微差别，但对于方差的贡献率、累计贡献率、主成分的

系数等基本完全一致，差别不大。

(2) 不能，前三个主成分的累计贡献率低于 70%，偏低。

(3) 选择前 6 个主成分，累计贡献率可达 90%。

第六章　因子分析

1. 解:

(1) 先计算 Σ 的特征值，为 $\lambda_1=\dfrac{4}{3},\lambda_2=1,\lambda_3=\dfrac{2}{3}$，对应的单位化后的特征向量为

$$\begin{cases} e_1=\left(0,\dfrac{\sqrt{2}}{2},\dfrac{\sqrt{2}}{2}\right)^T \\ e_2=(1,0,0)^T \\ e_3=\left(0,-\dfrac{\sqrt{2}}{2},\dfrac{\sqrt{2}}{2}\right)^T \end{cases}$$

则

$$\Sigma=\hat{A}\hat{A}^{\mathrm{T}}=\begin{pmatrix}1&0&0\\0&1&1/3\\0&1/3&1\end{pmatrix}=\begin{pmatrix}0&1&0\\\frac{\sqrt{6}}{3}&0&-\frac{\sqrt{3}}{3}\\\frac{\sqrt{6}}{3}&0&\frac{\sqrt{3}}{3}\end{pmatrix}\begin{pmatrix}0&\frac{\sqrt{6}}{3}&\frac{\sqrt{6}}{3}\\1&0&0\\0&-\frac{\sqrt{3}}{3}&\frac{\sqrt{3}}{3}\end{pmatrix},$$

累计贡献率为

$$\frac{\lambda_1}{\sum\limits_{i=1}^{3}\lambda_i}=\frac{4/3}{3}=\frac{4}{9}<75\%,$$
$$\frac{\lambda_1+\lambda_2}{\sum\limits_{i=1}^{3}\lambda_i}=\frac{4/3+1}{3}=\frac{7}{9}>75\%,$$
$$\frac{\lambda_1+\lambda_2+\lambda_3}{\sum\limits_{i=1}^{3}\lambda_i}=1>75\%.$$

累计贡献率大于 75% 的公因子为 f_1,f_2，则因子模型为

$$\begin{cases} X_1= & & f_2 & +\varepsilon_1 \\ X_2= & \frac{\sqrt{6}}{3}f_1 & & +\varepsilon_2 \\ X_3= & \frac{\sqrt{6}}{3}f_1 & & +\varepsilon_3 \end{cases}$$

(2) f_1,f_2,f_3 的方差贡献分别为 $\lambda_1=\dfrac{4}{3},\lambda_2=1,\lambda_3=\dfrac{2}{3}$。

(3) 由于 $\mathrm{Cov}(X_i,f_j)=a_{ij}$，所以

$$\begin{aligned} &\mathrm{Cov}(X_1,f_1)=0, &&\mathrm{Cov}(X_1,f_2)=1, &&\mathrm{Cov}(X_1,f_3)=0,\\ &\mathrm{Cov}(X_2,f_1)=\tfrac{\sqrt{6}}{3}, &&\mathrm{Cov}(X_2,f_2)=0, &&\mathrm{Cov}(X_2,f_3)=-\tfrac{\sqrt{3}}{3},\\ &\mathrm{Cov}(X_3,f_1)=\tfrac{\sqrt{6}}{3}, &&\mathrm{Cov}(X_3,f_2)=0, &&\mathrm{Cov}(X_3,f_3)=\tfrac{\sqrt{3}}{3}. \end{aligned}$$

2. 程序:

```
x <- read.table("ex6_2-data.txt", header=TRUE)
fact <- factanal(x, 3, scores="Bartlett", rotation="varimax"); fact
fact$scores
colMeans(x)
```

输出结果：

```
> fact
Call:
factanal(x = x, factors = 3, scores = "Bartlett", rotation = "varimax")

Uniquenesses:
  X1 X2 X3 X4 X5 X6 X7
0.030 0.005 0.188 0.005 0.005 0.005 0.241

Loadings:
  Factor1 Factor2 Factor3
X1 0.116 0.950 0.232
X2 0.379 0.921
X3 -0.788 0.280 0.335
X4 0.817 0.573
X5 0.982 0.171
X6 0.975 0.141 0.163
X7 0.530 -0.482 -0.496

            Factor1 Factor2 Factor3
SS loadings 2.834 2.073 1.617
Proportion Var 0.405 0.296 0.231
Cumulative Var 0.405 0.701 0.932

Test of the hypothesis that 3 factors are sufficient.
The chi square statistic is 57.11 on 3 degrees of freedom.
The p-value is 2.44e-12

> fact$scores
       Factor1 Factor2 Factor3
 [1,] 1.6529734 -1.65211154 0.09649973
 [2,] 0.5937706 0.49870759 -0.15396797
 [3,] -1.0507634 -0.33507998 0.82724465
 [4,] -1.3602507 0.59342604 1.01466103
 [5,] -0.4106409 0.52779334 0.81728819
 [6,] 0.1351855 -0.85757911 -0.02025507
 [7,] 0.9057612 -1.43130352 -1.35275397
 [8,] -0.7273935 0.47787621 0.82636880
 [9,] 2.1907486 0.54815313 1.76154678
[10,] 1.0709157 1.91452350 -2.38819512
[11,] 0.6707224 0.86157369 -1.19951381
```

```
[12,] -0.8950983 -0.02771697 0.38397967
[13,] -0.6271528 -1.02904389 -0.33345128
[14,] -0.1280872 -0.39128734 -1.37282935
[15,] -0.7317347 -2.27447887 -0.44110066
[16,] 0.0582647 -0.64983615 0.71251106
[17,] 0.8989055 0.64761904 1.67682588
[18,] -0.5093322 0.48905989 -0.11862349
[19,] 0.5303530 -1.17243219 0.27036622
[20,] -0.2864957 0.31907958 -1.22262548
[21,] 1.7792655 0.63655739 0.76786294
[22,] -0.9984357 1.18081053 -0.16626248
[23,] -1.0715159 -0.67719910 0.25958371
[24,] -0.5497015 0.71240982 -1.01847110
[25,] -1.1402634 1.09047892 0.37331109

> colMeans(x)
   X1 X2 X3 X4 X5 X6 X7
7.1000 4.7732 2.3488 9.1524 5.4584 7.1672 2.3460
```

主要结论:

可以得出因子模型为

$$
\begin{cases}
x_1 - 7.100 = 0.116 f_1 + 0.950 f_2 + 0.232 f_3 + \varepsilon_1 \\
x_2 - 4.7732 = 0 f_1 + 0.379 f_2 + 0.921 f_3 + \varepsilon_2 \\
x_3 - 2.3488 = -0.788 f_1 + 0.280 f_2 + 0.335 f_3 + \varepsilon_3 \\
x_4 - 9.1524 = 0 f_1 + 0.817 f_2 + 0.573 f_3 + \varepsilon_4 \\
x_5 - 5.4584 = 0.982 f_1 + 0.171 f_2 + 0 f_3 + \varepsilon_5 \\
x_6 - 7.1672 = 0.975 f_1 + 0.141 f_2 + 0.163 f_3 + \varepsilon_6 \\
x_7 - 2.3460 = 0.530 f_1 - 0.482 f_2 - 0.496 f_3 + \varepsilon_7
\end{cases}
$$

f_1 在 x_5, x_6 上的载荷很大，可以将 x_5, x_6 合为一个指标；f_2 在 x_1, x_4 上载荷较大，可以将 x_1, x_4 合为一个指标；f_3 在 x_2 上载荷很大；这样就可以将 7 个指标降成 3 个因子，方便后续操作处理。

3. 程序:

```
x <- read.table("ex6_3-data.txt", header=TRUE, row.names = 1)
std.x <- as.data.frame(scale(x))
prin1 <- princomp(std.x, cor = T)
screeplot(prin1, type="lines")
# 通过碎石图判断需要2个因子
fact <- factanal(x, 2, scores="Bartlett", rotation="varimax")
fact
fact$scores
```

```
colMeans(x)
```

输出结果:

```
> fact

Call:
factanal(x = x, factors = 2, scores = "Bartlett", rotation = "varimax")

Uniquenesses:
   X1  X2  X3  X4  X5  X6
0.005 0.014 0.068 0.005 0.032 0.008

Loadings:
   Factor1 Factor2
X1 0.789 0.610
X2 0.853 0.508
X3 0.455 0.852
X4 0.638 0.768
X5 0.839 0.514
X6 0.553 0.828

            Factor1 Factor2
SS loadings 2.975 2.895
Proportion Var 0.496 0.482
Cumulative Var 0.496 0.978

Test of the hypothesis that 2 factors are sufficient.
The chi square statistic is 9.64 on 4 degrees of freedom.
The p-value is 0.047
> fact$scores
           Factor1 Factor2
shanghai 1.61488810 3.28060030
nanjing 0.04603819 0.25383057
suzhou 2.71139789 -2.01206700
wuxi 0.78530751 -0.64125138
changzhou -0.33545512 -0.21289765
zhenjiang -0.68960491 -0.13452035
nantong -0.34352013 -0.11683543
yangzhou -0.80581667 0.04008038
tai4zhou -0.76307362 -0.08571657
hangzhou 0.62763381 -0.11163156
```

```
ningbo 0.24935155 0.02978948
jiaxing -0.29215815 -0.24665626
huzhou -0.87148041 0.03201790
shaoxing -0.26852669 -0.12364901
zhoushan -1.13400413 0.02154910
tai1zhou -0.53097720 0.02735747
> colMeans(x)
       X1 X2 X3 X4 X5 X6
 1487.42125 685.85438 10874.18750 449.13000 58.87375 62.83937
```

主要结论:

可以得出因子模型为

$$\begin{cases} x_1 - 1487.42125 = 0.789f_1 + 0.610f_2 + \varepsilon_1 \\ x_2 - 685.85438 = 0.853f_1 + 0.508f_2 + \varepsilon_2 \\ x_3 - 10874.18750 = 0.455f_1 + 0.852f_2 + \varepsilon_3 \\ x_4 - 449.13000 = 0.638f_1 + 0.768f_2 + \varepsilon_4 \\ x_5 - 58.87375 = 0.839f_1 + 0.514f_2 + \varepsilon_5 \\ x_6 - 62.83937 = 0.553f_1 + 0.828f_2 + \varepsilon_6 \end{cases}$$

f_1 在 x_2, x_5 上的载荷较大，可以将固定资产投资与外贸出口额合成一个指标；f_2 在 x_3, x_6 上的载荷较大，可以将货运总量与互联网上网人数合成一个指标；这样就可以将 6 个指标降成 2 个因子，方便后续操作处理。

4. 程序:

```
ex6_4 <- function(x, m) {
   p <- nrow(x)
   x.diag <- diag(x)
   sum.rank <- sum(x.diag)
   # 设置行名、列名
   rowname <- paste("X", 1:p, sep = "")
   colname <- paste("Factor", 1:m, sep = "")
   # 构造因子载荷矩阵A，初值设为0
   A <- matrix(0, nrow = p, ncol = m, dimnames = list(rowname, colname))
   # eig包含两个元素,values为特征根，vectors为特征向量
   eig <- eigen(x)
   for (i in 1:m) {
        # 填充矩阵A的值
        A[, i] <- sqrt(eig$values[i]) * eig$vectors[, i]
     }
   # 公共因子的方差
   var.A <- diag(A %*% t(A))
   rowname1 <- c("SS loadings", "Proportion Var", "Cumulative Var")
   # 构造输出结果的矩阵，初值设为0
```

```
    result <- matrix(0, nrow = 3, ncol = m, dimnames = list(rowname1, colname))
    for (i in 1:m) {
        # 计算各因子的方差
        result[1, i] <- sum(A[, i]^2)
        # 计算方差贡献率
        result[2, i] <- result[1, i] / sum.rank
        # 累计方差贡献率
        result[3, i] <- sum(result[1, 1:i]) / sum.rank
    }
    method <- c("Principal Component Method")
    # 输出计算结果
    list(method = method, loadings = A, var = cbind(common = var.A, specific = x.
        diag - var.A), result = result)
}
```

输出结果:

略。

主要结论:

略。

第七章　聚类分析

1. 解：

   最短距离法：

   (1) 聚类过程如下：

   六个样品自成一类 $G_k = \{\boldsymbol{x}_i^{\mathrm{T}}\}(k = 1, 2, \cdots, 6)$，先求距离矩阵 $\boldsymbol{D}_{(0)}$。

$\boldsymbol{D}_{(0)}$	G_1	G_2	G_3	G_4	G_5	G_6
G_1	0					
G_2	1	0				
G_3	*0.4*	1.4	0			
G_4	2.5	2.5	2.5	0		
G_5	5.5	6.5	5.1	6	0	
G_6	2	3	1.6	2.5	3.5	0

$\boldsymbol{D}_{(0)}$ 中最小非零元素为 0.4，所以将对应的 G_1 和 G_3 合并为新类 $G_7 = G_1 \cup G_3$，按公式 (7.3.2) 计算新类 G_7 与其他类的距离，即 $D_{7k} = \min\{D_{1k}, D_{3k}\}(k = 2, 4, 5, 6)$。其他类与类之间的距离不变，得到距离矩阵 $\boldsymbol{D}_{(1)}$。

$\boldsymbol{D}_{(1)}$	G_2	G_4	G_5	G_6	G_7
G_2	0				
G_4	2.5	0			
G_5	6.5	6	0		
G_6	3	2.5	3.5	0	
G_7	*1*	2.5	5.1	1.6	0

$\boldsymbol{D}_{(1)}$ 中最小非零元素为 1，所以将对应的 G_2 和 G_7 合并为新类 $G_8 = G_2 \cup G_7$，按公式 (7.3.2) 计算新类 G_8 与其他类的距离，即 $D_{8k} = \min\{D_{2k}, D_{7k}\}(k = 4, 5, 6)$。其他类与类之间的距离不变，得到距离矩阵 $\boldsymbol{D}_{(2)}$。

$\boldsymbol{D}_{(2)}$	G_4	G_5	G_6	G_8
G_4	0			
G_5	6	0		
G_6	2.5	3.5	0	
G_8	2.5	5.1	*1.6*	0

$\boldsymbol{D}_{(2)}$ 中最小非零元素为 1.6，所以将对应的 G_6 和 G_8 合并为新类 $G_9 = G_6 \cup G_8$，计算距离矩阵 $\boldsymbol{D}_{(3)}$。

$\boldsymbol{D}_{(3)}$	G_4	G_5	G_9
G_4	0		
G_5	6	0	
G_9	*2.5*	3.5	0

$\boldsymbol{D}_{(3)}$ 中最小非零元素为 2.5，所以将对应的 G_4 和 G_9 合并为新类 $G_{10}=G_4\cup G_9$，计算距离矩阵 $\boldsymbol{D}_{(4)}$。

$\boldsymbol{D}_{(4)}$	G_5	G_{10}
G_5	0	
G_{10}	*3.5*	0

最后，所有样品合并为一，聚类过程结束。

(2) 树形图如下：

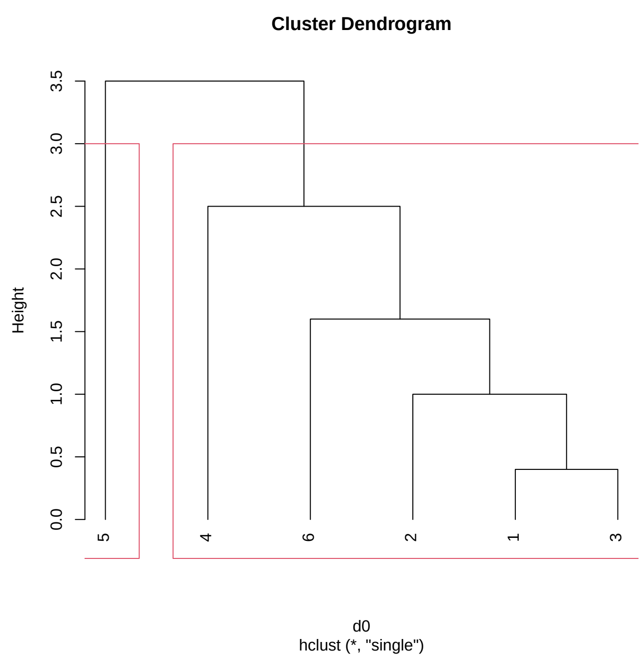


图 11: 题 1 最短距离法树形图

(3) $\boldsymbol{x}_5^{\mathrm{T}}=(\{3,5\})$ 为一类，其余样品为一类。

最长距离法：

(1) 聚类过程如下：

六个样品自成一类 $G_k=\{\boldsymbol{x}_i^{\mathrm{T}}\}(k=1,2,\cdots,6)$，先求距离矩阵 $\boldsymbol{D}_{(0)}$。

$\boldsymbol{D}_{(0)}$	G_1	G_2	G_3	G_4	G_5	G_6
G_1	0					
G_2	1	0				
G_3	*0.4*	1.4	0			
G_4	2.5	2.5	2.5	0		
G_5	5.5	6.5	5.1	6	0	
G_6	2	3	1.6	2.5	3.5	0

$\boldsymbol{D}_{(0)}$ 中最小非零元素为 0.4，所以将对应的 G_1 和 G_3 合并为新类 $G_7=G_1\cup G_3$，按公式 (7.3.4) 计算新

类 G_7 与其他类的距离，即 $D_{7k}=\max\{D_{1k},D_{3k}\}(k=2,4,5,6)$。其他类与类之间的距离不变，得到距离矩阵 $\boldsymbol{D}_{(1)}$。

$\boldsymbol{D}_{(1)}$	G_2	G_4	G_5	G_6	G_7
G_2	0				
G_4	2.5	0			
G_5	6.5	6	0		
G_6	3	2.5	3.5	0	
G_7	*1.4*	2.5	5.5	2	0

$\boldsymbol{D}_{(1)}$ 中最小非零元素为 1.4，所以将对应的 G_2 和 G_7 合并为新类 $G_8=G_2\cup G_7$，按公式 (7.3.4) 计算新类 G_8 与其他类的距离，即 $D_{8k}=\max\{D_{2k},D_{7k}\}(k=4,5,6)$。其他类与类之间的距离不变，得到距离矩阵 $\boldsymbol{D}_{(2)}$。

$\boldsymbol{D}_{(2)}$	G_4	G_5	G_6	G_8
G_4	0			
G_5	6	0		
G_6	2.5	3.5	0	
G_8	*2.5*	6.5	3	0

$\boldsymbol{D}_{(2)}$ 中最小非零元素为 2.5，所以将对应的 G_4 和 G_8 合并为新类 $G_9=G_4\cup G_8$，计算距离矩阵 $\boldsymbol{D}_{(3)}$。

$\boldsymbol{D}_{(3)}$	G_5	G_6	G_9
G_5	0		
G_6	3.5	0	
G_9	6.5	*3*	0

$\boldsymbol{D}_{(3)}$ 中最小非零元素为 3，所以将对应的 G_6 和 G_9 合并为新类 $G_{10}=G_6\cup G_9$，计算距离矩阵 $\boldsymbol{D}_{(4)}$。

$\boldsymbol{D}_{(4)}$	G_5	G_{10}
G_5	0	
G_{10}	*6.5*	0

最后，所有样品合并为一，聚类过程结束。

(2) 树形图如下：

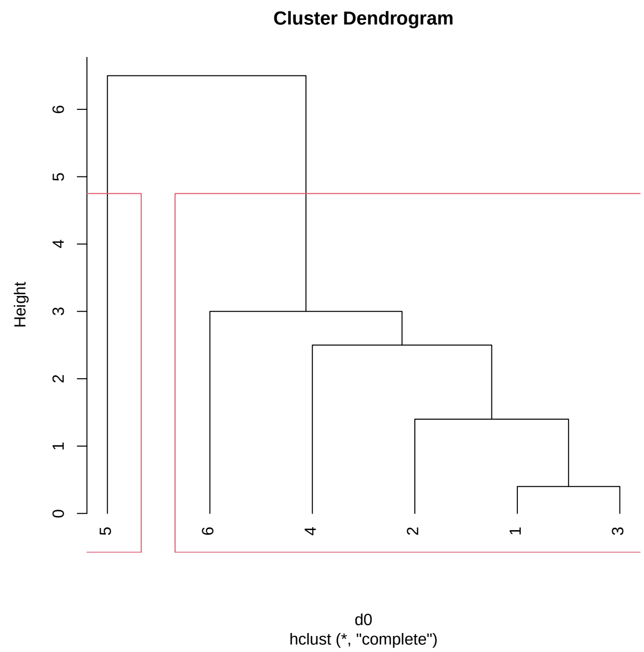


图 12: 题 1 最长距离法树形图

(3) 所以 $\boldsymbol{x}_5^{\mathrm{T}} = (\{3,5\})$ 为一类，其余样品为一类。

程序：

```
x <- c(1,0.5,1.2,2,3,2.5,1.5,1,1.7,0,5,2.0)
dim(x) <- c(6,2)
d0 <- dist(x, method="minkowski", diag=TRUE, upper=FALSE, p=1)
# 最短距离法
hcs <- hclust(d0, method="single")
plot(hcs, hang=-1)
rect.hclust(hcs, k=2, h=NULL, border=2)
# 最长距离法
hcs <- hclust(d0, method="complete")
plot(hcs, hang=-1)
rect.hclust(hcs, k=2, h=NULL, border=2)
```

输出结果：

输出结果见上。

主要结论：

主要结论见上。

2. 证：

$$\begin{aligned}
D_{rk}^2 &= \frac{1}{n_r n_k} \sum_{i\in G_r, j\in G_k} d_{ij}^2 \\
&= \frac{1}{n_r n_k} \left(\sum_{i\in G_p, j\in G_k} d_{ij}^2 + \sum_{i\in G_q, j\in G_k} d_{ij}^2 \right) \\
&= \frac{n_p}{n_r} \cdot \frac{1}{n_p n_k} \sum_{i\in G_p, j\in G_k} d_{ij}^2 + \frac{n_q}{n_r} \cdot \frac{1}{n_q n_k} \sum_{i\in G_q, j\in G_k} d_{ij}^2 \\
&= \frac{n_p}{n_r} D_{pk}^2 + \frac{n_q}{n_r} D_{qk}^2
\end{aligned}$$

3. 解：

最短距离法：

(1) 聚类过程如下：

$\boldsymbol{D}_{(0)}$	G_1	G_2	G_3	G_4	G_5	G_6	G_7
G_1	0						
G_2	4	0					
G_3	7	3	0				
G_4	12	8	5	0			
G_5	18	14	11	6	0		
G_6	19	15	12	7	*1*	0	
G_7	21	17	14	9	3	2	0

$\boldsymbol{D}_{(1)}$	G_1	G_2	G_3	G_4	G_7	$G_8 = G_5 \cup G_6$
G_1	0					
G_2	4	0				
G_3	7	3	0			
G_4	12	8	5	0		
G_7	21	17	14	9	0	
G_8	18	14	11	6	*2*	0

$\boldsymbol{D}_{(2)}$	G_1	G_2	G_3	G_4	$G_9 = G_7 \cup G_8$
G_1	0				
G_2	4	0			
G_3	7	*3*	0		
G_4	12	8	5	0	
G_9	18	14	11	6	0

$\boldsymbol{D}_{(3)}$	G_1	G_4	G_9	$G_{10}=G_2\cup G_3$
G_1	0			
G_4	12	0		
G_9	18	6	0	
G_{10}	*4*	5	11	0

$\boldsymbol{D}_{(4)}$	G_4	G_9	$G_{11}=G_1\cup G_{10}$
G_4	0		
G_6	6	0	
G_{11}	*5*	11	0

$\boldsymbol{D}_{(5)}$	G_9	$G_{12}=G_4\cup G_{11}$
G_9	0	
G_{12}	*6*	0

(2) 树形图如下：

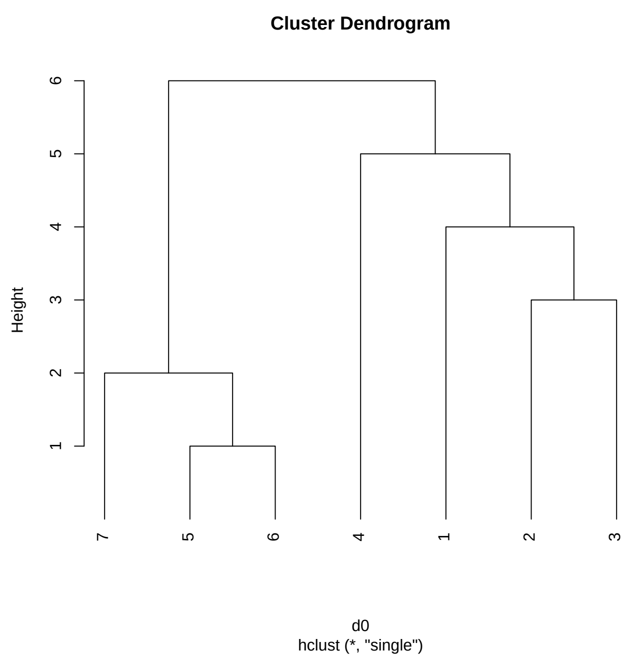


图 13: 题 3 最短距离法树形图

最长距离法：

(1) 聚类过程如下：

$\boldsymbol{D}_{(0)}$	G_1	G_2	G_3	G_4	G_5	G_6	G_7
G_1	0						
G_2	4	0					
G_3	7	3	0				
G_4	12	8	5	0			
G_5	18	14	11	6	0		
G_6	19	15	12	7	*1*	0	
G_7	21	17	14	9	3	2	0

$\boldsymbol{D}_{(1)}$	G_1	G_2	G_3	G_4	G_7	$G_8 = G_5 \cup G_6$
G_1	0					
G_2	4	0				
G_3	7	3	0			
G_4	12	8	5	0		
G_7	21	17	14	9	0	
G_8	19	15	12	7	*3*	0

$\boldsymbol{D}_{(2)}$	G_1	G_2	G_3	G_4	$G_9 = G_7 \cup G_8$
G_1	0				
G_2	4	0			
G_3	7	*3*	0		
G_4	12	8	5	0	
G_9	21	17	14	9	0

$\boldsymbol{D}_{(3)}$	G_1	G_4	G_9	$G_{10} = G_2 \cup G_3$
G_1	0			
G_4	12	0		
G_9	21	9	0	
G_{10}	*7*	8	17	0

$\boldsymbol{D}_{(4)}$	G_4	G_9	$G_{11} = G_1 \cup G_{10}$
G_4	0		
G_9	*9*	0	
G_{11}	12	21	0

$\boldsymbol{D}_{(5)}$	G_{11}	$G_{12} = G_4 \cup G_9$
G_{11}	0	
G_{12}	*21*	0

(2) 树形图如下：

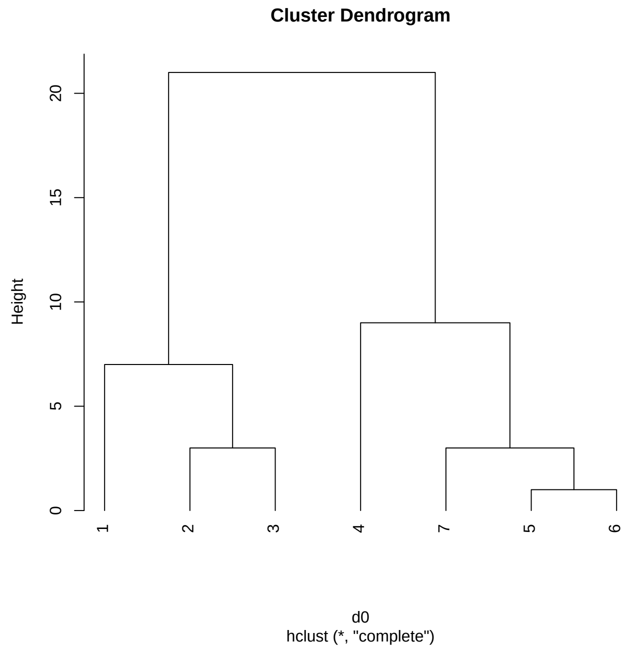


图 14: 题 3 最长距离法树形图

程序：

```
D <- matrix(c(0,4,7,12,18,19,21,
            0,0,3,8,14,15,17,
            0,0,0,5,11,12,14,
            0,0,0,0,6,7,9,
            0,0,0,0,0,1,3,
            0,0,0,0,0,0,2,
            0,0,0,0,0,0,0), nr=7)
D <- D + t(D)
rownames(D) <- seq(1,7)
d0 <- as.dist(D)
# 最短距离法
hcs <- hclust(d0, method="single")
plot(hcs, hang=-1)
# 最长距离法
hcs <- hclust(d0, method="complete")
plot(hcs, hang=-1)
```

输出结果：

输出结果如上。

主要结论：

主要结论见上。

4. 程序:

```r
x <- read.table("ex7_4-data.txt", header=TRUE, row.names = 1)
stdx <- scale(x, center=TRUE, scale=TRUE)
# 最短距离法
d0 <- dist(x, method="euclidean", diag=TRUE, upper=FALSE)
hcs <- hclust(d0, method="single")
plot(hcs, hang=-1)
# 最长距离法
d0 <- dist(x, method="euclidean", diag=TRUE, upper=FALSE)
hcs <- hclust(d0, method="complete")
plot(hcs, hang=-1)
# 类平均法, 聚为5类
d0 <- dist(x, method="euclidean", diag=TRUE, upper=FALSE)
hcs <- hclust(d0, method="average")
rect.hclust(hcs, k=5, h=NULL, border=2)
plot(hcs, hang=-1)
# 重心法, 聚为5类
kmeans(stdx, 5, iter.max=10, algorithm="MacQueen")
```

输出结果:

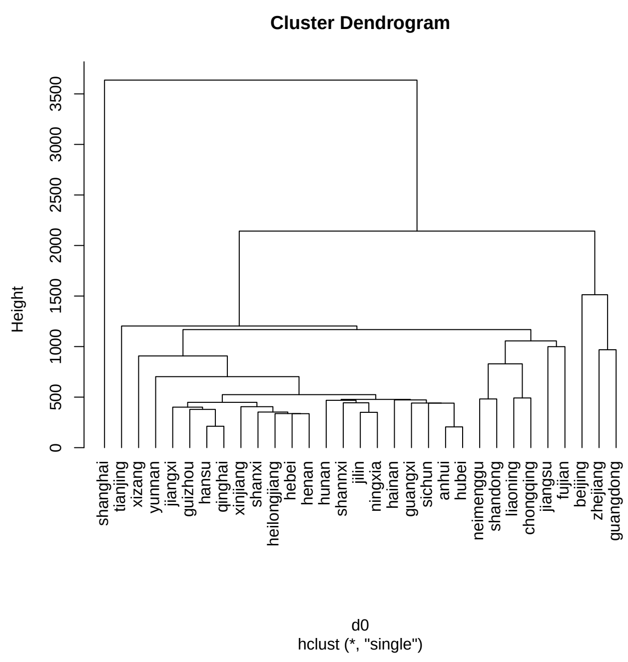


图 15: 题 4 最短距离法树形图

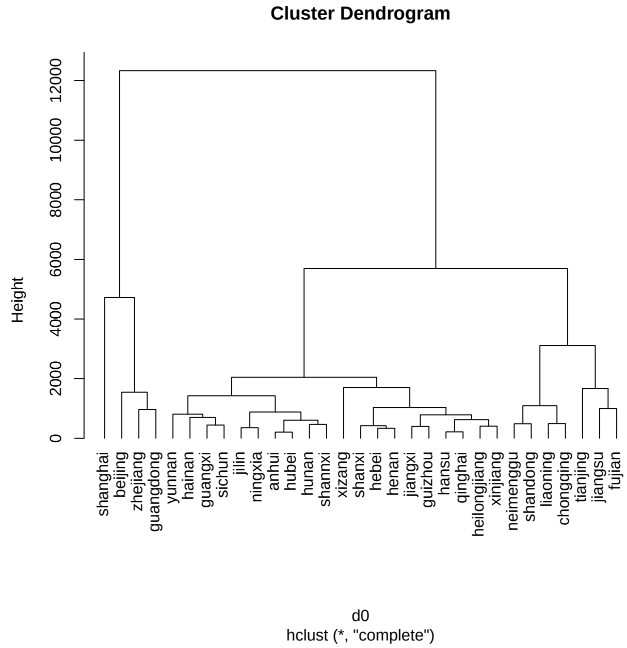


图 16: 题 4 最长距离法树形图

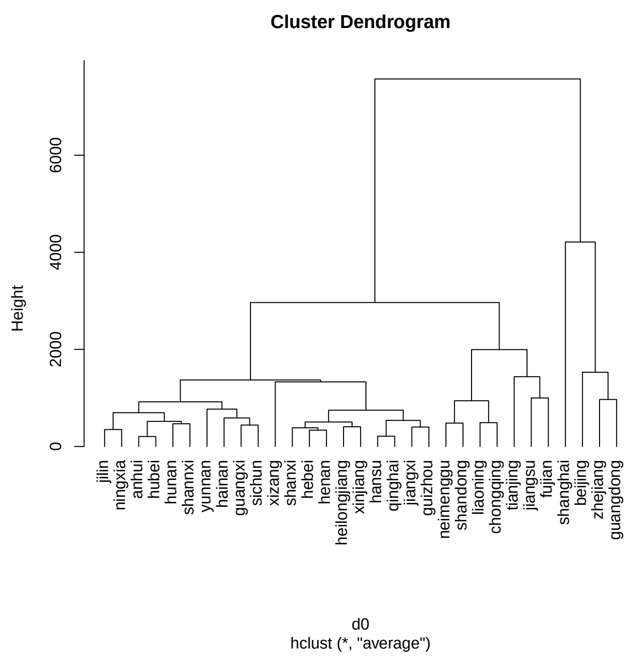


图 17: 题 4 类平均法树形图

```
> kmeans(stdx, 5, iter.max=10, algorithm="MacQueen")
K-means clustering with 5 clusters of sizes 10, 5, 3, 11, 2

Cluster means:
    xiaofei shipin yizhuo juzhu jiatingshebei yiliao jiaotong
```

```
1 -0.4688650 -0.1858709 -0.8724430 -0.5255167 -0.29786714 -0.7981605 -0.3631822
2 1.9568033 1.7878121 0.9215928 1.6390354 1.59656427 1.3068036 1.9033693
3 -0.2694473 -0.5776927 0.2481019 0.7341867 -0.78773623 0.4678536 -0.3847263
4 -0.2592457 -0.5170184 0.3971536 -0.2012223 0.07033978 0.2332705 -0.3693902
5 -0.7176609 0.1699644 -0.4982646 -1.4645628 -1.70733938 -1.2609747 -0.3337768
     jiaoyu
1 -0.4096537
2 1.8461115
3 -0.3023638
4 -0.1503188
5 -1.2867112

Clustering vector:
    beijing tianjing hebei shanxi neimenggu liaoning
         2 2 4 3 4 3
      jilin heilongjiang shanghai jiangsu zhejiang anhui
         3 4 2 4 2 1
     fujian jiangxi shandong henan hubei hunan
         1 1 4 4 1 4
  guangdong guangxi hainan chongqing sichun guizhou
         2 1 1 4 1 1
     yunnan xizang shannxi hansu qinghai ningxia
         5 5 4 1 1 4
   xinjiang
         4

Within cluster sum of squares by cluster:
[1] 18.717017 32.974531 2.251508 19.541294 1.161091
 (between_SS / total_SS = 68.9 %)

Available components:

[1] "cluster" "centers" "totss" "withinss" "tot.withinss"
[6] "betweenss" "size" "iter" "ifault"
```

主要结论：

主要结论如上。

5. 程序：

```
x <- read.table("ex7_5-data.txt", header=TRUE, row.names = 1)
stdx <- scale(x, center=TRUE, scale=TRUE)
set.seed(7)
```

```
kmeans(stdx, 5, iter.max=10, algorithm="MacQueen")
```

输出结果：

```
> kmeans(stdx, 5, iter.max=10, algorithm="MacQueen")
K-means clustering with 5 clusters of sizes 11, 7, 4, 2, 7

Cluster means:
       PM10 SO2 CO2 grade2
1 0.1901517 0.2683215 0.6502499 -0.1638951
2 -0.9179291 -0.3624441 0.2052909 1.0339058
3 1.4571533 0.5235103 0.5781826 -1.7822978
4 -1.9628525 -2.1530044 -1.5680164 1.3859312
5 0.3472753 0.2567914 -1.1094977 -0.1538808

Clustering vector:
    Beijing Tianjin Shijiazhuang Taiyuan Hohhot Shenyang
         3 1 5 5 2 5
     Dalian Harbin Shanghai Nanjing Hangzhou Hefei
         2 1 1 1 1 3
     Fuzhou Nanchang Jinan Zhengzhou Wuhan Changsha
         2 2 5 1 1 1
  Guangzhou Nanning Haikou Chongqing Chengdu Guiyang
         2 2 4 1 1 5
    Kunming Lhasa Xi'an      Lanzhou       Xining      Yinchuan
           2            4            1            3            5            5
      Urumqi
           3

Within cluster sum of squares by cluster:
[1]  6.4045158  5.6476870 16.9585359  0.3133693 10.4873904
 (between_SS / total_SS =  66.8 %)

Available components:

[1] "cluster"      "centers"      "totss"        "withinss"     "tot.withinss"
[6] "betweenss"    "size"         "iter"         "ifault"
```

主要结论：

为方便固定原始随机点，使用 set.seed() 播撒随机数种子，根据输出可得到以下分类：

(1) 第 1 类 (11 个)：天津、哈尔滨、上海、南京、杭州、郑州、武汉、长沙、重庆、成都、西安；

(2) 第 2 类 (6 个)：呼和浩特、福州、南昌、广州、南京、昆明；

(3) 第 3 类 (3 个)：北京、兰州、乌鲁木齐；

(4) 第 4 类 (2 个)：海口、拉萨；

(5) 第 5 类 (7 个)：石家庄、太原、沈阳、济南、贵阳、西宁、银川。

第八章　判别分析

1. 程序：

```
x1 <- read.table("ex8_1-data.txt", header=T);
x <- x1[,3:4];
G <- x1[,2];
source("distance.distinguish.R")
distance.distinguish(x, G)
```

输出结果：

```
> distance.distinguish(x, G)
   G distance blong
1  1 0.03013383 1
2  1 0.50815510 1
3  1 0.33524999 1
4  1 1.34555129 1
5  1 0.43868072 1
6  1 0.66384649 2
7  1 4.37422412 1
8  1 2.84346730 1
9  1 0.34564661 1
10 1 3.48167679 1
11 1 1.37803514 1
12 1 4.28252864 1
13 2 0.26495655 2
14 2 1.39664196 1
15 2 1.25568174 2
16 2 0.63012488 2
17 2 0.61739940 2
18 2 0.43868072 1
19 2 3.90040897 2
20 2 2.06443438 2
21 2 2.50173811 2
22 2 2.05946834 2
```

主要结论：

根据结果，6、14、18 被误判，误判率 $\hat{p} = \dfrac{3}{22} \approx 13.64\%$。

2. 程序：

```
library(MASS)
x <- data.matrix(iris)
```

```
colnames(x) <- c("X1", "X2", "X3", "X4", "C")
x <- data.frame(x)
discrim <- lda(C~X1+X2+X3+X4, x); discrim
p.discrim <- predict(discrim)
cbind(x[[5]], p.discrim$x, p.discrim$class)
table(x[[5]], p.discrim$class)
prop.table(table(x[[5]], p.discrim$class))
```

输出结果:

```
> discrim <- lda(C~X1+X2+X3+X4, x); discrim
Call:
lda(C ~ X1 + X2 + X3 + X4, data = x)

Prior probabilities of groups:
      1 2 3
0.3333333 0.3333333 0.3333333

Group means:
    X1 X2 X3 X4
1 5.006 3.428 1.462 0.246
2 5.936 2.770 4.260 1.326
3 6.588 2.974 5.552 2.026

Coefficients of linear discriminants:
        LD1 LD2
X1 0.8293776 0.02410215
X2 1.5344731 2.16452123
X3 -2.2012117 -0.93192121
X4 -2.8104603 2.83918785

Proportion of trace:
  LD1 LD2
0.9912 0.0088
> p.discrim <- predict(discrim)
> cbind(x[[5]], p.discrim$x, p.discrim$class)
          LD1 LD2
1 1 8.0617998 0.300420621 1
2 1 7.1286877 -0.786660426 1
3 1 7.4898280 -0.265384488 1
4 1 6.8132006 -0.670631068 1
5 1 8.1323093 0.514462530 1
```

```
6 1 7.7019467 1.461720967 1
7 1 7.2126176 0.355836209 1
8 1 7.6052935 -0.011633838 1
9 1 6.5605516 -1.015163624 1
10 1 7.3430599 -0.947319209 1
11 1 8.3973865 0.647363392 1
12 1 7.2192969 -0.109646389 1
13 1 7.3267960 -1.072989426 1
14 1 7.5724707 -0.805464137 1
15 1 9.8498430 1.585936985 1
16 1 9.1582389 2.737596471 1
17 1 8.5824314 1.834489452 1
18 1 7.7807538 0.584339407 1
19 1 8.0783588 0.968580703 1
20 1 8.0209745 1.140503656 1
21 1 7.4968023 -0.188377220 1
22 1 7.5864812 1.207970318 1
23 1 8.6810429 0.877590154 1
24 1 6.2514036 0.439696367 1
25 1 6.5589334 -0.389222752 1
26 1 6.7713832 -0.970634453 1
27 1 6.8230803 0.463011612 1
28 1 7.9246164 0.209638715 1
29 1 7.9912902 0.086378713 1
30 1 6.8294645 -0.544960851 1
31 1 6.7589549 -0.759002759 1
32 1 7.3749525 0.565844592 1
33 1 9.1263463 1.224432671 1
34 1 9.4676820 1.825226345 1
35 1 7.0620139 -0.663400423 1
36 1 7.9587624 -0.164961722 1
37 1 8.6136720 0.403253602 1
38 1 8.3304176 0.228133530 1
39 1 6.9341201 -0.705519379 1
40 1 7.6882313 -0.009223623 1
41 1 7.9179372 0.675121313 1
42 1 5.6618807 -1.934355243 1
43 1 7.2410147 -0.272615132 1
44 1 6.4144356 1.247301306 1
45 1 6.8594438 1.051653957 1
46 1 6.7647039 -0.505151855 1
```

```
47 1 8.0818994 0.763392750 1
48 1 7.1867690 -0.360986823 1
49 1 8.3144488 0.644953177 1
50 1 7.6719674 -0.134893840 1
51 2 -1.4592755 0.028543764 2
52 2 -1.7977057 0.484385502 2
53 2 -2.4169489 -0.092784031 2
54 2 -2.2624735 -1.587252508 2
55 2 -2.5486784 -0.472204898 2
56 2 -2.4299673 -0.966132066 2
57 2 -2.4484846 0.795961954 2
58 2 -0.2226665 -1.584673183 2
59 2 -1.7502012 -0.821180130 2
60 2 -1.9584224 -0.351563753 2
61 2 -1.1937603 -2.634455704 2
62 2 -1.8589257 0.319006544 2
63 2 -1.1580939 -2.643409913 2
64 2 -2.6660572 -0.642504540 2
65 2 -0.3783672 0.086638931 2
66 2 -1.2011726 0.084437359 2
67 2 -2.7681025 0.032199536 2
68 2 -0.7768540 -1.659161847 2
69 2 -3.4980543 -1.684956162 2
70 2 -1.0904279 -1.626583496 2
71 2 -3.7158961 1.044514421 3
72 2 -0.9976104 -0.490530602 2
73 2 -3.8352593 -1.405958061 2
74 2 -2.2574125 -1.426794234 2
75 2 -1.2557133 -0.546424197 2
76 2 -1.4375576 -0.134424979 2
77 2 -2.4590614 -0.935277280 2
78 2 -3.5184849 0.160588866 2
79 2 -2.5897987 -0.174611728 2
80 2 0.3074879 -1.318871459 2
81 2 -1.1066918 -1.752253714 2
82 2 -0.6055246 -1.942980378 2
83 2 -0.8987038 -0.904940034 2
84 2 -4.4984664 -0.882749915 3
85 2 -2.9339780 0.027379106 2
86 2 -2.1036082 1.191567675 2
87 2 -2.1425821 0.088779781 2
```

```
88 2 -2.4794560 -1.940739273 2
89 2 -1.3255257 -0.162869550 2
90 2 -1.9555789 -1.154348262 2
91 2 -2.4015702 -1.594583407 2
92 2 -2.2924888 -0.332860296 2
93 2 -1.2722722 -1.214584279 2
94 2 -0.2931761 -1.798715092 2
95 2 -2.0059888 -0.905418042 2
96 2 -1.1816631 -0.537570242 2
97 2 -1.6161564 -0.470103580 2
98 2 -1.4215888 -0.551244626 2
99 2 0.4759738 -0.799905482 2
100 2 -1.5494826 -0.593363582 2
101 3 -7.8394740 2.139733449 3
102 3 -5.5074800 -0.035813989 3
103 3 -6.2920085 0.467175777 3
104 3 -5.6054563 -0.340738058 3
105 3 -6.8505600 0.829825394 3
106 3 -7.4181678 -0.173117995 3
107 3 -4.6779954 -0.499095015 3
108 3 -6.3169269 -0.968980756 3
109 3 -6.3277368 -1.383289934 3
110 3 -6.8528134 2.717589632 3
111 3 -4.4407251 1.347236918 3
112 3 -5.4500957 -0.207736942 3
113 3 -5.6603371 0.832713617 3
114 3 -5.9582372 -0.094017545 3
115 3 -6.7592628 1.600232061 3
116 3 -5.8070433 2.010198817 3
117 3 -5.0660123 -0.026273384 3
118 3 -6.6088188 1.751635872 3
119 3 -9.1714749 -0.748255067 3
120 3 -4.7645357 -2.155737197 3
121 3 -6.2728391 1.649481407 3
122 3 -5.3607119 0.646120732 3
123 3 -7.5811998 -0.980722934 3
124 3 -4.3715028 -0.121297458 3
125 3 -5.7231753 1.293275530 3
126 3 -5.2791592 -0.042458238 3
127 3 -4.0808721 0.185936572 3
128 3 -4.0770364 0.523238483 3
```

```
129 3 -6.5191040 0.296976389 3
130 3 -4.5837194 -0.856815813 3
131 3 -6.2282401 -0.712719638 3
132 3 -5.2204877 1.468195094 3
133 3 -6.8001500 0.580895175 3
134 3 -3.8151597 -0.942985932 2
135 3 -5.1074897 -2.130589999 3
136 3 -6.7967163 0.863090395 3
137 3 -6.5244960 2.445035271 3
138 3 -4.9955028 0.187768525 3
139 3 -3.9398530 0.614020389 3
140 3 -5.2038309 1.144768076 3
141 3 -6.6530868 1.805319760 3
142 3 -5.1055595 1.992182010 3
143 3 -5.5074800 -0.035813989 3
144 3 -6.7960192 1.460686950 3
145 3 -6.8473594 2.428950671 3
146 3 -5.6450035 1.677717335 3
147 3 -5.1795646 -0.363475041 3
148 3 -4.9677409 0.821140550 3
149 3 -5.8861454 2.345090513 3
150 3 -4.6831543 0.332033811 3
> table(x[[5]], p.discrim$class)

    1 2 3
  1 50 0 0
  2 0 48 2
  3 0 1 49
> prop.table(table(x[[5]], p.discrim$class))

           1 2 3
  1 0.333333333 0.000000000 0.000000000
  2 0.000000000 0.320000000 0.013333333
  3 0.000000000 0.006666667 0.326666667
```

主要结论:

setosa 全部判定正确; versicolor 共 50 个, 48 个判定正确, 2 个误判的来自 virginica; virginica 共 50 个, 49 个判定正确, 1 个误判的来自 versicolor。

3. 程序:

```
# 距离判别分析
x1 <- read.table("data.exam8.1.1.txt", header=T);
```

```
x <- x1[,3:6];
G <- x1[,1];
source("distance.distinguish.R")
distance.distinguish(x, G)
# 逐步判别分析
source("step.distinguish.R")
Class <- factor(x1[,1])
X <- x1[,3:6]
step.distinguish(X, Class, 3, 2)
library(MASS)
discrim <- lda(类别~蛋白质.克.+碳水化合物.克., x1)
discrim.new <- predict(discrim, x1)$class; discrim.new
```

输出结果：

```
> # 距离判别分析
> distance.distinguish(x, G)
   G distance blong
1 1 4.3787100 2
2 1 1.0525988 1
3 1 4.8132482 1
4 1 2.8991588 1
5 1 5.1404677 1
6 1 0.6086319 1
7 1 4.9104902 1
8 2 1.1025444 2
9 2 1.2821348 2
10 2 1.3082985 2
11 2 2.0305928 2
12 2 1.2567908 2
13 2 0.7929908 3
14 2 6.2835284 4
15 2 1.0467843 2
16 2 2.1486510 2
17 2 1.4494785 2
18 2 1.9106217 3
19 2 1.5017083 3
20 2 0.5803590 2
21 2 2.2790242 2
22 2 1.6158079 2
23 2 7.5244729 2
24 2 3.7590954 1
```

```
25 2 16.3220759 2
26 2 1.1933420 2
27 3 1.5832875 2
28 3 0.6001095 2
29 3 0.7715365 2
30 3 0.9989526 3
31 3 1.9371579 2
32 3 6.0741457 3
33 3 5.0845694 3
34 3 1.3260521 2
35 4 2.8151348 2
36 4 6.4970138 4
37 4 4.1744166 4
38 4 3.4092930 4
39 4 1.2765777 4
40 4 2.3049596 4
41 4 7.1090074 4
42 4 0.6855894 4
43 4 1.7329603 4

> # 逐步判别分析
> step.distinguish(X, Class, 3, 2)
$Varible
[1] "蛋白质.克." "碳水化合物.克."

$F.Value
[1] 10.125172 9.326243

$Enter.exclude
[1] "Enter" "Enter"

$F.out.max
[1] 2.470404

$F.in.min
[1] 9.326243

$F.out
[1] 0.000000 2.470404 0.000000 1.035299
> discrim.new
 [1] 2 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 4 2 2 2 2 2 2 2 2 2 2 1 4 4 4
```

```
[40] 4 4 4 4
Levels: 1 2 3 4
```

主要结论：

(1) 距离判别分析：1、13、14、18、19、24、31、34、35 被误判。

(2) 逐步判别分析：1、23、24、25、27、28、29、30、31、32、33、34、35、36 被误判。

(3) 距离判别分析法在此题的情况下误判率低于逐步判别分析法。

4. 程序：

```
ex7_4 <- function(X.origin, X.predict) {
  if (length(unique(X.origin)) != 2) {
    print("Make sure there are only two categories in the data.");
    return(NULL)
  }
  if (length(X.origin) != length(X.predict)) {
    print("Make sure the two sets of data are consistent in length.");
    return(NULL)
  }
  flag <- 0;
  for (i in seq(1, length(X.origin))) {
    if (X.origin[i] != X.predict[i]) {
      flag = flag + 1;
    }
  }
  p <- flag / length(X.origin);
  return(p)
}
```

利用题 1 的数据进行输出

```
x1 <- read.table("ex8_1-data.txt", header=T);
x <- x1[,3:4];
X.origin <- x1[,2];
source("distance.distinguish.R")
X.new <- distance.distinguish(x, G)
X.new <- X.new[,3]
source("ex7_4.R")
ex7_4(X.origin, X.new)
```

输出结果：

```
> ex7_4(X.origin, X.new)
[1] 0.1363636
```

主要结论：

在题 1 的数据下，误判率大致为 13.64%。

5. 程序：

```
ex7_5 <- function(data, C) {
  # data是样本数据，C是样本所属的类别，使用Fisher线性判别
  # 判断是否只有2类
  if (length(unique(C)) != 2) {
    print("Make sure there are only two categories in the data.");
    return(NULL)
  }
  # 读取数据并进行基础的命名
  m <- ncol(data);
  cols <- sprintf("X%d", seq(1:m));
  x <- cbind(C, data);
  colnames(x) <- c("C", cols);
  n <- nrow(x);
  # 进行判别
  n_wrong = 0;
  for (i in seq(1:n)) {
    x_new <- rbind(x[1:i - 1,], x[i + 1:n,]);
    discrim <- lda(C ~ ., x_new);
    p.discrim <- predict(discrim, x[i,]);
    if (p.discrim$class != x[i, 1]) {
      n_wrong = n_wrong + 1;
    }
  }
  p <- n_wrong / n;
  return(p);
}
```

利用题 1 的数据进行输出

```
x1 <- read.table("ex8_1-data.txt", header=T);
data <- x1[,3:4];
C <- x1[,2];
source("ex7_5.R");
ex7_5(data, C);
```

输出结果：

```
> ex7_5(data, C);
[1] 0.2272727
```

主要结论：

在题 1 的数据下，误判率大致为 $22.73\% > 13.64\%$，高于回代法。在样本量偏小时，交叉确认估计误判率时准确率偏低，适用于样本量更大的情况。

第九章 相关分析

1. 程序:

```
data <- read.table("ex9_1-data.txt", header=T, row.names = 1);
cor(data)
cor.test(~X1+X2, data=data)
cor.test(~X1+X3, data=data)
cor.test(~X1+X4, data=data)
cor.test(~X1+X5, data=data)
cor.test(~X2+X3, data=data)
cor.test(~X2+X4, data=data)
cor.test(~X2+X5, data=data)
cor.test(~X3+X4, data=data)
cor.test(~X3+X5, data=data)
cor.test(~X4+X5, data=data)
```

输出结果:

```
> cor(data)
        X1 X2 X3 X4 X5
X1 1.0000000 0.33837901 0.5169851 0.8991527 0.72340083
X2 0.3383790 1.00000000 0.2772404 0.2701376 -0.05311005
X3 0.5169851 0.27724044 1.0000000 0.4255092 0.39922122
X4 0.8991527 0.27013758 0.4255092 1.0000000 0.76877136
X5 0.7234008 -0.05311005 0.3992212 0.7687714 1.00000000
> cor.test(~X1+X2, data=data)

      Pearson's product-moment correlation

data:  X1 and X2
t = 1.9365, df = 29, p-value = 0.06261
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.01813559  0.61855376
sample estimates:
     cor
0.338379

> cor.test(~X1+X3, data=data)

        Pearson's product-moment correlation
```

```
data: X1 and X3
t = 3.2524, df = 29, p-value = 0.002901
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.1991218 0.7364213
sample estimates:
     cor
0.5169851

> cor.test(~X1+X4, data=data)

	Pearson's product-moment correlation

data:  X1 and X4
t = 11.064, df = 29, p-value = 6.341e-12
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.7995557 0.9506198
sample estimates:
      cor
0.8991527

> cor.test(~X1+X5, data=data)

	Pearson's product-moment correlation

data: X1 and X5
t = 5.6423, df = 29, p-value = 4.266e-06
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.4962694 0.8578487
sample estimates:
     cor
0.7234008

> cor.test(~X2+X3, data=data)

	Pearson's product-moment correlation

data:  X2 and X3
t = 1.5539, df = 29, p-value = 0.1311
```

```
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.08549887  0.57508563
sample estimates:
      cor
0.2772404

> cor.test(~X2+X4, data=data)

        Pearson's product-moment correlation

data: X2 and X4
t = 1.5109, df = 29, p-value = 0.1416
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.09311562 0.56992422
sample estimates:
     cor
0.2701376

> cor.test(~X2+X5, data=data)

       Pearson's product-moment correlation

data:  X2 and X5
t = -0.28641, df = 29, p-value = 0.7766
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.3999240  0.3070076
sample estimates:
        cor
-0.05311005

> cor.test(~X3+X4, data=data)

        Pearson's product-moment correlation

data: X3 and X4
t = 2.5321, df = 29, p-value = 0.01701
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
```

```
 0.08380479 0.67767319
sample estimates:
     cor
0.4255092

> cor.test(~X3+X5, data=data)

      Pearson's product-moment correlation

data:  X3 and X5
t = 2.3448, df = 29, p-value = 0.02609
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.05227608 0.66017318
sample estimates:
      cor
0.3992212

> cor.test(~X4+X5, data=data)

        Pearson's product-moment correlation

data: X4 and X5
t = 6.4735, df = 29, p-value = 4.383e-07
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.5695918 0.8826672
sample estimates:
     cor
0.7687714
```

主要结论:

(1) 各指标间的相关系数为

	X_1	X_2	X_3	X_4	X_5
X_1	1	0.33837901	0.5169851	0.8991527	0.72340083
X_2	0.3383790	1	0.2772404	0.2701376	−0.05311005
X_3	0.5169851	0.27724044	1	0.4255092	0.39922122
X_4	0.8991527	0.27013758	0.4255092	1	0.76877136
X_5	0.7234008	−0.05311005	0.3992212	0.7687714	1

(2) X_1, X_2 显著性检验的 p 值为 $0.06261 > 0.05$，则接受原假设，认为 X_1, X_2 不具有显著的线性相关性；

(3) X_1, X_3 显著性检验的 p 值为 $0.002901 < 0.05$，则拒绝原假设，认为 X_1, X_3 具有显著的线性相关性；

(4) X_1, X_4 显著性检验的 p 值为 $6.341 \times 10^{-12} < 0.05$，则拒绝原假设，认为 X_1, X_4 具有显著的线性相关性；

(5) X_1, X_5 显著性检验的 p 值为 $4.266 \times 10^{-6} < 0.05$，则拒绝原假设，认为 X_1, X_5 具有显著的线性相关性；

(6) X_2, X_3 显著性检验的 p 值为 $0.1311 > 0.05$，则接受原假设，认为 X_2, X_3 不具有显著的线性相关性；

(7) X_2, X_4 显著性检验的 p 值为 $0.1416 > 0.05$，则接受原假设，认为 X_2, X_4 不具有显著的线性相关性；

(8) X_2, X_5 显著性检验的 p 值为 $0.7766 > 0.05$，则接受原假设，认为 X_2, X_5 不具有显著的线性相关性；

(9) X_3, X_4 显著性检验的 p 值为 $0.01701 < 0.05$，则拒绝原假设，认为 X_3, X_4 具有显著的线性相关性；

(10) X_3, X_5 显著性检验的 p 值为 $0.02609 < 0.05$，则拒绝原假设，认为 X_3, X_5 具有显著的线性相关性；

(11) X_4, X_5 显著性检验的 p 值为 $4.383 \times 10^{-7} < 0.05$，则拒绝原假设，认为 X_4, X_5 具有显著的线性相关性。

2. 程序：

```
# 消除X1影响，X2和X3的偏相关系数
x <- read.table("ex9_1-data.txt",head=TRUE);
x <- x[,2:6]
cor(x)
t.df <- nrow(x) - ncol(x)
ndata <- nrow(x)
nvar <- ncol(x)
r23 = (cor(x)[2, 3] - cor(x)[2, 1] * cor(x)[3, 1]) / (sqrt(1 - cor(x)[2, 1] ^ 2) *
    sqrt(1 - cor(x)[3, 1] ^ 2))
t <- r23 * sqrt(ndata - nvar) / sqrt(1 - r23 ^ 2)
p1 <- pt(t, t.df)
p2 <- p1 - 0.5
if (any(p2 <= 0)) p <- 2 * p1 else p <- 2 * (1 - p1)
data.frame(parial.coefs23 = r23, t = t, df = t.df, p_value = p)
# 消除X1影响，X4和X5的偏相关系数
r45 = (cor(x)[4, 5] - cor(x)[4, 1] * cor(x)[5, 1]) / (sqrt(1 - cor(x)[4, 1] ^ 2) *
    sqrt(1 - cor(x)[5, 1] ^ 2))
t <- r45 * sqrt(ndata - nvar) / sqrt(1 - r45 ^ 2)
p1 <- pt(t, t.df)
p2 <- p1 - 0.5
if (any(p2 <= 0)) p <- 2 * p1 else p <- 2 * (1 - p1)
data.frame(parial.coefs45 = r45, t = t, df = t.df, p_value = p)
```

输出结果：

```
> data.frame(parial.coefs23 = r23, t = t, df = t.df, p_value = p)
  parial.coefs23 t df p_value
1 0.1270064 0.6528951 26 0.5195558
```

```
> data.frame(parial.coefs45 = r45, t = t, df = t.df, p_value = p)
  parial.coefs45 t df p_value
1 0.3915981 2.170076 26 0.03932272
```

主要结论：
消除 X_1 影响时，X_2, X_3 的偏相关系数为 0.1270064；
消除 X_1 影响时，X_4, X_5 的偏相关系数为 0.3915981。

3. 程序：

```
x <- read.table("ex9_3-data.txt",head=TRUE,row.names=1);
x <- scale(x);
ca <- cancor(x[,1:2], x[,3:4]); ca
```

输出结果：

```
> ca
$cor
[1] 0.7885079 0.0537397

$xcoef
       [,1] [,2]
X1 0.1127152 -0.2789099
X2 0.1064583 0.2813576

$ycoef
       [,1] [,2]
X3 0.1029701 -0.3610078
X4 0.1098775 0.3589657

$xcenter
         X1 X2
 1.243450e-16 -6.049328e-16

$ycenter
         X3 X4
-3.380629e-16 -1.359746e-15
```

主要结论：
两对典型变量为

$$\begin{cases} \hat{U}_1 = 0.1127152X_1 + 0.1064583X_2, \\ \hat{V}_1 = 0.1029701X_3 + 0.1098775X_4. \end{cases}, \begin{cases} \hat{U}_2 = -0.2789099X_1 + 0.3589657X_2, \\ \hat{V}_2 = -0.3610078X_3 + 0.3589657X_4. \end{cases}$$

4. 程序：

```
# 偏相关系数
ex9_4_partial_cor <- function(without, with, data) {
  cols <- sprintf("X%d", c(without, with));
  x <- cbind(data[,without], data[,with]);
  colnames(x) <- cols;
  rho <- cor(x);
  Mab <- det(rho[-1 * with[1], -1 * with[2]]);
  Maa <- det(rho[-1 * with[1], -1 * with[1]]);
  Mbb <- det(rho[-1 * with[2], -1 * with[2]]);
  r <- Mab / (sqrt(Maa) * sqrt(Mbb));
  return(r);
}
# 复相关系数
ex9_4_multiple_cor <- function(without, with, data) {
  cols <- sprintf("X%d", c(without, with));
  x <- cbind(data[, without], data[, with]);
  colnames(x) <- cols;
  rho <- cor(x);
  Maa <- det(rho[-1 * with[1], -1 * with[1]]);
  r <- sqrt(1 - det(rho) / Maa);
  return(r);
}
```

利用题 2 的前半问数据进行输出

```
data <- read.table("ex9_1-data.txt", header=T, row.names = 1);
source("ex9_4_partial_cor.R");
ex9_4_partial_cor(c(1), c(2,3), data);
source("ex9_4_multiple_cor.R");
ex9_4_multiple_cor(c(1), c(2,3), data);
```

输出结果:

```
> ex9_4_partial_cor(c(1), c(2,3), data);
[1] 0.1270064
> ex9_4_multiple_cor(c(1), c(2,3), data);
[1] 0.3588649
```

主要结论:

利用题 2 的前半问数据进行输出，故消除 X_1 影响时，X_2, X_3 的偏相关系数为 0.1270064，X_2, X_3 的复相关系数为 0.3588649。

5. 程序:

```
ex9_5 <- function(r, n, p, q, alpha = 0.1) {
```

```r
  m <- length(r);
  Q <- rep(0, m);
  lambda <- 1
  for (k in m:1) {
    lambda <- lambda * (1 - r[k] ^ 2);
    Q[k] <- -log(lambda)
  }
  s <- 0;
  i <- m
  for (k in 1:m) {
    Q[k] <- (n - k + 1 - 1 / 2 * (p + q + 3) + s) * Q[k]
    chi <- 1 - pchisq(Q[k], (p - k + 1) * (q - k + 1))
    if (chi > alpha) {
      i <- k - 1;
      break
    }
    s <- s + 1 / r[k] ^ 2
  }
  i
}
```

输出结果:

略。

主要结论:

略。