Answers to the Exercises for Digital Signal Processing

1.

(a) $\omega_0 = \dfrac{72}{73}$, $\dfrac{2\pi}{\omega_0} = \dfrac{73\pi}{36}$, the signal is aperiodic.

(b) $\omega_0 = \dfrac{\pi}{8}$, $\dfrac{2\pi}{\omega_0} = 16$, the signal is periodic and the period is 16.

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2. 136

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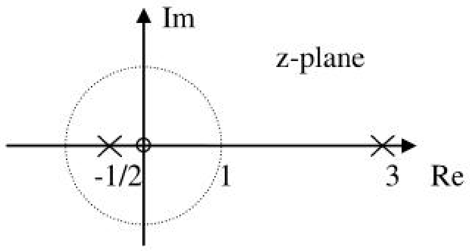
3. LPF

4.

$$H(z) = \frac{z^{-1}}{1 - \frac{5}{2}z^{-1} - \frac{3}{2}z^{-2}} = \frac{-\frac{2}{7}z}{z + \frac{1}{2}} + \frac{\frac{2}{7}z}{z - 3}.$$

(a) zeros: $z_1 = 0$, poles: $p_1 = -\dfrac{1}{2}$, $p_2 = 3$



(b) If the system is stable, ROC： $\dfrac{1}{2} < |z| < 3$，

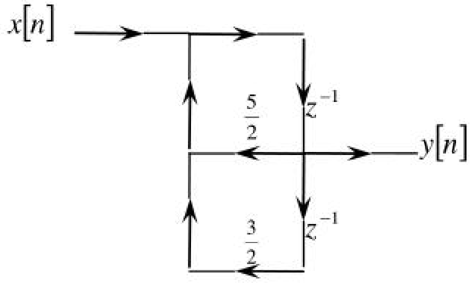
so the frequency response $H(e^{j\omega}) = H(z)\big|_{z=e^{j\omega}} = \dfrac{e^{-j\omega}}{1 - \frac{5}{2}e^{-j\omega} - \frac{3}{2}e^{-j2\omega}}$

the unit impulse response $h[n] = -\dfrac{2}{7}\left(-\dfrac{1}{2}\right)^n u[n] - \dfrac{2}{7}3^n u[-n-1]$

(c) If the system is causal, ROC： $|z| > 3$,

the unit impulse response $h[n] = \dfrac{2}{7}\left(3^n - \left(-\dfrac{1}{2}\right)^n\right)u[n]$

(d) Diagram of direct form 2 realization for this system.

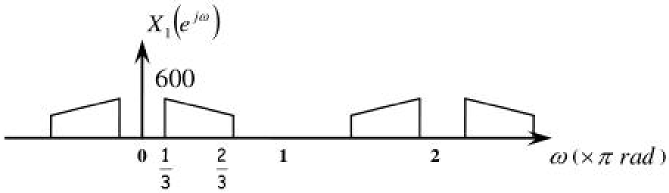


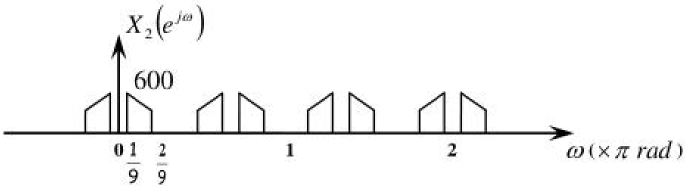
5.

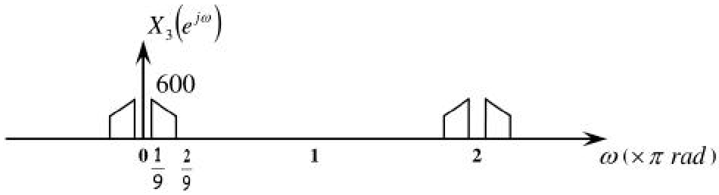
(a) $f_{s2}=f_{s1}$*3/2=900Hz

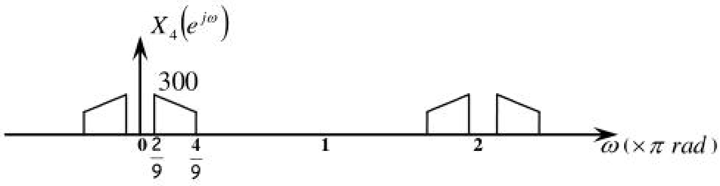
(b) $\omega_c = \dfrac{\pi}{3}$ rad ($\dfrac{2\pi}{9}$ ~ $\dfrac{4\pi}{9}$ rad)

(c) The spectra of sequence $x_1[n]$, $x_2[n]$, $x_3[n]$, $x_4[n]$.









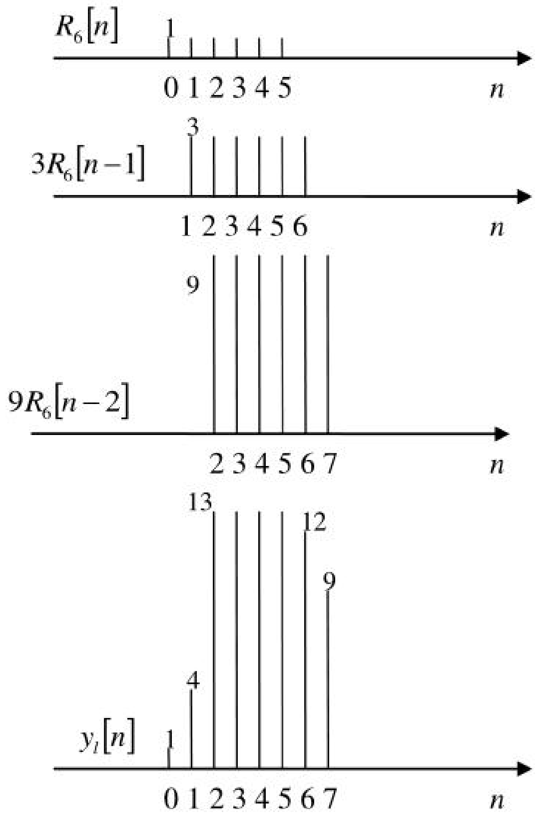
The amplitude and the unit of each figure.

6.

(a) The z.s. response $y_l[n]=x[n]*h[n]$

$$=3^n(u[n]-u[n-3])*R_6[n]=\sum_{m=-\infty}^{\infty}3^m(u[m]-u[m-3])R_6[n-m]$$

$$=\sum_{m=0}^{2}3^m R_6[n-m]=R_6[n]+3R_6[n-1]+9R_6[n-2]$$

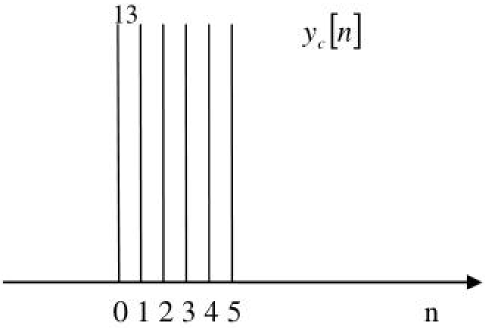


(b) If N=6, $Y_c(k)=X(k)H(k)$, k=0,1,….,5

$y_c[n]$ is 6-point circular convolution of $x[n]$ and $h[n]$, that is

$$y_c[n]=x[n] ⑥ h[n]$$

$$y_c[n]=y_l[[n]]_6 R_6[n]$$

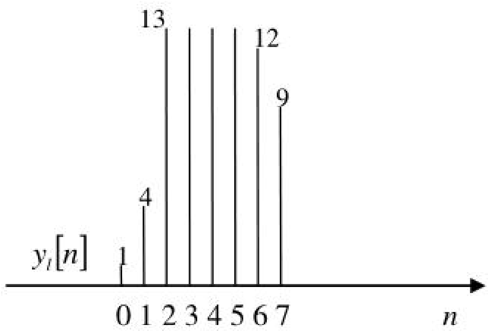


(c) If N=8, $Y_c(k)=X(k)H(k)$, k=0,1,…,7

$y_c[n]$ is 8-point circular convolution of $x[n]$ and $h[n]$, that is

$$y_c[n]=x[n] \circledast h[n]$$

$$y_c[n]=y_l[[n]]_8 R_8[n]=y_l[n]$$



7.

(a) $h_1[n]$ and $h_2[n]$ are both causal functions, so System A and System B are both causal.

(b) $y[n]=(x[n]*h_1[n]+x[n])*h_2[n]+x[n]=x[n]*h_1[n]*h_2[n]+x[n]*h_2[n]+x[n]$

$=(x[n]*h_1[n]+x[n])*h_2[n]+x[n]$

$=x[n]*(h_1[n]*h_2[n]+h_2[n]+\delta[n])$

So the unit impulse response of overall system:

$h[n]=h_1[n]*h_2[n]+h_2[n]+\delta[n]$

$$= 3\delta[n] + 4\delta[n-1] + 4\delta[n-2] + 2\delta[n-3]$$

(c) $\sum_{n=-\infty}^{\infty} |h[n]| = 13 < \infty$

The unit impulse response of overall system $h[n]$ is absolutely summable, so the overall system is stable.

8.

Consider these transfer functions for FIR filters shown as follows:

(i) $H_1(z) = 1 + 0.87z^{-1} + 1.1z^{-2} - 1.1z^{-4} - 0.87z^{-5} - z^{-6}$

$$h_1[n] = \{1, 0.87, 1.1, 0, -1.1, -0.87, -1\}$$

$h_1[n] = -h_1[6-n]$, the unit impulse response is odd symmetrical about $n = \frac{N-1}{2} = 3$, so this filter has a 2nd class linear phase characteristic, and its phase function

$$\theta(\omega) = -\frac{N-1}{2}\omega - \frac{\pi}{2} = -3\omega + \frac{\pi}{2} \qquad (\text{ or } -3\omega - \frac{\pi}{2})$$

(ii) $H_2(z) = 1 + 0.707z^{-2} + 0.54z^{-3} - 0.707z^{-4} - z^{-6}$

$$h_2[n] = \{1, 0, 0.707, 0.54, -0.707, 0, -1\}$$

The unit impulse response is not symmetrical about $n = \frac{N-1}{2} = 3$, so this filter has not linear phase characteristic

(iii) $H_3(z) = 1 + z^{-7}$

$$h_3[n] = \{1, 0, 0, 0, 0, 0, 0, 1\}$$

$h_3[n] = h_3[7-n]$, the unit impulse response is even symmetrical about $n = \frac{N-1}{2} = 3.5$, so this filter has a 1st class linear phase characteristic, and its phase function

$$\theta(\omega) = -\frac{N-1}{2}\omega = -3.5\omega$$

9.

(a) $H(z)=H_a(s)\Big|_{s=\frac{2(1-z^{-1})}{T(1+z^{-1})}}=\dfrac{1}{s^2+\sqrt{2}s+1}\Bigg|_{s=\frac{2(1-z^{-1})}{T(1+z^{-1})}}$

$$=\frac{1}{16\dfrac{(1-z^{-1})^2}{(1+z^{-1})^2}+4\sqrt{2}\dfrac{(1-z^{-1})}{(1+z^{-1})}+1}=\frac{z^2+2z+1}{(17+4\sqrt{2})z^2-30z+(17-4\sqrt{2})}$$

(b) the 3dB cutoff frequency for the digital LPF

$$\omega_c=2\tan^{-1}\left(\frac{\Omega_c T}{2}\right)=2\tan^{-1}\left(\frac{1}{4}\right)rad=0.49rad$$

10.

(a) The transition width $\Delta\omega$=0.15 π rad,

cut-off frequency $\omega_c=0.275\ \pi$ rad.

(b) The ideal frequency response

$$H(e^{j\omega})=\begin{cases} e^{j\frac{M}{2}\omega}, & |\omega|<0.275\pi \\ 0, & 0.275\pi<|\omega|<\pi \end{cases}$$

The ideal impulse response

$$h_d[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(e^{j\omega})e^{j\omega n}d\omega=\frac{\sin[0.275\pi(n-M/2)]}{\pi(n-M/2)},n=-\infty......\infty\ .$$

(c) $\alpha_s=60dB$, select the Blackman window.

According to Table 1, $M=\dfrac{11\pi}{0.15\pi}$=73.3$\rightarrow M=74$.

The length of the window is 75.

(d) $h[n]=h_d[n]w[n]=\dfrac{\sin[0.275\pi(n-37)]}{\pi(n-37)}\left(0.42-0.5\cos\left(\dfrac{\pi n}{37}\right)+0.08\cos\left(\dfrac{2\pi n}{37}\right)\right),n=0,1...,74$