# Frequency-domain Approach to LTI Systems

## 4.2 Frequency response of LTI systems

$x[n] = Ae^{j\Omega_0 n}$

$y[n] = \sum_{m=-\infty}^{+\infty} h[m]x[n-m] = \sum_{m=-\infty}^{+\infty} h[m] Ae^{j\Omega_0(n-m)} = \{A\sum_{m=-\infty}^{+\infty} h[m]e^{-j\Omega_0 m}\}e^{j\Omega_0 n} \triangleq Be^{j\Omega_0 n}$

$B = A\sum_{m=-\infty}^{+\infty} h[n]e^{-j\Omega_0 m}$

☆ $x[n] = Ae^{j\Omega_0 n} \longrightarrow y[n] = A\underbrace{H(e^{j\Omega_0})}_{\sum_{m=-\infty}^{+\infty} h(m)e^{-j\Omega_0 m}} e^{j\Omega_0 n}$

$x[n] = P_x\cos(\Omega_0 n+\phi_x) \longrightarrow y[n] = P_y\cos(\Omega_0 n+\phi_y)$

$P_y = P_x|H(e^{j\Omega_0})| \quad \phi_y = \phi_x + \phi(\Omega_0)$

☆ $x[n] = P_x\cos(\Omega_0 n+\phi_x) \longrightarrow y[n] = P_y\cos(\Omega_0 n+\phi_y)$

## 4.4 Frequency response of LTIs described LCCDES

$x(t) \xrightarrow{LTI} \boxed{h(t)} \longrightarrow y(t) \quad x[n] \to y[n]: \ y[n] + \sum_{k=1}^{N} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$

① $y(t) = x(t) * h(t)$

② $Y(j\omega) = X(j\omega)H(j\omega)$

$H(j\omega)$: ① LCCDE

② $x(t)$ $y(t)$ $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$

$\begin{cases} x[n-m] = e^{j\Omega(n-m)} = e^{j\Omega n}e^{-j\Omega m} \\ y[n-m] = H(e^{j\Omega})e^{j\Omega(n-m)} = H(e^{j\Omega})e^{j\Omega n}e^{-j\Omega m} \end{cases}$

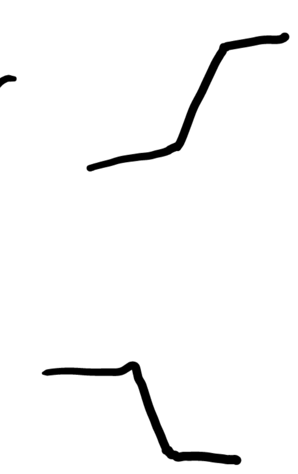
$H(e^{j\Omega}) = \frac{\sum_{m=0}^{M} b_m e^{-j\Omega m}}{1+\sum_{k=1}^{N} a_k e^{-j\Omega k}}$

$H(j\omega) = \frac{\sum_{k=0}^{M} \beta_k (j\omega)^{M-k}}{(j\omega)^N + \sum_{k=1}^{N} \alpha_k (j\omega)^{N-k}}$

$0.5^n u[n] = \frac{1}{1-\frac{1}{2}e^{-j\Omega}} \qquad (-0.5)^n u[n] = \frac{1}{1+\frac{1}{2}e^{-j\Omega}}$

## Bode Plot

$H(j\omega) = K\frac{1+j\frac{\omega}{\omega_1}}{1+j\frac{\omega}{\omega_2}}$



$\omega_1$小 $\omega_2$大

$\omega_1$大 $\omega_2$小

## 4.5 Frequency domain approach to system outputs

$\begin{cases} y[n] = \sum_{m=-\infty}^{+\infty} h[m]x[n-m] \\ y(t) = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)d\tau \end{cases} \Longleftrightarrow \begin{array}{l} Y(e^{j\Omega}) = X(e^{j\Omega})H(e^{j\Omega}) \\ Y(j\omega) = X(j\omega)H(j\omega) \end{array}$

$\Delta \begin{cases} y[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} H(e^{j\Omega})X(e^{j\Omega})e^{jn\Omega}d\Omega \\ y(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)H(j\omega)e^{j\omega t}d\omega \end{cases}$

## 4.3 Bode Plots for continuous-time LTI systems

$20\log_{10}\sqrt{2} = 3dB$

$H(j\omega) = K\frac{1+j\omega/\omega_1}{1+j\omega/\omega_2}$ with $K, \omega_1, \omega_2$ all constant

$20\log_{10}|H(j\omega)| = 20\log_{10}|K| + 20\log_{10}|1+j\omega/\omega_1| - 20\log_{10}|1+j\omega/\omega_2|$

$20\log_{10}|1+j\omega/\omega_k| \approx \begin{cases} 0, & 0 \leq \omega < |\omega_k| \\ 20\log_{10}\omega - 20\log_{10}|\omega_k|, & \omega \geq |\omega_k| \end{cases}$

20 dB -per-decade