Q1:

(a) If $x_1[n]=(n+1)u[n]$, $x_2[n]=u[n]-2u[n-1]+u[n-2]$, determine $x[n]=x_1[n]*x_2[n]$ and sketch $x[n]$ carefully.

(b) If $x_1(t)=e^{-2t}u(t)$, $x_2(t)=e^{-t}$, $-\infty<t<\infty$, and $x_1(t)*x(t)=x_2(t)$, determine $x(t)$ and sketch it carefully.

(Hint: use the derivative property of the convolution and $\frac{dx_1(t)}{dt}=-2e^{-2t}u(t)+\delta(t)$)

Q2: Three linear time-invariant discrete-time systems are connected as shown in Figure 1. System A has a unit impulse response $h_1[n]=3\delta[n+1]+2\delta[n-1]+5^{-n}u[n-3]$, System B has a unit impulse response $h_2[n]=-3\delta[n+1]+\delta[n-2]-5^{-n}u[n-3]$ and System C has a unit impulse response $h_3[n]=u[n]-u[n-4]$.

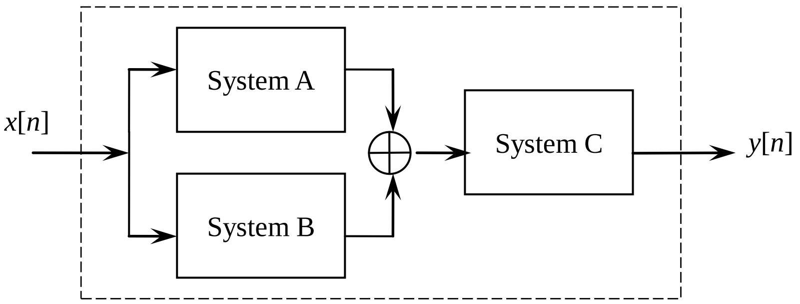

Figure 1

(a) Are all System A, System B and System C causal? Justify your answer.

(b) Determine the unit impulse response of the overall system.

(c) Is the overall system stable? Justify your answer.

Q3: Let $X_0(j\omega)$ be the Fourier transform of $x_0(t)$ depicted in Figure 2(a).

(a) Determine the Fourier Transform of $x_1(t)$ depicted by Figure 2(b) in terms of $X_0(j\omega)$.

(b) Figure 2(c) is a periodic signal $x(t)$ of period T with $x(t)$ satisfying

$$x(t)=x_0(t)+x_1(t),\ -T/2<t<T/2$$

Determine the Fourier Series coefficients of $x(t)$ in terms of $X_0(j\omega)$.

(c) Let $H(j\omega)=\begin{cases}\omega & |\omega|\le\dfrac{\pi}{T}\\ 0 & otherwise\end{cases}$ be the frequency response of an LTI system. Compute the output of the system when the input is $x(t)$ given in Part (b).

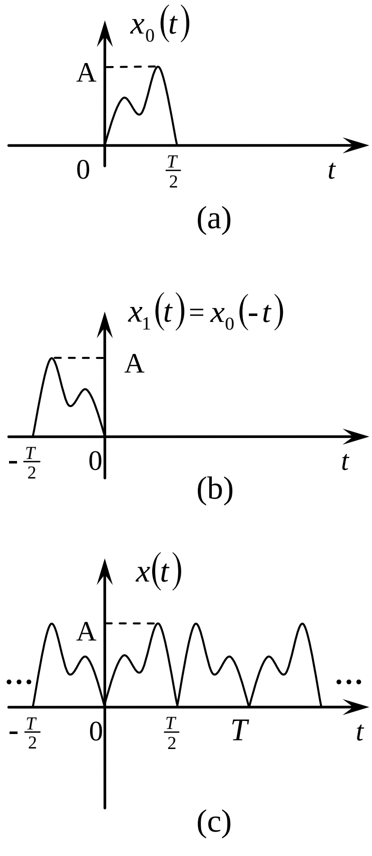

Figure 2

Q4: A causal LTI discrete system is characterized by the difference equation

$$y[n]-\frac{1}{6}y[n-1]-\frac{1}{6}y[n-2]=x[n-2]$$

(a) Determine the system function $H(z)$, draw the pole-zero plot and specify the corresponding region of convergence. Is it stable?

(b) Determine the unit impulse response $h[n]$.

(c) Determine the output $y[n]$ when the input signal is $x[n]=\cos\pi n,\ -\infty<n<+\infty$.

Q5: As shown in Figure 3, a continuous-time LTI system S is constructed by cascading two causal continuous-time LTI systems S_1 and S_2.

System S_1 has the differential equation

$$\frac{dw(t)}{dt}+3w(t)=x(t)$$

while System S_2 is characterized by

$$\frac{dy(t)}{dt}+7y(t)=\frac{dw(t)}{dt}+2w(t).$$

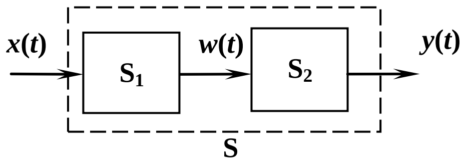

Figure 3

(a) Determine the transfer function $H(s)$ of the overall system S.

(b) Find out the differential equation of system S. If $y(0\text{-})=1$, $y'(0\text{-})=0$, and the excitation (input signal) $x(t)=2e^{-2t}u(t)$, determine system response, indicate the zero-input response and zero-state response of the system.

(c) With the overall system S obtained above, determine the transfer function between r(t) and y(t) of the system shown in Figure 4.

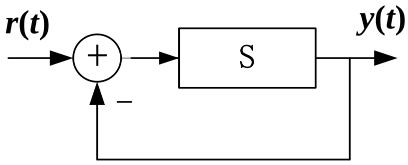

Figure 4

Is the system stable?