Chapter 6 STRUCTURES FOR DISCRETE-TIME SYSTEMS

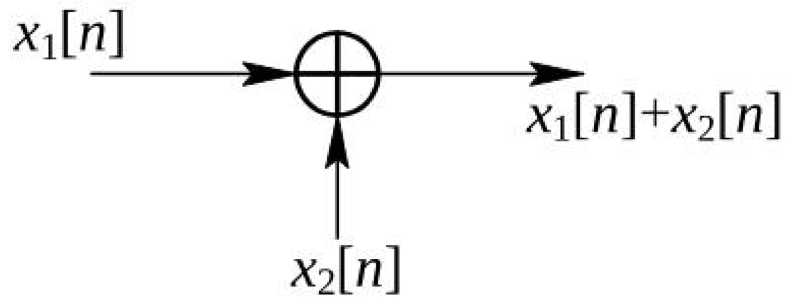
Main Topics

1. Signal flow graph representation of linear constant-coefficient difference equations
2. Basic structures for IIR system
3. Basic structures for FIR system

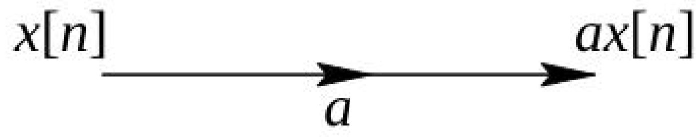
6.1 BLOCK DIAGRAM REPRESENTATION OF LINEAR CONSTANT-COEFFICIENT DIFFERENCE EQUATIONS

Basic elements

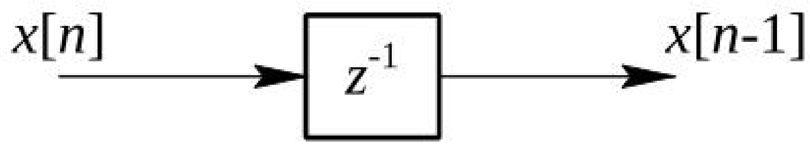
- Addition



- Multiplication by a constant



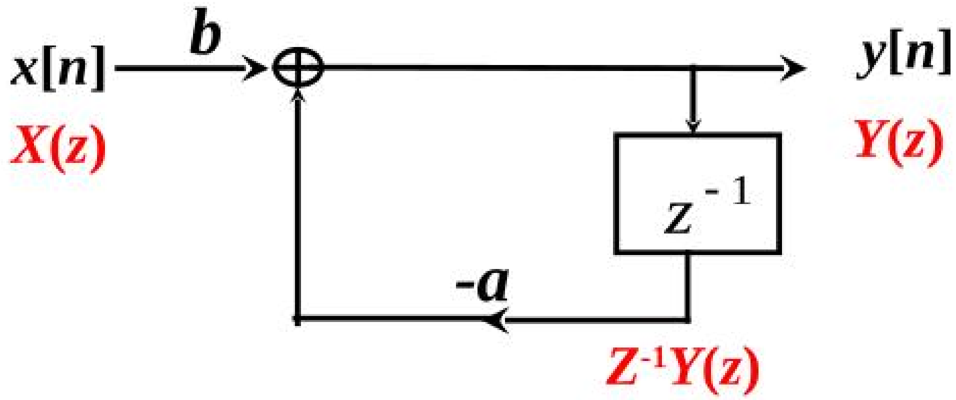
- Unit delay



e.g.

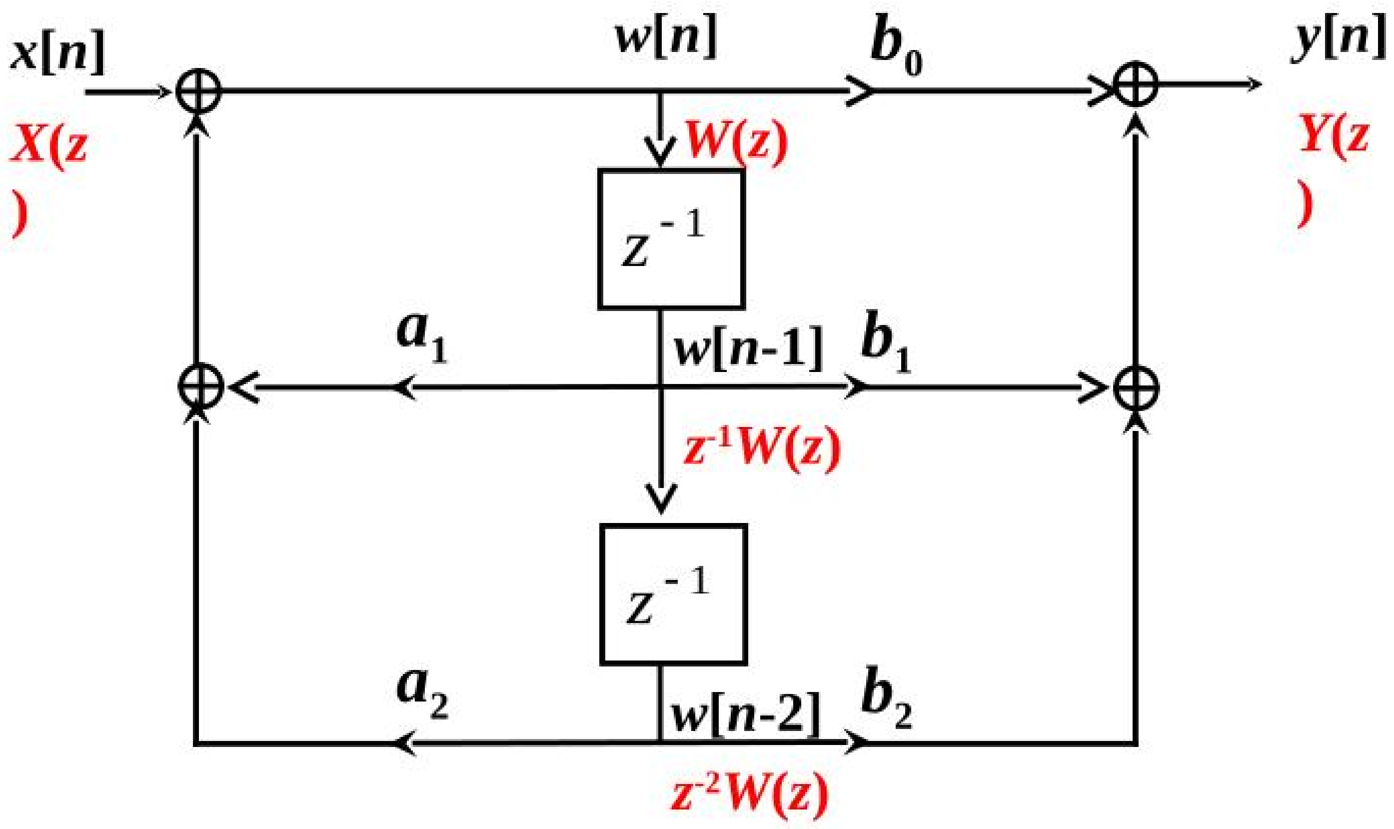
$$y[n]+ay[n-1]=bx[n]$$

addition delay multiplication



$$y[n]-a_1y[n-1]-a_2y[n-2]=b_0x[n]+b_1x[n-1]+b_2x[n-2]$$

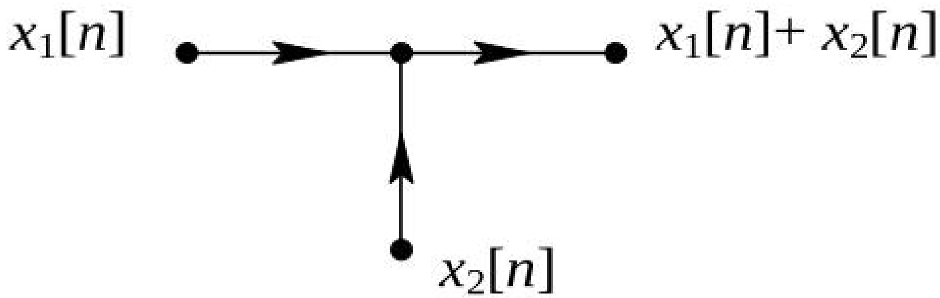
$$H(z)=\frac{b_0+b_1z^{-1}+b_2z^{-2}}{1-a_1z^{-1}-a_2z^{-2}}$$



6.2 SIGNAL FLOW GRAPH REPRESENTATION OF LINEAR CONSTANT-COEFFICIENT DIFFERENCE EQUATIONS

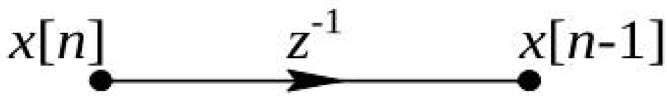
Basic elements

- Addition

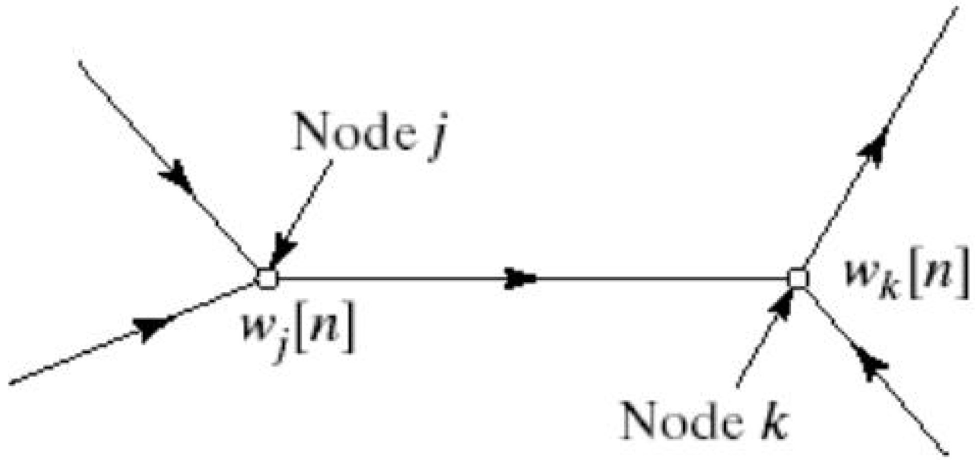


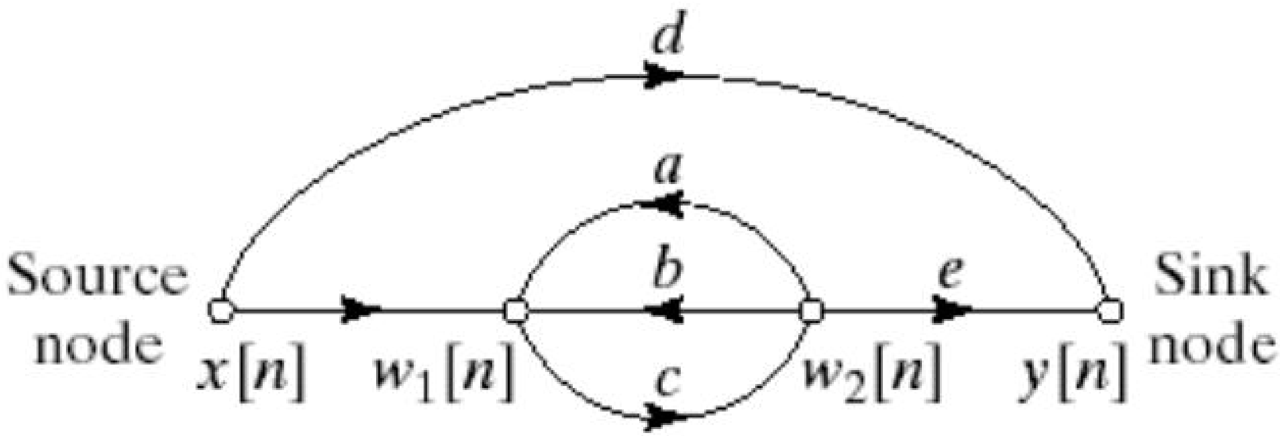
- Multiplication by a constant

- Unit delay



nodes and branches





node ： source 、 sink 、 network

branch ： constant 、 z^{-1} 、 1 、 -1

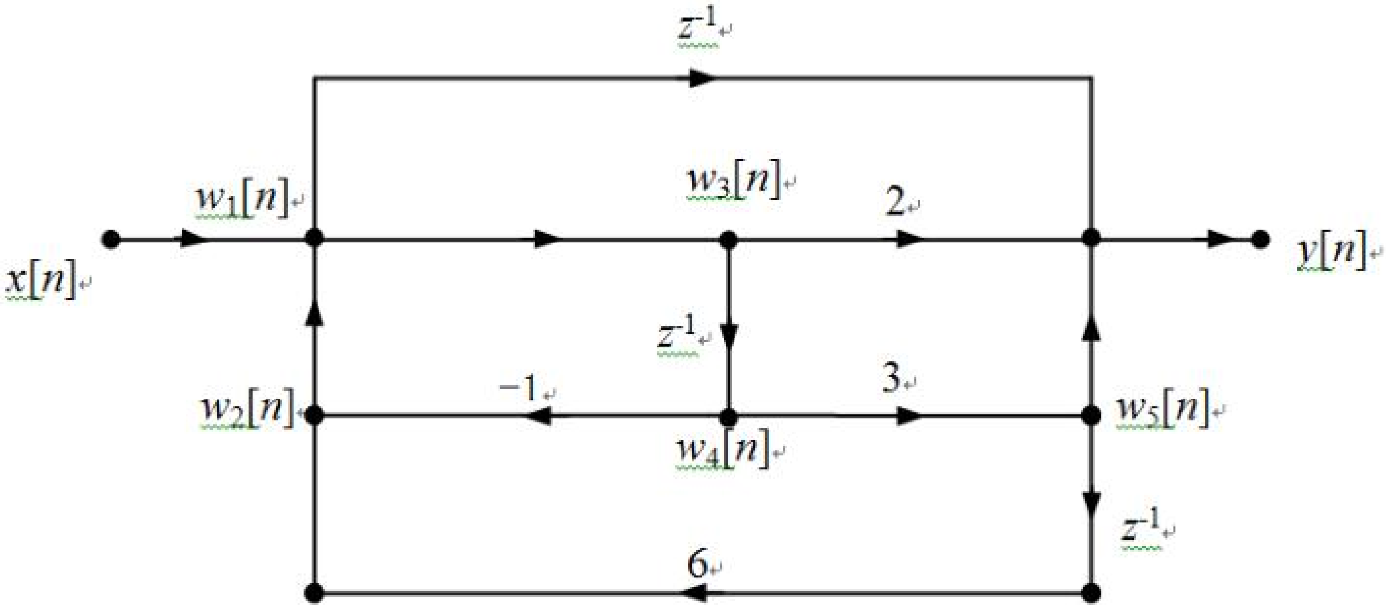
The value at each node in a graph is the sum of the outputs of all the branches entering the node.

$$w_1[n] = x[n] + aw_2[n] + bw_2[n]$$

$$w_2[n] = cw_1[n]$$

$$y[n] = ew_2[n] + dx[n]$$

e.g.



$$W_1(z) = X(z) + W_2(z)$$
$$W_2(z) = 6z^{-1}W_5(z) - W_4(z)$$
$$W_3(z) = W_1(z)$$
$$W_4(z) = z^{-1}W_3(z)$$
$$W_5(z) = 3W_4(z)$$
$$Y(z) = z^{-1}W_1(z) + 2W_3(z) + W_5(z)$$

$$Y(z) = X(z)\frac{(2+4z^{-1})}{1+z^{-1}-18z^{-2}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(2+4z^{-1})}{1+z^{-1}-18z^{-2}}$$

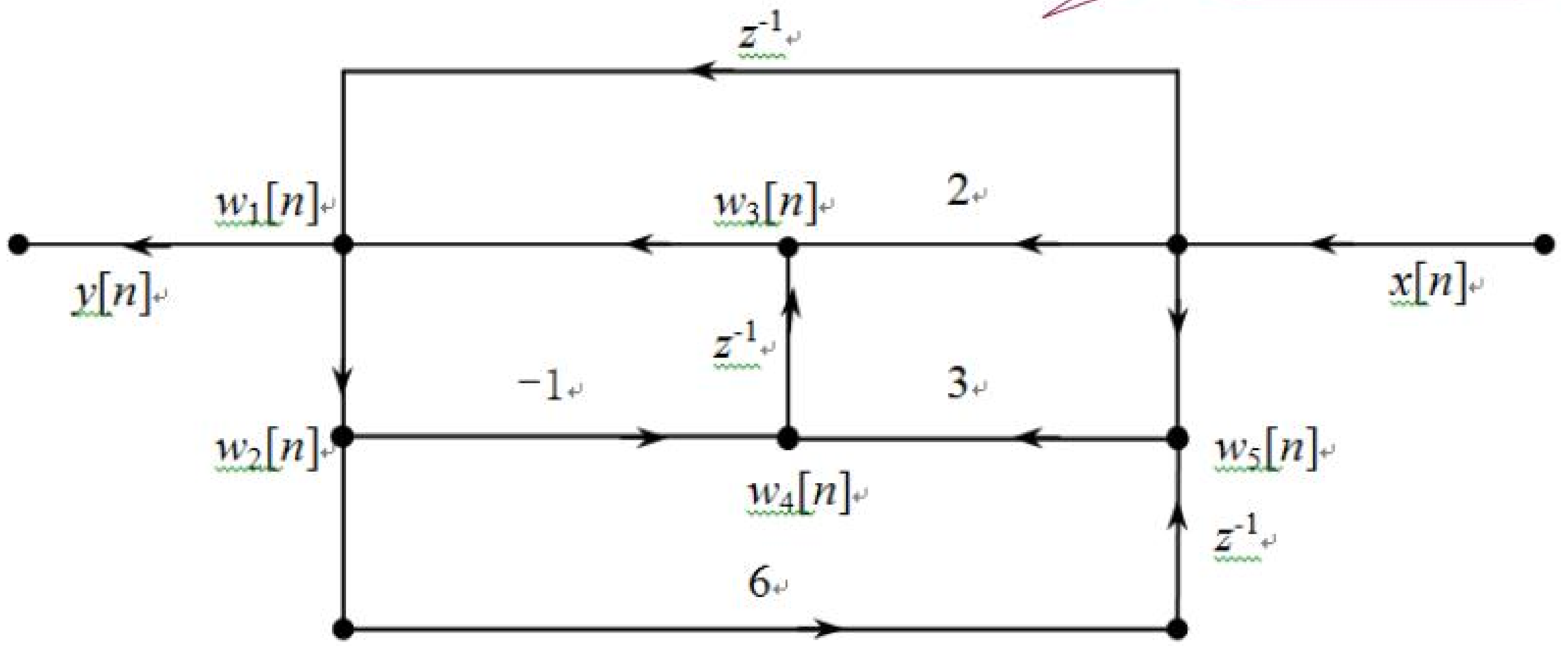
$$y[n] + y[n-1] - 18y[n-2] = 2x[n] + 4x[n-1]$$

Transpose： reverse the directions of all branches

reverse the roles of the input and output

The relationship between the input and output does not change.

Transposed form



6.3 BASIC STRUCTURES FOR IIR SYSTEMS

1. Direct Forms

Direct I

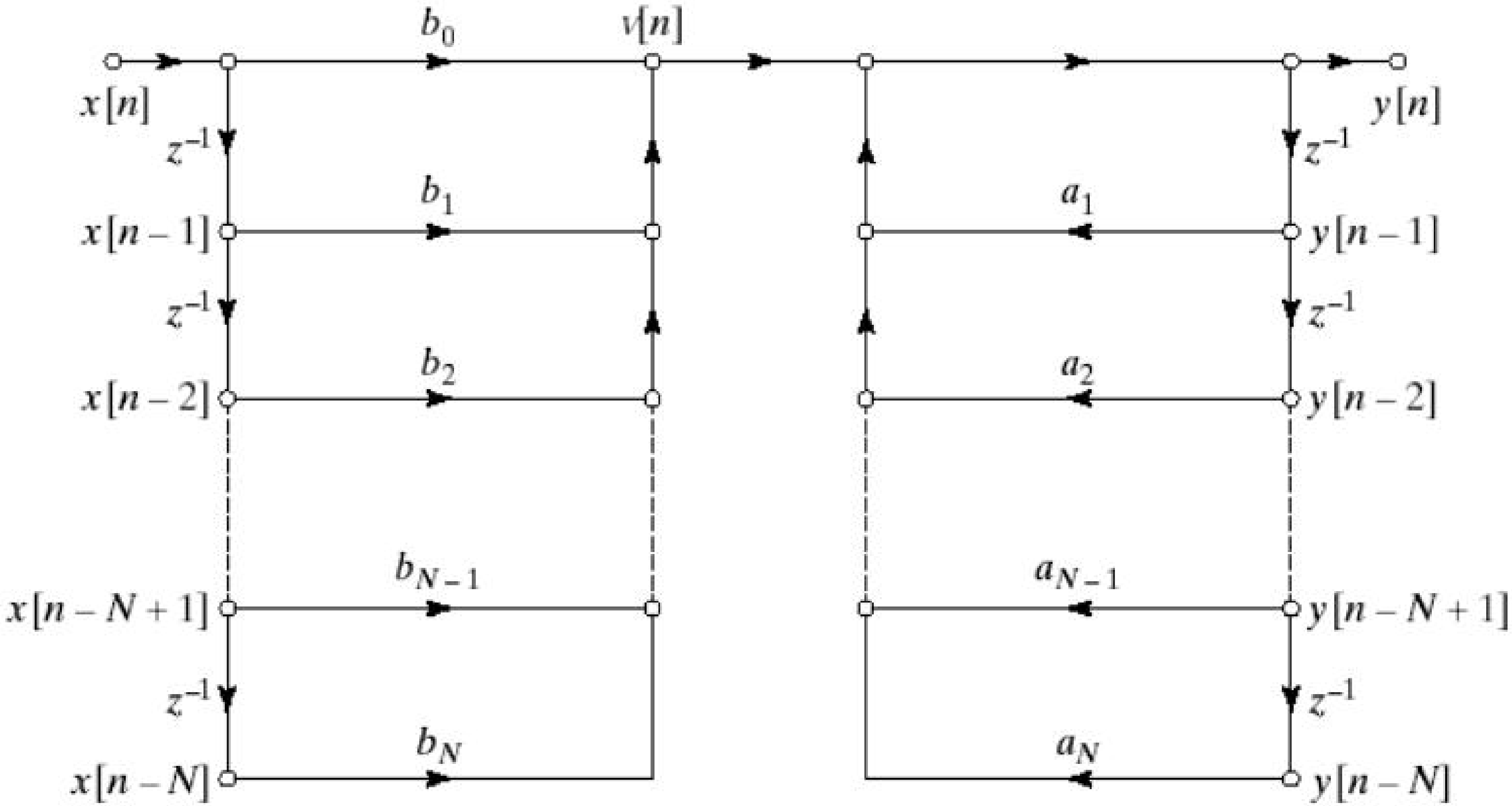
$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \sum_{k=0}^{M} b_k z^{-k} \frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}} = H_1(z) H_2(z)$$

$$H_1(z) = \frac{W(z)}{X(z)} = \sum_{k=0}^{M} b_k z^{-k} \quad , \quad H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

$$w[n] = \sum_{k=0}^{M} b_k x[n-k], \qquad y[n] = w[n] + \sum_{k=1}^{N} a_k y[n-k]$$

$$w[n] = \sum_{k=0}^{M} b_k x[n-k], \quad y[n] = w[n] + \sum_{k=1}^{N} a_k y[n-k]$$



Strongpoint ： simple ;

Shortcoming ： more delay ； be sensitive to word length ；
be inconvenient to adjust zeros and poles

Direct II （canonic direct form）

$$H(z)=\frac{\sum_{k=0}^{M} b_k z^{-k}}{1-\sum_{k=1}^{N} a_k z^{-k}}=H_1(z)H_2(z)=H_2(z)H_1(z)$$

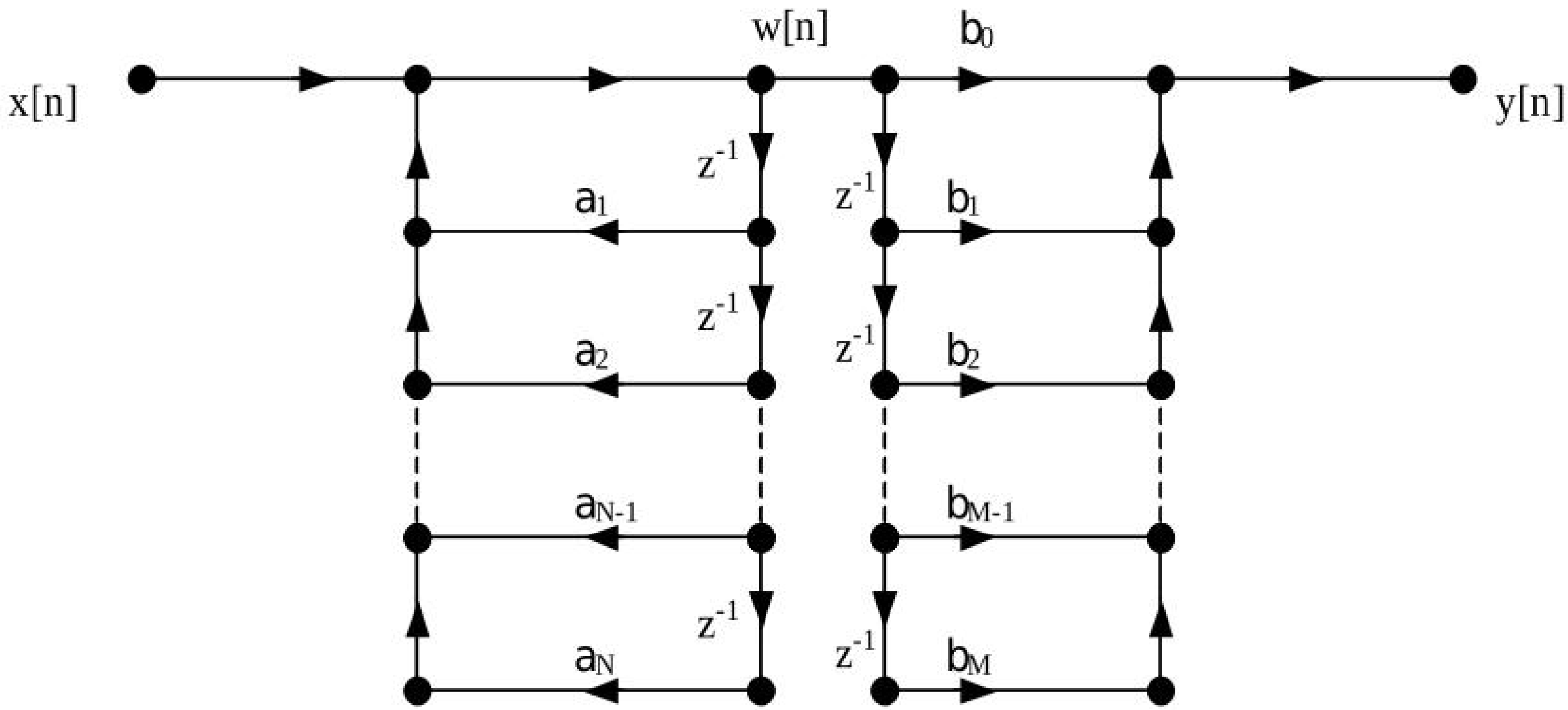
$$where: H_2(z)=\frac{W(z)}{X(z)}=\frac{1}{1-\sum_{k=1}^{N} a_k z^{-k}},$$

$$H_1(z)=\frac{Y(z)}{W(z)}=\sum_{k=0}^{M} b_k z^{-k}$$

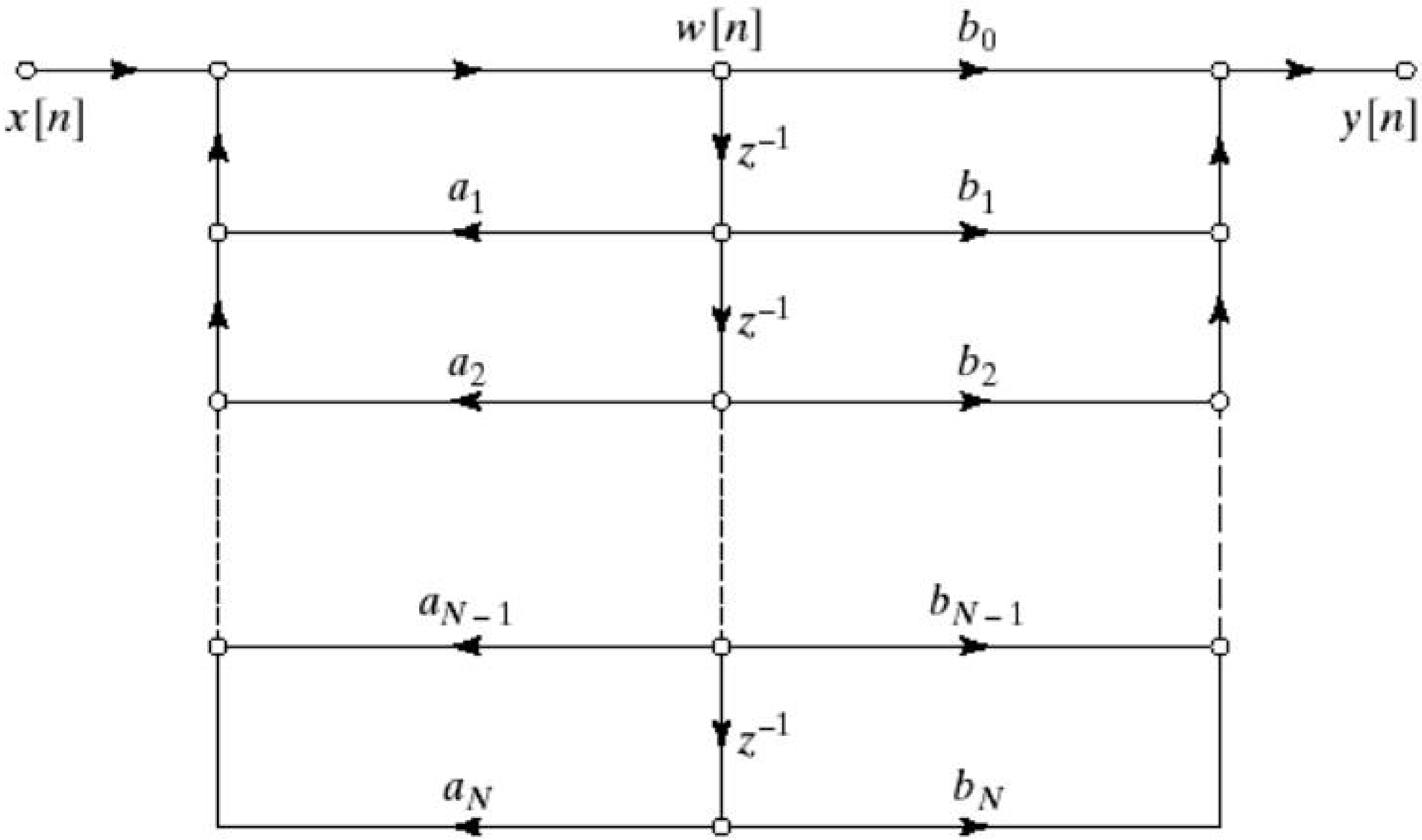
$$\therefore w[n]=x[n]+\sum_{k=1}^{N} a_k w[n-k]$$

$$y[n]=\sum_{k=0}^{M} b_k w[n-k]$$

$$w[n] = x[n] + \sum_{k=1}^{N} a_k w[n-k], \quad y[n] = \sum_{k=0}^{M} b_k w[n-k]$$



canonic direct form



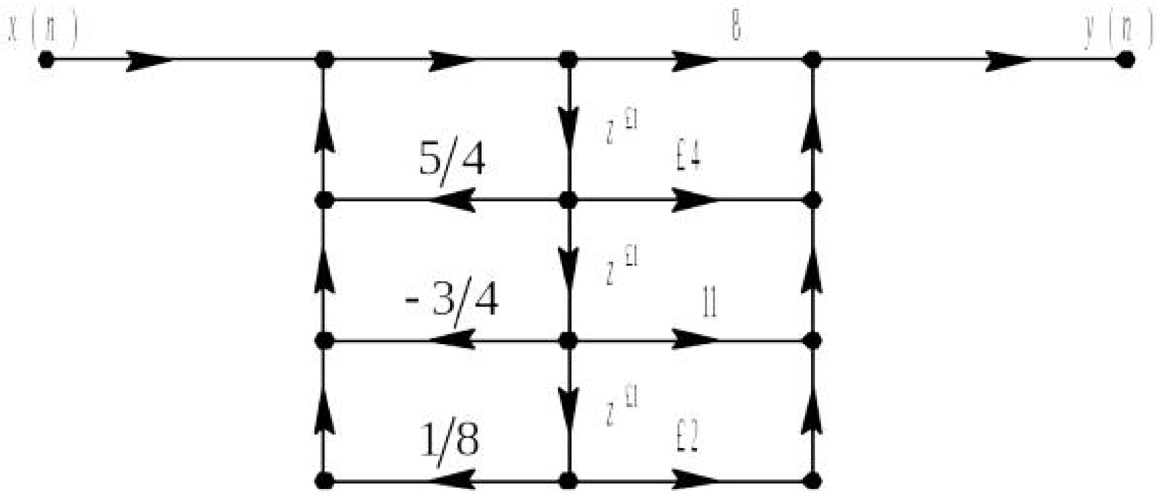
Strongpoint : delay is reduced half

e.g.

$$H(z)=\frac{8-4z^{-1}+11z^{-2}-2z^{-3}}{1-\frac{5}{4}z^{-1}+\frac{3}{4}z^{-2}-\frac{1}{8}z^{-3}}$$

draw the canonic direct form signal flow graph.

$$y[n]=\frac{5}{4}y[n-1]-\frac{3}{4}y[n-2]+\frac{1}{8}y[n-3]+8x[n]-4x[n-1]$$
$$+11x[n-2]-2x[n-3]$$



2. Cascade Forms

$$H(z)=\frac{\sum_{k=0}^{M} b_k z^{-k}}{1-\sum_{k=1}^{N} a_k z^{-k}}=\prod_{k=1}^{[MAX(M,N)+1]/2} H_k(z)=b_0\prod_{k=1}^{[MAX(M,N)+1]/2}\frac{1+\beta_{1k}z^{-1}+\beta_{2k}z^{-2}}{1-\alpha_{1k}z^{-1}-\alpha_{2k}z^{-2}}$$

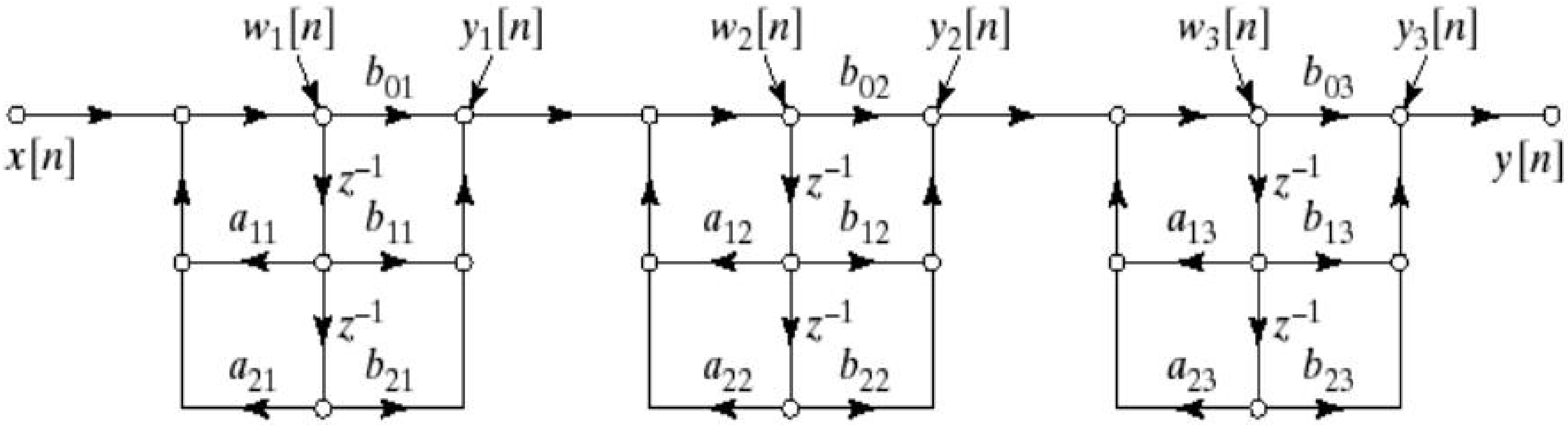
$$=\prod_{k=1}^{[MAX(M,N)+1]/2}\frac{b_{0k}+b_{1k}z^{-1}+b_{2k}z^{-2}}{1-\alpha_{1k}z^{-1}-\alpha_{2k}z^{-2}}$$

Reason of being nonuniform：

pairing manner of real zeros and poles；

order of cascade connection;

pairing manner of zeros and poles 。



Strongpoint：

Lower sensitivity to coefficient quantization than that of direct form;

Search the least-error ones because of the effects of limited word length;

Pairing manner of real zeros and poles；

Order of cascade connection;

Be convenient to adjust zeros and poles;

Time division multiplexing using a second order loop.

Shortcoming：

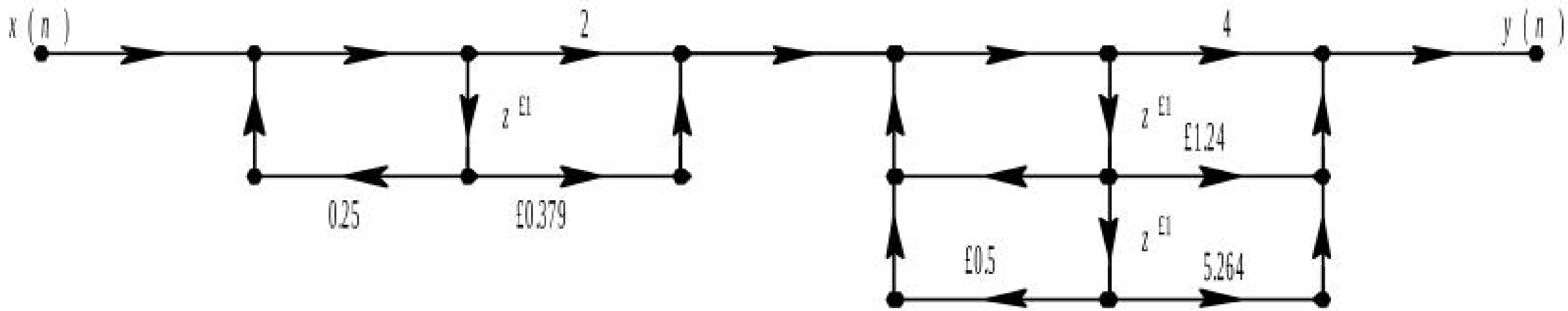
not as fast as parallel form

e.g.

$$H(z) = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - 1.25z^{-1} + 0.75z^{-2} - 0.125z^{-3}}$$

Draw the cascade form signal flow graph.

$$H(z) = \frac{(2 - 0.379z^{-1})(4 - 1.24z^{-1} + 5.264z^{-2})}{(1 - 0.25z^{-1})(1 - z^{-1} + 0.5z^{-2})}$$



3. Parallel Forms

$$H(z)=\frac{\sum_{k=0}^{M} b_k z^{-k}}{1-\sum_{k=1}^{N} a_k z^{-k}}=\sum_{k=0}^{M-N} C_k z^{-k}+\sum_{k=1}^{[\frac{N+1}{2}]}\frac{\gamma_{ok}+\gamma_{1k} z^{-1}}{1-\alpha_{1k} z^{-1}-\alpha_{2k} z^{-2}}$$

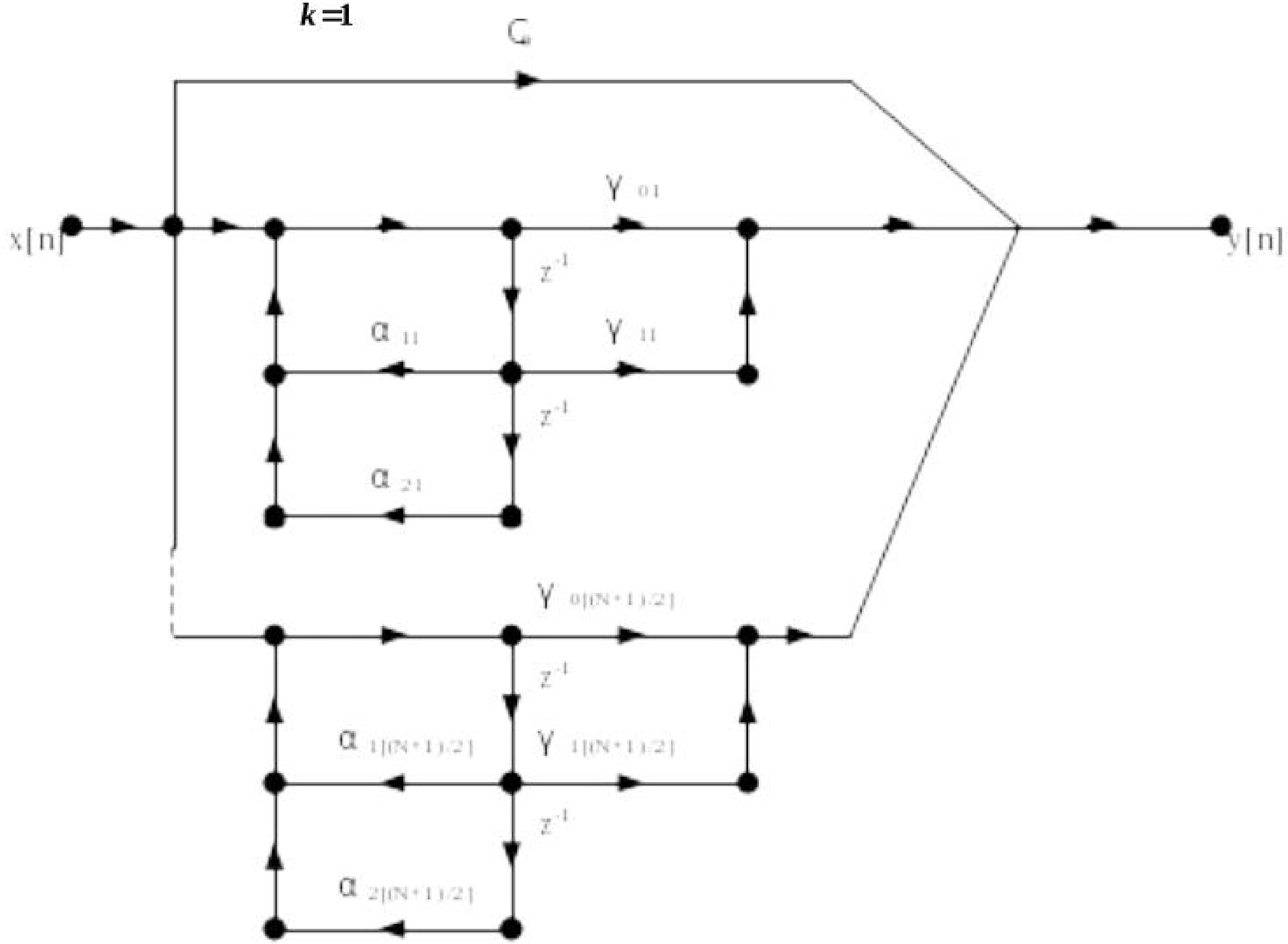
Strongpoint：

Lower sensitivity to coefficient quantization than that of direct form;
Less error because of the effects of limited word length;
Be convenient to adjust poles;
Fast hardware realization.

Shortcoming：

Can not adjust zeros;
Can not be used in the filters with high precision
Requirement of zero location, such as notch filter and narrowband bandstop filter.

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \sum_{k=0}^{M-N} C_k z^{-k} + \sum_{k=1}^{[\frac{N+1}{2}]} \frac{\gamma_{ok} + \gamma_{1k} z^{-1}}{1 - \alpha_{1k} z^{-1} - \alpha_{2k} z^{-2}}$$

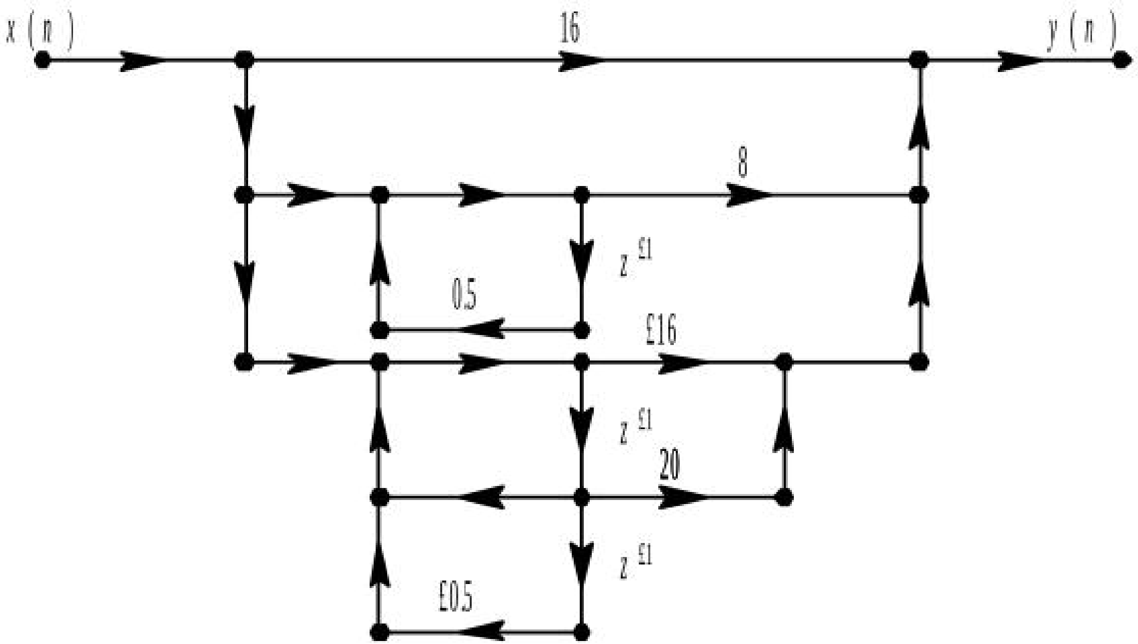


e.g.

$$H(z) = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - 1.25z^{-1} + 0.75z^{-2} - 0.125z^{-3}}$$

Draw the parallel form signal flow graph.

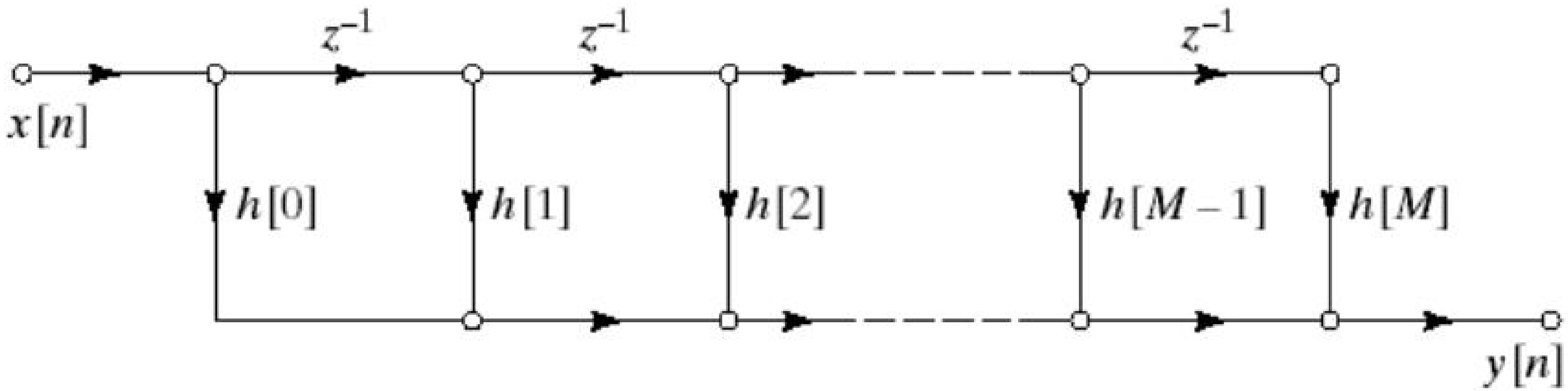
$$H(z) = 16 + \frac{8}{1 - 0.5z^{-1}} + \frac{-16 + 20z^{-1}}{1 - z^{-1} + 0.5z^{-2}}$$



6.3 BASIC STRUCTURES FOR FIR SYSTEM

1. Direct Forms

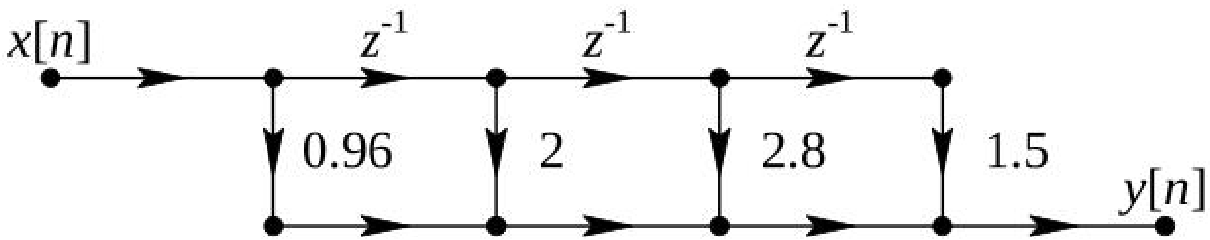
$$H(z) = \sum_{k=0}^{M} h[k] z^{-k}, \; y[n] = \sum_{k=0}^{M} h[k] x[n-k]$$



Transversal filter structure

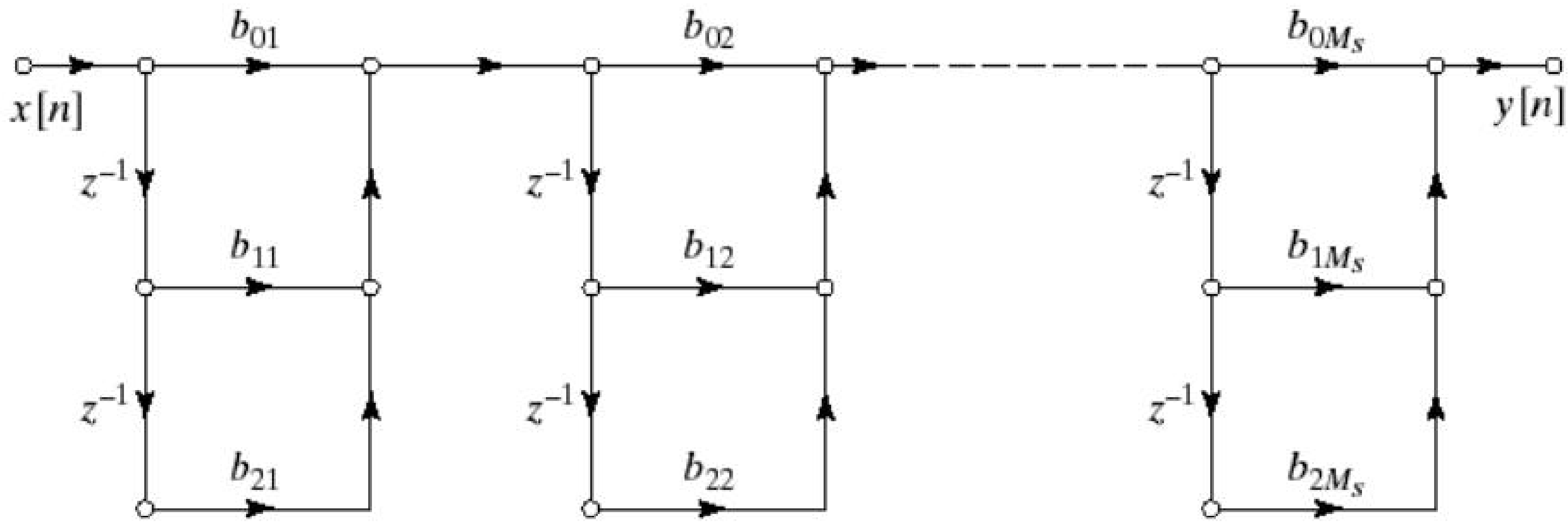
e.g. Draw the direct form signal flow graph.

$$H(z) = 0.96 + 2z^{-1} + 2.8z^{-2} + 1.5z^{-3}$$



2. Cascade Forms

$$H(z)=\sum_{k=0}^{M} h(k)z^{-k}=h[0]\prod_{k=1}^{[\frac{M+1}{2}]}\left(1+b_{1k}z^{-1}+b_{2k}z^{-2}\right)=\prod_{k=1}^{[\frac{M+1}{2}]}\left(b_{0k}'+b_{1k}'z^{-1}+b_{2k}'z^{-2}\right)$$



Strongpoint：

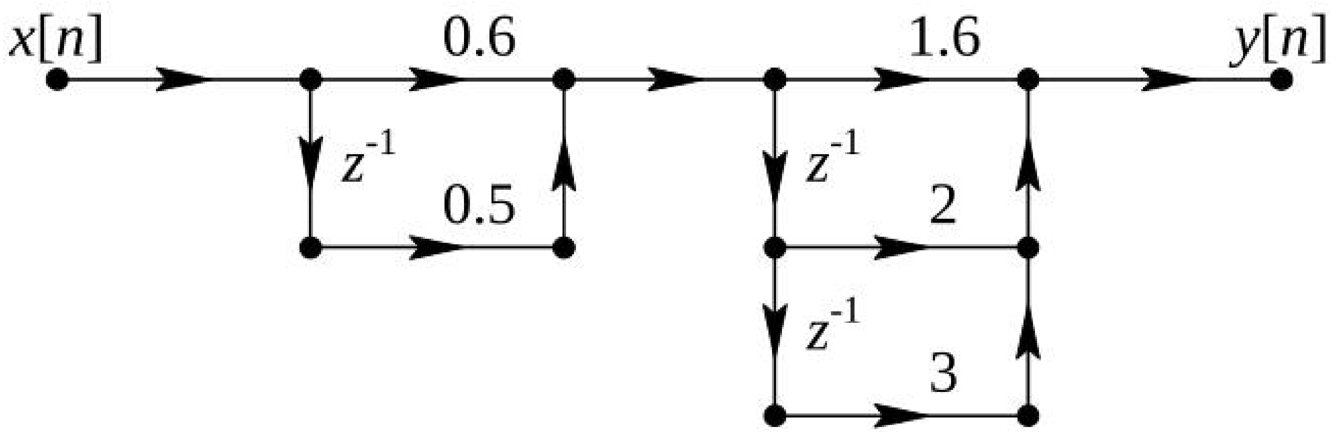
Be convenient to adjust zeros;

Time division multiplexing using a second order loop.

e.g. Draw the Cascade form signal flow graph.

$$H(z) = 0.96 + 2z^{-1} + 2.8z^{-2} + 1.5z^{-3}$$

$$H(z) = \left(0.6 + 0.5z^{-1}\right)\left(1.6 + 2z^{-1} + 3z^{-2}\right)$$



3. Structures For Linear-phase FIR System

(1) M is even

TYPE I ， III

$$y[n]=\sum_{k=0}^{M} h[k]x[n-k]$$

$$=\sum_{k=0}^{\frac{M}{2}-1} h[k]x[n-k]+h[\frac{M}{2}]x[n-\frac{M}{2}]+\sum_{k=\frac{M}{2}+1}^{M} h[k]x[n-k]$$

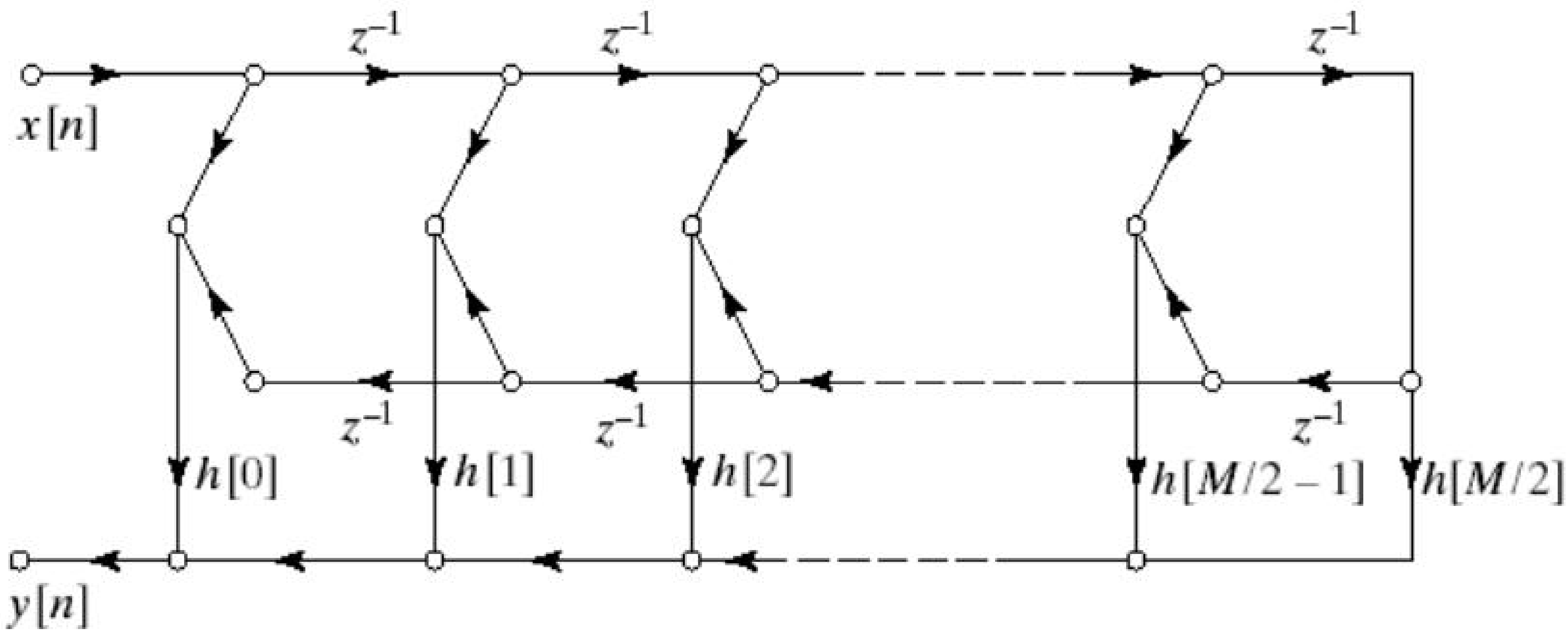
$$=\sum_{k=0}^{\frac{M}{2}-1} h[k]x[n-k]+h[\frac{M}{2}]x[n-\frac{M}{2}]+\sum_{k=0}^{\frac{M}{2}-1} h[M-k]x[n-M+k]$$

$$=\sum_{k=0}^{\frac{M}{2}-1} h[k](x[n-k]\pm x[n-M+k])+h[\frac{M}{2}]x[n-\frac{M}{2}]$$

(1) M is even

TYPE I ， III

$$y[n]=\sum_{k=0}^{\frac{M}{2}-1} h[k](x[n-k]\pm x[n-M+k])+h[\frac{M}{2}]x[n-\frac{M}{2}]$$



(2) M is odd

TYPE II , IV

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

$$= \sum_{k=0}^{\frac{M-1}{2}} h[k]x[n-k] + \sum_{k=\frac{M+1}{2}}^{M} h[k]x[n-k]$$

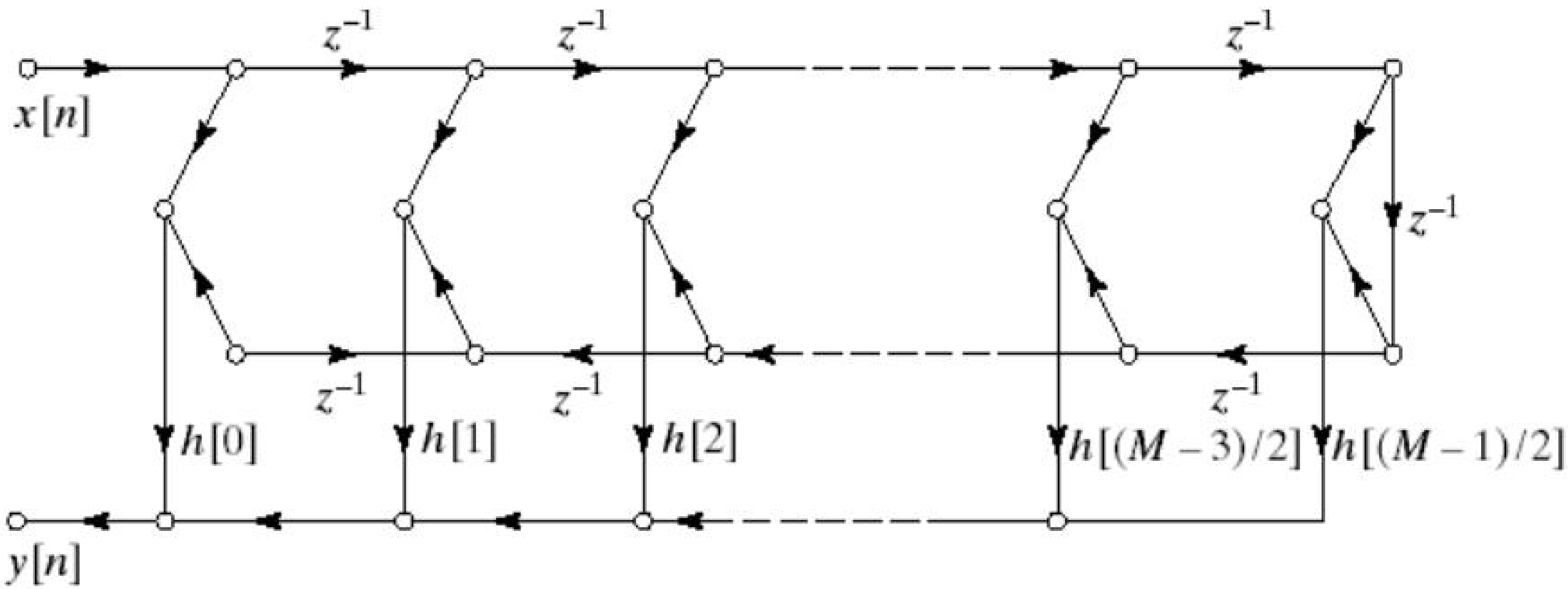
$$= \sum_{k=0}^{\frac{M-1}{2}} h[k]x[n-k] + \sum_{k=0}^{\frac{M-1}{2}} h[M-k]x[n-M+k]$$

$$= \sum_{k=0}^{\frac{M-1}{2}} h[k](x[n-k] \pm x[n-M+k])$$

(2) M is odd

TYPE II ， IV

$$y[n] = \sum_{k=0}^{\frac{M-1}{2}} h[k](x[n-k] \pm x[n-M+k])$$

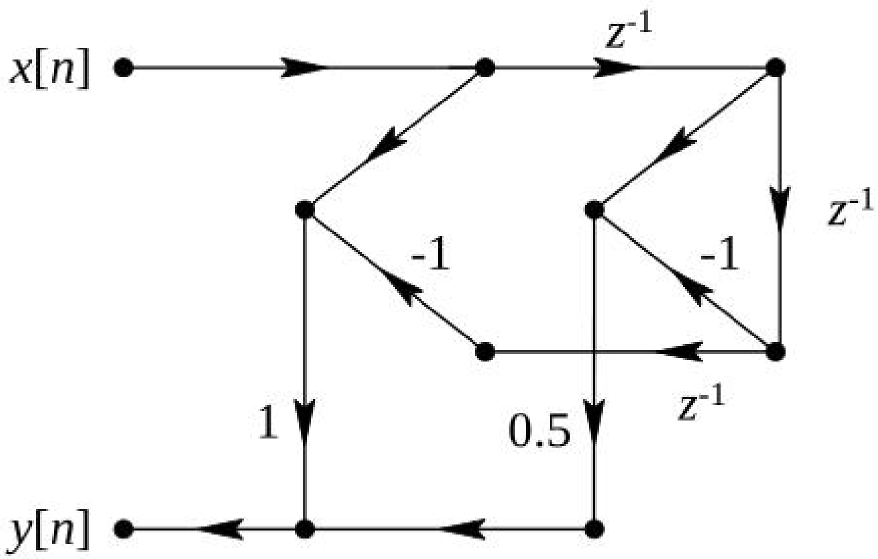


Strongpoint ： multiplication operation is reduced half

e.g. Draw the signal flow graph of linear-phase system.

$$H(z) = 1 + 0.5z^{-1} - 0.5z^{-2} - z^{-3}$$

linear phase type IV 。 $M=3$ ， odd

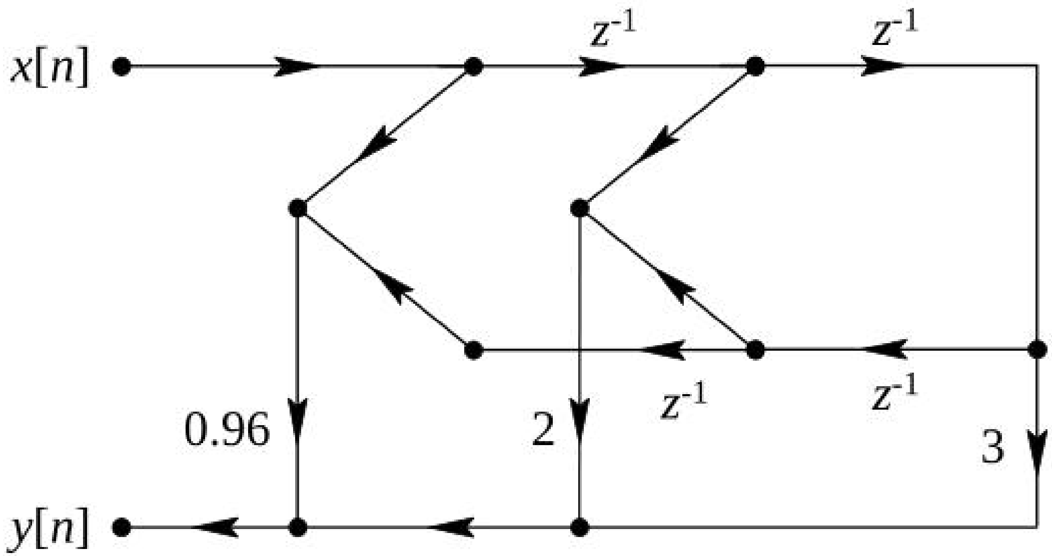


$$\theta(\omega) = -\frac{M}{2}\omega + \frac{\pi}{2} = -1.5\omega + \frac{\pi}{2}$$

e.g. Draw the signal flow graph of linear-phase system.

$$H(z) = 0.96 + 2z^{-1} + 3z^{-2} + 2z^{-3} + 0.96z^{-4}$$

linear phase type I 。 $M=4$ ， even



$$\theta(\omega) = -\frac{M}{2}\omega = -2\omega$$

6.4 effects of limited word length

Different infinite-precision realization structures ： the same result, different operation quantity, speed, storage space;
Different finite-precision realization structures ： different results, different error of frequency response, different difficulties to adjust frequency response.

Reasons of error：
1. coefficient quantity of filter's ： frequency response alters ， even is instable; the more dense zeros and poles are, the more sensitive to effects of limited word length
2. round in operations

High-order IIR should try to avoid using direct form;
FIR （generally, zeros distribute uniformly） use linear-phase direct form widely.

SUMMARY

1. Signal flow graph representation of linear constant-coefficient difference equations
2. Basic structures for IIR system
   - Direct forms
   - Cascade forms
   - Parallel forms
3. Basic structures for FIR system
   - Direct forms
   - Cascade forms
   - Structures for linear-phase FIR system

Exercises

6.6

6.7

6.10

6.15