Q1

Solution:

(a) Because $x_2[n]=u[n]-2u[n-1]+u[n-2]=\delta[n]-\delta[n-1]$

$$x[n]=x_1[n]*x_2[n]=(n+1)u[n]*(\delta[n]-\delta[n-1])$$
$$=(n+1)u[n]-nu[n-1]=u[n]$$

Figure of $x[n]$……

(b) $x_1'(t)*x(t)=x_2'(t)$

$$[-2e^{-2t}u(t)+\delta(t)]*x(t)=(-e)^{-t}$$

$$-2e^{-2t}u(t)*x(t)+x(t)=-e^{-t}$$
$$-2e^{-t}+x(t)=-e^{-t}$$

$$x(t)=e^{-t},\quad -\infty<t<\infty$$

Figure ……

Q2:

Solution

(a) $h_1[n]$ and $h_2[n]$ are not causal signals, so System A, System B are uncausal.

$h_3[n]$ is a causal signal, so System C is causal.

(b) the unit impulse response of the overall system $h[n]=(h_1[n]+h_2[n])*h_3[n]=$

$$2\delta[n-1]+3\delta[n-2]+3\delta[n-3]+3\delta[n-4]+\delta[n-5]$$

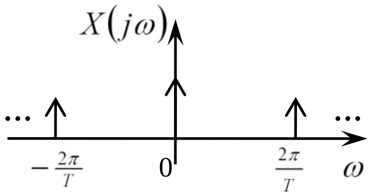
$\sum_{n=-\infty}^{\infty}|h[n]|=12<\infty$, $h[n]$ is absolutely summable, so the overall system is stable.

Q3:

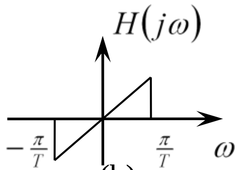
Solution

(a) The Fourier Transform of $x_1(t)$

$$X_1(j\omega) = X_0(-j\omega).$$



(a)



(b)

(b) $x(t) = x_0(t) + x_1(t),\ \ -T/2 < t < T/2$

the Fourier Series coefficients of $x(t)$

$$a_k = \frac{1}{T}\left(X_0\left(j\frac{2\pi k}{T} \right) + X_0\left(-j\frac{2\pi k}{T} \right) \right).$$

(c) $H(j\omega) = \begin{cases} \omega & |\omega| \le \dfrac{\pi}{T} \\ 0 & otherwise \end{cases}$ is the frequency response of an LTI system. The output of the system

$$Y(j\omega) = X(j\omega)H(j\omega) = 0.$$

Q4

Solution:

(a) Apply z-transform to the both sides of the equation:

$$Y(z) - \frac{1}{6}z^{-1}Y(z) - \frac{1}{6}z^{-2}Y(z) = z^{-2}X(z)$$

The system function

$$H(z) = \frac{z^{-2}}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}} = \frac{z^{-2}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{3}z^{-1})}$$

The zero is ∞. The poles are repeated at $\frac{1}{2},\ -\frac{1}{3}$

figure:…. of pole-zero plot……

Because the system is causal, the ROC is $|z| > \frac{1}{2}$

Yes, it is stable as ROC includes the unit circle.

(b) $H(z)=z^{-2}\left(\frac{3/5}{1-\frac{1}{2}z^{-1}}+\frac{2/5}{1+\frac{1}{3}z^{-1}}\right)$

$$h[n]=\frac{3}{5}\left(\frac{1}{2}\right)^{n-2}u[n-2]+\frac{2}{5}\left(-\frac{1}{3}\right)^{n-2}u[n-2]$$

(c) The frequency response of the system $H(e^{j\Omega})=\dfrac{e^{-2j\Omega}}{(1-\frac{1}{2}e^{-j\Omega})(1+\frac{1}{3}e^{-j\Omega})}$

the input signal is $x[n]=\cos\pi n,\ \infty-<n<+\infty$ with the frequency $\Omega=\pi$

$$H(e^{j\pi})=\frac{e^{-2j\pi}}{(1-\frac{1}{2}e^{-j\pi})(1+\frac{1}{3}e^{-j\pi})}=\frac{1}{(1+\frac{1}{2})(1-\frac{1}{3})}=1$$ So $y[n]=\cos\pi n,\ -\infty<n<+\infty$

Q5:

Solution

(a) the transfer function of the overall system S

$$H(s)=H_1(s)H_2(s)=\frac{1}{s+3}\frac{s+2}{s+7}=\frac{s+2}{s^2+10s+21}.$$

(b) the differential equation of system S

$$\frac{d^2y(t)}{dt^2}+10\frac{dy(t)}{dt}+21y(t)=\frac{dx(t)}{dt}+2x(t)$$

$$s^2Y(s)-sy(0-)-y'(0-)+10(sY(s)-y(0-))+21Y(s)=sX(s)-x(0-)+2X(s)$$

$$y(0-)=1,\quad y'(0-)\ =0,\quad X(s)=\frac{2}{s+2},\quad x(0-)=0$$

$$Y(s)=\frac{s+10}{s^2+10s+21}+\frac{2}{s^2+10s+21}=\frac{\frac{7}{4}}{s+3}+\frac{-\frac{3}{4}}{s+7}+\frac{\frac{1}{2}}{s+3}+\frac{-\frac{1}{2}}{s+7}$$

$$y(t)=\left(\frac{7}{4}e^{-3t}-\frac{3}{4}e^{-7t}\right)u(t)+\left(\frac{1}{2}e^{-3t}-\frac{1}{2}e^{-7t}\right)u(t)=\left(\frac{9}{4}e^{-3t}-\frac{5}{4}e^{-7t}\right)u(t)$$

$$y_{zi}(t)=\left(\frac{7}{4}e^{-3t}-\frac{3}{4}e^{-7t}\right)u(t),\quad y_{zs}(t)=\left(\frac{1}{2}e^{-3t}-\frac{1}{2}e^{-7t}\right)u(t)$$

(c) the transfer function between r(t) and y(t) of the system

$$x(t) = r(t) - y(t), \quad X(s) = R(s) - Y(s)$$

$$\begin{cases} Y(s) = H(s)X(s) \\ X(s) = R(s) - Y(s) \end{cases} \Rightarrow \frac{Y(s)}{R(s)} = \frac{H(s)}{1 + H(s)} = \frac{s+2}{s^2 + 11s + 23}.$$

the system is stable.