Discrete Processing of Analog. Signals

(模拟信号的离散化处理)

$x(t) \xleftrightarrow{FT} \tilde{X}(jw) = \int_{-\infty}^{+\infty} x(t) e^{-jwt} dt$

$\downarrow t = nT \qquad \downarrow ??$

$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\Omega n}$

$x[n] = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{X}(jw) e^{jwnT_s} dw$

采样频率 $w_k \triangleq \frac{2\pi k + \pi}{T_s}$, $k = \cdots, -2, -1, 0, 1, 2, \cdots$

$$\begin{cases} X(e^{j\Omega}) = \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} \tilde{X}\left(j\frac{\Omega + 2k\pi}{T_s}\right) \\ \left(w = \frac{\Omega}{T_s},\ w_s \triangleq \frac{2\pi}{T_s}\right) \\ X(e^{jT_s w}) = \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} \tilde{X}(j(w + kw_s)) \end{cases}$$

幅度 $\frac{1}{T_s}$，作 kw_s 的周期延拓.

采样定理 $w_s \triangleq \frac{2\pi}{T_s} > 2w_m \triangleq w_N$ (若要无失真恢复,必须大于2倍最高频率)

w_N: Nyquist rate

语音信号 3.4KHz → 2倍采样 6.8KHz 约 → 8KHz ⟹ 125μs 1帧

EXAMPLE.

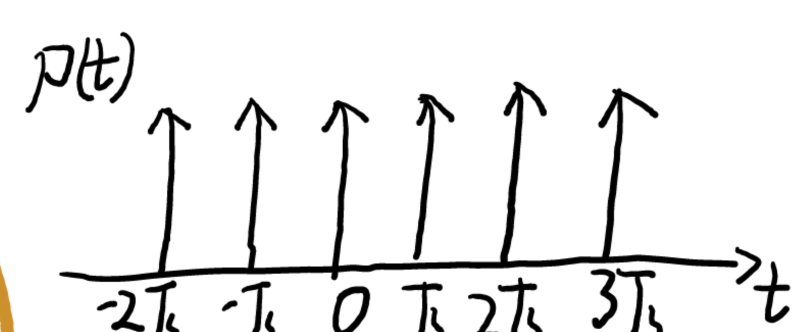
$x(t) \longrightarrow \otimes \longrightarrow x_p(t)$, with $p(t)$ into $\otimes$

$p(t) = \sum_{k=-\infty}^{+\infty} \delta(t + kT_s)$

$x(t) \downarrow X(jw)$ $\overset{?}{\longleftrightarrow}$ $x_p(t) \downarrow X_p(jw)$

$x_p(t) = x(t)p(t)$

$X_p(jw) = \frac{1}{2\pi} X(jw) * P(jw)$



$p(t) = \sum_{k=-\infty}^{+\infty} C[k] e^{jkw_s t}$, $w_s = \frac{2\pi}{T_s}$

$C[k] = \frac{1}{T_s} \int_{T_s} \delta(t) e^{-jkw_s t} dt = \frac{1}{T_s}$

$x(t) * \delta(t - t_0) = x(t - t_0)$

$P(jw) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{+\infty} \delta(w - kw_s)$

$p(t) = \sum_{k=-\infty}^{+\infty} \frac{1}{T_s} e^{jkw_s t}$ $\qquad 1 \leftrightarrow 2\pi\delta(w)$

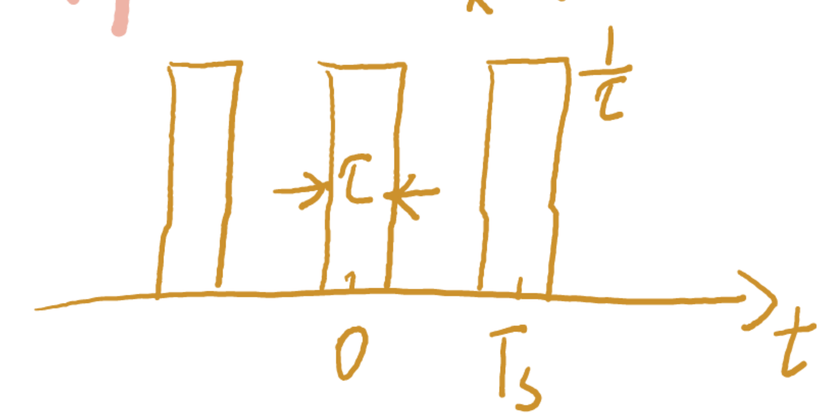
$\downarrow P(jw)$

$X_p(jw) = \frac{1}{2\pi} X(jw) * \frac{2\pi}{T_s} \sum_{k=-\infty}^{+\infty} \delta(w - kw_s)$

$= \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} X(j(w - kw_s))$

Ex 5.1

$p_\tau(t) = \sum_{k=-\infty}^{+\infty} \frac{1}{\tau} W_\tau(t - kT_s)$



$x(t) \longrightarrow \otimes \longrightarrow \tilde{x}(t) = x(t)p_\tau(t)$, with $p_\tau(t)$ into $\otimes$

求 $\tilde{X}(jw)$?

$$\begin{cases} \tilde{x}(t) = x(t)p_\tau(t) \\ \tilde{X}(jw) = \frac{1}{2\pi} X(jw) * P_\tau(jw) \end{cases}$$

$p_\tau(t) = \sum_{k=-\infty}^{+\infty} C[k] e^{jkw_s t}$, $w_s = \frac{2\pi}{T_s}$

方法① $C[k] = \frac{1}{T_s} \int_{T_s} p_\tau(t) e^{-jkw_s t} dt$

$= \frac{1}{T_s} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \frac{1}{\tau} e^{-jkw_s t} dt$

方法② $g(t) = \frac{1}{\tau} W_\tau(t)$

$p_\tau(t) = \sum_{k=-\infty}^{+\infty} g(t + kT_s)$

则 $C[k] = \frac{G(jkw_s)}{T_s}$ **(3.48, P112)**

$\boxed{W_\tau(t) \longleftrightarrow \tau \frac{\sin w\frac{\tau}{2}}{w\frac{\tau}{2}}}$

$G(jw) = \frac{1}{\tau} \boxed{\tau \frac{\sin w\frac{\tau}{2}}{w\frac{\tau}{2}}}$

$C[k] = \frac{1}{T_s} \frac{\sin kw\frac{\tau}{2}}{kw\frac{\tau}{2}}$