1.

Determine whether each of the discrete-time signals listed as follows is periodic or not. If it is periodic, find its period.

(a) $\frac{6}{5}\cos\left(\frac{72}{73}n\right)$

(b) $\frac{1}{5}e^{j\left(\frac{\pi n}{8}\right)}+1$

2.

The unit impulse response of a LTI system $h[n]$ has a length of 9, and the input of the system $x[n]$ has a length of 128. Then the zero-state response of the system should have a length of_________.

3.

A gain curve of a digital filter is shown in Figure 1. What type could it be, LPF, HPF, BPF, or BSF?___________

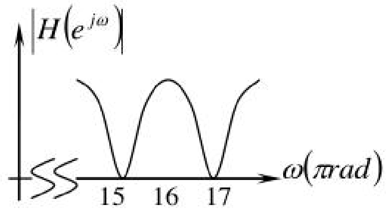

Figure 1

4.

Consider a discrete-time LTI system with transfer function:

$$H(z)=\frac{z^{-1}}{1-\frac{5}{2}z^{-1}-\frac{3}{2}z^{-2}}.$$

(a) Specify the poles and zeros of the system, and then sketch the pole-zero plots for the system on the z-plane.

(b) If the system is stable, determine the unit impulse response $h[n]$ and the frequency response of the system.

(c) What is the unit impulse response h[n] of the system if it is causal?

(d) Sketch the diagram of direct form 2 realization for this system.

5.

Suppose $x(t)$ is a continuous-time signal with the continuous-time Fourier transform $X(j\Omega)$ shown in Figure 2. We want to get a discrete-time sequence $x_4[n]$ from the system shown in Figure 3, where the gain of the Ideal Discrete-time LPF is 1 throughout the passband.

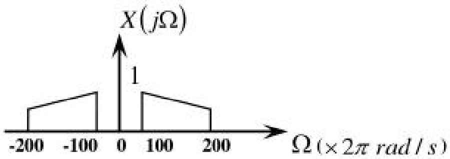

Figure 2

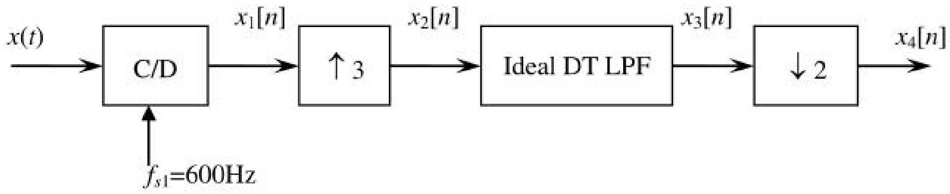

Figure 3

(a) The output signal $x_4[n]$ should be the sampling version of $x(t)$ at a new sampling frequency f_{s2}. Determine the value of f_{s2}.

(b) To avoid aliasing in f-domain and reverse mirror-image, determine the cut-off frequency of the ideal discrete-time LPF.

(c) Sketch the spectra of each sequence $x_1[n]$, $x_2[n]$, $x_3[n]$ and $x_4[n]$.

6.

A LTI system has a unit impulse response $h[n]=R_6[n]$, and the input of the system is $x[n]=3^n[u[n]-u[n-3]]$. Let $X(k)$ and $H(k)$ be the N-point DFT of $x[n]$ and $h[n]$ respectively, and given

$$y_c[n]=\text{IDFT}_N[\,X(k)H(k)\,],\quad n,k=0,12,3,\ldots,N\text{-}1$$

(a) Find the zero-state response of the system $y_l[n]$. Sketch and label it carefully.

(b) If N=6, determine $y_c[n]$, then sketch and label it carefully.

(c) If N=8, determine $y_c[n]$, then sketch and label it carefully.

7.

Two linear time-invariant discrete-time systems are connected as shown in Figure 4. System A has a unit impulse response $h_1[n]=\delta[n]+2\delta[n-1]$ and System B has a unit impulse response $h_2[n]=u[n]-u[n-3]$.

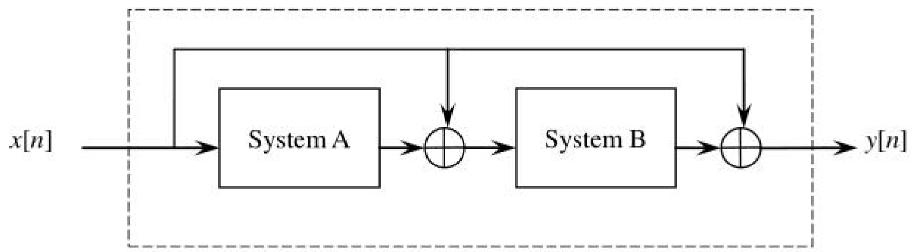

Figure 4

(d) Are both System A and System B causal? Justify your answer.

(e) Determine the unit impulse response of the overall system.

(f) Is the overall system stable? Justify your answer.

8.

Consider these transfer functions for FIR filters shown as follows:

(i) $H_1(z)=1+0.87z^{-1}+1.1z^{-2}-1.1z^{-4}-0.87z^{-5}-z^{-6}$

(ii) $H_2(z)=1+0.707z^{-2}+0.54z^{-3}-0.707z^{-4}-z^{-6}$

(iii) $H_3(z)=1+z^{-7}$

(a) Determine whether each filter has a linear phase characteristic or not, justify your answer.

(b) If the filter has a linear phase characteristic, find its phase function $\theta(\omega)$.

9.

The transfer function of a 2nd order analog Butterworth LPF is:

$$H_a(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

Let the sampling period T=0.5s, the cutoff frequency of the filter be $1 rad/\sec$, transform this analog filter to a digital filter by bilinear transformation method.

(a) Find the transfer function $H(z)$ for the digital LPF.

(b) Calculate the 3dB cutoff frequency for the digital LPF.

10.

Using windows method to design a FIR LPF with linear phase. Specifications are given as follows:

The pass band cut-off frequency ω_p=0.2π rad,

The stop band cut-off frequency ω_s=0.35π rad,

The minimum attenuation in stopband $\alpha_s = 60dB$,

The maximum attenuation in passband $\alpha_p = 3dB$,

(a) Find the transition width and cut-off frequency.

(b) Determine the frequency response $H_d\left(e^{j\omega}\right)$and impulse response $h_d[n]$ for the ideal filter.

(c) Choose the window type and length according to Table 1.

(d) Find the unit impulse response $h[n]$ for the FIR filter.

Table 1 Window function guidelines

Window Type	Window Function $0 \le n \le M$	Approximate Transition width	Peak Approximation Error，$\mathbf{20\log_{10}\delta}$ (dB)
Rectangular	1	$\frac{1.8\pi}{M}$	-21
Bartlett	$\begin{cases} \frac{2n}{M}, & 0 \le n \le \frac{M}{2} \\ 2-\frac{2n}{M}, & \frac{M}{2} \le n \le M \end{cases}$	$\frac{6.1\pi}{M}$	-25
Hanning	$0.5-0.5\cos\left(\frac{2\pi n}{M}\right)$	$\frac{6.2\pi}{M}$	-44
Hamming	$0.54-0.46\cos\left(\frac{2\pi n}{M}\right)$	$\frac{6.6\pi}{M}$	-53
Blackman	$0.42-0.5\cos\left(\frac{2\pi n}{M}\right)+0.08\cos\left(\frac{4\pi n}{M}\right)$	$\frac{11\pi}{M}$	-74