Final

2020年1月3日 11:00

一. Signals.

(1) the periodic signal T_0 N_0

FS: $\begin{cases} x(t)=\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}, & \omega_0=\frac{2\pi}{T_0} \\ a_k=\frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt \end{cases}$

$\sum_{m=-\infty}^{+\infty}\delta(t+mT_0)=\frac{1}{T_0}\sum_{k=-\infty}^{+\infty}e^{jk\frac{2\pi}{T_0}t}$

DFS: $\begin{cases} x[n]=\sum_{k=0}^{N_0-1} X_D[k]e^{jk\Omega_0 n} \\ X_D[k]=\frac{1}{N_0}\sum_{n=0}^{N_0-1}x[n]e^{-jk\Omega_0 n}, & \Omega_0=\frac{2\pi}{N_0} \end{cases}$

(2) nonperiodic signal

FT $\begin{cases} X(jw)=\int_{-\infty}^{+\infty}x(t)e^{-jwt}dt \\ x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(jw)e^{jwt}dw \end{cases}$

$e^{-\gamma t}u(t)\Longleftrightarrow\frac{1}{jw+\gamma}$, $Re\{\gamma\}>0$

$-e^{-\beta t}u(-t)\Longleftrightarrow\frac{1}{jw+\beta}$, $Re\{\beta\}>0$

$\delta(t)\Longleftrightarrow 1$

$1\Longleftrightarrow 2\pi\delta(w)$

☆ $\begin{cases} W_\tau(t)\Longleftrightarrow\tau\frac{\sin w\frac{\tau}{2}}{\frac{\tau}{2}} \\ \frac{\sin w_0 t}{\pi t}\Longleftrightarrow W_{2w_0}(jw) \end{cases}$

$u(t)\Longleftrightarrow\frac{1}{jw}+\pi\delta(w)$

$e^{-\alpha|t|}\Longleftrightarrow\frac{2\alpha}{w^2+\alpha^2}$

$\frac{1}{jw+\alpha}-\frac{1}{jw-\alpha}=\frac{jw-\alpha-(jw+\alpha)}{(jw+\alpha)(jw-\alpha)}=\frac{-2\alpha}{-w^2-\alpha^2}$

DTFT $\begin{cases} X(e^{j\Omega})=\sum_{n=-\infty}^{+\infty}x[n]e^{-j\Omega n} \\ x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\Omega})e^{j\Omega n}d\Omega \end{cases}$

$\gamma^n u[n]\Longleftrightarrow\frac{1}{1-\gamma e^{-j\Omega}}$, $|\gamma|<1$

$-\gamma^n u[-n-1]\Longleftrightarrow\frac{1}{1-\gamma e^{-j\Omega}}$, $|\gamma|>1$

$\delta[n]\Longleftrightarrow 1$

$g(t)=G(jw)$

$x(t)=\sum_{k=-\infty}^{+\infty}g(t+kT_0)$ 周期延拓后

$x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jkw_0 t}$, $w_0=\frac{2\pi}{T_0}$

(FT

$X(jw)=2\pi\sum_{k=-\infty}^{+\infty}a_k\delta(w-kw_0)$

$a_k=\frac{1}{T_0}G(jw)|_{w=kw_0}$

(3) laplace transform

$X(s)=\int_{-\infty}^{+\infty}x(t)e^{-st}dt$

ROC $X(jw)=X(s)|_{s=jw}$

$e^{\alpha t}u(t)\Longleftrightarrow\frac{1}{s-\alpha}$, $Re\{s\}>Re\{\alpha\}$

$-e^{\beta t}u(-t)\Longleftrightarrow\frac{1}{s-\beta}$, $Re\{s\}<Re\{\beta\}$

$te^{\alpha t}u(t)\Longleftrightarrow\frac{1}{(s-\alpha)^2}$, $Re\{s\}>Re\{\alpha\}$

$u(t)\Longleftrightarrow\frac{1}{s}$

Z transform

$X(z)=\sum_{n=-\infty}^{+\infty}x[n]z^{-n}$

ROC $X(e^{j\Omega})=X(z)|_{z=e^{j\Omega}}$

$\alpha^n u[n]\Longleftrightarrow\frac{1}{1-\alpha z^{-1}}$, $|z|>|\alpha|$

$-\alpha^n u[-n-1]\Longleftrightarrow\frac{1}{1-\alpha z^{-1}}$, $|z|<|\alpha|$

$n\alpha^n u[n]\Longleftrightarrow\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$, $|z|>|\alpha|$

$u[n]\Longleftrightarrow\frac{1}{1-z^{-1}}$

(4) unilateral

$X(s)=\int_{0_-}^{+\infty}x(t)e^{-st}dt$

☆ $\begin{cases} \frac{dx(t)}{dt}\Longleftrightarrow sX(s)-x(0_-) \\ \frac{d^2x(t)}{dt^2}\Longleftrightarrow s[sX(s)-x(0_-)]-\frac{dx(t)}{dt}|_{t=0_-} \end{cases}$

$X(z)=\sum_{n=0_-}^{+\infty}x[n]z^{-n}$

$\begin{cases} x[n-1]\Longleftrightarrow z^{-1}X(z)+x[-1] \\ x[n+1]\Longleftrightarrow zX(z)-zx[0] \end{cases}$

二. Systems

(1)

$h(t)$, $h[n]$: unit impulse response

$H(jw)$, $H(e^{j\Omega})$: frequency response

$H(s)$, $H(z)$: system function.

☆ $\begin{cases} y(t)=x(t)*h(t) \\ y[n]=x[n]*h[n] \end{cases}$

$x[n-n_0]*\delta[n-n_1]=x[n-n_0-n_1]$

$Y(jw)=H(jw)X(jw)$

$Y(s)=H(s)X(s)$ LCCDE.

(2) properties...

(3) the complete response

= Zero-input response $y_{zi}(t)$

+ Zero-state response $y_{zs}(t)$

$x(t)$

$y(t)=y_{zs}(t)+y_{zi}(t)$

$\tilde{x}(t)=Ax(t-t_0)$ 初始状态不变.

$y(t)=y_{zi}(t)+Ay_{zs}(t-t_0)$

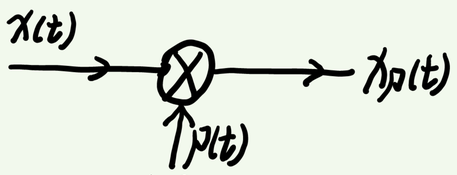
Bode plot 近似条件

structure

三. Sampling. 采样

Band-limited signal W_m

$W_s\geq 2W_m$



$p(t)=\sum_{k=-\infty}^{+\infty}\delta(t+kT_0)$

$x_p(t)=x(t)p(t)$

$X_p(jw)=\frac{1}{T_0}\sum_{k=-\infty}^{+\infty}X(j(w+kw_s))$, $w_s=\frac{2\pi}{T_0}$