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Time-domain Analysis of LTI Systems

Unit Impulse

decomposion (分解)

Unit impulse response (单位冲激响应)

$x[n] \rightarrow y[n]$ excited by $\delta[n]$

$\delta[n] \rightarrow \delta[n]$ no matter LTI or not

$t\delta(t)=0$

different system may have the same impulse

1. $y[n]=x[n-n_0] \Longrightarrow h[n]=\delta[n-n_0]$
2. $y[n]=\sum_{k=-\infty}^{n} x[k] \Longrightarrow h[n]=\sum_{k=-\infty}^{n}\delta[k]=u[n]$
3. $y[n]=\sum_{k=-\infty}^{n} x^2[k] \Longrightarrow h[n]=\sum_{k=-\infty}^{n}\delta^2[k]=u[n]$
4. $y(t)=\int_{-\infty}^{t} x(\tau)d\tau \Longrightarrow h(t)=\int_{-\infty}^{t}\delta(\tau)\,d\tau=u(t)$
5. $y(t)=\int_{-\infty}^{t} x(\tau)d\tau+t\,x(t) \Longrightarrow h(t)=\int_{-\infty}^{t}\delta(\tau)d\tau+\underbrace{t\delta(t)}_{0}=u(t)$

The convolution sum (卷积)

$$\begin{cases} y[n]=\sum_{k=-\infty}^{+\infty} x[k]h[n-k] \triangleq x[n]*h[n] \\ y(t)=\int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau \triangleq x(t)*h(t) \end{cases}$$

proof: (证明)

cascade (串联)

$x[n]\triangleq \alpha_1 x_1[n]+\alpha_2 x_2[n]$ $\quad$ $\tilde{x}[n]\triangleq x[n-n_0]$

$\hookrightarrow y[n]=\alpha_1 y_1[n]+\alpha_2 y_2[n]$ $\quad$ $\hookrightarrow \tilde{y}[n]=y[n-n_0]$

$x[n]=\delta[n] \longrightarrow y[n]=\delta[n]*h[n]=h[n]$

并联相加

串联卷积

Properties

$$\begin{cases} \delta[n-n_0]*w[n]=w[n-n_0],\ \forall n_0\in\mathbb{Z}. \\ \delta(t-t_0)*w(t)=w(t-t_0),\ \forall t_0\in\mathbb{R} \end{cases}$$

① Commutativity $\quad y=x*h=h*x$

② Distributivity $\quad y=x*(h_1+h_2)=x*h_1+x*h_2$

③ Associativity $\quad y=x*\{h_1*h_2\}=\{x*h_1\}*h_2$

(commutativity) $\Downarrow$

$y=x*\{h_2*h_1\}=\{x*h_2\}*h_1$

平方系统不是LTI

$\rightarrow x^2 \rightarrow x_2$

$\rightarrow x_2 \rightarrow x^2$ $\quad$ ✗

④ Derivative property

denote $\dot{f}(t)\triangleq \frac{d\,f(t)}{dt}$ $\quad$ If $y(t)=x(t)*h(t)$ Then $\dot{y}(t)=\dot{x}(t)*h(t)$

$$\begin{cases} \delta(t)=\frac{du(t)}{dt} \\ h(t)=\frac{d\delta(t)}{dt} \end{cases}$$

⑤ Integration property

denote $f_{int}(t)\triangleq\int_{-\infty}^{t} f(\tau)d\tau$ $\quad$ If $y(t)=h(t)*x(t)$ Then $y_{int}(t)=x_{int}(t)*h(t)$

Ex2.6: Note $x(t)=u(t-1)-u(t-3)$, $h(t)=(t+2)[u(t+2)-u(t+1)]$

Solution: $\int_{-\infty}^{t} f(\tau)\,u(\tau+t_0)\,d\tau=\int_{-t_0}^{t} f(\tau)\,u(\tau+t_0)\,d\tau$

$\tau+t_0\geq 0$

$\tau\geq -t_0$

$=\int_{-t_0}^{t} f(\tau)\,d\tau\ \ u(t+t_0)$

$u(\tau+t_0)=0$ for satisfying $t<\tau<-t_0$, i.e., $t<-t_0$.

so $h_{int}(t)=\int_{-\infty}^{t} h(\tau)\,d\tau=\int_{-\infty}^{t}(\tau+2)[u(\tau+2)-u(\tau+1)]d\tau$

$=\int_{-\infty}^{t}(\tau+2)u(\tau+2)\,d\tau-\int_{-\infty}^{t}(\tau+2)u(\tau+1)d\tau$

$=\int_{-2}^{t}(\tau+2)d\tau\ u(t+2)-\int_{-1}^{t}(\tau+2)\,d\tau\ u(t+1)$

that is $h_{int}(t)=\frac{1}{2}(t+2)^2u(t+2)-\frac{1}{2}[(t+2)^2-1]u(t+1)$

代入

since $\dot{x}(t)=\delta(t-1)-\delta(t-3)$ and $\delta(t-t_0)*v(t)=v(t-t_0)$

so $y(t)=x(t)*h(t)=\dot{x}(t)*h_{int}(t)=h_{int}(t-1)-h_{int}(t-3)$

$=\frac{1}{2}(t+1)^2u(t+1)-\frac{1}{2}[(t+1)^2-1]u(t)-\frac{1}{2}(t-1)^2u(t-1)+\frac{1}{2}[(t-1)^2-1]u(t-2)$

Causality and Stability of LTI

$$\triangle\begin{cases} h(t)=0,\ \forall t<0 \\ h[n]=0,\ \forall n<0 \end{cases} \Longrightarrow \text{causal}\ (\text{因果})$$

$$\triangle\begin{cases} \int_{-\infty}^{+\infty}|h(t)|dt<+\infty \\ \sum_{n=-\infty}^{+\infty}|h[n]|<+\infty \end{cases} \Longrightarrow \text{stable}\ (\text{稳定})$$

$x(t)\,\delta(t-t_0)=x(t_0)\,\delta(t-t_0)$

$\int_{-\infty}^{+\infty} x(t)\,\delta(t-t_0)\,dt=x(t_0)$

例: $\delta(t)*\delta(t-2)$

$=\int_{-\infty}^{+\infty}\delta(\tau)\,\delta(t-2-\tau)\,d\tau$

$=\delta(t-2)$

$\delta(t-1)*\delta(t-2)$

$=\int_{-\infty}^{+\infty}\delta(\tau-1)\,\delta(t-2-\tau)d\tau$

$=\delta(t-2-1)$

$=\delta(t-3)$