Laplace Transform 'Properties

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Property	L.T. pair $x(t) \leftrightarrow X(s),\ x_k(t) \leftrightarrow X_k(s)$	ROC
Linearity	$\sum_k \alpha_k x_k(t) \leftrightarrow \sum_k \alpha_k X_k(s)$	$\supseteq \bigcap_k ROC_{x_k}$
Time shift	$x(t-\tau) \leftrightarrow X(s)e^{-s\tau}$	$ROC_x \cap \{-\infty < Re(s) < \infty\}$
Time scaling	$x(\alpha t) \leftrightarrow \frac{1}{\lvert\alpha\rvert} X(\frac{s}{\alpha}),\ \alpha \neq 0$	$\sigma_1 < Re(\frac{s}{\alpha}) < \sigma_2$
Multiplication by e^{ct}	$x(t)e^{ct} \leftrightarrow X(s-c)$	ROC_x shifted with c
$\frac{d}{dt}$	$\frac{dx(t)}{dt} \leftrightarrow sX(s)$	$\supseteq ROC_x \cap \{\lvert s\rvert < \infty\}$
$\frac{d}{ds}$	$tx(t) \leftrightarrow -\frac{dx(s)}{ds}$	ROC_x
$x(t) = x_1(t) * x_2(t)$	$X(s) = X_1(s)X_2(s)$	$\supseteq ROC_{x_1} \cap ROC_{x_2}$

$-\frac{dX(s)}{d(jw)} \rightarrow \frac{dX(jw)}{dw}$

① $\delta(t) \Leftrightarrow 1$ s-Plane

$\delta(t-1) \Leftrightarrow e^{-s}$ 不包含 $-\infty$

$\delta(t+1) \Leftrightarrow e^{s}$ 不包含 $+\infty$

② $te^{\alpha t}u(t) \Leftrightarrow \frac{1}{(s-\alpha)^2}$ ROC: $Re(s) > Re(\alpha)$

$-te^{\alpha t}u(-t) \Leftrightarrow \frac{1}{(s-\alpha)^2}$ ROC: $Re(s) < Re(\alpha)$

$$\begin{cases} t^m e^{\alpha t}u(t) \Leftrightarrow \frac{m!}{(s-\alpha)^{m+1}}, & \{s: Re(s) > Re(\alpha)\} \\ -t^m e^{\alpha t}u(-t) \Leftrightarrow \frac{m!}{(s-\alpha)^{m+1}}, & \{s: Re(s) < Re(\alpha)\} \end{cases}$$

③ $u(t) \Leftrightarrow \frac{1}{s}$

$tu(t) \Leftrightarrow \frac{1}{s^2}$