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Time-Domain Equalization for Single-Frequency Full-Duplex Wireless Relay Using $\mathcal{H}^2$ Optimal Control

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Abstract

In this article, we propose a design method of a digital filter that suppresses self-interference arising at a single-frequency full-duplex wireless relay, and also compensates frequency selective fading from a source station to the relay station. The signals and the wireless relaying system can be modeled as random processes and a discrete-time linear time-invariant system respectively. We formulate the filter design problem based on the minimum-mean-square error (MMSE) criterion. It is shown that the filter design problem is equal to an H^2 optimal control problem. The effectiveness of the proposed method is shown by simulations.

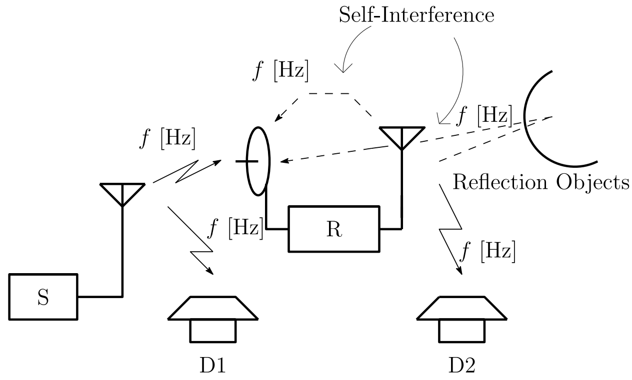

Fig. 1: Self-interference

1 Introduction

Single-frequency full-duplex relaying is regarded as a key technology for the next generation mobile networks (5G) [1]. One of the most important problems to realize such relaying is suppression of *self-interference* [2, 3], which causes oscillation and instability. Fig. 1 illustrates the basic scenario of self-interference. In the figure, S denotes the source station, R denotes a relay station, D1, and D2 are the destinations. Because of the self-interference, the communication performance is significantly deteriorated. To tackle with this problem, the authors have recently proposed a self-interference cancellation method using $\mathcal{H}^\infty$ control theory [4, 5]. In [4], the relay station is regarded as a closed-loop system and an $\mathcal{H}^\infty$ design is proposed for self-interference cancelers that stabilize the closed-loop system and also suppress the effects of self-interference based on robust control theory. It is shown that the stability can be guaranteed with the designed filter even if the amplifier gain in the relay station is very large, e.g. 1000 (60 dB).

Meanwhile, frequency-domain equalization has been most often used to cope with frequency-selective fading generated by multipath channels [6]. In particular, the idea of orthogonal frequency division multiplexing (OFDM) [7], which is based on frequency-domain equalization schemes, has been employed in several wireless standards such as a local area network (LAN) standard, the IEEE 802.11a [8], a metropolitan area network (MAN) standard, the IEEE 802.16 [9], and a standard for wireless communication of high-speed data for mobile phones, long-term evolution (LTE) [10]. One of the features of OFDM is the presence of a guard interval that eliminates intersymbol interference from the received signal. When the impulse response of the channel is longer than the length of the guard interval, then the communication performance is significantly deteriorated [11]. On the other hand, a longer length of the guard interval may lead to lower speed of the data transmission. Thus, the length of the GI is usually set to be in some degree longer than the length of the channel impulse response in usual communications.

In amplify-and-forward wireless relaying, in which transmitted signals are not decoded to bits in the relay station, the impulse response of the overall channel from source to destination is given by the convolution of the two channel responses: from a source station to a relay station and from the relay station to a destination station. Thus, the overall channel length may be longer than one assumed in usual communications. Therefore, when we employ frequency-domain equalization for such amplify-and-forward wireless relaying, then the overall channel length may be longer than the guard length and hence the communication performance extremely drops. In this article, we propose a design method of a digital filter that not only suppresses self-interference but also compensates the frequency selective fading channel from the source station to the relay station in the time-domain. It is shown

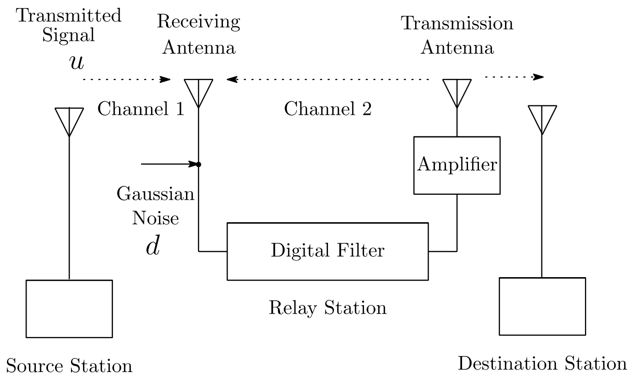

Fig. 2: Wireless relaying system

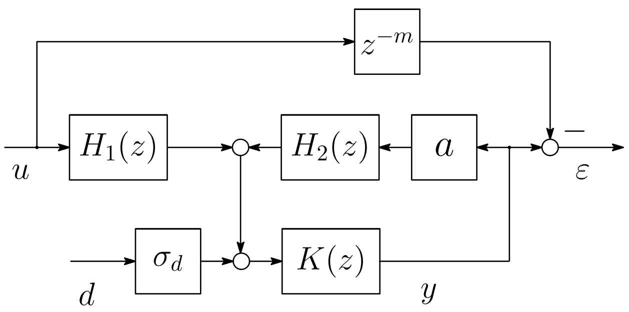

Fig. 3: Error system for the filter design

that the design problem is formulated as an $\mathcal{H}^2$ optimal control problem with the minimum-mean-square error (MMSE) criterion for additive white Gaussian noise (AWGN). The effectiveness of the proposed method is shown by simulations.

2 Filter Design

Fig. 2 shows the wireless relaying system considered in this study. The source station transmits u, which is received by the receiving antenna of the relay station. The received signal with noise is then processed by the digital filter and the amplifier. The amplified signal is re-transmitted from the transmission antenna of the relay station to the destination station. Simultaneously, the transmitted signal is partly fed back to the receiving antenna of the relay station, which may cause self-interference. The purpose of the digital filter is to suppress self-interference and to equalize the channel from the source station to the relay station.

Fig. 3 depicts the error system for the digital filter design. Let the transmitted (input) signal u be an independent and identically distributed (i.i.d.) discrete-time random process and d denotes an i.i.d. Gaussian noise with zero mean and variance of σ_d^2. The output ε is the error signal and $a \in \mathbb{R}$ is the gain of the amplifier at the relay station. When the length of the overall response from the source to the destination is within an allowable range, the filter output y is allowed to be delayed against the input u, and hence we consider the delay step $m \in \mathbb{N}$. We assume the channels to be frequency selective, and $H_1(z)$ and $H_2(z)$ represent the transfer functions of the channels from the source station to the relay station (Channel 1) and from the transmission antenna to the receiving antenna of the relay station (Channel 2) respectively. The purpose is to design the digital filter $K(z)$ that minimizes the error signal with the error system in Fig. 3.

We formulate the filter design problem based on the MMSE criterion as follows:

Problem 1 *For $n \in \mathbb{N}$, let J be the cost function:*

$$J[n] \triangleq E\{\mathrm{tr}\left[\varepsilon[n]\varepsilon^H[n]\right]\} \ n = 0, 1, 2, \ldots.$$

Find a digital filter $K(z)$ that minimizes $J[n]$, where $E, \mathrm{tr}[\cdot]$, and $(\cdot)^H$ denote the ensemble average, the trace, and the complex conjugate transpose respectively.

Note that $E\{\mathrm{tr}\left[\varepsilon[n]\varepsilon^H[n]\right]\}$ is constant since u and d are i.i.d. random processes and the systems in Fig. 3 are all time-invariant. After this, we omit the index number n from $J[n]$.

From the relationship of the autocorrelation and the power spectrum of ε we have,

$$\begin{aligned} J &= \mathrm{tr}\left[R(0)\right] \\ &= \mathrm{tr}\left[\frac{1}{2\pi}\int_0^{2\pi} S(e^{j\omega})d\omega\right] \\ &= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{tr}\left[S(e^{j\omega})\right]d\omega, \end{aligned}$$

where $R(0)$ is the value of the autocorrelation at the origin and $S(e^{j\omega})$ is the spectrum density of ε [12]. Further $T(z)$ is defined as the system from $[u\ d]^\top$ to ε, where $[\cdot]^\top$ stands for the transpose. With $T(z)$, we obtain

$$\mathrm{tr}\left[S(e^{j\omega})\right] = \mathrm{tr}\left[T(e^{j\omega})T(e^{j\omega})^H\right].$$

Thus, the following correspondence between the cost function J and T

$$J = \|T\|_2^2$$

is satisfied, where $\|\cdot\|_2$ denotes the $\mathcal{H}^2$ norm of the system [13].

After all, the problem can be transformed into finding $K(z)$ that stabilizes the error system and minimizes the $\mathcal{H}^2$ norm of the system. Therefore, it is equivalent to the following standard $\mathcal{H}^2$ optimal control problem:

Problem 2 *Let $T(z)$ be the transfer function of the system from $[u\ d]^\top$ to ε. Find a digital filter $K(z)$ which minimizes $\|T\|_2^2$, where $\|T\|_2$ is the $\mathcal{H}^2$ norm of the system T defined by*

$$\|T\|_2 \triangleq \sqrt{\mathrm{tr}\left[\frac{1}{2\pi}\int_0^{2\pi} T(e^{j\omega})T(e^{j\omega})^H d\omega\right]}.$$

This problem can be solved with `MATLAB` [14], for example.

Table 1: Simulation parameters

Baseband modulation	QPSK
Process delay	$m = 6$
Channel model	3 path Rayleigh fading
# of channel realizations	1000
# of symbols	1000

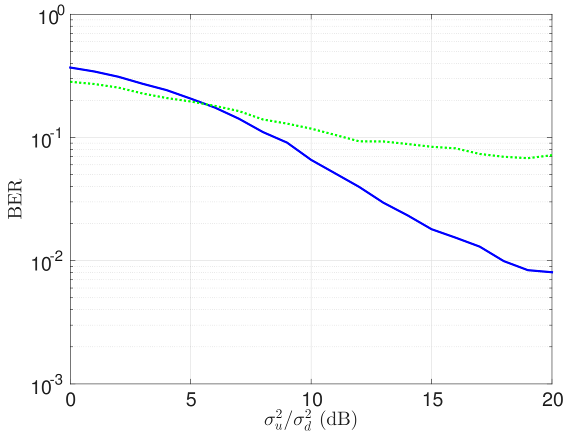

Fig. 4: BER vs SNR that is defined by σ_u^2/σ_d^2 (dB) when the amplifier gain in the relay station $a = 3$: solid line: the proposed method, dashed line: the $\mathcal{H}^\infty$ method.

3 Simulations

Table 1 shows the simulation parameters. The quadrature phase shift keying (QPSK) [16] is employed as the baseband modulation scheme. The allowable process delay is set to be 6. The channels H_1 and H_2 are modeled as 3 path Rayleigh fading channels [15], that is, they have frequency selective fading. The $\mathcal{H}^2$ norms of the channels H_1 and H_2 are normalized to 1. For each simulation, we evaluate the performance by averaging the results obtained in 1000 trials, where the number of the symbols is set to be 1000.

Fig. 4 illustrates the bit error rate (BER) vs the signal-to-noise ratio (SNR) σ_u^2/σ_d^2 when the amplifier gain $a = 3$. In this simulation, we utilize y in Fig. 3 for decoding in order to evaluate the quality of the relayed signal. The solid line represents the BER by the proposed method while the dashed line shows the $\mathcal{H}^\infty$ method [4]. From the figure, we can see that self-interference cancellation and equalization are simultaneously realized by the proposed method when the amplifier gain is 3, which is less than usual gains of practical systems. Fig. 5 shows the error value $|\varepsilon[n]|^2$ and the variance σ_d^2 to evaluate the performance of the self-interference cancellation when $a = 3$. It is shown that the self-interference is sufficiently suppressed in the case of that the closed-loop system is stable without any self-interference cancellation.

In practice, the amplifier gain of such a relay station is more than 60 dB. Thus, it is very important

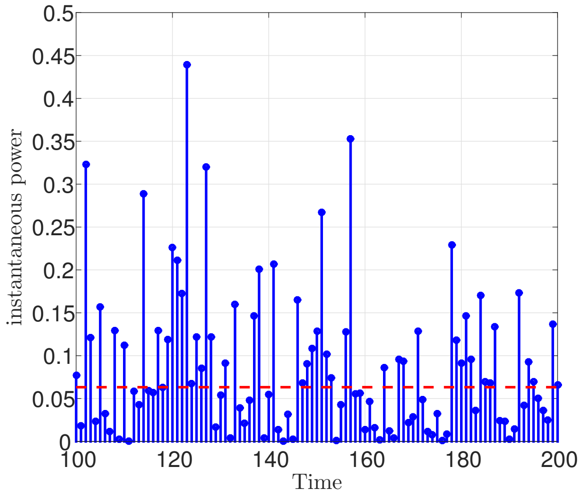

Fig. 5: Performance of the self-interference cancellation with the proposed method when the amplifier gain $a = 3$: solid line: the error value $|\varepsilon[n]|^2$ with the proposed method, dashed line: the variance σ_d^2 of the noise d.

to consider the case that the amplifier gain a is much larger. Fig. 6 illustrates the bit error rate (BER) vs the signal-to-noise ratio (SNR) σ_u^2/σ_d^2 when $a = 3000$. The lines represent the BER by the same methods as the previous simulations. From the figure, we can see that the proposed method is effective under the condition that a high amplifier gain is used at the relay station. Fig. 7 shows $|\varepsilon[n]|^2$ and σ_d^2 when $a = 3000$. It is shown that the self-interference is sufficiently suppressed even when the amplifier gain is high.

4 Conclusions

We have proposed a design method of a digital filter that not only suppresses self-interference but also compensates frequency selective fading. We have formulated the filter design problem based on the MMSE criterion and have transformed it to an $\mathcal{H}^2$ optimal control problem. Simulation results have shown the effectiveness of the proposed method with regard to the BER vs SNR characteristic and the error reduction performance. We have seen that the self-interference has been sufficiently suppressed even when the amplifier gain is very high, e.g. 3000.

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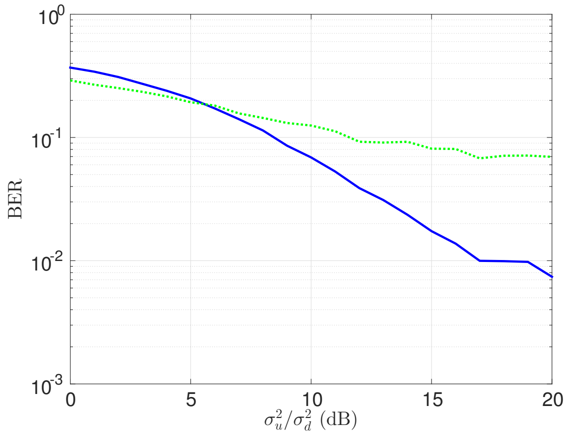

Fig. 6: BER vs SNR that is defined by σ_u^2/σ_d^2 (dB) when the amplifier gain in the relay station $a = 3000$: solid line: the proposed method, dashed line: $\mathcal{H}^\infty$ method.

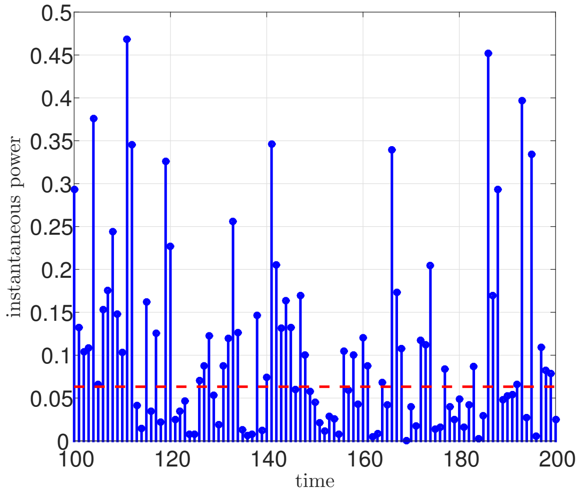

Fig. 7: Performance of the self-interference cancellation with the proposed method when the amplifier gain $a = 3000$: solid line: the error value $|\varepsilon[n]|^2$ with the proposed method, dashed line: the variance σ_d^2 of the noise d.

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