Q1:

Solution:

(a) $y(t)=t\cdot x(1-2t)$

(1) Let $t=1$, $y(1)=x(-1)$, so it's with memory.

(2) Assuming $x_1(t)\to y_1(t)=t\cdot x_1(1-2t)$

Let $x_2(t)=x_1(t-t_0)\to y_2(t)=t\cdot x_2(1-2t)=t\cdot x_1(1-2t-t_0)$,

However, $y(t-t_0)=(t-t_0)\cdot x(1-2(t-t_0))\neq y_2(t)$

so it is time varying,

(3) Assuming $x_1(t)\to y_1(t)=t\cdot x_1(1-2t)$ and $x_2(t)\to y_2(t)=t\cdot x_2(1-2t)$

Let $x_3(t)=ax_1(t)+bx_2(t)\to y_3(t)=t\cdot x_3(1-2t)=t\cdot[ax_1(1-2t)+bx_2(1-2t)]$,

While $ay_1(t)+by_2(t)=at\cdot x_1(1-2t)+bt\cdot x_2(1-2t)=y_3(t)$,

so it is linear,

(4) Let $t=-1$, $y(-1)=-x(3)$, so it's noncausal,

(5) If $|x(t)|\le M$, $|y(t)|=|t\cdot x(1-2t)|=|t||x(1-2t)|\le M|t|\overset{t\to\infty}{=}\infty$, so it's unstable.

(b) The system is not linear time-invariant. Because the input and the output signal of a LTI system should have the same frequency.

(c) $x(t)$ is a signal of spectrum limited to ω_0 rad/s,

$y(t)=5x(4t-3)$, so $y(t)$ is a signal of spectrum limited to $4\omega_0$ rad/s.

If $y[n]$ contains the same information as $y(t)$ does, then

$$T_s<\frac{\pi}{4\omega_0}$$

Q2:

Solution:

(a) The frequency response of the system is

$$\boldsymbol{H(j\omega)=\frac{1}{j\omega+1}}$$

The magnitude frequency response: $|H(j\omega)|=\dfrac{1}{\sqrt{\omega^2+1}}$

The phase frequency response: $\varphi(\omega)=-arc\tan\omega$

(b) Based on the magnitude frequency response $|H(j\omega)|$, the system is a low pass filter.

And it is NOT a linear phase response.

(c) For $x(t)=\cos(t)$, $y(t)=|H(j1)|\cos(t+\varphi(1))=\dfrac{\sqrt{2}}{2}\cos(t-\pi/4)$

Q3:

Solution:

(a) Based on the system structure, we can obtain

$$(X(s)-X(s)H_1(s))H_2(s)=Y(s)$$

$$H(s)=\frac{Y(s)}{X(s)}=(1-H_1(s))H_2(s)$$

(b) Since $H_1(s)=e^{-s}$ and $H_2(s)=\dfrac{1}{s}$ and the system is causal, the system function

$$H(s)=\frac{1}{s}(1-e^{-s}) \quad \Re_e(s)>0$$

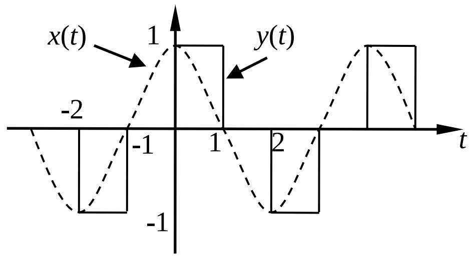
Then $h(t)=u(t)-u(t-1)$

(c) From (b), we can see that the system is Zero-order hold reconstruction system.

$$y(t)=s(t)p(t)*h(t)=s(t)\sum_{n=-\infty}^{+\infty}\delta(t-n)*h(t)$$

$$=\sum_{n=-\infty}^{+\infty}s(n)\delta(t-n)*h(t)=\sum_{n=-\infty}^{+\infty}s(n)h(t-n)$$

Figure:



Q4:

Solution:

(a) $H(s)=\dfrac{A(s+1)}{(s+2)(s+3)}=A(\dfrac{-1}{s+2}+\dfrac{2}{s+3})$

The system is causal, so the region of convergence is $\Re_e(s)>-2$

$h(t)=A(2e^{-3t}-e^{-2t})u(t)$

Based on $h[0+]=3$, we can obtain $A=3$

So the system function $H(s)=\dfrac{3(s+1)}{(s+2)(s+3)}$ with ROC $\Re_e(s)>-2$

(b) Yes. Because the ROC of $H(s)$ includes the $j\omega$-axis.

(c) From (a), the input $x(t)$ and output $y(t)$ satisfies the LCCDE:

$$\frac{d^2y(t)}{dt^2}+5\frac{dy(t)}{dt}+6y(t)=3\frac{dx(t)}{dt}+3x(t)$$

Apply Unilateral LT to both sides of the equation

$s^2Y(s)-s+1+5sY(s)-5+6Y(s)=3(s+1)X(s)$

$Y(s)=\dfrac{3(s+1)}{s^2+5s+6}X(s)+\dfrac{s+5-1}{s^2+5s+6}$

$X(s)=\dfrac{1}{s+4}\qquad \Re_e(s)>-4$

$$Y(s)=\frac{3(s+1)}{(s+2)(s+3)(s+4)}+\frac{s+4}{(s+2)(s+3)}\qquad \Re_e(s)>-2$$

$$=3(\frac{-1/2}{s+2}+\frac{2}{s+3}-\frac{3/2}{s+4})+\frac{2}{s+2}-\frac{1}{s+3}$$

So

$$y(t) = (-\frac{3}{2}e^{-2t} + 6e^{-3t} - \frac{9}{2}e^{-4t})u(t) + (2e^{-2t} - e^{-3t})u(t)$$
$$= (\frac{1}{2}e^{-2t} + 5e^{-3t} - \frac{9}{2}e^{-4t})u(t)$$

The zero-state response is $y_{zs}(t) = (-\frac{3}{2}e^{-2t} + 6e^{-3t} - \frac{9}{2}e^{-4t})u(t)$

The zero-input response is $y_{zi}(t) = (2e^{-2t} - e^{-3t})u(t)$

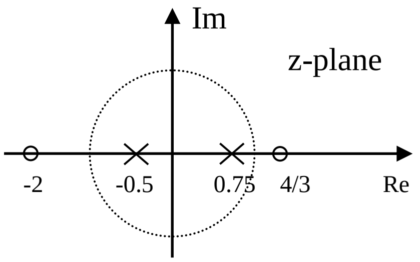
Q5:

Transfer function:

$$H(z) = \frac{z^{-2} - \frac{1}{4}z^{-1} - \frac{3}{8}}{1 - \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}} \qquad \text{ROC: } |z| > 0.80.$$

(a) pole: $p_1 = 0.75$, $p_2 = -0.5$

zero: $z_1 = \frac{4}{3}$, $z_2 = -2$



The ROC includes the unit circle, so the system is stable.

(b) $$H(z) = -\frac{8}{3} + \frac{\frac{77}{120}}{1 - \frac{3}{4}z^{-1}} + \frac{\frac{33}{20}}{1 + \frac{1}{2}z^{-1}} \qquad \text{ROC: } |z| > 0.80$$

the unit impulse response

$$h[n] = -\frac{8}{3}\delta[n] + \left(\frac{77}{120}\left(\frac{3}{4}\right)^n + \frac{33}{20}\left(-\frac{1}{2}\right)^n\right)u[n]$$

(c) $$F(z) = \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$$

$$1 - \alpha e^{-j\Omega} = M(\alpha, \Omega)e^{j\phi(\alpha,\Omega)}$$

α is real and $|\alpha|<1$, so $M(\alpha,\Omega)$ is even about Ω, and $\phi(\alpha,\Omega)$ is odd about Ω.

$$M(\alpha,\Omega)=M(\alpha,-\Omega),\quad \phi(\alpha,\Omega)=-\phi(\alpha,-\Omega)$$

$$F\left(e^{j\Omega}\right)=\frac{e^{-j\Omega}-\alpha}{1-\alpha e^{-j\Omega}}=\frac{\left(1-\alpha e^{j\Omega}\right)e^{-j\Omega}}{1-\alpha e^{-j\Omega}}$$

the magnitude response

$$\left|F\left(e^{j\Omega}\right)\right|=\left|\frac{e^{-j\Omega}-\alpha}{1-\alpha e^{-j\Omega}}\right|=\frac{\left|e^{-j\Omega}-\alpha\right|}{\left|1-\alpha e^{-j\Omega}\right|}=\frac{\left|1-\alpha e^{j\Omega}\right|\left|e^{-j\Omega}\right|}{\left|1-\alpha e^{-j\Omega}\right|}=\frac{M(\alpha,-\Omega)}{M(\alpha,\Omega)}=1,$$

phase response

$$\angle F\left(e^{j\Omega}\right)=arg\left(e^{-j\Omega}-\alpha\right)-arg\left(1-\alpha e^{-j\Omega}\right)$$

$$=arg\left(1-\alpha e^{j\Omega}\right)-\Omega-arg\left(1-\alpha e^{-j\Omega}\right)$$

$$=\phi(\alpha,-\Omega)-\Omega-\phi(\alpha,\Omega)$$

$$=-\Omega-2\phi(\alpha,\Omega).$$

Based on the result just obtained for $F(z)$,

$$H(z)=\frac{\left(z^{-1}-\frac{3}{4}\right)\left(z^{-1}+\frac{1}{2}\right)}{\left(1-\frac{3}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}=F(z)\Big|_{\alpha=\frac{3}{4}}F(z)\Big|_{\alpha=-\frac{1}{2}}$$

the magnitude response

$$\left|H\left(e^{j\Omega}\right)\right|=1,$$

the phase response

$$\angle H\left(e^{j\Omega}\right)=-\Omega-2\phi\left(\frac{3}{4},\Omega\right)-\Omega-2\phi\left(-\frac{1}{2},\Omega\right)=-2\Omega-2\phi\left(\frac{3}{4},\Omega\right)-2\phi\left(-\frac{1}{2},\Omega\right).$$

Q6:

Solution:

(a) The sampling frequency $\omega_s=\frac{2\pi}{T_s}=\frac{4}{3}\omega_m<2\omega_m$ which is smaller than the Nyquist frequency, therefore we **can not** recover $x(t)$ from its samples $x[n]=x(nT_s)$

(b) $p(t)$ is a periodic signal with the period of 2Δ, the FS coefficient is

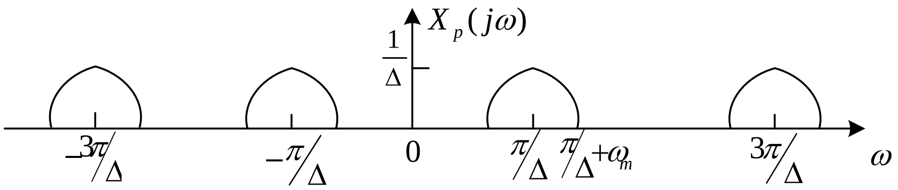
$$c[k]=\frac{1}{2\Delta}\int_T p(t)e^{-jk\frac{2\pi}{2\Delta}t}dt=\frac{1}{2\Delta}\int_T[\delta(t)-\delta(t-\Delta)]e^{-jk\frac{2\pi}{2\Delta}t}dt=\frac{1}{2\Delta}(1-e^{-jk\pi})=\begin{cases}\frac{1}{\Delta}, & k \text{ is odd}\\ 0, & k \text{ is even}\end{cases}$$

And $P(j\omega)=\sum_k 2\pi c[k]\delta(\omega-k\frac{\pi}{\Delta})=\sum_k \frac{2\pi}{\Delta}\delta(\omega-k\frac{\pi}{\Delta})$ where k is odd.

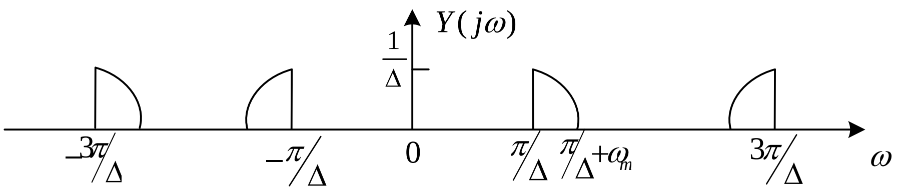
(c)

(i) $X_p(j\omega)=\frac{1}{2\pi}X(j\omega)*P(j\omega)=\frac{1}{2\pi}X(j\omega)*\sum_{k\text{ is odd}}\frac{2\pi}{\Delta}\delta(\omega-k\frac{\pi}{\Delta})=\frac{1}{\Delta}\sum_{k\text{ is odd}}X(j\omega-jk\frac{\pi}{\Delta})$

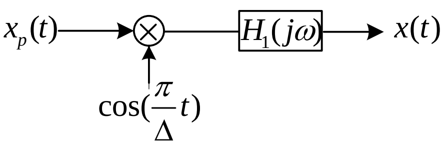
We have $\Delta<\frac{\pi}{2\omega_m}$, $\frac{\pi}{\Delta}>2\omega_m$, and the spectrum $X_p(j\omega)$ is depicted as



$Y(j\omega)=X_p(j\omega)H(j\omega)$, the spectrum is depicted as



(ii) the system below can realize to reconstruct $x(t)$ from $x_p(t)$

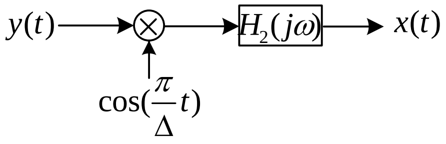


Where $H_1(j\omega)$ is a low-pass filter,

$$H_1(j\omega)=\begin{cases}\Delta & |\omega|<\omega_m\\ 0 & \text{otherwise}\end{cases}$$

As $x_p(t)\cos(\frac{\pi}{\Delta}t)\Rightarrow\frac{1}{2}[X_p(j\omega+j\frac{\pi}{\Delta})+X_p(j\omega-j\frac{\pi}{\Delta})]$

(iii) the system below can realize to reconstruct $x(t)$ from $y(t)$



where $H_2(j\omega)$ is a low-pass filter,

$$H_2(j\omega) = \begin{cases} 2\Delta & |\omega| < \omega_m \\ 0 & \text{otherwise} \end{cases}$$