03－04(二)期终试卷答案

一．1、$\dfrac{z}{x-z}$；2、$2\sqrt{3}$；3、$\int_{\frac{p}{4}}^{\frac{p}{3}} dq \int_0^{\frac{p}{4}} dj \int_0^1 f(g^2) g^2 \sin j\, dg$　4、2；　5、$\dfrac{(-1)^{n-1}}{n}$

二．A；B；C；B；D

三．1、$\dfrac{\partial z}{\partial x} = 2f_u + yf_v$，

$$\frac{\partial z}{\partial y} = f_u + xf_v,$$

$$dz = (2f_u + yf_v)dx + (f_u + xf_v)dy$$

2、$\int_0^a dx \int_{-x}^{x} f(x,y)dy + \int_a^{2a} dx \int_{-\sqrt{2ax-x^2}}^{\sqrt{2ax-x^2}} f(x,y)dy$

$$\int_0^a dy \int_y^{a+\sqrt{a^2-y^2}} f(x,y)dx + \int_{-a}^0 dy \int_{-y}^{a+\sqrt{a^2-y^2}} f(x,y)dx$$

3、　$\Sigma : z = \sqrt{a^2 - x^2 - y^2}$　　$D_{xy} : x^2 + y^2 \le a^2$

$$ds = \sqrt{1 + \frac{x^2}{z^2} + \frac{y^2}{z^2}}dxdy = \frac{a}{z}dxdy \text{ 或 } \left(= \frac{a}{\sqrt{a^2-x^2-y^2}}dxdy \right)$$

$$\iint_{\Sigma} zds = \iint_{D_{xy}} z \cdot \frac{a}{z} dxdy$$

$$= pa^3$$

四．1、　$\lim\limits_{n\to\infty} \left| \dfrac{u_{n+1}}{u_n} \right| = \lim\limits_{n\to\infty} \dfrac{1}{p} \cdot \dfrac{\sin \dfrac{p}{n+1}}{\sin \dfrac{p}{n}}$

$$= \lim_{n\to\infty} \frac{1}{p} \cdot \frac{n}{n+1} = \frac{1}{p} < 1$$

所以绝对收敛。

2、　$\sum\limits_{n=1}^{\infty} b_n \sin(np)$ 是 x 在 $(-p, p)$ 上的付氏级数

因为 x 是奇函数，　所以 $a_n = 0$

$$b_n = \frac{2}{p} \int_0^p x \sin(nx)dx$$

$$= -\frac{2}{np} \left(x\cos nx \Big|_0^p - \frac{1}{n} \sin(nx) \Big|_0^p \right)$$

$$= \frac{2(-1)^{n-1}}{n}$$

3、 $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right|=\lim\limits_{n\to\infty}\dfrac{n+2}{2^{n+1}(n+1)!}\cdot\dfrac{2^n n!}{n+1}=0$ **所以收敛区间** $(-\infty,\infty)$

$$s(x)=\sum_{n=1}^{\infty}\frac{1}{(n-1)!}\left(\frac{x}{2}\right)^n+\sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{x}{2}\right)^n$$

$$=\frac{x}{2}\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{x}{2}\right)^n+e^{x/2}-1$$

$$=\left(\frac{x}{2}+1\right)e^{x/2}-1$$

五、(1) $\begin{cases}\dfrac{\partial z}{\partial x}=14-8y-4x=0\\ \dfrac{\partial z}{\partial y}=32-8x-20y=0\end{cases}$ **驻点** $\begin{cases}x=1.5\\ y=1\end{cases}$

因为 $\dfrac{\partial^2 z}{\partial x^2}=-4$**，** $\dfrac{\partial^2 z}{\partial y\partial x}=-8$**，** $\dfrac{\partial^2 z}{\partial y^2}=-20$ **所以** $AC-B^2=16>0.\quad A<0$

所以当 $x=1.5, y=1$ **时，** z **取唯一极大值，为最大值。**

(2) $F(x,y,l)=15+14x+32y-8xy-2x^2-10y^2+l(x+y-1.5)$

$$\begin{cases}F_x=14-8y-4x+l=0\\ F_y=32-8x-20y+l=0\\ x+y=1.5\end{cases}$$

解得 $x=0,\ y=1.5$**，即将全部广告费用于报纸。**

六、加 $\sum_1$ **：**Z=1， $(x^2+y^2\le 1)$ **的上侧，用高斯公式**

$$\oiint\limits_{\Sigma+\Sigma_1}=\iiint\limits_{\Omega}3\,dv$$

$$=3\int_0^{2p}dq\int_0^1 r\,dr\int_{r^2}^1 dz\ \left(\text{或}\ 3\int_0^1 dz\iint\limits_{x^2+y^2\le z}dxdy\right)$$

$$=\frac{3}{2}p$$

$\iint\limits_{\Sigma_1}(2x+z)dydz=0$ **，** $\therefore\iint\limits_{\Sigma_1}=\iint\limits_{\Sigma_1}zdxdy=p$

$$\therefore\iint\limits_{\Sigma}=\oiint\limits_{\Sigma_1+\Sigma}-\iint\limits_{\Sigma_1}=\frac{p}{2}$$

七、曲面上任一点(x_0, y_0, z_0)，（其中$z_0 = x_0 f(\frac{y_0}{x_0})$）

所以切平面法方向$\vec{n} = \{f(\frac{y_0}{x_0}) - \frac{y_0}{x_0} f'(\frac{y_0}{x_0}), f'(\frac{y_0}{x_0}), -1\}$

切平面方程 $(f - \frac{y_0}{x_0} f')(x - x_0) + f' \cdot (y - y_0) - (z - z_0) = 0$

将原点代入

左边$= x_0 f + y_0 f' - y_0 f' + z_0$

$\because z_0 = x_0 f(\frac{y_0}{x_0})$ $\therefore$ 左边 $= 0$

又(x_0, y_0, z_0)是任意的

所以 所有的切平面过原点

八、$\frac{\partial p}{\partial y} = \frac{\partial}{\partial y}(\frac{1}{y} + yf(xy)) = -\frac{1}{y^2} + f(xy) + xyf'(xy)$

$\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}(xf(xy) - \frac{x}{y^2}) = f(xy) + xyf'(xy) - \frac{1}{y^2}$

$\therefore \frac{\partial p}{\partial y} = \frac{\partial Q}{\partial x}$ ，积分与路径无关

沿折线积分

$$\int_L = \int_a^c (\frac{1}{b} + bf(bx))dx + \int_b^d (cf(cy) - \frac{c}{y^2})dy$$

$$= \frac{c}{b} - \frac{a}{b} + \int_a^c f(bx)d(bx) + \frac{c}{d} - \frac{c}{b} + \int_b^d f(cy)d(cy)$$

$$= \frac{c}{d} - \frac{a}{b} + \int_{ab}^{cb} f(t)dt + \int_{bc}^{dc} f(t)dt$$

$\because ab = dc,\quad \int_{ab}^{cb} + \int_{bc}^{dc} = 0$

$\therefore I = \frac{c}{d} - \frac{a}{b}$