Chapter1

2019年11月6日 18:54

Continous-time signals / discrete signals

CT 连续时间信号 t, τ, f

DT 离散时间信号 i, k, n, m, N, q

Energy signals/power signals

- Energy E_x: (能量信号)

$$\begin{cases} E_x(T) \triangleq \int_{-T}^{T} |x(t)|^2 dt & \text{— CT} \\ E_x[N] \triangleq \sum_{n=-N}^{N} |x[n]|^2 & \text{— DT} \end{cases}$$

for $T>0$, $N>0$

$$\begin{cases} E_x \triangleq \lim_{T\to+\infty} E_x(T) = \int_{-\infty}^{+\infty} & \text{— CT} \\ E_x \triangleq \lim_{N\to+\infty} E_x[N] = \sum_{n=-\infty}^{+\infty} |x[n]|^2 & \text{— DT} \end{cases}$$

- Power P_x: (功率信号)

$$\begin{cases} P_x \triangleq \lim_{T\to+\infty} \frac{E_x(T)}{2T} & \text{— CT} \\ P_{x^0} \triangleq \lim_{N\to+\infty} \frac{E_x[N]}{2N+1} & \text{— DT} \end{cases}$$

$$\begin{cases} 0 < E_x < +\infty \Rightarrow \text{energy signal} \Rightarrow P_x = 0 \\ 0 < P_x < +\infty \Rightarrow \text{power signal} \Rightarrow E_x = +\infty \end{cases}$$

$C = Re(c) + j\,Im(c) \qquad c = \rho e^{j\theta}$

$|e^{j\theta}| = 1 \qquad e^{j\theta} = \cos\theta + j\sin\theta$

$$\begin{cases} Re(c) = \rho\cos\theta \\ Im(c) = \rho\sin\theta \end{cases} \qquad ☆ \begin{cases} \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \\ \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \end{cases}$$

不可能同时是能量又是功率
但可以两个都不是

周期信号 $x(t) = x(t+T), \forall t \in R. \quad f_0 \triangleq \frac{1}{T_0}, \omega_0 = 2\pi f_0$

$x[n] = x[n+N], \forall n \in Z. \quad F_0 \triangleq \frac{1}{N_0}, \Omega_0 = 2\pi F_0$

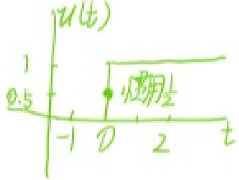
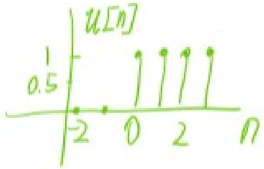
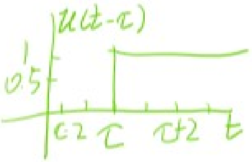
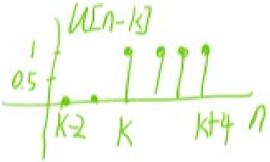
$$\sum_{k=0}^{N-1} \alpha^k = \begin{cases} N, & \alpha = 1 \\ \frac{1-\alpha^N}{1-\alpha}, & \alpha \neq 1 \end{cases}$$

Elementary signals（基本信号）

① Unit step signals（单位阶跃信号）

$$u(t) \triangleq \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

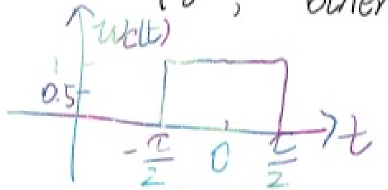
$$u[n] \triangleq \begin{cases} 1 & t\geq 0 \\ 0 & n<0 \end{cases}$$

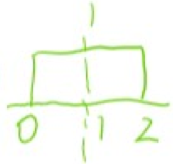
② Window signals（窗信号）

$$w[n]=u[n-N_1]-u[n-N_2-1]$$

$$W_\tau(t) \triangleq \begin{cases} 1 , & -\frac{\tau}{2}<t<\frac{\tau}{2} \\ 0 , & \text{otherwise} \end{cases}$$

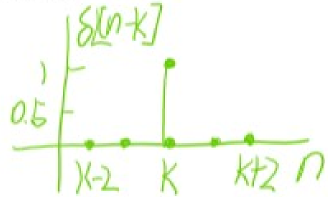


τ为总宽度 如 $W_2(t-1)$



③ Unit impulse signals（单位冲激信号）

$$\delta[n] \triangleq \begin{cases} 1 , & n=0 \\ 0 , & \text{otherwise} \end{cases}$$

单位冲激

$$\delta(t) \triangleq \lim_{\tau\to 0} \frac{1}{\tau} W_\tau(t)$$

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = \int_{-\sigma}^{\sigma} \delta(t)\,dt = 1, \forall \sigma>0$$

$$\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$$

$$\delta(t) = \delta(-t)$$

$$x(\xi)\delta(t-\xi) \equiv x(t)\delta(t-\xi)$$

$$\int_{-\infty}^{+\infty} x(\xi)\delta(t-\xi)\,d\xi = x(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

$$\delta(t) = \frac{du(t)}{dt}$$

⟹ 推导

$$u(t) = \int_{-\infty}^{+\infty} u(\xi)\delta(\xi-t)\,d\xi \overset{\tau=\xi-t}{=\!=\!=} \int_{-\infty}^{+\infty} u(\tau+t)\delta(\tau)\,d\tau \underset{\tau>-t}{\overset{\tau+t>0}{=\!=\!=}} \int_{-t}^{+\infty} \delta(\tau)\,d\tau = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

⟹ 例

$$(t-2)[\delta(t-2)-\delta(t-3)]$$
$$= (t-2)\delta(t-2) - (t-2)\delta(t-3)$$
$$= (2-2)\delta(t-2) - (3-2)\delta(t-3)$$
$$= 0 - \delta(t-3) = -\delta(t-3)$$

Properties of systems

① Memoryless / with memory (记忆) 无记忆 → 只与当前 t/n 有关.   with memory $y(t)=x(t\pm t_0)$、$x(t^2)$、积、微

② Causality (因果) $\begin{cases} y=x(t-t_0) \\ y=x(t+t_0) \end{cases}$ $t_0>0$ 因果 / 非因果   或用ROC: $|s|>s_0$. 只与现在和过去时刻有关. 或 若任意时刻 $t<0$ $y=0$.

③ Invertibility (可逆) 不同输入(一定要不同)导致不同的输出. 类似于单调函数.

④ Stability (稳定) 输入有界 ⇒ 输出有界. 或 ROL包含1 或 绝对可积/绝对可和

⑤ Time-invariance (时不变)

例: $y[n]=n\,x[n]+\alpha$ (判断 $\hat{y}[n]$ 与 $y[n-n_0]$)

则 $\hat{x}[n]=x[n-n_0]$ 时移

$\hat{y}[n]=n\hat{x}[n]+\alpha$ 又 $y[n-n_0]=(n-n_0)x[n-n_0]+\alpha$

$=n\,x[n-n_0]+\alpha$

得 $\hat{y}[n]\neq y[n-n_0]$ Not TI.

$y(t)=x(at)$ $a\neq1$时 时变

$y(t)=f(t)\cdot x(t)$ $f(t)$为关于t函数 时变

$y(t)=x(t-t_0)$ 参数为时 时不变.

⑥ Linearity (线性)

例: $y[n]=n\,x[n]+\alpha$ (判断 $\hat{y}[n]$ 与 $\alpha_1 y_1[n]+\alpha_2 y_2[n]$)

则 $\hat{x}[n]=\alpha_1 x_1[n]+\alpha_2 x_2[n]$ 线性组合

$\hat{y}[n]=n\,\hat{x}[n]+\alpha$ ← 代入

$=n\alpha_1 x_1[n]+n\alpha_2 x_2[n]+\alpha$

$\alpha_1 y_1[n]+\alpha_2 y_2[n]=n\alpha_1 x_1[n]+n\alpha_2 x_2[n]+\alpha(\alpha_1+\alpha_2)$

得 $\hat{y}[n]\neq\alpha_1 y_1[n]+\alpha_2 y_2[n]$ Not linear