Q1:

(a) Consider a continuous-time system described by the following input-output relationship:

$$y(t) = t\,x(1-2t)$$

where $y(t)$ is the system output, corresponding to the input $x(t)$. Determine which of the properties listed below hold and which do not hold for this system:

(1) Memoryless, (2) Time invariant, (3) Linear, (4) Causal, (5) Stable.

Justify your answers.

(b) A system's output is $y[n] = 10\cos(\frac{3\pi n}{4} - \frac{\pi}{3})$ when the input is $x[n] = \cos(\frac{3\pi n}{5} - \frac{2\pi}{3})$.

Is this system linear time-invariant? Justify your answer.

(c) Let $y(t) = 5x(4t-3)$, where $x(t)$ is a signal of spectrum limited to ω_0 (in rad/s). If $y[n]$ is obtained by sampling $y(t)$ with sampling period T_s. What is the constraint on T_s such that $y[n]$ contains the same information as $y(t)$ does?

Q2: Consider a causal LTI continuous-time system, whose input $x(t)$ and output $y(t)$ are related by

$$\frac{dy(t)}{dt} + y(t) = x(t)$$

(a) Find out the magnitude frequency response and phase frequency response of the system.

(b) What is the type of this system? Is it of linear phase response?

(c)When $x(t) = \cos(t)$, determine the output $y(t)$.

Q3: Consider a system depicted in Fig. 1.

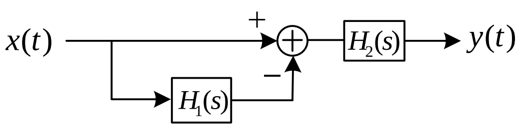

Fig. 1

where $H_1(s) = e^{-s}$ and $H_2(s) = \frac{1}{s}$.

Let $H(s)$ be the transfer function of the system between $x(t)$ and $y(t)$.

(a) Find out $H(s)$ in terms of $H_1(s)$ and $H_2(s)$.

(b) What is the unit impulse response $h(t)$ of this system?

(c) If the input signal $x(t)$ is given by $x(t)=s(t)p(t)$, where $s(t)=\cos\frac{\pi}{2}t$ and $p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-n)$ with n an integer. Determine and sketch $y(t)$.

Q4: A causal LTI continuous-time system satisfies the following conditions:

(i) The system function $H(s)$ is of the form given below

$$H(s)=\frac{A(s+1)}{(s+2)(s+3)}$$

where A is a constant.

(ii) The initial value of the unit impulse response $h(t)$ is $h(0_+)=3$.

Answer the following questions:

(a) Determine the system function $H(s)$ and its corresponding region of convergence.

(b) Is the system stable?

(c) Determine the output $y(t),\ t>0\_$ when the input is $x(t)=e^{-4t}u(t)$ and the initial conditions are $\frac{dy(t)}{dt}|_{t=0\_}=-1,\ y(0\_)=1,$ and indicate the zero-input and zero-state responses of the system.

Q5: Consider a discrete-time LTI system with transfer function given by:

$$H(z)=\frac{z^{-2}-\frac{1}{4}z^{-1}-\frac{3}{8}}{1-\frac{1}{4}z^{-1}-\frac{3}{8}z^{-2}}$$

with a region of convergence containing $|z|>0.80$.

(a) Sketch the pole-zero plot for the system on the *z*-plane. Is the system stable?

(b) Determine the unit impulse response $h[n]$ of the system.

(c) Let $F(z)=\frac{z^{-1}-\alpha}{1-\alpha z^{-1}}$ be the transfer function of an LTI system, where α is real and $|\alpha|<1$.

Assume $1-\alpha e^{-j\Omega} = M(\alpha,\Omega)e^{j\phi(\alpha,\Omega)}$ with both $M(\alpha,\Omega)>0$, $\phi(\alpha,\Omega)$ given. Determine the magnitude response and phase response of the system. Based on the result just obtained for $F(z)$, compute those of the system given by $H(z)$.

Q6: Consider a system shown in Fig. 2(a), where the signal $p(t)$ is an impulse-train as shown in Fig. 2(b). The Fourier transform (i.e., spectrum) $X(j\omega)$ of the input signal $x(t)$ and the system frequency response $H(j\omega)$ is shown in Fig. 2 (c) and (d), respectively. Answer the following questions:

(a) If $x(t)$ is sampled with a sampling period of $T_s = \dfrac{3\pi}{2\omega_m}$ (seconds), can we recover $x(t)$ from its samples $x[n]=x(nT_s)$?

(b) Determine the FS of $p(t)$. What is the spectrum $P(j\omega)$ of $p(t)$?

(c) Suppose $\Delta < \dfrac{\pi}{2\omega_m}$, consider the following problems:

(i) Sketch the spectrum of $x_p(t)$ and the spectrum of $y(t)$.

(ii) Design a system to reconstruct $x(t)$ from $x_p(t)$.

(iii) Suggest a system to recover $x(t)$ from $y(t)$.

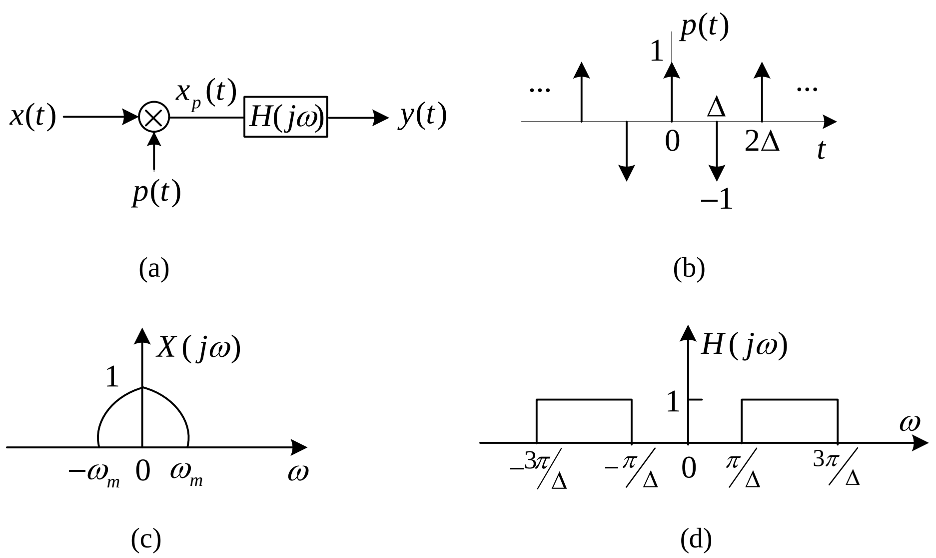

Fig.2