

浙江工业大学 2013 - 2014 学年第二学期
概率论与数理统计参考答案

一. 填空题 (每空 3 分, 共 30 分)

1. $\frac{1}{3}$

2. $\frac{3}{7}$

3. $\frac{1}{4}$

4. 1

5. 9 , 13

6. $\frac{1}{2}$

7. 0.9544

8. $(2, 1)$, 1

二. 选择题 (每题 2 分, 共 10 分)

1. C

2. D

3. A

4. B

5. B

三. 解答题 (共 60 分)

1. 解:

1) $P(X = \frac{\sqrt{3}}{4}) = \frac{6}{C_6^3} = 0.3;$

2)

X	$\frac{\sqrt{3}}{4}$	$\frac{2\sqrt{3}}{4}$	$\frac{3\sqrt{3}}{4}$
p	0.3	0.6	0.1

3) $EX = \frac{\sqrt{3}}{4}[1 \times 0.3 + 2 \times 0.6 + 3 \times 0.1] = \frac{9\sqrt{3}}{20};$

4) $EX^2 = \frac{3}{16}[1 \times 0.3 + 4 \times 0.6 + 9 \times 0.1] = \frac{27}{40};$

$Var(X) = EX^2 - (EX)^2 = \frac{27}{400}.$

2. 解:

$$1) \quad 1 = \int_0^1 Cx(1-x)dx = C(\frac{1}{2} - \frac{1}{3}) \Rightarrow C = 6;$$

$$2) \quad x < 0, F(x) = 0; \quad x > 1, F(x) = 1; \quad 0 \leq x \leq 1, \quad F(x) = \int_0^x 6(s-s^2)ds = 3x^2 - 2x^3;$$

$$3) \quad 0 < y < 1, \quad F_Y(y) = P(Y \leq y) = P((2X-1)^2 \leq y) = P(\frac{1-\sqrt{y}}{2} \leq X \leq \frac{1+\sqrt{y}}{2}) = \frac{3}{2}\sqrt{y} - \frac{1}{2}y\sqrt{y}; \quad \text{从而}$$

$$f_Y(y) = \begin{cases} \frac{3}{4} \frac{1-y}{\sqrt{y}}, & 0 < y < 1 \\ 0, & \text{其它} \end{cases}$$

3. 解:

$$1) \quad 1 = \int_0^1 \int_0^1 Cx(1-y)dxdy = \frac{C}{4} \Rightarrow C = 4;$$

2)

$$f_X(x) = \int_0^1 4x(1-y)dy = 2x, \quad 0 < x < 1$$

$$f_Y(y) = \int_0^1 4x(1-y)dx = 2(1-y), \quad 0 < y < 1$$

$$f(x, y) = f_X(x)f_Y(y)$$

因此, X, Y 独立。

3)

$$\begin{aligned} P(X < Y) &= \int_0^1 \int_0^y 4x(1-y)dxdy \\ &= \int_0^1 2y^2(1-y)dy = 2(\frac{1}{3} - \frac{1}{4}) = \frac{1}{6} \end{aligned}$$

4. 解:

$$\begin{aligned} \text{矩估计: } EX &= 0 \times (1-\theta) + 2 \times \theta - \theta^2 + 3 \times \theta^2 = \theta^2 + 2\theta, \\ \theta &= \sqrt{1+EX} - 1, \quad \hat{\theta} = \sqrt{1+\bar{X}} - 1 = \sqrt{\frac{12}{5}} - 1; \end{aligned}$$

极大似然估计: $L(\theta) = (1-\theta)^2(\theta-\theta^2)^2\theta^2 = \theta^4(1-\theta)^4$, 极大似然估计 $\hat{\theta}$ 为最大值点 $\frac{1}{2}$ 。

5. 解:

$$H_0 : \mu = \mu_0 = 20, \quad H_1 : \mu \neq \mu_0$$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = 2;$$

拒绝域为 $(-\infty, -2.1315) \cup (2.1315, \infty)$;

t 的值不在拒绝域中，认为该机器生产的螺丝长度正常。