Polynomial Approximation Wave Propagation in One Space Dimension

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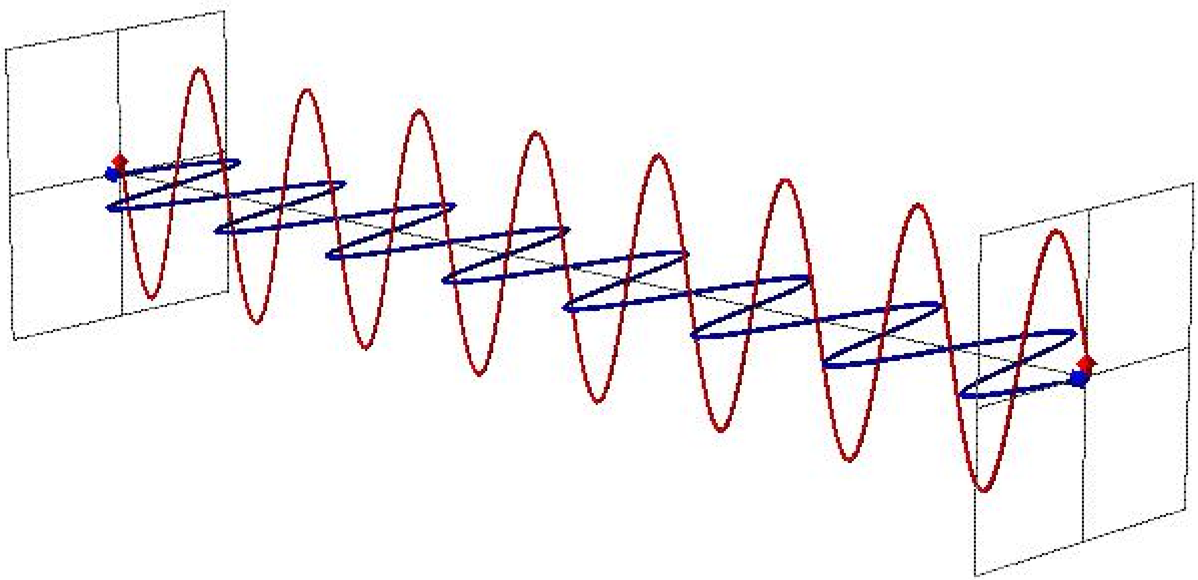

Motivation

What is wave propagation?

Design of floodbank

Possible method:

1. Natural Cubic Spline
2. Newton's Method

Problem Statement

$$u(x,t) = \frac{1}{2}[\phi(x+ct) + \phi(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} \psi(s)\,ds.$$

with $u(x,0) = \phi(x) \qquad u_t(x,0) = \psi(x),$

Want to approximate u(x,t) with polynomial.

This gives a simplfied model of wave propagation which is easier to compute.

Methods

How to implement and solve?

Plug points of u(t,x) with fixed x into python function *NaturalCubicSpline* to find corresponding coefficient.

Why cubic spline?

Becasue the interpolant is not only continuously differentiable but also has a continuous second derivative.

How to validate?

Compute additional points using u(t,x) and compare the difference.

Result

For initial condition $u(x,0) = \phi(x)$ =0

$$u_t(x,0) = \psi(x) = \cos(x)$$

u(x,t)=[sin(x+t)-sin(x-t)]/2

Our interpolation is closer to the actual u(x,t) as we take more points. Moreover, the closer the point to xn, the more accurate it is.

We can also reach desired accuracy by taking more points.

For more complex initial condition, the advantage of evaluating using polynomial is more significant.