Linear Algebra: solution

- <u>5.1</u> The eigenvectors are the columns of S, the eigenvalues are the diagonal entries of Λ.

$$S_A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \Lambda_A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}; \quad S_B = \begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}, \Lambda_B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

- <u>5.6</u> The eigenvalue of A is $2, 4$, with eigenvectors $x_1 = [1, -1]^T, x_2 = [1, 1^T]$. Therefore, the general solution is $u = c_1 e^{2t} x_1 + c_2 e^{4t} x_2$.

$$\begin{cases} c_1 = \frac{1}{2} = c_2 \text{ for } u_0 = [1, 0]^T & \Rightarrow u_a(t) = \frac{1}{2}(e^{2t}x_1 + e^{4t}x_2) \\ c_1 = \frac{-1}{2} = -c_2 \text{ for } u_0 = [0, 1]^T & \Rightarrow u_b(t) = \frac{-1}{2}(e^{2t}x_1 - e^{4t}x_2) \end{cases}$$

$$e^{At} = \begin{bmatrix} u_a(t) & u_b(t) \end{bmatrix}$$

- <u>5.9</u> One can set up the recursive equation as

$$u_{-(k+1)} = \begin{bmatrix} F_{-(k+2)} \\ F_{-(k+1)} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} u_{-k}.$$

  Solving the difference equation, one gets the result of

$$F_{-k} = (-1)^{k+1} F_k, \quad k > 0.$$

  (This result can be obtaied via observation, but I hope you can set up the problem in ways you have just learned.)

- <u>5.13</u>

  a. $\frac{d}{dt} e^{At} X(0) e^{Bt} = \frac{de^{At}}{dt} X(0) e^{Bt} + e^{At} X(0) \frac{de^{Bt}}{dt} = AX + XB$

  b. From (a), $X(t) = e^{At} X(0) e^{-At}$.

$$\det(X(t) - \lambda I) = 0 \Rightarrow \det(X(0) - \lambda I) = 0.$$

- <u>5.17</u>

  a. $\lambda_1 = 1, \lambda_2 = -1; x_1 = [4, 1]^T, x_2 = [-4, 1]^T$.

  b. $u(t) = 62.5 e^t x_1 + 32.5 e^{-t} x_2$.

  c. 4