Weighted Finite-State Transducer Algorithms

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Abstract

- Weighted finite-state transducers are used in many applications such as text, speech and image processing.
- This chapter gives an overview of several recent weighted transducer algorithms, including composition of weighted transducers, determinization of weighted automata, a weight pushing algorithm, and minimization of weighted automata.
- It briefly describes these algorithms, discusses their running time complexity and conditions of application, and shows examples illustrating their application.

Introduction

- Weighted transducers 用在很多應用,例如: text, speech and image processing
- 同樣的字有不同的發音，針對不同的發音有不同的機率並利用不同的權重去表示
- 這篇paper 概述一些weighted transducer演算法,包括composition,determinization,weight pushing, and minimization

Preliminaries

- 這個章節在介紹定義和符號的使用
- 半環(Semiring)是一個代數結構，表示成

  $(\mathbb{K}, \oplus, \otimes, \overline{0}, \overline{1})$

Semiring Example

SEMIRING	SET	$\oplus$	$\otimes$	$\bar{0}$	$\bar{1}$
Boolean	$\{0, 1\}$	$\vee$	$\wedge$	0	1
Probability	$\mathbb{R}_+$	$+$	$\times$	0	1
Log	$\mathbb{R} \cup \{-\infty, +\infty\}$	$\oplus_{\log}$	$+$	$+\infty$	0
Tropical	$\mathbb{R} \cup \{-\infty, +\infty\}$	min	$+$	$+\infty$	0

Table 1: *Semiring examples.* $\oplus_{\log}$ *is defined by:* $x \oplus_{\log} y = -\log(e^{-x} + e^{-y})$.

Notation

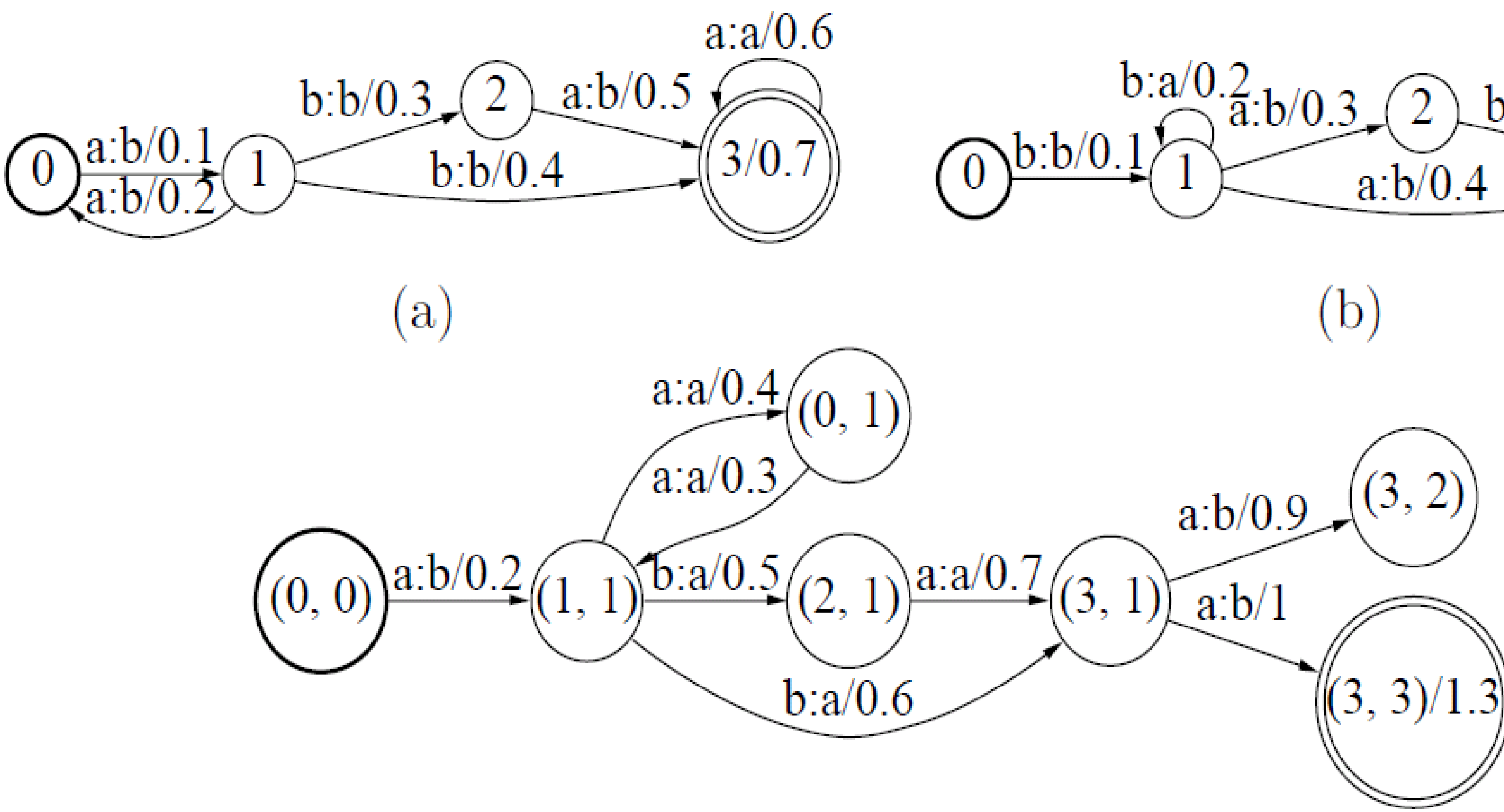
- 符號定義T=(A, B, Q, I, F, E, λ, ρ)
- T:有限狀態轉換器(在代數結構K中)
- A/B:輸入/輸出字母
- Q/I/F:有限/初始/結束狀態集合
- E:轉換路徑(Qx(A∪ε)x(B ∪ ε)xKxQ)
- λ: I->K 初始權重函式
- ρ : F->K 結束權重函式
- |T|: 狀態數
- p[e]/n[e]代表轉換路徑e的原本/下個狀態
- w[e]代表轉換路徑e的權重
- P(q, x, y, q’)代表從q到q’的路徑集合
- x/y代表輸入/輸出標籤

Composition

- Composition 是用來結合不同的轉換器
- 例:C = A。B
  - Transducers :A(x/z)、B(z/y)
  - 輸出成單一的 transducer C(x/y),並且給所有可能的序列和原本分開的transducers相同的權重

$$[\![T_1 \circ T_2]\!](x,y) = \bigoplus_{z} T_1(x,z) \otimes T_2(z,y)$$

Composition Example



Determinization

- 每個input最多只有一個轉換
- 沒有空集合的輸入
- 用det(C)表示
- 目的是減少找路徑的時間

Determinization Example

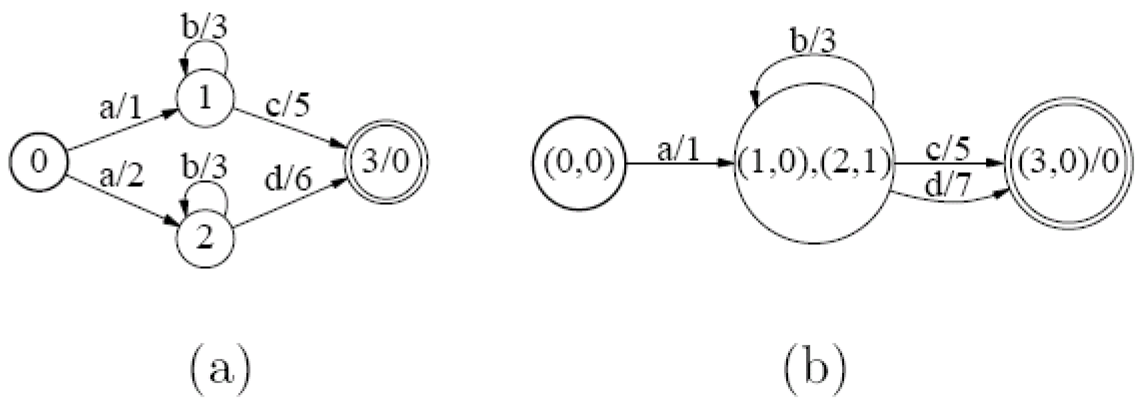


Figure 2: Determinization of weighted automata. (a) Weighted automaton over the tropical semiring A. (b) Equivalent weighted automaton B obtained by determinization of A.

Weight Pushing

先利用algorithm算每一狀態到結尾狀態的最短路徑

$$d[q] = \bigoplus_{\pi \in P(q,F)} (w[\pi] \otimes \rho[n[\pi]])$$

再依下列算式改變各個權重

$$\begin{aligned}
\forall e \in E \text{ s.t. } d[p[e]] \neq \overline{0}, w[e] &\leftarrow d[p[e]]^{-1} \otimes w[e] \otimes d[n[e]] \\
\forall q \in I, \lambda[q] &\leftarrow \lambda[q] \otimes d[q] \\
\forall q \in F, \text{ s.t. } d[q] \neq \overline{0}, \rho[q] &\leftarrow d[q]^{-1} \otimes \rho[q]
\end{aligned}$$

Weight Pushing Example

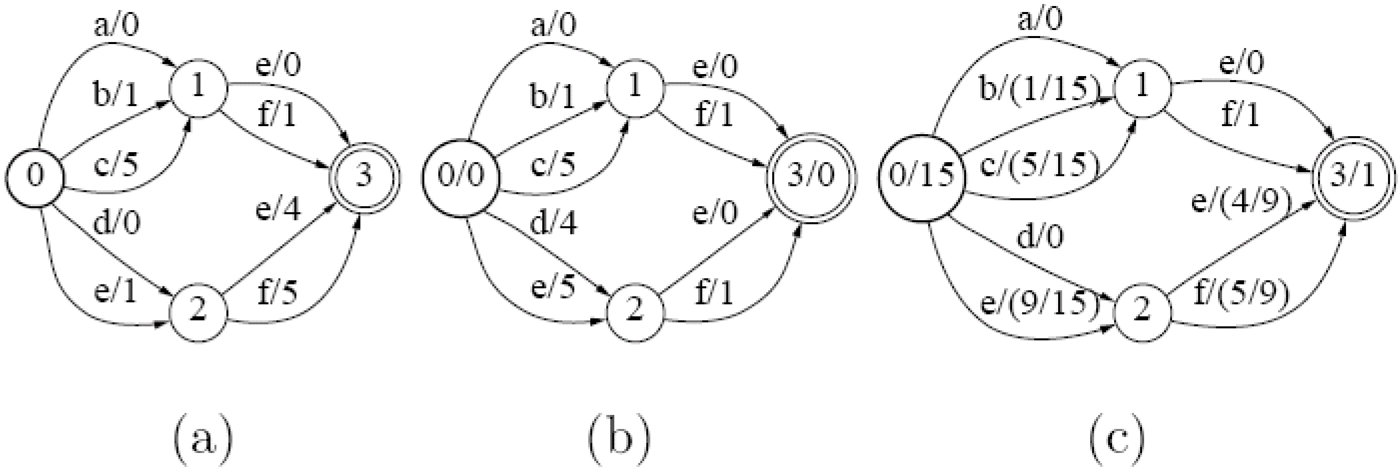


Figure 3: Weight pushing algorithm. (a) Weighted automaton A. (b) Equivalent weighted automaton B obtained by weight pushing in the tropical semiring. (c) Weighted automaton C obtained from A by weight pushing in the probability semiring.

Minimization

- 如果final state 字串的集合相同且權重也相同就稱這兩個deterministic weighted automaton爲等價
- 這兩個等價的狀態被merge不會影響到結果

Minimization Example

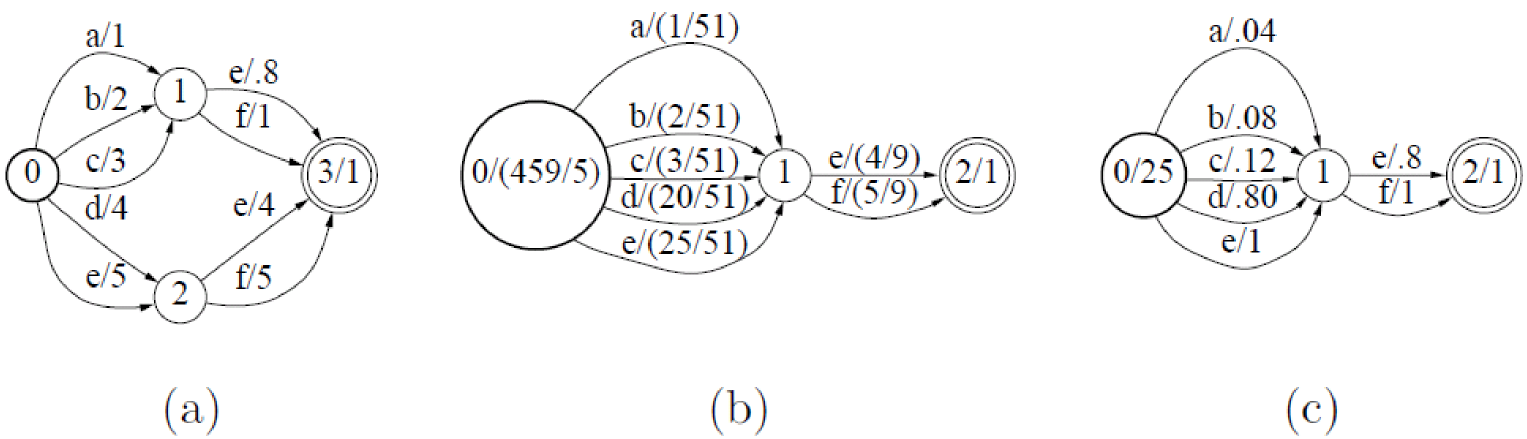


Figure 4: Minimization of weighted automata. (a) Weighted automaton A' over the probability semiring. (b) Minimal weighted automaton B' equivalent to A'. (c) Minimal weighted automaton C' equivalent to A'.

結語

- 這些演算法被使用在各種應用去創造有效率且複雜的系統
- 他們被用於幾十億狀態的權重轉換器,去創造大量詞彙的語音辨識系統
- 其他演算法像是 ε-removal and synchronization of weighted transducers也在大規模的系統扮演重要的角色