3.6

$\because$orthogonal matrix $(v_i, v_i) = 1$, every element= $\frac{1}{16}$

$\therefore$ the matrix is 16 by 16

3.7

$\because Q^T * Q = I$

$\therefore (Q^3)^T * Q^3 = (QQQ)^T QQQ = Q^T Q^T Q^T QQQ = I$

3.12

Using Gram-Schmidt Process

$u_1 = (1, 0, 0)$

$u_2 = (1, 1, 0) - \frac{(v_2, u_1)}{(u_1, u_1)}(1, 0, 0)$

$= (1, 1, 0) - (1, 0, 0) = (0, 1, 0)$

projection of b=$\frac{(b,u_1)}{(u_1,u_1)}u_1 + \frac{(b,u_2)}{(u_2,u_2)} = (1, 1, 0)$

3.14

$$\Rightarrow basis = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ -1 & 0 & 1 \end{bmatrix}$$

Solve $Ax = 0 \Rightarrow x_1 = x_3 \Rightarrow N(A) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

3.21

basis= $\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} = A$

$$P = A(A^T A)^{-}1 A^T = \begin{bmatrix} \frac{6}{4} & \frac{3}{4} & \frac{-3}{4} \\ \frac{3}{4} & \frac{1}{2} & \frac{-1}{4} \\ \frac{-3}{4} & \frac{-1}{4} & \frac{1}{2} \end{bmatrix} \Rightarrow N(P) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

3.23

(1)

$$a = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, c = 8 \Rightarrow dist. = \frac{c}{\|a\|} = \frac{8}{\sqrt{4}} = 4$$

(2)

vertical vector $= (1, 1, -1, -1)$

assume the point of the plane $(0, 0, 0, 0) + a(1, 1, -1, -1) = (a, a, -a, -a)$

Get $\Rightarrow a = 2 \Rightarrow$ the nearest point $(2, 2, -2, -2)$

3.18

$\int x^4 dx = \frac{1}{5}$

3.36

$$\begin{cases} 6 = C + D \\ 4 = C + 2D \\ 0 = C + 4D \end{cases} \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

$\Rightarrow A^T Ax = A^T b$

$$\Rightarrow \begin{bmatrix} C \\ D \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 21 & -7 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 14 \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 112 \\ -28 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$