5.5-15

U and V are unitary matrixs $\because U^HU = I$,$UU^H = I$, $V^HV = I$, $VV^H = I$
$\therefore (UV)^HUV = V^HU^HUV = V^HIV = I$
$UV(UV)^H = UVV^HU^H = U^HIU = I$
UV is unitary matrix

5.5-38

$A + iB$ is Hermitian matrix Both A and B are real matrixs
$\therefore A + iB = (A + iB)^H$
Both A and B are m by n matrix
$A + iB = A^H - B^Hi = A^T - B^Ti$
$A = A^T$, $B = -B^T$
A and B are square matrixs, so $m = n$.
$\begin{bmatrix} A & -B \\ B & A \end{bmatrix} = C$ is 2n by 2n matrix

$$C^T = \begin{bmatrix} A & -B \\ B & A \end{bmatrix}^T = \begin{bmatrix} A^T & B^T \\ -B^T & A^T \end{bmatrix} = \begin{bmatrix} A & -B \\ B & A \end{bmatrix} = C$$

5.6-4

(a)
$\because B$ is similar to A, C is similar to B
$\therefore B \sim A$, $C \sim B$
$B = M^{-1}AM$, $C = N^{-1}BN$
$C = N^{-1}BN = N^{-1}M^{-1}AMN = (MN)^{-1}AMN$
C is similar to A
(b)
Let S is similar to I
It exists invertable matrix P such that $P^{-1}SP = I$
$S = PIP^{-1} = I$, so $S = I$
only I is similar to I

5.6-20

(a)
$det(A - \lambda I) = 0$
$(\lambda - 2)(\lambda - 1) = 0$
e_1 is eigenvector when $\lambda = 2$
e_2 is eigenvector when $\lambda = 1$

$$e_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

U_1 is unitary matrix
The first column of U_1 is unit eigenvector $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$(when $\lambda = 2$)

$AU_1 = U_1T_1 = U_1 \begin{bmatrix} 2 & \star \\ 0 & \star \end{bmatrix}$
we restrict U_1 as symmetric and orthogonal matrix
$\therefore U_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & b \end{bmatrix}$
, $b = -\frac{1}{\sqrt{2}}$
$U_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$
$U_1^{-1}AU_1 = T_1 = \begin{bmatrix} 2 & 7 \\ 0 & 1 \end{bmatrix}$
The unitary matrix is U_1
The triangular matrix is T_1
(b)
$det(A - \lambda I) = 0 \; -\lambda^3 = 0$
e is unit eigenvector when $\lambda = 0$
$e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
$AU_1 = U_1N \begin{bmatrix} 0 & \star & \star \\ 0 & \star & \star \\ 0 & \star & \star \end{bmatrix}$
we restrict U_1 as symmetric and orthongonal matrix
The first column of U_1 is the unit eigenvector $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ when $\lambda = 0(A)$
$U_1 = \begin{bmatrix} 0 & a & e \\ 0 & b & f \\ 1 & c & g \end{bmatrix}$
By the restriction:
a=0, e=1, b=1 or -1, c=0, f=0, g=0
$U_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
$U_1^{-1}AU_1 = N = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
$M_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ is 2 by 2 submatrix of N
M_1 have eigenvalues 0 and 0, where corresponding unit eigenvector is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Find unitary matrix U_2 such that $NU_2 = U_2T = U_2 \begin{bmatrix} 0 & \star & \star \\ 0 & 0 & \star \\ 0 & 0 & \star \end{bmatrix}$

U_2 have this format $\begin{bmatrix} 1 & 0 & 0 \\ 0 & a & c \\ 0 & b & d \end{bmatrix}$

$M_2 = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ is the 2 by 2 submatrix of U_2

The first column of M_2 is unit eigenvector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ of M_1

we restrict M_2 as symmetric and orthogonal matrix, so c=1 d=0

$\therefore U_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$U_2^{-1} N U_2 = U_2^{-1} U_1^{-1} A U_1 U_2 = T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

The unitary matrix is $U_1 U_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

The triaugular matrix is $T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

5.5-26

Let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $B = M^{-1} A M$

$MB = AM$, $ad - bc \neq 0$

(a)

$MB = \begin{bmatrix} 0 & a+b \\ 0 & c+d \end{bmatrix}$, $AM = \begin{bmatrix} a & b \\ a & b \end{bmatrix}$

$M = \begin{bmatrix} 0 & b \\ c & b-c \end{bmatrix}$, $bc \neq 0$

(b)

$MB = \begin{bmatrix} a-b & b-a \\ c-d & d-c \end{bmatrix}$, $AM = \begin{bmatrix} a+c & b+d \\ a+c & b+d \end{bmatrix}$

$M = \begin{bmatrix} a & -c \\ c & -a \end{bmatrix}$, $a^2 \neq c^2$

(c)

$MB = \begin{bmatrix} 4a+2b & 3a+b \\ 4c+2d & 3c+d \end{bmatrix}$, $AM = \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$

$M = \begin{bmatrix} \frac{2}{3}d & c-d \\ c & d \end{bmatrix}$, $\frac{2}{3}d^2 \neq c(c-d)$