3.1-3.3 Homework Solution

- 3.1-48
  (1)
  parallel
  (2)
  $b_2 = 2b_1$
  (3)
  $(b_1, b_2) = (1, 2)$
  In that case $(b_1, b_2) = (1, 2)$ is perpendicular to the vector (2,-1) or (-2,1)
  (4)
  $3x + y = 0$
  (5)
  (1,-3) or (-1,3)

- 3.2-15
  (a)
  $P^T = P \ P^2 = P$
  $(Px)^T y = x^T P^T y = x^T Py$
  (b)
  No,
  $\cos\theta_{xa} = \frac{a^T x}{\|a\|\|x\|} = \frac{1}{\sqrt{15}}$
  $\cos\theta_{ya} = \frac{a^T y}{\|a\|\|y\|} = \frac{1}{3\sqrt{3}}$
  (c)
  $(Px)^T Py = x^T P^T Py = x^T Py$
  $Px = \frac{1}{3}(1, 1, -1) = (\frac{1}{3}, \frac{1}{3}, \frac{-1}{3})$
  $Py = \frac{1}{3}(1, 1, -1) = (\frac{1}{3}, \frac{1}{3}, \frac{-1}{3})$
  $\cos\theta_{PxPy} = 1 \ \theta_{PxPy} = 0$

- 3.2-21
  $P_1 = \frac{1}{9} \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} -1 & 2 & 2 \end{bmatrix}$
  $= \begin{bmatrix} \frac{1}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{-2}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{-2}{9} & \frac{4}{9} & \frac{4}{9} \end{bmatrix}$

$P_2 = \frac{1}{9}\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}\begin{bmatrix} 2 & 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} \frac{4}{9} & \frac{4}{9} & \frac{-2}{9} \\ \frac{4}{9} & \frac{4}{9} & \frac{-2}{9} \\ \frac{-2}{9} & \frac{-2}{9} & \frac{1}{9} \end{bmatrix}$

$P_1P_2 = 0$ because $a_1 \perp a_2$

- 3.3-6

$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$

$A^TA = \begin{bmatrix} 1 & 1 & -2 \\ 1 & -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -8 \\ -8 & 18 \end{bmatrix}$

$(A^TA)^{-1} = \begin{bmatrix} \frac{9}{22} & \frac{2}{11} \\ \frac{2}{11} & \frac{3}{22} \end{bmatrix}$

$p = A\hat{x} = A(A^TA)^{-1}A^Tb$

$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} \frac{9}{22} & \frac{2}{11} \\ \frac{2}{11} & \frac{3}{22} \end{bmatrix}\begin{bmatrix} 1 & 1 & -2 \\ 1 & -1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{23}{11} \\ \frac{-14}{11} \\ \frac{65}{11} \end{bmatrix}$

$b = p + q = \begin{bmatrix} \frac{23}{11} \\ \frac{-14}{11} \\ \frac{65}{11} \end{bmatrix} + \begin{bmatrix} \frac{-12}{11} \\ \frac{36}{11} \\ \frac{12}{11} \end{bmatrix}$

q is the left nullspace.

- 3.3-28

(a)
$\hat{x} = \frac{a^Tb}{a^Ta} = \frac{b1+b2+...+bm}{m}$

(b)
The variance is $\|e\|^2 = \sum_{i=1}^{m}(b_i - \hat{x})^2$

(c)
e=(-2,-1,3)

$p^T e = -6 - 3 + 9 = 0 \; p \perp e$

$P = \frac{aa^T}{a^T a} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$