6.1.9

(a)

at $(0,0)$

$f_{xx} = 4e^x = 4$ and $f_{yy} = 5xsiny + 12 = 12$ and $f_{xy} = -5cosy = -5$

$\because 4 * 12 - (-5)^2 > 0$ and $f_{xx} > 0$

$\therefore (0,0)$ is minimum

(b)

at $(1,\pi)$

$f_{xx} = 2cosy = -2$ and $f_{yy} = -x^2cosy + 2xcosy = -1$ and $f_{xy} = -2xsiny + 2siny = 0$

$\because (-2)(-1) - 0 > 0$ and $f_{xx} < 0$

$\therefore (1,\pi)$ is maximum

6.1.22

(1)

$f_{xx} = 3x^2 + 2y$ and $f_{yy} = 2$ and $f_{xy} = 2x$

$\Rightarrow A_1 = \begin{bmatrix} 3x^2 + 2y & 2x \\ 2x & 2 \end{bmatrix}$

$$\begin{cases} 2x^2 + 4y > 0 \\ 3x^2 + 2y > 0 \\ f_x = x^3 + 2xy = 0 \\ f_y = x^2 + 2y = 0 \end{cases} \Rightarrow \text{not exist}$$

(2)

$f_{xx} = 6x$ and $f_{yy} = 0$ and $f_{xy} = 1$

$\Rightarrow A_1 = \begin{bmatrix} 6x & 1 \\ 1 & 0 \end{bmatrix}$

$$\begin{cases} f_x = 3x^2 + y - 1 = 0 \\ f_y = x = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 1 \end{cases}$$

6.2.21

(1)

$$\begin{cases} s > 0 \\ s^2 - 16 > 0 \\ s^3 - 48s - 128 > 0 \end{cases} \Rightarrow s > 8$$

(2)

$$\begin{cases} t > 0 \\ t^2 - 9 > 0 \\ t^3 - 25t > 0 \end{cases} \Rightarrow t > 5$$

6.2.32

$$A = L\sqrt{D}\sqrt{D}L^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow L\sqrt{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$

$$B = L\sqrt{D}\sqrt{D}L^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow L\sqrt{D} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & \sqrt{5} \end{bmatrix}$$

6.3.6

$$A^T A = \begin{bmatrix} {\sigma_1}^2 & \cdots & 0 & \\ 0 & {\sigma_2}^2 & \cdots & 0 \\ \vdots & & & \\ 0 & \cdots & 0 & {\sigma_n}^2 \end{bmatrix} \text{ and } x_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \vdots x_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \Sigma = \begin{bmatrix} \sigma_1 & \cdots & 0 & \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & & & \\ 0 & \cdots & 0 & \sigma_n \end{bmatrix} V = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$\Rightarrow u_1 = \frac{1}{\sigma_1} A \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \frac{1}{\sigma_1} w_1 \Rightarrow u_2 = \frac{1}{\sigma_2} A \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} = \frac{1}{\sigma_2} w_2 \Rightarrow u_n = \frac{1}{\sigma_n} A \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} = \frac{1}{\sigma_n} w_n$$

$$\Rightarrow U = \begin{bmatrix} \frac{w_1}{\sigma_1} & \frac{w_2}{\sigma_2} & \cdots & \frac{w_n}{\sigma_n} \end{bmatrix}$$

6.3.17

$A^T A = O_{nn} \Rightarrow \lambda_1 = \lambda_2 = \cdots = \lambda_n = 0$

$\Rightarrow \sigma_1 = \sigma_2 = \cdots = \sigma_n = 0 \Rightarrow \Sigma = O_{mn}$

$$x_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \cdots x_n = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \Rightarrow V = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}_{nn}$$

$U = O_{mm}$

$0^+ = V\Sigma^+ U^T$

6.4.4

$R_A(x) = \frac{x^T A x}{x^T x}$

$R_{A+B}(x) = \frac{x^T (A+B) x}{x^T x} = \frac{x^T A x + x^T B x}{x^T x} = R_A(x) + R_B(x)$

$\because Bispositivedefinite \Rightarrow R_B(x) > 0$

$\therefore R_{A+B}(x) = R_A(x) + R_B(x) > R_A(x)$

6.4.11

$R_A(x) = \frac{x^T A x}{x^T x} \geq \lambda_1$

$R_B(x) = \frac{x^T B x}{x^T x} \geq \mu_1$

$R_{A+B}(x) = \frac{x^T (A+B) x}{x^T x} = \frac{x^T A x + x^T B x}{x^T x} = R_A(x) + R_B(x) \geq \lambda_1 + \mu_1$