Linear Algebra: solution[1]

- 2.3

  i. False. $dim(S) \leq m$.
  ii. True. The zero vector must be in the intersection.
  iii. False. Both x and y are in the nullspace of A, yet they can be different.
  iv. False. A basis of a vector space is not unique.
  v. True. $A^2x = 0 \Rightarrow A(Ax) = 0 \Rightarrow Ax = 0 \Rightarrow x = 0$. So A^2 is invertible so it has independent columns.

- 2.6

  - A: row space $\{[1\ 2]\}$; nullspace $\{[2\ -1]\}$; column space $\{[1\ 3]\}$; left nullspace $\{[3\ -1]\}$;
  - B: row space $\{[1\ 2]\}$; nullspace $\{[2\ -1]\}$; column space $\{[0\ 1]\}$; left nullspace $\{[1\ 0]\}$;
  - C: row space $\{[1\ 1\ 0\ 0], [0\ 1\ 0\ 1]\}$; nullspace $\{[0\ 0\ 1\ 0], [1\ -1\ 0\ 1]\}$; column space $\{[1\ 0], [1\ 1]\}$; left nullspace $\{[0\ 0]\}$;

- 2.9

  a. No. $A - A$ is of rank 0
  b. symmetric matrices
  c. the entire space (as explained in class)
  d. the entire space

- 2.12 $\quad n - 1 - (n-2) = 1$

- 2.15 Let A be of size $m \times n$, and r be the rank of A.

  i. $r = n$. No solution if $b \notin \mathcal{R}(A)$, 1 solution otherwise.
  ii. $r = m < n$
  iii. $m < n$. No solution if $b \notin \mathcal{R}(A)$, infinitely many solutions otherwise.
  iv. $r = m = n$

[1] Generally speaking, the solutions provided here serve as answer keys. In many instances, there are details to be filled in to get full credits. Some solutions will be omitted here if the main ideas are already presented.

- 2.18

$$A^2 = A \Rightarrow A(A - I) = 0 \Rightarrow A - I = 0 \Rightarrow A = I,$$

since the nullspace of A contains only the 0 vector.

- 2.21 (1) rank = 1 (2) rank = 2 for a checkerboard matrix.

- 2.24 The rank of the matrix is 3. It does have a right inverse, such as $A^T(AA^T)^{-1}$. It does not have a left inverse.

- 2.30

  a. $Ax = 0 \Rightarrow A(Ax) = 0$, so any vector in the nullspace of A is in the nullspace of A^2.

  b. The columns of A^2 are linear combinations of the columns of A, so the column space of A^2 is contained in the column space of A.