Linear Algebra: solution

- 5.5

  a. Since $\det(A + \lambda I) = 0$ must have solution in the complex domain for any A, there is no A that $A + cI$ is invertible for every complex c.

  b. The eigenvalues can all be non-real, in which case $A + rI$ is invertible for all real r. Example $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

- 5.10 The eigenvalues are $0, \pm\sqrt{2}i$, so the general solution is

$$x(t) = c_1 x_1 + c_2 e^{i\sqrt{2}t} x_2 + c_3 e^{-i\sqrt{2}t} x_3,$$

  where x_i is the ith eigenvector. This solution is periodic in time and the period $T = \frac{2\pi}{\sqrt{2}}$.

- 5.14 The general solution for $\frac{du}{dt} = Au$ is

$$u(t) = e^{At} u_0 = S e^{\Lambda t} S^{-1} u_0.$$

  The general solution for $u_{k+1} = Au_k$ is

$$u_k = A^k u_0 = S \Lambda^k S^{-1} u_0.$$

  The rest is straightforward with the given eigenvalues and eigenvectors.

- 5.23

$$\lambda_2 x^T y = x^T A^T y = (Ax)^T y = \lambda_1 x^T y \Rightarrow x^T y = 0.$$

- 5.30

$$\begin{bmatrix} \frac{1}{3}(a+b) \\ \frac{2}{3}(a+b) \end{bmatrix}$$