2.1

(a)

$$a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, c = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, d = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow$ S=span{a,b,c,d}, but dim(S)=3

(b)

$$True$$

(c)

$$False$$

$$Assume\ A = \begin{bmatrix} 3 & 0 \\ 1 & 0 \end{bmatrix}, x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, y = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$Ax = 0\ and\ Ay = 0,\ but\ x \neq y$$

(d)

$$True$$

(e)

$$True$$

2.4

(1)

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \Rightarrow rand = 2 \Rightarrow N(A) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} t$$

(2)

$$B = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -2 \end{bmatrix} \Rightarrow\ rank = 2,\ N(B) = s \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix},\ s, t \in R$$

2.6

(a)

True

(b)

In 2 by 2 matrices, the permutation matrix is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, so the subspace is symmetric matrix

(c)

Not exist, $\forall \vec{v}$ $\in$positive matrices, but $-\vec{v} \notin$ positive matrix

(d)

Not exist, $0 \notin$ invertible matrix

2.10

(a)

$$\begin{bmatrix} 2 & 2 & -4 \\ 3 & 5 & -4 \\ 1 & 3 & -2 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 5 & -10 \\ -1 & 2 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 5 \\ 1 & 5 \\ 2 & 10 \end{bmatrix}$$

(d)

Yes

2.13

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2+t \\ -1-2t \\ t \end{bmatrix}$$

2.22

$$Assume\ x = \begin{bmatrix} x^1 \\ x^2 \\ \vdots \\ x^n \end{bmatrix}, y = \begin{bmatrix} y^1 \\ y^2 \\ \vdots \\ y^n \end{bmatrix},$$

then

$$x^T y = x^1 y^1 + x^2 y^2 + \cdots + x^n y^n = 0$$

We set that

$$y = \begin{bmatrix} 1 \\ 0 \\ \cdots \\ 0 \end{bmatrix} \Rightarrow x^T y = x_1 = 0 \Rightarrow x_1 = 0$$

$$y = \begin{bmatrix} 0 \\ 1 \\ \cdots \\ 0 \end{bmatrix} \Rightarrow x^T y = x_2 = 0 \Rightarrow x_2 = 0$$

$$\cdots x_n = 0 \Rightarrow x = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = 0$$

2.24

$$\begin{bmatrix} 1 & 2 & 0 & 3 & b1 \\ 0 & 0 & 0 & 0 & b2 \\ 2 & 4 & 0 & 1 & b3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 & 3 & b1 \\ 0 & 0 & 0 & 0 & b2 \\ 0 & 0 & 0 & -5 & b3 - 2b1 \end{bmatrix}$$

(a)

$b3 - 2b1 \neq 0$ and $b2 = 0$

(b)

$$\Rightarrow basis = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ t \\ 0 \end{bmatrix}, t \in R$$

(c)

$$x_1 + 2x_2 = \frac{3b_3 - b_1}{5},\ x_4 = \frac{2b_1 - b_3}{5}$$

(d)

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 4 \\ 3 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix} \Rightarrow basis = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} and \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

(e)

$$rank = 2$$

2.25

(1) rank=1

(2) rank=2

2.26

We know that $rank(A) = n$, so A invertible

$$A^{-1}A^2 = A^{-1}A = I, \ \Rightarrow A = I$$

2.31

(a)

$$x_1 + x_2 = x_3 + x_4 = tx_3 + x_4 = x_5 + x_6 = t$$
$$\Rightarrow t \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + u \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + v \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + w \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & 1 & 0 & 0 & -1 & -1 \\ 2 & 2 & -1 & -1 & -1 & -1 \\ -1 & -1 & 2 & 2 & -1 & -1 \\ -1 & -1 & -1 & -1 & 2 & 2 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$