# Linear Algebra: solution

- 5.19

$$Q^T = [(I-K)(I+K)^{-1}]^T = [(I+K)^T]^{-1}(I-K)^T = (I-K)^{-1}(I+K)$$

$$Q^TQ = (I-K)^{-1}(I+K)(I-K)(I+K)^{-1} = (I-K)^{-1}(I-K)(I+K)(I+K)^{-1} = I$$

$$Q = \frac{1}{5}\begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix}$$

- 5.20

  a. $1$ is not an eignevalue of $K$

  b. $K$ has the same eigenvectors as an Hermitian matrix and therefore it is diagonalizable by a unitary matrix.

  c. $(e^{\Lambda t})^H e^{\Lambda t} = e^{-\Lambda t}e^{\Lambda t} = I$

  d. $(e^{Kt})^H e^{Kt} = U(e^{\Lambda t})^H U^H U e^{\Lambda t} U^H = I$

- 5.22

$$Ax = \lambda x \Rightarrow A^2 x = \lambda^2 x = -x \Rightarrow \lambda = \pm i$$

Let $n_i$ be the multiplicity of $i$ and $n_{-i}$ be the multiplicity of $-i$. Then $\sum \lambda = (n_i - n_{-i})i = tr(A)$ is a real number. Therefore $n_i = n_{-i}$). It follows $n$ is even.

- 5.26

  a.

$$P = \frac{1}{9}\begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{bmatrix}$$

  b. $\lambda = 1, x = [2, 1, 2]$

  c. $P^k = P, k \geq 1$, so $u_k = Pu_0 = [6, 3, 6], k \geq 1$.

- 5.28

For $A$, $\lambda_1 = 2,$ eigenvector is $x_1; \lambda_2 = 1,$ eigenvector is $x_2$.
For $B$, $\lambda_1 = 2,$ eigenvector is $x_1; \lambda_2 = 1,$ eigenvector is $-3x_1 + x_2$.