Linear Algebra: solution 2

- <u>2.1.3</u>

$$\mathcal{R}(A) = \{(x_1\ x_2)^T | x_2 = 0\},\ \mathcal{N}(A) = \{(x_1\ x_2)^T | x_1 - x_2 = 0\},$$
$$\mathcal{R}(B) = \{(0\ 0)^T\},\ \mathcal{N}(B) = R^3$$

- <u>2.1.8</u> The solution set is a line, a subspace (of R^3), and is the nullspace of A

- <u>2.1.9</u> The closure property under addition is violated:

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix},\quad \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}.$$

- <u>2.2.8</u> Apply the Gauss elimination,

$$\begin{bmatrix}1 & 1 & 2 & 2\\2 & 3 & -1 & 5\\3 & 4 & 1 & c\end{bmatrix} \to \begin{bmatrix}1 & 1 & 2 & 2\\0 & 1 & -5 & 1\\0 & 0 & 0 & c-7\end{bmatrix}.$$

If $c \neq 7$, there is no solution.

- <u>2.2.9</u> Since the column space of A is R^2, any b will be in the column space and the equation is solvable. To find the nullspace of A via $Ax = 0$, apply the Gauss elimination

$$\begin{bmatrix}1 & 2 & 0 & 3\\2 & 4 & 0 & 7\end{bmatrix} \to \begin{bmatrix}1 & 2 & 0 & 3\\0 & 0 & 0 & 1\end{bmatrix}.$$

Two independent vectors in the nullspace of A can be found by setting $x_2 = 1, x_3 = 0$ and $x_2 = 0, x_3 = 1$ respectively in solving $Ax = 0$. So $\mathcal{N}(A) = \{x | x = c_1(-2\ 1\ 0\ 0)^T + c_2(0\ 0\ 1\ 0)^T\}$. A particular solution by setting $x_2 = x_3 = 0$ is $(7b_1 - 3b_2\ 0\ 0\ b_2 - 2b_1)^T$. Finally, the general solution is

$$x = c_1(-2\ 1\ 0\ 0)^T + c_2(0\ 0\ 1\ 0)^T + (7b_1 - 3b_2\ \ 0\ \ 0\ \ b_2 - 2b_1)^T.$$

- <u>2.2.11</u> The rank of A is n, since the columns are independent.

- <u>2.3.1</u>

$$Vc = \begin{bmatrix}1 & 1 & 0 & 0\\1 & 0 & 0 & 1\\0 & 1 & 1 & 0\\0 & 0 & 1 & 1\end{bmatrix} \begin{bmatrix}c_1\\c_2\\c_3\\c_4\end{bmatrix} = 0 \to c = \alpha \begin{bmatrix}-1\\1\\-1\\1\end{bmatrix},$$

so the v's are not linearly independent. Furthermore, they do not span R^4 as $Vc = (0\ 0\ 0\ 1)^T$ has no solution. That is, $(0\ 0\ 0\ 1)^T$ is *not* a linear combination of the v's.

- 2.3.5 The row vectors are linearly independent if and only if there are no rows of zeros after the Gauss elimination. For the v's in 2.3.1,

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

  so they are not linearly independent.

- 2.3.13
  - (a) 3
  - (b) 0
  - (c) 16

- 2.3.21 There are 6 and 53 independent vectors satisfying $Ax = 0$ and $A^T y = 0$, respectively. Note that these are the numbers of free variables in the respective system of equations.