5.6-5.Review Homework Solution

- 5.6-41

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} d & c \\ b & a \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} b & a \\ d & c \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

- 5.6-42

(a)10x10
B:4x6 ; BA=4x4; AB=6x6

$\begin{bmatrix} I & -A \\ 0 & I \end{bmatrix}$: 10x10
$\begin{bmatrix} I & A \\ 0 & I \end{bmatrix}$: 10x10
(b)6-4=2
F is similar to G, so F and G have same eigenvalues.
the eigenvalues of F: the eigenvalues of AB and 4 zeros
the eigenvalues of G: the eigenvalues of BA and 6 zeros
the eigenvalues of AB: the eigenvalues of BA and 2 zeros

- 5.6-44

(a)True
$A^2 = (MBM^{-1})(MBM^{-1}) = MB^2M^{-1}$
(b)True
$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
$B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$
$B^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
A^2 and B^2 are similar, but A and B are not similar.

(c)True
$$\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
$$M = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$
(d)True
M^{-1} does not exist.
(e)True
$$B = \begin{bmatrix} 0 & 1 & ... \\ 1 & 0 & ... \\ . & . & . \\ . & . & . \end{bmatrix} A \begin{bmatrix} 0 & 1 & ... \\ 1 & 0 & ... \\ . & . & . \\ . & . & . \end{bmatrix}$$ M exists, so A and B are similar.

- 5.r-16
  an orthogonal matrix: $QQ^T = I$ and $Q^TQ = I$
  $Q^T = ((I-K)(I+K)^{-1})^T = ((I+K)^(-1))^T(I-K)^T = (I-K)^{-1}(I+K)$

  $Q^TQ = ((I-K)(I+K)^{-1})^T((I-K)(I+K)^{-1})$
  $= (I-K)^{-1}(I+K)(I-K)(I+K)^{-1}$
  $= (I-K)^{-1}(I-K)(I+K)(I+K)^{-1} = I$

  $QQ^T = (I-K)(I+K)^{-1}((I-K)(I+K)^{-1})^T$
  $= (I-K)(I+K)^{-1}(I-K)^{-1}(I+K)$
  $= (I-K)((I-K)(I+K))^{-1}(I+K)$
  $= (I-K)((I+K)(I-K))^{-1}(I+K)$
  $= (I-K)(I-K)^{-1}(I+K)^{-1}(I+K) = I$

  $$K = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$
  $$Q = \tfrac{1}{5}\begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix}$$

- 5.r-20
  (a)
  $Kx = \lambda x$
  $(K-I)x = (\lambda - 1)x$
  $\det K - I \neq 0$

  (b)
  $KS = S\Lambda$

$K = S\Lambda S^{-1}$
The eigenvectors are orthogonal.
$S = U$, $S^{-1} = U^{-1} = U^H$
$K = U\Lambda U^H$

(c)
$(e^{\Lambda t})^H e^{\Lambda t} = e^{\Lambda t - \Lambda t} = e^0 = I$
(The eigenvalues are imaginary, so $\Lambda^H = -\Lambda$)

(d)
$(e^{Kt})^H e^{Kt} = e^{Kt - Kt} = e^0 = I$
$(K^H = -K)$