1.(a)The difference equation

$$y[n]-\frac{1}{2}y[n-1]=x[n]+2x[n-1]+x[n-2]$$

Taking the Fourier transform of both sides,

$$Y(e^{j\omega})[1-\frac{1}{2}e^{-j\omega}]=X(e^{j\omega})[1+2e^{-j\omega}+e^{-j2\omega}]$$

The frequency response is

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1+2e^{-j\omega}+e^{-j2\omega}}{1-\frac{1}{2}e^{-j\omega}}$$

(b) $H(e^{j\omega})=\dfrac{1-\frac{1}{2}e^{-j\omega}+e^{-j3\omega}}{1+\frac{1}{2}e^{-j\omega}+\frac{3}{4}e^{-j2\omega}}=\dfrac{Y(e^{j\omega})}{X(e^{j}\omega)}$

$$X(e^{j\omega})[1-\frac{1}{2}e^{-j\omega}+e^{-j3\omega}]=Y(e^{j}\omega)[1+\frac{1}{2}e^{-j\omega}+\frac{3}{4}e^{-j2\omega}]$$

$$y[n]+\frac{1}{2}y[n-1]+\frac{3}{4}y[n-2]=x[n]-\frac{1}{2}x[n-1]+x[n-3]$$

2. $y[n]=\sum_{k=-\infty}^{\infty}a^{-k}u[-k]u[n-k]$

for $n\le 0$ $\quad y[n]=\sum_{k=-\infty}^{\infty}a^{-k}=\sum_{k=-n}^{\infty}a^{k}=\frac{a^{-n}}{1-a}$

for $n>0$ $\quad y[n]=\sum_{k=-\infty}^{0}a^{-k}=\sum_{k=0}^{\infty}a^{k}=\frac{1}{1-a}$

3.(a)

$$h[0]=0$$
$$h[1]=1$$
$$h[2]=\frac{1}{a}$$
$$h[3]=(\frac{1}{a})^{2}$$
$$\vdots$$
$$h[n]=(\frac{1}{a})^{n-1}u[n-1]$$

(b)h[n] is summable if $|\frac{1}{a}|<1$ or $|a|>1$

$$Y(e^{j\omega})-\frac{5}{6}e^{-j\omega}Y(e^{j\omega})+\frac{1}{6}e^{-2j\omega}Y(e^{j\omega})=\frac{1}{3}e^{-2j\omega}X(e^{j\omega})$$

4.frequency response:

$$H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{\frac{1}{3}e^{-2j\omega}}{1-\frac{5}{6}e^{-j\omega}+\frac{1}{6}e^{-2j\omega}}$$

$$H(e^{j\omega})=\frac{-2}{1-\frac{1}{3}e^{-j\omega}}+\frac{2}{1-\frac{1}{2}e^{-j\omega}}$$

impulse response:

$$h[n]=-2(\frac{1}{3})^{n}u[n]+2(\frac{1}{2})^{n}u[n]$$

step response:

$$s[n]=\sum_{k=-\infty}^{\infty}h[k]u[n-k]\sum_{k=-\infty}^{\infty}h[k]=-2\frac{1-(1/3)^{n+1}}{1-1/3}u[n]+2\frac{1-(1/2)^{n+1}}{1-1/2}u[n]=(1+(\frac{1}{3})^{n}-2(\frac{1}{2})^{n})u[n]$$

5.(a)F

(b)T

(c)T