Linear Algebra: solution[1]

- <u>3.1</u> $||a||^2 = 2^2 + (-2)^2 + 1^1 = 9 \Rightarrow ||a|| = 3$. To find two independent vectors perpendicular to a, solve the equation $2x - 2y + z = 0$ for two independent solutions, for example, $(1, 1, 0)$ and $(-1, 0, 2)$.
- <u>3.4</u> $p = Pb = a\frac{a^T b}{a^T a} = 0$, as $a^T b = 0$.
- <u>3.6</u> Solve for the least squares solution

$$A^T A\bar{x} = A^T b \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \bar{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow \bar{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow p = A\bar{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

- <u>3.9</u>

$$P = A(A^T A)^{-1} A^T = \frac{1}{14} \begin{bmatrix} 10 & 6 & -2 \\ 6 & 5 & 3 \\ -2 & 3 & 13 \end{bmatrix}$$

- <u>3.12</u> The permutation matrices ($P^T P = I$). There are 6 of them.
- <u>3.14</u> By the Gram-Schmidt process

$$\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & 0 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \begin{bmatrix} 1 & \sin\theta\cos\theta \\ 0 & \sin^2\theta \end{bmatrix}$$

- <u>3.19</u> The equality follows from $QQ^T = I$ and the block matrix multiplication. Alternatively, one can show that $\sum_{i=1}^n v_i v_i^T x = x$ for any x.
- <u>3.22</u> The answer is the projection point of b to the nullspace of $A = [1\ 1\ -1]$. Details omitted.
- <u>3.33</u>

  a. $\{(\frac{1}{10}(1, 3, 4, 5, 7), \frac{1}{10}(-7, 3, 4-5, 1)\}$

  b.

$$\begin{bmatrix} 1 & -6 \\ 3 & 6 \\ 4 & 8 \\ 5 & 0 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & \frac{-7}{10} \\ \frac{3}{10} & \frac{3}{10} \\ \frac{4}{10} & \frac{4}{10} \\ \frac{5}{10} & \frac{-5}{10} \\ \frac{7}{10} & \frac{1}{10} \end{bmatrix} \begin{bmatrix} 10 & 10 \\ 0 & 10 \end{bmatrix}$$

[1]Generally speaking, the solutions provided here serve as answer keys. In many instances, there are details to be filled in to get full credits. Some solutions will be omitted here if the main ideas are already presented.

c. $\bar{x} = (b^T q_1, b^T q_2)$. Plug in the b and $q's$.

- <u>3.34</u>

  a. $V \cap W = \{0\}, V + W = R^n$

  b. $V \cap W =$ the set of diagonal matrices, $V + W =$ the set of all matrices