4.1.

$$
\begin{aligned}
x[n] &= x_c(nT) \\
&= \sin\left(2\pi(100)n\frac{1}{400}\right) \\
&= \sin\left(\frac{\pi}{2}n\right)
\end{aligned}
$$

4.2. The discrete-time sequence

$$x[n] = \cos(\frac{\pi n}{4})$$

results by sampling the continuous-time signal

$$x_c(t) = \cos(\Omega_o t).$$

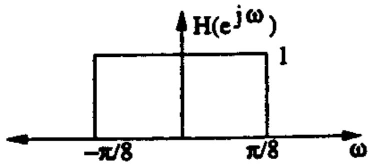
Since $\omega = \Omega T$ and $T = 1/1000$ seconds, the signal frequency could be:

$$\Omega_o = \frac{\pi}{4} \cdot 1000 = 250\pi$$

or possibly:

$$\Omega_o = (2\pi + \frac{\pi}{4}) \cdot 1000 = 2250\pi.$$

4.5. A plot of $H(e^{j\omega})$ appears below.



(a)

$$x_c(t) = 0, \quad , |\Omega| \geq 2\pi \cdot 5000$$

The Nyquist rate is 2 times the highest frequency. $\Rightarrow T = \frac{1}{10,000}$ sec. This avoids all aliasing in the C/D converter.

(b)

$$
\begin{aligned}
\frac{1}{T} &= 10kHz \\
\omega &= T\Omega \\
\frac{\pi}{8} &= \frac{1}{10,000}\Omega_c \\
\Omega_c &= 2\pi \cdot 625\text{rad/sec} \\
f_c &= 625Hz
\end{aligned}
$$

(c)

$$
\begin{aligned}
\frac{1}{T} &= 20kHz \\
\omega &= T\Omega \\
\frac{\pi}{8} &= \frac{1}{20,000}\Omega_c \\
\Omega_c &= 2\pi \cdot 1250\text{rad/sec} \\
f_c &= 1250Hz
\end{aligned}
$$

4.7. The continuous-time signal contains an attenuated replica of the original signal with a delay of τ_d.

$$x_c(t) = s_c(t) + \alpha s_c(t - \tau_d)$$

(a) Taking the Fourier transform of the analog signal:

$$X_c(j\Omega) = S_c(j\Omega) \cdot (1 + \alpha e^{-j\tau_d \Omega})$$

Note that $X_c(j\Omega)$ is zero for $|\Omega| > \pi/T$. Sampling the continuous-time signal yields the discrete time sequence, $x[n]$. The Fourier transform of the sequence is

$$\begin{aligned} X(e^{j\omega}) &= \frac{1}{T} \sum_{r=-\infty}^{\infty} S_c(\frac{j\omega}{T} + j\frac{2\pi r}{T}) \\ &+ \frac{\alpha}{T} \sum_{r=-\infty}^{\infty} S_c(\frac{j\omega}{T} + j\frac{2\pi r}{T}) e^{-j\tau_d(\frac{\omega}{T} + \frac{2\pi r}{T})}. \end{aligned}$$

(b) The desired response:

$$H(j\Omega) = \begin{cases} 1 + \alpha e^{-j\tau_d \Omega}, & \text{for } |\Omega| \leq \frac{\pi}{T} \\ 0, & \text{otherwise} \end{cases}$$

Using $\omega = \Omega T$, we obtain a discrete-time system which simulates the above response:

$$H(e^{j\omega}) = 1 + \alpha e^{-j\frac{\tau_d \omega}{T}}$$

(c) We need to take the inverse Fourier transform of the discrete-time impulse response of part (b).

$$\begin{aligned} h[n] &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n}\, d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} (1 + \alpha e^{-j\frac{\tau_d \omega}{T}}) e^{j\omega n}\, d\omega \end{aligned}$$

(i) Consider the case when $\tau_d = T$:

$$\begin{aligned} h[n] &= \frac{1}{2\pi} \int_{-\pi}^{\pi} (e^{j\omega n} + \alpha e^{j\omega(n-1)})\, d\omega \\ &= \frac{\sin(\pi n)}{\pi n} + \frac{\alpha \sin[\pi(n-1)]}{\pi(n-1)} \\ &= \delta[n] + \alpha\delta[n-1] \end{aligned}$$

(ii) For $\tau_d = T/2$:

$$\begin{aligned} h[n] &= \frac{1}{2\pi} \int_{-\pi}^{\pi} (e^{j\omega n} + \alpha e^{j\omega(n-\frac{1}{2})})\, d\omega \\ &= \frac{\sin(\pi n)}{\pi n} + \frac{\alpha \sin[\pi(n-\frac{1}{2})]}{\pi(n-\frac{1}{2})} \\ &= \delta[n] + \frac{\alpha \sin[\pi(n-\frac{1}{2})]}{\pi(n-\frac{1}{2})} \end{aligned}$$

4.9. (a) Since $X(e^{j\omega}) = X(e^{j(\omega-\pi)})$, $X(e^{j\omega})$ is periodic with period π.

(b) Using the inverse DTFT,

$$\begin{aligned}
x[n] &= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}d\omega \\
&= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j(\omega-\pi)})e^{j\omega n}d\omega \\
&= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j(\omega+\pi)n}d\omega \\
&= \frac{1}{2\pi}e^{j\pi n}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}d\omega \\
&= (-1)^n x[n].
\end{aligned}$$

All odd samples of $x[n] = 0$, because $x[n] = -x[n]$. Hence $x[3] = 0$.

(c) Yes, $y[n]$ contains all even samples of $x[n]$, and all odd samples of $x[n]$ are 0.

$$x[n] = \begin{cases} y[n/2], & n \text{ even} \\ 0, & \text{otherwise} \end{cases}$$

4.13. (a)

$$\begin{aligned}
x_c(t) &= \sin(\frac{\pi}{20}t) \\
y_c(t) &= \sin(\frac{\pi}{20}(t-5)) \\
&= \sin(\frac{\pi}{20}t - \frac{\pi}{4}) \\
y[n] &= \sin(\frac{\pi n}{2} - \frac{\pi}{4})
\end{aligned}$$

(b) We get the same result as before:

$$\begin{aligned}
x_c(t) &= \sin(\frac{\pi}{10}t) \\
y_c(t) &= \sin(\frac{\pi}{10}(t-2.5)) \\
&= \sin(\frac{\pi}{10}t - \frac{\pi}{4}) \\
y[n] &= \sin(\frac{\pi n}{2} - \frac{\pi}{4})
\end{aligned}$$

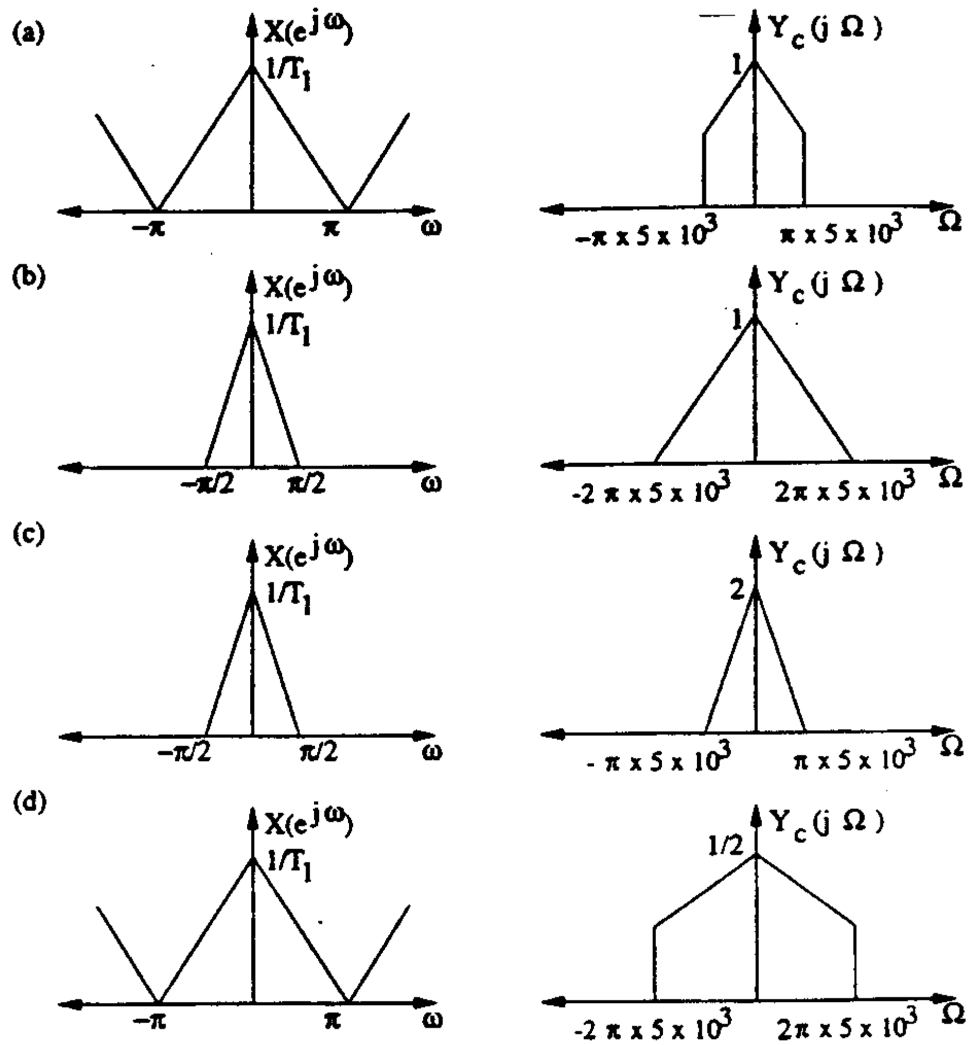
(c) The sampling period T is not limited by the continuous time system $h_c(t)$.

4.14. There is no loss of information if $X(e^{j\omega/2})$ and $X(e^{j(\omega/2-\pi)})$ do not overlap. This is true for (b), (d), (e).

4.20. (a) The Nyquist sampling property must be satisfied: $T \leq \pi/\Omega_0 \Longrightarrow F_s \geq 2000$.

(b) We'd have to sample so that $X(e^{j\omega})$ lies between $|\omega| < \pi/2$. So $F_s \geq 4000$.

4.24. The Fourier transform of $y_c(t)$ is sketched below for each case.



4.35. The frequency response $H(e^{j\omega}) = H_c(j\Omega/T)$. Finding that

$$H_c(j\Omega) = \frac{1}{(j\Omega)^2 + 4(j\Omega) + 3},$$

$$\begin{aligned} H(e^{j\omega}) &= \frac{1}{(10j\omega)^2 + 4(10j\omega) + 3} \\ &= \frac{1}{-100\omega^2 + 3 + 40j\omega} \end{aligned}$$