2.4-2.5 Homework Solution

- 2.4-3

$$
\begin{aligned}
&\dim(\mathrm{RS(A)}) = 1, \mathrm{basis}: \begin{bmatrix} 0 & 1 & 4 & 0 \end{bmatrix} \\
&\dim(\mathrm{CS(A)}) = 1, \mathrm{basis}: \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\
&\dim(\ker(\mathrm{A})) = 4 - 1 = 3, \mathrm{basis}: \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\
&\dim(\ker(\mathrm{A^T})) = 2 - 1 = 1, \mathrm{basis}: \begin{bmatrix} -2 \\ 1 \end{bmatrix} \\
&\dim(\mathrm{RS(U)}) = 1, \mathrm{basis}: \begin{bmatrix} 0 & 1 & 4 & 0 \end{bmatrix} \\
&\dim(\mathrm{CS(U)}) = 1, \mathrm{basis}: \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
&\dim(\ker(\mathrm{U})) = 4 - 1 = 3, \mathrm{basis}: \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\
&\dim(\ker(\mathrm{U^T})) = 2 - 1 = 1, \mathrm{basis}: \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\end{aligned}
$$

- 2.4-9
  (a)

  Because m < n so only right inverse.

$$
\begin{aligned}
&C = A^T(AA^T)^{-1} \\
&= \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \right)^{-1} \\
&= \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} \end{bmatrix}
\end{aligned}
$$

$$= \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

(b)

Because m > n so only left inverse.

$$B = (MM^T)^{-1}M^T$$
$$= \left(\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

(c)

T is a invertible matrix.

$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & a & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} a & 0 & 1 & -\frac{b}{a} \\ 0 & a & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{a} & -\frac{b}{a^2} \\ 0 & 1 & 0 & \frac{1}{a} \end{array}\right]$$

so

$$T^{-1} = \begin{bmatrix} \frac{1}{a} & -\frac{b}{a^2} \\ 0 & \frac{1}{a} \end{bmatrix}$$

- 2.4-40
  dim(ker(A)) = n − r, and columns of B are in $ker(A)$,
  $\therefore$ rank(B) $\leq$ n − r
  $\Rightarrow$ rank(A) + rank(B) $\leq$ n

- 2.5-11

  (a)

  9 nodes - 12 edges + 4 loops = 1
  (b)

  7 nodes - 12 edges + 6 loops = 1

- 2.5-19
  A complete graph is a simple graph in which every pair of distinct vertices is connected by an edge.
  so

  The number of edges $= C_2^n = \frac{n(n-1)}{2}$