Automatic Speech Recognition
Question Set 1

1. A finite-duration discrete-time sequence $x[n]$ is given by

$$x[n] = \begin{cases} n, & 0 \leq n \leq 7 \\ 0, & \text{otherwise} \end{cases}$$

Compute

(a) the DFT of $x[n]$

(b) the IDFT of the DFT of $x[n]$

solution

This problem is meant to be solved via computer programs. Simply write routines for DFT and IDFT and pass the input signal. As a sanity check for your routines, the IDFT of the DFT of a sequence is the same as the original sequence. Another sanity check is that for real sequences, $X[k]$ and $X[N-k]$ are complex conjugate to each other.

2. A discrete-time sequence is *left-sided* if it vanishes after a finite time index. It is *right-sided* if it vanishes before a finite time index. It is *finite-duration* if it is both left-sided and right-sided. It is *two-sided* if it is neither left-sided nor right-sided.

The *region of convergence* of a discrete-time sequence is the region in the z-plane where its z-transform converges.

Find the regions of convergence for the following discrete-time signals.

(a) a left-sided signal $l[n] = a^n\ u[N_1 - n]$

(b) a right-sided signal $r[n] = b^n\ u[n - N_2]$

(c) a two-sided signal $t[n] = c^n$

(d) a finite-duration signal $f[n] = d^n\ (u[n - N_3] - u[n - N_4])$

Here $u[n]$ is the unit step sequence, defined by

$$u[n] = \begin{cases} 1 & n \geq 0, \\ 0 & \text{otherwise}, \end{cases}$$

a, b, c, d are given real numbers and N_1, N_2, N_3, N_4 are given integers.

solution

Since

$$|X(z)| = \left| \sum_{n=-\infty}^{\infty} x[n]z^{-n} \right| \leq \sum_{n=-\infty}^{\infty} |x[n]z^{-n}|,$$

$X(z)$ converges whenever $\sum |x[n]z^{-n}|$ converges.

(a) For the left-sided sequence,

$$\sum_{n=-\infty}^{\infty} l[n]z^{-n} = \sum_{n=-\infty}^{N_1} (az^{-1})^n.$$

For this sum to be finite, we need $|az^{-1}|$ to be larger than 1, since the summation goes to $-\infty$. Therefore $|z| < |a|$, which is a disk. When $N_1 \geq 0$, the point $z = 0$ should be excluded. This possible exception at $z = 0$ will not be explained later.

(b) For the right-sided sequence, $|z| > |b|$.

(c) For the two-sided sequence the region of convergence is empty since $c \neq 0$ (when $c = 0$, the sequence is finite-duration.) Note that had $t[n] = l[n] + r[n]$, and $|b| < |a|$, then the ROC is a ring $|b| < |z| < |a|$.

(d) For the finite-duration sequence, the region of convergence is the entire z-plane.