The empirical mode decomposition

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- Intrinsic mode functions
- The empirical mode decomposition method: the sifting process
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Introduction

- Data analysis is a necessary part in research and practical applications.
- A new method for analyzing nonlinear and non-stationary data has been developed.
- The method is the empirical mode decomposition (EMD).

IMF

- An intrinsic mode function (IMF) is a function that satisfies two conditions:

  (1)in the whole data set, **the number of extrema** and **the number of zero crossings** must either **equal** or **differ at most by one**

  (2) at any point, **the mean value** of the envelope defined by the local maxima and the envelope defined by the local minima is **zero**.

IMF

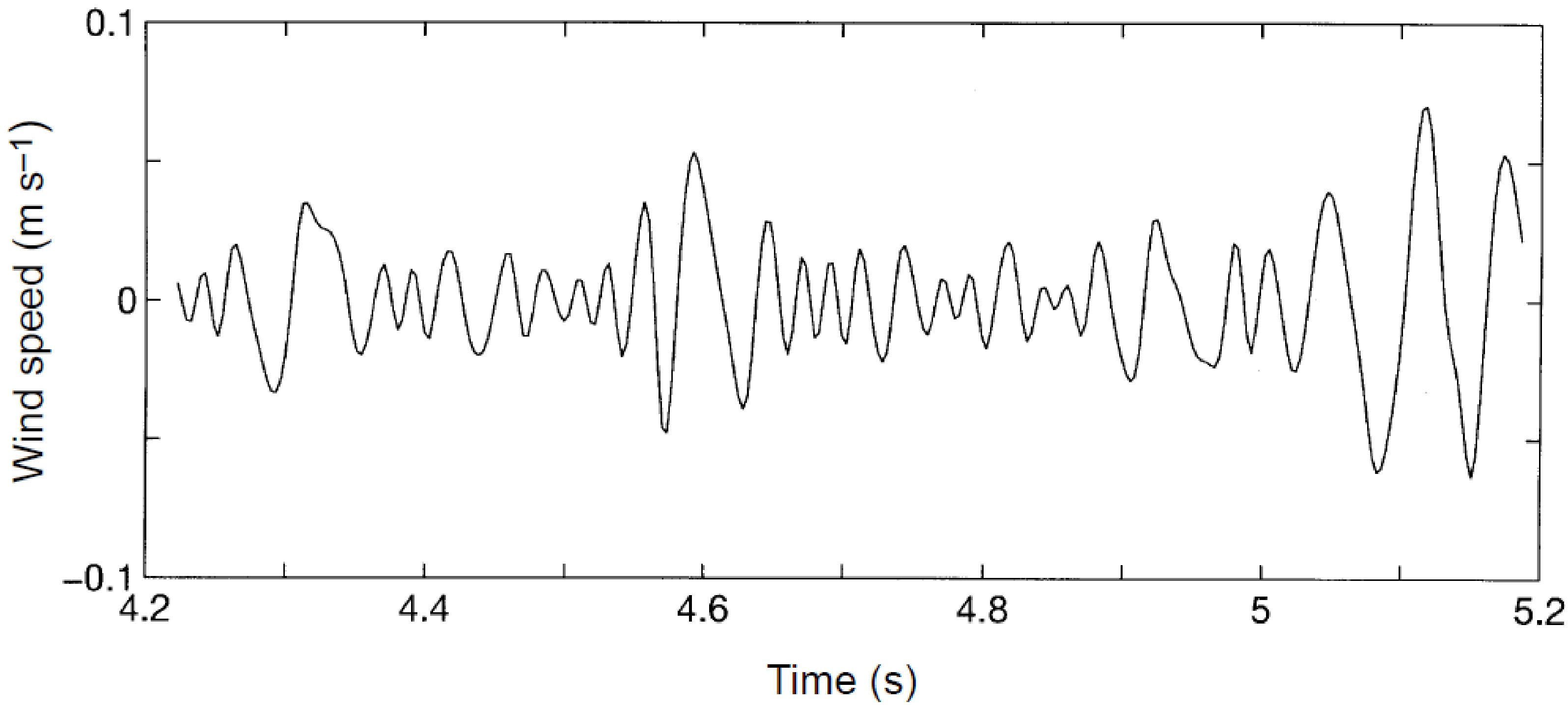


Figure 2. A typical intrinsic mode function with the same numbers of zero crossings and extrema, and symmetry of the upper and lower envelopes with respect to zero.

Cubic Spline

- If $k=\{x_0,\dots,x_m\}$ => given *m+1* points and *m* intervals

$$S_i(x)=a_i(x-x_i)^3+b_i(x-x_i)^2+c_i(x-x_i)+d_i$$

  - (1) $S_i(x_i)=y_i \quad for\, i=0:m-1$ and $S_{m-1}(x_m)=y_m$
  - (2) $S_i(x_{i+1})=S_{i+1}(x_{i+1}) \quad for\, i=0:m-2$
  - (3) $S_i^{'}(x_{i+1})=S^{'}{}_{i+1}(x_{i+1}) \quad for\, i=0:m-2$
  - (4) $S_i^{''}(x_{i+1})=S_{i+1}^{''}(x_{i+1}) \quad for\, i=0:m-2$

- We can define two extra boundary conditions.

- Natural Spline:

$$S_0''(x_0) = 0 \qquad S_{m-1}''(x_m) = 0$$

- Not-a-Knot Spline:

$$S_0'''(x_1) = S_1'''(x_1)$$

$$S_{m-2}'''(x_{m-1}) = S_{m-1}'''(x_{m-1})$$

The empirical mode decomposition method: the sifting process

- The decomposition is based on the assumptions:

  (1) the signal has at least two extrema: one maximum and one minimum

  (2) the characteristic time scale is defined by the time lapse between the extrema

  (3) if the data were totally devoid of extrema but contained only inflection points, then it can be differentiated once or more times to reveal the extrema.

EMD

1) Identification of all the extrema (maxima and minima) of the series $X(t)$.

2) Generation of the upper $u_{jk}(t)$ and lower envelope $l_{jk}(t)$ via **cubic spline** interpolation among all the maxima and minima, respectively.

3) Averaging of the two envelopes to compute a local mean series $m_{jk}(t)$.

$$m_{jk}(t) = (u_{jk}(t) + l_{jk}(t))/2$$

EMD

4) Subtraction of $m(t)$ from the data to obtain a IMF candidate

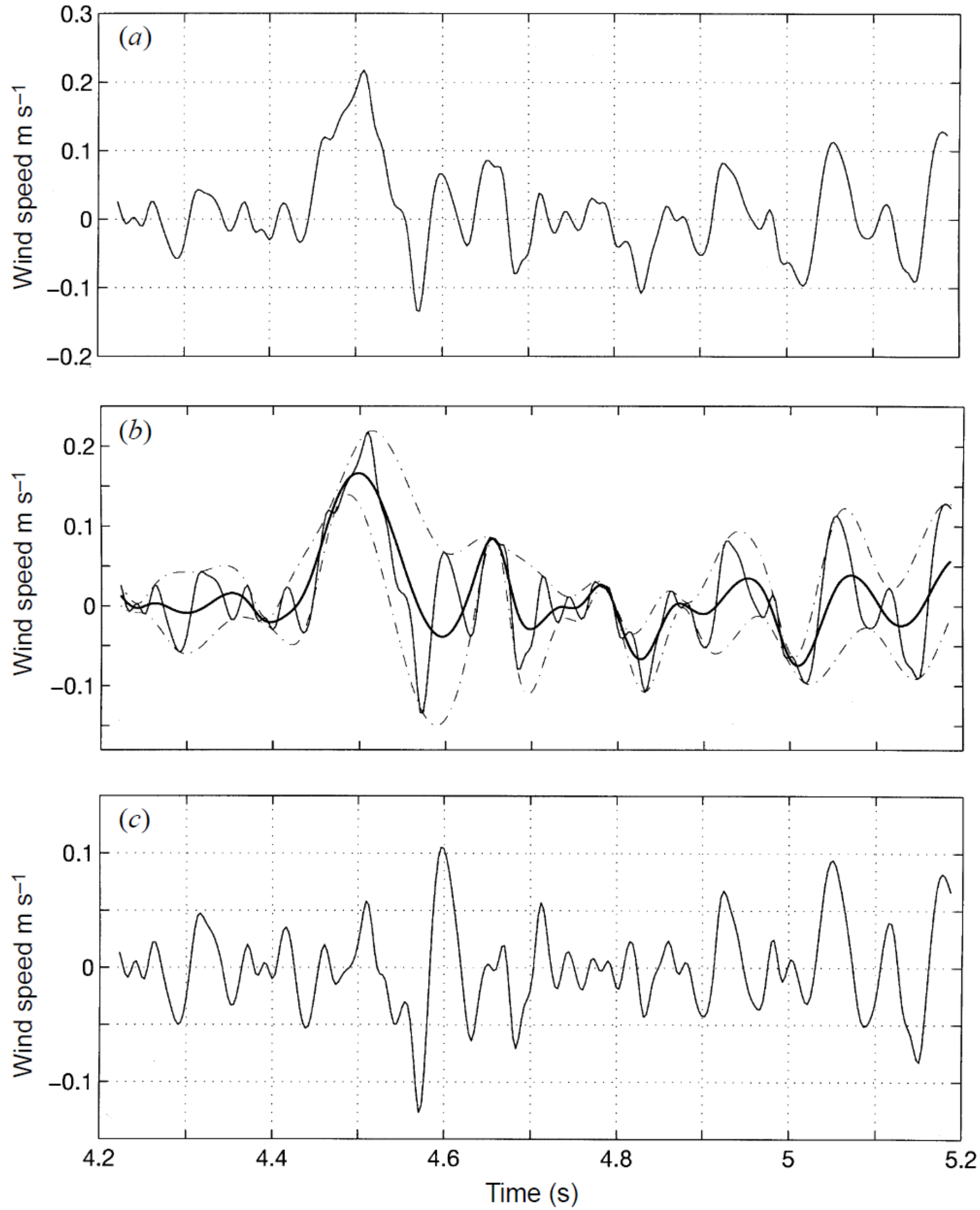
$h_{jk}(t) = x(t) - m_{jk}(t)$.

5) Check the properties of $h_{jk}(t)$:

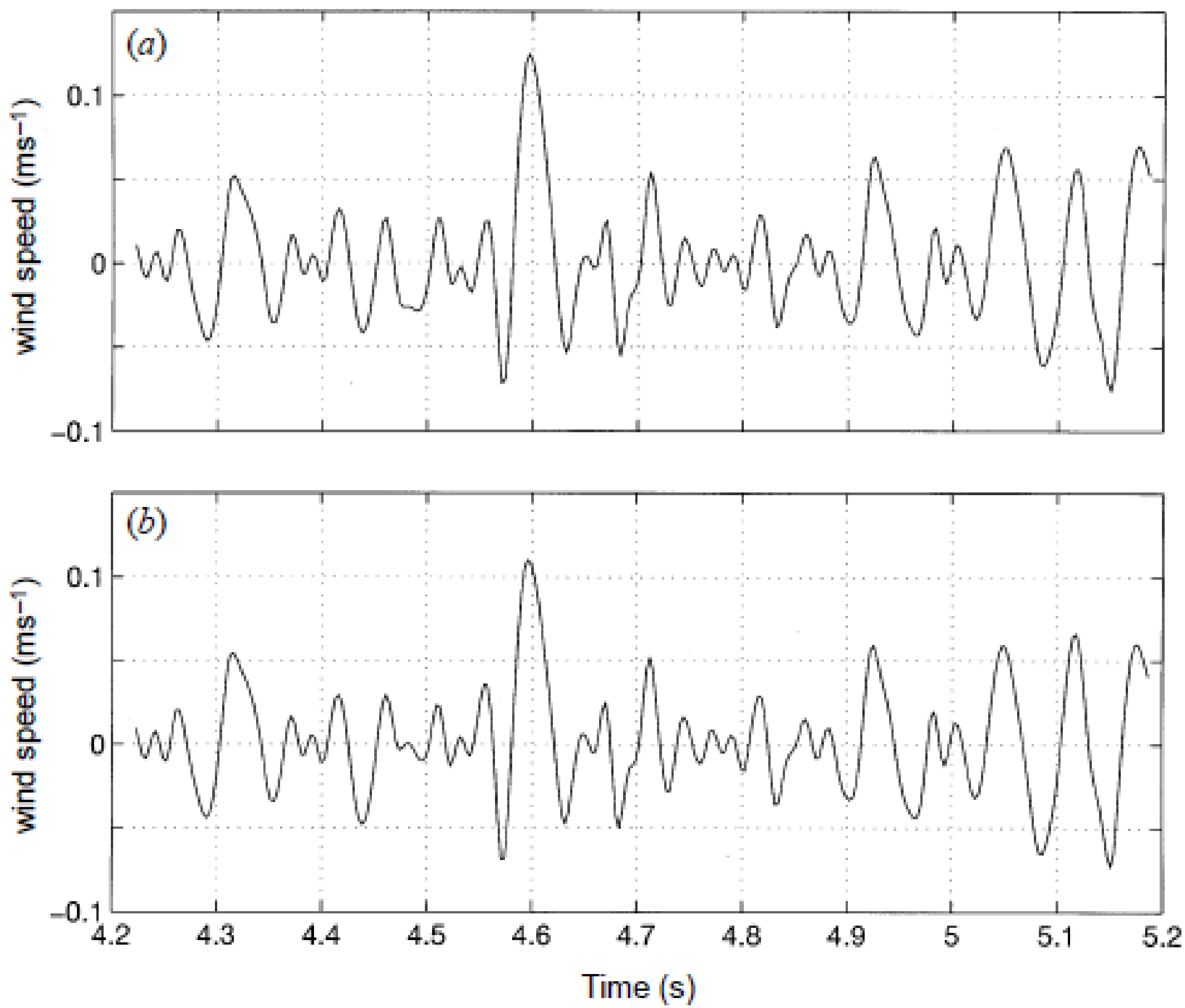
If $h_{jk}(t)$ is not a IMF: Repeat steps 1-4 until satisfy definition of he IMF.

If $h_{jk}(t)$ is a IMF :$c_j(t) = h_{jk}(t)$. Evaluate the $r_j(t) = X(t) - c_j(t)$.

Repeat the procedure from step 1 to step 5 by sifting the signal $r_j(t)$ (Sifting process).



(a)
(b)
(c)
Wind speed m s⁻¹
0.3
0.2
0.1
0
−0.1
−0.2
4.2
4.4
4.6
4.8
5
5.2
Time (s)


(a)
(b)
wind speed (ms⁻¹)
0.1
0
−0.1
4.2
4.3
4.4
4.5
4.6
4.7
4.8
4.9
5
5.1
5.2
Time (s)

EMD

- A typical value for standard deviation (SD) can be set between *0.2* and *0.3*.

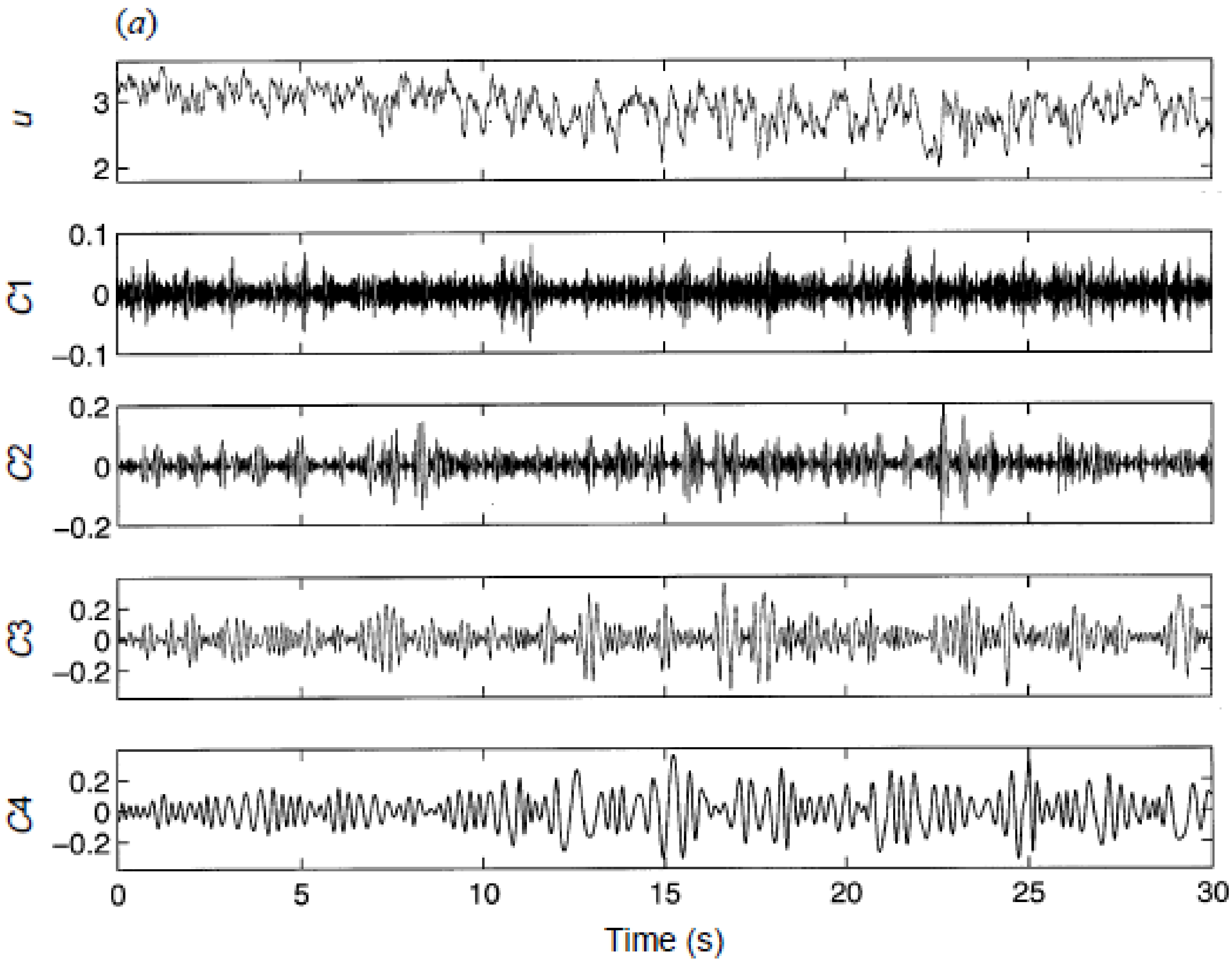
$$SD = \sum_{t=0}^{T} \left[\frac{\left| h_{k-1}(t) - h_k(t) \right|^2}{h^2{}_{k-1}(t)} \right]$$

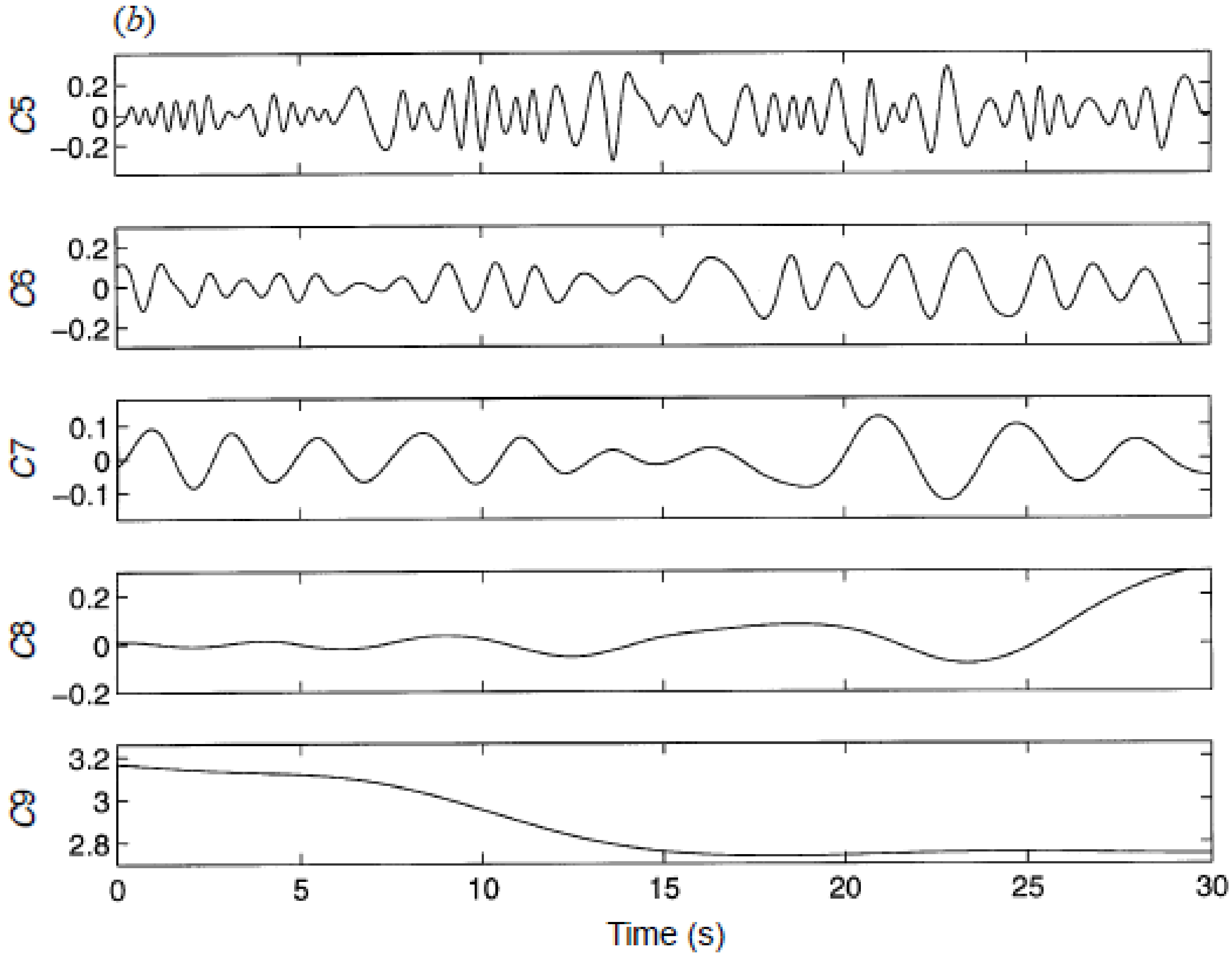
- If given $X(t) - c_1 = r_1$

$$r_1 - c_2 = r_2, \ldots, r_{n-1} - c_n = r_n$$

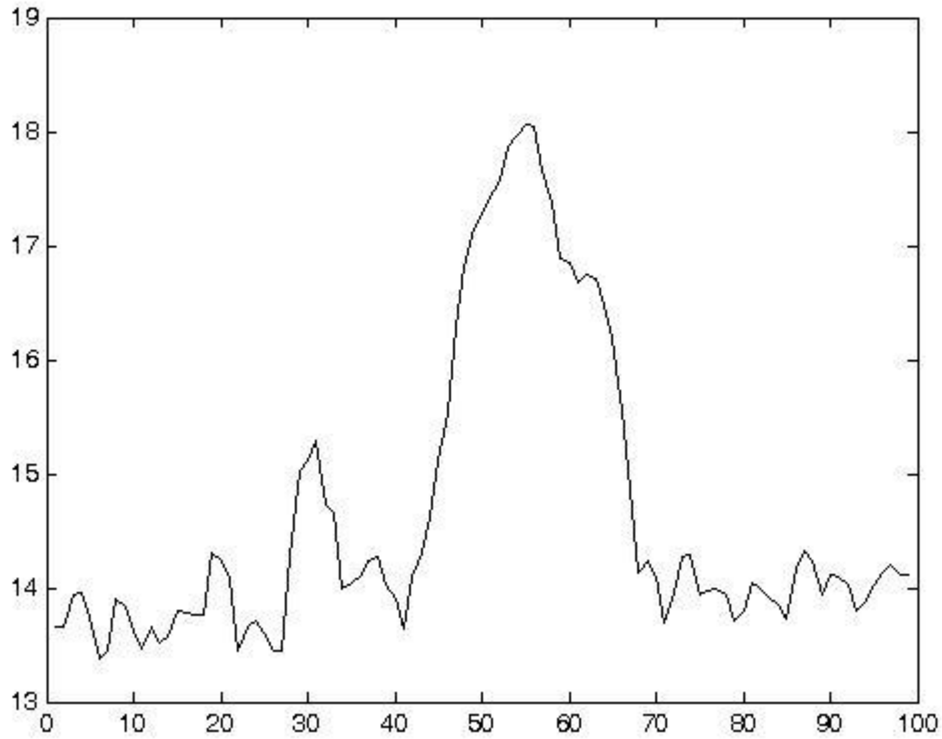
- r_n becomes a monotonic function from which no more IMF can be extracted.

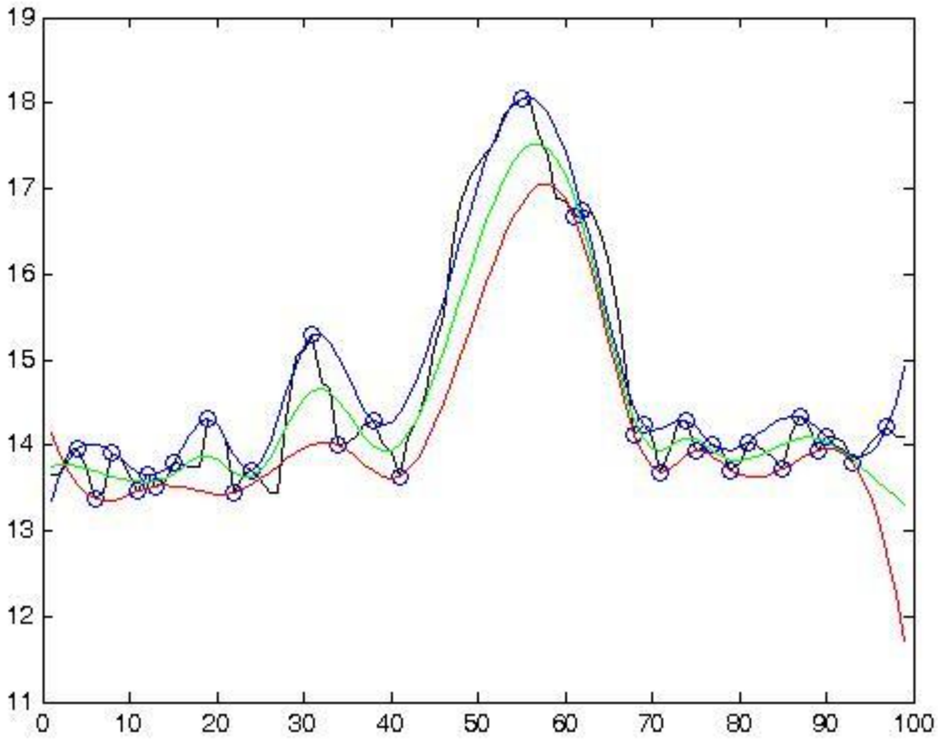
$$X(t) = \sum_{i=1}^{n} c_i + r_n$$


(a)
u
3
2
C1
0.1
0
−0.1
C2
0.2
0
−0.2
C3
0.2
0
−0.2
C4
0.2
0
−0.2
0
5
10
15
20
25
30
Time (s)

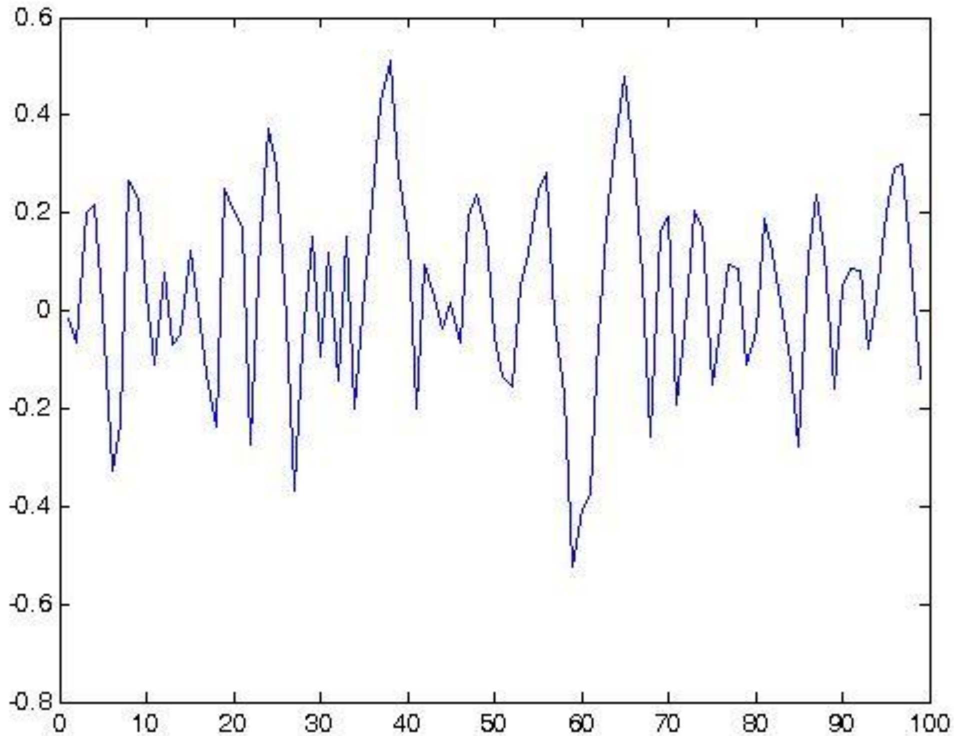

(b)
C5
C6
C7
C8
C9
0.2
0
−0.2
0.1
−0.1
3.2
3
2.8
0
5
10
15
20
25
30
Time (s)

$X(t)$

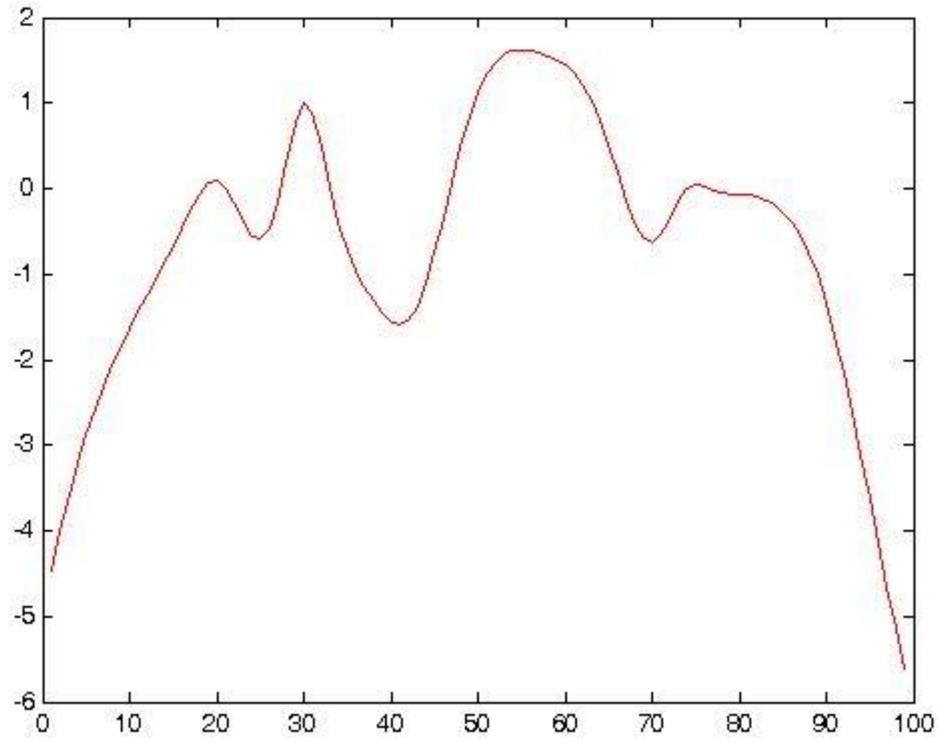



19
18
17
16
15
14
13
12
11
0
10
20
30
40
50
60
70
80
90
100

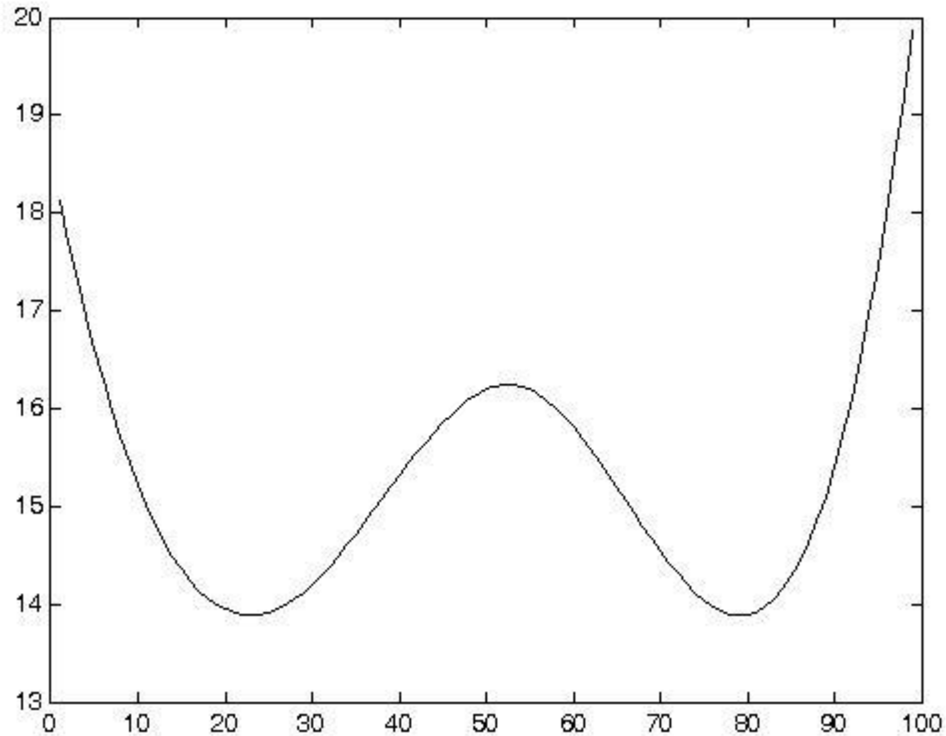
m_1

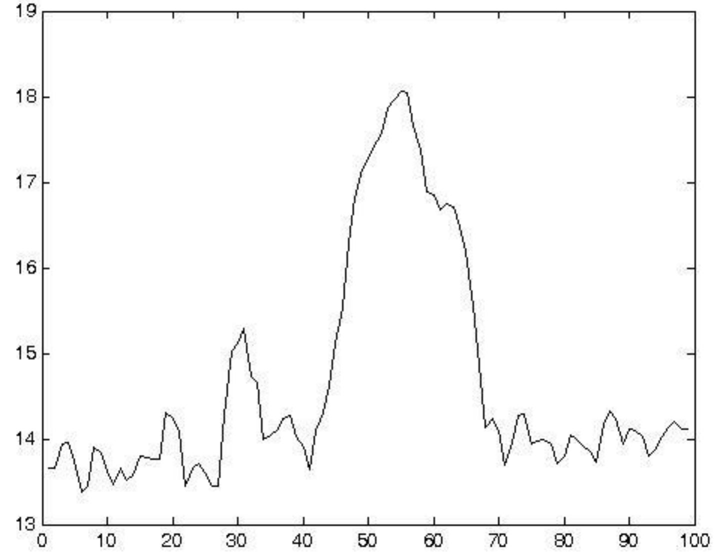


m_2



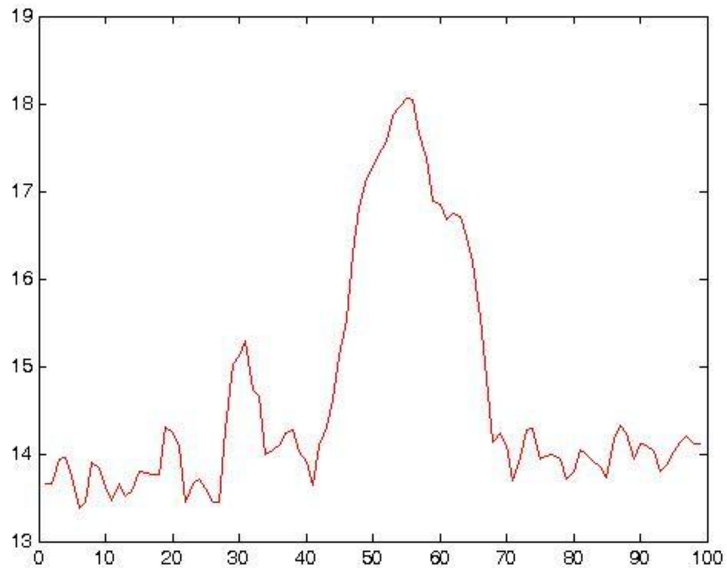
r_2





19
18
17
16
15
14
13
0
10
20
30
40
50
60
70
80
90
100

$X(t)$



19
18
17
16
15
14
13
0
10
20
30
40
50
60
70
80
90
100

$$X(t) = m_1 + m_2 + r_2$$