Linear Algebra Quiz2

1. (40%) Find the general solution of the following system of linear equations

$$\begin{aligned} u + 3v + 7w - 11y &= 6 \\ 2u - 4v + w + y &= 9 \\ u + 2v - 5w + 2y &= -5. \end{aligned}$$

   (hint: find the homogeneous solution and a particular solution)

   A= $\begin{bmatrix} 1 & 3 & 7 & -11 & | & 6 \\ 2 & -4 & 1 & 1 & | & 9 \\ 1 & 2 & -5 & 2 & | & -5 \end{bmatrix}$

   $\rightarrow by\ row\ operation\ for\ A \rightarrow \begin{bmatrix} 1 & 3 & 7 & -11 & | & 6 \\ 0 & 0 & 107 & -107 & | & 107 \\ 0 & -1 & -12 & 13 & | & -11 \end{bmatrix}$

   $so\ we\ can\ get\ the\ linear\ equations\ and\ solve:$

$$\begin{matrix} u + & 3v + & 7w - & 11y & = 6 & & u + 3v = 4y - 1 & & u = y + 2 \\ & & 107w - & 107y & = 107 & \rightarrow & w = 1 + y & \rightarrow & w = 1 + y \\ - & v - & 12w + & 13y & = -11 & & v = y - 1 & & v = y - 1 \end{matrix}$$

$$\therefore general\ solution = \left\{ \begin{bmatrix} t+2 \\ t-1 \\ t+1 \\ t \end{bmatrix} | t \in R \right\}$$

2. (40%) Given the following edge-node incidence matrix

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix},$$

   answer the following questions.

(a) (5%) How many nodes? How many edges?

(b) (5%) Draw the graph.

(c) (5%) What are the dimensions of the row space?

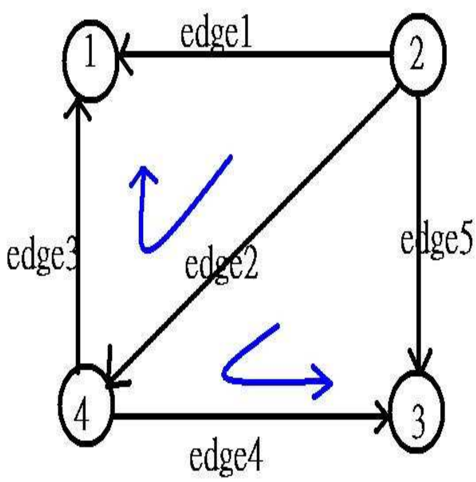
(d) (5%) How many independent loops are there in the graph?

(e) (10%)What is the nullspace?

(f) (10%) What is the column space?

(a) node=4,edge=5

(b)



(c) A = $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix} \rightarrow row\ operation\ for\ A$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore dim(RS(A)) = rank(A) = 3$

(d) by(b)

there are 2 independent loops in graph.

(e) by(c) will get nallspace of (A)

$$\begin{aligned} w - x &= 0 \\ -x + y &= 0 \rightarrow \ w = x = y = z \ \therefore nullsapce\ of\ (A) = \ \{ \begin{bmatrix} t \\ t \\ t \\ t \end{bmatrix} |t \in R \} \\ -y + z &= 0 \end{aligned}$$

(f) by(c)

$\because pivot\ at\ 1, 2, 3\ column$

$$\therefore CS(A) = span\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ -1 \\ 0 \end{bmatrix} \}$$

3. (40%) Suppose a linear transformation T maps vectors a, b, c to d, e, f, where

$$a = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \ b = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \ c = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \ d = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \ e = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \ f = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

Find the matrix M such that

$$T(x) = Mx \text{ for any } x \in R^3.$$

assume X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ =m $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ + n $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ +p $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$\rightarrow\ m = \frac{x+2y+z}{6},\ n = \frac{-x+z}{2},\ p = \frac{x-y+z}{3}$

$\therefore T(X) = MX = T(\begin{bmatrix} x \\ y \\ z \end{bmatrix})$

$= (\frac{x+2y+z}{6})T(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix})\ +\ (\frac{-x+z}{2})T(\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix})\ +\ (\frac{x-y+z}{3})T(\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix})$

$= (\frac{x+2y+z}{6})\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}\ +\ (\frac{-x+z}{2})\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}\ +\ (\frac{x-y+z}{3})\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} x \\ \frac{-x+y+2z}{3} \\ \frac{2x+y-z}{3} \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/3 & 1/3 & 2/3 \\ 2/3 & 1/3 & -1/3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\therefore M = \begin{bmatrix} 1 & 0 & 0 \\ -1/3 & 1/3 & 2/3 \\ 2/3 & 1/3 & -1/3 \\ 0 & 0 & 1 \end{bmatrix}$