Automatic Speech Recognition
Question Set 2

1. Given
$$Y = T\ X,$$
where X is an n-dimension random vector with mean μ_X and covariance Σ_X, T is an $m \times n$ matrix.

   (a) Show that $\mu_Y = T\mu_x$ and $\Sigma_Y = T\Sigma_X T'$, where T' is the transpose of T. Note that the covariance matrix of a random vector Z is defined as
   $$\Sigma_Z = E(Z - \mu_Z)(Z - \mu_Z)',$$
   where E is the expectation value operator.

   (b) Show that if X has a normal distribution, then so is Y. In other words, a linear transformation of a Gaussian vector is another Gaussian vector.

   (c) Show that if the row vectors of T consist of m eigenvectors of Σ_X with distinct eigenvalues, then Σ_Y is diagonal and the m components of Y become independent random variables.

2. Given N random samples, $\{X_1, X_2, \ldots, X_N\}$, the sample mean vector M is defined as
$$M = \frac{1}{N}\sum_{i=1}^{N} X_i,$$
and the (biased) sample covariance matrix is given by
$$V = \frac{1}{N}\sum_{i=1}^{N}(X_i - M)(X_i - M)',$$
These data are assumed to be independent samples of random vector X with mean μ_X and covariance Σ_X.

   (a) Show that
   $$E(M) = \mu_X, \text{ and } E(V) = \frac{N-1}{N}\Sigma_X.$$

   (b) Suppose X is normal. Is M the maximum likelihood estimator for μ_X? How about V for Σ_X?