4.1

(1)

$cof(3) = 9$, $cof(5) = -6$, $cof(6) = -5$, $cof(9) = 3$

inverse = $\begin{bmatrix} -3 & \frac{5}{3} \\ 2 & -1 \end{bmatrix}$

(2)

$cof(a_{11}) = \cos\theta$, $cof(a_{12}) = -\sin\theta$, $cof(a_{21}) = \sin\theta$, $cof(a_{22}) = \cos\theta$

inverse = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(3)

$cof(a_{11}) = b, cof(a_{12}) = -a, cof(a_{21}) = -b, cof(a_{22}) = a$

$N(A) = N(\begin{bmatrix} a & b \\ a & b \end{bmatrix}) \Rightarrow N(A) = ax + by = 0 = \begin{bmatrix} \frac{-b}{a} \\ 1 \end{bmatrix}$

4.4

$P_1 + P_2 = P_1({P_1}^T + {P_2}^T)P_2$

$det(P_1 + P_2) = det(P_1)det({P_1}^T + {P_2}^T)det(P_2)$

(P_1:even permutation $\Rightarrow$ $det(P_1) = 1$)

(P_2:odd permutation $\Rightarrow$ $det(P_2) = -1$)

$det(P_1 + P_2) = -det({P_1}^T + {P_2}^T) = -det((P_1 + P_2)^T) = -det(P_1 + P_2)$

$2det(P_1 + P_2) = 0 \Rightarrow det(P_1 + P_2) = 0$

4.10

$\begin{bmatrix} x & y & 1 \\ 2 & 8 & 1 \\ 4 & 7 & 1 \end{bmatrix}$ can row operation to $\begin{bmatrix} x & y & 1 \\ 2-x & 8-y & 0 \\ 4-x & 7-y & 0 \end{bmatrix}$

$det = 0$ $\Rightarrow$2nd and 3th row parelle and then we can say $(x, y), (2, 8)$ and $(4, 7)$ on the same line

and $det \begin{pmatrix} x & y & 1 \\ 2 & 8 & 1 \\ 4 & 7 & 1 \end{pmatrix} = x + 2y - 18$

4.12

(1)

$det = -6$

(2)

$det = 0$

4.15

$$\Rightarrow P = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots \\ 1 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$det(P) = (-1)^{n-1} \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix} = (-1)^{n-1}$$

4.16

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

$det = 6$

n is odd $det = n + 1$

n is even $det = -(n + 1)$

5.4

(1)

eigenvalue $=1, 3$

$N(1) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$N(3) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$s = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

(2)

eigenvalue = $2, 1$

$N(1) = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$N(2) = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$

$s = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

5.7

$u(t) = e^{\lambda t} X \Rightarrow A e^{\lambda t} X = \lambda e^{\lambda t} X$

$\lambda = 0, \pm\sqrt{-2}$

$\lambda(0) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \lambda(\sqrt{-2}) \begin{bmatrix} -1 \\ \sqrt{-2} \\ 1 \end{bmatrix}, \lambda(-\sqrt{-2}) = \begin{bmatrix} 1 \\ \sqrt{-2} \\ -1 \end{bmatrix}$

$u(t) = c_1 e^0 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c - 2e^{\sqrt{-2}t} \begin{bmatrix} -1 \\ \sqrt{-2} \\ 1 \end{bmatrix} + c_3 e^{-\sqrt{-2}} \begin{bmatrix} 1 \\ \sqrt{-2} \\ -1 \end{bmatrix}$

5.11

$u(t) = c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2$

$c_1 = 1, c_2 = 2 \Rightarrow u(t) = e^t \begin{bmatrix} 5 \\ 2 \end{bmatrix} + 2e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\Rightarrow u(0) = \begin{bmatrix} 5 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

$u_k = \begin{bmatrix} 5 \\ 2 \end{bmatrix} + 3^k \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

5.27 $\Rightarrow \lambda = 1, 0.1$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$$

as $k \to \infty$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$$

$$A \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{a+b}{3} \\ \frac{2a+2b}{3} \end{bmatrix}$$