linear algebra: solution 1

- 1.1

$$A = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix},\ B = \begin{pmatrix} 1.00000 & 0.50000 & 0.33333 \\ 2.00000 & 1.00000 & 0.66667 \\ 3.00000 & 1.50000 & 1.00000 \end{pmatrix},$$

$$AB = \begin{pmatrix} -8.00000 & -4.00000 & -2.66667 \\ -2.00000 & -1.00000 & -0.66667 \\ 4.00000 & 2.00000 & 1.33333 \end{pmatrix},$$

$$BA = \begin{pmatrix} 1.16667 & -0.66667 & -2.50000 \\ 2.33333 & -1.33333 & -5.00000 \\ 3.50000 & -2.00000 & -7.50000 \end{pmatrix},$$

$$A^2 = \begin{pmatrix} -5 & -2 & 1 \\ -2 & -2 & -2 \\ 1 & -2 & -5 \end{pmatrix}$$

- 1.4

$$\begin{pmatrix} 1 & 0 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 1 & 1 & 1 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & 0 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\to w = 3,\ v = 2,\ u = 1$$

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -2 & 6 \end{pmatrix}$$

$$\to w = -3,\ v = 3,\ u = 3$$

- 1.6 (a) 6 (b) more likely to be invertible (via simulation)

- 1.11

$$E = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix},\ E^2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix},\ E^8 = \begin{pmatrix} 1 & 0 \\ 8 & 1 \end{pmatrix},\ 8E = \begin{pmatrix} 8 & 0 \\ 8 & 8 \end{pmatrix}$$

- 1.13

$$c = (0\ 0\ 1)^T,\ x = (1\ -3\ 1)^T, \text{which is the last column of } A^{-1}$$

- 1.16

$$k = \begin{cases} 1, & \text{infinitely many solutions} \\ -1, & \text{no solution} \\ \text{otherwise}, & \text{unique solution} \end{cases}$$

- 1.18

$$A = \begin{pmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{v_3}{v_2} & 1 & 0 \\ 0 & \frac{v_4}{v_2} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = LU$$

For A^{-1}, work out the details of Gauss-Jordan,

$$\begin{pmatrix} A & | & I \end{pmatrix} \rightarrow \begin{pmatrix} I & | & A^{-1} \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} 1 & -\frac{v_1}{v_2} & 0 & 0 \\ 0 & \frac{1}{v_2} & 0 & 0 \\ 0 & -\frac{v_3}{v_2} & 1 & 0 \\ 0 & -\frac{v_4}{v_2} & 0 & 1 \end{pmatrix}$$

- 1.20 (a) 24, $n!$ (b) $k = 6$
- 1.27 (1) true (property 1I) (2) true, $A(A+I) = (A+I)A = I$ (3) false
- 1.29

$$(a) \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \ (b) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ (c) \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \ (d) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$