Linear Algebra: solution 3

- <u>2.4.3</u> Some answers can be read off directly from A or U. Note that the bases are not unique.
  - row space: dimension = 2, basis = $\{[1\ 2\ 0\ 1], [0\ 1\ 1\ 0]\}$
  - nullspace: dimension = 2, basis = $\{[2\ \ -1\ 1\ 0], [-1\ 0\ 0\ 1]\}$
  - column space: dimension = 2, basis = $\{[1\ 0\ 1]^T, [2\ 1\ 1]^T\}$
  - left nullspace: dimension = 1, basis = $\{[-1\ 0\ 1]^T\}$
- <u>2.4.10</u>
  - $\begin{bmatrix} 1 & 2 & 4 \end{bmatrix}$
  - $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \end{bmatrix}, abc \neq 0$
- <u>2.4.13</u>

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \end{bmatrix}, \quad \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix}.$$

- <u>2.4.18</u> A simply uses the spanning set of V as the rows.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 5 & 0 \end{bmatrix}.$$

  B must be a rank-1 matrix, since the dimension of V is 2. B can be $[0\ 0\ 1]$, so the row is obviously orthogonal to all vectors in the spanning set of V.

- <u>2.4.19</u>
  - The row space of A is the same as the row space of U, so a basis is $\{[0\ 1\ 2\ 3\ 4], [0\ 0\ 0\ 1\ 2]\}$.
  - The nullspace of A is the same as the nullspace of U, so a basis is $\{[1\ 0\ 0\ 0\ 0], [0\ \ -2\ 1\ 0\ 0], [0\ 2\ 0\ \ -2\ 1]\}$.
  - A basis of the column space of A consists of the same columns corresponding to independent columns in U, so a basis is $\{[1\ 1\ 0]^T, [3\ 4\ 1]^T\}$.
  - The last $m - r$ rows of L^{-1} is a basis for the left nullspace of A, so a basis is $\{[1\ \ -1\ 1]^T\}$.