2.6-5

(a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 90^\circ & -\sin 90^\circ \\ 0 & \sin 90^\circ & \cos 90^\circ \end{bmatrix} \begin{bmatrix} \cos 90^\circ & 0 & -\sin 90^\circ \\ 0 & 1 & 0 \\ \sin 90^\circ & 0 & \cos 90^\circ \end{bmatrix} \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 180^\circ & -\sin 180^\circ \\ 0 & \sin 180^\circ & \cos 180^\circ \end{bmatrix} \begin{bmatrix} \cos 180^\circ & 0 & -\sin 180^\circ \\ 0 & 1 & 0 \\ \sin 180^\circ & 0 & \cos 180^\circ \end{bmatrix} \begin{bmatrix} \cos 180^\circ & -\sin 180^\circ & 0 \\ \sin 180^\circ & \cos 180^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.6-17

(a)

$let\ v = (1,1),\ w = (1,0)\ v\ ,w \in R^{1x2}\ c \in R$

$T(v+w) = \frac{(2,1)}{\sqrt{2^2+1^2}} = \frac{1}{\sqrt{5}}(2,1)$

$T(v) + T(w) = \frac{1}{\sqrt{2}}(1,1) + (1,0)$

$\therefore T(v+w) \neq T(v) + T(w)$

$T(cv) = \frac{cv}{||cv||} = \frac{cv}{|c|*||v||} \neq c\frac{v}{||v||}$

(b)

$let\ v = (v_1, v_2, v_3),\ w = (w_1, w_2, w_3)\ c \in R$

$T(v+w) = v_1 + w_1 + v_2 + w_2 + v_3 + w_3 = (v_1 + v_2 + v_3) + (w_1 + w_2 + w_3) = T(v) + T(w)$

$T(cv) = cv_1 + cv_2 + cv_3 = cT(v)$

(c)

$let\ v = (v_1, v_2, v_3),\ w = (w_1, w_2, w_3)\ c \in R$

$T(v+w) = (v_1 + w_1, 2(v_2 + w_2), 3(v_3 + w_3))$

$T(v)+T(w) = (v_1, 2v_2, 3v_3)+(w_1, 2w_2, 3w_3) = (v_1+w_1, 2(v_2+w_2), 3(v_3+w_3))$

$\therefore T(v+w) = T(v) + T(w)$

$T(cv) = (cv_1, 2cv_2, 3cv_3) = c(v_1, 2v_2, 3v_3) = cT(v)$

(d)

$let\ v = [1\ 2],\ w = [2\ 1]\ c = -3$

$T(v+w) = 3$

$T(v) + T(w) = 4$

$\therefore T(v+w) \neq T(v) + T(w)$

$T(cv) = -3$

$cT(v) = -6$

$\therefore T(cv) \neq cT(v)$

$(b), (c)$ satisfy $T(v+w) = T(v) + T(w)$ and $T(cv) = cT(v)$

2.6-20

$let v = (v_1, v_2)\ ,\ w = (w_1, w_2),\ z = (0, 0),\ c \in R$
(a)

(i)

$T(z) = (0, 0)$

(ii)

$T(cv) = (cv_2, cv_1) = cT(v)$

(iii)

$T(v + w) = (v_2 + w_2, v_1 + w_1) = (v_2, v_1) + (w_2, w_1) = T(v) + T(w)$

$T(v) = (v_2, v_1)$ is linear

(b)

(i)

$T(z) = (0, 0)$

(ii)

$T(cv) = (cv_1, cv_1) = cT(v)$

(iii)

$T(v + w) = (v_1 + w_1, v_1 + w_1) = (v_1, v_1) + (w_1, w_1) = T(v) + T(w)$

$T(v) = (v_1, v_1)$ is linear

(c)

(i)

$T(z) = (0, 0)$

(ii)

$T(cv) = (0, cv_1) = cT(v)$

(iii)

$T(v + w) = (0, v_1 + w_1) = (0, v_1) + (0, w_1) = T(v) + T(w)$

$T(v) = (0, v_1)$ is linear

(d)

(i)

$T(z) = (0,1) \neq (0,0)$

$T(v) = (0,1)$ is not linear

2.6-22

$v = (v_1, v_2)$, $c \in R$, $T(v) = v$, except that $T(0, v_2) = (0,0)$

(1) $v_1 \neq 0$

$T(cv) = cv = cT(v)$

(2) $v_1 = 0$

$T(cv_1, cv_2) = T(0, cv_2) = (0,0) = cT(0, v_2)$

when $v_2 \neq 0$

$T((v_1, 0) + (0, v_2)) = (v_1, v_2)$

$T((v_1, 0)) + T((0, v_2)) = (v_1, 0)$

$T((v_1, 0) + (0, v_2)) \neq T((v_1, 0)) + T((0, v_2))$ when $v_2 \neq 0$

2.6-38

(a)

$H^T H = 4I$ and $HH^T = 4I$

$$H^{-1} = \frac{1}{4}H^T = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

(b)

$H^{-1}(7,5,3,1)^T = (4,1,2,0)$