5.3-5.4 Homework Solution

- 5.3-14

  (a)

  The columns are linearly dependent, so one of the eigenvalue is 0. And the column 1 + column2 = 2(column 3) the eigenvector is

  $$\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

  Ans:$\lambda_1 = 0$ the eigenvector of λ_1 is $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

  (b)

  It's Markov so the sum of the eigenvaluse equals the trace of A.

  $$tr(A) = 0.8$$

  It's Markov so one of the eigenvalue = 1

  $$1 + 0 + \lambda_2 = 0.8 \Rightarrow \lambda_2 = -0.2$$

  Ans: $\lambda_2 = -0.2$ $\lambda_3 = 1$

  (c)

  When $\lambda_2 = -0.2$ the eigenvector of λ_2 is $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

  When $\lambda_3 = 1$ the eigenvector of λ_3 is $\begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$

$A = PDP^{-1}$

$\Rightarrow A^k = PD^kP^{-1} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0^k & 0 & 0 \\ 0 & (-0.2)^k & 0 \\ 0 & 0 & 1^k \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-3}{10} \\ \frac{1}{2} & \frac{1}{-2} & 0 \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \end{bmatrix}$

$\Rightarrow \lim_{k\to\infty} A^k = P\left(\lim_{k\to\infty} D^k\right)P^{-1} = \begin{bmatrix} \frac{3}{10} & \frac{3}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{3}{10} & \frac{3}{10} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \end{bmatrix}$

$\Rightarrow \lim_{k\to\infty} A^k u_0 = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$

Ans: $\begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$

- 5.3-24

$\mathrm{AB} = \mathrm{S}\Lambda_\mathrm{A}\mathrm{S}^{-1}\mathrm{S}\Lambda_\mathrm{B}\mathrm{S}^{-1}$
$= \mathrm{S}\Lambda_\mathrm{A}\Lambda_\mathrm{B}\mathrm{S}^{-1}$
$= \mathrm{S}\Lambda_\mathrm{B}\Lambda_\mathrm{A}\mathrm{S}^{-1}$
$= \mathrm{S}\Lambda_\mathrm{B}\mathrm{S}^{-1}\mathrm{S}\Lambda_\mathrm{A}\mathrm{S}^{-1}$
$= \mathrm{BA}$

- 5.4-1

$A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$(A - \lambda I)\,x = 0$
$\lambda^2 + 2\lambda = 0$
$\lambda = 0, -2$

if $\lambda = 0$ eigenvector

$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

if $\lambda = -2$ eigenvector

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}, S = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$e^{At} = Se^{\Lambda t}S^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} e^0 & \\ & e^{-2t} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1+e^{-2t} & 1-e^{-2t} \\ 1-e^{-2t} & 1+e^{-2t} \end{bmatrix}$$

- 5.4-18

Every eigenvalues are real number when $(trace)^2 - 4\,(\det) \geq 0$,

$$(0)^2 - 4\,(-a^2 - b^2 + c^2) \geq 0$$
$$\Rightarrow 4a^2 + 4b^2 - 4c^2 \geq 0$$
$$\Rightarrow a^2 + b^2 \geq c^2$$

- 5.4-36

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$$

$$(A - \lambda I)\,x = 0$$
$$(1-\lambda)(3-\lambda) = 0$$
$$\lambda = 1, 3$$

if $\lambda = 1$ eigenvector

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

if $\lambda = 3$ eigenvector

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, S = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$e^{At} = Se^{\Lambda t}S^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} e^t & \\ & e^{3t} \end{bmatrix} \begin{bmatrix} 1 & \frac{-1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}$$
$$= \begin{bmatrix} e^t & \frac{1}{2}(e^{3t} - e^t) \\ 0 & e^{3t} \end{bmatrix}$$

When t = 0

$$\Rightarrow e^{At} = \begin{bmatrix} e^0 & \frac{1}{2}(e^0 - e^0) \\ 0 & e^0 \end{bmatrix} = I$$