3.4-3.Review Homework Solution

- 3.4-1

  (a)

$$p_1 = \frac{a_1^T b}{a_1^T a_1} a_1 = 2 \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{-1}{3} \end{bmatrix} = \begin{bmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{-2}{3} \end{bmatrix}$$

  (b)

$$p_2 = \frac{a_2^T b}{a_2^T a_2} a_2 = 2 \begin{bmatrix} \frac{-1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{-2}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{bmatrix}$$

  (c)

  projection p onto the plane of a_1 and a_2

$$p = p_1 + p_2 = \begin{bmatrix} \frac{2}{3} \\ \frac{8}{3} \\ \frac{2}{3} \end{bmatrix}$$

- 3.4-9

$$a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, c = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

  using Gram-Schmidt

$$q_1 = \frac{a}{\|a\|} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$B = b - \left(q_1^T b\right) q_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$q_2 = \frac{B}{\|B\|} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$C = c - \left(q_1^T c\right) q_1 - \left(q_2^T c\right) q_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$q_3 = \frac{C}{\|C\|} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$QR = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix} \begin{bmatrix} q_1^T a & q_1^T b & q_1^T c \\ & q_2^T b & q_2^T c \\ & & q_3^T c \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

- 3.4-32

$$A = a = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$B = b - \frac{a^T b}{a^T a} a = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{-1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -1 \\ 0 \end{bmatrix}$$

$$C = c - \frac{a^T c}{a^T a} a - \frac{B^T c}{B^T B} B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{-1}{3} \\ \frac{-1}{3} \\ \frac{2}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ -1 \end{bmatrix}$$

- 3.review-31

  we replace the plane $x + y - z = 0$ by the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

$$p = A(A^T A)^{-1} A^T b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

- 3.review-37

  (a)

for column space

$$a = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}, b = \begin{bmatrix} -6 \\ 6 \\ 8 \\ 0 \\ 8 \end{bmatrix}$$

$$q_1 = \frac{a}{\|a\|} = \frac{1}{10} \begin{bmatrix} 1 \\ 3 \\ 4 \\ 5 \\ 7 \end{bmatrix}$$

$$B = b - (q_1^T b) q_1 = \begin{bmatrix} -6 \\ 6 \\ 8 \\ 0 \\ 8 \end{bmatrix} - 10 \begin{bmatrix} \frac{1}{10} \\ \frac{3}{10} \\ \frac{4}{10} \\ \frac{5}{10} \\ \frac{7}{10} \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 4 \\ -5 \\ 1 \end{bmatrix}$$

$$q_2 = \frac{B}{\|B\|} = \frac{1}{10} \begin{bmatrix} -7 \\ 3 \\ 4 \\ -5 \\ 1 \end{bmatrix}$$

Orthonormal basis

$$Q = \begin{bmatrix} \frac{1}{10} & \frac{-7}{10} \\ \frac{3}{10} & \frac{3}{10} \\ \frac{2}{5} & \frac{2}{5} \\ \frac{1}{2} & \frac{-1}{2} \\ \frac{7}{10} & \frac{1}{10} \end{bmatrix}$$

(b)

$$QR = \begin{bmatrix} \frac{1}{10} & \frac{-7}{10} \\ \frac{3}{10} & \frac{3}{10} \\ \frac{2}{5} & \frac{2}{5} \\ \frac{1}{2} & \frac{-1}{2} \\ \frac{7}{10} & \frac{1}{10} \end{bmatrix} \begin{bmatrix} 10 & 10 \\ 0 & 10 \end{bmatrix}$$

(c)

$$R\hat{x} = Q^T b = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\hat{x} = R^{-1} \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & \frac{-1}{10} \\ 0 & \frac{1}{10} \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}$$