

1.2-7

(1)

$$\because (2) - (3) - (1)0 = 1$$

$\therefore \rightarrow \leftarrow$  contradictory. The system is singular.

(2)

replace 0 with -1

(3)

$$(u, v, w) = (3 + t, -1 - 2t, t) \quad t \in R$$

Let  $t = 0$

$$u = 3 \quad v = -1 \quad w = 0$$

1.3-11

(1)

$$d = 10$$

(2)

$$\begin{array}{rcl} 2x + 5y + & z & = 0 \\ y - & z & = 3 \\ - & z & = 2 \end{array}$$

(3)

$$d = 11$$

1.4-13

$$(A + B)^2 = (A + B)(B + A) = A^2 + AB + BA + B^2 = A(A + B) + B(A + B)$$

1.5-7

$$Ax = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix}$$

$$Lc = b \Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

$$Ux = c$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix} x = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

1.6-24

Suppose that  $I - AB$  is invertible.

Prove that  $B(I - AB)^{-1}A + I$  is the inverse of  $I - BA$

$$\begin{aligned} (I - BA)(B(I - AB)^{-1}A + I) &= (I - BA)B(I - AB)^{-1}A + I - BA \\ &= B(I - AB)(I - AB)^{-1}A + I - BA \\ &= BA + I - BA \\ &= I \end{aligned}$$

we can also prove  $(B(I - AB)^{-1}A + I)(I - BA) = I$

$I - BA$  is invertible.