2.1-2.3 Homework Solution

- 2.1-24
  (a)

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & b1 \\ 2 & 8 & 4 & b2 \\ -1 & -4 & -2 & b3 \end{array}\right] \to r_{12}^{(-2)} r_{13}^{1} \to \left[\begin{array}{ccc|c} 1 & 4 & 2 & b1 \\ 0 & 0 & 0 & b2-2b1 \\ 0 & 0 & 0 & b3+b1 \end{array}\right]$$

  當$b2 = 2b1$, $b3 = -b1$時有無限多組解
  (b)

$$\left[\begin{array}{cc|c} 1 & 4 & b1 \\ 2 & 9 & b2 \\ -1 & -4 & b3 \end{array}\right] \to r_{12}^{(-2)} r_{13}^{1} \to \left[\begin{array}{cc|c} 1 & 4 & b1 \\ 0 & 1 & b2-2b1 \\ 0 & 0 & b3+b1 \end{array}\right]$$

  當$b3 = -b1$時有1組解

- 2.2-32
  (a)

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix} \to r_{12}^{(-2)} r_{13}^{(-1)} \to \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & c-1 & 0 & 0 \end{bmatrix} = R$$

$$Ax = 0 \to Rx = 0$$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & c-1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix} = 0$$

  - (1) $c = 1$

$$x1 + x2 + 2x3 + 2x4 = 0$$

$$N(A) = \begin{bmatrix} -x2-2x3-2x4 \\ x2 \\ x3 \\ x4 \end{bmatrix} = x2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x3 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x4 \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$N = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- (2) $c \neq 1$

  $(c-1)x2 = 0 \rightarrow x2 = 0$

  $$N(A) = \begin{bmatrix} -(2x3 + 2x4) \\ 0 \\ x3 \\ x4 \end{bmatrix} = x3 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x4 \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

  $$N = \begin{bmatrix} -2 & -2 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1-c & 2 \\ 0 & 2-c \end{bmatrix}$$

- (1)$c = 1$

  $$A = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \rightarrow R = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

  $$Rx = 0 \rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} = 0$$

  $$N(A) = x1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

  $$N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- (2)$c = 2$

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} = R$$

$$Rx = 0 \rightarrow -x1 + 2x2 = 0$$

$$N(A) = x2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

– (3)$c \neq 1$ and $c \neq 2$

$$A = \begin{bmatrix} 1-c & 2 \\ 0 & 2-c \end{bmatrix} \rightarrow R = \begin{bmatrix} 1-c & 2 \\ 0 & 2-c \end{bmatrix}$$

$$Rx = 0 \rightarrow \begin{cases} (1-c)x1 + 2x2 = 0 & => \quad x1 = 0 \\ \quad (2-c)x2 = 0 & => \quad x2 = 0 \end{cases}$$

$$N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- 2.2-50

– (a)

$$\left[\begin{array}{ccc|c} 3 & 0 & 6 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow r_{21}^{(-3)} \rightarrow \left[\begin{array}{ccc|c} 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

解爲(0, a, 0), 其中a爲任意數

– (b)

$$\left[\begin{array}{ccc|c} 3 & 0 & 6 & 9 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

因爲$row_3 \neq 0$,所以無解

- 2.3-7

$$\begin{bmatrix} v_1^T \\ v_2^T \\ v_3^T \\ v_4^T \\ v_5^T \\ v_6^T \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

v1,v2 and v3 are independent vectors.
This number is the dimension of the space spanned by the v's.

- 2.3-34

the basis: $\begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} = 1/5 * \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 1/2 * \begin{bmatrix} 4 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 5 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$