第 2 章

Problem 2.3.

Suppose that you enter into a short futures contract to sell July silver for $17.20 per ounce. The size of the contract is 5,000 ounces. The initial margin is $4,000, and the maintenance margin is $3,000. What change in the futures price will lead to a margin call? What happens if you do not meet the margin call?

There will be a margin call when $1,000 has been lost from the margin account. This will occur when the price of silver increases by 1,000/5,000 = $0.20. The price of silver must therefore rise to $17.40 per ounce for there to be a margin call. If the margin call is not met, your broker closes out your position.

Problem 2.10.

Explain how margin accounts protect futures traders against the possibility of default.

Margin is money deposited by a trader with his or her broker. It acts as a guarantee that the trader can cover any losses on the futures contract. The balance in the margin account is adjusted daily to reflect gains and losses on the futures contract. If losses lead to the balance in the margin account falling below a certain level, the trader is required to deposit a further margin. This system makes it unlikely that the trader will default. A similar system of margin accounts makes it unlikely that the trader's broker will default on the contract it has with the clearing house member and unlikely that the clearing house member will default with the clearing house.

Problem 2.23.

Suppose that on October 24, 2018, a company sells one April 2019 live-cattle futures contracts. It closes out its position on January 21, 2019. The futures price (per pound) is 121.20 cents when it enters into the contract, 118.30 cents when it closes out its position, and 118.80 cents at the end of December 2018. One contract is for the delivery of 40,000 pounds of cattle. What is the total profit? How is it taxed if the company is (a) a hedger and (b) a speculator? Assume that the company has a December 31 year end.

The total profit is

$$40{,}000 \times (1.2120 - 1.1830) = \$1{,}160$$

If the company is a hedger this is all taxed in 2019. If it is a speculator

$$40{,}000 \times (1.2120 - 1.1880) = \$960$$

is taxed in 2018 and

$$40{,}000 \times (1.1880 - 1.1830) = \$200$$

is taxed in 2019.

Problem 2.31.

Suppose that there are no storage costs for crude oil and the interest rate for borrowing or lending is 4% per annum. How could you make money if the June and December futures contracts for a particular year trade at $50 and $56?

You could go long one June oil contract and short one December contract. In June you take delivery of the oil borrowing $50 per barrel at 4% to meet cash outflows. The interest accumulated in six months is about 50×0.04×1/2 or $1 per barrel. In December the oil is sold for $56 per barrel which is more than the $51 that has to be repaid on the loan. The strategy therefore leads to a profit. Note that this profit is independent of the actual price of oil in June and December. It will be slightly affected by the daily settlement procedures.

第 3 章

Problem 3.3.

Explain what is meant by a perfect hedge. Does a perfect hedge always lead to a better outcome than an imperfect hedge? Explain your answer.

A *perfect hedge* is one that completely eliminates the hedger's risk. A perfect hedge does not always lead to a better outcome than an imperfect hedge. It just leads to a more certain outcome.

Consider a company that hedges its exposure to the price of an asset. Suppose the asset's price movements prove to be favorable to the company. A perfect hedge totally neutralizes the company's gain from these favorable price movements. An imperfect hedge, which only partially neutralizes the gains, might well give a better outcome.

Problem 3.7.

A company has a $20 million portfolio with a beta of 1.2. It would like to use futures contracts on a stock index to hedge its risk. The index futures is currently standing at 1080, and each contract is for delivery of $250 times the index. What is the hedge that minimizes risk? What should the company do if it wants to reduce the beta of the portfolio to 0.6?

The formula for the number of contracts that should be shorted gives

$$1.2 \times \frac{20,000,000}{1080 \times 250} = 88.9$$

Rounding to the nearest whole number, 89 contracts should be shorted. To reduce the beta to 0.6, half of this position, or a short position in 44 contracts, is required.

Problem 3.13.

"If the minimum-variance hedge ratio is calculated as 1.0, the hedge must be perfect." Is this statement true? Explain your answer.

The statement is not true. The minimum variance hedge ratio is

$$\rho \frac{\sigma_S}{\sigma_F}$$

It is 1.0 when $\rho = 0.5$ and $\sigma_S = 2\sigma_F$. Since $\rho < 1.0$ the hedge is clearly not perfect.

Problem 3.30.

It is July 16. A company has a portfolio of stocks worth $100 million. The beta of the portfolio is 1.2. The company would like to use the December futures contract on a stock index to change beta of the portfolio to 0.5 during the period July 16 to November 16. The index is currently 2,000, and each contract is on $250 times the index.

a) *What position should the company take?*
b) *Suppose that the company changes its mind and decides to increase the beta of the portfolio from 1.2 to 1.5. What position in futures contracts should it take?*

a) The company should short

$$\frac{(1.2-0.5)\times 100{,}000{,}000}{2{,}000\times 250}$$

or 140 contracts.

b) The company should take a long position in

$$\frac{(1.5-1.2)\times 100{,}000{,}000}{2{,}000\times 250}$$

or 60 contracts.

第 5 章

Problem 5.6.

Explain carefully the meaning of the terms convenience yield and cost of carry. What is the relationship between futures price, spot price, convenience yield, and cost of carry?

Convenience yield measures the extent to which there are benefits obtained from ownership of the physical asset that are not obtained by owners of long futures contracts. The *cost of carry* is the interest cost plus storage cost less the income earned. The futures price, F_0, and spot price, S_0, are related by

$$F_0 = S_0 e^{(c-y)T}$$

where c is the cost of carry, y is the convenience yield, and T is the time to maturity of the futures contract.

Problem 5.9.

A one-year long forward contract on a non-dividend-paying stock is entered into when the stock price is $40 and the risk-free rate of interest is 5% per annum with continuous compounding.

a) *What are the forward price and the initial value of the forward contract?*
b) *Six months later, the price of the stock is $45 and the risk-free interest rate is still 5%. What are the forward price and the value of the forward contract?*

a) The forward price, F_0, is given by equation (5.1) as:

$$F_0 = 40e^{0.05\times 1} = 42.05$$

or $42.05. The initial value of the forward contract is zero.

b) The delivery price K in the contract is $42.05. The value of the contract, f, after six months is given by equation (5.5) as:

$$f = 45 - 42.05e^{-0.05\times 0.5} = 3.99$$

i.e., it is $3.99. The forward price is:

$$45e^{0.05\times 0.5} = 46.14$$

or $46.14.

Problem 5.14.

The two-month interest rates in Switzerland and the United States are, respectively, 1% and 2% per annum with continuous compounding. The spot price of the Swiss franc is $1.0500. The futures price for a contract deliverable in two months is also $1.0500. What arbitrage opportunities does this create?

The theoretical futures price is

$$1.0500e^{(0.02-0.01)\times 2/12} = 1.0518$$

The actual futures price is too low. This suggests that a Swiss arbitrageur should sell Swiss francs for US dollars and buy Swiss francs back in the futures market.

Problem 5.15.

The spot price of silver is $25 per ounce. The storage costs are $0.24 per ounce per year payable quarterly in advance. Assuming that interest rates are 5% per annum for all maturities, calculate the futures price of silver for delivery in nine months.

The present value of the storage costs for nine months are

$$0.06+0.06e^{-0.05\times 0.25}+0.06e^{-0.05\times 0.5}= 0.178$$

or $0.178. The futures price is from equation (5.11) given by F_0 where

$$F_0 = (25.000 + 0.178)e^{0.05\times 0.75} = 26.14$$

i.e., it is $26.14 per ounce.

Problem 5.30.

A stock is expected to pay a dividend of $1 per share in two months and in five months. The stock price is $50, and the risk-free rate of interest is 8% per annum with continuous compounding for all maturities. An investor has just taken a short position in a six-month forward contract on the stock.

a) *What are the forward price and the initial value of the forward contract?*
b) *Three months later, the price of the stock is $48 and the risk-free rate of interest is still 8% per annum. What are the forward price and the value of the short position in the forward contract?*

a) The present value, I, of the income from the security is given by:

$$I = 1\times e^{-0.08\times 2/12} + 1\times e^{-0.08\times 5/12} = 1.9540$$

From equation (5.2) the forward price, F_0, is given by:

$$F_0 = (50 - 1.9540)e^{0.08\times 0.5} = 50.01$$

or $50.01. The initial value of the forward contract is (by design) zero. The fact that the forward price is very close to the spot price should come as no surprise. When the compounding frequency is ignored the dividend yield on the stock equals the risk-free rate of interest.

b) In three months:

$$I = e^{-0.08\times 2/12} = 0.9868$$

The delivery price, K, is 50.01. From equation (5.6) the value of the short forward contract, f, is given by

$$f = -(48 - 0.9868 - 50.01e^{-0.08\times 3/12}) = 2.01$$

and the forward price is

$$(48 - 0.9868)e^{0.08\times 3/12} = 47.96$$

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第 10 章：

Problem 10.11、10.12

第 11 章：

Problem 11.11、11.13、11.14、11.16、11.18

第 12 章：

Problem 12.8、12.23、12.25、12.26、12.28

√Problem 10.11.

Describe the terminal value of the following portfolio: a newly entered-into long forward contract on an asset and a long position in a European put option on the asset with the same maturity as the forward contract and a strike price that is equal to the forward price of the asset at the time the portfolio is set up. Show that the European put option has the same value as a European call option with the same strike price and maturity.

The terminal value of the long forward contract is:

$$S_T - F_0$$

where S_T is the price of the asset at maturity and F_0 is the forward price of the asset at the time the portfolio is set up. (The delivery price in the forward contract is also F_0.)

The terminal value of the put option is:

$$\max(F_0 - S_T, 0)$$

The terminal value of the portfolio is therefore

$$S_T - F_0 + \max(F_0 - S_T, 0)$$

$$= \max(0, S_T - F_0)$$

This is the same as the terminal value of a European call option with the same maturity as the forward contract and an exercise price equal to F_0. This result is illustrated in the Figure S10.5.

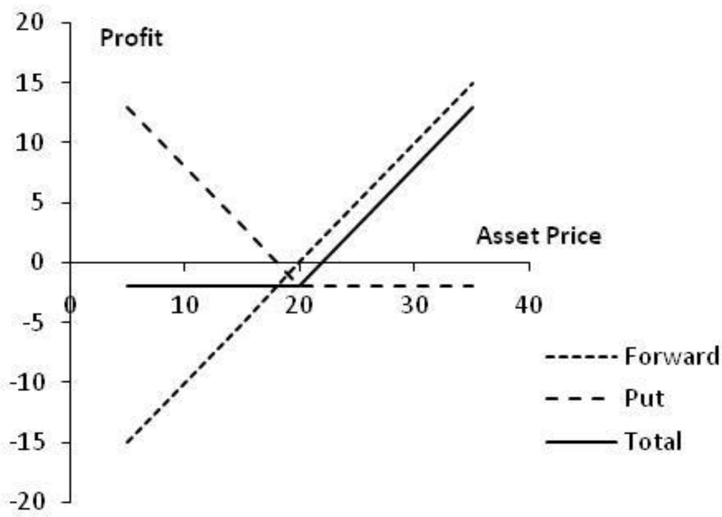

Figure S10.5: Profit from portfolio in Problem 10.11

We have shown that the forward contract plus the put is worth the same as a call with the same strike price and time to maturity as the put. The forward contract is worth zero at the time the portfolio is set up. It follows that the put is worth the same as the call at the time the portfolio is set up.

√Problem 10.12.

A trader buys a call option with a strike price of $45 and a put option with a strike price of $40. Both options have the same maturity. The call costs $3 and the put costs $4. Draw a diagram showing the variation of the trader's profit with the asset price.

Figure S10.6 shows the variation of the trader's position with the asset price. We can divide the alternative asset prices into three ranges:

a) When the asset price less than $40, the put option provides a payoff of $40-S_T$ and the call option provides no payoff. The options cost $7 and so the total profit is $33-S_T$.

b) When the asset price is between $40 and $45, neither option provides a payoff. There is a net loss of $7.

c) When the asset price greater than $45, the call option provides a payoff of S_T-45 and the put option provides no payoff. Taking into account the $7 cost of the options, the total profit is S_T-52.

The trader makes a profit (ignoring the time value of money) if the stock price is less than $33 or greater than $52. This type of trading strategy is known as a strangle and is discussed in Chapter 12.

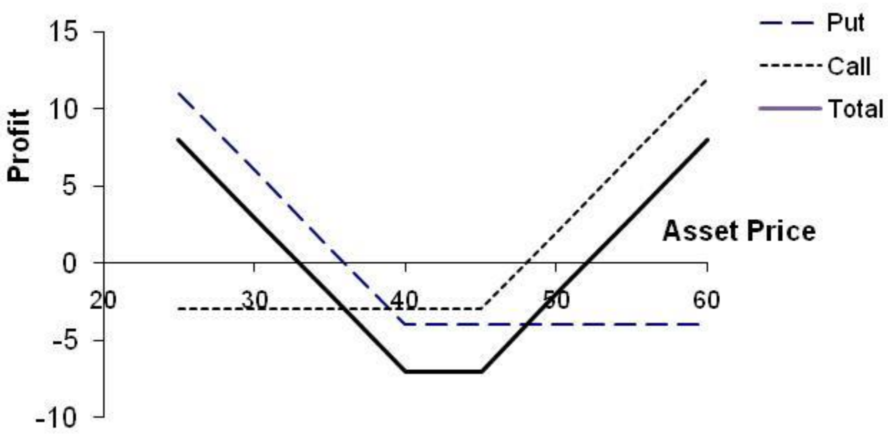

Figure S10.6: Profit from trading strategy in Problem 10.12

√Problem 11.11.

A four-month European call option on a dividend-paying stock is currently selling for \$5. The stock price is \$64, the strike price is \$60, and a dividend of \$0.80 is expected in one month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?

The present value of the strike price is $60e^{-0.12\times 4/12} = \$57.65$. The present value of the dividend is $0.80e^{-0.12\times 1/12} = 0.79$. Because

$$5 < 64 - 57.65 - 0.79$$

the condition in equation (11.8) is violated. An arbitrageur should buy the option and short the stock. This generates $64 - 5 = \$59$. The arbitrageur invests \$0.79 of this at 12% for one month to pay the dividend of \$0.80 in one month. The remaining \$58.21 is invested for four months at 12%. Regardless of what happens a profit will materialize.

If the stock price declines below \$60 in four months, the arbitrageur loses the \$5 spent on the option but gains on the short position. The arbitrageur shorts when the stock price is \$64, has to pay dividends with a present value of \$0.79, and closes out the short position when the stock price is \$60 or less. Because \$57.65 is the present value of \$60, the short position generates at least $64 - 57.65 - 0.79 = \$5.56$ in present value terms. The present value of the arbitrageur's gain is therefore at least $5.56 - 5.00 = \$0.56$.

If the stock price is above \$60 at the expiration of the option, the option is exercised. The arbitrageur buys the stock for \$60 in four months and closes out the short position. The present value of the \$60 paid for the stock is \$57.65 and as before the dividend has a present value of \$0.79. The gain from the short position and the exercise of the option is therefore exactly $64 - 57.65 - 0.79 = \$5.56$. The arbitrageur's gain in present value terms is exactly $5.56 - 5.00 = \$0.56$.

√Problem 11.13.

Give an intuitive explanation of why the early exercise of an American put becomes more attractive as the risk-free rate increases and volatility decreases.

The early exercise of an American put is attractive when the interest earned on the strike price is greater than the insurance element lost. When interest rates increase, the value of the interest earned on the strike price increases making early exercise more attractive. When volatility decreases, the insurance element is less valuable. Again this makes early exercise more attractive.

√Problem 11.14.

The price of a European call that expires in six months and has a strike price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in two months and again in five months. Risk-free interest rates (all maturities) are 10%. What is the price of a European put *option that expires in six months and has a strike price of $30?*

Using the notation in the chapter, put-call parity [equation (11.10)] gives

$$c + Ke^{-rT} + D = p + S_0$$

or

$$p = c + Ke^{-rT} + D - S_0$$

In this case

$$p = 2 + 30e^{-0.1\times 6/12} + (0.5e^{-0.1\times 2/12} + 0.5e^{-0.1\times 5/12}) - 29 = 2.51$$

In other words the put price is $2.51.

√Problem 11.16.

The price of an American call on a non-dividend-paying stock is $4. The stock price is $31, the strike price is $30, and the expiration date is in three months. The risk-free interest rate is 8%. Derive upper and lower bounds for the price of an American put on the same stock with the same strike price and expiration date.

From equation (11.7)

$$S_0 - K \le C - P \le S_0 - Ke^{-rT}$$

In this case

$$31 - 30 \le 4 - P \le 31 - 30e^{-0.08\times 0.25}$$

or

$$1.00 \le 4.00 - P \le 1.59$$

or

$$2.41 \le P \le 3.00$$

Upper and lower bounds for the price of an American put are therefore $2.41 and $3.00.

√Problem 11.18.

Prove the result in equation (11.7). (Hint: For the first part of the relationship consider (a) a portfolio consisting of a European call plus an amount of cash equal to K and (b) a portfolio consisting of an American put option plus one share.)

As in the text we use c and p to denote the European call and put option price, and C and P to denote the American call and put option prices. Because $P \ge p$, it follows from put–call parity that

$$P \ge c + Ke^{-rT} - S_0$$

and since $c = C$,

$$P \geq C + Ke^{-rT} - S_0$$

or

$$C - P \leq S_0 - Ke^{-rT}$$

For a further relationship between C and P, consider

Portfolio I: One European call option plus an amount of cash equal to K.

Portfolio J: One American put option plus one share.

Both options have the same exercise price and expiration date. Assume that the cash in portfolio I is invested at the risk-free interest rate. If the put option is not exercised early portfolio J is worth

$$\max(S_T, K)$$

at time T. Portfolio I is worth

$$\max(S_T - K, 0) + Ke^{rT} = \max(S_T, K) - K + Ke^{rT}$$

at this time. Portfolio I is therefore worth more than portfolio J. Suppose next that the put option in portfolio J is exercised early, say, at time τ. This means that portfolio J is worth K at time τ. However, even if the call option were worthless, portfolio I would be worth $Ke^{r\tau}$ at time τ. It follows that portfolio I is worth at least as much as portfolio J in all circumstances. Hence

$$c + K \geq P + S_0$$

Since $c = C$,

$$C + K \geq P + S_0$$

or

$$C - P \geq S_0 - K$$

Combining this with the other inequality derived above for $C - P$, we obtain

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$$

√Problem 12.8.

Use put–call parity to relate the initial investment for a bull spread created using calls to the initial investment for a bull spread created using puts.

A bull spread using calls provides a profit pattern with the same general shape as a bull spread using puts (see Figures 12.2 and 12.3 in the text). Define p_1 and c_1 as the prices of put and call with strike price K_1 and p_2 and c_2 as the prices of a put and call with strike price K_2. From put-call parity

$$p_1 + S = c_1 + K_1 e^{-rT}$$

$$p_2 + S = c_2 + K_2 e^{-rT}$$

Hence:

$$p_1 - p_2 = c_1 - c_2 - (K_2 - K_1)e^{-rT}$$

This shows that the initial investment when the spread is created from puts is less than the initial investment when it is created from calls by an amount $(K_2 - K_1)e^{-rT}$. In fact as mentioned in the text the initial investment when the bull spread is created from puts is negative, while the initial investment when it is created from calls is positive.

The profit when calls are used to create the bull spread is higher than when puts are used by $(K_2 - K_1)(1 - e^{-rT})$. This reflects the fact that the call strategy involves an additional risk-free investment of $(K_2 - K_1)e^{-rT}$ over the put strategy. This earns interest of $(K_2 - K_1)e^{-rT}(e^{rT} - 1) = (K_2 - K_1)(1 - e^{-rT})$.

√Problem 12.23.

Three put options on a stock have the same expiration date and strike prices of $55, $60, and $65. The market prices are $3, $5, and $8, respectively. Explain how a butterfly spread can be created. Construct a table showing the profit from the strategy. For what range of stock prices would the butterfly spread lead to a loss?

A butterfly spread is created by buying the $55 put, buying the $65 put and selling two of the $60 puts. This costs $3 + 8 - 2 \times 5 = \$1$ initially. The following table shows the profit/loss from the strategy.

Stock Price	*Payoff*	*Profit*
$S_T \geq 65$	0	-1
$60 \leq S_T < 65$	$65 - S_T$	$64 - S_T$
$55 \leq S_T < 60$	$S_T - 55$	$S_T - 56$
$S_T < 55$	0	-1

The butterfly spread leads to a loss when the final stock price is greater than $64 or less than $56.

√Problem 12.25.

Draw a diagram showing the variation of an investor's profit and loss with the terminal stock price for a portfolio consisting of

a. *One share and a short position in one call option*
b. *Two shares and a short position in one call option*
c. *One share and a short position in two call options*
d. *One share and a short position in four call options*

In each case, assume that the call option has an exercise price equal to the current stock price.

The variation of an investor's profit/loss with the terminal stock price for each of the four strategies is shown in Figure S12.5. In each case the dotted line shows the profits from the components of the investor's position and the solid line shows the total net profit.

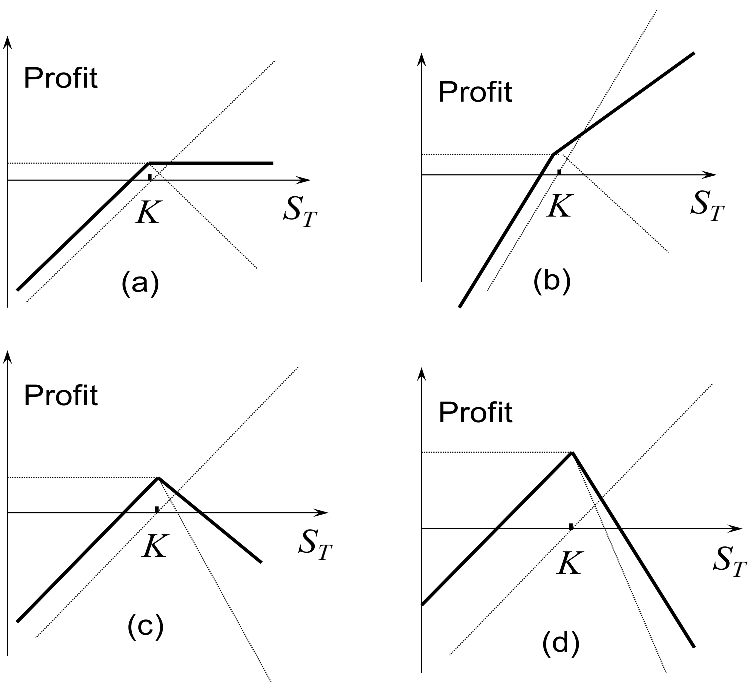

Figure S12.5 Answer to Problem 12.24

√Problem 12.26.

Suppose that the price of a non-dividend-paying stock is $32, its volatility is 30%, and the risk-free rate for all maturities is 5% per annum. Use DerivaGem to calculate the cost of setting up the following positions. In each case provide a table showing the relationship between profit *and final stock price. Ignore the impact of discounting.*

a. *A bull spread using European call options with strike prices of \$25 and \$30 and a maturity of six months.*

b. *A bear spread using European put options with strike prices of \$25 and \$30 and a maturity of six months*

c. *A butterfly spread using European call options with strike prices of \$25, \$30, and \$35 and a maturity of one year.*

d. *A butterfly spread using European put options with strike prices of \$25, \$30, and \$35 and a maturity of one year.*

e. *A straddle using options with a strike price of \$30 and a six-month maturity.*

f. *A strangle using options with strike prices of \$25 and \$35 and a six-month maturity.*

In each case provide a table showing the relationship between profit and final stock price. Ignore the impact of discounting.

(a) A call option with a strike price of 25 costs 7.90 and a call option with a strike price of 30 costs 4.18. The cost of the bull spread is therefore $7.90-4.18=3.72$. The profits ignoring the impact of discounting are

Stock Price Range	*Profit*
$S_T \leq 25$	-3.72
$25 < S_T < 30$	$S_T - 28.72$
$S_T \geq 30$	1.28

(b) A put option with a strike price of 25 costs 0.28 and a put option with a strike price of 30 costs 1.44. The cost of the bear spread is therefore $1.44-0.28=1.16$. The profits ignoring the impact of discounting are

Stock Price Range	*Profit*
$S_T \leq 25$	$+3.84$
$25 < S_T < 30$	$28.84 - S_T$
$S_T \geq 30$	-1.16

(c) Call options with maturities of one year and strike prices of 25, 30, and 35 cost 8.92, 5.60, and 3.28, respectively. The cost of the butterfly spread is therefore $8.92+3.28-2\times 5.60=1.00$. The profits ignoring the impact of discounting are

Stock Price Range	*Profit*
$S_T \leq 25$	-1.00
$25 < S_T < 30$	$S_T - 26.00$
$30 \leq S_T < 35$	$34.00 - S_T$

(d) Put options with maturities of one year and strike prices of 25, 30, and 35 cost 0.70, 2.14, 4.57, respectively. The cost of the butterfly spread is therefore $0.70 + 4.57 - 2 \times 2.14 = 0.99$. Allowing for rounding errors, this is the same as in (c). The profits are the same as in (c).

(e) A call option with a strike price of 30 costs 4.18. A put option with a strike price of 30 costs 1.44. The cost of the straddle is therefore $4.18 + 1.44 = 5.62$. The profits ignoring the impact of discounting are

Stock Price Range	*Profit*
$S_T \leq 30$	$24.38 - S_T$
$S_T > 30$	$S_T - 35.62$

(f) A six-month call option with a strike price of 35 costs 1.85. A six-month put option with a strike price of 25 costs 0.28. The cost of the strangle is therefore $1.85 + 0.28 = 2.13$. The profits ignoring the impact of discounting are

Stock Price Range	*Profit*
$S_T \leq 25$	$22.87 - S_T$
$25 < S_T < 35$	-2.13
$S_T \geq 35$	$S_T - 37.13$

√Problem 12.28. (Excel file)

Describe the trading position created in which a call option is bought with strike price K_1 and a put option is sold with strike price K_2 when both have the same time to maturity and $K_2 > K_1$. What does the position become when $K_1 = K_2$?

The position is as shown in the diagram below (for $K_1 = 25$ and $K_2 = 35$). It is known as a range forward and is discussed further in Chapter 17. When $K_1 = K_2$, the position becomes a regular long forward.

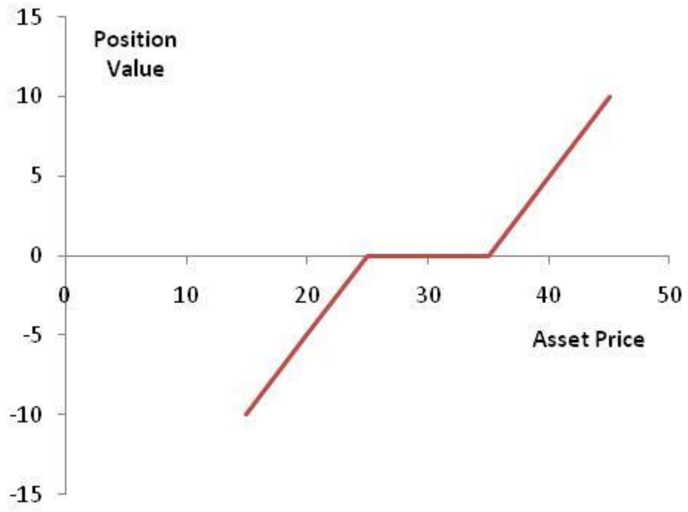

Figure S12.6: Trading position in Problem 12.27

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第 13 章

Problem 13.1、13.4（这两题，每题分别用无套利技术、风险中性定价技术、组合复制技术这三种方法求解）。**13.5, 13.6, 13.13**

第 14 章

Problem 14.2，14.4，14.5，14.9，14.10，14.11，14.17

第 15 章

Problem 15.6，15.7，15.8

√Problem 13.1.

A stock price is currently $40. It is known that at the end of one month it will be either $42 or $38. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-month European call option with a strike price of $39?

Consider a portfolio consisting of
-1: Call option
$+\Delta$: Shares
If the stock price rises to $42, the portfolio is worth $42\Delta - 3$. If the stock price falls to $38, it is worth 38Δ. These are the same when

$$42\Delta - 3 = 38\Delta$$

or $\Delta = 0.75$. The value of the portfolio in one month is 28.5 for both stock prices. Its value today must be the present value of 28.5, or $28.5e^{-0.08\times 0.08333} = 28.31$. This means that

$$-f + 40\Delta = 28.31$$

where f is the call price. Because $\Delta = 0.75$, the call price is $40\times 0.75 - 28.31 = \$1.69$. As an alternative approach, we can calculate the probability, p, of an up movement in a risk-neutral world. This must satisfy:

$$42p + 38(1-p) = 40e^{0.08\times 0.08333}$$

so that

$$4p = 40e^{0.08\times 0.08333} - 38$$

or $p = 0.5669$. The value of the option is then its expected payoff discounted at the risk-free rate:

$$[3\times 0.5669 + 0\times 0.4331]e^{-0.08\times 0.08333} = 1.69$$

or $1.69. This agrees with the previous calculation.

√Problem 13.4.

A stock price is currently $50. It is known that at the end of six months it will be either $45 or $55. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a six-month European put option with a strike price of $50?

Consider a portfolio consisting of
-1: Put option
$+\Delta$: Shares
If the stock price rises to $55, this is worth 55Δ. If the stock price falls to $45, the portfolio is worth $45\Delta - 5$. These are the same when

$$45\Delta - 5 = 55\Delta$$

or $\Delta = -0.50$. The value of the portfolio in six months is -27.5 for both stock prices. Its value today must be the present value of -27.5, or $-27.5e^{-0.1\times 0.5} = -26.16$. This means that

$$-f + 50\Delta = -26.16$$

where f is the put price. Because $\Delta = -0.50$, the put price is $1.16. As an alternative approach we can calculate the probability, p, of an up movement in a risk-neutral world. This must satisfy:

$$55p + 45(1-p) = 50e^{0.1\times 0.5}$$

so that

$$10p = 50e^{0.1\times 0.5} - 45$$

or $p = 0.7564$. The value of the option is then its expected payoff discounted at the risk-free rate:

$$[0\times 0.7564 + 5\times 0.2436]e^{-0.1\times 0.5} = 1.16$$

or $1.16. This agrees with the previous calculation.

√Problem 13.5.

A stock price is currently $100. Over each of the next two six-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-year European call option with a strike price of $100?

In this case $u = 1.10$, $d = 0.90$, $\Delta t = 0.5$, and $r = 0.08$, so that

$$p = \frac{e^{0.08\times 0.5} - 0.90}{1.10 - 0.90} = 0.7041$$

The tree for stock price movements is shown in Figure S13.1. We can work back from the end of the tree to the beginning, as indicated in the diagram, to give the value of the option as $9.61. The option value can also be calculated directly from equation (13.10):

$$[0.7041^2 \times 21 + 2\times 0.7041\times 0.2959\times 0 + 0.2959^2 \times 0]e^{-2\times 0.08\times 0.5} = 9.61$$

or $9.61.

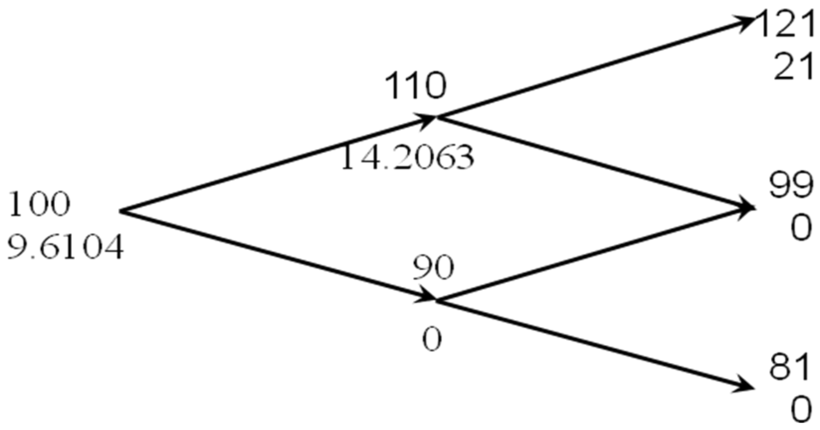


Figure S13.1: Tree for Problem 13.5

√Problem 13.6.

For the situation considered in Problem 13.5, what is the value of a one-year European put option with a strike price of $100? Verify that the European call and European put prices satisfy put–call parity.

Figure S13.2 shows how we can value the put option using the same tree as in Problem 13.5. The value of the option is $1.92. The option value can also be calculated directly from equation (13.10):

$$e^{-2\times0.08\times0.5}[0.7041^2\times0+2\times0.7041\times0.2959\times1+0.2959^2\times19]=1.92$$

or $1.92. The stock price plus the put price is $100+1.92=\$101.92$. The present value of the strike price plus the call price is $100e^{-0.08\times1}+9.61=\$101.92$. These are the same, verifying that put–call parity holds.

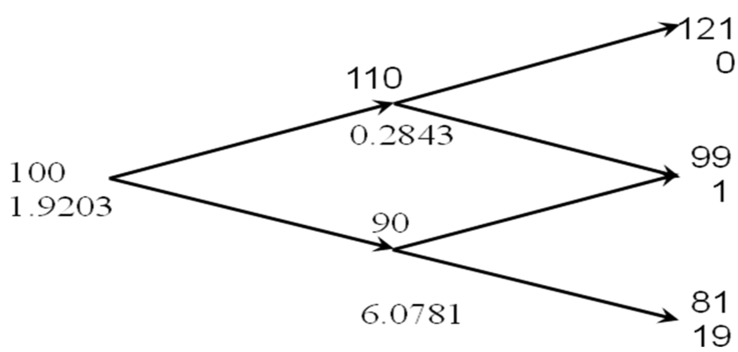
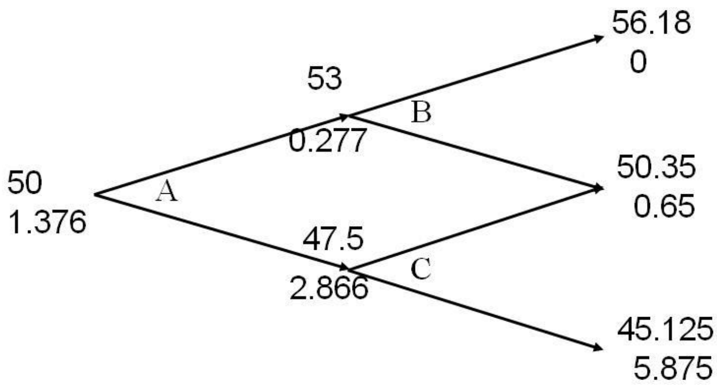


Figure S13.2: Tree for Problem 13.6



√Problem 13.13.

For the situation considered in Problem 13.12, what is the value of a six-month European put option with a strike price of $51? Verify that the European call and European put prices satisfy put–call parity. If the put option were American, would it ever be optimal to exercise it early at any of the nodes on the tree?

The tree for valuing the put option is shown in Figure S13.4. We get a payoff of $51-50.35=0.65$ if the middle final node is reached and a payoff of $51-45.125=5.875$ if the lowest final node is reached. The value of the option is therefore

$$(0.65\times 2\times 0.5689\times 0.4311+5.875\times 0.4311^2)e^{-0.05\times 6/12}=1.376$$

This can also be calculated by working back through the tree as indicated in Figure S13.4.
The value of the put plus the stock price is

$$1.376+50=51.376$$

The value of the call plus the present value of the strike price is

$$1.635+51e^{-0.05\times 6/12}=51.376$$

This verifies that put–call parity holds
To test whether it worth exercising the option early we compare the value calculated for the option at each node with the payoff from immediate exercise. At node C the payoff from immediate exercise is $51-47.5=3.5$. Because this is greater than 2.8664, the option should be exercised at this node. The option should not be exercised at either node A or node B.

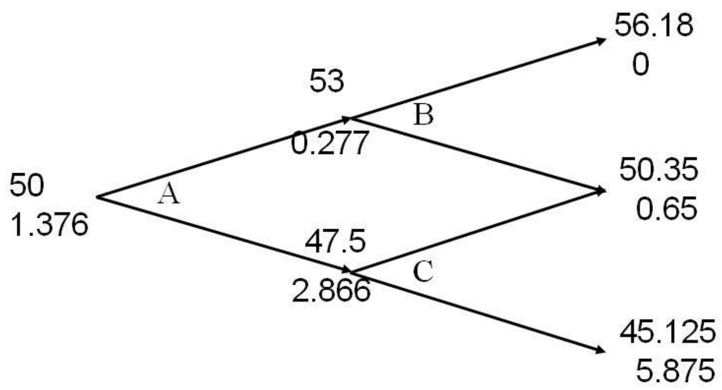


Figure S13.4: Tree for Problem 13.13

√Problem 14.2.

Can a trading rule based on the past history of a stock's price ever produce returns that are consistently above average? Discuss.

The first point to make is that any trading strategy can, just because of good luck, produce above average returns. The key question is whether a trading strategy *consistently* outperforms the market when adjustments are made for risk. It is certainly possible that a trading strategy could do this. However, when enough investors know about the strategy and trade on the basis of the strategy, the profit will disappear.

As an illustration of this, consider a phenomenon known as the small firm effect. Portfolios of stocks in small firms appear to have outperformed portfolios of stocks in large firms when appropriate adjustments are made for risk. Research was published about this in the early 1980s and mutual funds were set up to take advantage of the phenomenon. There is some evidence that this has resulted in the phenomenon disappearing.

√Problem 14.4.

Variables X_1 and X_2 follow generalized Wiener processes with drift rates μ_1 and μ_2 and variances σ_1^2 and σ_2^2. What process does X_1+X_2 follow if:

(a) *The changes in X_1 and X_2 in any short interval of time are uncorrelated?*

(b) *There is a correlation ρ between the changes in X_1 and X_2 in any short interval of time?*

(a) Suppose that X_1 and X_2 equal a_1 and a_2 initially. After a time period of length T, X_1 has the probability distribution

$$\varphi(a_1+\mu_1 T, \sigma_1^2 T)$$

and X_2 has a probability distribution

$$\varphi(a_2 + \mu_2 T, \sigma_2^2 T)$$

From the property of sums of independent normally distributed variables, $X_1 + X_2$ has the probability distribution

$$\varphi\left(a_1 + \mu_1 T + a_2 + \mu_2 T, \sigma_1^2 T + \sigma_2^2 T\right)$$

i.e.,

$$\varphi\left[a_1 + a_2 + (\mu_1 + \mu_2)T, (\sigma_1^2 + \sigma_2^2)T\right]$$

This shows that $X_1 + X_2$ follows a generalized Wiener process with drift rate $\mu_1 + \mu_2$ and variance rate $\sigma_1^2 + \sigma_2^2$.

(b) In this case the change in the value of $X_1 + X_2$ in a short interval of time Δt has the probability distribution:

$$\varphi\left[(\mu_1 + \mu_2)\Delta t, (\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2)\Delta t\right]$$

If μ_1, μ_2, σ_1, σ_2 and ρ are all constant, arguments similar to those in Section 14.2 show that the change in a longer period of time T is

$$\varphi\left[(\mu_1 + \mu_2)T, (\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2)T\right]$$

The variable, $X_1 + X_2$, therefore follows a generalized Wiener process with drift rate $\mu_1 + \mu_2$ and variance rate $\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$.

√Problem 14.5.

Consider a variable, S, that follows the process

$$dS = \mu\, dt + \sigma\, dz$$

For the first three years, $\mu = 2$ and $\sigma = 3$; for the next three years, $\mu = 3$ and $\sigma = 4$. If the *initial value of the variable is 5, what is the probability distribution of the value of the variable at the end of year six?*

The change in S during the first three years has the probability distribution

$$\varphi(2\times 3,\ 9\times 3)=\varphi(6,\ 27)$$

The change during the next three years has the probability distribution

$$\varphi(3\times 3,\ 16\times 3)=\varphi(9,\ 48)$$

The change during the six years is the sum of a variable with probability distribution $\varphi(6,27)$ and a variable with probability distribution $\varphi(9,48)$. The probability distribution of the change is therefore

$$\varphi(6+9, 27+48)$$

$$=\varphi(15,75)$$

Since the initial value of the variable is 5, the probability distribution of the value of the variable at the end of year six is

$$\varphi(20,75)$$

√Problem 14.9.

It has been suggested that the short-term interest rate, r, follows the stochastic process

$$dr=a(b-r)\,dt+rc\,dz$$

where a, b, and c are positive constants and dz is a Wiener process. Describe the nature of this process.

The drift rate is $a(b-r)$. Thus, when the interest rate is above b the drift rate is negative and, when the interest rate is below b, the drift rate is positive. The interest rate is therefore continually pulled towards the level b. The rate at which it is pulled toward this level is a. A volatility equal to c is superimposed upon the "pull" or the drift.

Suppose $a=0.4$, $b=0.1$ and $c=0.15$ and the current interest rate is 20% per annum. The interest rate is pulled towards the level of 10% per annum. This can be regarded as a long run average. The current drift is -4% per annum so that the expected rate at the end of one year is about 16% per annum. (In fact it is slightly greater than this, because as the interest rate decreases, the "pull" decreases.) Superimposed upon the drift is a volatility of 15% per annum.

√Problem 14.10.

Suppose that a stock price, S, follows geometric Brownian motion with expected return μ and volatility σ:

$$dS = \mu S\, dt + \sigma S\, dz$$

What is the process followed by the variable S^n? Show that S^n also follows geometric Brownian motion.

If $G(S,t) = S^n$ then $\partial G / \partial t = 0$, $\partial G / \partial S = nS^{n-1}$, and $\partial^2 G / \partial S^2 = n(n-1)S^{n-2}$. Using Itô's lemma:

$$dG = [\mu n G + \frac{1}{2} n(n-1)\sigma^2 G]\, dt + \sigma n G\, dz$$

This shows that $G = S^n$ follows geometric Brownian motion where the expected return is

$$\mu n + \frac{1}{2} n(n-1)\sigma^2$$

and the volatility is $n\sigma$. The stock price S has an expected return of μ and the expected value of S_T is $S_0 e^{\mu T}$. The expected value of S_T^n is

$$S_0^n e^{[\mu n + \frac{1}{2} n(n-1)\sigma^2]T}$$

√Problem 14.11.

Suppose that x is the yield to maturity with continuous compounding on a zero-coupon bond that pays off \$1 at time T. Assume that x follows the process

$$dx = a(x_0 - x)\, dt + sx\, dz$$

where a, x_0, and s are positive constants and dz is a Wiener process. What is the process followed by the bond price?

The process followed by B, the bond price, is from Itô's lemma:

$$dB = \left[\frac{\partial B}{\partial x} a(x_0 - x) + \frac{\partial B}{\partial t} + \frac{1}{2} \frac{\partial^2 B}{\partial x^2} s^2 x^2 \right] dt + \frac{\partial B}{\partial x} sxdz$$

Since:

$$B = e^{-x(T-t)}$$

the required partial derivatives are

$$\frac{\partial B}{\partial t} = xe^{-x(T-t)} = xB$$
$$\frac{\partial B}{\partial x} = -(T-t)e^{-x(T-t)} = -(T-t)B$$
$$\frac{\partial^2 B}{\partial x^2} = (T-t)^2 e^{-x(T-t)} = (T-t)^2 B$$

Hence:

$$dB = \left[-a(x_0 - x)(T-t) + x + \frac{1}{2} s^2 x^2 (T-t)^2 \right] Bdt - sx(T-t)Bdz$$

√Problem 14.17.

A stock price is currently 50. Its expected return and volatility are 12% and 30%, respectively. What is the probability that the stock price will be greater than 80 in two years? (Hint $S_T > 80$ *when* $\ln S_T > \ln 80$ *.)*

The variable $\ln S_T$ is normally distributed with mean $\ln S_0 + (\mu - \sigma^2 / 2)T$ and standard deviation $\sigma\sqrt{T}$. In this case $S_0 = 50$, $\mu = 0.12$, $T = 2$, and $\sigma = 0.30$ so that the mean and standard deviation of $\ln S_T$ are $\ln 50 + (0.12 - 0.3^2 / 2)2 = 4.062$ and $0.3\sqrt{2} = 0.424$, respectively. Also, $\ln 80 = 4.382$. The probability that $S_T > 80$ is the same as the probability that $\ln S_T > 4.382$. This is

$$1 - N\left(\frac{4.382 - 4.062}{0.424} \right) = 1 - N(0.754)$$

where $N(x)$ is the probability that a normally distributed variable with mean zero and standard deviation 1 is less than x. Because $N(0.754) = 0.775$, the required probability is 0.225.

√Problem 15.6.

What is implied volatility? How can it be calculated?

The implied volatility is the volatility that makes the Black–Scholes-Merton price of an option equal to its market price. The implied volatility is calculated using an iterative procedure. A simple approach is the following. Suppose we have two volatilities one too high (i.e., giving an option price greater than the market price) and the other too low (i.e., giving an option price lower than the market price). By testing the volatility that is half way between the two, we get a new too-high volatility or a new too-low volatility. If we search initially for two volatilities, one too high and the other too low we can use this procedure repeatedly to bisect the range and converge on the correct implied volatility. Other more sophisticated approaches (e.g., involving the Newton-Raphson procedure) are used in practice.

√Problem 15.7.

A stock price is currently $40. Assume that the expected return from the stock is 15% and its volatility is 25%. What is the probability distribution for the rate of return (with continuous compounding) earned over a two-year period?

In this case $\mu = 0.15$ and $\sigma = 0.25$. From equation (15.7) the probability distribution for the rate of return over a two-year period with continuous compounding is:

$$\varphi\left(0.15 - \frac{0.25^2}{2}, \frac{0.25^2}{2}\right)$$

i.e.,

$$\varphi(0.11875, 0.03125)$$

The expected value of the return is 11.875% per annum and the standard deviation is 17.7% per annum.

√Problem 15.8.

A stock price follows geometric Brownian motion with an expected return of 16% and a volatility of 35%. The current price is $38.

a) *What is the probability that a European call option on the stock with an exercise price of $40 and a maturity date in six months will be exercised?*
b) *What is the probability that a European put option on the stock with the same exercise price and maturity will be exercised?*

a) The required probability is the probability of the stock price being above \$40 in six months time. Suppose that the stock price in six months is S_T

$$\ln S_T \sim \varphi\left[\ln 38+\left(0.16-\frac{0.35^2}{2}\right)0.5, 0.35^2\times 0.5\right]$$

i.e.,

$$\ln S_T \sim \varphi(3.687, 0.247^2)$$

Since $\ln 40 = 3.689$, we require the probability of $\ln(S_T)>3.689$. This is

$$1-N\left(\frac{3.689-3.687}{0.247}\right)=1-N(0.008)$$

Since N(0.008) = 0.5032, the required probability is 0.4968.

b) In this case the required probability is the probability of the stock price being less than \$40 in six months time. It is

$$1-0.4968=0.5032$$

2022-4 作业（Hull 英文第十版，见 QQ 群文件）

第 17 章

Problem: 17.9、17.10、17.17

第 18 章

Problem: 18.10、18.12

第 19 章

Problem: 19.24、19.25

第 22 章

Problem 22.1

√Problem 17.9.

A foreign currency is currently worth \$1.50. The domestic and foreign risk-free interest rates are 5% and 9%, respectively. Calculate a lower bound for the value of a six-month call option on the currency with a strike price of \$1.40 if it is (a) European and (b) American.

Lower bound for European option is

$$S_0 e^{-r_f T} - Ke^{-rT} = 1.5e^{-0.09\times 0.5} - 1.4e^{-0.05\times 0.5} = 0.069$$

Lower bound for American option is

$$S_0 - K = 0.10$$

√Problem 17.10.

Consider a stock index currently standing at 250. The dividend yield on the index is 4% per annum, and the risk-free rate is 6% per annum. A three-month European call option on the index with a strike price of 245 is currently worth \$10. What is the value of a three-month put option on the index with a strike price of 245?

In this case $S_0 = 250$, $q = 0.04$, $r = 0.06$, $T = 0.25$, $K = 245$, and $c = 10$. Using put–call parity

$$c + Ke^{-rT} = p + S_0 e^{-qT}$$

or

$$p = c + Ke^{-rT} - S_0 e^{-qT}$$

Substituting:

$$p = 10 + 245e^{-0.25\times 0.06} - 250e^{-0.25\times 0.04} = 3.84$$

The put price is 3.84.

√Problem 17.17.

Consider again the situation in Problem 17.16. Suppose that the portfolio has a beta of 2.0, the risk-free interest rate is 5% per annum, and the dividend yield on both the portfolio and the index is 3% per annum. What options should be purchased to provide protection against the value of the portfolio falling below \$54 million in one year's time?

When the value of the portfolio falls to \$54 million the holder of the portfolio makes a capital loss of 10%. After dividends are taken into account the loss is 7% during the year. This is 12% below the risk-free interest rate. According to the capital asset pricing model, the expected excess return of the portfolio above the risk-free rate equals beta times the expected excess return of the market above the risk-free rate.

Therefore, when the portfolio provides a return 12% below the risk-free interest rate, the market's expected return is 6% below the risk-free interest rate. As the index can be assumed to have a beta of 1.0, this is also the excess expected return (including dividends) from the index. The expected return from the index is therefore -1% per annum. Since the index provides a 3% per annum dividend yield, the expected movement in the index is -4%. Thus when the portfolio's value is $54 million the expected value of the index is $0.96\times1200=1152$. Hence European put options should be purchased with an exercise price of 1152. Their maturity date should be in one year.
The number of options required is twice the number required in Problem 17.16. This is because we wish to protect a portfolio which is twice as sensitive to changes in market conditions as the portfolio in Problem 17.16. Hence options on $100,000 (or 1,000 contracts) should be purchased. To check that the answer is correct consider what happens when the value of the portfolio declines by 20% to $48 million. The return including dividends is -17%. This is 22% less than the risk-free interest rate. The index can be expected to provide a return (including dividends) which is 11% less than the risk-free interest rate, i.e. a return of -6%. The index can therefore be expected to drop by 9% to 1092. The payoff from the put options is $(1152-1092)\times100,000=\$6$ million. This is exactly what is required to restore the value of the portfolio to $54 million.

√Problem 18.10.

Consider a two-month futures call option with a strike price of 40 when the risk-free interest rate is 10% per annum. The current futures price is 47. What is a lower bound for the value of the futures option if it is (a) European and (b) American?

Lower bound if option is European is

$$(F_0-K)e^{-rT}=(47-40)e^{-0.1\times2/12}=6.88$$

Lower bound if option is American is

$$F_0-K=7$$

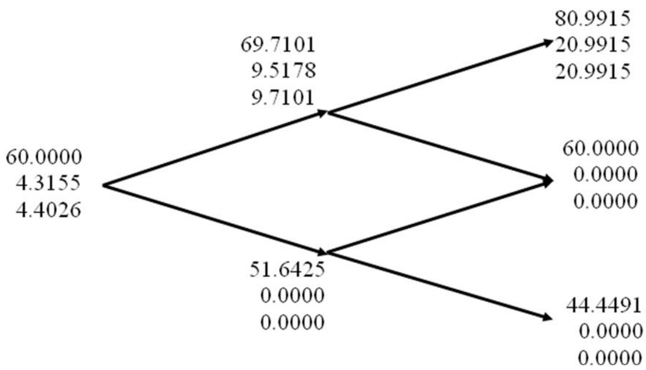
√Problem 18.12.

A futures price is currently 60 and its volatility is 30%. The risk-free interest rate is 8% per annum. Use a two-step binomial tree to calculate the value of a six-month European call option on the futures with a strike price of 60? If the call were American, would it ever be worth exercising it early?

In this case $u=e^{0.3\times\sqrt{1/4}}=1.1618$; $d=1/u=0.8607$; and

$$p=\frac{1-0.8607}{1.1618-0.8607}=0.4626$$

In the tree shown in Figure S18.1 the middle number at each node is the price of the European option and the lower number is the price of the American option. The tree shows that the value of the European option is 4.3155 and the value of the American option is 4.4026. The American option should sometimes be exercised early.



√Problem 19.24.

A financial institution has the following portfolio of over-the-counter options on sterling:

Type	*Position*	*Delta of Option*	*Gamma of Option*	*Vega of Option*
Call	*−1,000*	*0.5*	*2.2*	*1.8*
Call	*−500*	*0.8*	*0.6*	*0.2*
Put	*−2,000*	*-0.40*	*1.3*	*0.7*
Call	*−500*	*0.70*	*1.8*	*1.4*

A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

(a) *What position in the traded option and in sterling would make the portfolio both gamma neutral and delta neutral?*

(b) *What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral? Assume that all implied volatilities change by the same amount so that vegas can be aggregated.*

The delta of the portfolio is

$$-1,000\times0.50-500\times0.80-2,000\times(-0.40)-500\times0.70=-450$$

The gamma of the portfolio is

$$-1,000\times2.2-500\times0.6-2,000\times1.3-500\times1.8=-6,000$$

The vega of the portfolio is

$$-1,000\times1.8-500\times0.2-2,000\times0.7-500\times1.4=-4,000$$

(a) A long position in 4,000 traded options will give a gamma-neutral portfolio since the long position has a gamma of $4,000\times1.5=+6,000$. The delta of the whole portfolio (including traded options) is then:

$$4,000 \times 0.6 - 450 = 1,950$$

Hence, in addition to the 4,000 traded options, a short position of 1,950 in sterling is necessary so that the portfolio is both gamma and delta neutral.

(b) A long position in 5,000 traded options will give a vega-neutral portfolio since the long position has a vega of $5,000 \times 0.8 = +4,000$. The delta of the whole portfolio (including traded options) is then

$$5,000 \times 0.6 - 450 = 2,550$$

Hence, in addition to the 5,000 traded options, a short position of 2,550 in sterling is necessary so that the portfolio is both vega and delta neutral.

√Problem 19.25.

Consider again the situation in Problem 19.24. Suppose that a second traded option with a delta of 0.1, a gamma of 0.5, and a vega of 0.6 is available. How could the portfolio be made delta, gamma, and vega neutral?

Let w_1 be the position in the first traded option and w_2 be the position in the second traded option. We require:

$$6,000 = 1.5w_1 + 0.5w_2$$

$$4,000 = 0.8w_1 + 0.6w_2$$

The solution to these equations can easily be seen to be $w_1 = 3,200$, $w_2 = 2,400$. The whole portfolio then has a delta of

$$-450 + 3,200 \times 0.6 + 2,400 \times 0.1 = 1,710$$

Therefore the portfolio can be made delta, gamma and vega neutral by taking a long position in 3,200 of the first traded option, a long position in 2,400 of the second traded option and a short position of 1,710 in sterling.

√Problem 22.1.

Consider a position consisting of a \$100,000 investment in asset A and a \$100,000 investment in asset B. Assume that the daily volatilities of both assets are 1% and that the coefficient of correlation between their returns is 0.3. Estimate the 5-day 99% VaR and ES for the portfolio assuming normally distributed returns.

The standard deviation of the daily change in the investment in each asset is \$1,000. The variance of the portfolio's daily change is

$$1,000^2 + 1,000^2 + 2 \times 0.3 \times 1,000 \times 1,000 = 2,600,000$$

The standard deviation of the portfolio's daily change is the square root of this or \$1,612.45. The standard deviation of the 5-day change is

$$1,612.45 \times \sqrt{5} = \$3,605.55$$

Because $N^{-1}(0.01) = 2.326$, 1% of a normal distribution lies more than 2.326 standard deviations below the mean. The 5-day 99 percent value at risk is therefore 2.326×3,605.55 = \$8,388. The 5-day 99% ES is

$$\frac{3605.55 \times e^{-2.326^2/2}}{\sqrt{2\pi} \times 0.01} = 9,617$$