电路分析与电子线路

课程要点复习

课程内容回顾

1. 线性电阻网络分析(基本网络定理和分析方法)

线性VS非线性

2. 非线性电阻电路分析(包括二极管、MOS管、BJT)

3. 放大电路分析（MOS管、BJT、运算放大器）有源VS无源

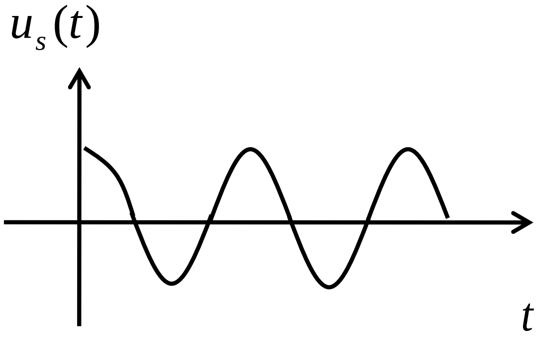
4. 动态电路瞬态分析（一阶和二阶电路）

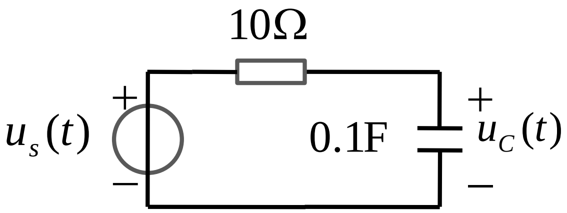
动态VS稳态

5. 正弦稳态电路分析(激励信号为稳定的交流信号)

正弦稳态响应

➢ 激励为正弦信号





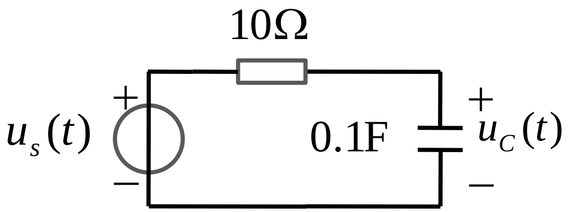
$$u_s(t) = 10\cos 2t\varepsilon(t)$$

$$\begin{cases} RC\dfrac{du_C}{dt} + u_C = 10\cos 2t \\ u_C(0) = 0 \end{cases}$$

正弦稳态响应

$$\frac{du_C}{dt}+u_C=10\cos 2t$$

$$u_C(0)=0$$



$$s+1=0$$

$$s=-1$$

$$u_C(t)=Ke^{-t}$$

$$u_C^*(t)=A\sin 2t+B\cos 2t$$ 特解为正弦项和余弦项的组合

$$\frac{d}{dt}(A\sin 2t+B\cos 2t)+A\sin 2t+B\cos 2t=10\cos 2t$$

$$2A\cos 2t-2B\sin 2t+A\sin 2t+B\cos 2t=10\cos 2t$$

正弦稳态响应

$$2A\cos 2t - 2B\sin 2t + A\sin 2t + B\cos 2t = 10\cos 2t$$

$$2A + B = 10$$
$$-2B + A = 0$$

$$A = 4$$
$$B = 2$$

$$u_C^*(t) = 4\sin 2t + 2\cos 2t$$

$$u_C(t) = Ke^{-t} + 4\sin 2t + 2\cos 2t$$

$$u_C(0) = K + 2 = 0$$
$$K = -2$$

$$u_C(t) = -2e^{-t} + 4\sin 2t + 2\cos 2t$$

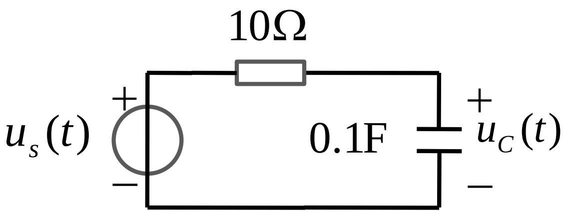
$$u_C(t) = \underset{\text{瞬态响应}}{\mathbf{-2e^{-t}}} + \underbrace{\mathbf{4\sin 2t + 2\cos 2t}}_{\text{稳态响应}}$$

$$t \to \infty$$

$$u_C(t) = 4\sin 2t + 2\cos 2t$$

- 求解涉及微分方程
- 求解涉及对三角函数的求导
- 有没有简便的方法求解稳态响应?

复指数函数激励



$$u_s(t) = 10\cos 2t$$

$$RC\frac{du_C}{dt} + u_C = 10\cos 2t$$

$$RC\frac{d(U_C e^{j\omega t})}{dt} + U_C e^{j\omega t} = U_S e^{j\omega t}$$

$$RC\frac{d(U_C e^{j2t})}{dt} + U_C e^{j2t} = 10e^{j2t}$$

- 复指数函数激励

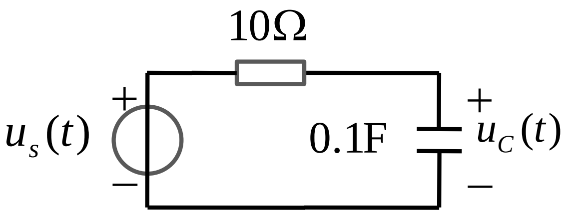
$$U_m e^{j\omega t} = U_m e^{st} \qquad s=j\omega$$

- 复指数信号工程上是没有的
- 复指数信号是一种很好的数学分析工具
- 欧拉公式
- 复指数信号将微分方程简化为代数方程

$$U_m e^{j\omega t} = U_m\cos(\omega t) + jU_m\sin(\omega t)$$

$$U_m\cos(\omega t) = \mathrm{Re}[U_m e^{j(\omega t)}]$$

复指数函数激励



$$RC\frac{d(U_C e^{j2t})}{dt}+U_C e^{j2t}=10e^{j2t}$$

$$2jRCU_C e^{j2t}+U_C e^{j2t}=10e^{j2t}$$

$$2jRCU_C+U_C=10$$

$$U_C=\frac{10}{2jRC+1}=\frac{10}{2j+1}$$

$$U_C e^{j2t}=\frac{10}{2j+1}e^{j2t}$$

$$U_C e^{j2t}=\frac{10}{\sqrt{5}}e^{j\varphi}e^{j2t}\qquad \varphi=\arctan(-2)$$

$$\mathrm{Re}(U_C e^{j2t})=\frac{10}{\sqrt{5}}\cos(2t+\varphi)$$

$$=4\sin 2t+2\cos 2t$$

相量

- 复指数函数激励简化了求解过程
- 求解微分方程变成了求解代数方程

相量

$$U_m \cos(\omega t+\varphi) = \mathrm{Re}[U_m e^{j(\omega t+\varphi)}]$$
$$= \mathrm{Re}[U_m e^{j\omega t+j\varphi}]$$
$$= \mathrm{Re}[U_m e^{j\omega t} e^{j\varphi}]$$
$$= \mathrm{Re}[U_m e^{j\varphi} e^{j\omega t}]$$

$U_m e^{j\varphi}$

- 定义为相量
- 复指数
- 与时间无关

三角函数		相量
$U_m \cos(\omega t+\varphi)$	$\leftrightarrow$	$U_m e^{j\varphi}$
$U_m \cos(\omega t+\varphi)$	$\leftrightarrow$	$U_m \angle \varphi$

相量基本运算

$$U_m\cos(\omega t+\varphi) \quad \leftrightarrow \quad U_m\angle\varphi$$

$$\frac{d}{dt}U_m\cos(\omega t+\varphi) \quad \leftrightarrow \quad j\omega U_m\angle\varphi$$

$$i(t) \quad \leftrightarrow \quad \dot{I}$$

$$\frac{d}{dt}i(t) \quad \leftrightarrow \quad j\omega\dot{I}$$

$$\frac{d^2}{dt^2}i(t) \quad \leftrightarrow \quad (j\omega)^2\dot{I}$$

$$\int_{-\infty}^{t}i(\tau)d\tau \quad \leftrightarrow \quad \frac{\dot{I}}{j\omega}$$

正弦信号的表达

$$u(t) = U_m \sin(\omega t + \varphi)$$

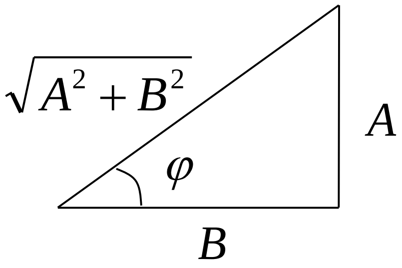
$$u(t) = U_m \cos(\omega t + \varphi)$$

$$u(t) = A\sin \omega t + B\cos \omega t$$

$$u(t) = \sqrt{A^2 + B^2}\left(\frac{A}{\sqrt{A^2 + B^2}}\sin \omega t + \frac{B}{\sqrt{A^2 + B^2}}\cos \omega t\right)$$

$$u(t) = \sqrt{A^2 + B^2}(\sin \varphi \sin \omega t + \cos \varphi \cos \omega t)$$

$$u(t) = \sqrt{A^2 + B^2}\cos(\omega t + \varphi) \qquad \tan \varphi = \frac{A}{B}$$



$$\sin \varphi = \frac{A}{\sqrt{A^2 + B^2}}$$

$$\cos \varphi = \frac{B}{\sqrt{A^2 + B^2}}$$

欧拉公式

$$e^{j\theta} = \cos\theta + j\sin\theta$$

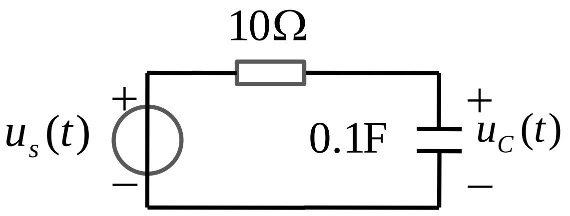
$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$e^{j(\omega t+\varphi)} = \cos(\omega t+\varphi) + j\sin(\omega t+\varphi)$$

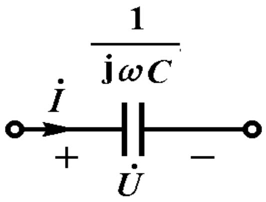
$$\mathrm{Re}[e^{j(\omega t+\varphi)}] = \cos(\omega t+\varphi)$$

$$\mathrm{Im}[e^{j(\omega t+\varphi)}] = \sin(\omega t+\varphi)$$

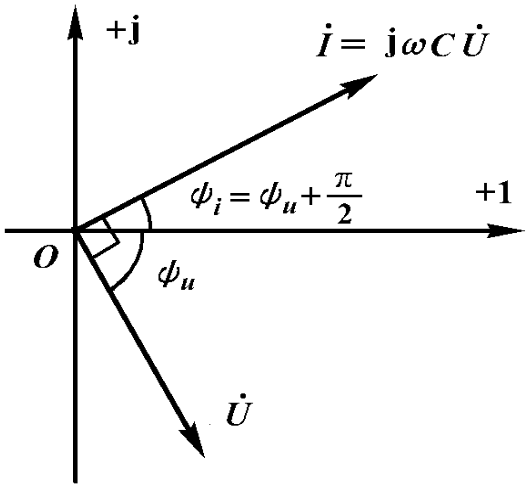
电容的阻抗



$$U_C = \frac{10}{2jRC+1} = \frac{10}{R+\dfrac{1}{2jC}}\frac{1}{2jC}$$



(a)



(b)

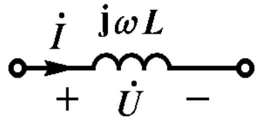
电容的阻抗： $\frac{1}{2jC} \rightarrow \frac{1}{j\omega C}$

电容电流的相位超前于电容电压的相位90°

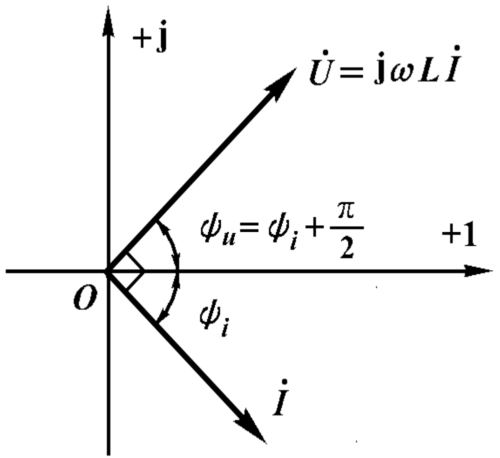
$$\boxed{\dot{I} = \mathrm{j}\omega C\dot{U}}$$

电感的阻抗

$$\boxed{\dot{U} = \mathrm{j}\omega L\dot{I}}$$



(a)



(b)

电感两端电压的相位超前于电感电流的相位90°

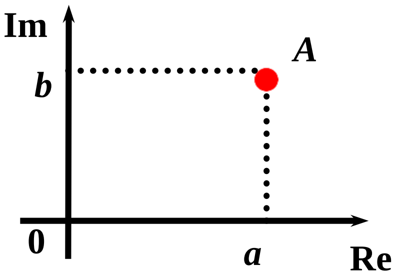
阻抗总结

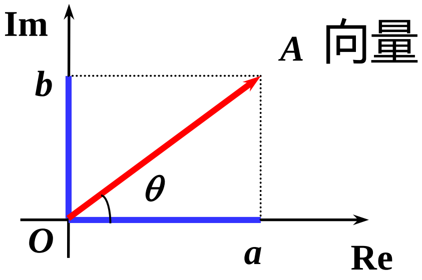
	阻抗：		导纳：
R	R	**R**	$\dfrac{1}{R}$
L	$j\omega L$	**L**	$\dfrac{1}{j\omega L}$
C	$\dfrac{1}{j\omega C}$	**C**	$j\omega C$

- 阻抗的概念比电阻的宽泛
- 电阻是纯电阻，阻抗为复数，实部为电阻，虚部为电抗（容抗或感抗）
- 导纳是阻抗的倒数（复数的倒数）

向量与复数

直角坐标形式　　　复数表示形式：$A=a+jb$





$$\begin{cases} a=|A|\cos\theta = \mathrm{Re}[A] \\ b=|A|\sin\theta = \mathrm{Im}[A] \end{cases}$$

模值　$|A|=\sqrt{a^2+b^2}$

幅角　$\theta=\mathrm{arctag}\dfrac{b}{a}$

三角函数形式：　$A=|A|(\cos\theta+j\sin\theta)$

极坐标形式(指数形式)：　$A=|A|e^{j\theta}=|A|\angle\theta$

欧拉公式：　$e^{j\theta}=\cos\theta+j\sin\theta$

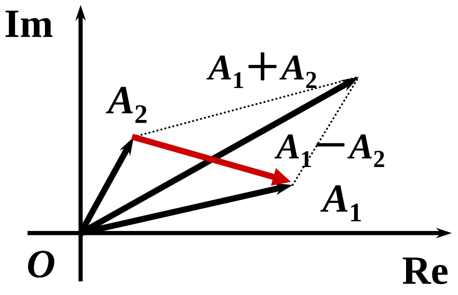
复数运算

(1)加减运算——直角坐标

$A_1=a_1+jb_1$， $A_2=a_2+jb_2$

$A_1 \pm A_2=(a_1 \pm a_2)+j(b_1 \pm b_2)$

加减法可用图解法



(2)乘除运算——极坐标

$A_1=|A_1|\ \theta_1$

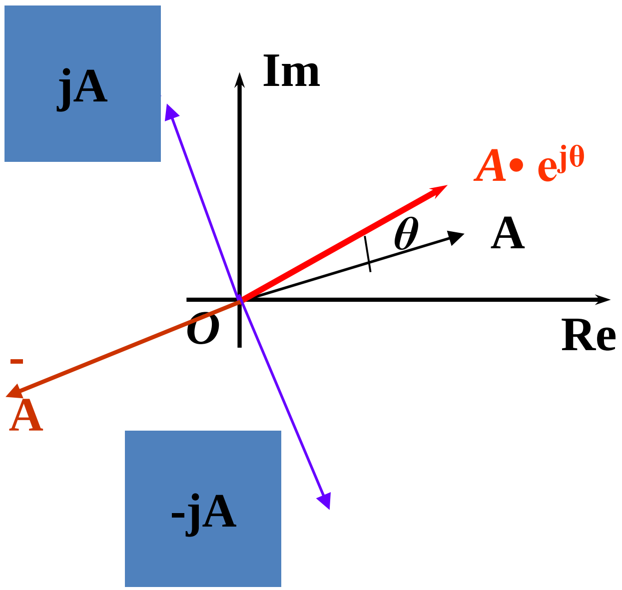
$A_2=|A_2|\ \theta_2$

乘法：模相乘，角相加 $A_1 A_2 =|A_1||A_2|\ \ \theta_1+\theta_2$

除法：模相除，角相减 $$\frac{A_1}{A_2}=\frac{|A_1|\angle\theta_1}{|A_2|\angle\theta_2}=\frac{|A_1|\,\mathrm{e}^{\mathrm{j}\theta_1}}{|A_2|\,\mathrm{e}^{\mathrm{j}\theta 2}}=\frac{|A_1|}{|A_2|}\mathrm{e}^{\mathrm{j}(\theta_1-\theta_2)}=\frac{|A_1|}{|A_2|}\angle(\theta_1-\theta_2)$$

旋转因子

$A\bullet e^{j\theta}$ 相当于A逆时针旋转一个角度θ，而模不变。



$$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = j$$

$$e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$

$$e^{j\pi} = \cos\pi + j\sin\pi = -1$$

+j, –j, -1 都可以看成旋转因子。

$e^{j\omega t}$ 模为1幅角为ωt，旋转向量

复数 $e^{j\theta} = 1\angle\theta$

相量法（利用相量法求正弦稳态解）

前提条件：线性非时变电路，同一个频率的正弦信号

分析步骤：

1. 建立相量模型电路：

- 独立源用相量表示
- 元件用阻抗形式表示
- 电压和电流变量用相量描述

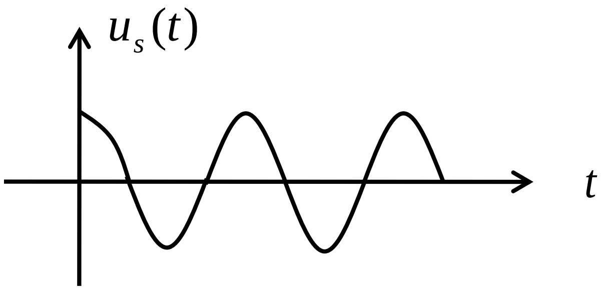
2. 建立方程（含复数的代数方程），并求解方程

3. 映射，并求出正弦稳态解

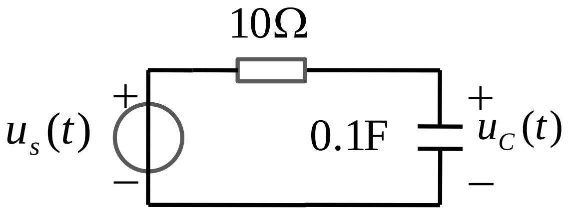
相量法（利用相量法求正弦稳态解）

动态电路	$\longrightarrow$	相量模型
$\downarrow$		$\downarrow$
微分方程	$\dashrightarrow$	含复数的代数方程
$\downarrow$		$\downarrow$
电路的解	$\longleftarrow$	对应相量解
$u_k(t)$ $i_k(t)$		$\dot{U}_k$ $\dot{I}_k$

正弦信号的响应



$$u_s(t) = 10\cos 2t\varepsilon(t)$$

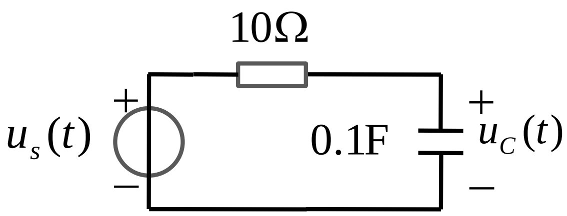


$$\begin{cases} RC\dfrac{du_C}{dt} + u_C = 10\cos 2t \\ u_C(0) = 0 \end{cases}$$

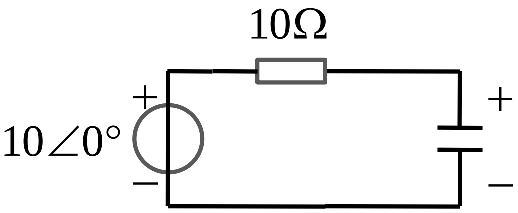
$$u_C(t) = -2e^{-t} + \boxed{4\sin 2t + 2\cos 2t}$$

正弦稳态解

相量分析法



电路图



$$\frac{1}{j\omega C} = -j5$$

相量模型

求解微分方程

$$u_C(t) = 4\sin 2t + 2\cos 2t$$

$$-j4 + 2 \;= \sqrt{4^2 + 2^2}\angle -63.4°$$
$$= 4.47\angle -63.4°$$

$$u_C(t) = 4.47\cos(2t - 63.4°)$$

求解代数方程

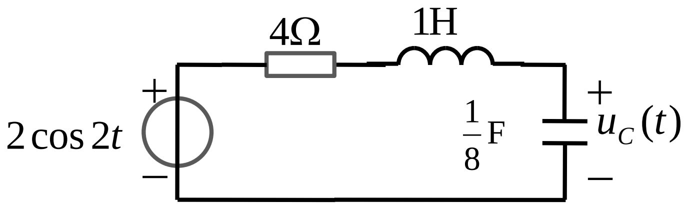
$$\dot{U}_C = \frac{-j5}{10 - j5}10 = \frac{10\angle 180°}{2 - j}$$

$$\dot{U}_C = \frac{10\angle -90°}{\sqrt{5}\angle -26.56°} = \frac{10}{\sqrt{5}}\angle -63.4°$$

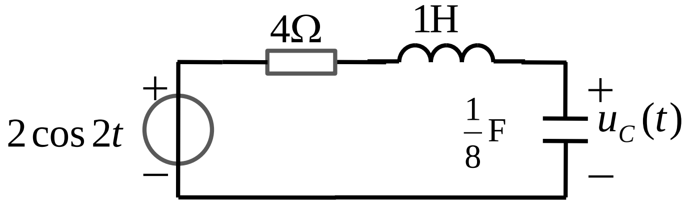
$$\dot{U}_C = 4.47\angle -63.4°$$

课堂练习

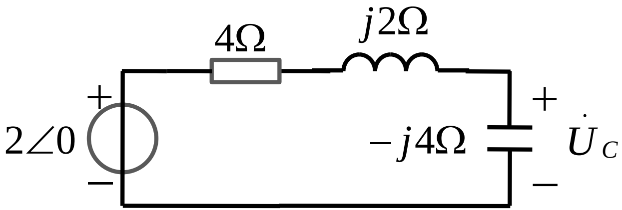
电路如下图所示，求电路在稳态时的$u_c(t)$



课堂练习——相量法



1. 画出相量模型



2. 建立代数方程

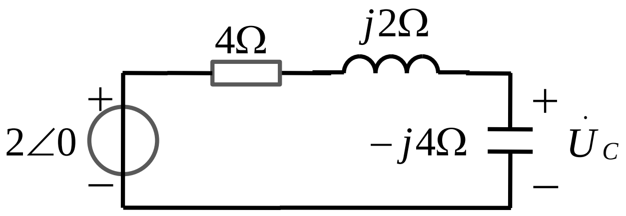
$$4\dot{I}+j2\dot{I}-j4\dot{I}=2$$

$$4\dot{I}-j2\dot{I}=2$$

$$\dot{I}=\frac{2}{4-j2}=\frac{1}{2-j}=\frac{1}{\sqrt{5}}\angle 26.6^\circ$$

$$\dot{U}_C=-j4\dot{I}=4\angle -90^\circ\frac{1}{\sqrt{5}}\angle 26.6^\circ=\frac{4}{\sqrt{5}}\angle -63.4^\circ$$

课堂练习——相量法



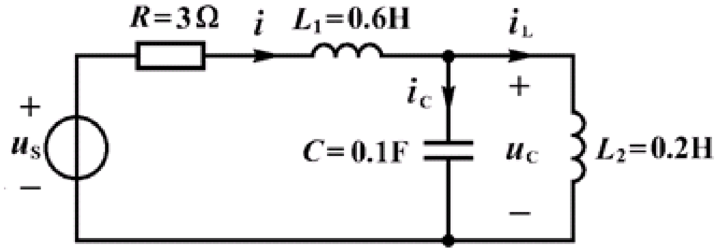
3. 求对应正弦稳态解

$$\dot{U}_C = \frac{4}{\sqrt{5}}\angle -63.4°$$

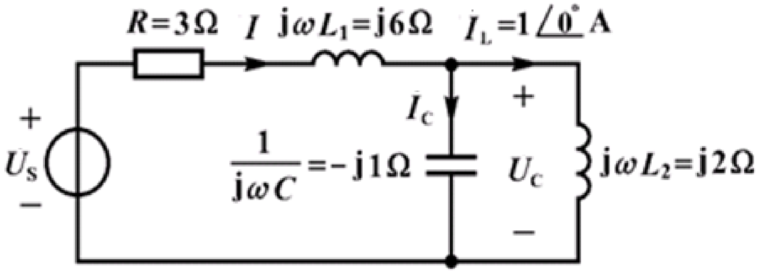
$$u_C(t) = \frac{4}{\sqrt{5}}\cos(2t - 63.4°)$$

课堂练习——相量法

$i_L(t)=\sqrt{2}\cos\omega t$ A, $\omega=10\text{rad/s}$ 求电流$i(t)$, 电压$u_C(t)$和$u_S(t)$。



相量模型



$$\dot{U}_C=\dot{U}_L=j\omega L_2\dot{I}_L=j2\text{V}$$

$$\dot{I}_C=\frac{\dot{U}_C}{1/j\omega C}=\frac{j2}{-j1}\text{A}=-2\text{A}$$

$$\dot{I}=\dot{I}_L+\dot{I}_C=(1-2)\text{A}=-1\text{A}$$

$$\dot{U}_S=R\dot{I}+j\omega L_1\dot{I}+\dot{U}_C$$

$$=3\times(-1)\text{V}+j6\times(-1)\text{V}+j2\text{V}$$

$$=(-3-j4)\text{V}=5\angle-126.9^\circ\text{V}$$

$$i(t)=\sqrt{2}\cos(10t+180^\circ)\text{A}$$

$$u_C(t)=2\sqrt{2}\cos(10t+90^\circ)\text{V}$$

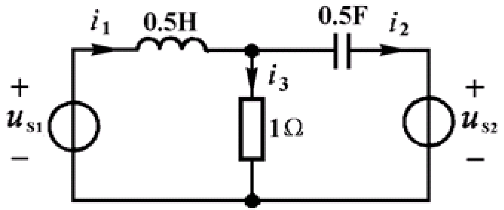
$$u_S(t)=5\sqrt{2}\cos(10t-126.9^\circ)\text{V}$$

课堂练习——相量法

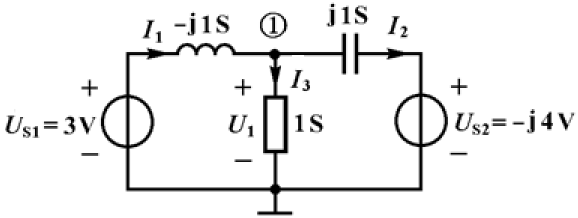
$u_{S1}(t) = 3\sqrt{2}\cos\omega t\ \text{V},\ u_{S2}(t) = 4\sqrt{2}\sin\omega t\ \text{V},\ \omega = 2\text{rad/s}$

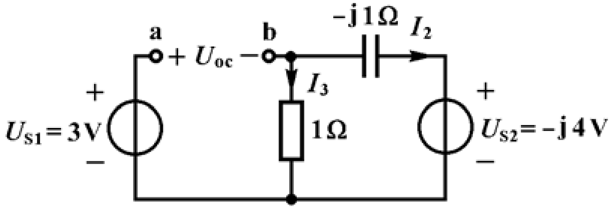
(1) 用节点分析法求电流$i_1(t)$

(2) 用戴维南等效电路求电流$i_1(t)$



相量模型





$$(1-\text{j}1+\text{j}1)U_1-(-\text{j}1)\times 3-\text{j}1\times(-\text{j}4)=0$$

$$U_1=-\text{j}1\times 3\text{V}+\text{j}1\times(-\text{j}4)\text{V}=(4-\text{j}3)\text{V}=5\angle-36.9^\circ\text{V}$$

$$I_1=-\text{j}1\times(U_{S1}-U)=(3+\text{j}1)\text{A}=3.162\angle 18.43^\circ\text{A}$$

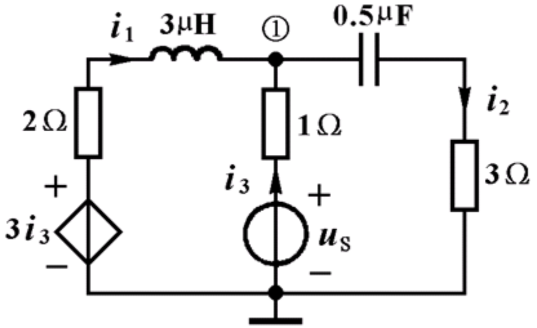
$$i_1(t)=3.162\sqrt{2}\cos(2t+18.43^\circ)\text{A}$$

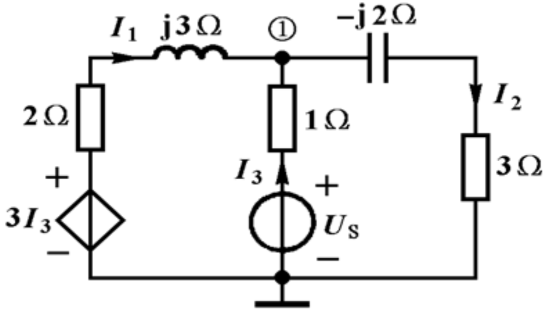
$$U_{oc}=U_{S1}-\frac{1}{1-\text{j}1}\times U_{S2}=(1+\text{j}2)\text{V}$$

$$Z_o=\frac{1\times(-\text{j}1)}{1-\text{j}1}\Omega=(0.5-\text{j}0.5)\Omega \qquad I_1=\frac{U_{oc}}{Z_o+1j}$$

课堂练习——相量法

$u_S(t) = 5\cos(\omega t + 30^\circ)\text{V},\ \omega = 10^6\,\text{rad/s}$ 分别用节点分析法和戴维南求电流$i_2(t)$





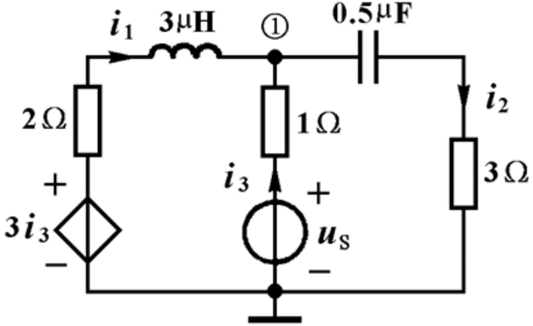
$$\left(\frac{1}{2+\text{j}3} + 1 + \frac{1}{3-\text{j}2}\right)\boldsymbol{U}_1 - \frac{3\boldsymbol{I}_3}{2+\text{j}3} - \frac{5\angle 30^\circ}{1} = 0$$

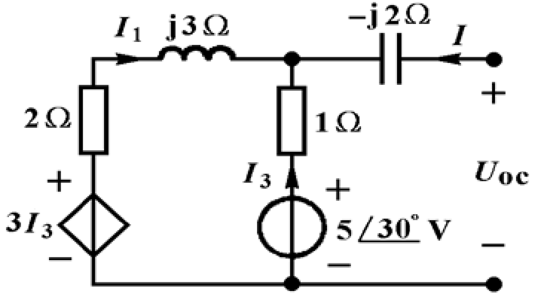
$$\boldsymbol{I}_3 = \frac{\boldsymbol{U}_S - \boldsymbol{U}_1}{1} = 5\angle 30^\circ\,\text{V} - \boldsymbol{U}_1$$

$$\boldsymbol{U}_1 = 4.043\angle 27.27^\circ\,\text{V} \qquad \boldsymbol{I}_2 = \frac{\boldsymbol{U}_1}{3-\text{j}2} = 1.12\angle 60.96^\circ\,\text{A}$$

课堂练习——相量法

$u_S(t) = 5\cos(\omega t + 30^\circ)\text{V},\ \omega = 10^6\,\text{rad/s}$　分别用节点分析法和戴维南求电流$i_2(t)$





$$3I_3 + (2\Omega)I_3 + (3j\Omega)I_3 + (1\Omega)I_3 - U_S = 0$$

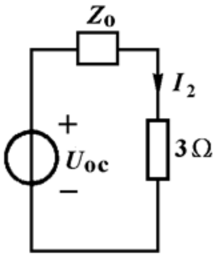
$$I_3 = 0.744 + 0.0447j$$

$$U_{oc} = -(1\Omega)I_3 + U_S = 4.346\angle 34.4^\circ \text{V}$$

$$\begin{cases} (-2j\Omega)I - (3\Omega)I_3 = U \\ (I + I_3)(3j + 2)\Omega + (3\Omega)I_3 = U \end{cases}$$

$$Z_o = \frac{U}{I} = \frac{8 - j9}{6 + j3}\Omega = 1.795\angle -74.93^\circ\,\Omega$$

$$I_2 = \frac{U_{oc}}{Z_o + 3}$$



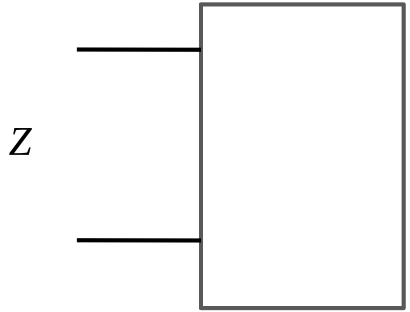
单口网络相量模型

无源单口网络

$$Z=\frac{\dot U}{\dot I} \qquad Y=\frac{\dot I}{\dot U}$$

$$Z=R+jX$$

$$Y=\frac{1}{Z}=\frac{1}{R+jX}$$

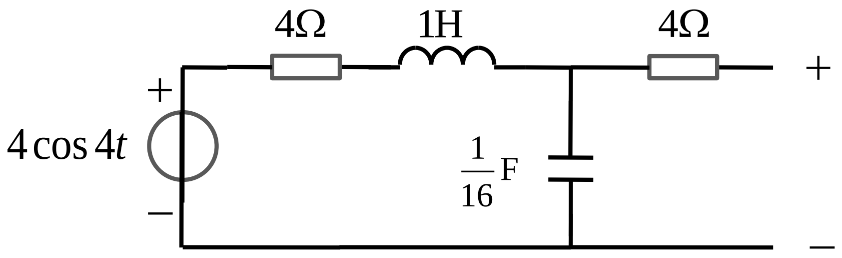


含独立源单口网络

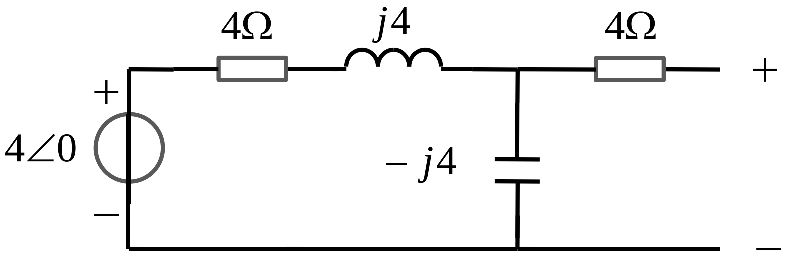
满足相量模型的
戴维宁定理

课堂练习

求端口的戴维宁等效电路



相量模型如下：

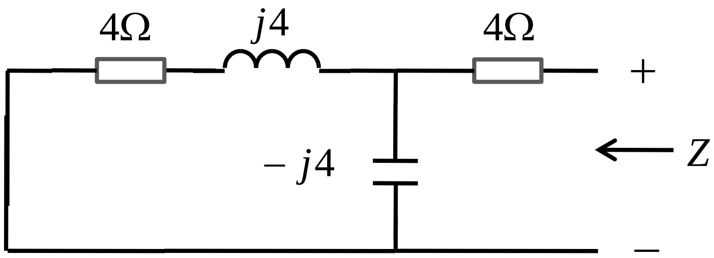


（1）开路电压

$$\dot{U}_{oc}=\frac{-j4}{4+j4-j4}4\angle 0=-j4$$

$$=4\angle -90^{\circ}$$

课堂练习

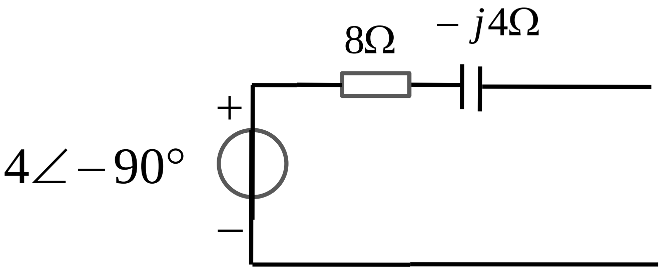


（2）等效阻抗

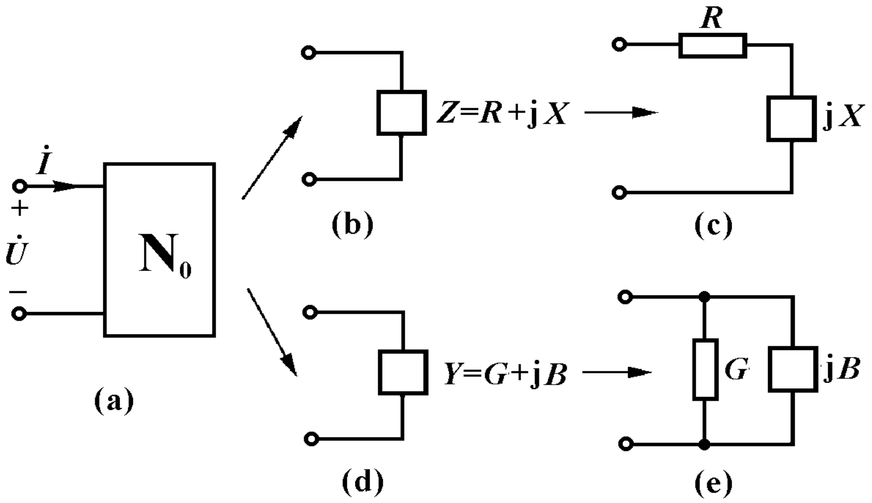
$$Z = 4 + \frac{-j4(4+j4)}{4+j4-j4} = 4+4-j4$$

$$= 8 - j4$$

（3）相量模型的戴维宁等效



阻抗与导纳的变换



已知导纳，求阻抗

$$Z = R + \mathrm{j}X = \frac{1}{Y} = \frac{1}{G+\mathrm{j}B} = \frac{G}{G^2+B^2} + \frac{-\mathrm{j}B}{G^2+B^2}$$

$$R = \frac{G}{G^2+B^2} \qquad X = \frac{-B}{G^2+B^2}$$

已知阻抗，求导纳

$$Y = G + \mathrm{j}B = \frac{1}{Z} = \frac{1}{R+\mathrm{j}X}$$

$$= \frac{R}{R^2+X^2} + \frac{-\mathrm{j}X}{R^2+X^2}$$

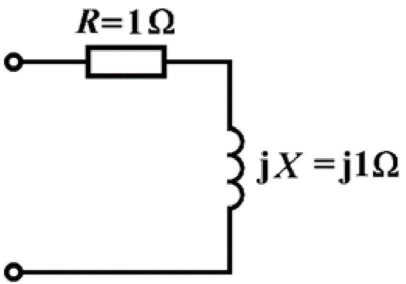
$$G = \frac{R}{R^2+X^2} \qquad B = \frac{-X}{R^2+X^2}$$

容易犯错！

$$R \neq \frac{1}{G} \qquad X \neq \frac{1}{B}$$

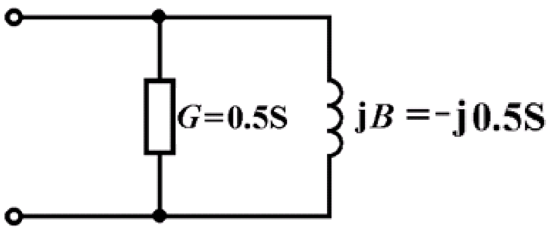
课堂练习

根据下图所示电阻和电抗串联单口网络，求电导和电纳并联的等效电路。



$$G=\frac{R}{R^2+X^2}=\frac{1}{1+1}\text{S}=0.5\text{S}$$

$$B=\frac{-X}{R^2+X^2}=\frac{-1}{1+1}\text{S}=-0.5\text{S}$$

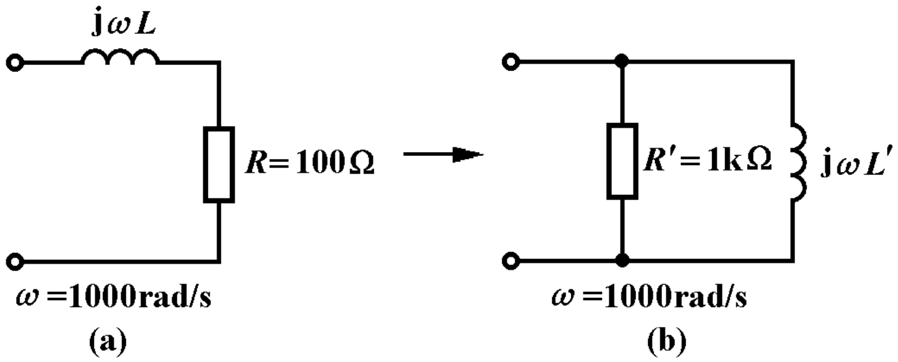


$$R=1\neq\frac{1}{G}=\frac{1}{0.5}=2$$

$$\text{j}X=\text{j}1\neq\frac{1}{\text{j}B}=\frac{1}{-\text{j}0.5}=\text{j}2$$

课堂练习

已知图 (a)所示电阻R=100Ω和电感L串联单口网络可以等效变换为图(b)所示电阻R' =1000Ω和电感L'并联的单口网络，试求电感L



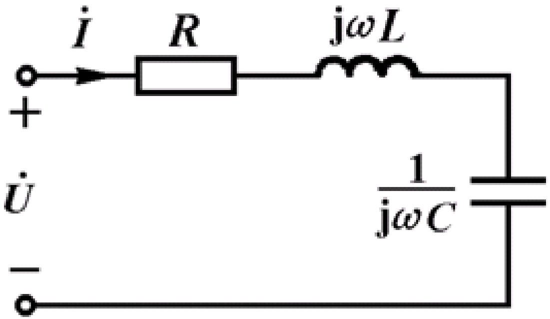
$$Y=\boxed{\frac{1}{R'}}+\frac{1}{\mathrm{j}\omega L'}=\frac{1}{R+\mathrm{j}\omega L}=\boxed{\frac{R}{R^2+(\omega L)^2}}+\frac{-\mathrm{j}\omega L}{R^2+(\omega L)^2}$$

$$\omega L=\sqrt{RR'-R^2}=\sqrt{10^5-10^4}\Omega=\sqrt{9\times10^4}\Omega=300\Omega$$

$$L=\frac{300\Omega}{\omega}=\frac{300}{10^3}\mathrm{H}=0.3\mathrm{H}$$

RLC串联谐振

RLC串联谐振电路，利用相量模型求端口阻抗



$$Z(\omega)=\frac{\dot{U}}{\dot{I}}=R+\mathrm{j}(\omega L-\frac{1}{\omega C})=|Z(\omega)|\angle\theta(\omega)$$

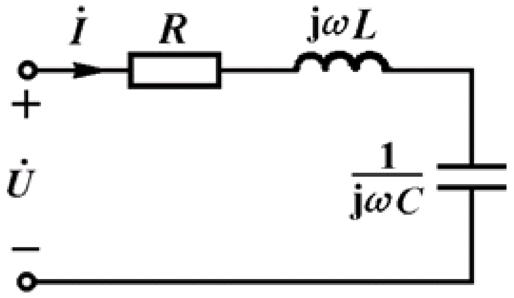
$$|Z(\omega)|=\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}\qquad \omega_0=\frac{1}{\sqrt{LC}}$$

$$\theta(\omega)=\arctan(\frac{\omega L-\frac{1}{\omega C}}{R})$$

谐振电路：含有电感、电容和电阻元件的单口网络。在谐振频率点，电容电感抵消，端口等效为纯电阻R，即端口电压和电流的相位相同

谐振条件

当$\omega L - \dfrac{1}{\omega C} = 0$，即$\omega = \dfrac{1}{\sqrt{LC}}$时，$\theta(\omega)=0$,



$|Z(\omega)|=R$,电压u(t)与电流i(t)相位相同，电路发生谐振。

RLC串联电路的谐振条件为

$$f = f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad \boxed{\omega = \omega_0 = \frac{1}{\sqrt{LC}}}$$

固有谐振角频率

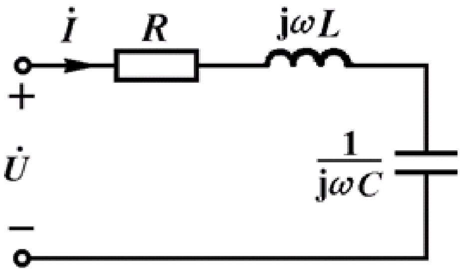
RLC串联电路在谐振时的感抗和容抗在量值上相等，其值称为谐振电路的特性阻抗，用ρ表示

$$\boxed{\rho = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}}$$

特性阻抗

RLC串联谐振

RLC串联电路发生谐振时，阻抗的电抗分量



$$X = \omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$Z(\omega_0) = R$$ 阻抗呈现纯电阻，达到最小值

$$\dot{I} = \frac{\dot{U}_S}{Z} = \frac{\dot{U}_S}{R}$$ 电流达到最大值

$$\dot{U}_R = R\dot{I} = \dot{U}_S \quad \dot{U}_C = \frac{1}{\mathrm{j}\omega_0 C}\dot{I} = -\mathrm{j}\frac{1}{\omega_0 RC}\dot{U}_S = -\mathrm{j}Q\dot{U}_S \quad \dot{U}_L = \mathrm{j}\omega_0 L\dot{I} = \mathrm{j}\frac{\omega_0 L}{R}\dot{U}_S = \mathrm{j}Q\dot{U}_S$$

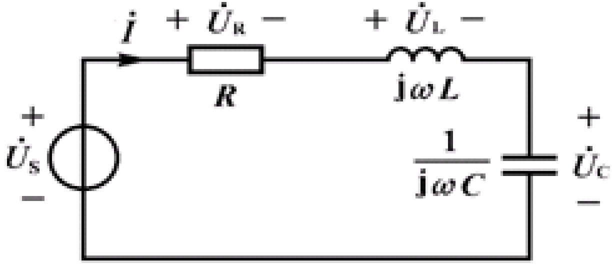
$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{\rho}{R}}$$

品质因数:谐振时感抗或容抗与电阻之比

RLC串联谐振

电阻电压与电压源电压相等

$$\dot{U}_R = \dot{U}_S$$

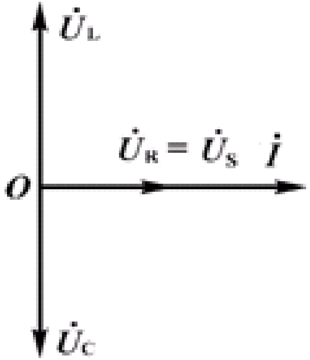


电感电压与电容电压之和为零

$$\dot{U}_L + \dot{U}_C = 0$$

电感电压或电容电压的幅度为电压源电压幅度的Q倍

$$U_L = U_C = QU_S = QU_R$$

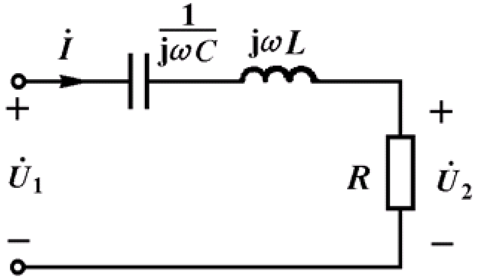


若Q>>1,则U_L=U_C>>U_S=U_R，称为电压谐振

串联谐振电路的频率特性（带通滤波器特性）

转移电压比

$$H(\omega)=\frac{\dot U_2}{\dot U_1}=\frac{R}{R+\mathrm{j}\left(\omega L-\dfrac{1}{\omega C}\right)}=\frac{1}{1+\mathrm{j}\left(\dfrac{\omega L}{R}-\dfrac{1}{\omega RC}\right)}$$



$$H(\omega)=\frac{\dot U_2}{\dot U_1}=\frac{1}{1+\mathrm{j}Q\left(\omega/\omega_0-\omega_0/\omega\right)}$$

$$\boxed{Q=\frac{\omega_0 L}{R}=\frac{1}{R\omega_0 C}}$$

谐振频率点感抗或容抗与电阻的比值

$$|H(\omega)|=\frac{1}{\sqrt{1+Q^2\left(\omega/\omega_0-\omega_0/\omega\right)^2}}$$

当ω=0或ω=∞时，|H(jω)|=0

当 $\omega=\omega_0=\dfrac{1}{\sqrt{LC}}$ 时，电路发生谐振，|H(jω)|=1达到最大值，该电路具有带通滤波特性。

串联谐振电路的频率特性（带通滤波器特性）

为求出通频带的宽度，先计算与 $|H(\mathrm{j}\omega)| = \frac{1}{\sqrt{2}}$ (即-3dB)对应的频率ω_+和ω_-

$$|H(\omega)| = \frac{1}{\sqrt{1+Q^2\left(\omega/\omega_0-\omega_0/\omega\right)^2}}$$

$$Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\pm 1$$

$$\omega_+ - \omega_- = \frac{\omega_0}{Q}$$

$$\omega^2 - \frac{\omega_0}{Q}\omega - \omega_0^2 = 0$$

$$\omega^2 + \frac{\omega_0}{Q}\omega - \omega_0^2 = 0$$

$$\omega_+ = \frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + 4\omega_0^2}$$

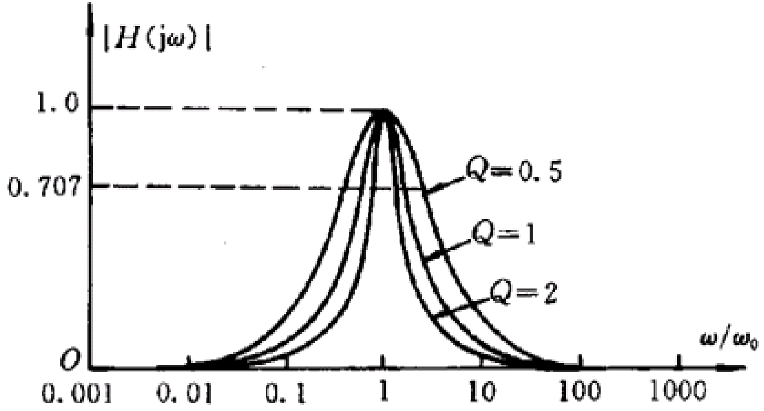
$$\omega_- = -\frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + 4\omega_0^2}$$

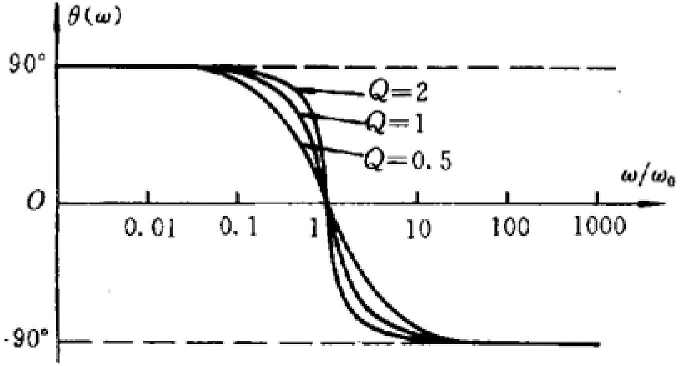
串联谐振电路的频率特性（带通滤波器特性）

由此求得3dB带宽

$$\Delta\omega=\omega_{+}-\omega_{-}=\frac{\omega_0}{Q}$$

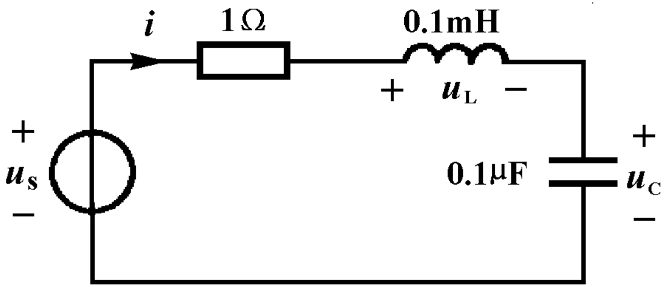
带宽$\Delta\omega$与品质因数Q成反比，Q越大，$\Delta\omega$越小，通带越窄曲线越尖锐，对信号的选择性越好





课堂练习1

已知 $u_S(t) = 10\cos\omega t\ \mathrm{V}$



求: (1) 频率ω为何值时，电路发生谐振。

(2)电路谐振时, U_L和U_C为何值。

解：(I)电压源的角频率应为

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-4} \times 10^{-8}}}\mathrm{rad/s} = 10^6\,\mathrm{rad/s}$$

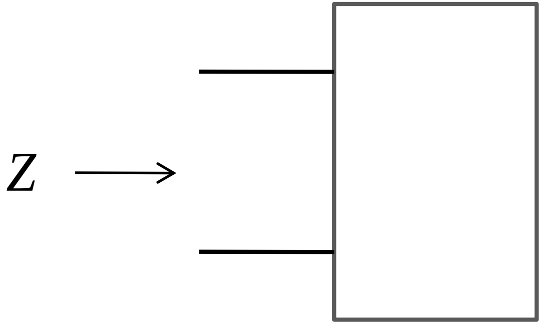
(2)电路的品质因数为

$$Q = \frac{\omega_0 L}{R} = 100 \qquad U_L = U_C = QU_S = 100 \times 10\mathrm{V} = 1000\mathrm{V}$$

单口网络的阻抗与导纳

阻抗： $Z(\omega)=\dot{U}/\dot{I}$

导纳： $Y(\omega)=\dot{I}/\dot{U}$



$$Z(\omega)=A(\omega)+jB(\omega)$$

$$Z(\omega)=\left|Z(\omega)\right|\angle\theta(\omega)$$

幅频特性　　相频特性

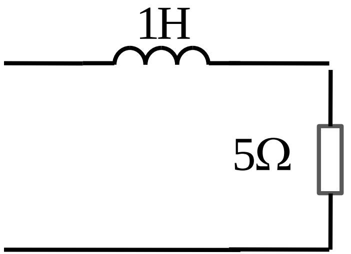
单口网络的频率响应

阻抗：$Z(\omega) = 5 + j\omega$

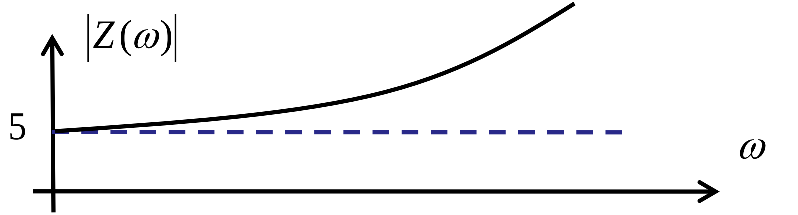
$$|Z(\omega)| = \sqrt{5^2 + \omega^2}$$

$$\theta(\omega) = ang\tan\frac{\omega}{5}$$

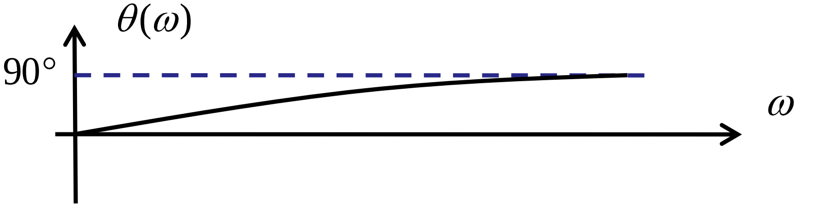
$$Z(\omega) \longrightarrow |Z(\omega)|\angle\theta(\omega)$$



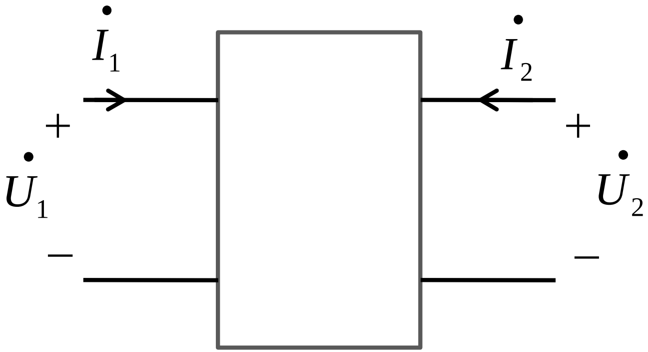
幅频特性



相频特性



双口网络



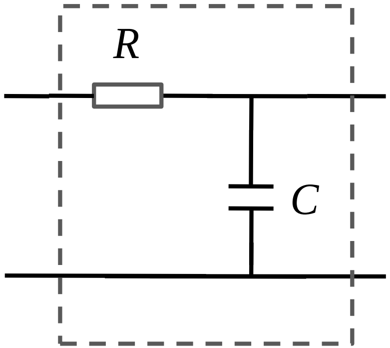
输入阻抗 $Z(\omega)=\dot{U}_1/\dot{I}_1$

输出阻抗 $Z(\omega)=\dot{U}_2/\dot{I}_2$

传输函数 $H(\omega)=\dot{U}_2/\dot{U}_1$

- 传递函数H：网络输出复幅值与输入复幅值的比值（频率的函数）
- 单口、双口网络都可以定义传递函数，输入、输出可以是电压或电流
- 频率响应：传递函数H的幅值和相位（频率的函数）的图形

双口RC电路的传递函数和频率响应



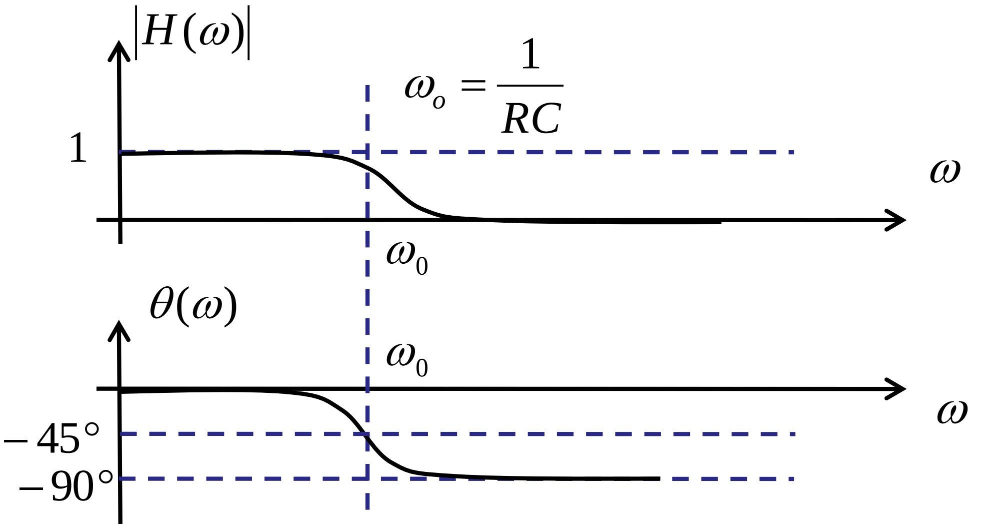
$$\dot{U}_2 = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \dot{U}_1$$

$$H(\omega) = \frac{\dot{U}_2}{\dot{U}_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

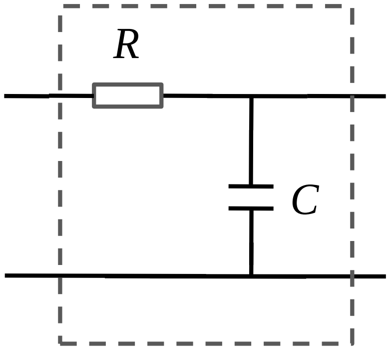
$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\theta(\omega) = 0 - ang\tan\omega RC$$

$$H(\omega) = |H(\omega)| \angle \theta(\omega) = \frac{1}{\sqrt{1 + (\omega RC)^2}} \angle - ang\tan\omega RC$$



双口RC电路的传递函数和频率响应

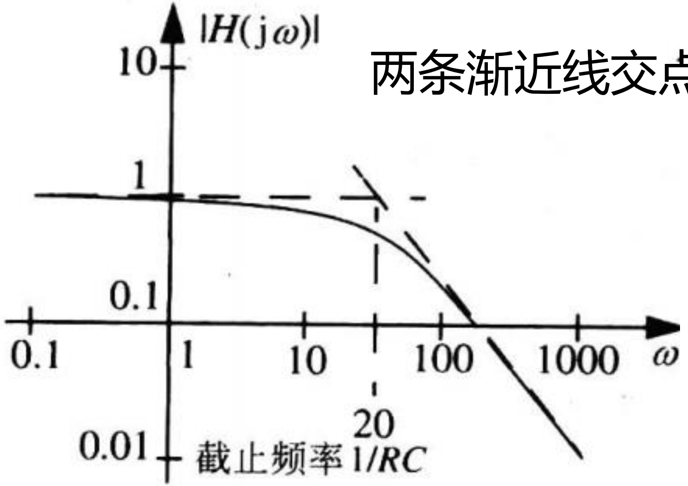


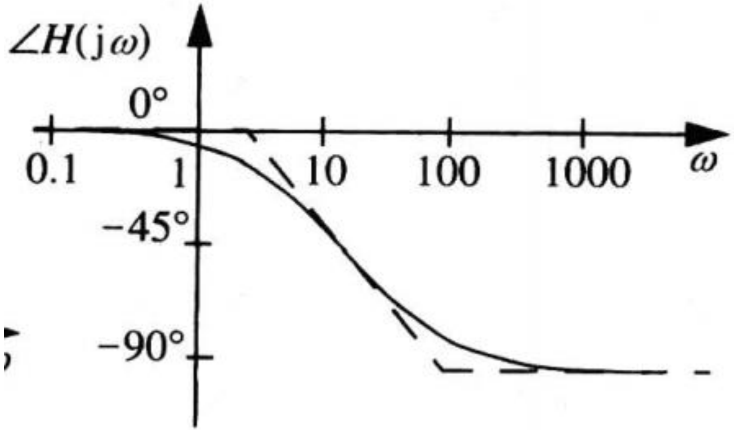
$$H(\omega)=\frac{1}{1+j\omega RC}$$

低频时 $\left|H(\mathrm{j}\omega)\right|\approx 1$ 对数坐标下平行直线

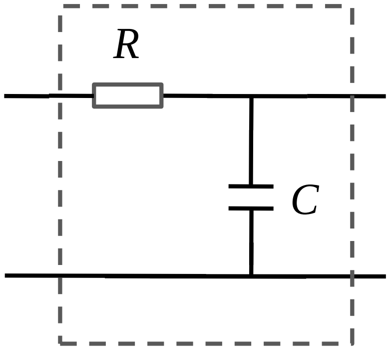
高频时 $\left|H(\mathrm{j}\omega)\right|\approx\frac{1}{\omega RC}\approx 0$ 对数坐标下是斜率为-1的直线

两条渐近线交点





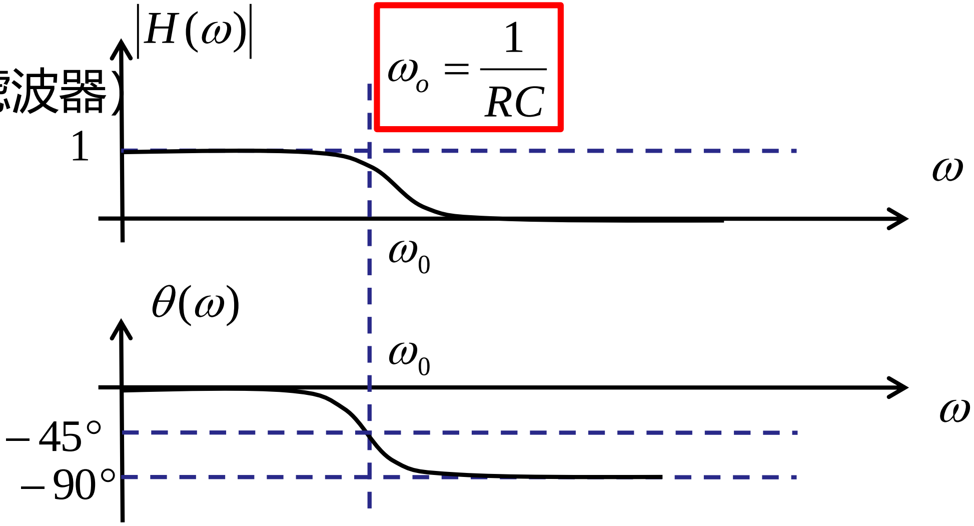
双口RC电路的传递函数和频率响应



$$H(\omega)=|H(\omega)|\angle\theta(\omega)=\frac{1}{\sqrt{1+(\omega RC)^2}}\angle-ang\tan\omega RC$$

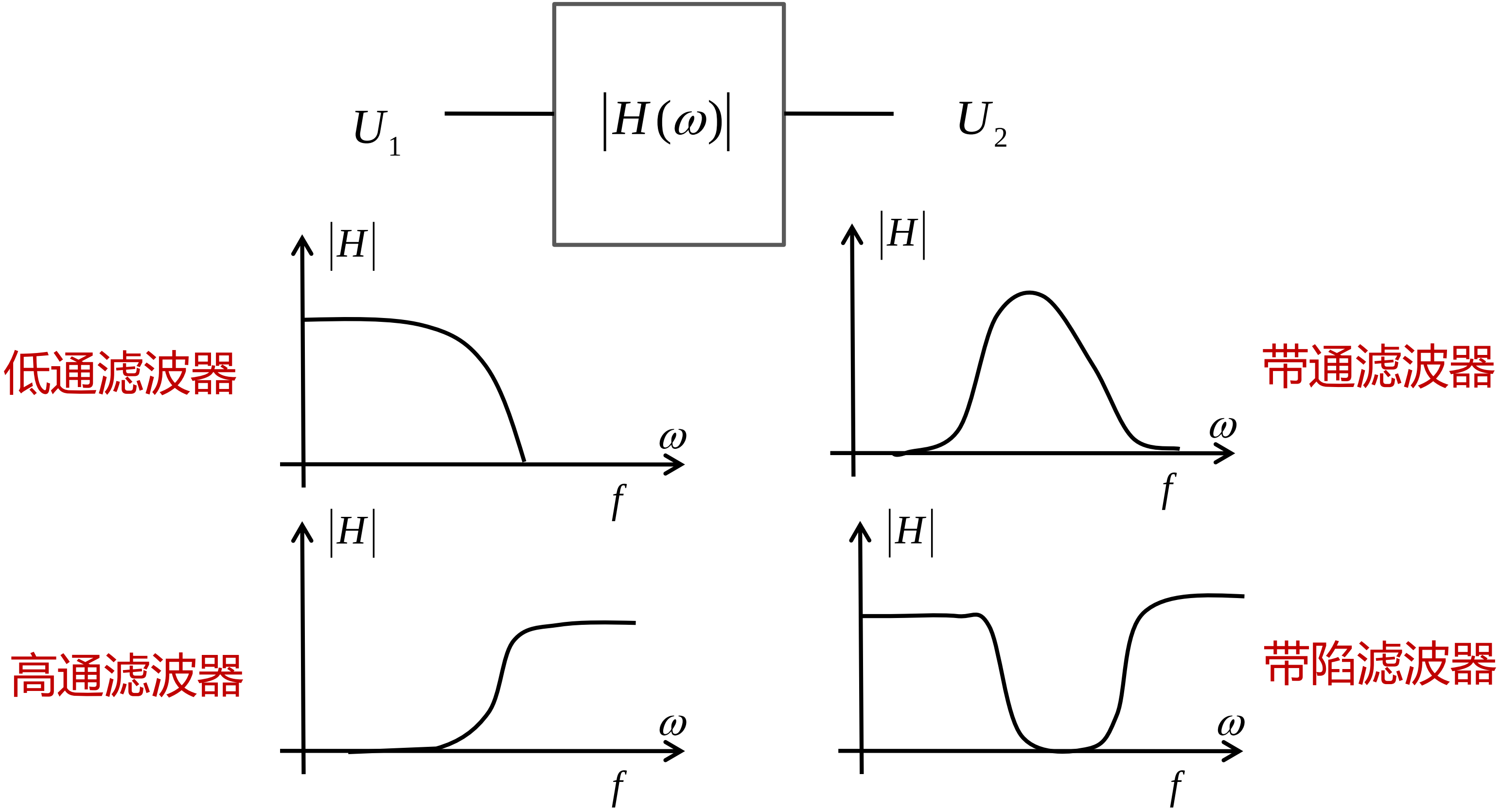
- 低频信号通过，高频信号衰减（低通滤波器）
- 在转折频率处，幅度变为0.707倍

$$\boxed{\begin{aligned}&20\log 10(0.707)\approx-3dB\\&10\log 10(0.5)\approx-3dB\end{aligned}}$$

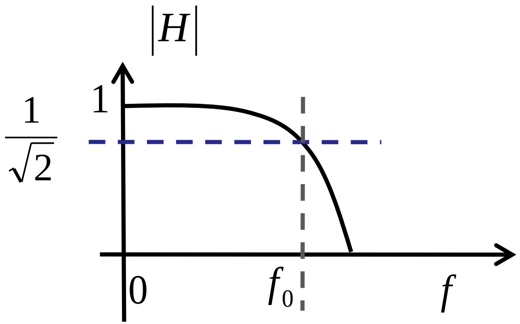


- 转折频率也称为-3dB频率或半功率点

滤波器频率响应

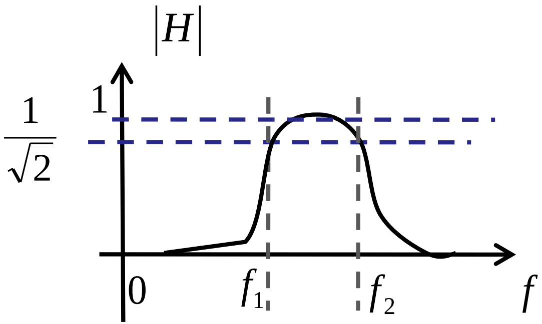


滤波器频率响应



低通滤波器:

带宽：以传输函数幅度的0.707倍对应的频率f_0为通带边界（转折频率）



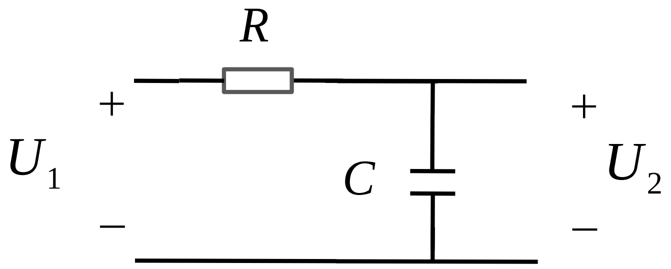
带通滤波器

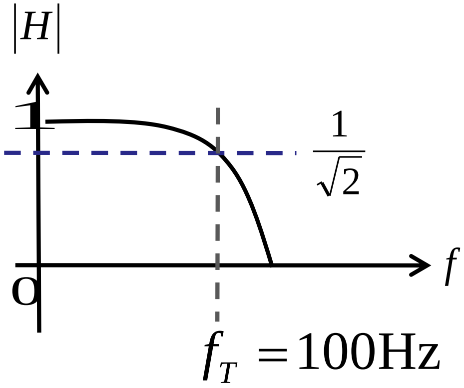
带宽：以传输函数幅度的0.707倍对应的频率f_1和f_2分别为通带的上边界和下边界

$\Delta f = f_2 - f_1$

RC低通滤波器的设计

RC低通滤波器如下图所示，请设计截止频率 f_T 为100Hz.





$$H(\omega)=\frac{1}{1+j\omega RC}$$

$$\left|H(\omega)\right|=\frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\left|H(\omega)\right|=\frac{1}{\sqrt{1+(\omega RC)^2}}=\frac{1}{\sqrt{2}}$$

$$\omega RC=1$$

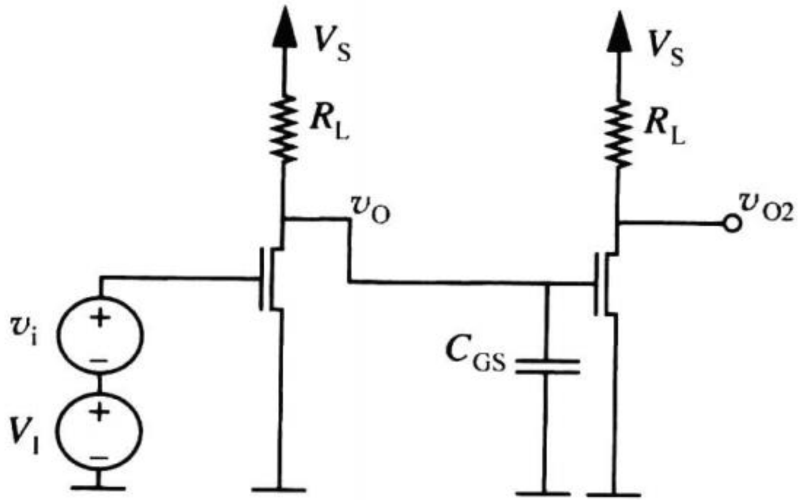
$$f_0=\frac{1}{2\pi RC}=100(Hz)$$

$$R=1000\Omega$$

$$C=\frac{1}{100\times 2\pi\times 1000}=1.6\times 10^{-6}=1.6(\mu\mathrm{F})$$

课堂练习

讨论下图电路中第二级MOSFET的栅极电容C_{GS}对电路放大功能的影响



1）如果C_G不存在，即C_G =0

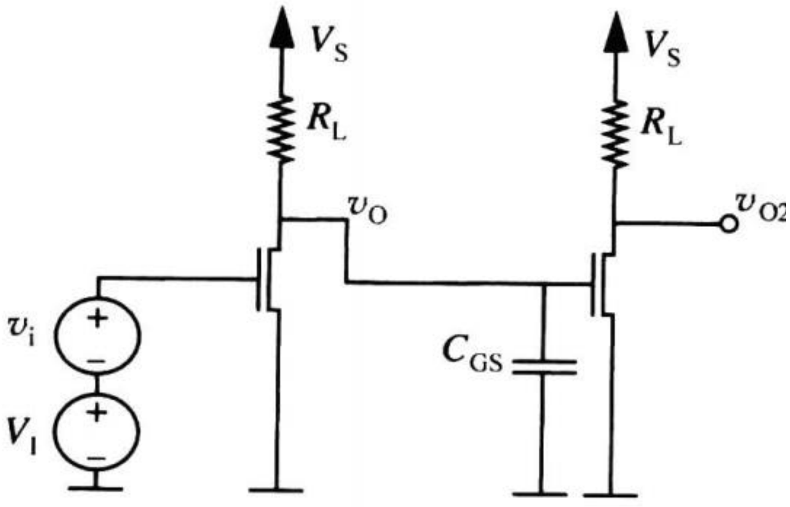
$$v_o = -K(V_i - V_T)R_L v_i = -g_m R_L v_i$$

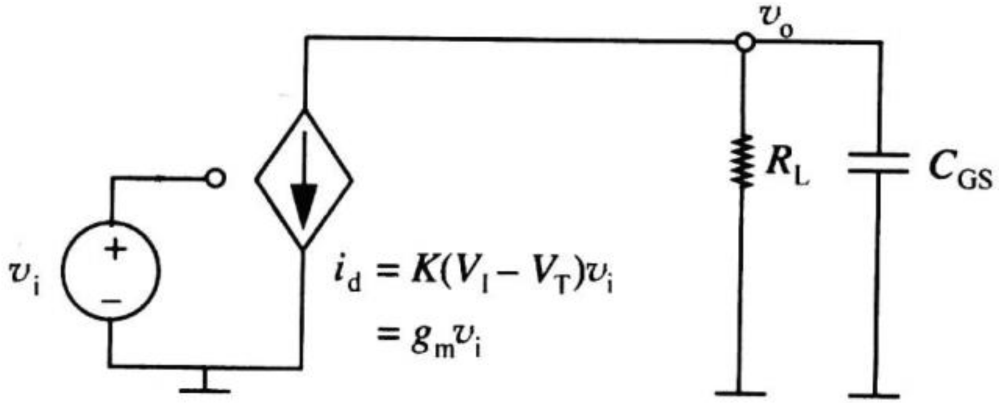
$$v_i = V_i \cos(\omega t)$$

$$v_o = -g_m R_L V_i \cos\omega t$$

课堂练习

2）如果$C_G \neq 0$，画出小信号模型





$$Z_L = R_L \parallel \frac{1}{sC_{GS}} = \frac{R_L}{1 + sR_L C_{GS}}$$

$$V_o = -g_m \frac{R_L}{1 + j\omega R_L C_{GS}} V_i$$

结论：

- 低频时，电压增益和$C_G = 0$的情况一样
- 高频时，电压增益随着频率增加而下降

课堂练习

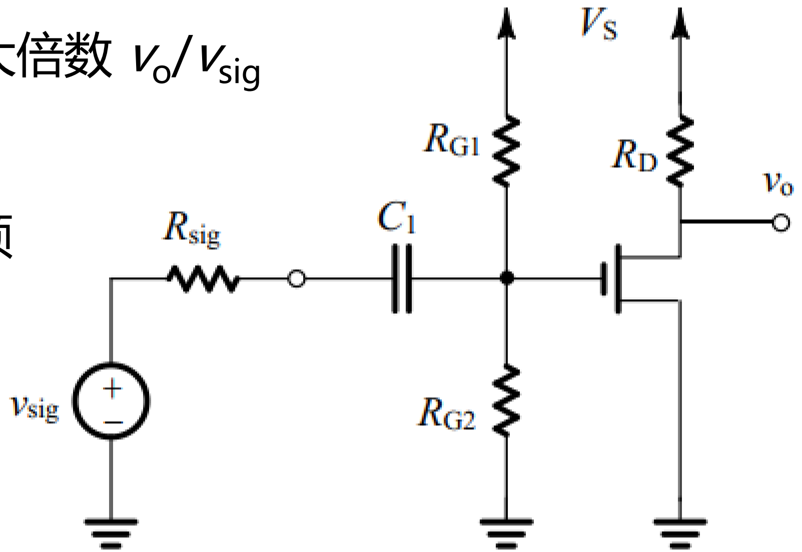
如下图所示 MOSFET 放大电路，已知 $V_S=5V$，$R_{G1}=350k\Omega$，$R_{G2}=150k\Omega$，$R_{sig}=5k\Omega$， $R_D=5k\Omega$，$C_1=10\mu F$， $V_T=1V$， $K=4mA/V^2$， 试分析求解：

1）试分析求解该电路的静态工作点， 即 V_{GS}、 V_{DS}、 I_D。

2）画出小信号模型，并求解该电路的电压放大倍数 v_o/v_{sig}

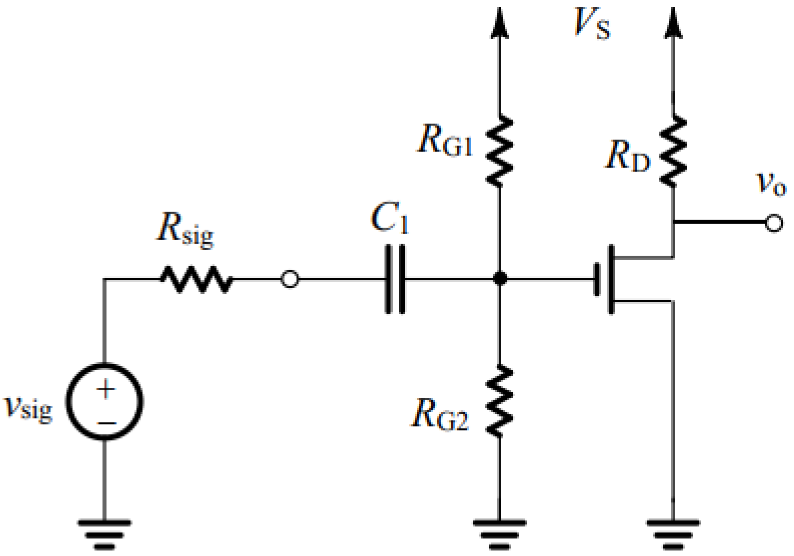
3）试定性的画出该电路的幅频特性曲线

4）说明C_1的作用，并求解该电路的下限转折频率（即 C_1 引起的转折频率)



课堂练习

1）试分析求解该电路的静态工作点， 即 V_{GS}、 V_{DS}、 I_D。



$$V_G = \frac{R_{G2}}{R_{G1} + R_{G2}} V_S = 1.5V$$

假设电路工作在电流饱和状态

$$I_D = \frac{1}{2} K (V_{GS} - V_T)^2 = \frac{1}{2} \times 4 \times (1.5 - 1)^2 = 0.5mA$$

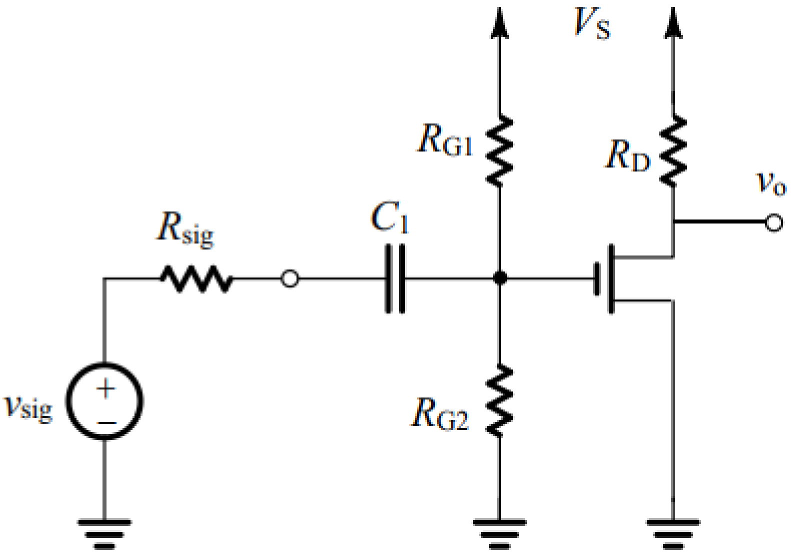
$$V_{DS} = V_S - I_D R_D = 5V - 0.5mA \times 5k\Omega = 2.5V$$

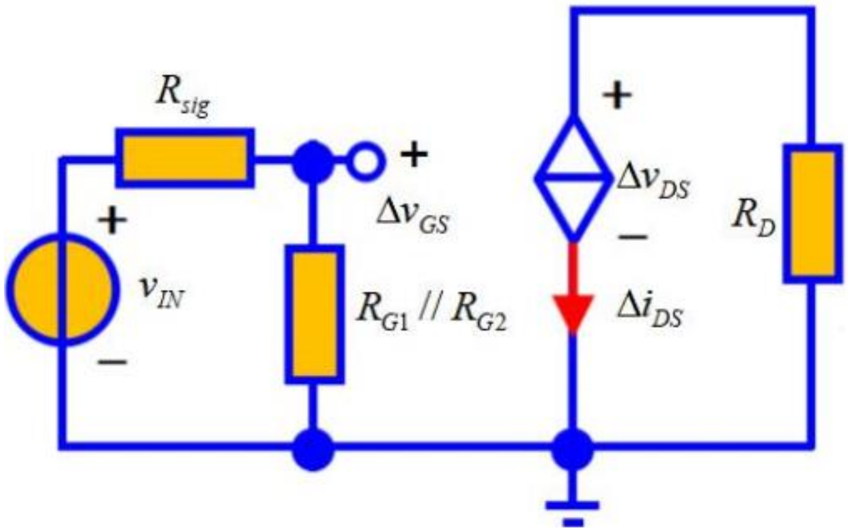
$$v_{GS} \geq V_T \text{，} \quad v_O \geq v_{IN} - V_T$$

假设成立

课堂练习

2）画出小信号模型，并求解该电路的电压放大倍数 v_o/v_{sig}



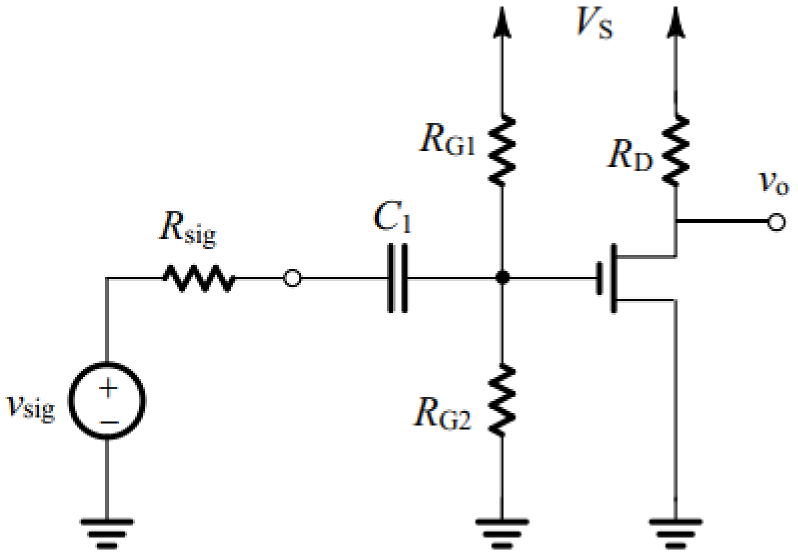


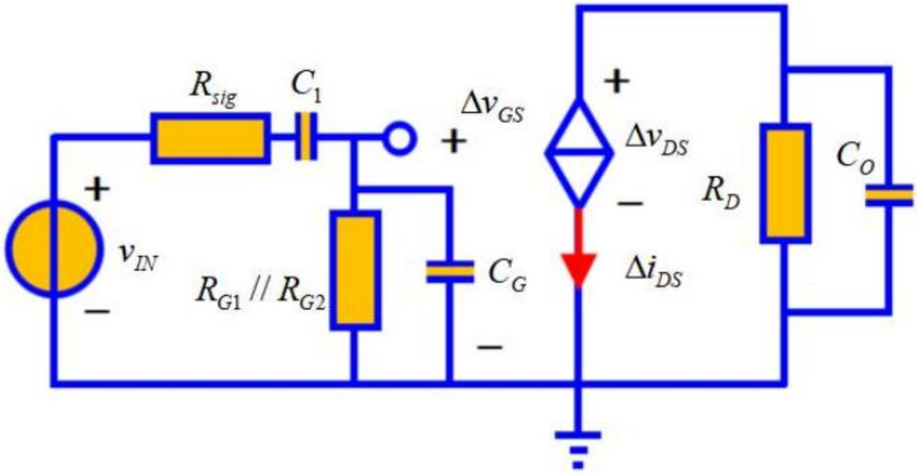
$$A_{vs} = -g_m R_D \frac{R_{G1} // R_{G2}}{R_{sig} + R_{G1} // R_{G2}} \approx -9.5$$

课堂练习

3）试定性的画出该电路的幅频特性曲线

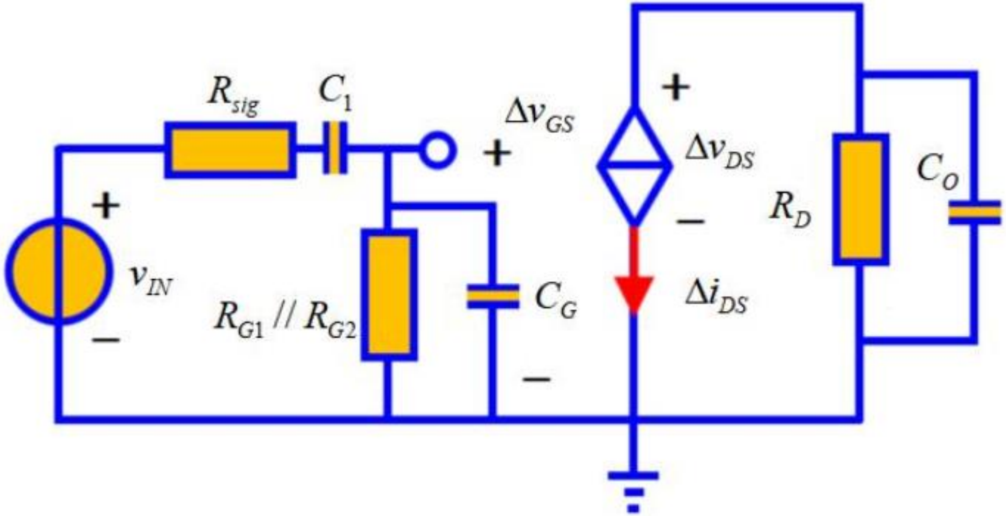
4）说明C_1的作用，并求解该电路的下限转折频率（即 C_1 引起的转折频率）





小信号模型

课堂练习



- 在低频段（ω→0）C_1、 C_G 和 C_O阻抗很大， 输入 MOS管栅极的信号频率越低幅度越小
- 在高频段（ω→∞）C_1、C_G和C_O阻抗很小，C_1的作用可以忽略
- C_G 和 C_O 分C_G是 MOS 管栅极电容，C_O是输出端分布电容，C_1 是耦合电容（隔直通交）
- 别并联在 MOS 管的栅极和输出端，将使放大器输入、 输出阻抗减小
- 输出信号随频率越高幅度越小， 可见电路具有带通特性
- 低频端的转折频率将由($R_{sig}+R_{G1}//R_{G1}$)和 C_1 确定

高通特性

$$f_c \approx \frac{1}{2\pi RC} = \frac{1}{2\pi 110k \times 10\mu} = 0.14\text{Hz}$$