HW1

Due Time: 2022/9/30 23:59:59;

Use word/latex to typeset your solutions and convert the document into pdf format with the filename of “StudentName_HW1”. Please send your pdf file to whxiong@uestc.edu.cn as the email attachment. The subject of your email shall be the same as the attached pdf file.

1. For the DC level in AWGN detection problem, we wish to have P_FA = 10^-4, and P_D =0.99. If the SNR, $10\log_{10}(A^2/\sigma^2)$ is -30dB, determine the necessary number of samples N

2. Design the perfect detector (no miss detection and false alarm) for the problem

   H0: $x \sim U[-c, c]$

   H1: $x \sim U[1-c, 1+c]$

   Where C>0 and U[a,b] denotes for uniform distribution in the range of [a,b]. By choosing the value of c, the detector has P_FA=0, and P_D = 1.

3. Find the MAP detector for

   H0: $x \sim N(0,1)$

   H1: $x \sim N(0,2)$

   when P(H0) = 1/2, and P(H0) =3/4. Draw the decision region for each case.

4. The observed data follows Gaussian distribution as $\mathbf{x} \sim N(\boldsymbol{\mu}, \mathbf{C})$, where $\mathbf{C} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$

   Consider the binary hypothesis test problem where under H_0, $\boldsymbol{\mu} = [0,1]^T$, and H_1 $\boldsymbol{\mu} = [1,1]^T$. Find the NP detector and discuss the what happens when $\rho = 0$

DUE TIME: 2022/10/16 midnight

Use word/latex to typeset your solutions and covert the document into pdf format with the filename of "StudentName_HW2(in Chinese)" and sent to Teaching Assistant via email. The subject of your email shall be the same as the attached pdf file.

1. If we want to detect the known signal of $s[n] = Ar^n$ for n=0,1…N-1 in WGN with variance of σ^2. Find the NP detector and its detection performance (P_D). Explain what happens when $N \to \infty$

2. Find the prewhitener D for the covariance matrix

$$\mathbf{C} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

3. A binary communication system with $\mathbf{s_0} = [1, -1]^T$ and $\mathbf{s}_1 = [1,1]^T$, the received signal is with WGN of variance of $\sigma^2 = 1$. Draw the decision on R^2 that minimize the probability of error. (Note: $P(S_0)$ may not equal to $P(S_1)$)

4. In the pulse amplitude modulation (PAM) system, we transmit M level of pulse such that

$$S_i[n] = A_i \qquad\qquad n = 0,1,\ldots N-1$$

For i=0,1…M-1. If each signal is equally likely to be transmitted, what is the optimal receiver for WGN with variance of σ^2? When M=2, with the signal energy fixed, how should A_1, and A_0 be chosen?

5. When we try to detect the signal of $s[n] = A\cos(\omega n + \phi)$ in AWGN (AWGN samples are of zero mean and variance of σ^2). Assume the value of the amplitude A is known, but ϕ (not random) is unknown .

(1) Design the NP detector for the signal s[n]

(2) If the detector wrongly assumes that $\phi = 0$ when the true value of ϕ is $\phi = \pi/6$, find the NP detector under this case, and discuss the detector's performance

HW4: Due data: 2022/10/30 23:59:59

1. the data $x[n] = Ar^n + w[n]$ for $n = 0,1, \ldots N-1$ are observed. $w[n]$ is AWGN with variance σ^2 known. r is also known. Find the CRLB of the unknown parameter A, and show that an efficient estimator of A exists and find its variance. What happens for $N \to \infty$ for various value of r.

2. The data $x[n] = r^n + w[n]$ for $n = 0,1, \ldots N-1$ are observed. $w[n]$ is AWGN with only the variance σ^2known. Find the efficient estimator and the CRLB of unknown r

3. If the unknown parameter θ is estimated via the linear model as $\hat{\theta} = (H^T H)^{-1} H^T X$, now what is the distribution of $\hat{\theta}$. If the another unknown $S = A\theta$ needs to be estimated. What is the estimator of S, and what distribution S follows?
4. Derive the estimator for linear model $x = H\theta + W$ when W follows N(0,C) (e.g., W is colored noise)
5. The i.i.d observations x[n] for n=0,1,⋯N-1 have the exponential PDF

$$p(x[n]; \lambda) = \begin{cases} \lambda \exp(-\lambda x[n]), & x[n] > 0 \\ 0, & x < 0 \end{cases}$$

Find the sufficient statistics for λ

6. We have the signal of $x[n] = \cos(2\pi f_0 n) + w[n]$, n=0,1,⋯N-1, where w[n] is AWGN with variance of σ^2, try to find the sufficient statistics for the frequency f0.

1. If $x[n] = Ar^n + w[n]$ for n=0,1,…N-1, where A is unknown, r is a known constant, and w[n] is AWGN with zero mean. Find the BLUE for A, and the variance of the estimated A. As $N \to \infty$ does the variance of the estimation approach to zero?

2. Assume $E(x[n]) = \theta S[n] + \beta$ where θ is unknown parameter, and β is a known constant. The data vector x has the covariance matrix of C. Prove that the modified BLUE for θ is

$$\hat{\theta} = \frac{\boldsymbol{S}^T \boldsymbol{C}^{-1}(\boldsymbol{X} - \boldsymbol{\beta}\mathbf{1})}{\boldsymbol{S}^T \boldsymbol{C}^{-1} \boldsymbol{S}}$$

Where **1** is an all one vector.

3. We observe N I.I.D samples from the pdf
   a) Gaussian

$$p(x; \mu) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(x[n] - \mu)^2\right]$$

   b) Exponential

$$p(x; \lambda) = \lambda \exp(-\lambda x[n]) \quad for\ x > 0$$

for each case find the MLE of the μ or λ.

4. For the signal model (8.3)

$$s[n] = \begin{cases} A & 0 \le n \le M \\ -A & M < n \le N-1 \end{cases}$$

Find the LSE of A and the minimum estimation error.

5. Prove the following properties of the projection matrix $P = H(H^T H)^{-1} H^T$

   a) $P^2 = P$
   b) P is positive semi-definite
   c) The eigenvalues of P is either 1 or 0