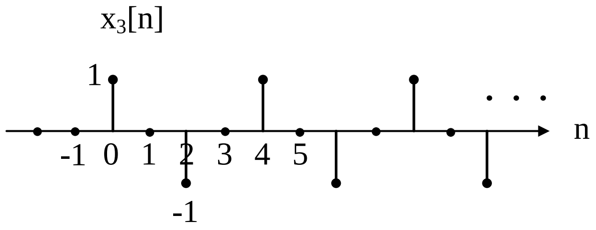
20-21-1 信号与系统期末 A 卷参考解答

一、计算题（12 分）

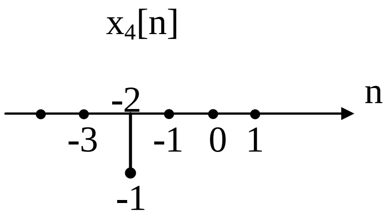
Solutions:

(1) $x_1[n]$ is periodic signal. The fundamental period is N=4. (2 分)

(2)

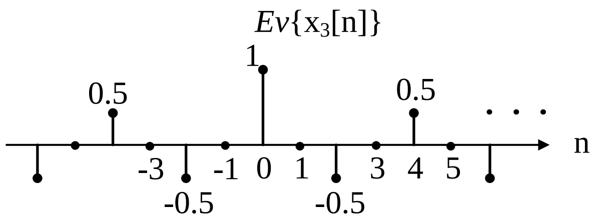


(2 分)

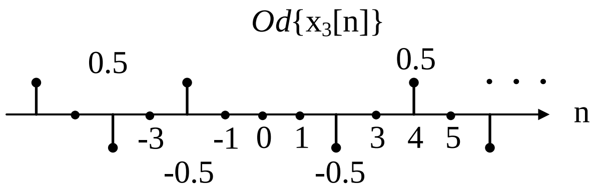


(2 分)

(3)



(3 分)



(3 分)

二、计算题（10 分）

Solutions:

$$y[n] = x[n]*h_1[n]*h_2[n]$$

$$= x[n]*h_2[n]*h_1[n] \quad (3\text{ 分})$$

$$= \{\delta[n] + a\delta[n-1]\}*\cos(2n) \quad (4\text{ 分})$$

$$= \cos(2n) + a\cos(2n-2) \quad (3\text{ 分})$$

三、计算题（16 分）

Solutions:

(a) $y(t) = x\left(\frac{1}{2}t - 1\right) \longleftrightarrow Y(j\omega) = 2X(2j\omega)e^{-2j\omega}$

$\omega_{\mathrm{Y}} = \omega_{\mathrm{M}}/2$, $\omega_{\mathrm{s}} = \omega_{\mathrm{M}}$ (4 分)

(b) $y(t) = x(t) + x^*(-t) \longleftrightarrow Y(j\omega) = X(j\omega) + X^*(j\omega)$

$\omega_{\mathrm{Y}} = \omega_{\mathrm{M}}$, $\omega_{\mathrm{s}} = 2\omega_{\mathrm{M}}$ (4 分)

(c) $y(t) = 3x^2(t) \longleftrightarrow Y(j\omega) = \frac{3}{2\pi}X(j\omega)*X(j\omega)$

$\omega_{\mathrm{Y}} = 2\omega_{\mathrm{M}}$, $\omega_{\mathrm{s}} = 4\omega_{\mathrm{M}}$ (4 分)

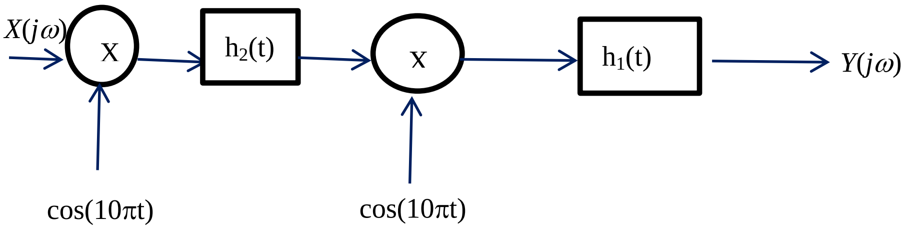
(d) $y(t) = x(t)\cos(\omega_M t) \longleftrightarrow \frac{1}{2}X\left[j(\omega+\omega_M)\right] + \frac{1}{2}X\left[j(\omega-\omega_M)\right]$

$\omega_{\mathrm{Y}} = 2\omega_{\mathrm{M}}$, $\omega_{\mathrm{s}} = 4\omega_{\mathrm{M}}$ (4 分)

四、计算题（14 分）

Solutions:

一种参考答案: $\omega_c = 10\pi$



(3 分)　(3 分)　(4 分)　　(4 分)

五、计算题（16 分）

Solutions:

$$u(t+1)-u(t-1) \longleftrightarrow \frac{2\sin\omega}{\omega} \quad (2\ 分)$$

$$x_1(t) = t\left[u(t+1)-u(t-1)\right] \longleftrightarrow X_1(j\omega) = j\frac{2\omega\cos\omega - 2\sin\omega}{\omega^2} \quad (2\ 分)$$

$$x_2(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\frac{\pi}{2}t}，\ a_k = \frac{1}{4}X_1(j\omega)\Big|_{\omega=k\pi/2} = \frac{j\sin(k\pi/2)}{2(k\pi/2)^2} \quad (4\ 分)$$

$$H(j\omega) = \begin{cases} 1/2 & 1<|\omega|<3 \\ 0 & \text{others} \end{cases}$$

$$H(jk\pi/2) = \begin{cases} 1/2 & k=\pm 1 \\ 0 & \text{others} \end{cases} \quad (4\ 分)$$

$$y(t) = \frac{1}{2}\times\frac{2j}{\pi^2}e^{j\frac{\pi}{2}t} - \frac{1}{2}\times\frac{2j}{\pi^2}e^{-j\frac{\pi}{2}t} = -\frac{2}{\pi^2}\sin\frac{\pi}{2}t \quad (4\ 分)$$

六、计算题（16 分）

(a) The block-diagram shows that $H(s)=\dfrac{Ks+4}{(s+2)(s+1)}$ (3 分)

From $h(0^+)=3=\lim\limits_{s\to\infty} sH(s)$ **,** K=3 **.**(2 分)

The causality gives that ROC: $\mathrm{Re}\{s\}>-1$ **.**

$\therefore H(s)=\dfrac{3s+4}{(s+2)(s+1)}$ **,** **ROC:** $\mathrm{Re}\{s\}>-1$ **.**(2 分)

The ROC includes the $j\omega$**-axis, so the system is stable.** (2 分)

(b) $\because H(s)=\dfrac{2}{s+2}+\dfrac{1}{s+1}$ **,** **ROC:** $\mathrm{Re}\{s\}>-1$ **,**

$\therefore h(t)=2e^{-2t}u(t)+e^{-t}u(t)$ **.**(3 分)

(c) $y(t)=1\cdot H(0)=2$ **.** (3 分)

七、计算题（16 分）

Solution:

(a)
$$\because x_1[n]=u[n]+\mathrm{L}u[n-1] \xleftrightarrow{z} X_1(z)=\frac{1}{1-z^{-1}}+\mathrm{L}\frac{z^{-1}}{1-z^{-1}} =\frac{1+\mathrm{L}z^{-1}}{1-z^{-1}},\ \text{ROC: } |z|>1$$, (2 分)

$$y_1[n]=-\frac{2}{3}(-2)^n u[-n-1]+\frac{1}{3}u[n] \xleftrightarrow{z} Y_1(z)=\frac{2}{3}\frac{1}{1-(-2)z^{-1}}+\frac{1}{3}\frac{1}{1-z^{-1}} =\frac{1}{(1+2z^{-1})(1-z^{-1})},\ \text{ROC: } 2>|z|>1$$,(2 分)

$$\therefore H(z)=\frac{Y_1(z)}{X_1(z)}=\frac{1}{(1+2z^{-1})(1+\mathrm{L}z^{-1})}$$,

$\because y_2[n]=\dfrac{2}{3}(-1)^{n+1}=H(-1)\cdot(-1)^n$ **,** $\therefore H(-1)=-\dfrac{2}{3}$ **,** $\therefore \mathrm{L}=-\dfrac{1}{2}$ **.**(2 分)

$$\therefore H(z)=\frac{Y(z)}{X(z)}==\frac{1}{(1+2z^{-1})(1-\frac{1}{2}z^{-1})},\ \text{ROC: } 2>|z|>\frac{1}{2}$$ **.**(4 分)

(b) The ROC shows that the system is stable but not causal. (2 分)

(c) $\because H(z)=\dfrac{1}{(1+2z^{-1})(1-z^{-1}/2)}=\dfrac{4}{5}\dfrac{1}{1+2z^{-1}}+\dfrac{1}{5}\dfrac{1}{1-z^{-1}/2}$, ROC: $2>|z|>\dfrac{1}{2}$ **,**

$\therefore h[n]=-\dfrac{4}{5}(-2)^n u[-n-1]+\dfrac{1}{5}\left(\dfrac{1}{2}\right)^n u(n)$ **.**(4 分)