19-20-2 信号与系统期末 A 卷参考解答

一、计算题（10 分）

Solutions:

(a) The system is not memoryless. (2 points)

(b) The system is time invariant. (2 points)

(c) The system is not linear. (2 points)

(d) The system is causal. (2 points)

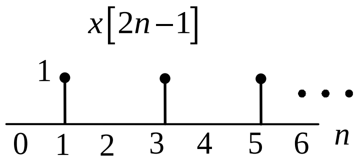
(e) The system is stable. (2 points)

二、计算题（8 分）

Solutions:

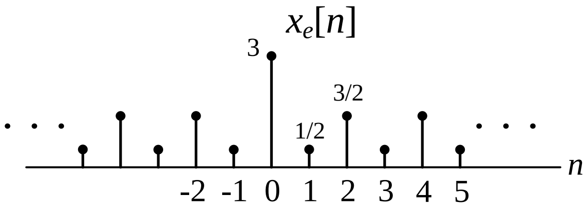
(a) $x[n]=[2+(-1)^n]\cdot u[n]$ is not periodic. (2 points)

(b) $\frac{1}{2}\left[1-\left(-1\right)^n\right]x[n]=\begin{cases}0 & , n \text{ is even} \\ x[n], & n \text{ is odd}\end{cases}$

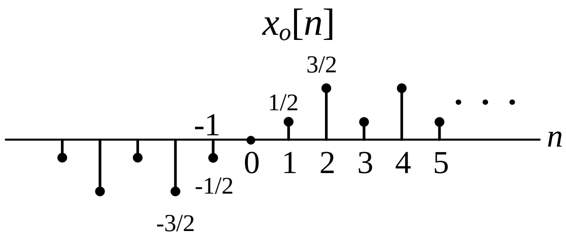


(2 points)

(3)



(2 points)

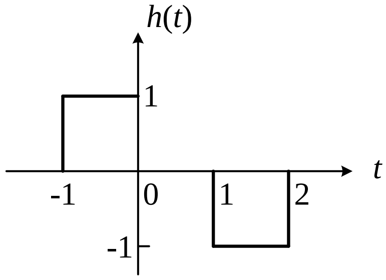


(2 points)

三、计算题（12 分）

Solutions:

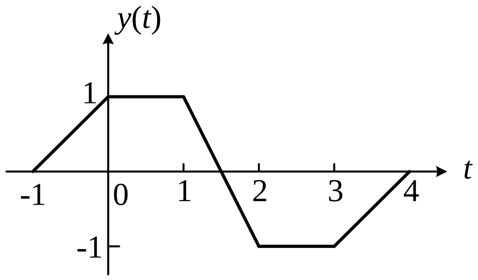
(a) $h(t)=\frac{ds(t)}{dt}=u(t+1)-u(t)-u(t-1)+u(t-2)$



(4 points)

(b) The system is not causal, but it is stable. (3 points)

(c) $y(t) = x(t) * h(t)$



(5 points)

四、计算题（12 分）

Solutions:

$x(t-1) \longleftrightarrow b_k = a_k e^{-jk\frac{\pi}{2}}$ is odd (2 points)

$x(t-1)$ is real odd signal. (2 points)

$a_0 = \frac{1}{4}\int_{<T>} x(t)dt = 0$ (2 points)

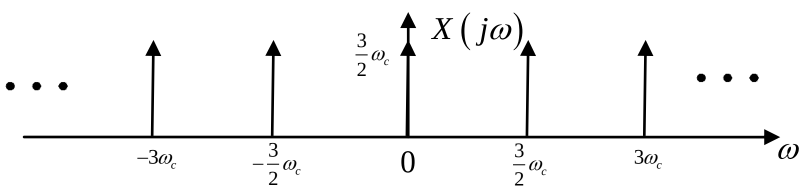
$|a_1|^2 + |a_{-1}|^2 = \frac{1}{4}\int_{<T>} |x(t)|^2 dt = \frac{1}{2}$, $a_1 = \pm\frac{j}{2}, a_{-1} = \mp\frac{j}{2}$ (3 points)

$x(t) = \pm\frac{j}{2}e^{j\frac{\pi}{2}t} \mp \frac{j}{2}e^{-j\frac{\pi}{2}t} = \pm\sin\frac{\pi}{2}t$ (3 points)

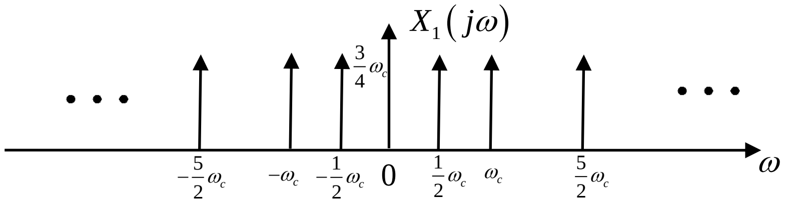
五、计算题（14 分）

Solutions:

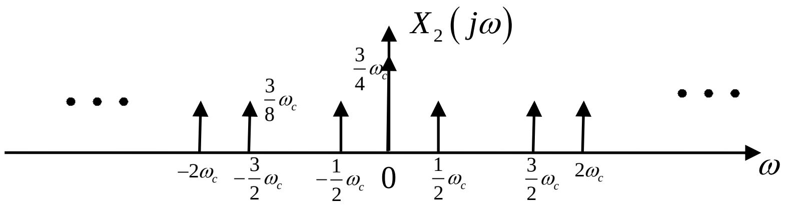
$$X(j\omega) = \sum_{k=-\infty}^{+\infty} \frac{3\omega_c}{2}\delta\left(\omega - \frac{3k\omega_c}{2}\right)$$



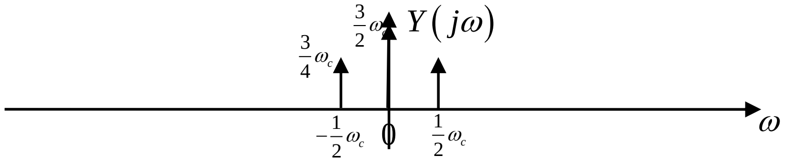
(3 points)



(3points)



(3 points)



(2 points)

$$y(t)=\frac{3}{4\pi}+\frac{3}{4\pi}\cos\frac{\omega_c}{2}t \quad \text{(3 points)}$$

六、计算题（12 分）

(a) $X(j\omega)\big|_{\omega=0}=\int_{-\infty}^{+\infty}x(t)dt=2$ (4 points)

(b) $Ev\{x(t)\}\longleftrightarrow \mathrm{Re}\{X(j\omega)\}$ (2 points)

$$\mathrm{Re}\{X(j\omega)\}=\frac{\sin 2\omega}{\omega} \quad \text{(2 points)}$$

(c) $\int_{-\infty}^{+\infty}\left|\mathrm{Re}\{X(j\omega)\}\right|^2 d\omega=2\pi\int_{-\infty}^{+\infty}\left|x(t)\right|^2 dt=2\pi$ (4 points)

七、计算题（16 分）

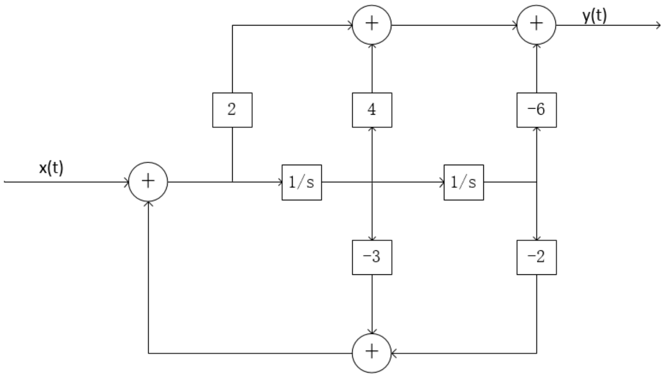
(a) $H(s)=\dfrac{2s^2+4s-6}{s^2+3s+2}$ (2 points)

ROC: Re{s}> -1(2 points), stable. (2 points)

(b)

$$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+2y(t)=2\frac{d^2x(t)}{dt}+4\frac{dx(t)}{dt}-6x(t) \quad \text{(3 points)}$$

(c) (4 points)



(d) $h(t)=2\delta(t)-8e^{-t}u(t)+6e^{-2t}u(t)$ (3points)

八、计算题（16 分）

Solution:

(a) From the figure, we get

$$H[z]=\frac{1-\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}} \qquad \text{(4 points)}$$

As the system is causal，$|z|>\frac{|k|}{3}$ (2 points)

(b) For the system to be stable, the ROC of H(z) must include the unit circle. (1 point)

This possible only if |k|/3< 1. This implies that |k| < 3. (1point)

(c)If k=1， $H[z]=\frac{1-\frac{1}{4}z^{-1}}{1+\frac{1}{3}z^{-1}}=\frac{1}{1+\frac{1}{3}z^{-1}}-\frac{\frac{1}{4}z^{-1}}{1+\frac{1}{3}z^{-1}}$ (2 points)

$h[n]=\left(\frac{1}{3}\right)^{n}u[n]-\frac{1}{4}\left(\frac{1}{3}\right)^{n-1}u[n-1]$ (2 points)

(d) If $k=1$ and $x[n]=(\frac{2}{3})^{n}$, $H[z]|_{z=\frac{2}{3}}=\frac{1-\frac{1}{4}z^{-1}}{1+\frac{1}{3}z^{-1}}|_{z=\frac{2}{3}}=\frac{5}{12}$, (1 point)

$y[n]=\left(\frac{2}{3}\right)^{n}H(\frac{2}{3})=\frac{5}{12}\left(\frac{2}{3}\right)^{n}$ (2 points)