

10、求 13.15 所示电路处于稳态时，电感的储能为多大？

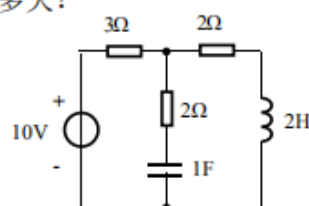


图 13.15

图 13.15
 \therefore 电路处于稳态时电容开路，电感短路
 $\therefore I = 10V / 5\Omega = 2A$
 \therefore 电感储能 $E_L = \frac{1}{2}LI^2 = \frac{1}{2} \times 2 \times 2^2 = 4J$

14、图 13.19 所示电路开关断开已经很久， $t=0$ 时开关闭合， $i(0_+) = ?$

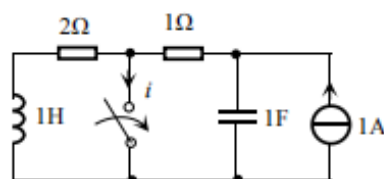


图 13.19

图 13.19
 $\therefore t < 0$ 时，直流输入，电感短路，电容开路
 电容上电压 $u_C(0_-) = 3V$
 电感上电流 $i_L(0_-) = 1A$
 $\therefore u_C(0_+) = u_C(0_-) = 3V$
 $i_L(0_+) = i_L(0_-) = 1A$
 $\therefore t = 0_+$ 时电路可等效为

\therefore 根据叠加定理 $i(0_+) = 2A$

- 1、如图 14.10 所示电路， $t=0$ 时，开关导通，求 $t>0$ 时，电容上电压 $u_C(t) = ?$

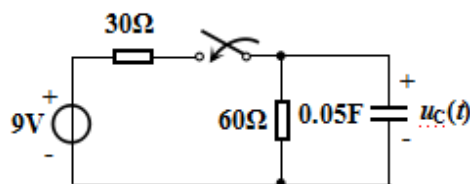


图 14.10

\therefore 初始条件 $U_0 = 0$ ，直流输入 $U_S \neq 0$
 \therefore 为零状态响应
 $\therefore u_C(t) = U_S - U_S e^{-\frac{1}{\tau}t}, (t > 0)$
 \therefore 当 $t > 0$ 时，戴维宁等效电路为
 $U_0 = 6V$
 $R_0 = \frac{30 \times 60}{30 + 60} = 20\Omega$
 $\therefore \tau = RC = 20 \times 0.05 = 1s$
 $\therefore u_C(t) = 6 - 6e^{-t}, (t \geq 0)$

- 6、如图 14.15 所示电路， $t < 0$ 时，双刀开关置 a，电路达到稳态；当 $t=0$ 时，开关从 a 置到 b，求 $t \geq 0$ 时，电感上电流 $i_L(t) = ?$

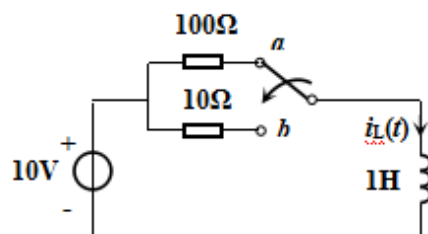


图 14.15

$\therefore i_L(0-) = 10V / 100\Omega = 0.1A$
 $\therefore i_L(0+) = i_L(0-) = 0.1A$
 $\therefore t \geq 0$ 时， $i_L(t) = I_S + (I_0 - I_S)e^{-\frac{1}{\tau}t}$
 ~~$\tau = L/R = 1/10s$~~
 \therefore 诺顿等效电路为
 $\tau = L/R = 0.1s$
 $\therefore i_L(t) = 1 - 0.9e^{-10t}, (t \geq 0)$

9、图 14.18 示电路原来已经稳定， $t=0$ 时闭合开关，求 $t>0$ 的电容电压 $u_c(t)$

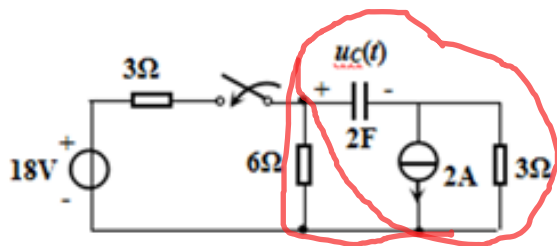


图 14.18

$\therefore t < 0$ 时，电路达到稳定，电容开路

$$\therefore u_c(0_-) = 2 \times 3 = 6V$$

$$\therefore u_c(0_+) = u_c(0_-) = 6V$$

\therefore 当 $t > 0$ 时，电容两端看入的戴维宁等效为

$$u_{c1} = \frac{6}{6+3} \times 18 = 12V$$

$$u_{c2} = -2 \times 3 = -6V$$

$$\therefore u_0 = 12 + 6 = 18V$$

$$R_0 = \frac{3 \times 6}{3+6} + 3 = 5\Omega$$

$$18V \text{ 源与 } 5\Omega \text{ 电阻串联，再与 } 2F \text{ 电容并联，} u_c(t), \tau = R_0 C = 10s$$

\therefore 全响应为

$$\begin{aligned} u_c(t) &= U_s + (U_0 - U_s) e^{-\frac{t}{\tau}} \\ &= 18 + (6 - 18) e^{-\frac{t}{10}} (V), (t > 0) \\ &= 18 - 12 e^{-\frac{1}{10}t} V, (t > 0) \end{aligned}$$

2、图 15.8 所示电路，已知电路的初始条件为： $u_C(0) = 1V$ ， $i_L(0) = 0A$ ，试求 $t \geq 0s$ 电容电压的响应？

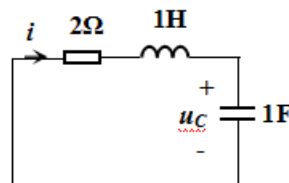


图 15.8

列出微分方程 $LC \frac{d^2 u_C}{dt^2} + RC \frac{du_C}{dt} + u_C = 0$

即 $\begin{cases} \frac{d^2 u_C}{dt^2} + 2 \frac{du_C}{dt} + u_C = 0 \\ u_C(0+) = 1V \\ i_C(0+) = 0A \end{cases}$

求特征根： $s^2 + 2s + 1 = 0$

$s_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2} = -1$

$s_{1,2}$ 是两个相等负实数，为临界阻尼情况

通解： $u_C(t) = K_1 e^{s_1 t} + K_2 t e^{s_2 t}$

$\therefore u_C(0+) = 1V$

$\therefore K_1 = 1$

$\therefore i_C(0+) = C[K_1 s_1 e^{s_1 t} + K_2 e^{s_2 t} + K_2 s_2 t] = 0$

$\therefore K_1 s_1 + K_2 = 0$

$\therefore K_2 = 1$

全解为

$u_C(t) = e^{s_1 t} + t e^{s_2 t}, t \geq 0$

3、图 15.9 所示电路，已知 $U_s = 1V$ ，电容和电感的初始状态均为 0，试求 $t \geq 0s$ 电容电压的响应？

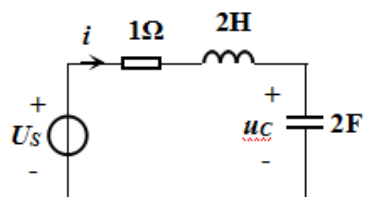


图 15.9

微分方程

$$\begin{cases} 4 \frac{d^2 u_c}{dt^2} + 2 \frac{du_c}{dt} + u_c = U_s \\ u_c(0+) = 0 \\ i_c(0+) = 0 \end{cases}$$

特征方程: $s^2 + 0.5s + 0.25 = 0$

$$s_{1,2} = \frac{-0.5 \pm \sqrt{0.25 - 1}}{2}, \text{ 为欠阻尼情况}$$

$$= -\frac{1}{4} \pm j \frac{\sqrt{3}}{4}$$

通解: $u_c(t) = e^{-\frac{1}{4}t} [k_1 \sin(\frac{\sqrt{3}}{4}t) + k_2 \cos(\frac{\sqrt{3}}{4}t)]$

特解: $u_c^*(t) = U_s$

全解: $u_c(t) = U_s + e^{-\frac{1}{4}t} [k_1 \sin(\frac{\sqrt{3}}{4}t) + k_2 \cos(\frac{\sqrt{3}}{4}t)]$

$\therefore u_c(0+) = 0$

$\therefore 1 + k_2 = 0 \Rightarrow k_2 = -1$

$\therefore i_c(0+) = 0$

$\therefore -\frac{1}{4} e^{-\frac{1}{4}t} [k_1 \sin(\frac{\sqrt{3}}{4}t) + k_2 \cos(\frac{\sqrt{3}}{4}t)]$
 $+ e^{-\frac{1}{4}t} [\frac{\sqrt{3}}{4} k_1 \cos(\frac{\sqrt{3}}{4}t) - \frac{\sqrt{3}}{4} k_2 \sin(\frac{\sqrt{3}}{4}t)] = 0$
 $= -\frac{1}{4} k_2 + \frac{\sqrt{3}}{4} k_1 = 0 \Rightarrow k_1 = -\frac{\sqrt{3}}{3}$

$\therefore u_c(t) = 1 + e^{-\frac{1}{4}t} [-\frac{\sqrt{3}}{3} \sin(\frac{\sqrt{3}}{4}t) - \cos(\frac{\sqrt{3}}{4}t)], t \geq 0$

7、图 15.13 所示电路，已知 $3AU(t)$ ，试求 $t \geq 0s$ 电感的电流响应？

$$I_s = 3AU(t)$$

$U(t)$ 为阶跃函数

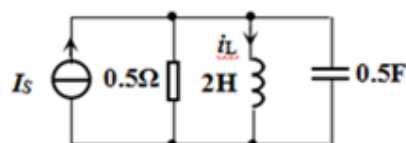


图 15.13

微分方程 $\begin{cases} LC \frac{d^2 i_L}{dt^2} + \frac{L}{R} \frac{di_L}{dt} + i_L = I_s \\ i_L(0) = 1A \\ u_L(0) = 1V \end{cases}$

求特征根: $s^2 + 4s + 1 = 0, s_{1,2} = -2 \pm \sqrt{3}$

$s_{1,2}$ 为两个不相等负实数, 为过阻尼情况

通解为: $i_L(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$, 特解 $i_L^*(t) = I_s$

$\therefore i_L(0) = 1$ 全解 $i_L(t) = I_s + k_1 e^{s_1 t} + k_2 e^{s_2 t}$

$\therefore k_1 + k_2 = -2$

$\therefore u_L(0) = 1$

$\therefore k_1 s_1 + k_2 s_2 = \frac{1}{2}$

$\therefore k_1 \approx -2.01, k_2 \approx 0.01$

\therefore 全解为

$$i_L(t) = 3 + 2.01 e^{(-2+\sqrt{3})t} + 0.01 e^{(-2-\sqrt{3})t}, t \geq 0$$

10、图 15.16 所示电路，试求 $u_C(t)$ 的响应。

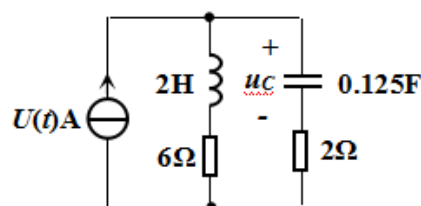


图 15.16

微分方程 $i_C(t) = C \frac{du_C}{dt}$
 $i_L(t) = I_s - C \frac{du_C}{dt}$
 $L \frac{di_L}{dt} + (I_s - C \frac{du_C}{dt}) R_1 = u_C + C \frac{du_C}{dt} R_2$
 $-LC \frac{d^2 u_C}{dt^2} - C(R_1 + R_2) \frac{du_C}{dt} + I_s R_1 - u_C = -I_s R_1$
 $\frac{d^2 u_C}{dt^2} + 4 \frac{du_C}{dt} + 4 u_C = 24$

求特征根: $s^2 + 4s + 4 = 0, s_{1,2} = -2$
 $s_{1,2}$ 为相等负实数, 为临界阻尼情况

通解: $u_C(t) = k_1 e^{s_1 t} + k_2 t e^{s_2 t}$

特解: $u_C^*(t) = 6V$

全解: $u_C(t) = 6 + k_1 e^{-2t} + k_2 t e^{-2t}$

$\therefore u_C(0) = 0$

$\therefore 6 + k_1 = 0, k_1 = -6$

$\therefore i_C(0) = 0$

$\therefore -2k_1 + k_2 = 0, k_2 = -12$

\therefore 全解为 $u_C(t) = 6 - 6e^{-2t} - 12te^{-2t}, t \geq 0$