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 $h_j = \sum b_{4j+i} \cdot 2^j$

3e2.18 2.20

Therefore,
$$B = \sum_{i=0}^{4n-1} b_i \cdot 2^i = \sum_{i=0}^{n-1} h_i \cdot 16^i$$

$$-B = 2^{4n} - \sum_{i=0}^{4n-1} b_i \cdot 2^i = 16^n - \sum_{i=0}^{n-1} h_i \cdot 16^i$$

Suppose a 3n-bit number B is represented by an n-digit octal number Q. Then the two's-complement of B is represented by the 8's-complement of Q.

3e2.22 Starting with the arrow pointing at any number, adding a positive number causes overflow if the arrow is advanced through the +7 to -8 transition. Adding a negative number to any number causes overflow if the arrow is not advanced through the +7 to -8 transition.

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3e2.24 2.26 Let the binary representation of X be $x_{n-1}x_{n-2}...x_1x_0$. Then we can write the binary representation of Y as $x_mx_{m-1}...x_{1x_0}$, where m=n-d. Note that x_{m-1} is the sign bit of Y. The value of Y is

$$Y = -2^{m-1} \cdot x_{m-1} + \sum_{i=0}^{m-1} x_i \cdot 2^i$$

The value of X is

$$\begin{split} X &= -2^{n-1} \cdot x_{n-1} + \sum_{i=0}^{n-2} x_i \cdot 2^i \\ &= -2^{n-1} \cdot x_{n-1} + Y + 2^{m-1} \cdot x_{m-1} + \sum_{i=m-1}^{n-2} x_i \cdot 2^i \\ &= -2^{n-1} \cdot x_{n-1} + Y + 2 \cdot 2^{m-1} + \sum_{i=0}^{n-2} x_i \cdot 2^i \end{split}$$

Case 1 $(x_{m-1} = 0)$ In this case, X = Y if and only if $-2^{n-1} \cdot x_{n-1} + \sum_{i=m}^{n-2} x_i \cdot 2^i = 0$, which is true if and only if all of the discarded bits $(x_m \dots x_{n-1})$ are 0, the same as x_{m-1} .

Case 2 $(x_{m-1} = 1)$ In this case, X = Y if and only if $-2^{n-1} \cdot x_{n-1} + 2 \cdot 2^{m-1} + \sum_{i=m}^{n-2} x_i \cdot 2^i = 0$, which is true if and only if all of the discarded bits $(x_m \dots x_{n-1})$ are 1, the same as x_{m-1} .

- 3e2.25 2.27 If the radix point is considered to be just to the right of the leftmost bit, then the largest number is $1.11\cdots 1$ and the 2's complement of D is obtained by subtracting it from 2 (singular possessive). Regardless of the position of the radix point, the 1s' complement is obtained by subtracting D from the largest number, which has all 1s (plural).
- 3e2.28 2.30

$$B = -b_{n-1} \cdot 2^{n-1} + \sum_{i=0}^{n-2} b_i \cdot 2^i$$

$$2B = -b_{n-1} \cdot 2^n + \sum_{i=0}^{n-2} b_i \cdot 2^{i+1}$$

Case 1 $(b_{n-1} = 0)$ First term is 0, summation terms have shifted coefficients as specified. Overflow if $b_{n-2} = 1$.

Case 2 $(b_{n-1} = 1)$ Split first term into two halves; one half is cancelled by summation term $b_{n-2} \cdot 2^{n-1}$ if $b_{n-2} = 1$. Remaining half and remaining summation terms have shifted coefficients as specified. Overflow if $b_{n-2} = 0$.

- 4e2.33 2.35 001-010, 011-100, 101-110, 111-000.
- 2.38 The manufacturer's code fails every 4th time, for a total of 2^{n-2} for an *n*-bit encoding disc. A standard binary code fails when the LSB changes from 1 to 0, which changes the next bit (and possibly others), guaranteeing a problem. So, half the boundaries in a standard binary code are bad, a total of 2^{n-1} for an *n*-bit encoding disc. The manufacturer's code is only half as bad as a standard binary code.
- 3e2.36 2.41 In the string representation, each position may have a 0, a 1, or an x, a total of three possibilities per position, and 3^n combinations in all.