

heap

- complete binary tree (review)
- heap and priority queues (Chapter 9)
- binary heap and min-heap
- max-heap demo
- *max-heap coding*
- heap sort (Chapter 7)

heap coding: heap.h

Heap ADT: A **one - based** and **one dimensional array** is used to simplify parent and child calculations.

```
struct Heap {
    int *nodes;          // an array of nodes
    int capacity;        // array size of node or key, item
    int N;               // the number of nodes in the heap
    bool (*comp)(Heap*, int, int);
    Heap(int capa = 2) {
        capacity = capa;
        nodes = new int[capacity];
        N = 0;
        comp = nullptr;
    };
    ~Heap() {}
};

using heap = Heap*;
```

heap coding: heap.h

```
void clear(heap hp);           // deallocate heap
int size(heap hp);             // return nodes in heap currently
int level(int n);              // return level based on num of nodes
int capacity(heap hp);         // return its capacity (array size)
int reserve(heap hp, int capa); // reserve the array size (= capacity)
int full(heap hp);             // return true/false
int empty(heap hp);            // return true/false
void grow(heap hp, int key);   // add a new key
void trim(heap hp);           // delete a queue
int heapify(heap hp);           // convert a complete BT into a heap

// helper functions to support grow/trim functions
int less(heap hp, int i, int j); // used in max heap
int more(heap hp, int i, int j); // used in min heap
void swim(heap hp, int k);      // bubble up
void sink(heap hp, int k);      // tickle down
// helper functions to check heap invariant
int heapOrdered(heap hp);       // is heap[1..N] a heap?
```

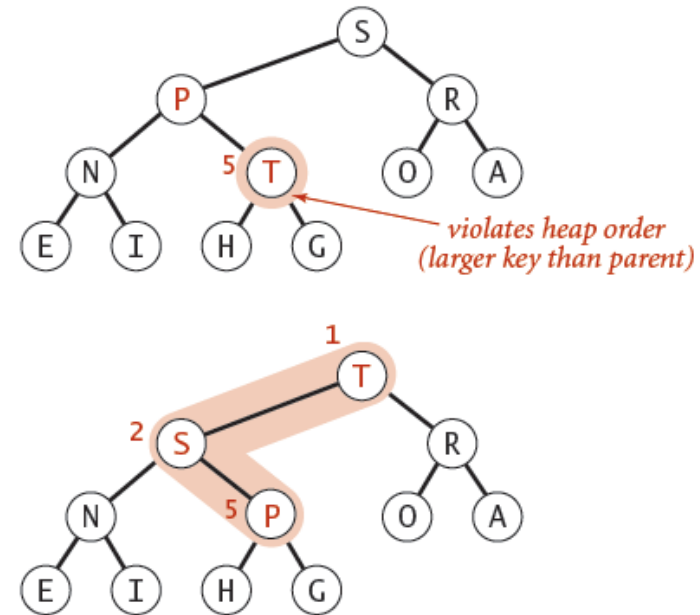
heap coding

Promotion in a heap: swim

- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until **heap order** restored.

swim up
or
sink down

This is a maxheap example.



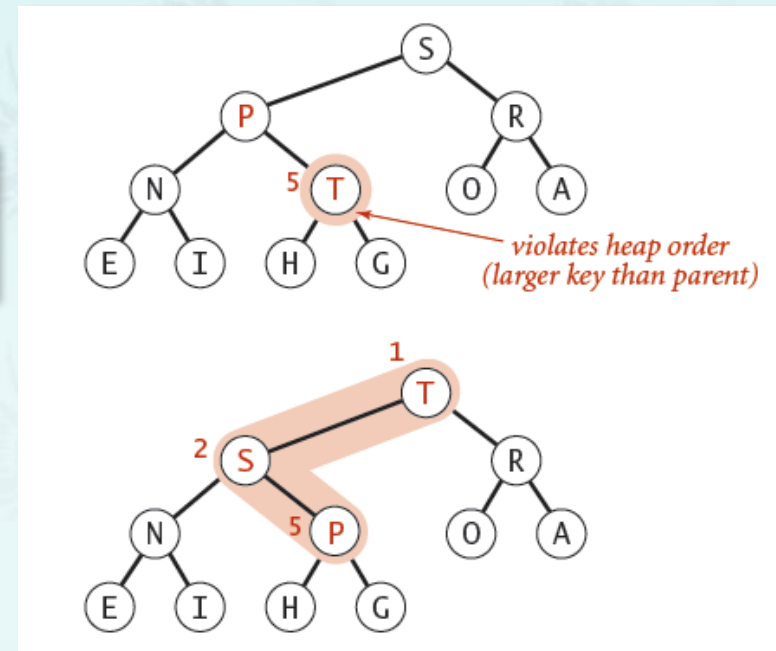
heap coding

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- To eliminate the violation:
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guess a name?

```
bool     (heap h, int p, int c) {  
    return h->nodes[p] < h->nodes[c];  
}
```



heap coding

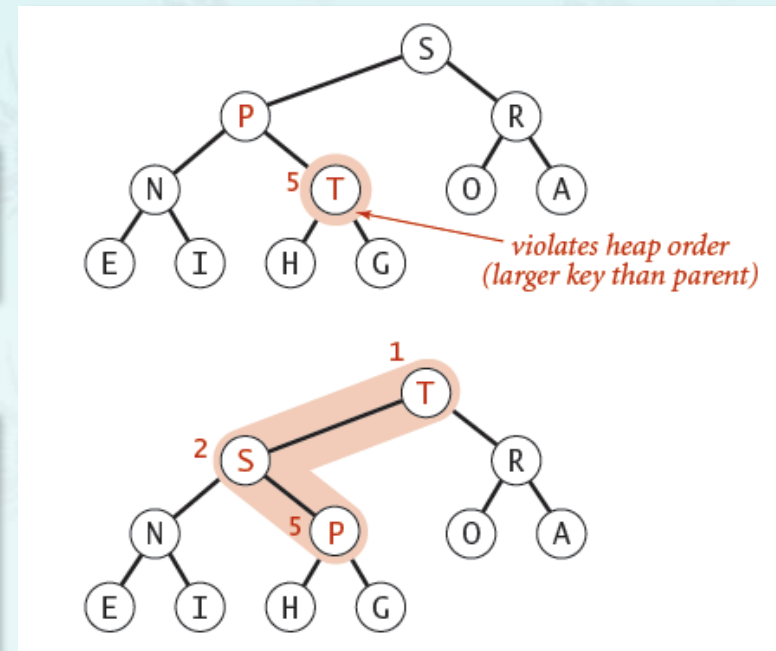
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}
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```
void     (heap h, int p, int c) {  
    int item = h->nodes[p];  
    h->nodes[p] = h->nodes[c];  
    h->nodes[c] = item;  
}
```



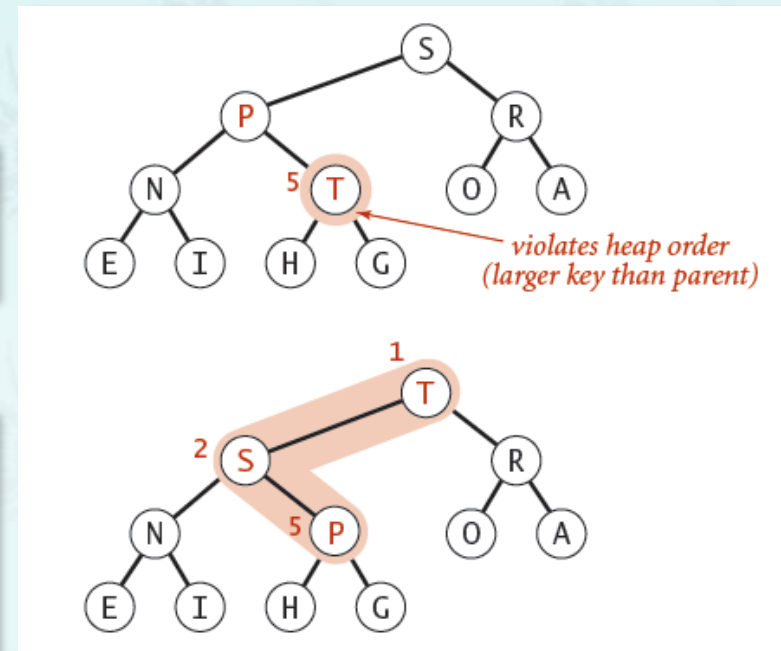
heap coding

Promotion in a heap: swim

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 - Repeat until heap order restored.

```
bool less(heap h, int p, int c) {  
    return h->nodes[p] < h->nodes[c];  
}
```

```
void swap(heap h, int p, int c) {  
    int item = h->nodes[p];  
    h->nodes[p] = h->nodes[c];  
    h->nodes[c] = item;  
}
```



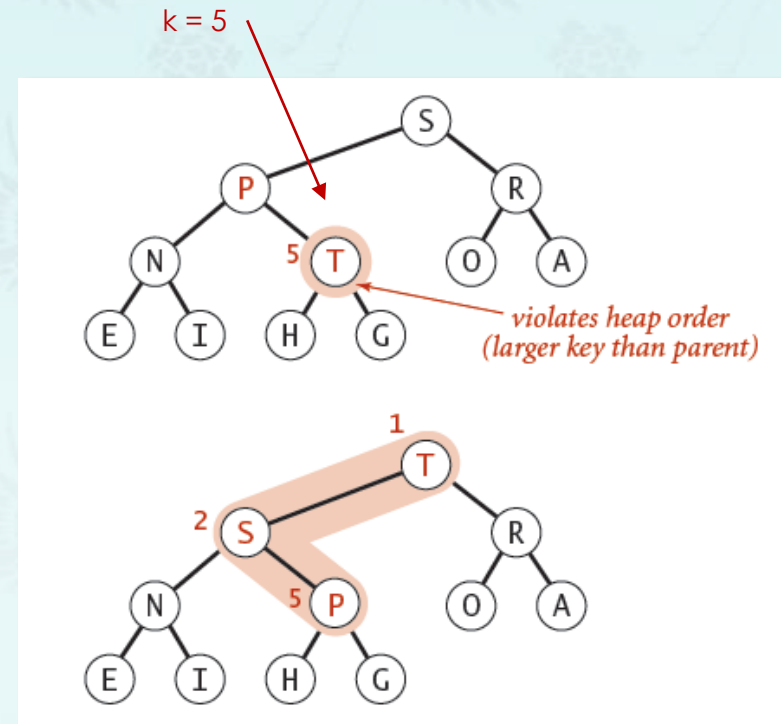
heap coding

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guess a name?

```
void       (heap h, int k)
{
    while (not reached at root &&
           k's parent key < k's key)
    {
        
    }
}
```

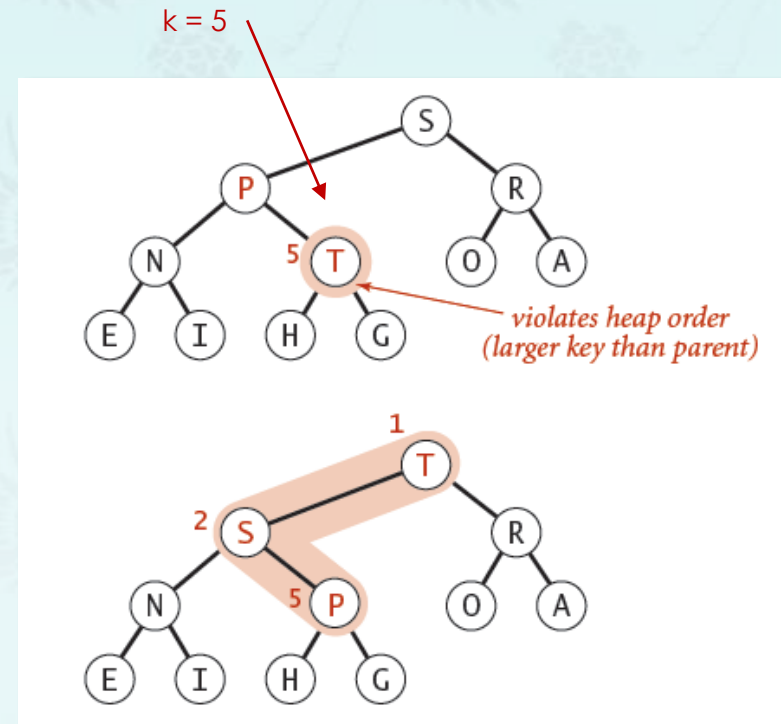


heap coding

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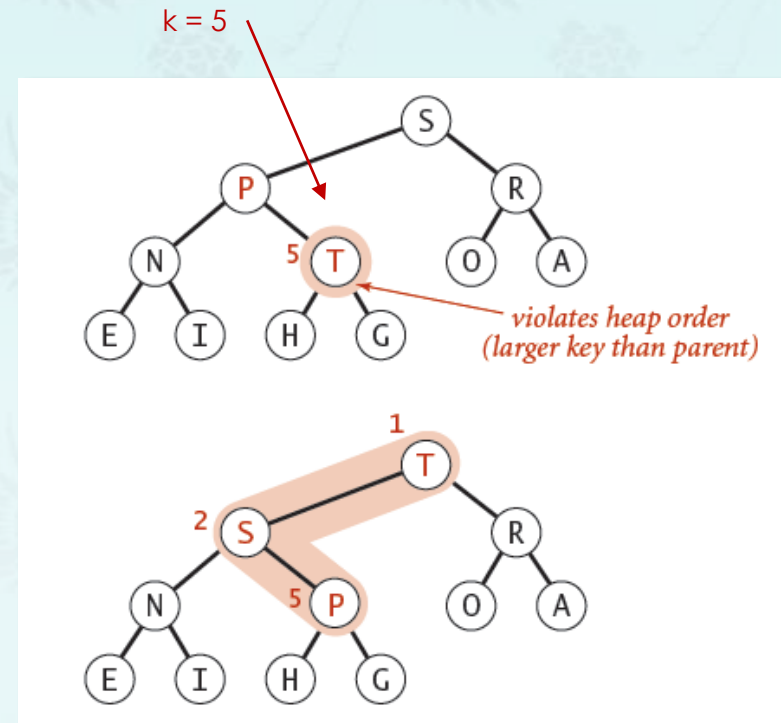


heap coding

Promotion in a heap: swim

- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.

```
void swim(heap h, int k)
{
    while (not reached at root &&
           k's parent key < k's key)
    {
        swap k and its parent
        go for the next
    }
}
```



heap coding

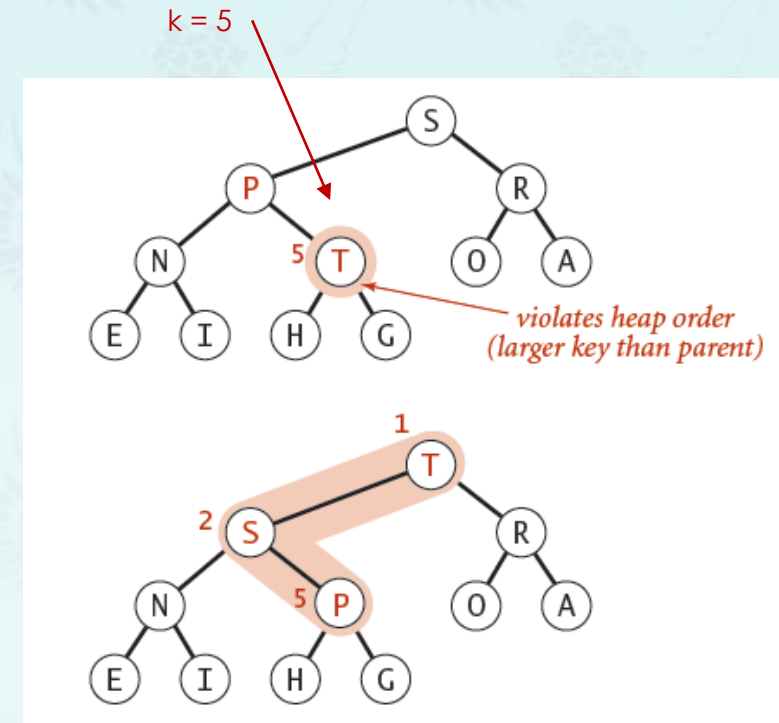
Promotion in a heap: swim

- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.

```
void swim(heap h, int k)
{
    while (k > 1)
    {
        // swap h[k] with h[k/2]
        // ...
    }
}
```

not reached at root

parent of k



heap coding

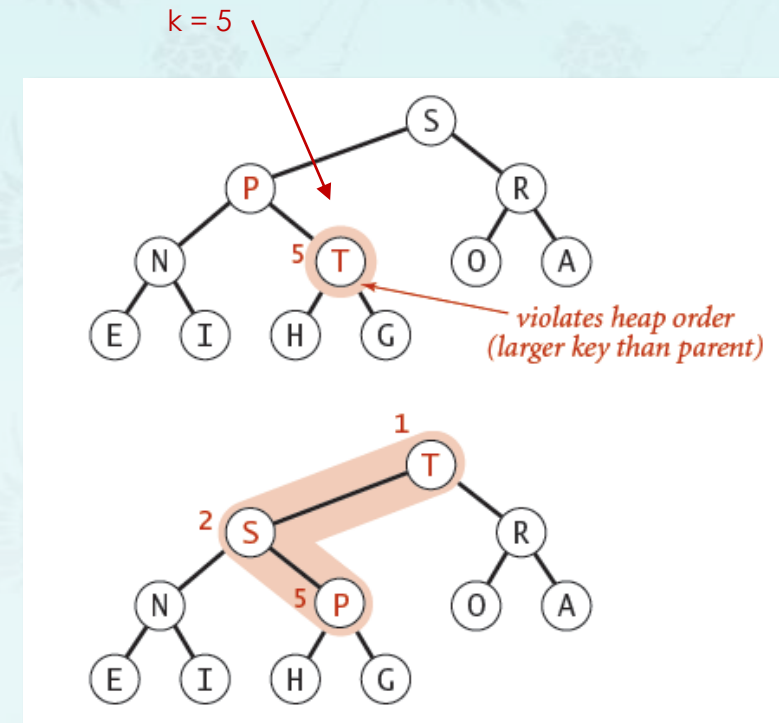
Promotion in a heap: swim

- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.

```
void swim(heap h, int k)
{
    while (k > 1 &&         )
    {
                
    }
}
```

not reached at root

parent(k/2) is less than its child(k)



heap coding

Promotion in a heap: swim

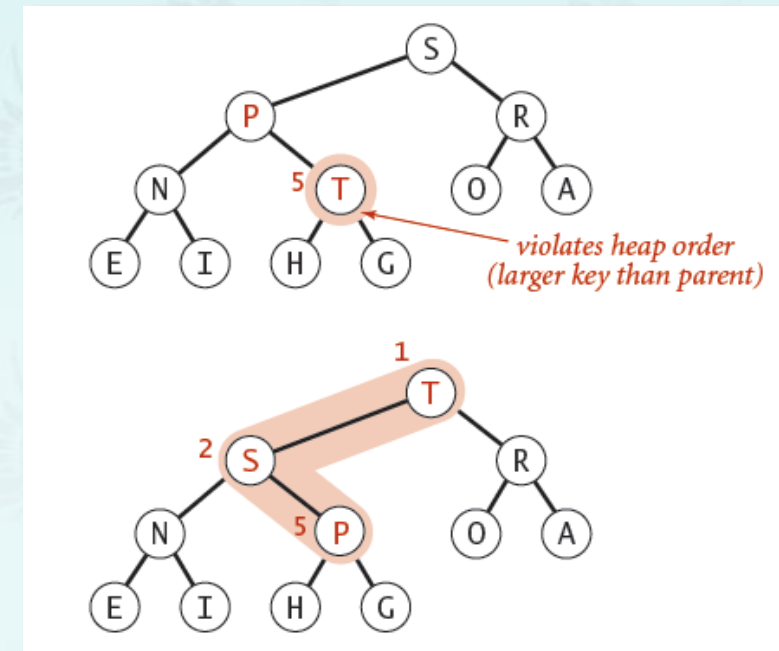
- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.

```
void swim(heap h, int k)
{
    while (k > 1 && less(h, k / 2, k))
    {
        // swap parent(k/2) and its child(k)
    }
}
```

not reached at root

parent(k/2) is less its child(k)

swap parent(k/2) and its child(k)



heap coding

Promotion in a heap: swim

- To eliminate the violation:
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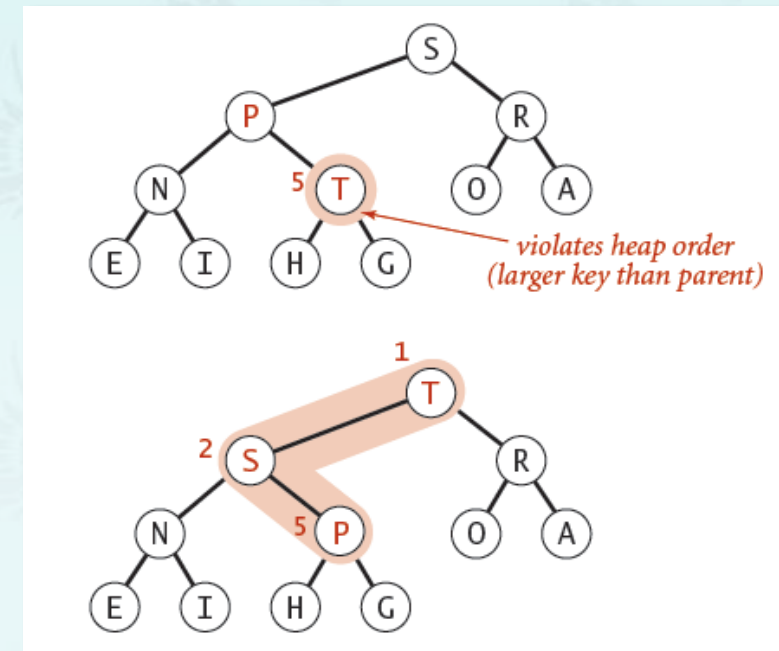
```
void swim(heap h, int k)
{
    while (k > 1 && less(h, k / 2, k))
    {
        swap(h, k / 2, k);
    }
}
```

not reached at root

parent(k/2) is less its child(k)

swap parent(k/2) and its child(k)

move up one level



heap coding

Promotion in a heap: swim

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 - Swap key in child with key in parent.
 - Repeat until heap order restored.

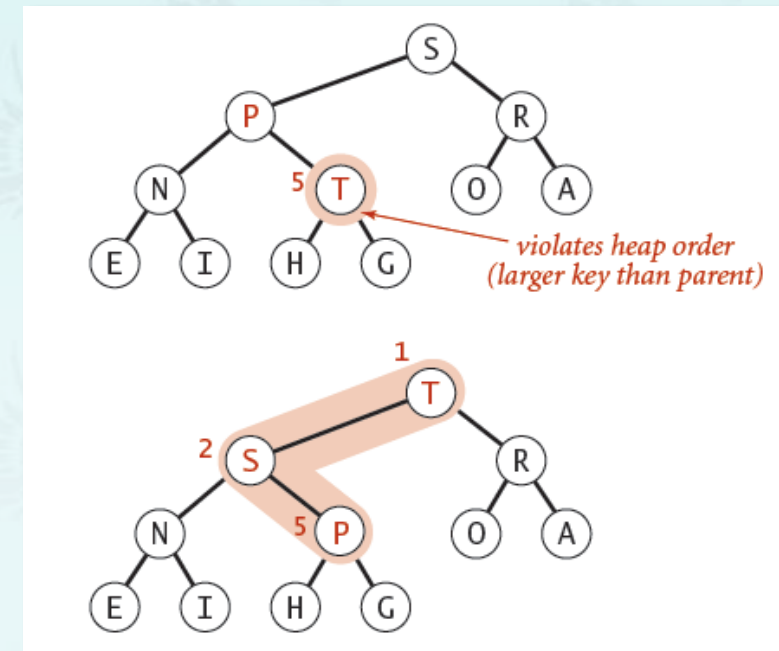
```
void swim(heap h, int k)
{
    while (k > 1 && less(h, k / 2, k))
    {
        swap(h, k / 2, k);
        k = k / 2;
    }
}
```

not reached at root

parent(k/2) is less its child(k)

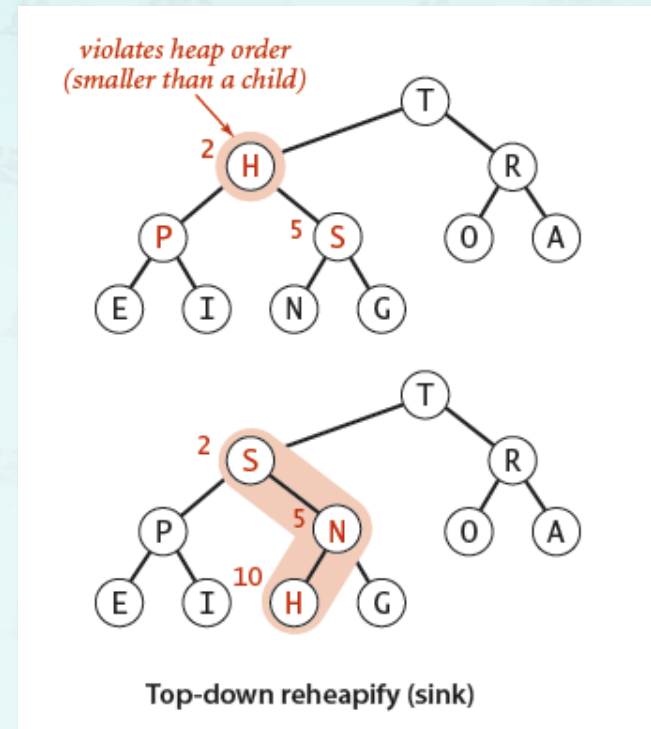
swap parent(k/2) and its child(k)

move up one level



heap coding

swim up
or
sink down



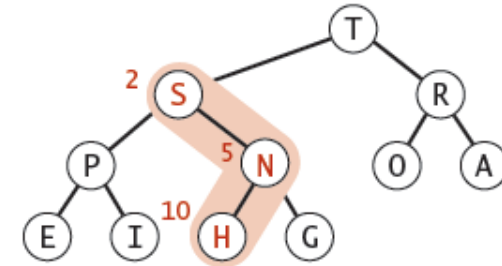
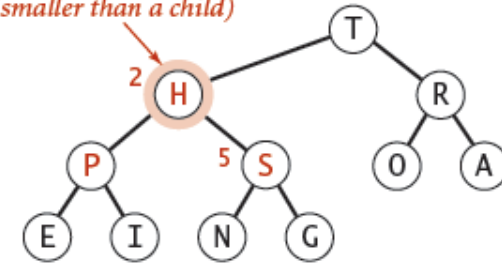
heap coding

Demotion in a heap: sink

- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation:
 - Swap key in parent with key in **larger** child (of two)
 - Repeat until heap order restored.

Why not smaller child?

violates heap order
(smaller than a child)



Top-down reheapify (sink)

This is a maxheap example.

heap coding

Demotion in a heap: sink

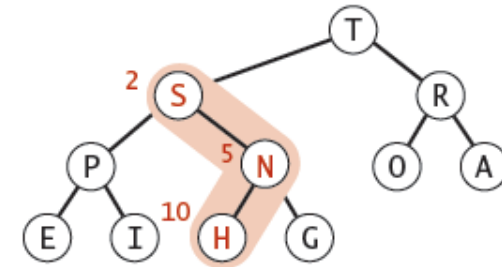
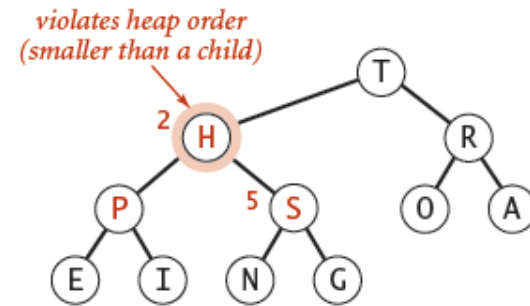
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Why not smaller child?

```
void sink(heap h, int k)
{
    while (k's child not reached the last)
    {
        find the larger child of k, let it be j. (j = 5)

        if k's key is not less than j's key, break;
        swap k and j since k's key > j's key
        set k to the next node wh
    }
}
```

k = 2



Top-down reheapify (sink)

heap coding

Demotion in a heap: sink

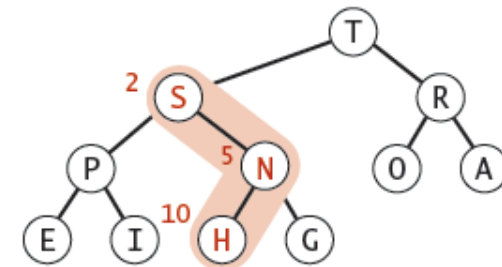
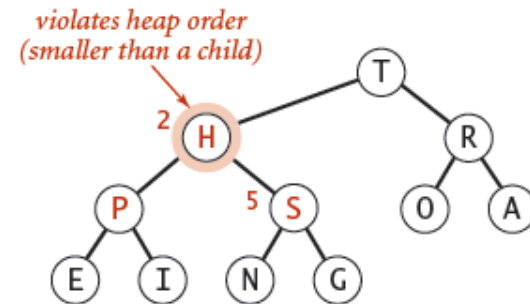
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        swap k and j since k's key > j's key
        set k to the next node which is j.
    }
}
```

k = 2



Top-down reheapify (sink)

heap coding

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```

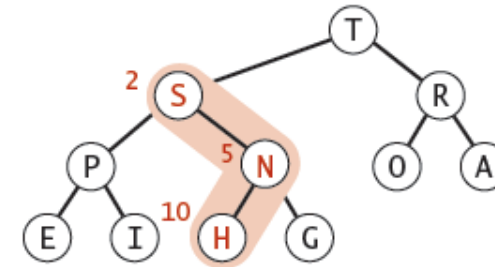
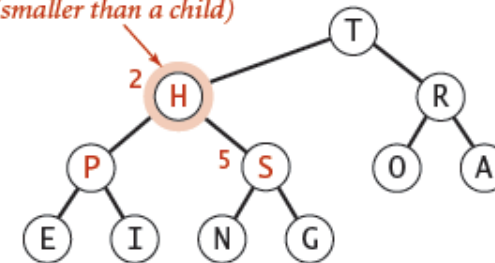
```
    while (k's child not reached the last)
```

```
    {
```

```
        find the larger child of k, let it be j. (j = 5)
```

```
    }
```

violates heap order
(smaller than a child)



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heap coding

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Why not smaller child?

```
void sink(heap h, int k)
```

```
{
```

```
  while (2 * k <= h->N)
```

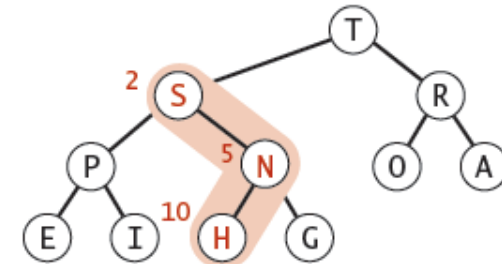
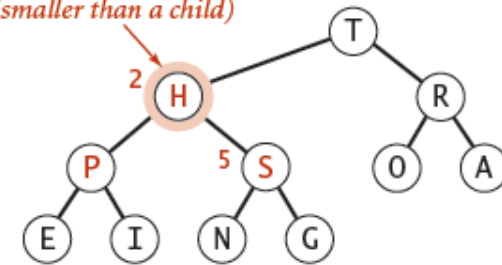
```
  {
```

```
    find the larger child of k, let it be j. (j = 5)
```

```
  }
```

children of node **k**
are **2k** and **2k+1**

violates heap order
(smaller than a child)



Top-down reheapify (sink)

heap coding

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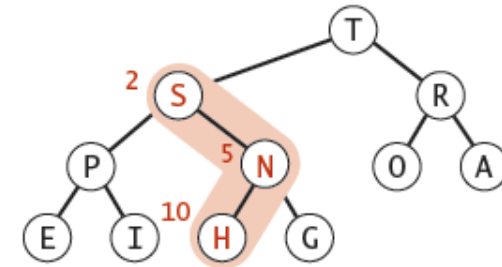
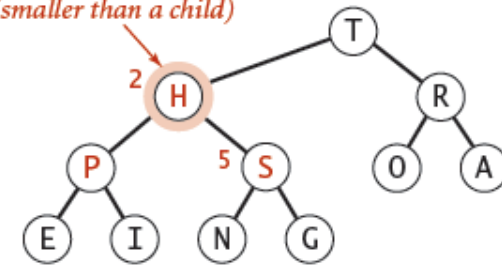
```
{  
  while (2 * k <= h->N)  
  {  
    int j = 2 * k;
```

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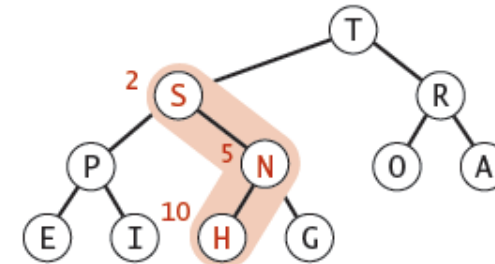
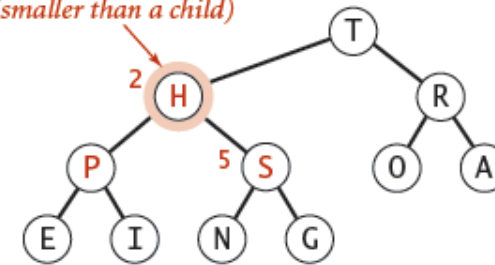
```
    int j = 2 * k;
```

```
    if (j < h->N && less(h, j, j + 1)) j++;
```

```
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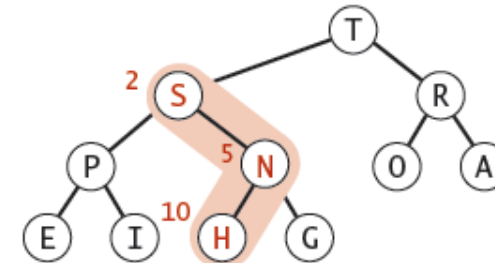
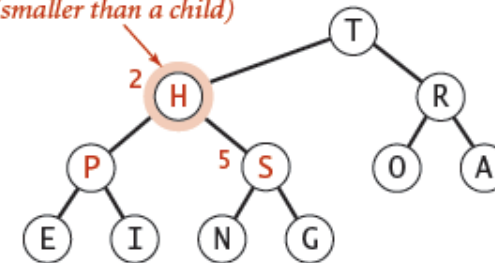
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```
    if k's key is not less than j's key, break;  
    swap k and j since k's key > j's key  
    set k to the next node (which is j.)
```

```
}
```

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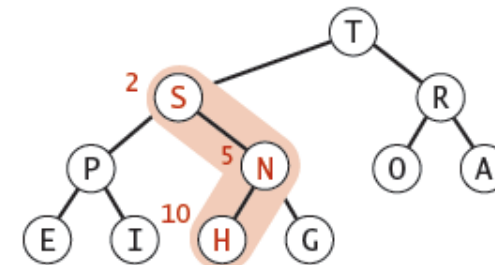
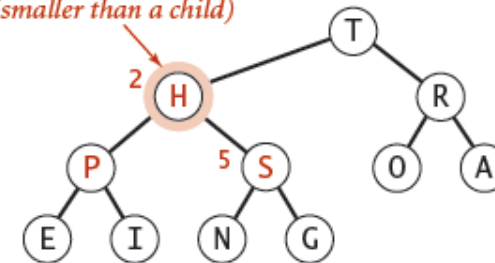
```
    if (!less(h, k, j)) break;
```

```
    swap k and j since k's key > j's key  
    set k to the next node (which is j.)
```

```
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Top-down reheapify (sink)

heap coding

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```
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```

```
    swap(h, k, j);
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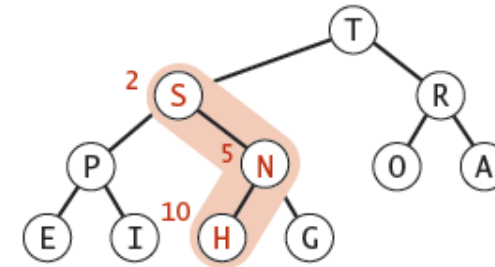
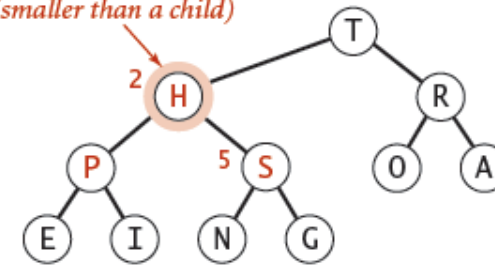
```
    set k to the next node (which is j.)
```

```
  }
```

```
}
```

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violates heap order
(smaller than a child)



Top-down reheapify (sink)

heap coding

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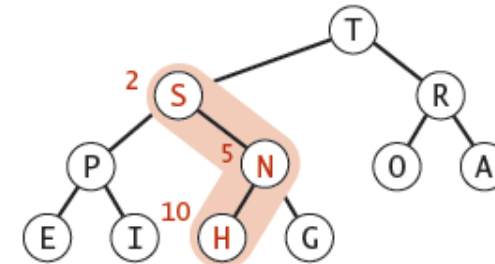
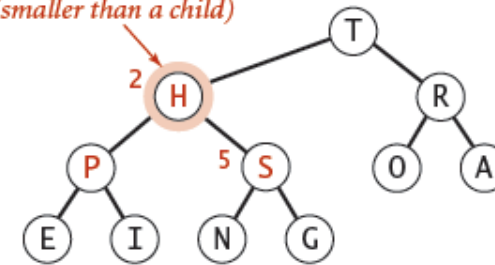
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    {
        int j = 2 * k;

        if (j < h->N && less(h, j, j + 1)) j++;
        if (!less(h, k, j)) break;
        swap(h, k, j);
        k = j;
    }
}
```

children of node **k**
are **2k** and **2k+1**

violates heap order
(smaller than a child)



Top-down reheapify (sink)

heap coding

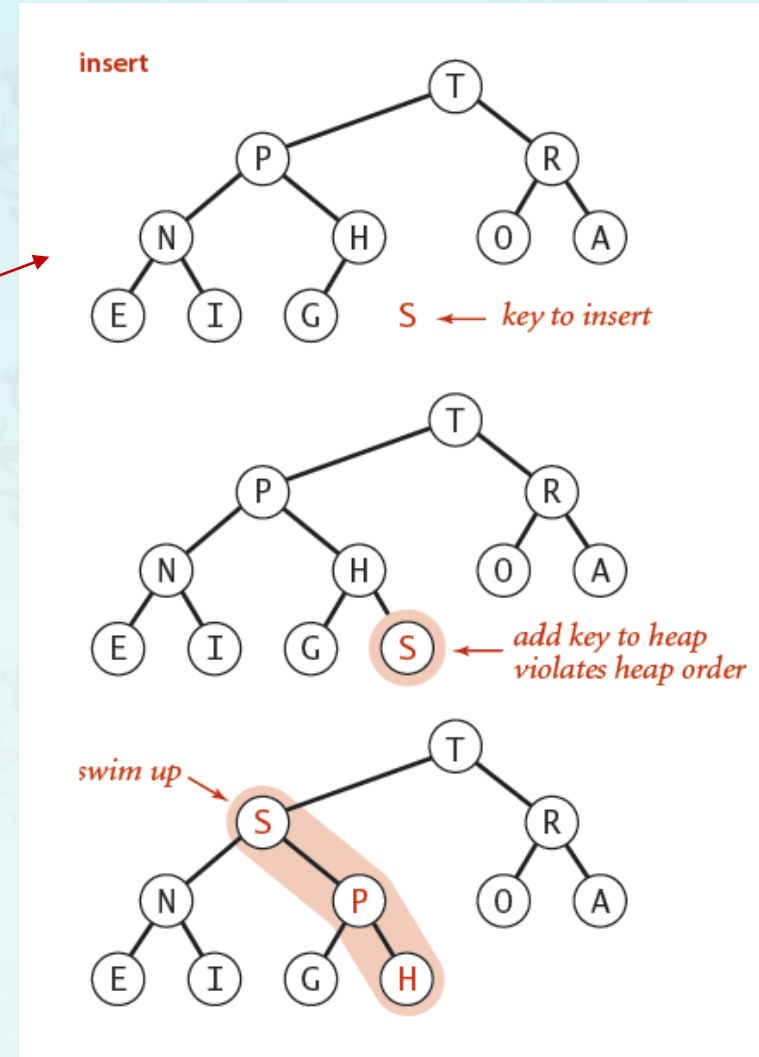
- Insert: Add node at end, then **swim** it up.
 - Cost: At most $1 + \log N$ compares.

Insert

What is N now?

Step 1

Step 2



heap coding

Insertion in a heap:

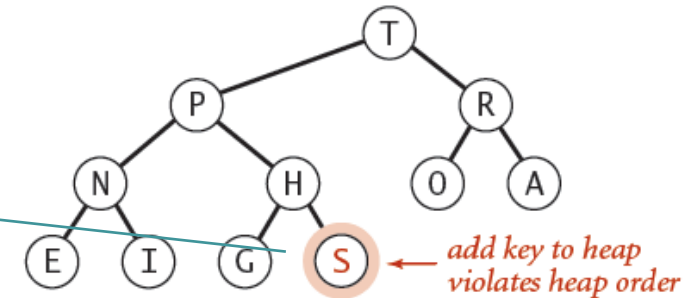
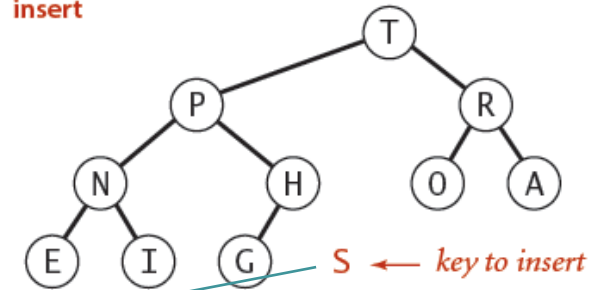
- Insert: Add node at end, then **swim** it up.
 - Cost: At most $1 + \log N$ compares.

```
void insert(heap h, int key)
{
    
}
```

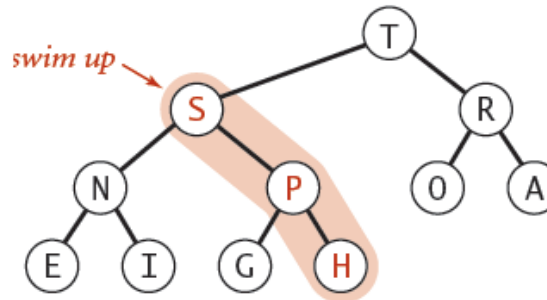
```
struct Heap {
    int *nodes;           // an array of nodes
    int capacity;         // array size of node or key, item
    int N;                // the number of nodes in the heap
    //
};
using heap = *Heap;
```

What is N now?

insert



swim up



heap coding

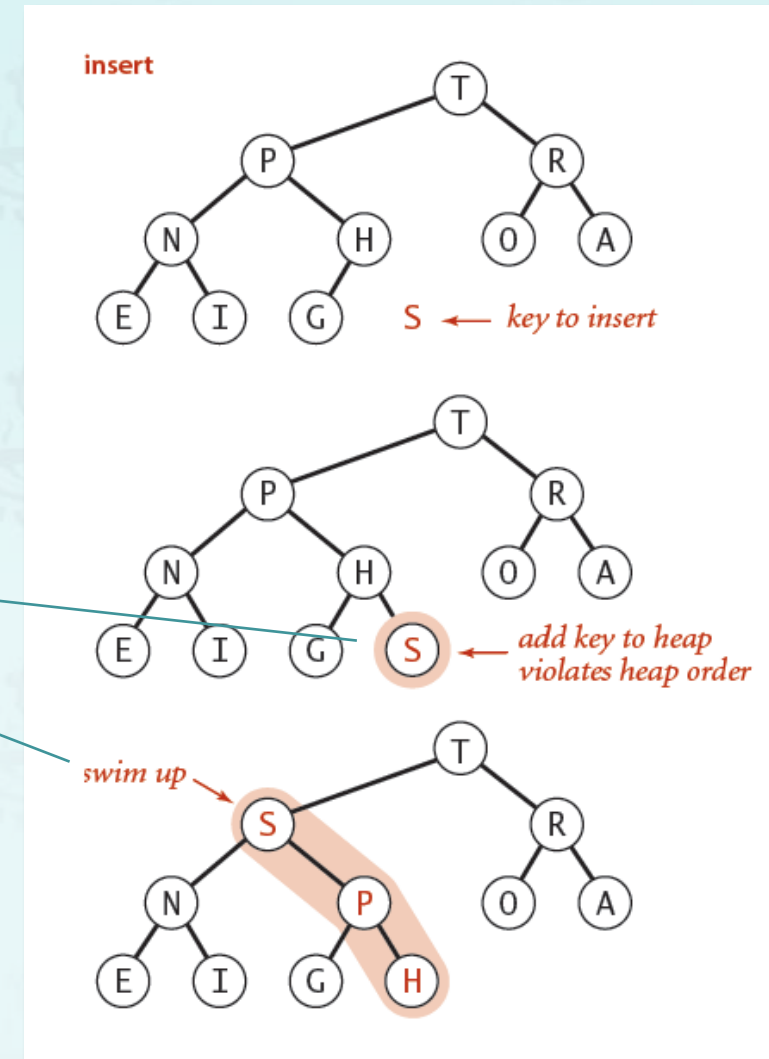
Insertion in a heap:

- Insert: Add node at end, then **swim** it up.
 - Cost: At most $1 + \log N$ compares.

```
void insert(heap h, int key)
{
    h->nodes[++h->N] = key;
    
}
```

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struct Heap {
    int *nodes;           // an array of nodes
    int capacity;         // array size of node or key, item
    int N;                // the number of nodes in the heap
    //
};
using heap = *Heap;
```

```
void swim(heap h, int k)
void sink(heap h, int k)
```



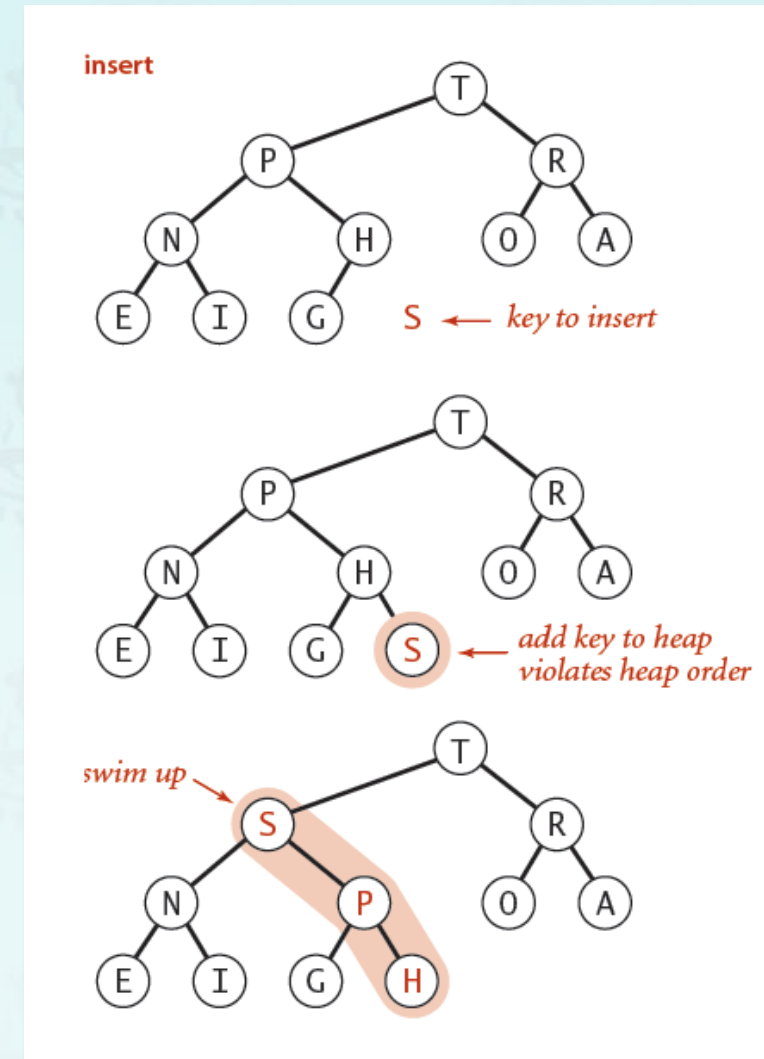
heap coding

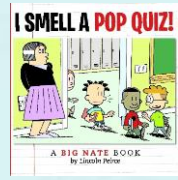
Insertion in a heap:

- Insert: Add node at end, then **swim** it up.
 - Cost: At most $1 + \log N$ compares.

```
void insert(heap h, int key)
{
    h->nodes[++h->N] = key;
    swim(h, h->N);
}
```

```
struct Heap {
    int *nodes;        // an array of nodes
    int capacity;      // array size of node or key, item
    int N;             // the number of nodes in the heap
    //
};
using heap = *Heap;
```



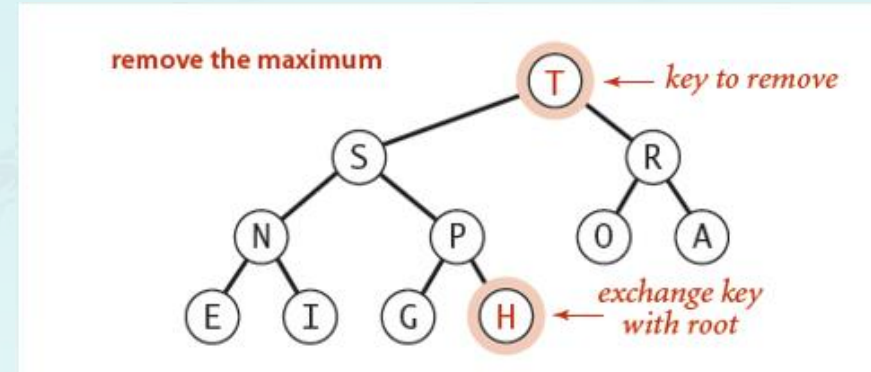


heap coding

(1) Delete the root (max or min) in a heap:

(2) How many times do comparisons occur for n nodes ? (select one):

n , $2n$, n^2 , $2 \log n$, $n \log n$, $\log n$



```
void delete(heap h) {  
  
  
  
}
```

← 2 or 3 lines of code

```
void swim(heap h, int k)  
void sink(heap h, int k)  
  
bool less(heap h, int p, int c)  
void swap(heap h, int p, int c)
```

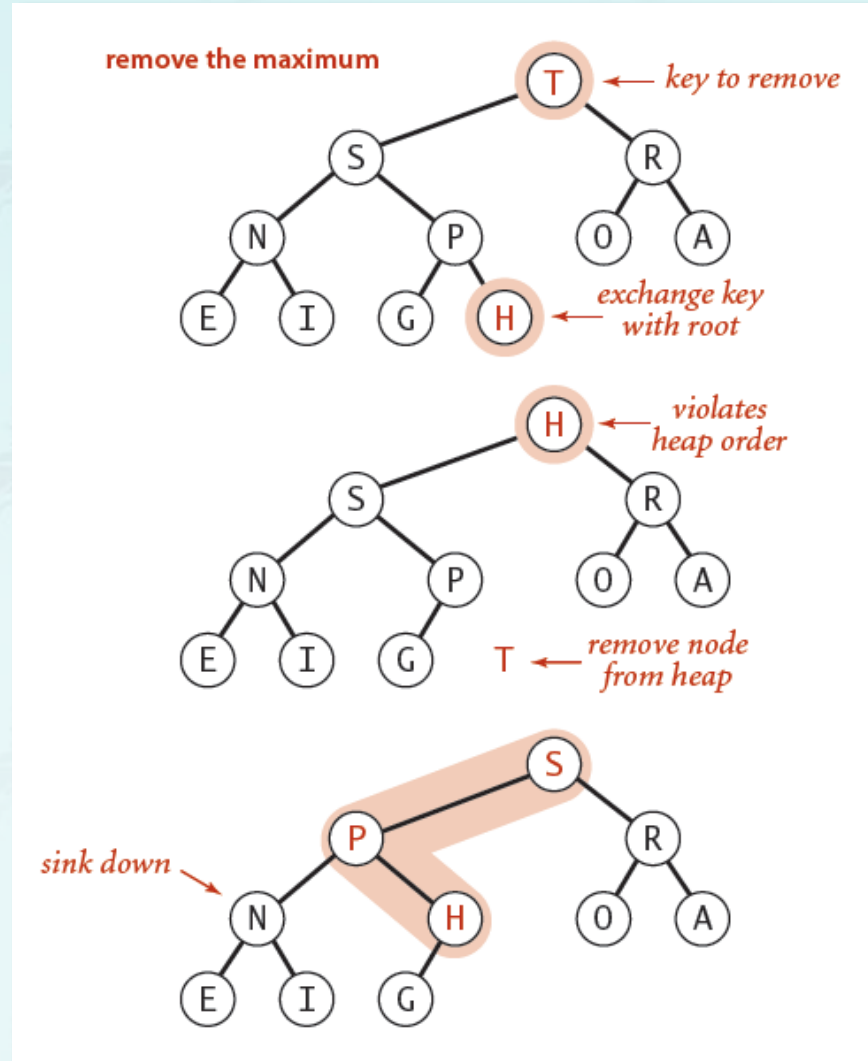

heap coding

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most $2 \log N$ compares.

```
void delete(heap h) {  
  
}  
  
    swap(h, ..., ... );  
    sink(h, ...);  
}
```

```
void swim(heap h, int k)  
void sink(heap h, int k)  
  
bool less(heap h, int p, int c)  
  
void swap(heap h, int p, int c)
```



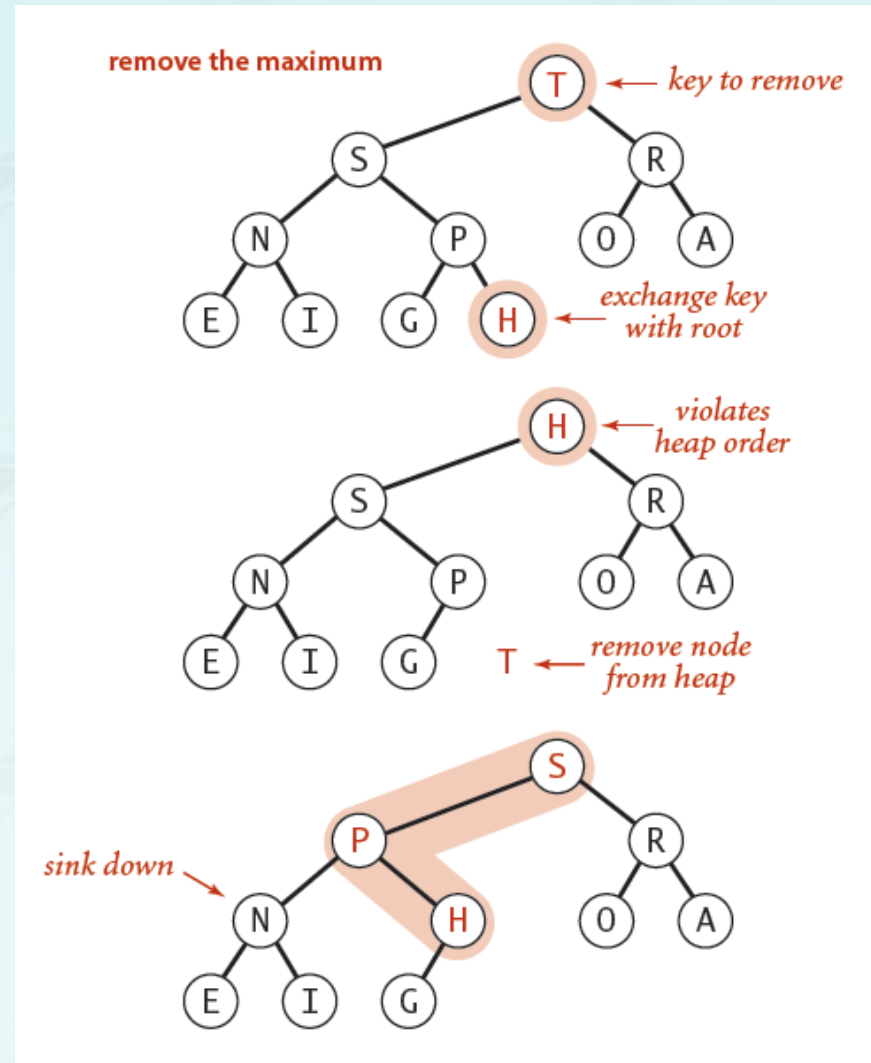
heap coding

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most $2 \log N$ compares.

```
void delete(heap h) {  
      
}  
}
```

```
void swim(heap h, int k)  
void sink(heap h, int k)  
bool less(heap h, int p, int c)  
void swap(heap h, int p, int c)
```



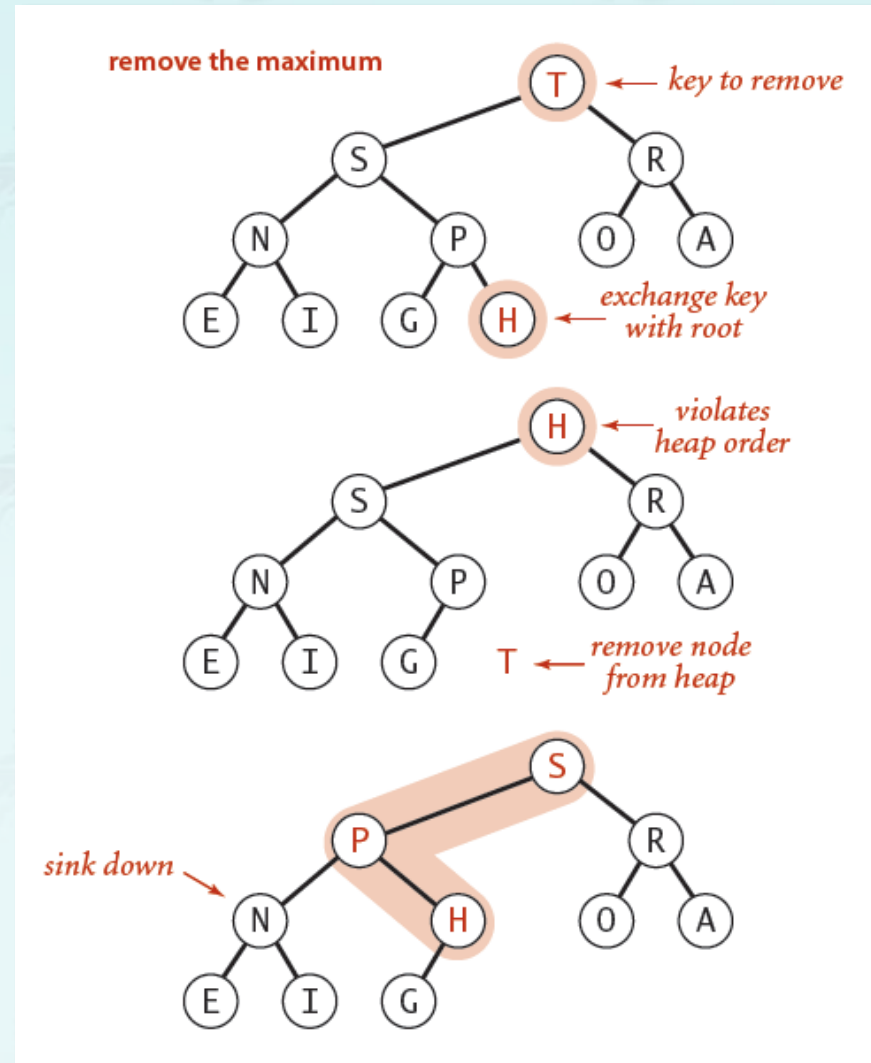
heap coding

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most $2 \log N$ compares.

```
void delete(heap h) {  
    swap(h, 1, h->N--);  
      
}
```

```
void swim(heap h, int k)  
void sink(heap h, int k)  
  
bool less(heap h, int p, int c)  
void swap(heap h, int p, int c)
```



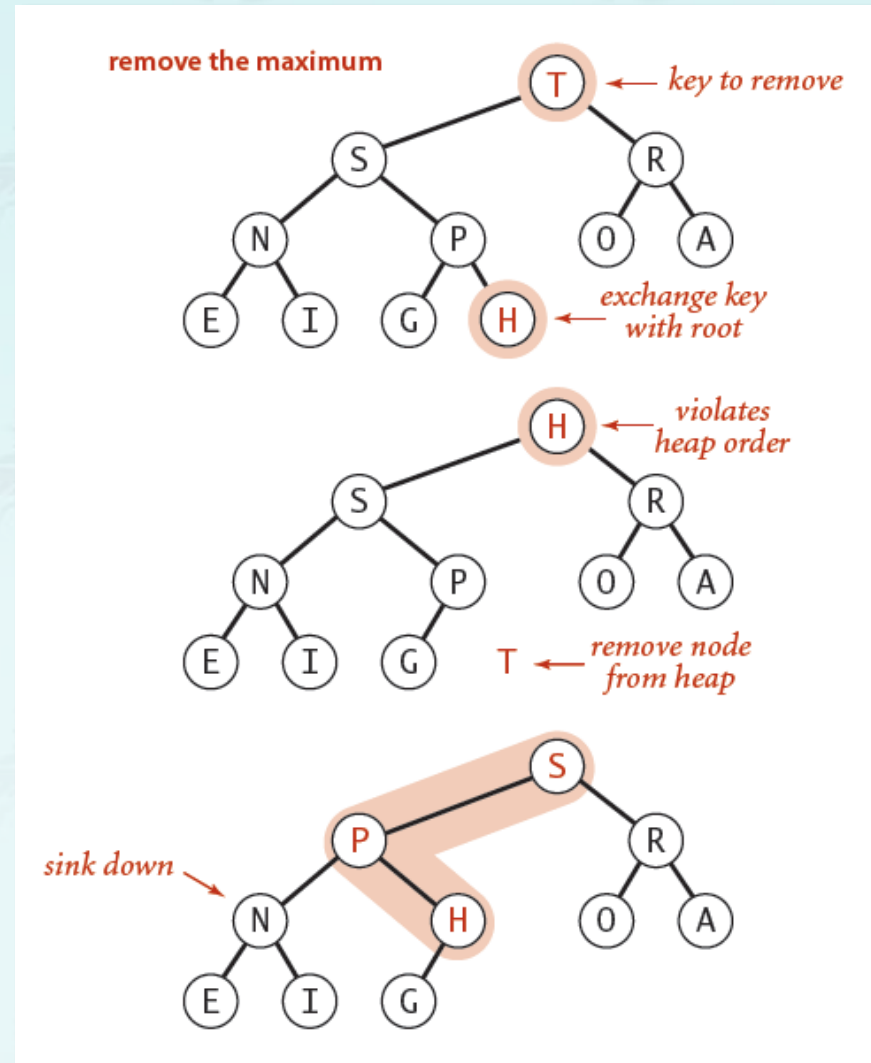
heap coding

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most $2 \log N$ compares.

```
void delete(heap h) {  
    swap(h, 1, h->N--);  
    sink(h, 1);  
}
```

```
void swim(heap h, int k)  
void sink(heap h, int k)  
bool less(heap h, int p, int c)  
void swap(heap h, int p, int c)
```



heap coding: heap.h

```
void clear(heap hp);           // deallocate heap
int size(heap hp);             // return nodes in heap currently
int level(int n);              // return level based on num of nodes
int capacity(heap hp);         // return its capacity (array size)
int reserve(heap hp, int capa); // reserve the array size (= capacity)
int full(heap hp);             // return true/false
int empty(heap hp);            // return true/false
void grow(heap hp, int key);    // add a new key
void trim(heap hp);            // delete a queue
int heapify(heap hp);          // convert a complete BT into a heap

// helper functions to support grow/trim functions
int less(heap hp, int i, int j); // used in max heap
int more(heap hp, int i, int j); // used in min heap
void swim(heap hp, int k);        // bubble up
void sink(heap hp, int k);        // tickle down
// helper functions to check heap invariant
int heapOrdered(heap hp);        // is heap[1..N] a heap?
```

heap coding: heap.cpp

```
// return the number of items in heap
int size(heap hp) {
    return heap->N;
}
```

```
// Is this heap empty?
int empty(heap hp) {
    return (heap->N == 0) ? true : false;
}
```

```
// Is this heap full?
int full(heap hp) {
    return (heap->N == heap->capacity - 1) ? true : false;
}
```

heap coding: heap.cpp

```
int less(heap hp, int i, int j) {  
    return heap->nodes[i] < heap->nodes[j];  
}
```

```
void swap(heap hp, int i, int j) {  
    int t = heap->nodes[i];  
    heap->nodes[i] = heap->nodes[j];  
    heap->nodes[j] = t;  
}
```

```
void swim(heap hp, int k) {  
  
}
```

```
void sink(heap hp, int k) {  
  
}
```

heap coding: heap.cpp

```
void grow(heap hp, int key) {  
    cout << "YOUR CODE HERE\n";  
  
    // add key @ ++heap->N  
  
    // swim up @ heap->N  
  
}
```

```
void trim(heap hp) {  
    if (empty(heap)) return;  
  
    cout << "YOUR CODE HERE\n";  
  
}
```


heap coding: heap.cpp

| | |
|------------|----------------------------------------------------------------------------------------------------------|
| newCBT() | with a given array, instantiate a new complete binary tree its result is neither maxheap nor minheap. |
| heapify() | make a complete binary tree into a (max) heap |
| heapsort() | use max/min-heap to sort elements in heap |

heap coding: heap.cpp

```
// instantiates a CBT with given data and its size.
heap newCBT(int *a, int n) {
    int capa = ?

    heap p = new Heap{ capa };

    p->N = n;
    for (int i = 0; i < n; i++)
        p->nodes[i + 1] = a[i];
    return p;
}
```

heap

- complete binary tree (review)
- heap and priority queues (Chapter 9)
- binary heap and min-heap
- max-heap demo
- *max-heap coding*
- heap sort (Chapter 7)