Linked List

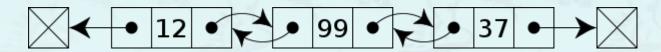
Data Structures C++ for C Coders

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Doubly Linked List with sentinel nodes

Doubly Linked lists

Doubly linked list: each node contains, besides the next-node link, a second link field pointing to the previous node in the sequence. The two links may be called forward and backward, or next and prev(ious).



Type definition

```
struct Node {
   int item;
   Node* prev;
   Node* next;
};
using pNode = Node*;
```

Q. Doubly linked list, Why?

Doubly Linked lists

Q. Array vs. Singly linked list vs. Doubly linked list, Why?

Advantages of linked list:

- Dynamic structure (Memory Allocated at run-time)
- Have more than one data type.
- Re-arrange of linked list is easy (Insertion-Deletion).
- <u>It doesn't waste memory.</u>

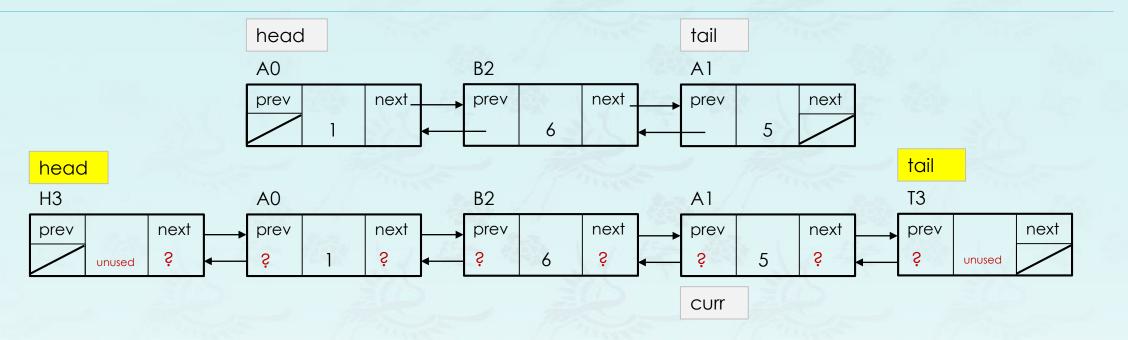
Disadvantages of linked list:

- In linked list, if we want to access any node it is difficult.
- It is occupying more memory.

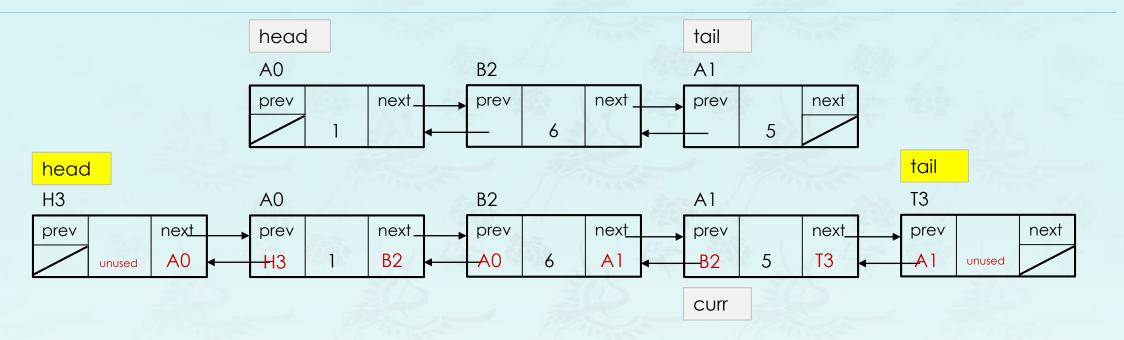
Advantages of doubly linked list:

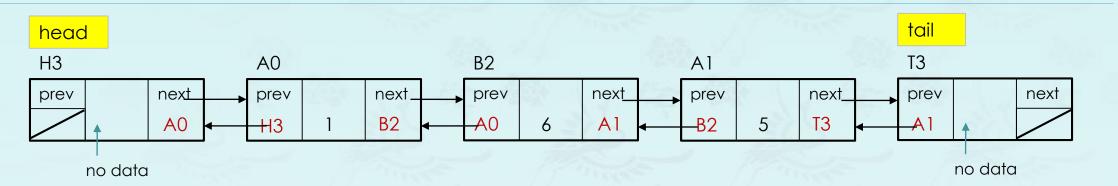
- A doubly linked list can be **traversed in both directions** (forward and backward). A singly linked list can only be traversed in one direction.
- most operations are O(1) instead of O(n)

Linked list with sentinel nodes



Fill the blanks(?) with mnemonics to form a linked list as shown.





- These extra nodes are known as sentinel nodes. The node at the front is known as head node, and the node at the end as a tail node. The head and tail nodes are created when the doubly linked list is initialized. The purpose of these nodes is to simply the insert, push/pop front and back, remove methods by eliminating all need for special-case code when the list empty, or when we insert at the head or tail of the list. This would greatly simplify the coding unbelievably.
- For instance, if we do not use a head node, then removing the first node becomes a special case, because we must reset the list's link to the first node during the remove and because the remove algorithm in general needs to access the node prior to the node being removed (and without a head node, the first node does not have a node prior to it).



An **empty** doubly linked list with sentinel nodes

doubly linked list – begin() and end()

```
// returns the first node which list::head points to in the container.
  pNode begin(pList p) {
    return p->head->next;
  // returns the tail node referring to the past -the last- node in the list.
  // The past -the last- node is the sentinel node which is used only as a sentinel
  // that would follow the last node. It does not point to any node next, and thus
  // shall not be dereferenced. Because the way we are going use during the iteration,
  // we don't want to include the node pointed by this. this function is often used
  // in combination with List::begin to specify a range including all the nodes in
  // the list. This is a kind of simulated used in STL. If the container is empty,
  // this function returns the same as List::begin.
  pNode end(pList p) {
   return p->tail;
(1)
   return p->tail->next;
        H3
                         A0
                                                                            T3
                                                           A1
                                                                                          tail
  head
                 next_
                         prev
                                                                                     next
       prev
                                                                    next
                                                                            prev
                 Α0
                                                                5
                                                                    T3
```

```
// returns the first node which list::head points to in the container.
pNode begin(pList p) {
  return p->head->next;
// returns the tail node referring to the past -the last- node in the list.
// The past -the last- node is the sentinel node which is used only as a sentinel
// that would follow the last node. It does not point to any node next, and thus
// shall not be dereferenced. Because the way we are going use during the iteration,
// we don't want to include the node pointed by this. this function is often used
// in combination with List::begin to specify a range including all the nodes in
// the list. This is a kind of simulated used in STL. If the container is empty,
// this function returns the same as List::begin.
pNode end(pList p) {
  return p->tail;
                  // not tail->next
```

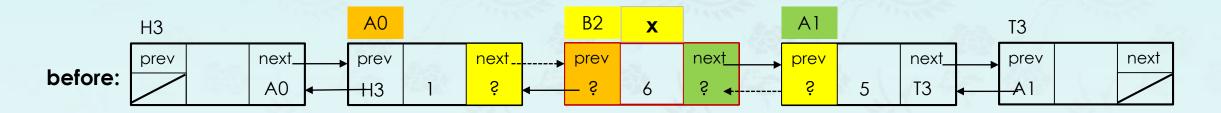
```
// returns the first node which list::head points to in the container.
pNode begin(pList p) {
 return p->head->next:
                     With the container given below, what does
// returns the tail no begin(p) and end(p) return respectively?
  The past -the last Answer in mnemonic:
                                                                           hel
// that would follow the last node. It does not point to any node next, and thus
// shall not be dereferenced. Because the way we are going use during the iteration,
// we don't want to include the node pointed by this. this function is often used
// in combination with List::begin to specify a range including all the nodes in
// the list. This is a kind of simulated used in STL. If the container is empty,
// this function returns the same as List::begin.
pNode end(pList p) {
 return p->tail; // not tail->next
```



- Let us supposed B2 is removed. Then,
- Which nodes are changed and where?

Express it in mnemonically or using node addresses (H3, A0, B2, A1 etc.):

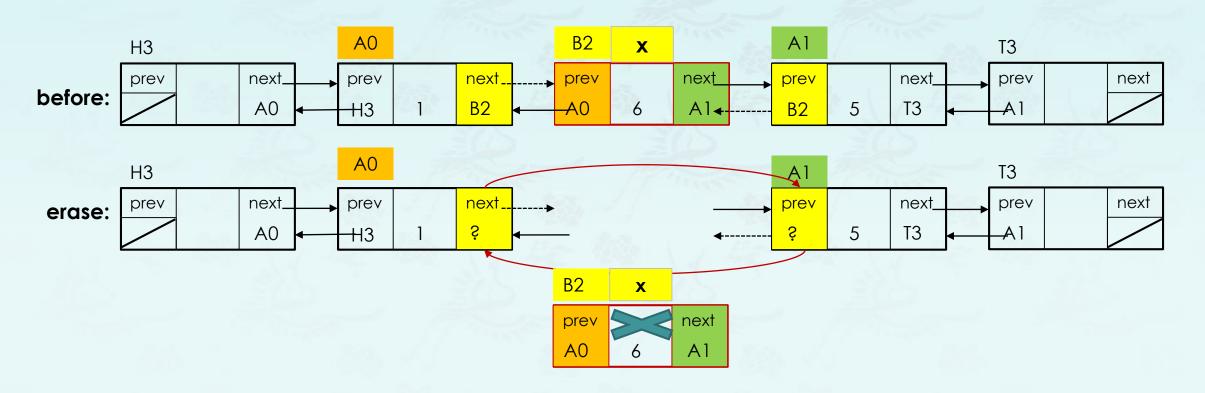
_



- Let us supposed B2 is removed. Then,
- Which nodes are changed and where?

Express it in mnemonically or using node addresses (H3, A0, B2, A1 etc.):

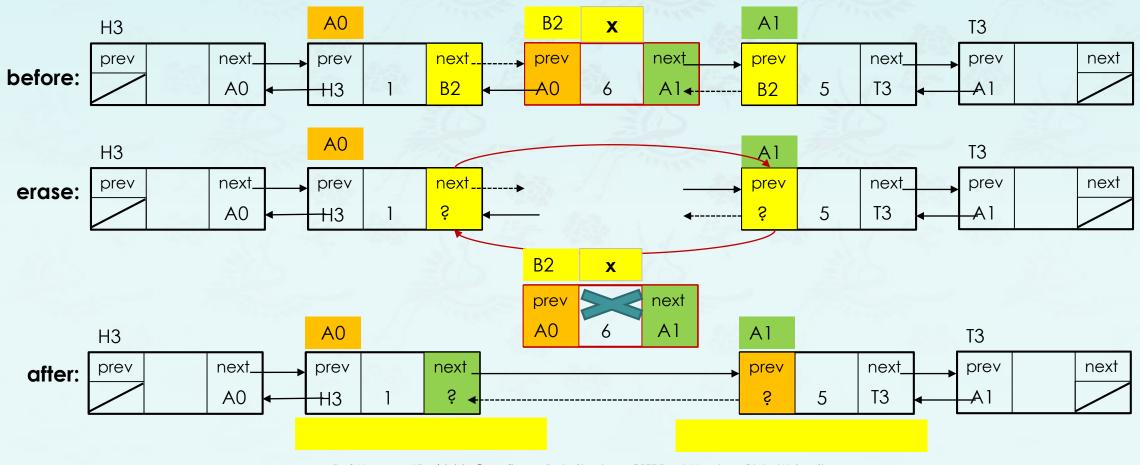
- B2 at A0→next should be A1;
- B2 at A1→prev should be A0;



- Let us supposed B2 is removed. Then,
- Which nodes are changed and where?

Express it in mnemonically or using node addresses (H3, A0, B2, A1 etc.):

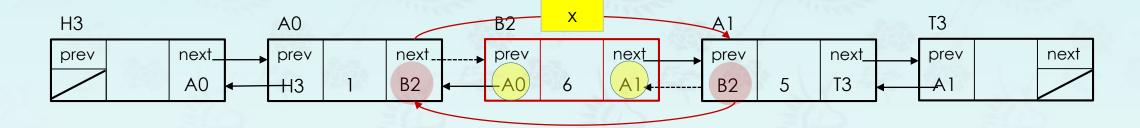
- B2 at A0→next should be A1;
- B2 at A1→prev should be A0;



- Let us supposed B2 is removed. Then,
- Which nodes are changed and where?

Express it in mnemonically or using node addresses (H3, A0, B2, A1 etc.):

- B2 at A0→next should be A1;
- B2 at A1→prev should be A0;



In coding:

```
void erase(pNode x) {
    x->prev->next = x->next;
    x->next->prev = x->prev;
    delete x;
}
```

Rewrite the following mnemonics in coding while x is given:

- A0:x→prev
- A1:x→next
- B2:x

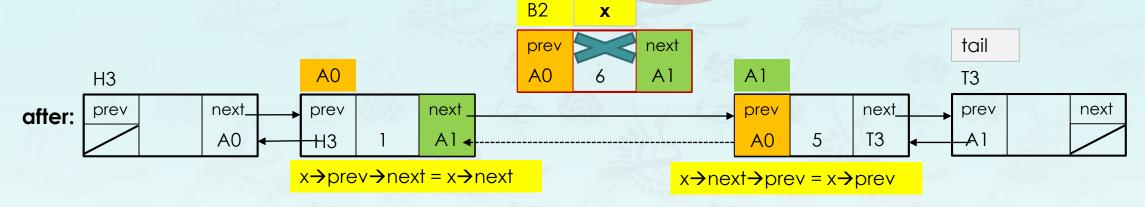
Check the values these pointers points.

- $-x \rightarrow prev \rightarrow next : B2$
- $x\rightarrow$ next \rightarrow prev : B2

- Let us supposed B2 is removed. Then,
- Which nodes are changed and where?

Express it in mnemonically or using node addresses (H3, A0, B2, A1 etc.):

- B2 at A0→next should be A1;
- B2 at A1 \rightarrow prev should be A0;



In coding:

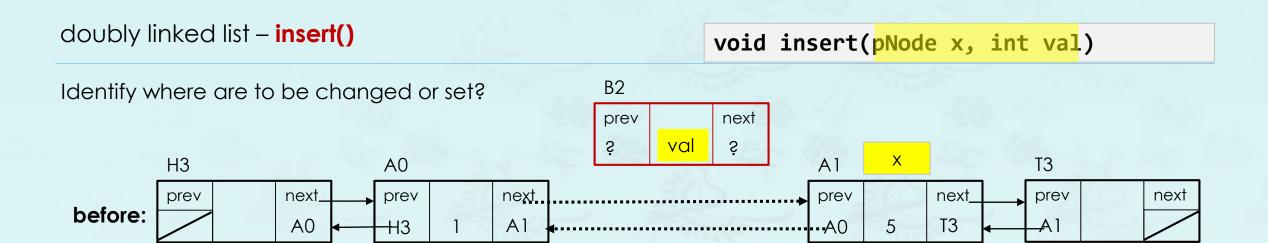
```
void erase(pNode x) {
    x->prev->next = x->next;
    x->next->prev = x->prev;
    delete x;
} need to be improved....
```

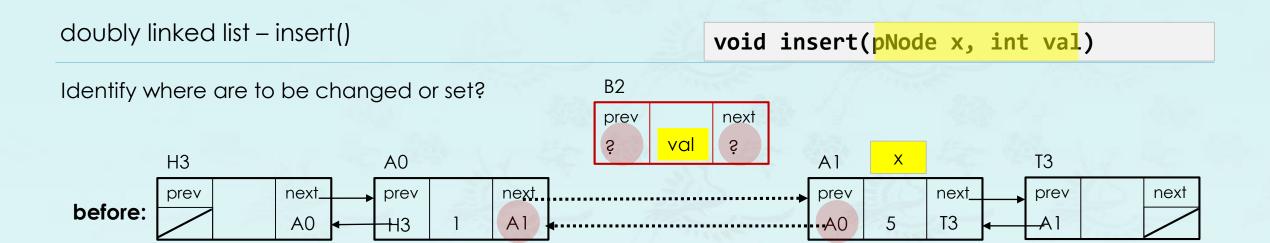
Rewrite the following mnemonics in coding while x is given:

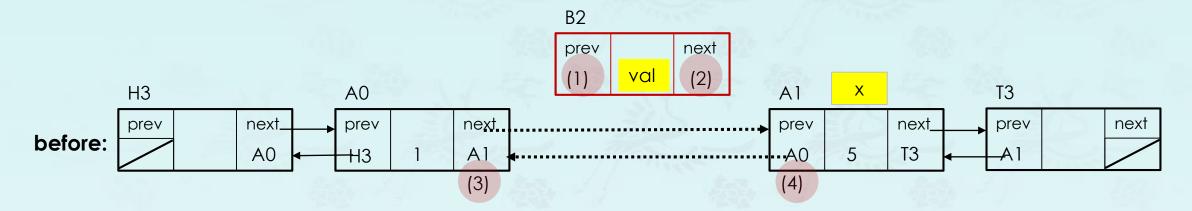
- A0:x→prev
- A1:x→next
- B2:x

Check the values these pointers points.

- $x \rightarrow prev \rightarrow next : B2$
- -x→next→prev:B2

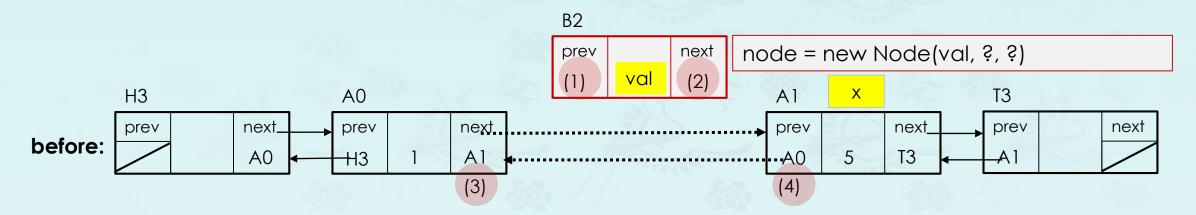






Express (1) \sim (4) in mnemonics and real code:

- (1) B2→prev
- (2) B2→next
- (3) A0→next or A1
- (4) A1→prev or A0



Express (1) \sim (4) in mnemonics and real code:

(1) B2→prev

(1) node→prev

(2) B2→next

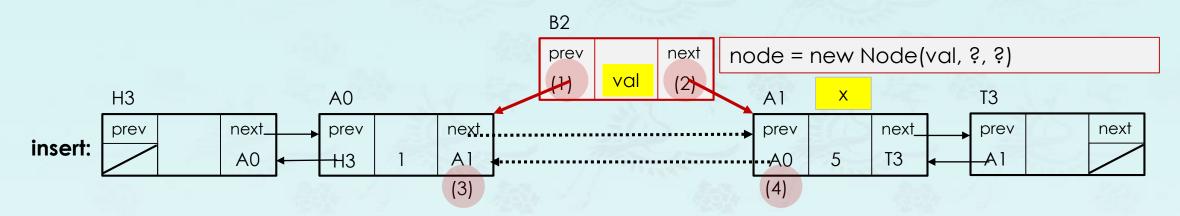
(2) node→next

(3) A0→next or A1

(3) $x \text{ or } x \rightarrow \text{prev} \rightarrow \text{next}$

(4) A1→prev or A0

(4) $x \rightarrow prev$



Express (1) \sim (4) in mnemonics and real code:

(1) B2→prev

(1) node→prev

(2) B2→next

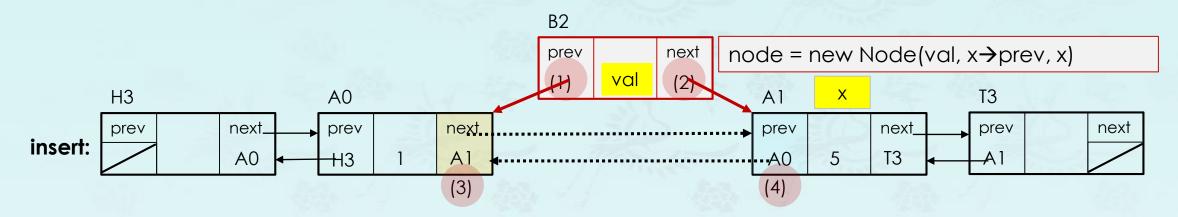
(2) node→next

(3) A0→next or A1

(3) $x \text{ or } x \rightarrow \text{prev} \rightarrow \text{next}$

(4) A1→prev or A0

(4) $x \rightarrow prev$



Link 1,2: Link the new node to A0 and A1 nodes pNode node = new Node(val, x→prev, x);

Express (1) ~ (4) in mnemonics and real code:

(1) B2→prev

(1) node→prev

(2) B2→next

(2) node→next

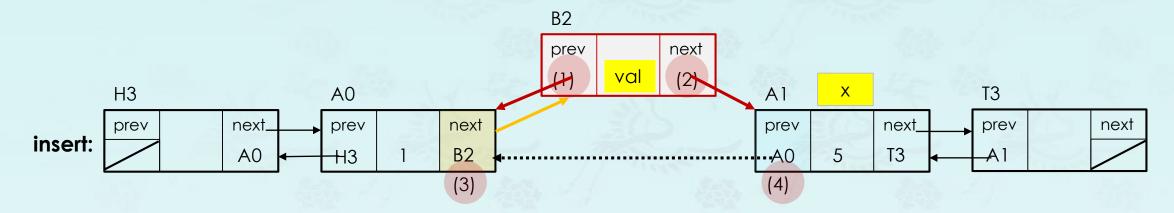
(3) $A0\rightarrow next or A1$

(3) x or $x \rightarrow prev \rightarrow next$

(4) A1→prev or A0

(4) x→prev

void insert(pNode x, int val)



Express (1) ~ (4) in mnemonics and real code:

(1) B2→prev

(1) node→prev

(2) B2→next

(2) node→next

(3) A0→next or A1

(3) $x \text{ or } x \rightarrow \text{prev} \rightarrow \text{next}$

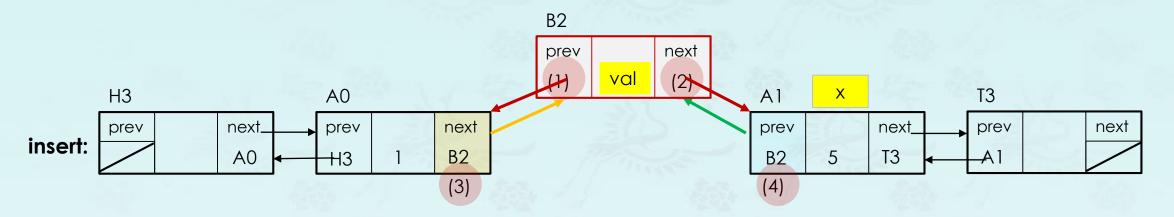
(4) A1 \rightarrow prev or A0

(4) x→prev

Link 1,2: Link the new node to A0 and A1 nodes pNode node = new Node(val, x→prev, x);

Link 3: Set A0 \rightarrow next to the new node $x\rightarrow$ prev \rightarrow next = node;

void insert(pNode x, int val)



Express (1) \sim (4) in mnemonics and real code:

(1) B2→prev

(1) node→prev

(2) B2→next

(2) $node \rightarrow next$

(3) $A0 \rightarrow next \text{ or } A1$

(3) $x \text{ or } x \rightarrow \text{prev} \rightarrow \text{next}$

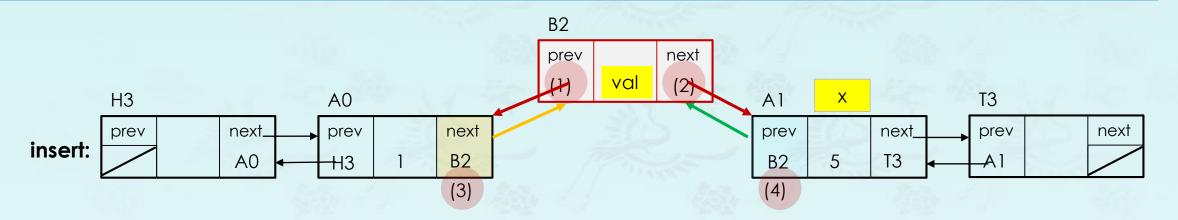
(4) A1→prev or A0

(4) x→prev

Link 1,2: Link the new node to A0 and A1 nodes pNode node = new Node(val, x→prev, x);

Link 3: Set A0 \rightarrow next to the new node $x\rightarrow$ prev \rightarrow next = node;

Link 4: Set A1 \rightarrow prev to the new node $x\rightarrow$ prev = node;

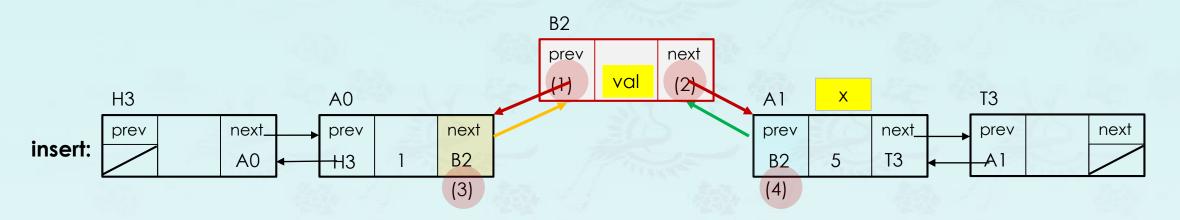


```
void insert(pNode x, int val) {
  pNode node = new Node{val, x->prev, x};
  x->prev = x->prev->next = node;
}
```

Link 1,2: Link the new node to A0 and A1 nodes pNode node = new Node(val, x→prev, x);

Link 3: Set A0 \rightarrow next to the new node $x\rightarrow$ prev \rightarrow next = node;

Link 4: Set A1→prev to the new node x→prev = node;



```
void insert(pNode x, int val) {
  pNode node = new Node{val, x->prev, x};
  x->prev = x->prev->next = node;
}
```

insert() extends the list by inserting a
new node with val before the node at the
specified position x.

For example, if **begin(p)** is specified as an insertion position, the new node becomes the first one in the list.

doubly linked list - push_front()

```
// Inserts a new node at the beginning of the list, right before its
// current first node. The content of item is copied(or moved) to the
// inserted node. This effectively increases the container size by one.
void push_front(pList p, int val) {
  insert(begin(p), val);
}
```



doubly linked list - pop_front()

```
// Removes the first node in the list container, effectively reducing
// its size by one. This destroys the removed node.
void pop_front(pList p) {
  if (!empty(p)) erase(begin(p));
}
```



doubly linked list – find(), _more(), and _less()

```
// returns the first node with a value,
// tail or end(p) node otherwise.
pNode find(pList p, int val) {
  pNode c = begin(p);
  for (; c != end(p); c = c->next)
    if (c->item == val) return c;
  return c;
}
```

doubly linked list – find(), _more(), and _less()

```
// returns the first node with a value,
// tail or end(p) node otherwise.
pNode find(pList p, int val) {
  pNode c = begin(p);
  for (; c != end(p); c = c->next)
    if (c->item == val) return c;
  return c;
}
```

```
// returns the node of which val is greater
// than x firstly encountered.
pNode _more(pList p, int val) {
  pNode c = begin(p);
  for (; c != end(p); c = c->next)
    if (c->item > val) return c;
  return c;
}
```

doubly linked list – find(), _more(), and _less()

```
// returns the first node with a value,
// tail or end(p) node otherwise.
pNode find(pList p, int val) {
  pNode c = begin(p);
  for (; c != end(p); c = c->next)
    if (c->item == val) return c;
  return c;
}
```

```
// returns the node of which val is greater
// than x firstly encountered.
pNode _more(pList p, int val) {
  pNode c = begin(p);
  for (; c != end(p); c = c->next)
    if (c->item > val) return c;
  return c;
}
```

```
// returns the node of which val is smaller
// than x firstly encountered
pNode _less(pList p, int val) {
  pNode c = begin(p);
  for (; c != end(p); c = c->next)
    if (c->item < val) return c;
  return c;
}</pre>
```

```
void erase(pNode x){
  x->prev->next = x->next;
  x->next->prev = x->prev;
  delete x;
}

void pop(pList p, int val){
  erase(find(p, val));
```

This code may not work some cases. How can you fix it?

```
void erase(pNode x){
  x->prev->next = x->next;
  x->next->prev = x->prev;
  delete x;
}

void pop(pList p, int val){
  erase(find(p, val));
}
```

This code may not work some cases. How can you fix it?

```
pNode find(pList p, int val){
   pNode curr = begin(p);
   while(curr != end(p)) {
      if (curr->item == val) return curr;
      curr = curr->next;
   }
   return curr;
}
```

```
pNode find(pList p, int val){
  pNode x = begin(p);
  for (; x != end(p); x = x->next;)
   if (x->item == val) return x;
  return x;
}
```

What does find() returns if val not found?

```
void erase(pNode x){
  x->prev->next = x->next;
  x->next->prev = x->prev;
  delete x;
}

void pop(pList p, int val){
  erase(find(p, val));
}
```

This code may not work some cases. How can you fix it?

```
pNode find(pList p, int val){
   pNode curr = begin(p);
   while(curr != end(p)) {
     if (curr->item == val) return curr;
     curr = curr->next;
   }
   return curr;
}
```

```
pNode find(pList p, int val){
  pNode x = begin(p);
  for (; x != end(p); x = x->next;)
   if (x->item == val) return x;
  return x;
}
```

What does find() returns if val not found?

The "end" node which is not nullptr.

```
void erase(pNode x){
  x - prev - next = x - next;
  x->next->prev = x->prev;
  delete x;
void pop(pList p, int val){
  erase(find(p, val));
                                          Is this good?
       This code may not work some cases.
       How can you fix it?
void pop(pList p, int val){
  pNode node = find(p, val);
  if (node == p->tail || node == p->head) return;
  erase(node);
```

```
void erase(pList p, pNode x){
void erase(pNode x){
                                      if (x == end(p) || x == p->head) return;
  x->prev->next = x->next;
                                      x->prev->next = x->next;
  x->next->prev = x->prev;
                                      x->next->prev = x->prev;
  delete x;
                                      delete x;
void pop(pList p, int val){
  erase(find(p, val));
       This code may not work some cases.
       How can you fix it?
void pop(pList p, int val){
  pNode node = find(p, val);
  if (node == p->tail | node == p->head) return;
  erase(node);
```

```
void erase(pNode x){
  x->prev->next = x->next;
  x->next->prev = x->prev;
  delete x;
}

void pop(pList p, int val){
  erase(find(p, val));
}
```

This code may not work some cases. How can you fix it?

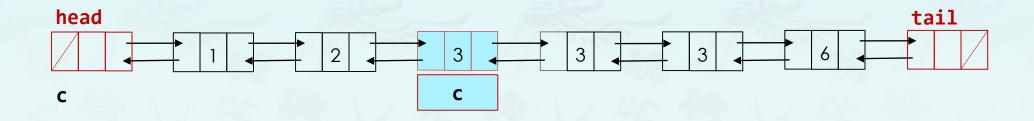
```
void erase(pList p, pNode x){
  if (x == end(p)|| x == p->head) return;
  x->prev->next = x->next;
  x->next->prev = x->prev;
  delete x;
}
```

```
void pop(pList p, int val){
  erase(p, find(p, val));
}
```

```
void pop(pList p, int val){
  pNode node = find(p, val);
  if (node == p->tail || node == p->head) return;
  erase(node);
}
```

doubly linked list - pop_all()*

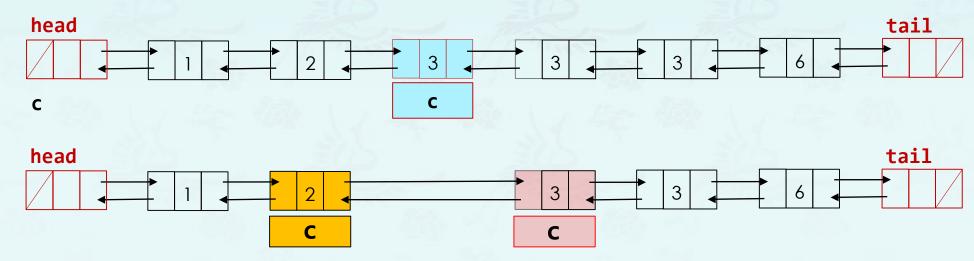
```
// remove all occurrences of nodes with val given in the list.
void pop_all(pList p, int val) {
  while (find(p, val) != end(p)) {
    pop(p, val);
  }
} version 1 with a problem
```



What is the time complexity of the function?

doubly linked list - pop_all()*

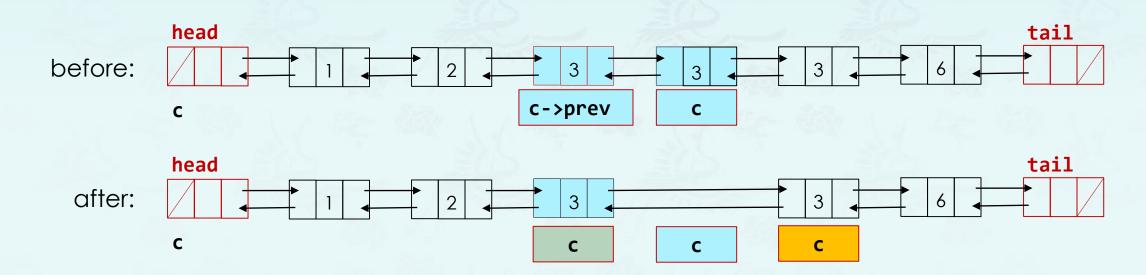
```
// remove all occurrences of nodes with val given in the list.
void pop_all(pList p, int val) {
  for (pNode c = begin(p); c != end(p); c = c->next)
    if (c->item == val)
       erase(p, c);
} version 2 with a bug
```



Where should c be pointing right after erase(p, c), 2 or 4?

doubly linked list - unique()*

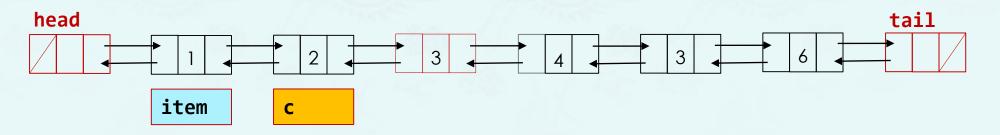
```
// removes extra nodes that have duplicate values from the list.
void unique(pList p) {
  if (size(p) <= 1) return;
  for (pNode c = begin(p); c != end(p); c = c->next)
    if (c->item == c->prev->item)
      erase(p, c);
} version 1 with a bug
```



Where should c be pointing right after the first erase(p, c)?

doubly linked list - sorted()

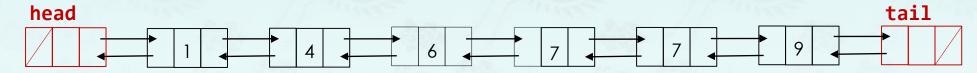
```
// returns true if the list is sorted either ascending or descending.
bool sorted(pList p) {
  return sorted(p, ascending) || sorted(p, descending);
bool sorted(pList p, int(*comp)(int a, int b)) {
  if (size(p) <= 1) return true;</pre>
  int item = "set it to 1<sup>st</sup> node value
  for (pNode c = "starts from second node"; c != end(p); c = c->next) {
    // your code here; return false as soon as out of order found
    // compare item and c->item
    item = c->item;
  return true;
```



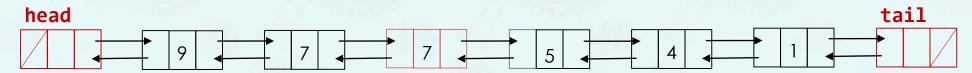
doubly linked list - push_sorted()

```
// inserts a new node with val in sorted order
void push_sorted(pList p, int val) {
  if sorted(p, "ascending order")
    insert("find a node _more() than val", val);
  else
    insert("find a node _less() than val", val);
}
```

Which node should be located to invoke insert() if val = 4?

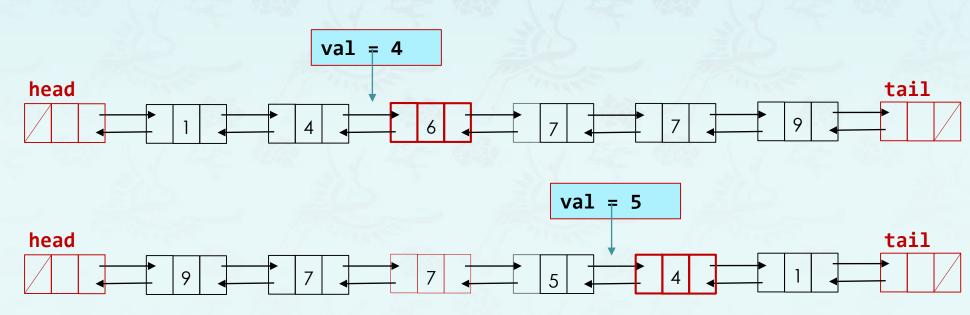


Which node should be located to invoke insert() if val = 5?



doubly linked list - push_sorted()

```
// inserts a new node with val in sorted order
void push_sorted(pList p, int val) {
  if sorted(p, "ascending order")
     insert("find a node _more() than val", val);
  else
    insert("find a node _less() than val", val);
}
```

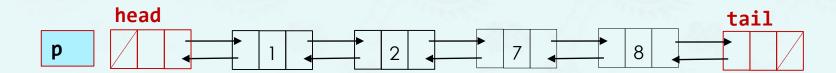


doubly linked list - push_sortNlog(pList p, int N)

```
// merges two sequences of numbers. One exists as a sorted linked list,
// the other exists as an array of randomly generated number.
void push_sortNlog(pList p, int N) {
}
```

```
N = 5 randomly generated numbers in the range of [0..(N + size(p))] \rightarrow [0..9]
```

$$vals[N] = \{ 1, 2, 4, 5, 9 \}; //[0..(N + size(p))] \rightarrow [0..9]$$



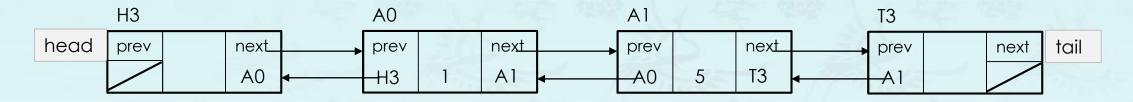
```
void bubbleSort(pList p, int(*comp)(int, int)) {
  // if (sorted(p)) { reverse(p); return; }
  pNode curr;
  for (pNode i = begin(p); i != end(p); i = i->next) {
    for (curr = begin(p); curr->next != end(p); curr = curr->next) {
      if (comp(curr->item, curr->next->item) > 0)
        swap(curr->item, curr->next->item);
    tail = curr;
```

doubly linked list - reverse()**

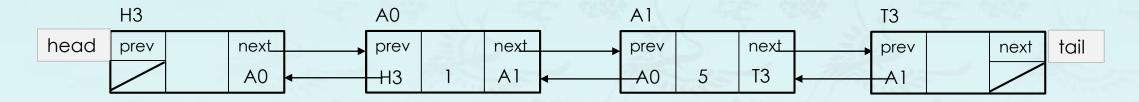
```
// reverses the order of the nodes in the list container. O(n)
void reverse(pList p) {
  if (size(p) <= 1) return;
  // hint: swap prev and next in every node including head & tail
  // then, swap head and tail.
  // hint: use while loop, don't use begin()/end()
  // your code here
}</pre>
```



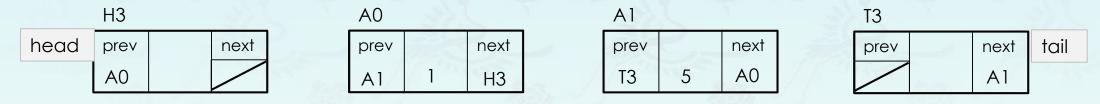
doubly linked list - reverse()**



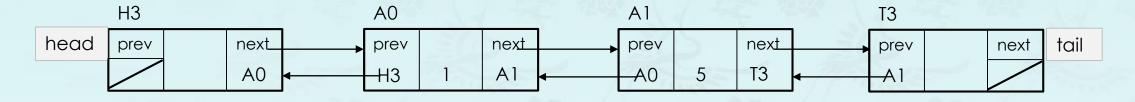
step 1: swap prev and next in every node including head & tail.



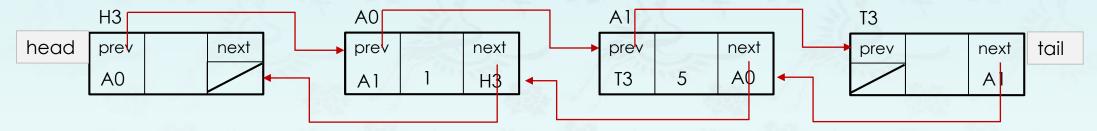
step 1: swap prev and next in every node including head & tail.



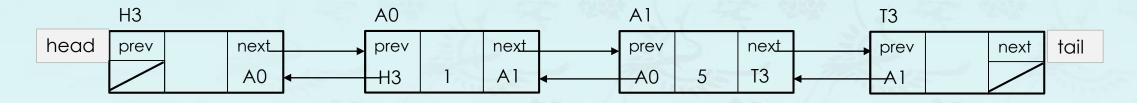
step 2: swap head and tail node.



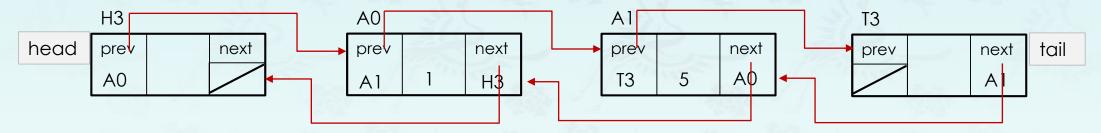
step 1: swap prev and next in every node including head & tail.



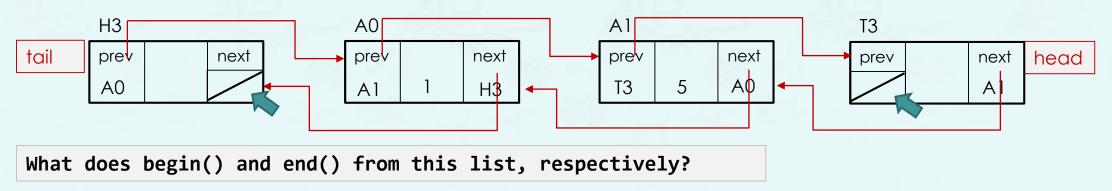
step 2: swap head and tail node.



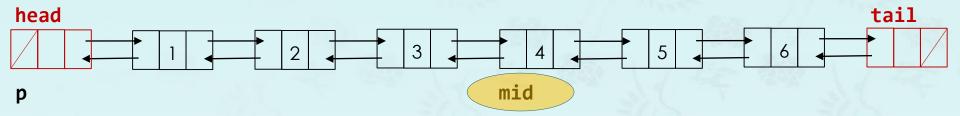
step 1: swap prev and next in every node including head & tail.



step 2: swap head and tail node.

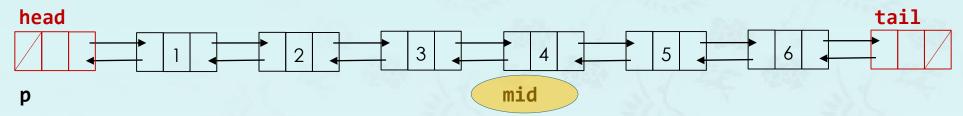


doubly linked list - half()



 Method 1: Count how many nodes there are in the list, then scan to the halfway point, breaking the last link followed.

```
pNode half(pList p) {
  int N = size(p);
  // go through the list
  // break at the halfway point
  // return the current pointer
}
```

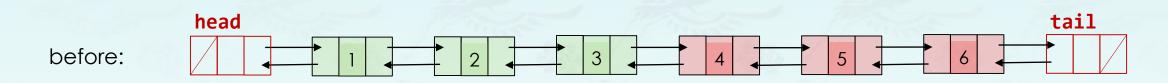


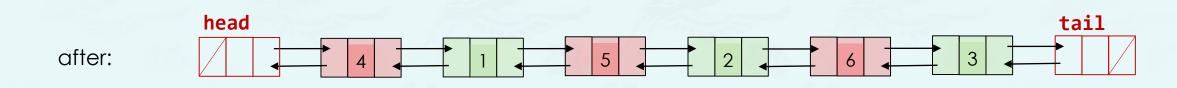
- Method 1: Count how many nodes there are in the list, then scan to the halfway point, breaking
 the last link followed.
- Method 2: It works by sending rabbit and turtle down the list: turtle moving at speed one, and
 rabbit moving at speed two. As soon as the rabbit hits the end, you know that the turtle
 is at the halfway point as long as the rabbit gets asleep at the halfway.

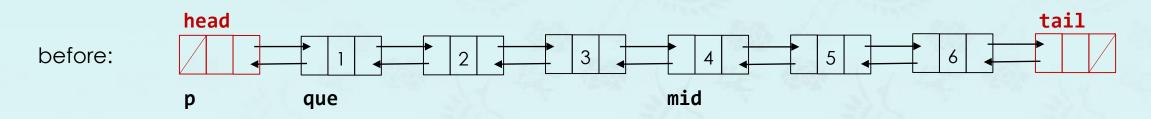
```
// returns so called "perfectly shuffled" list.
// The first half and the second half are interleaved each other.
// The shuffled list begins with the second half of the original.
// For example, 1234567890 returns 6172839405.
Algorithm:
1) find the mid node of the list p to split it into two lists at the mid node.
2) remove the 1st half from the list p, and keep it as a list "que" to add.
3) set the list p head such that it points the "mid" of the list p.
4) keep on interleaving nodes until the "que" is exhausted.
     save away next pointers of mid and que.
     interleave nodes in the "que" into "mid" in the list of p.
      (insert the first node in "que" at the second node in "mid".)
```

Algorithm:

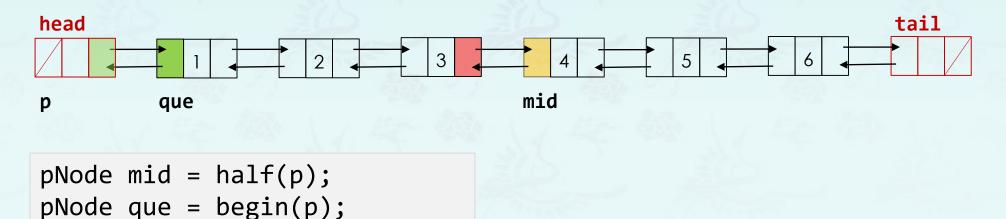
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 - save away next pointers of mid and que.
 - interleave nodes in the "que" into "mid" in the list of p. (start inserting the first node in "que" at the second node in "mid".)



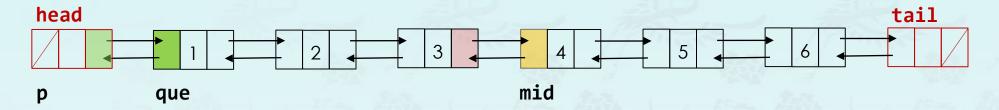


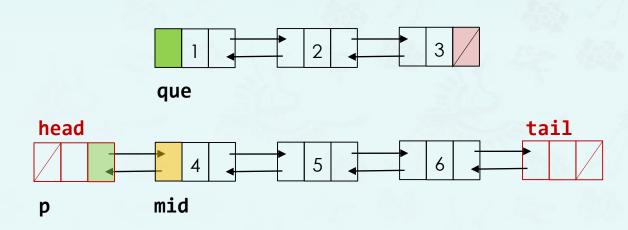


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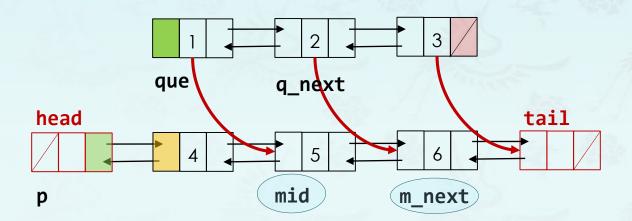


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- 4) keep on interleaving nodes until the "que" is exhausted.
 - save away next pointers of mid and que.
 - interleave nodes in the "que" into "mid" in the list of p.
 (start inserting the fist node in "que" at the second node in "mid".)



```
mid = begin(p)->next;
while (que != nullptr) {
  pNode q_next = que->next;
 pNode m_next = mid->next;
  // que is inserted at mid.
  mid = m next;
  que = q_next;
```

Goal: push sorted N – O(n log n)

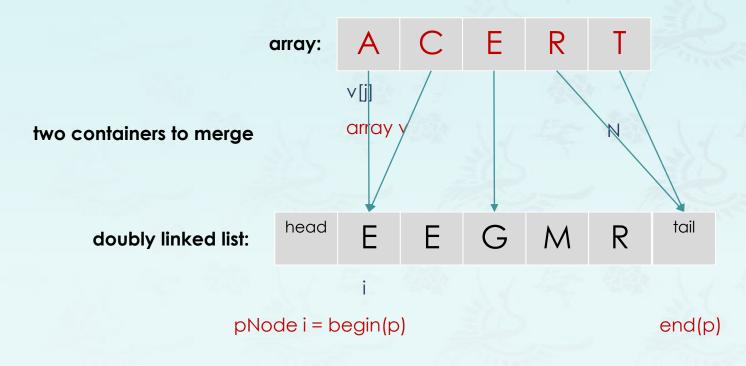


doubly linked list:
$$E E G M R$$

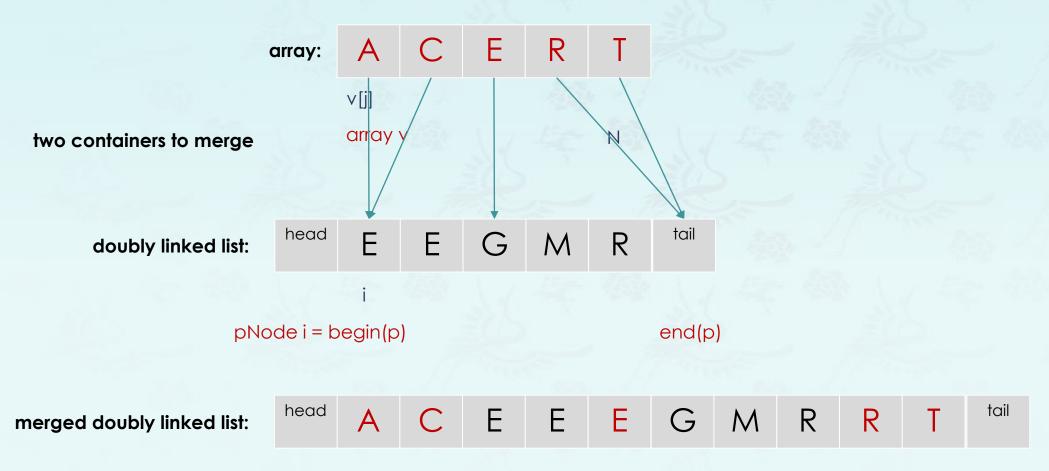
i

pNode $i = begin(p)$ end(p)

Goal: push sorted N – O(n log n)



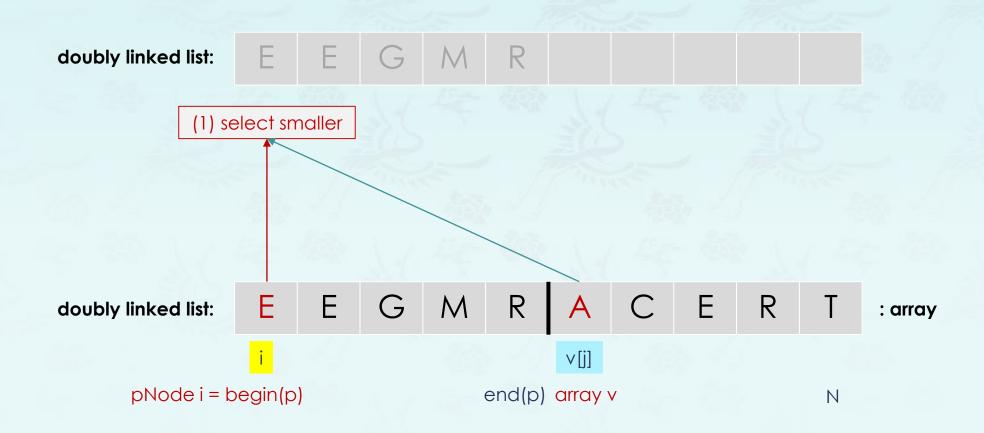
Goal: push sorted N – O(n log n)

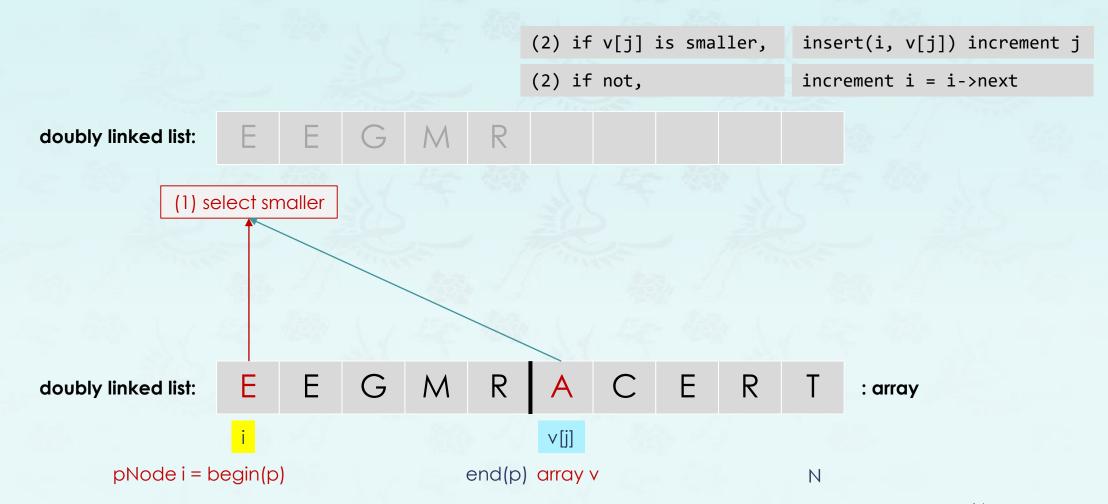


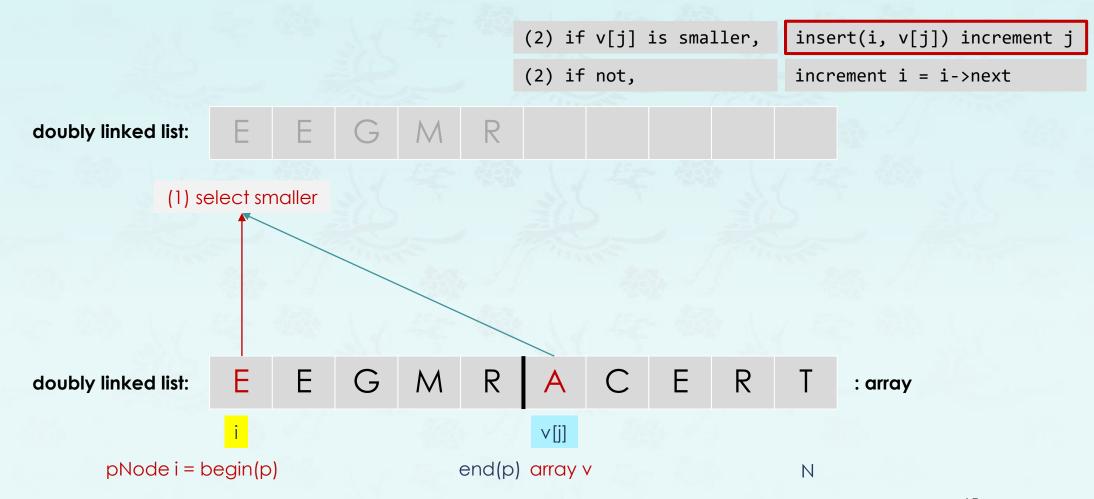
Goal: push sorted N – O(n log n)

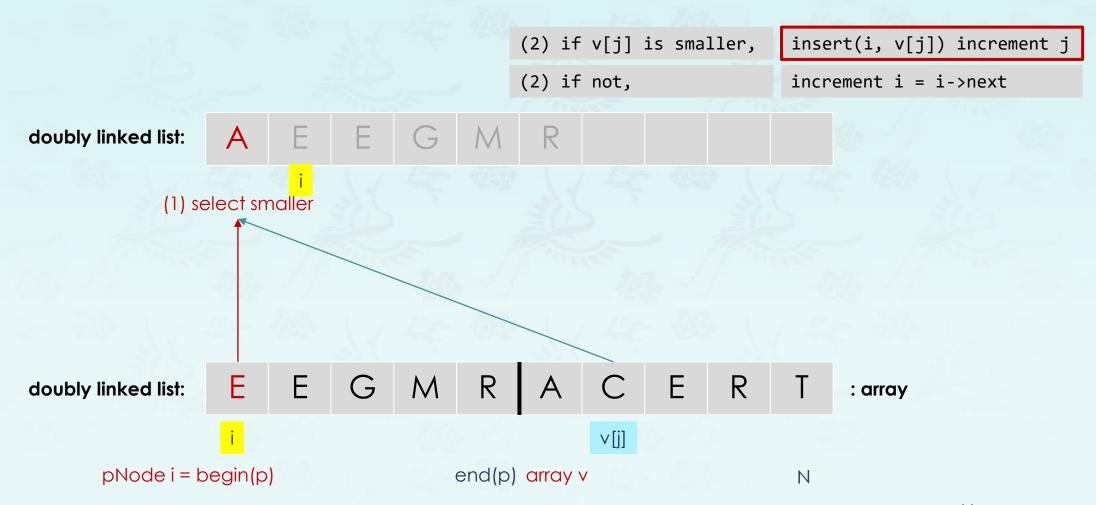


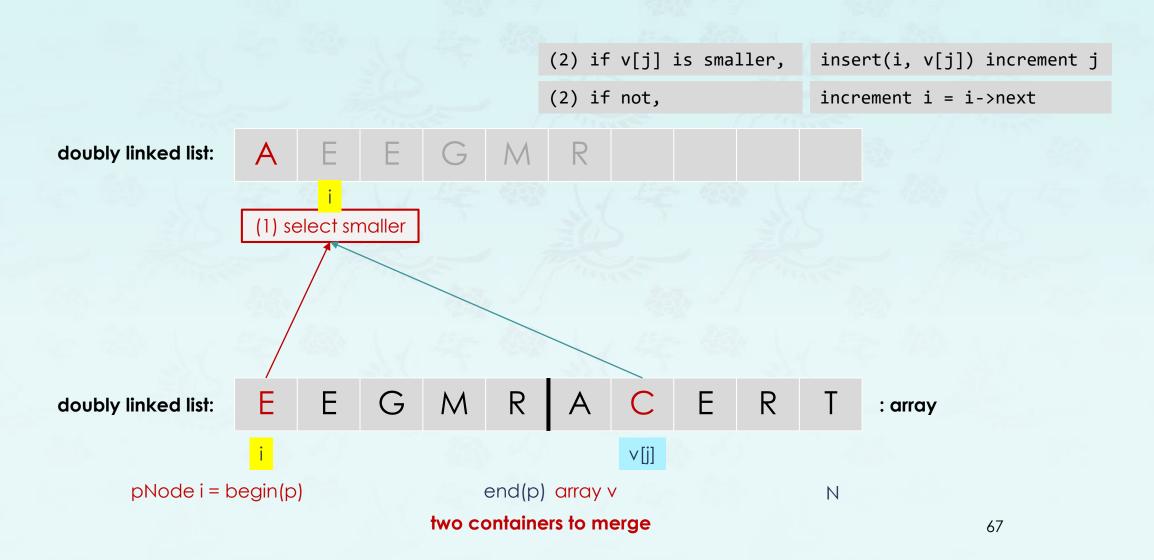


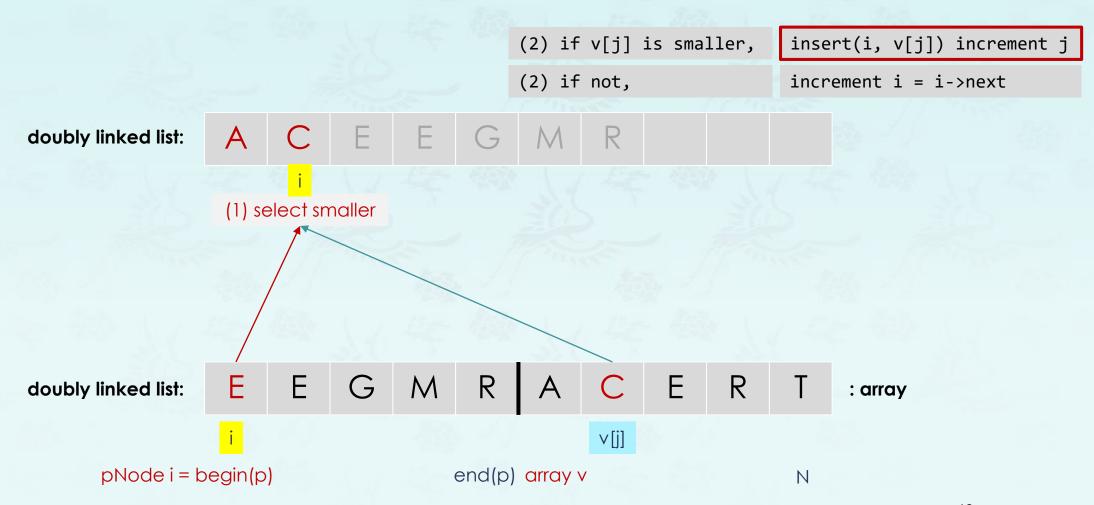


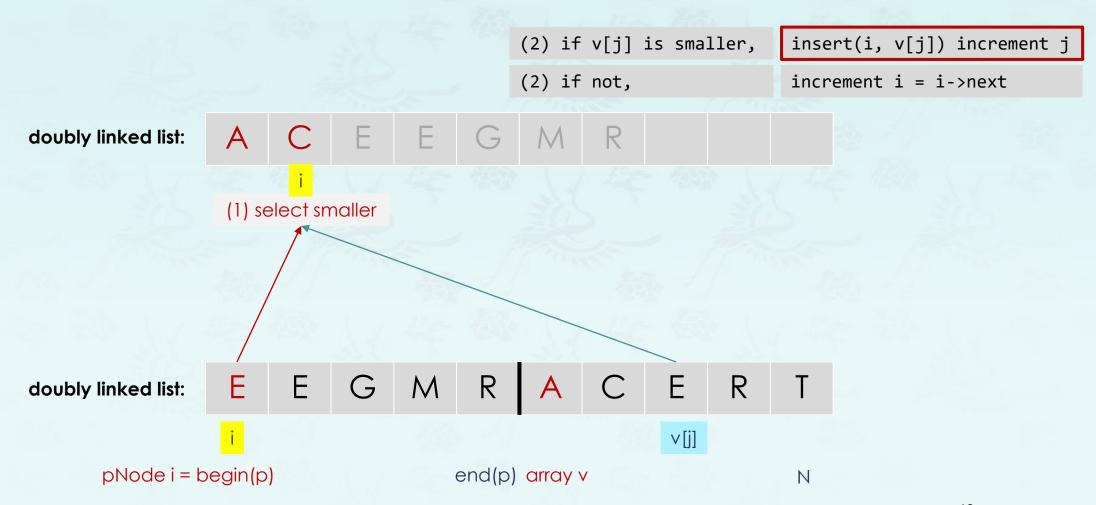


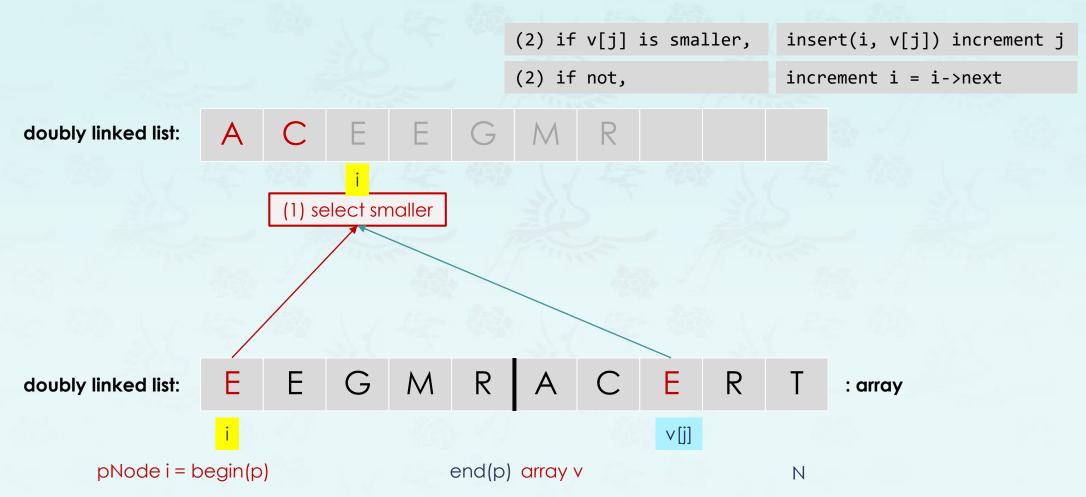


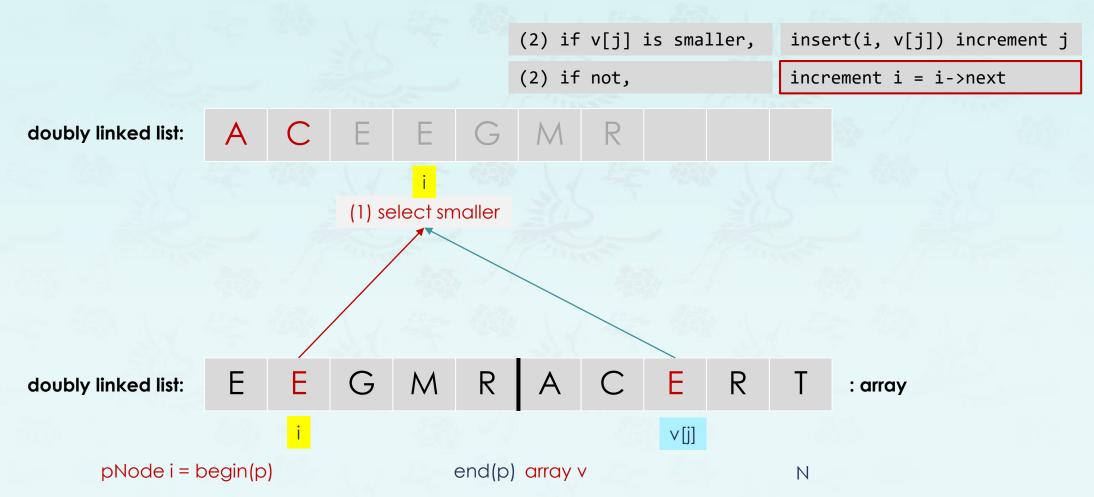


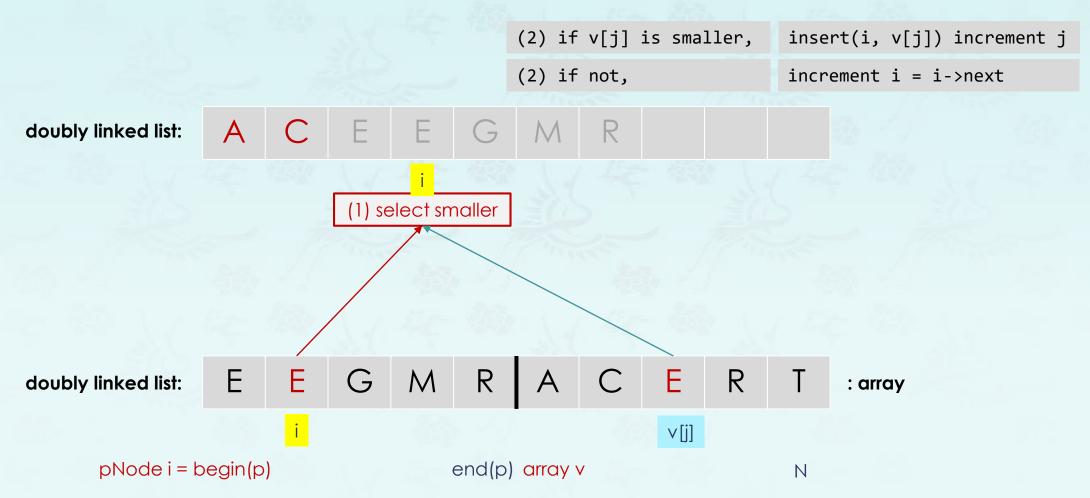


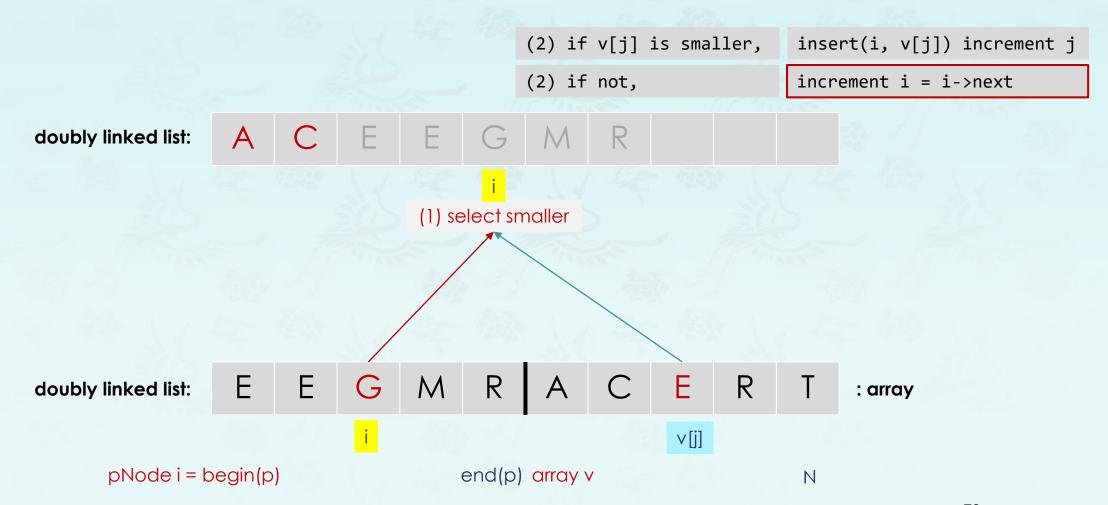


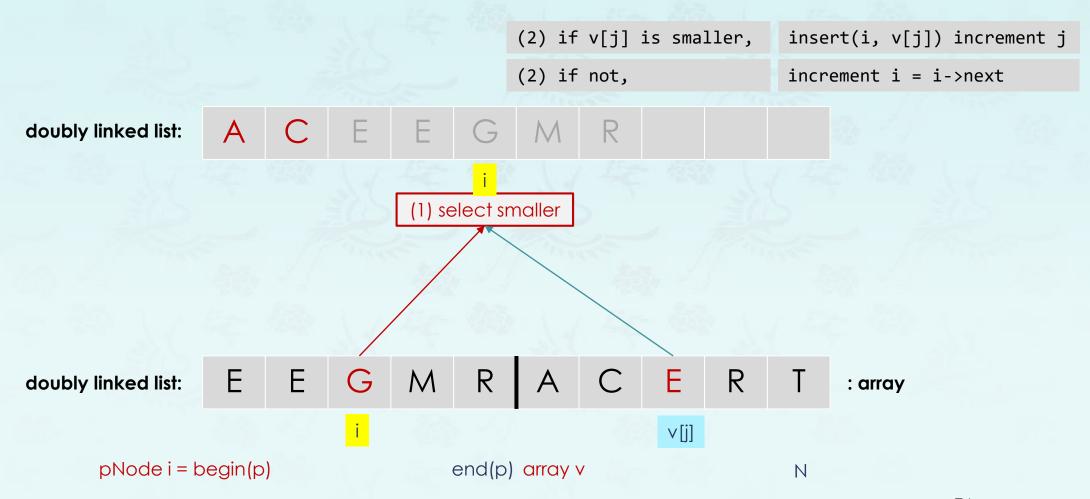


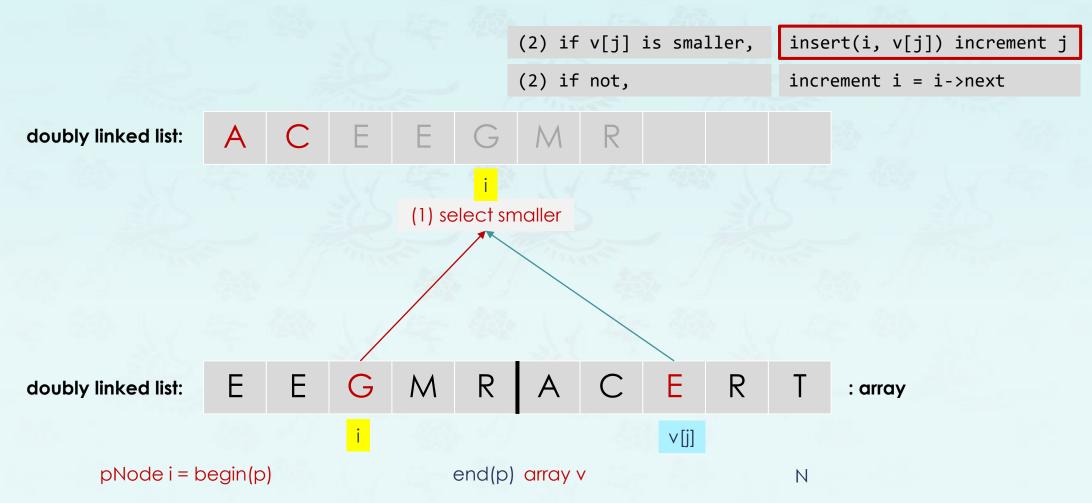


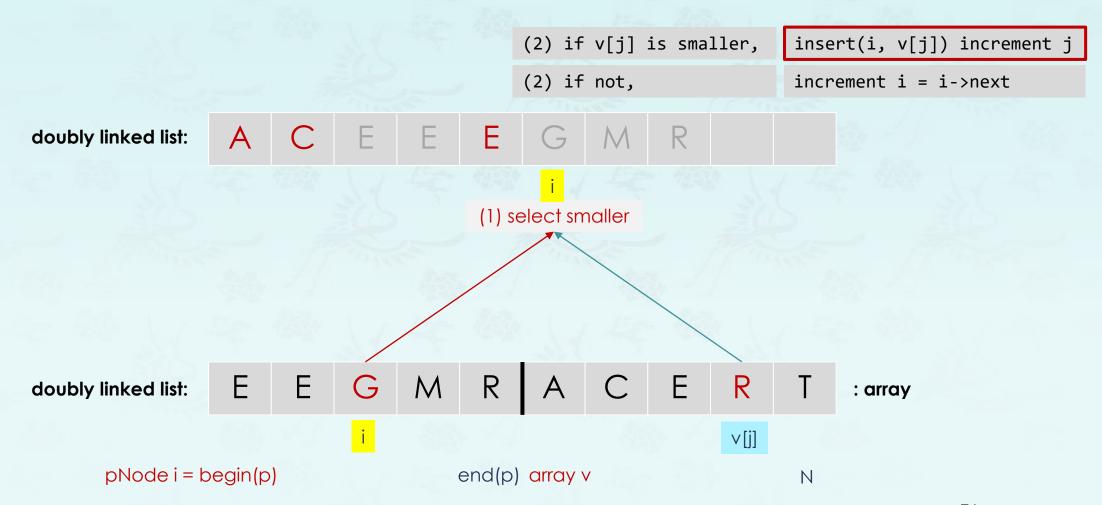


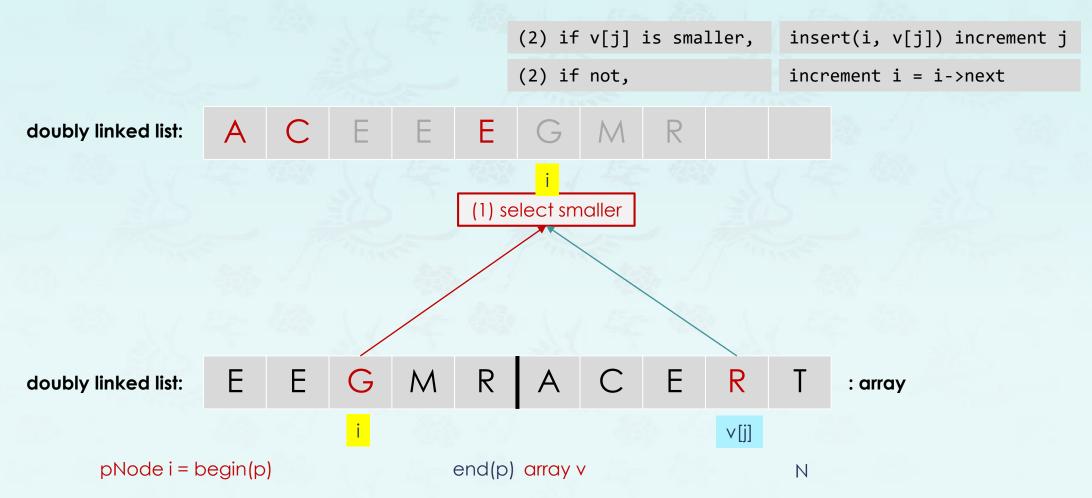


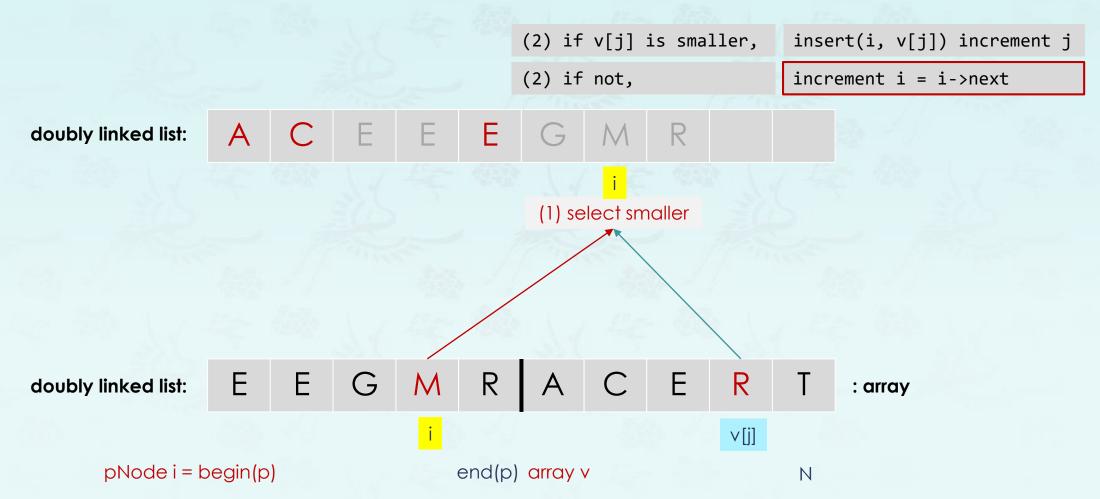


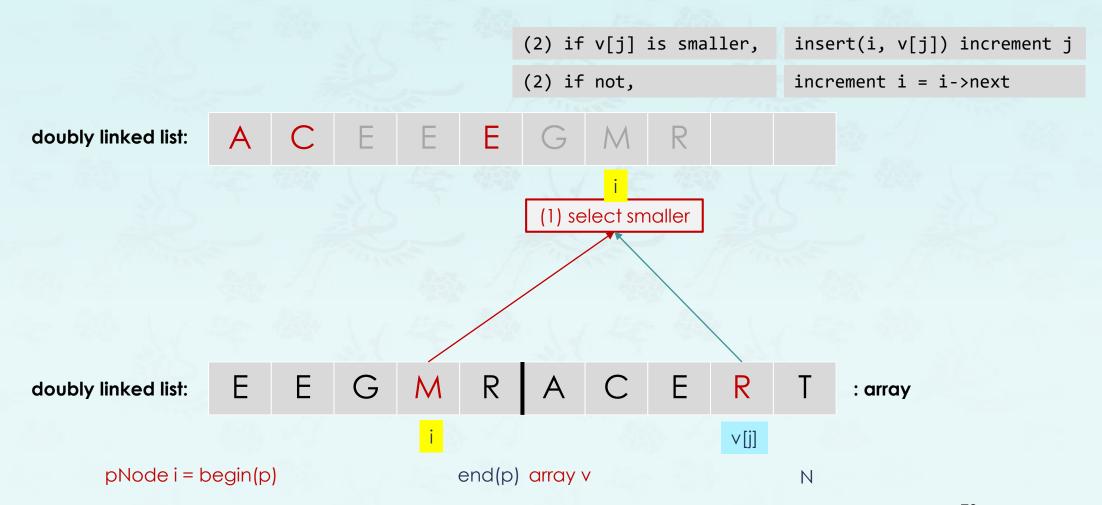


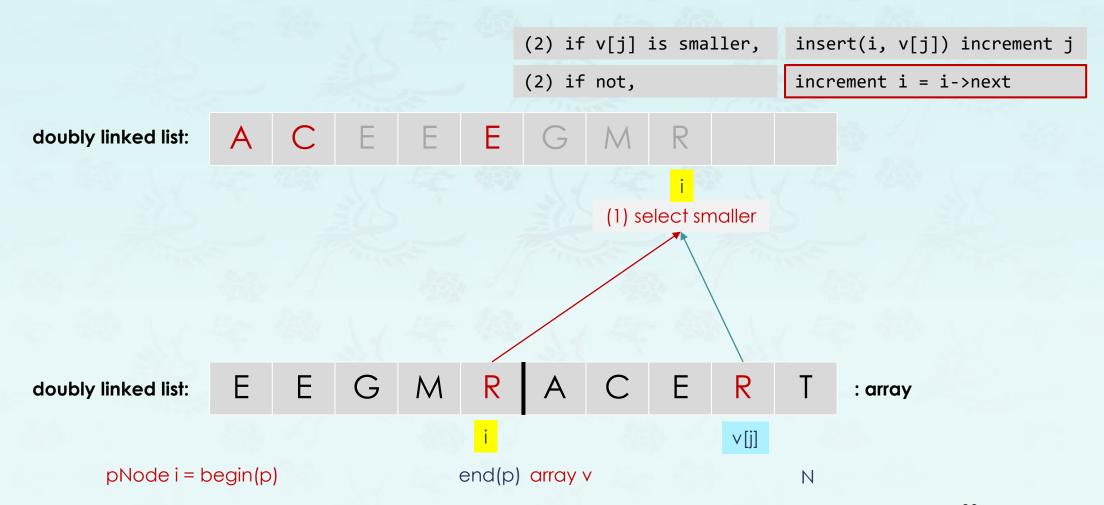


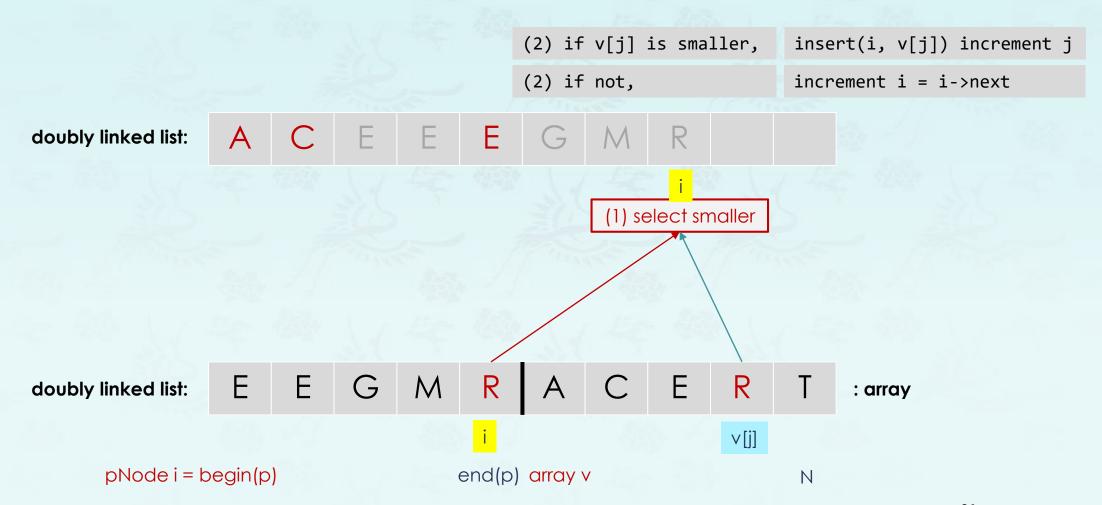


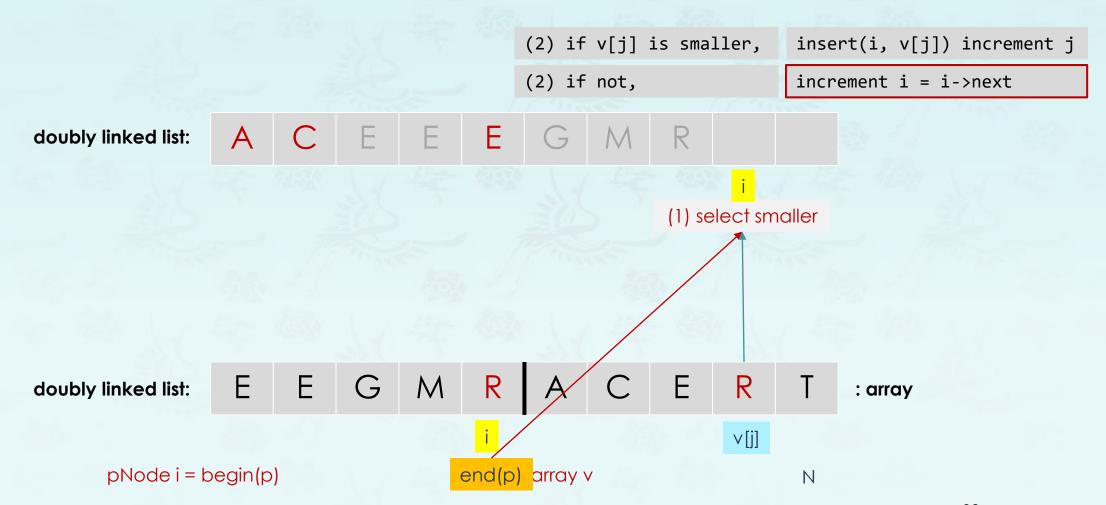


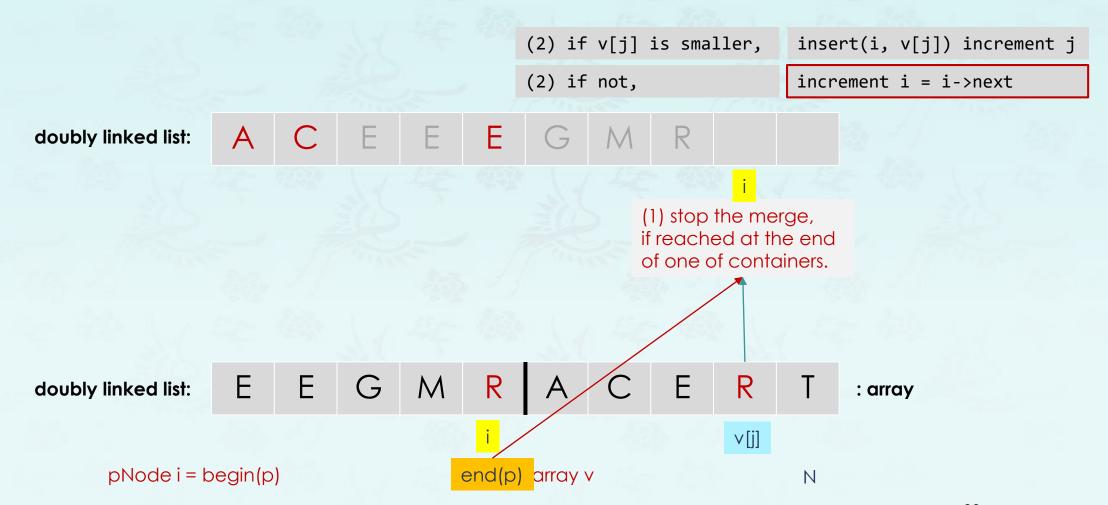


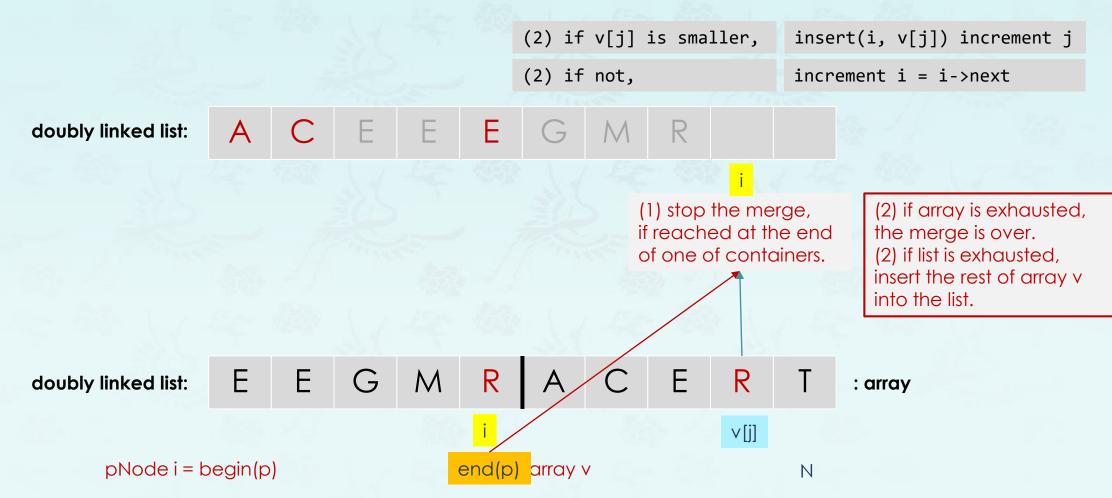


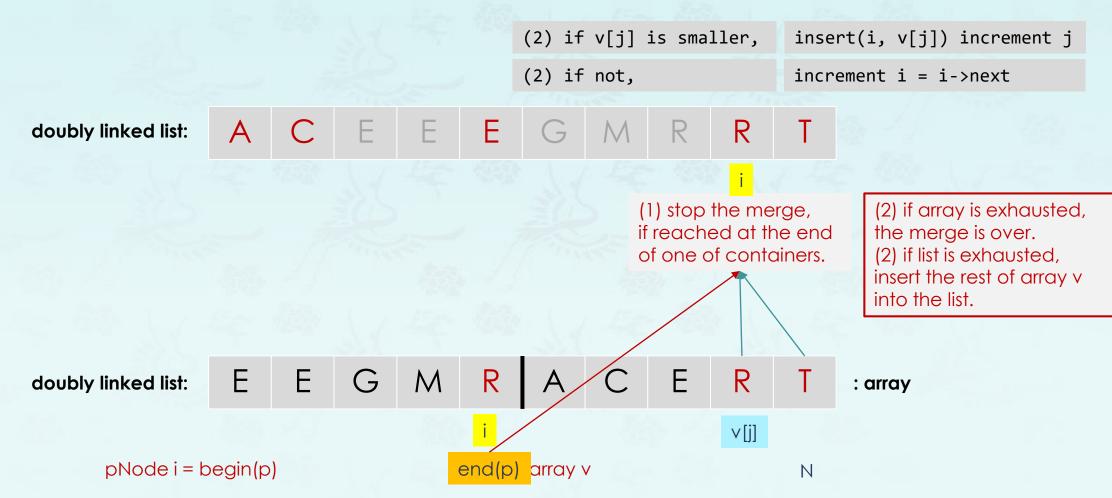




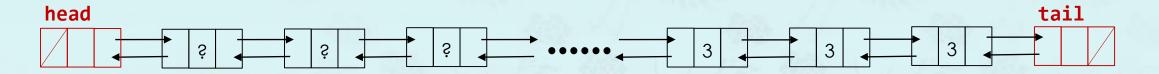








doubly linked list - testing



- 1. Running push_backN() once will be enough for most cases. For "pop vals" or "unique", you may run it twice as shown below.
 - 1) create 50,000 nodes filled with random values at the first run.
 - 2 create 50,000 nodes filled with a fixed value such as 100,000 or 3 at the second run.
- 2. Now you are ready to use this vector to test O(n), $O(n \log n)$, and $O(n^2)$
- 3. You may test your code with 1,000,000 nodes and compare them with listdblx.exe.

doubly linked list – timing test

N		10,000	100,000	1 Million	
push sorted O(n)	my code		7		
	listdblx		授發 \ /		
sort (selection) O(n^2)	my code	3377 ,	3)/		takes too long unless use quicksort
	listdblx				
reverse O(n)	my code	// "	17 1188	* **** -//	
	listdblx				
pop vals O(n)	my code		## \ /		/ 集 類 / /
	listdblx	37/65	7.65		
unique O(n)	my code				
	listdblx	11 11000	1/ 1998	the same of the	1166
shuffle O(n)	my code				
	listdblx				
push sorted N O(n^2)	my code	37/ 1	3)\		N = 10, 000, 100,000, 1,000,000
	listdblx				
push sorted N O(n log n)	my code	7.7	/// ***s		N = 10, 000, 100,000, 1,000,000
	listdblx	72	Y = 5 = 1		
	my code				
	listdblx				



Summary

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