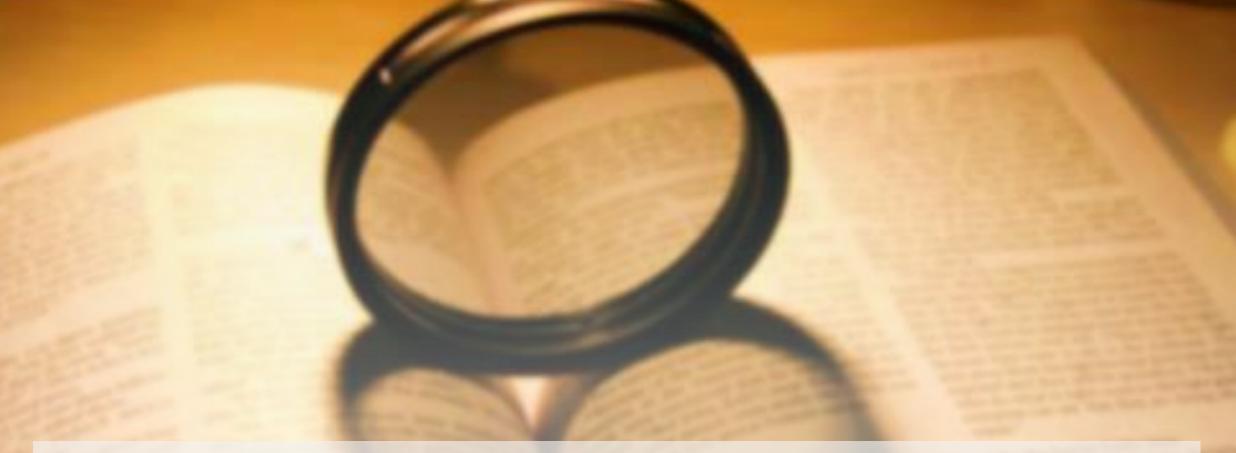
Data Structures Chapter 7: Graph

- 1. Introduction
 - Terminology, Representation, ADT
- 2. Basic Operations
 - DFS, CC, BFS, Processing
- 3. Digraph and Applications
- 4. Minimum Spanning Tree(MST)



네가 만일 네 입으로 예수를 주로 시인하며 또 하나님께서 그를 죽은 자 가운데서 살리신 것을 네 마음에 믿으면 구원을 받으리라 사람이 마음으로 믿어 의에 이르고 입으로 시인하여 구원에 이르느니라 (롬10:9-10)

죄의 삯은 사망이요 하나님의 은사는 그리스도 예수 우리 주 안에 있는 영생이니라 (롬 6:23)

모든 사람이 죄를 범하였으매 하나님의 영광에 이르지 못하더니 그리스도 예수 안에 있는 속량으로 말미암아하나님의 은혜로 값없이 의롭다 하심을 얻은 자 되었느니라 (롬 3:23-24)

Connectivity Queries

- Def.: Vertices v and w are connected if there is a path between them.
- Goal: Preprocess graph to answer queries of the form "is v connected to w?" in constant time.

| | Connected Component | |
|------|-------------------------|--------------------------------|
| | CC(Graph g) | find connected component in g |
| bool | connected(int v, int w) | are v and w connected" |
| int | count() | member of connected components |
| int | id(int v) | component identifier for v |

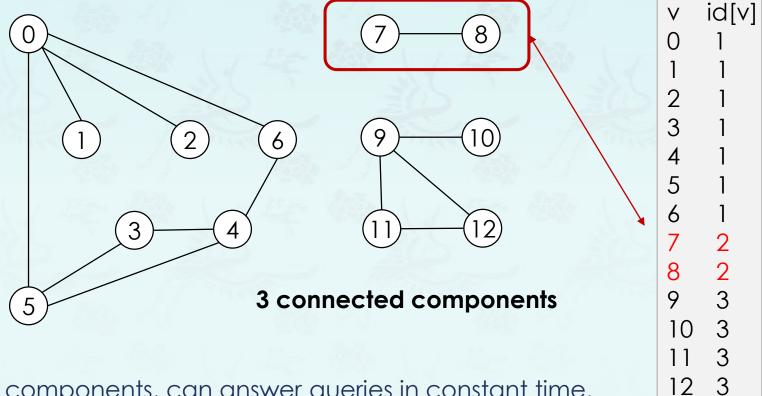
Depth-first search? Yes ...

The relation "is connected to" is equivalence relation:

Reflexive: v is connected to v.

Symmetric: if v is connected to w, then w is connected v.

Transitive: if v connected to w and w connected to x, then v connected to x



Remark:

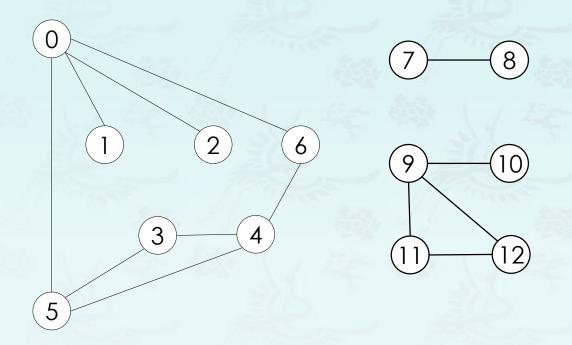
Given connected components, can answer queries in constant time.

Goal: Partition vertices into connected components.

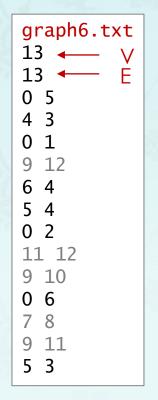
- Initialize all vertices v as unmarked.
- For each unmarked vertex v, run DFS to identify all vertices discovered as part of the same component.

To visit a vertex v:

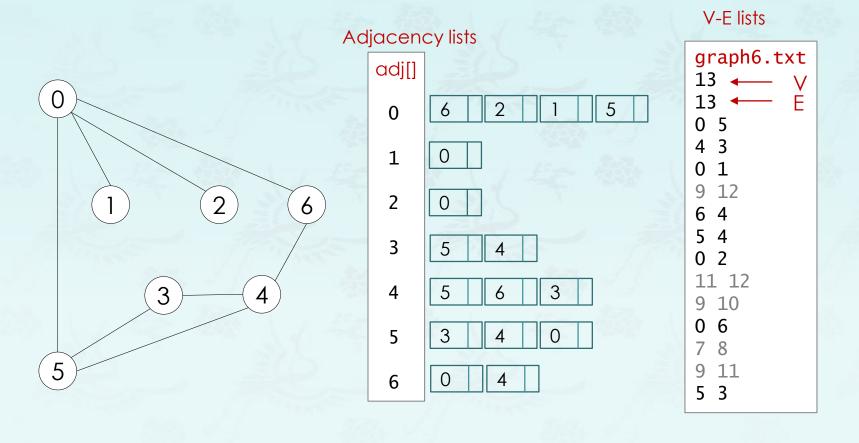
- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



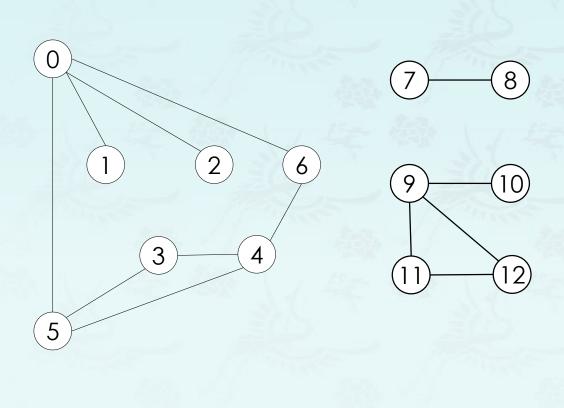
V-E lists



Graph g:

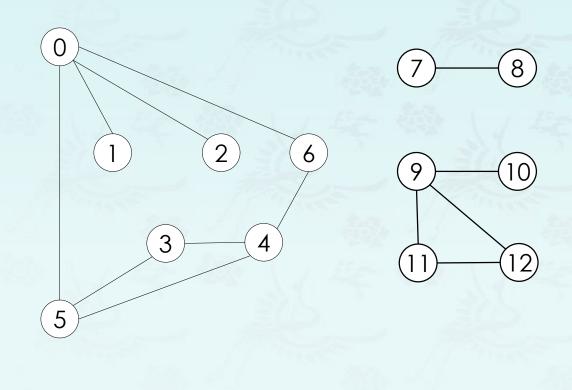


Graph g:



| v | marked[] | id[] |
|----|----------|------|
| 0 | F | _ |
| 1 | F | - |
| 2 | F | - |
| 3 | F | - |
| 4 | F | - |
| 5 | F | - |
| 6 | F | - |
| 7 | F | - |
| 8 | F | - |
| 9 | F | - |
| 10 | F | - |
| 11 | F | - |
| 12 | F | - |
| | | |

Graph g:



| v | marked[] | id[] |
|----|----------|------|
| 0 | Т | 0 |
| 1 | Т | 0 |
| 2 | Т | 0 |
| 3 | Т | 0 |
| 4 | Т | 0 |
| 5 | Т | 0 |
| 6 | Т | 0 |
| 7 | Т | 1 |
| 8 | Т | 1 |
| 9 | Т | 2 |
| 10 | Т | 2 |
| 11 | Т | 2 |
| 12 | Т | 2 |

Graph g:

Connected Components - Coding

```
// returns true if v and w are connected.
bool connected(graph g, int v, int w) {
  if (empty(g)) return true;

DFS_CCs(g);

return g->CCID[v] == g->CCID[w];
}
```

```
// returns number of connected components.
int nCCs(graph g) {
  int id = g->CCID[0];
  int count = 1;
  for (int i = 0; i < V(g); i++)
    if (id != g->CCID[i]) {
     id = g->CCID[i];
     count++;
    }
  return id == 0 ? 0 : count;
}
```



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Design pattern for graph processing

- Design pattern: Decouple graph data type
- Idea: Mimic maze exploration

DFS (to visit a vertex v)

- Mark v as visited.
- Recursively visit all unmarked vertices w adjacent to v.

Typical applications:

- Find all vertices connected to a given source vertex.
- Find a path between two vertices.

Challenge:

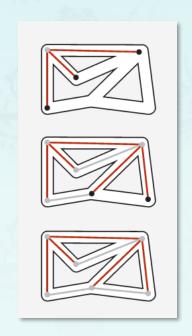
How to implement?

Breadth-first search

- Depth-first search: Put unvisited vertices on a stack.
- Breadth-first search: Put unvisited vertices on a queue.
- Shortest path: Find path from s to t that uses fewest number of edges.

BFS: (from source vertex s)

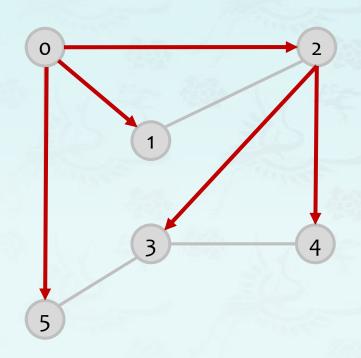
- Put s onto a FIFO queue, and mark s as visited.
- Repeat until the queue is empty:
 - remove the least recently added vertex v
 - add each of v's unvisited neighbors to the queue,
 - and mark them as visited.



Intuition: BFS examines vertices in increasing distance from s.

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



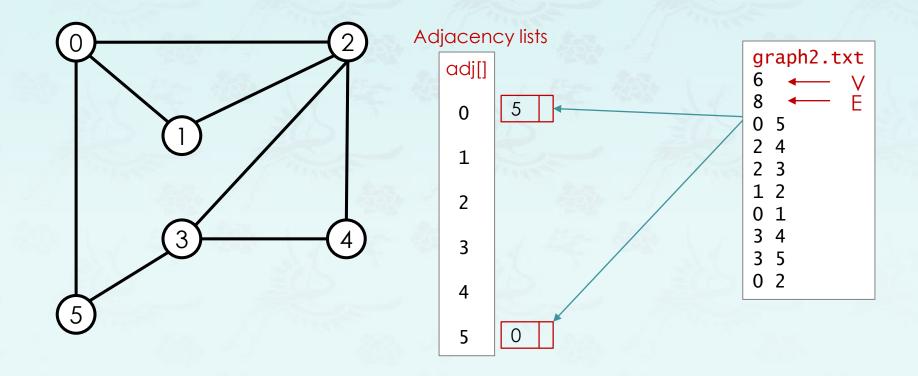
graph2.txt

| v p | parent[| v] distTo[] |
|-----|---------|-------------|
| 0 | 1-4 | 0 |
| 1 | 0 | 1 |
| 2 | 0 | 1 |
| 3 | 2 | 2 |
| 4 | 2 | 2 |
| 5 | 0 | 1 |

done

Repeat until queue is empty:

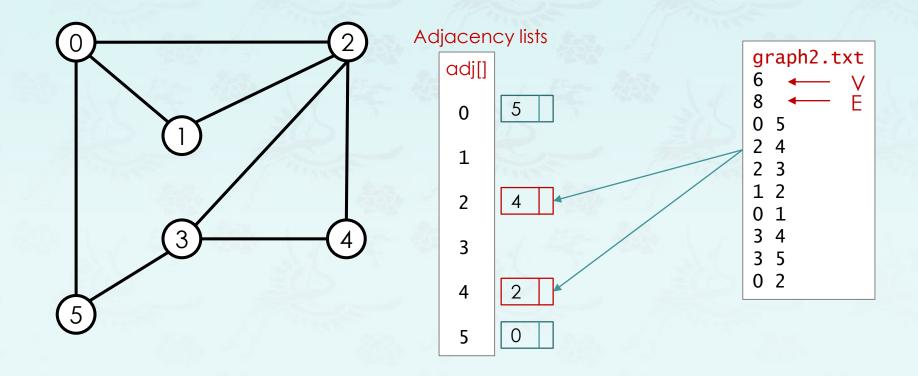
- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Graph g:

Repeat until queue is empty:

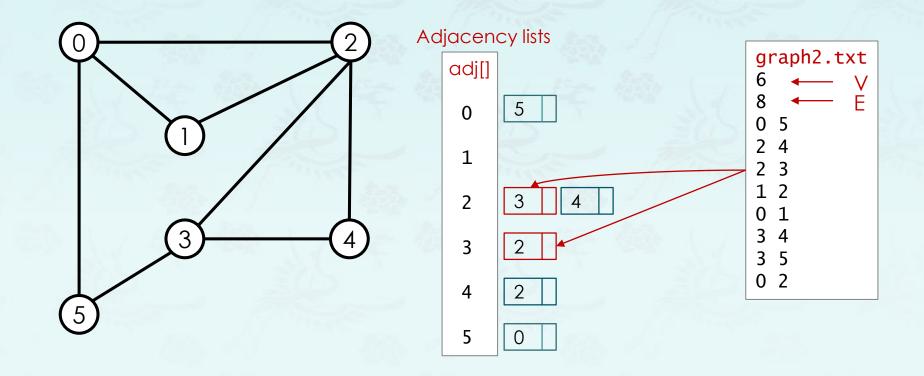
- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Graph g:

Repeat until queue is empty:

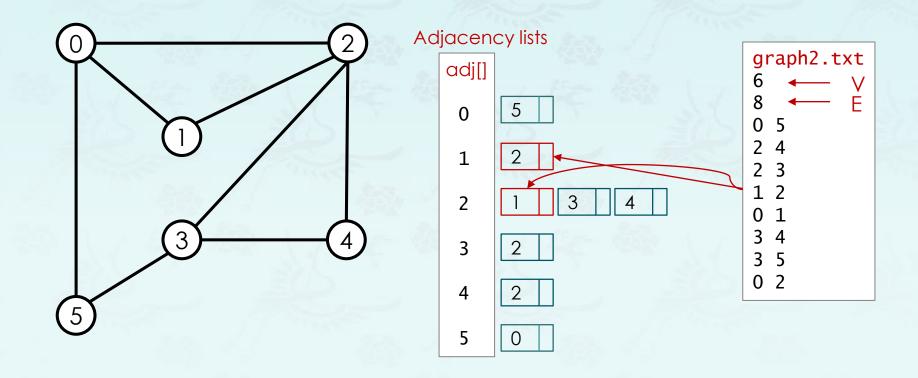
- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Graph g:

Repeat until queue is empty:

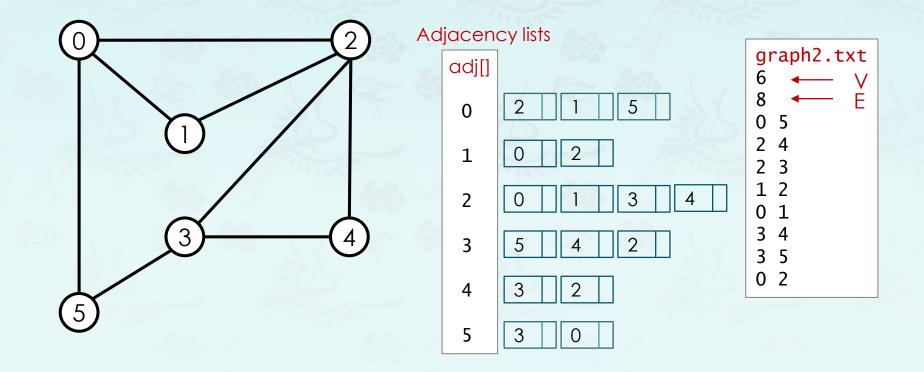
- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Graph g:

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.

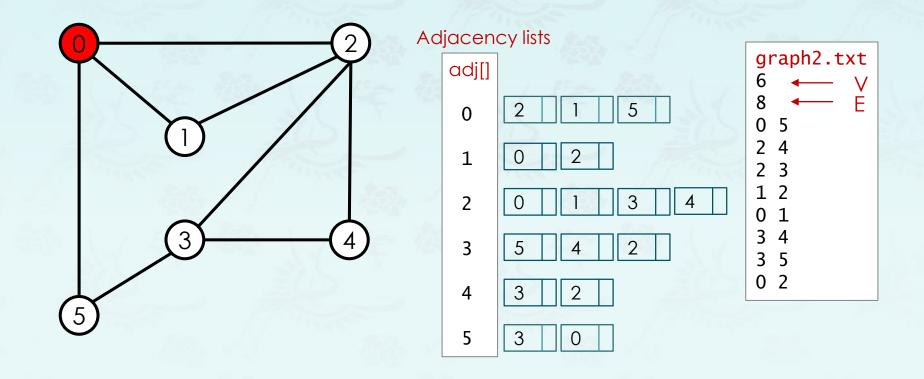


Graph g:

Challenge: build adjacency lists? Job done

Repeat until queue is empty:

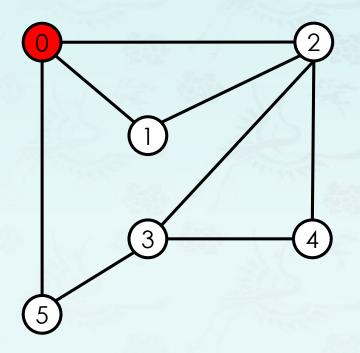
- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Graph g:

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.

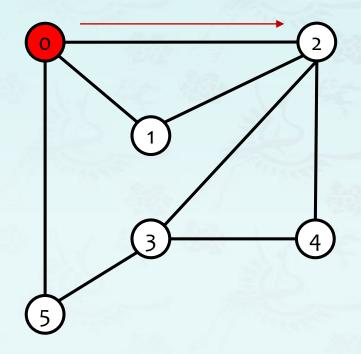


| v parent[v] distTo[| | | |
|---------------------|-----------------------|---------------------------------|--|
| 0 | 1-4 | 0 | |
| 1 | <u> </u> | _ | |
| 2 | _ | | |
| 3 | _ | _ | |
| 4 | - 2 | - / I | |
| 5 | - | | |
| | 0 1 2 3 4 | 0 - 1 - 2 - 3 - 4 - | |

add 0 to queue:

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.

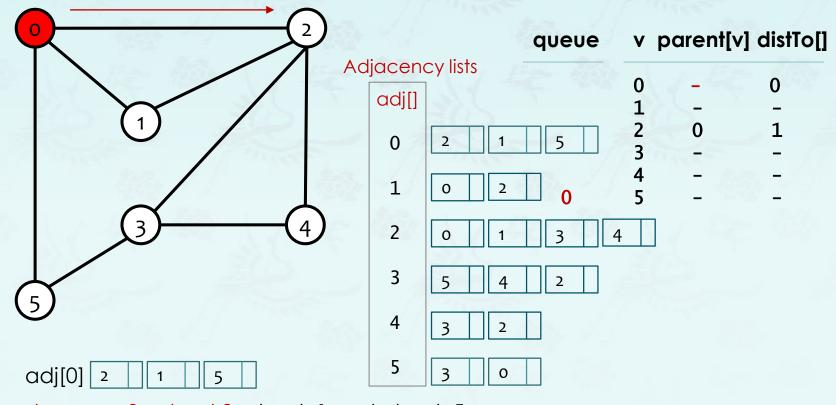


| queue | v parent[v] distTo[] | | |
|-------|----------------------|----------|------------|
| | 0 | 1-4 | 0 |
| | 1 | <u> </u> | _ |
| | 2 | 0 | 1 |
| | 3 | - | - |
| | 4 | - | - J |
| 0 | 5 | - | * <u>-</u> |

adj[0] 2 1 5

Repeat until queue is empty:

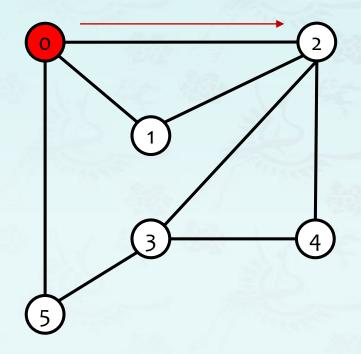
- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



dequeue 0: check2, check 1 and check 5

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.

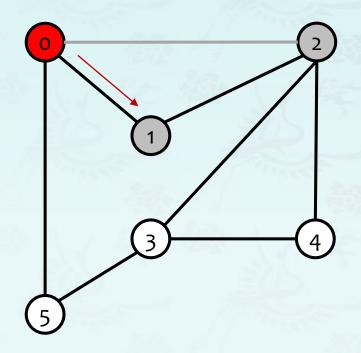


| queue | v parent[v] distTo[] | | |
|-------|----------------------|------------|-----|
| | 0 | 1-4 | 0 |
| | 1 | \ <u>-</u> | _ |
| | 2 | 0 | 1 |
| | 3 | - | - / |
| | 4 | - 2 | |
| 2 | 5 | - | |
| | | | |

adj[0] 2 1 5

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.

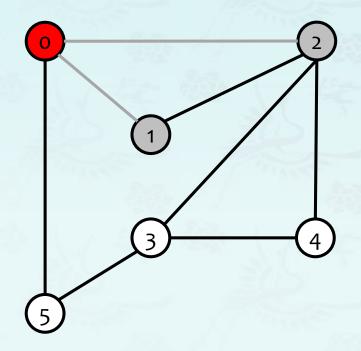


| queue | v parent[v] distTo[] | | |
|-------|----------------------|------------|-----|
| | 0 | 1-4 | 0 |
| | 1 | \ <u>-</u> | _ |
| | 2 | 0 | 1 |
| | 3 | - | - / |
| | 4 | - 2 | |
| 2 | 5 | - | |
| | | | |

adj[0] 2 1 5

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.

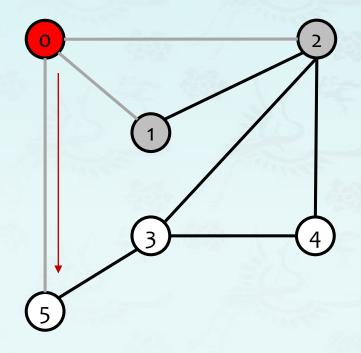


| queue | v parent[v] distTo[| | | |
|-------|---------------------|-----|-----|--|
| | 0 | 1-4 | 0 | |
| | 1 | 0 | 1 | |
| | 2 | 0 | 1 | |
| | 3 | - | - | |
| 1 | 4 | - 0 | -// | |
| 2 | 5 | - | - E | |
| | | | | |

adj[0] 2 1 5

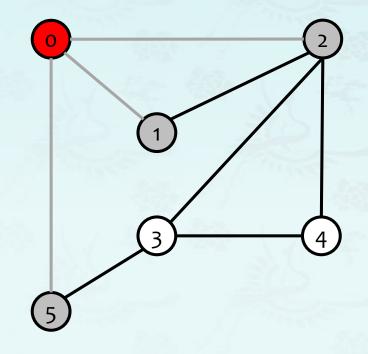
Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



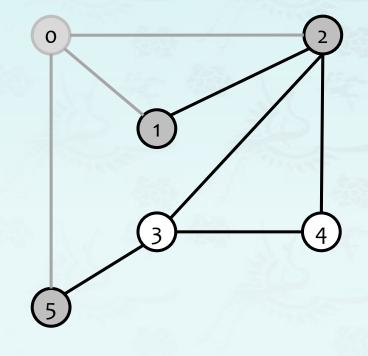
| queue | v parent[v] distTo[] | | | |
|-------|----------------------|-----|-----|--|
| | 0 | 1-4 | 0 | |
| | 1 | 0 | 1 | |
| | 2 | 0 | 1 | |
| | 3 | - | _ | |
| 1 | 4 | | - / | |
| 2 | 5 | _ | _ | |
| | | | | |

adj[0] 2 1 5



| queue | v parent[v] distTo[] | | | |
|-------|----------------------|-----|-----|--|
| | 0 | 1-4 | 0 | |
| | 1 | 0 | 1 | |
| | 2 | 0 | 1 | |
| 5 | 3 | - | - | |
| 1 | 4 | - 2 | - / | |
| 2 | 5 | 0 | 1 | |

adj[0] 2 1 5

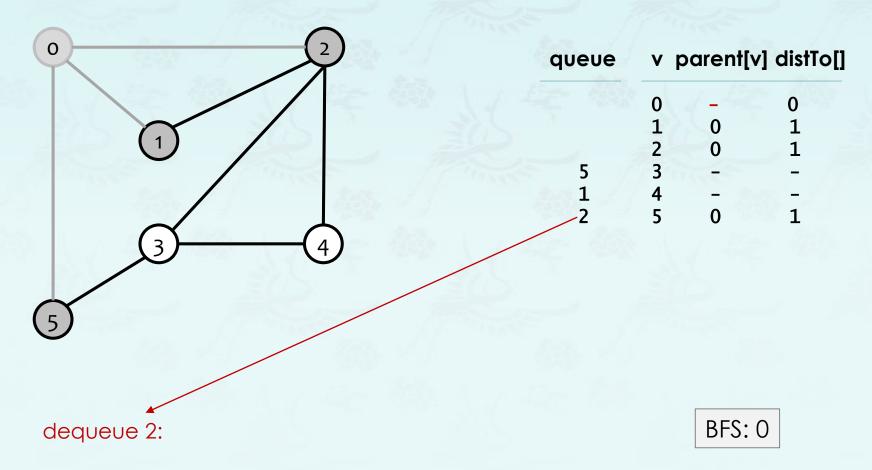


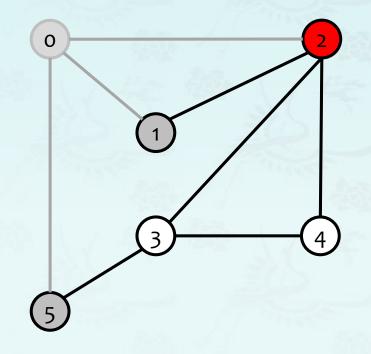
| queue | v parent[v] distTo[] | | |
|--------------------|----------------------|-----|-------|
| | 0 | 1-4 | 0 |
| | 1 | 0 | 1 |
| | 2 | 0 | 1 |
| 5 | 3 | - | - |
| 201 | 4 | - 2 | - / I |
| 2 | 5 | 0 | 1 |
| PERSONAL PROPERTY. | 4 5 | 0 | 1 |

adj[0] 2 1 5

0 done

BFS: 0





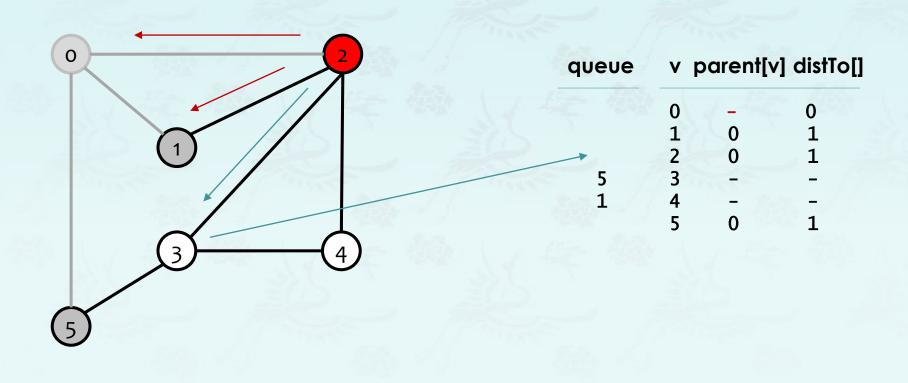
| queue | v parent[v] distTo[] | | | |
|-------|----------------------|-----|-----|--|
| | 0 | 1-4 | 0 | |
| | 1 | 0 | 1 | |
| | 2 | 0 | 1 | |
| 5 | 3 | - | - | |
| 201 | 4 | - 0 | -// | |
| | 5 | 0 | 1 | |
| | | | | |

adj[2] 0 1 3 4

dequeue 2: check 0, check 1, check 3 and check 4

BFS: 0

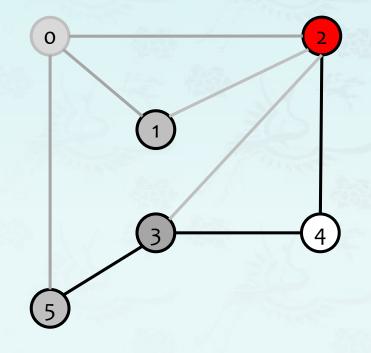
adj[2] o



32

BFS: 0

dequeue 2: check 0, check 1, check 3 and check 4

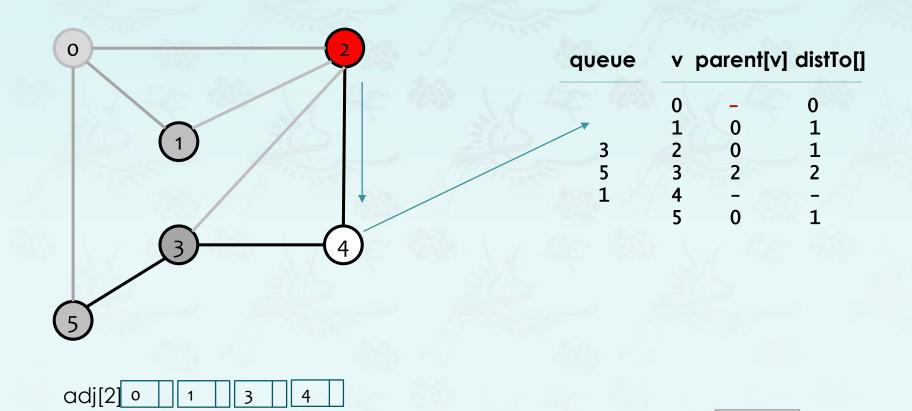


| queue | v parent[v] distTo[] | | | |
|-------|----------------------|-----|---|--|
| | 0 | 1-4 | 0 | |
| | 1 | 0 | 1 | |
| 3 | 2 | 0 | 1 | |
| 5 | 3 | 2 | 2 | |
| 1 | 4 | - 2 | | |
| | 5 | 0 | 1 | |

adj[2] 0 1 3 4

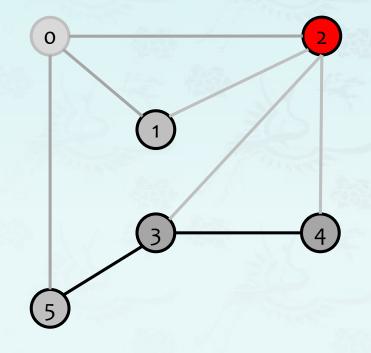
dequeue 2: check 0, check 1, check 3 and check 4

BFS: 0



BFS: 0

dequeue 2: check 0, check 1, check 3 and check 4

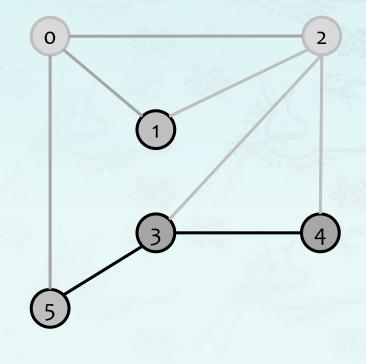


| queue | v parent[v] distTo[] | | | |
|-------|----------------------|-----|-----|--|
| | 0 | 1-4 | 0 | |
| 4 | 1 | 0 | 1 | |
| 3 | 2 | 0 | 1 | |
| 5 | 3 | 2 | 2 | |
| 1 | 4 | - 0 | - / | |
| | 5 | 0 | 1 | |

adj[2] 0 1 3 4

dequeue 2: check 0, check 1, check 3 and check 4

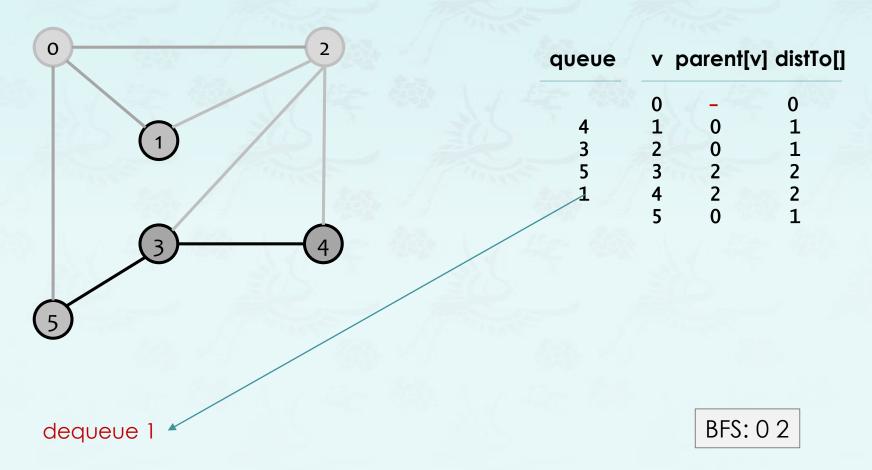
BFS: 0

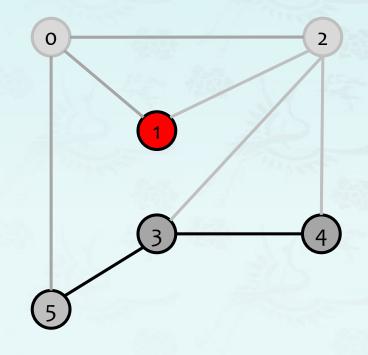


| queue | v parent[v] distTo[] | | | |
|-------|----------------------|-----|---|--|
| | 0 | 1-4 | 0 | |
| 4 | 1 | 0 | 1 | |
| 3 | 2 | 0 | 1 | |
| 5 | 3 | 2 | 2 | |
| 1/ | 4 | 2 | 2 | |
| | 5 | 0 | 1 | |

adj[2] 0 1 3 4 2 done

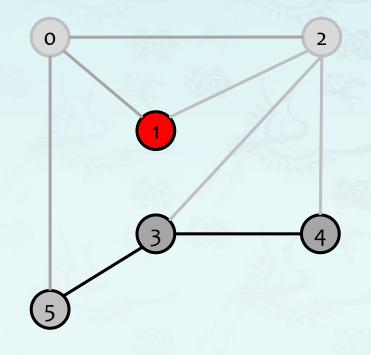
BFS: 0 2





| queue | v p | arent[| v] distTo[] |
|-------|-----|--------|-------------|
| | 0 | 1-4 | 0 |
| 4 | 1 | 0 | 1 |
| 3 | 2 | 0 | 1 |
| 5 | 3 | 2 | 2 |
| | 4 | 2 | 2 |
| | 5 | 0 | 1 |

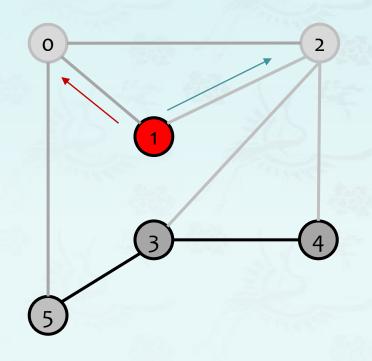
dequeue 1



| queue | v parent[v] distTo[] | | |
|-------|----------------------|-----|---|
| | 0 | 1-4 | 0 |
| 4 | 1 | 0 | 1 |
| 3 | 2 | 0 | 1 |
| 5 | 3 | 2 | 2 |
| | 4 | 2 | 2 |
| | 5 | 0 | 1 |

adj[1] 0 2

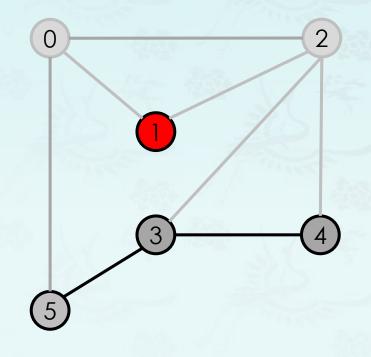
dequeue 1: check 0, and check 2



| queue | v parent[v] distTo[] | | |
|-------|----------------------|-----|---|
| | 0 | 1-4 | 0 |
| 4 | 1 | 0 | 1 |
| 3 | 2 | 0 | 1 |
| 5 | 3 | 2 | 2 |
| | 4 | 2 | 2 |
| | 5 | 0 | 1 |

adj[1] o 2

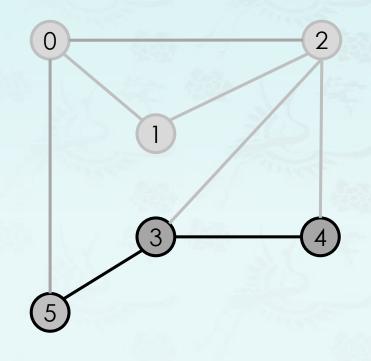
dequeue 1: check 0, and check 2



| queue | v parent[v] distTo[] | | | |
|-------|----------------------|-----|---|--|
| | 0 | 1-4 | 0 | |
| 4 | 1 | 0 | 1 | |
| 3 | 2 | 0 | 1 | |
| 5 | 3 | 2 | 2 | |
| | 4 | 2 | 2 | |
| | 5 | 0 | 1 | |

adj[1]0 2

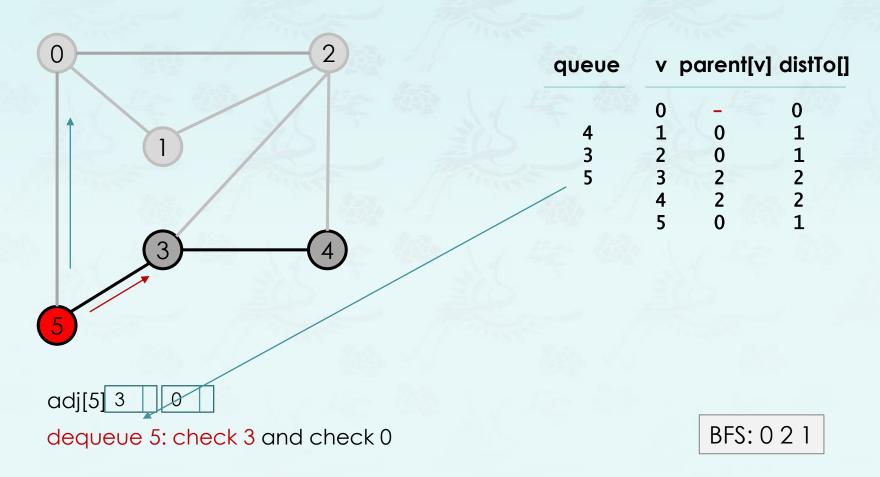
dequeue 1: check 0, and check 2



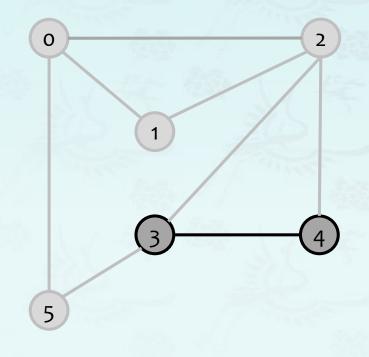
| queue | v parent[v] distTo[] | | | |
|-------|----------------------|-----|---|--|
| | 0 | 1-4 | 0 | |
| 4 | 1 | 0 | 1 | |
| 3 | 2 | 0 | 1 | |
| 5 | 3 | 2 | 2 | |
| | 4 | 2 | 2 | |
| | 5 | 0 | 1 | |



BFS: 0 2 1



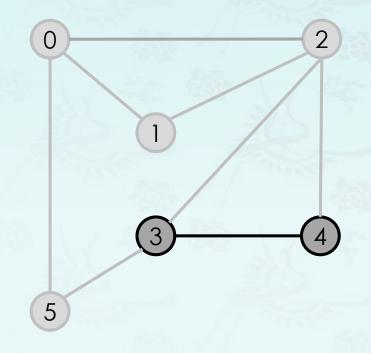
43



| queue | v parent[v] distTo[] | | |
|-------|----------------------|-----|---|
| | 0 | 1-4 | 0 |
| 4 | 1 | 0 | 1 |
| 3 | 2 | 0 | 1 |
| | 3 | 2 | 2 |
| | 4 | 2 | 2 |
| | 5 | 0 | 1 |

adj[5] 3 0 5 done

BFS: 0 2 1 5

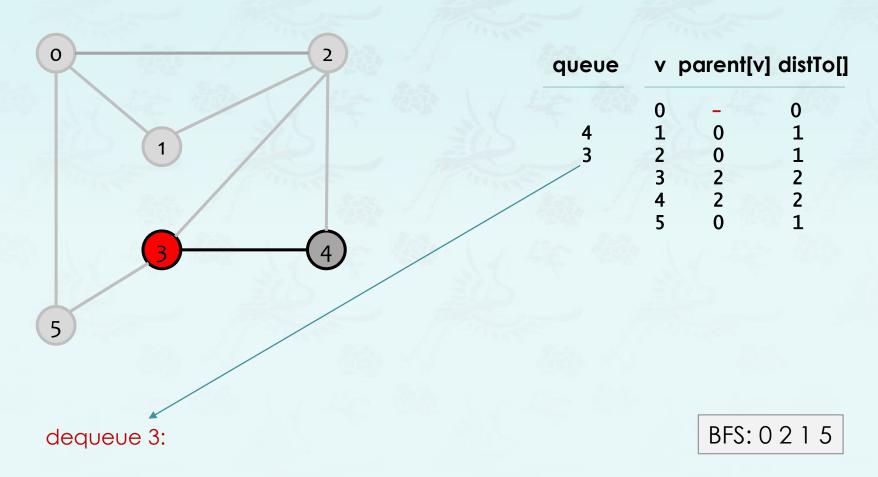


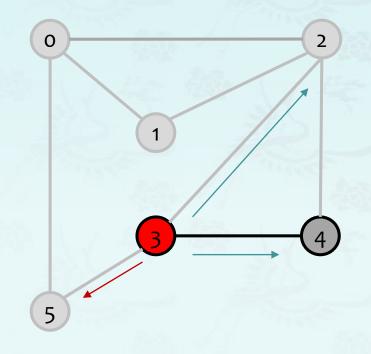
| queue | v parent[v] distTo[] | | |
|-------|----------------------|-----|---|
| | 0 | 1-4 | 0 |
| 4 | 1 | 0 | 1 |
| 3 | 2 | 0 | 1 |
| | 3 | 2 | 2 |
| | 4 | 2 | 2 |
| | 5 | 0 | 1 |
| | | | |

adj[3] 5 4 2

dequeue 3: Check 5, Check 4, and Check 2

BFS: 0 2 1 5



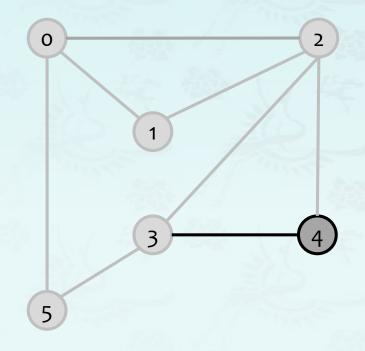


| queue | v parent[v] distTo[] | | |
|-------|----------------------|-----|---|
| | 0 | 1-4 | 0 |
| 4 | 1 | 0 | 1 |
| | 2 | 0 | 1 |
| | 3 | 2 | 2 |
| | 4 | 2 | 2 |
| | 5 | 0 | 1 |

adj[3] 5 4 2

dequeue 3: Check 5, Check 4, and Check 2

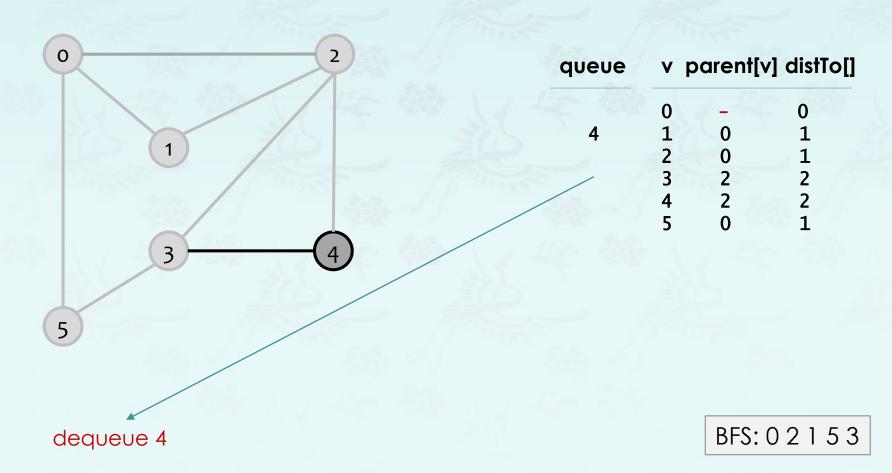
BFS: 0 2 1 5



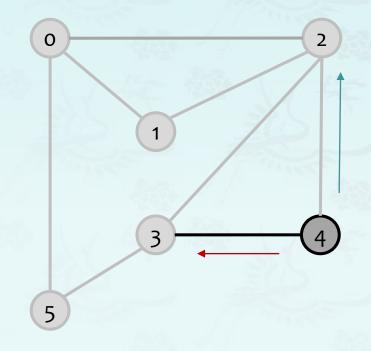
| queue | v parent[v] distTo[] | | |
|-------|----------------------|-----|---|
| | 0 | 1-4 | 0 |
| 4 | 1 | 0 | 1 |
| | 2 | 0 | 1 |
| | 3 | 2 | 2 |
| | 4 | 2 | 2 |
| | 5 | 0 | 1 |

adj[3] 5 4 2 3 done

BFS: 0 2 1 5 3



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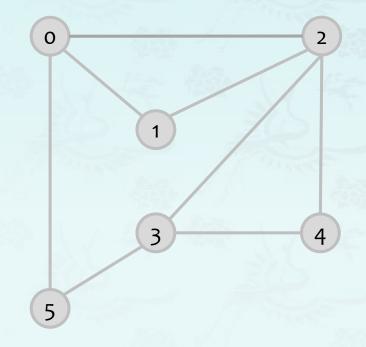


| queue | v parent[v] distTo[] | | |
|-------|----------------------|-----|---|
| | 0 | 1-4 | 0 |
| | 1 | 0 | 1 |
| | 2 | 0 | 1 |
| | 3 | 2 | 2 |
| | 4 | 2 | 2 |
| | 5 | 0 | 1 |
| | | | |
| | | | |

adj[4] 3 2

dequeue 4: Check 3 and Check 2

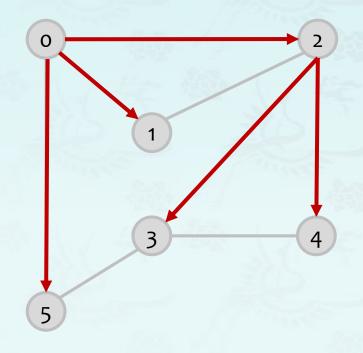
BFS: 0 2 1 5 3



| queue | v parent[v] distTo[] | | |
|-------|----------------------------|----------------------------|----------------------------|
| | 0 1 2 3 4 5 | - 0 0 2 2 2 | 0 1 1 2 2 1 |
| | | | |

4 done

BFS: 0 2 1 5 3 4



v parent[v] distTo[]

| 0 | 1-4 | 0 |
|---|-----|---|
| 1 | 0 | 1 |
| 2 | 0 | 1 |
| 3 | 2 | 2 |
| 4 | 2 | 2 |
| 5 | 0 | 1 |

done

BFS: 0 2 1 5 3 4

Breadth-first search

- Depth-first search: Put unvisited vertices on a stack.
- Breadth-first search: Put unvisited vertices on a queue.
- Shortest path: Find path from s to t that uses fewest number of edges.

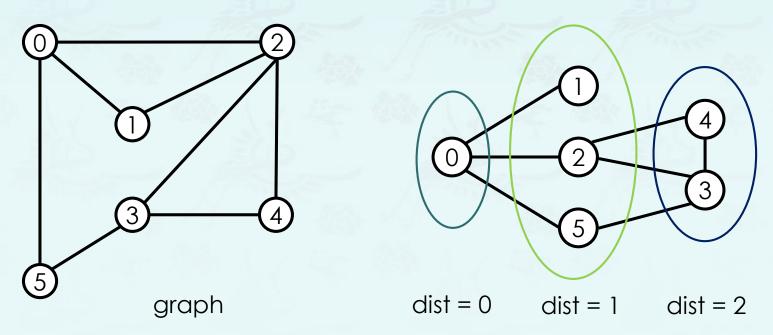
BFS: (from source vertex s)

- Put s onto a FIFO queue, and mark s as visited.
- Repeat until the queue is empty:
 - remove the least recently added vertex v
 - add each of v's unvisited neighbors to the queue, and mark them as visited.

Intuition: BFS examines vertices in increasing distance from s.

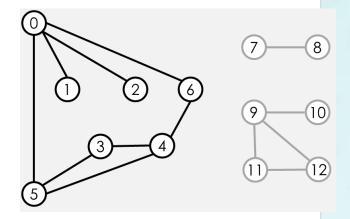
Breadth-first search properties

- Proposition: BFS computes shortest paths (fewest number of edges) from s to all other vertices in a graph in time proportional to E + V.
- Proof: [correctness] Queue always consists of zero or more vertices of distance k from s, followed by zero or more vertices of distance k + 1.
- Proof: [running time] Each vertex connected to s is visited once.



```
// runs BFS at v and produces BFS0[], distTo[] & parentBFS[]
void BFS(graph g, int v) {
 queue<int> que; // to process each vertex
 queue<int> sav;  // BFS result saved
 for (int i = 0; i < V(g); i++) g->marked[i] = false;
 g->distTo[v] = 0; g->BFSv = {};
             sav.push(v);
 que.push(v);
 while (!que.empty()) {
   int cur = que.front(); que.pop(); // remove it since processed
   for (gnode w = g->adj[cur].next; w; w = w->next) {
     if (!g->marked[w->item]) {
      g->marked[w->item] = true;
      que.push(w->item);  // queued to process next
      sav.push(w->item);  // save the result
      cout << "your code here"; // set parentBFS[] & distTo[]</pre>
 g->BFSv = sav;
                           // save the result at v
 setBFS0(g, v, sav);
```

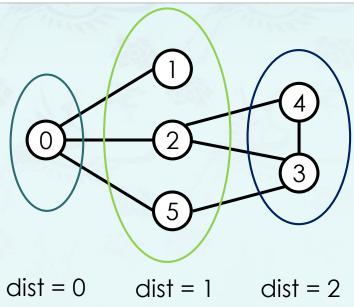
```
// runs BFS for all vertices or all connected components
// It begins with the first vertex 0 at the adjacent list.
// It produces BFS0[], distTo[] & parentBFS[].
void BFS_CCs(graph g) {
  if (empty(g)) return;
  for (int i = 0; i < V(g); i++) {
        g->marked[i] = false;
        g->parentBFS[i] = -1;
        g \rightarrow BFSO[i] = -1;
        g->distTo[i] = -1;
  BFS(g, 0);
                         BFS() with a shortcoming
  q \rightarrow BFSV = \{\};
```



```
// returns the number of edges in a shortest path between v and w
int distTo(graph g, int v, int w) {
  if (empty(g)) return 0;
  if (!connected(g, v, w)) return 0;

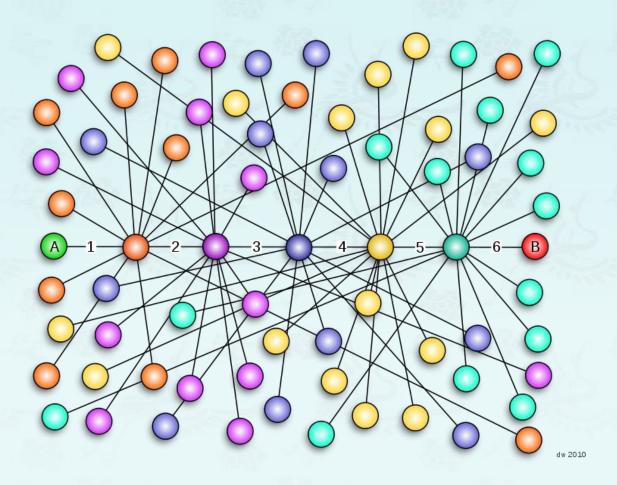
  BFS(g, v);

  cout << "your code here\n";
  return 0;
}</pre>
```



```
// returns a path from v to w using the BFS result or parentBFS[].
// It has to use a stack to retrace the path back to the source.
// Once the client(caller) gets a stack returned,
void BFSpath(graph g, int v, int w, stack<int>& path) {
  if (empty(g)) return;
  BFS(g, v);
                              // g->BFSv updated already.
  path = {};
                              // clear path
  cout << "your code here\n";</pre>
```

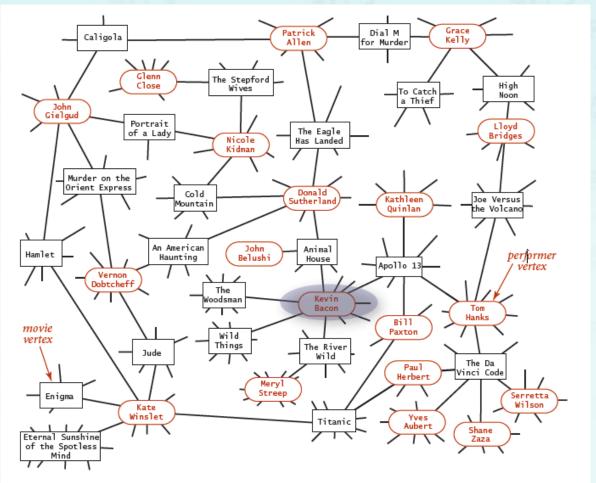
Breadth-first search application: Kevin Bacon numbers



six degrees of separation?

Breadth-first search application: Kevin Bacon numbers

- Include one vertex for each performer and one for each movie.
- Connect a movie to all performers that appear in that movie.
- Compute shortest path from s = Kevin Bacon.





http://www.bbc.co.uk/newsbeat/article/35500398/how-facebook-updated-six-degrees-of-separation-its-now-357

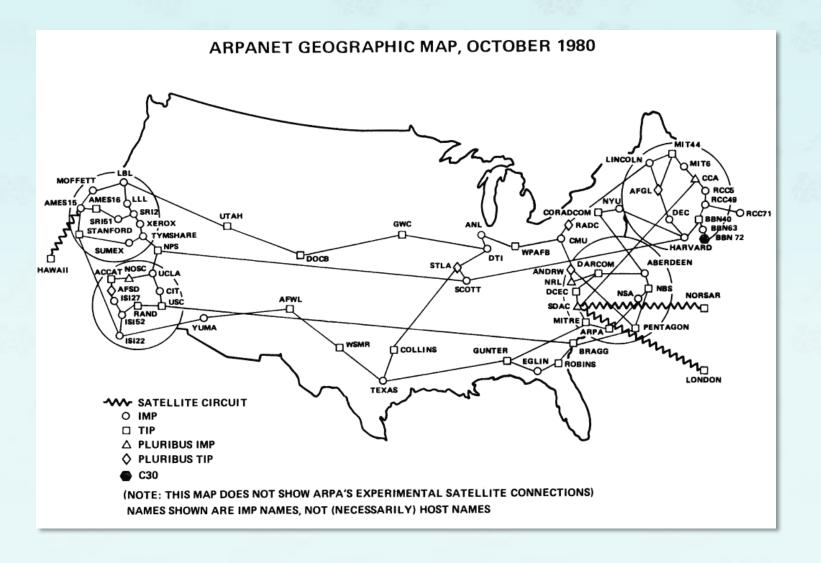


2008: 5.28 \rightarrow 2011: 4.74 \rightarrow 2016.2: 3.57

http://www.bbc.co.uk/newsbeat/article/35500398/how-facebook-updated-six-degrees-of-separation-its-now-357

Breadth-first search application: routing

Fewest number of hops in a communication network.



Data Structures Chapter 7: Graph

- 1. Introduction
 - Terminology, Representation, ADT
- 2. Basic Operations
 - DFS, CC, BFS, Processing
- 3. Digraph and Applications
- 4. Minimum Spanning Tree(MST)