Graph

- **Introduction**
- **Graph API**
- Elementary Graph Operations
  - DFS: Depth first search
  - BFS: Breadth first search
  - CC: Connected components

Major references:
1. Fundamentals of Data Structures by Horowitz, Sahni, Anderson-Freed,
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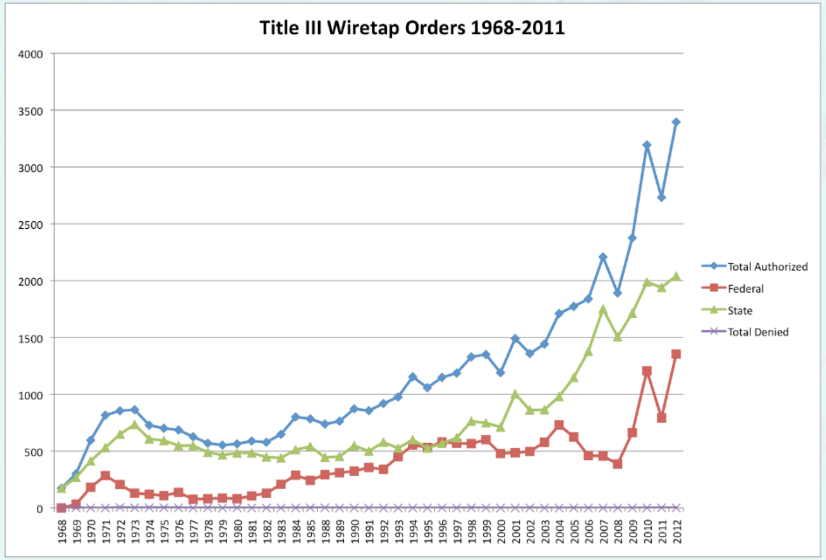
Undirected graphs

Graph: Set of vertices connected pairwise by edges.



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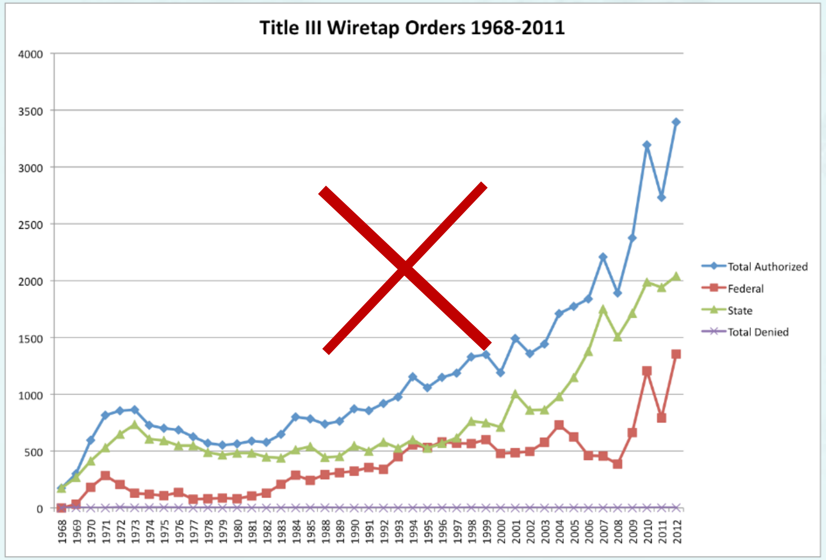

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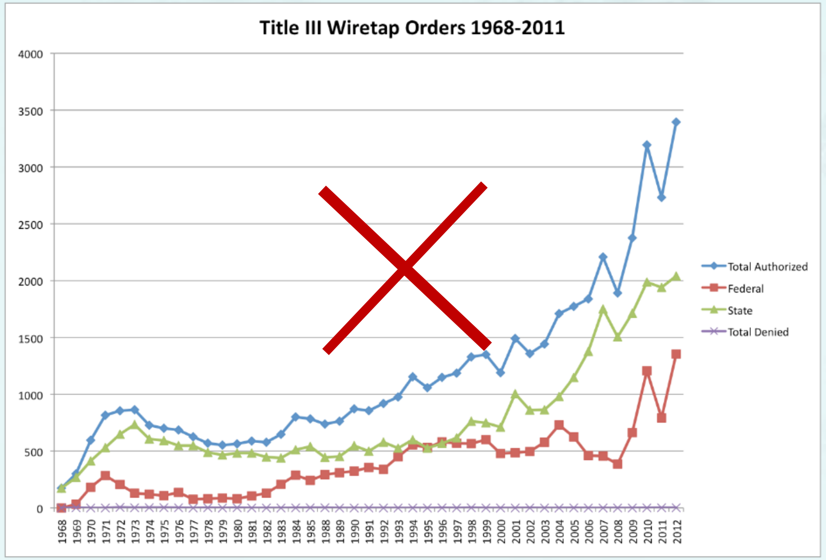
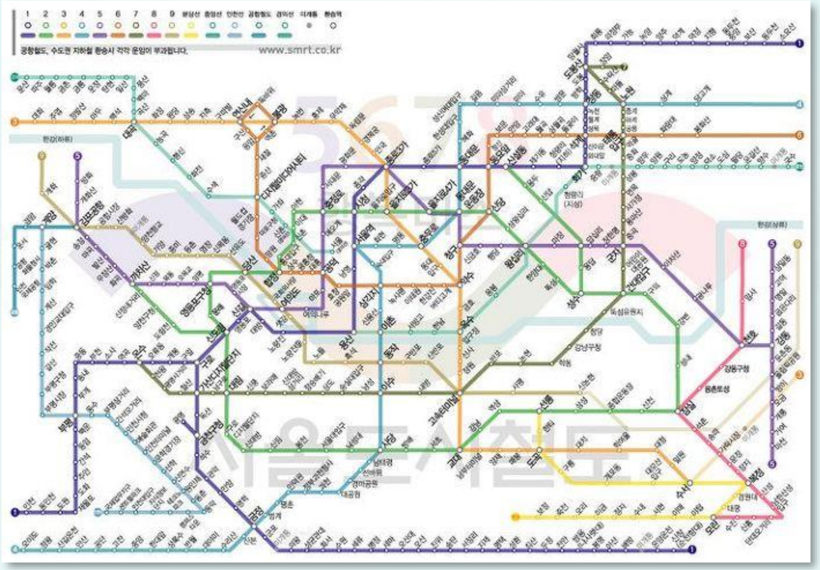


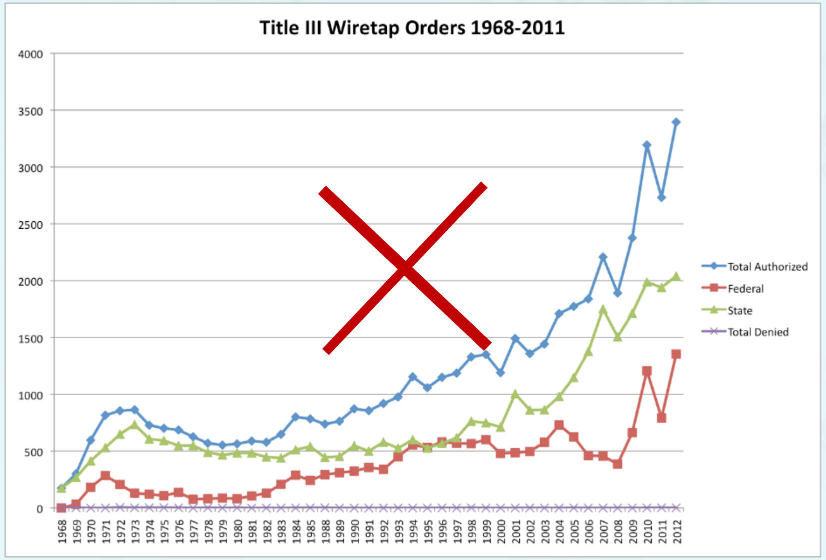


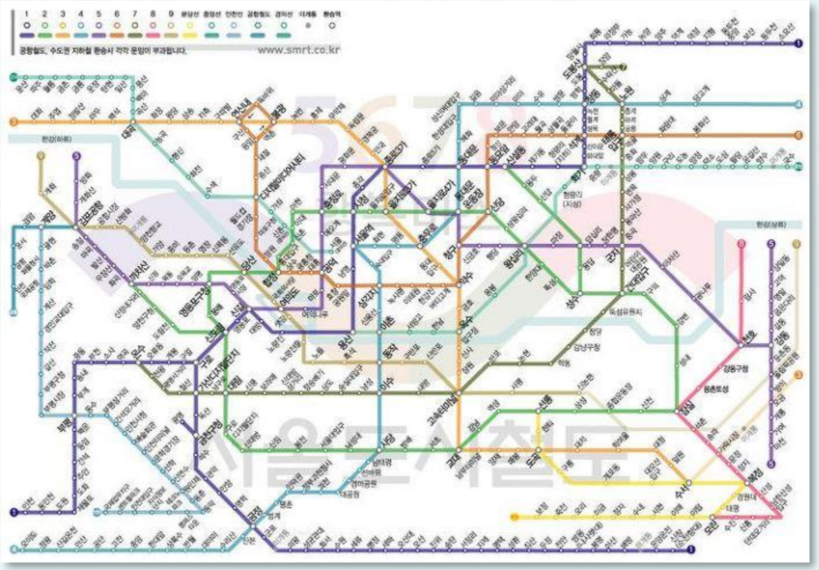
Prof. Youngsup Kim, idebtor@gmail.com, Data Structures, CSEE Dept., Handong Global University

Undirected graphs

Graph: Set of vertices connected pairwise by edges.

- *Why study graph algorithms?*

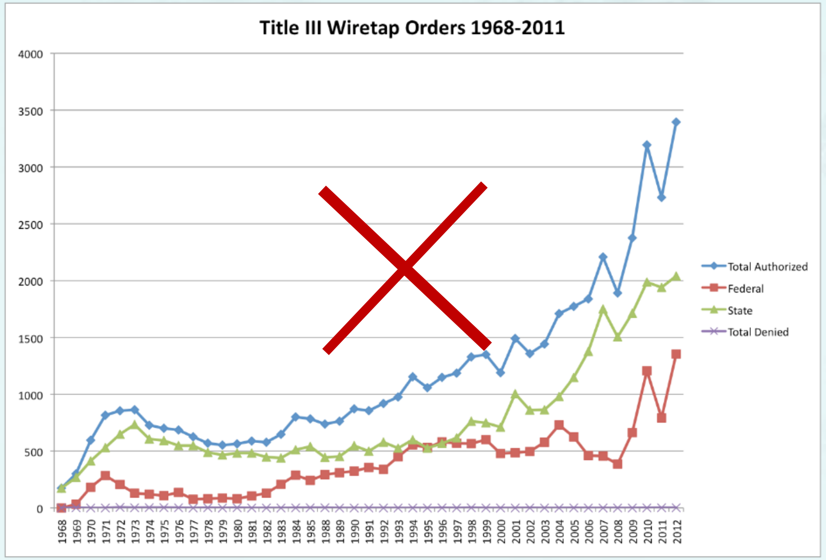


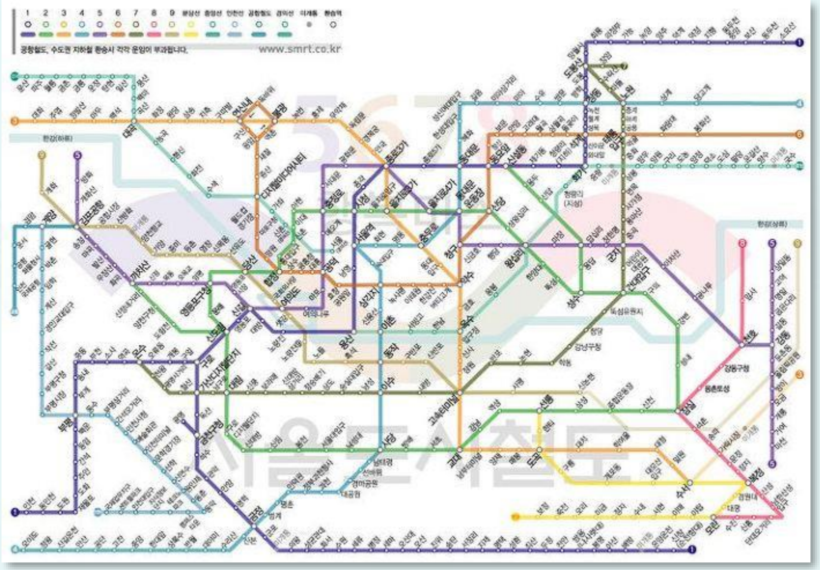


Undirected graphs

Graph: Set of vertices connected pairwise by edges.

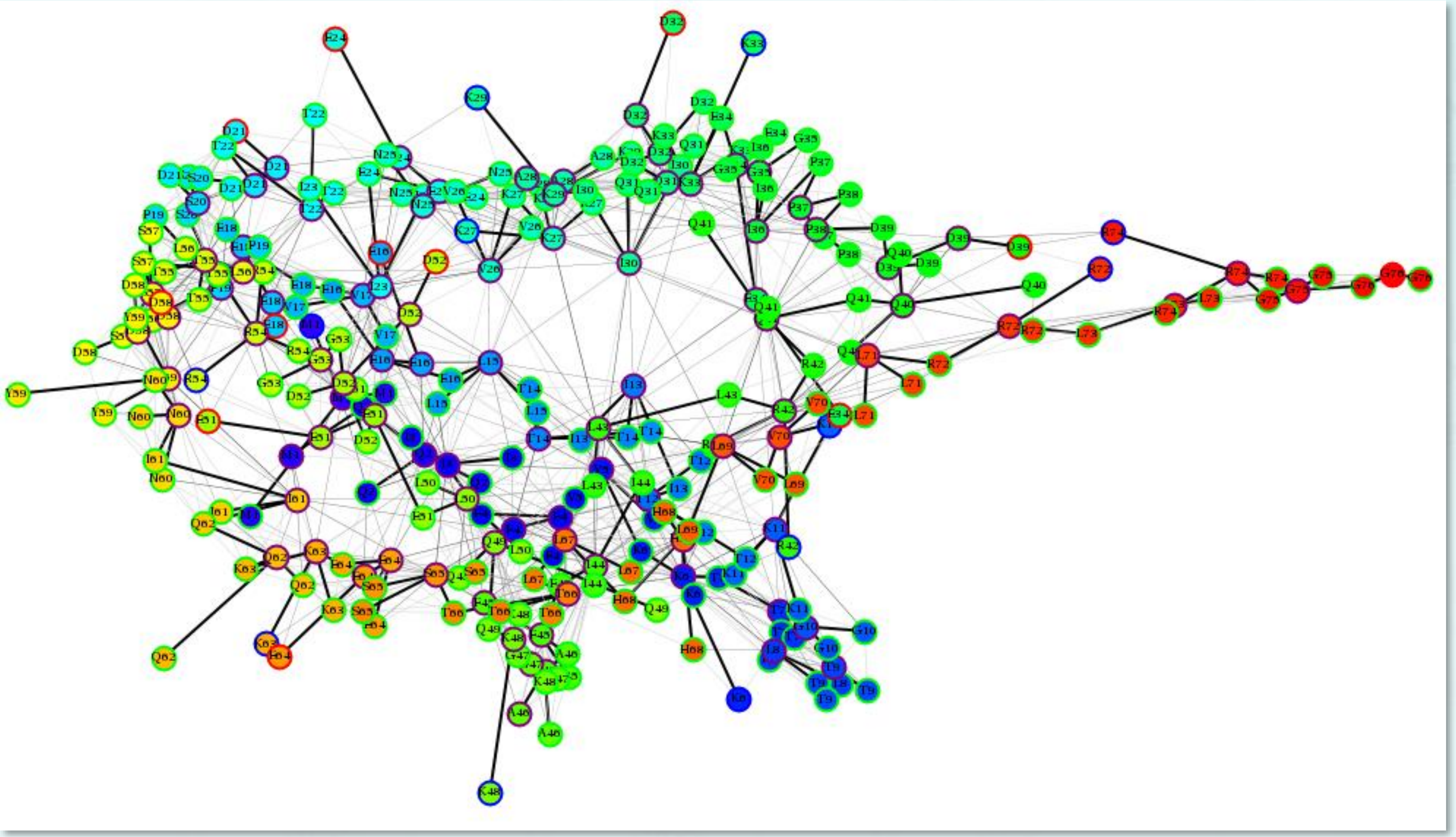
- *Why study graph algorithms?*
  - Thousands of practical applications.
  - Hundreds of graph algorithms known.
  - Interesting and broadly useful abstraction.
  - Challenging branch of computer science and discrete math.





Undirected graphs

Chemical Environments: Protein Graphs



Reference: **Benson NC**, Daggett V (2012) A comparison of methods for the analysis of molecular dynamics simulations. *J. Phys. Chem. B* **116**(29): 8722-31.

http://www.nejm.org/doi/full/10.1056/NEJMsa066082

The Spread of Obesity in a Large Social Network over 32 Years

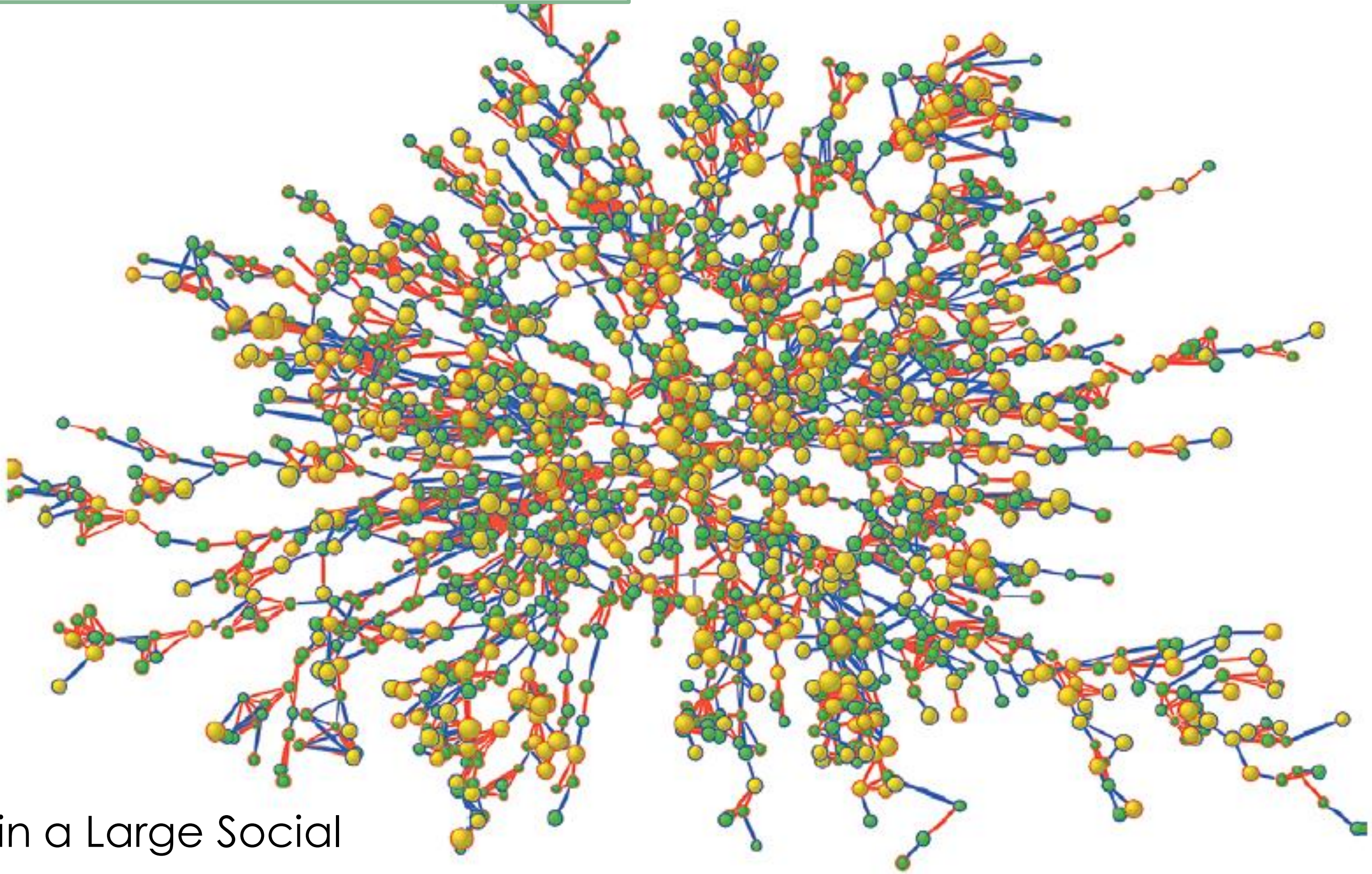


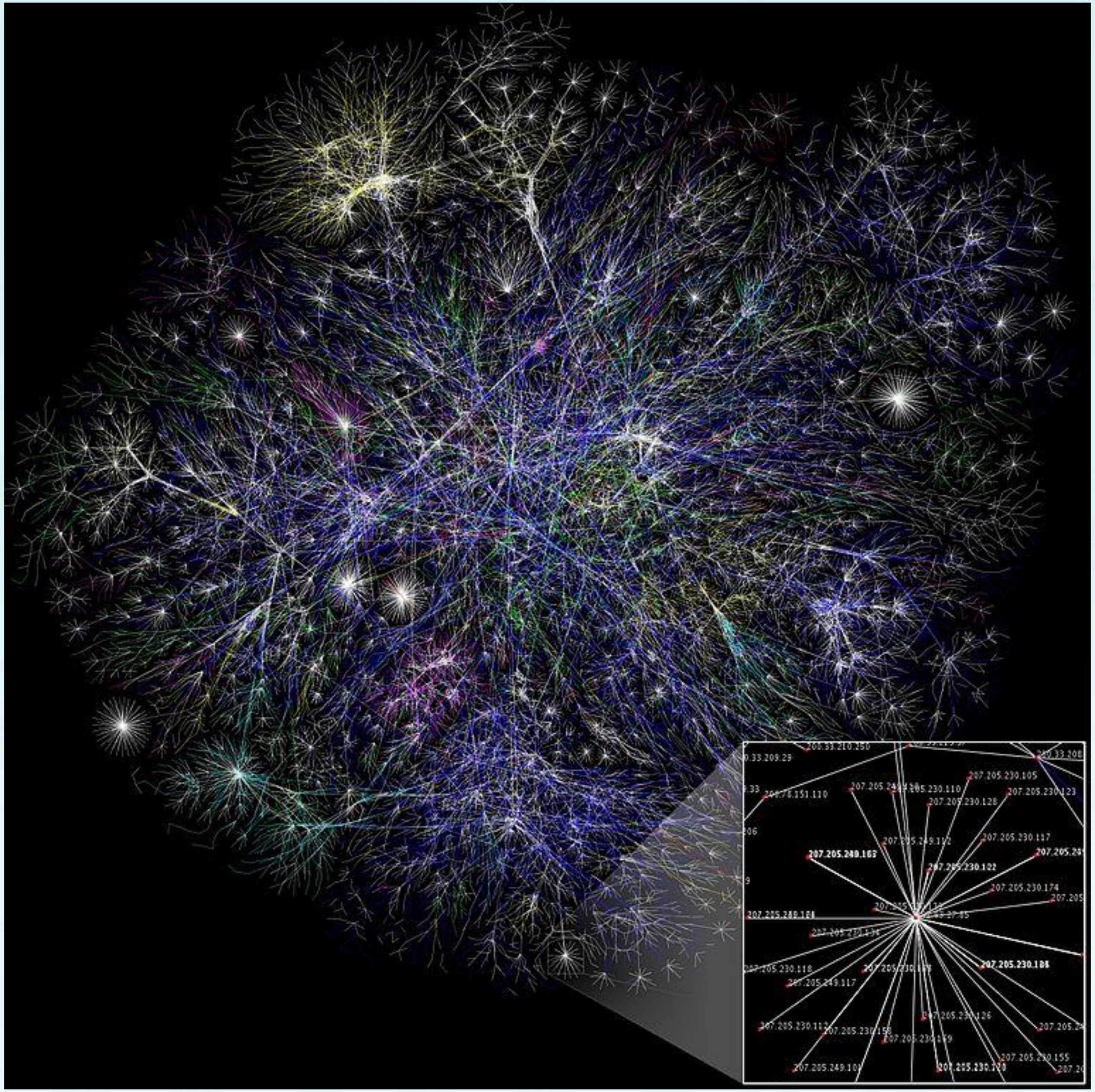
Figure 1. Largest Connected Subcomponent of the Social Network in the Framingham Heart Study in the Year 2000.

Each circle (node) represents one person in the data set. There are 2200 persons in this subcomponent of the social network. Circles with red borders denote women, and circles with blue borders denote men. The size of each circle is proportional to the person's body-mass index. The interior color of the circles indicates the person's obesity status: yellow denotes an obese person (body-mass index, ≥30) and green denotes a nonobese person. The colors of the ties between the nodes indicate the relationship between them: purple denotes a friendship or marital tie and orange denotes a familial tie.

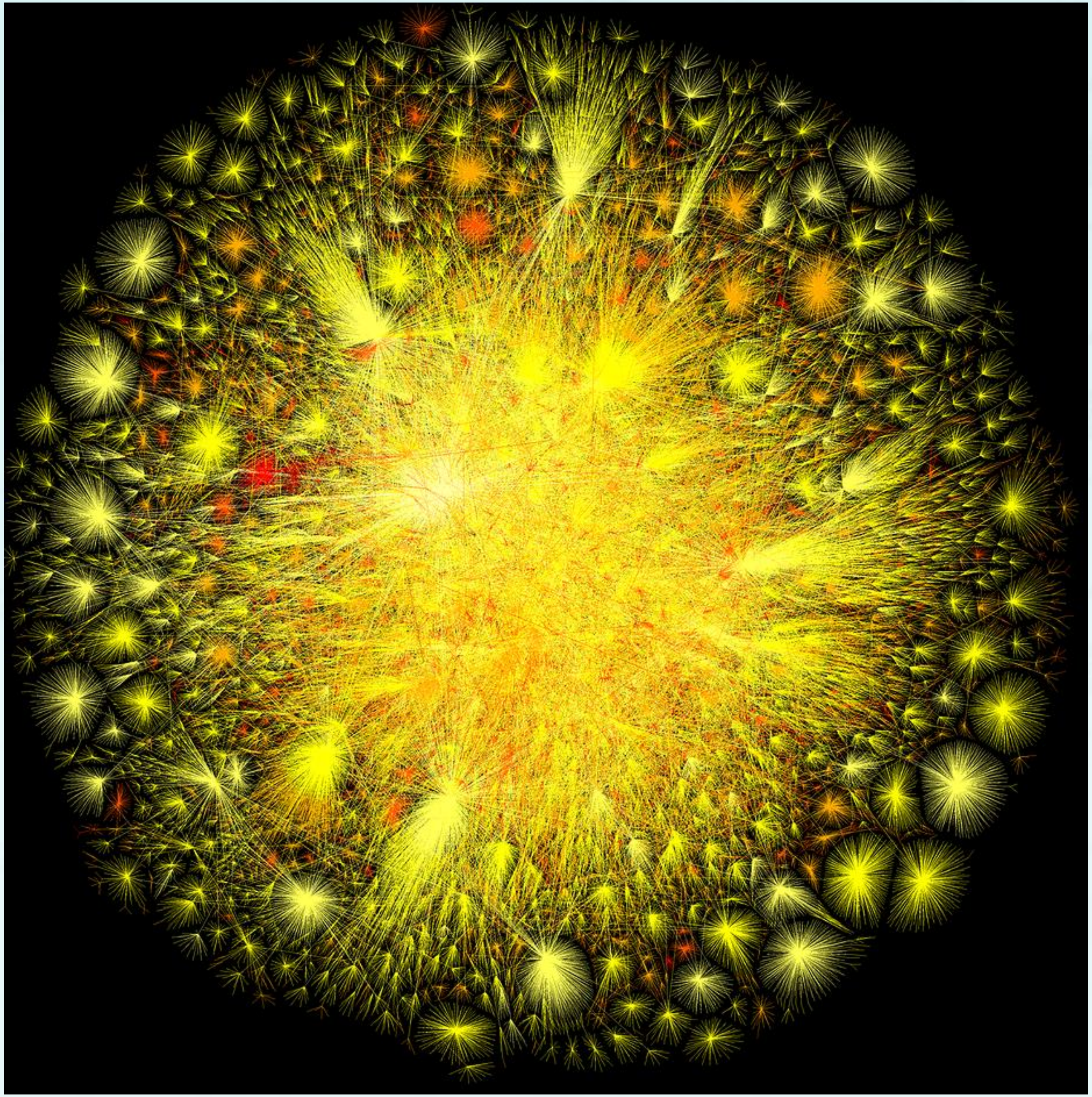
The Spread of Obesity in a Large Social Network over 32 Years

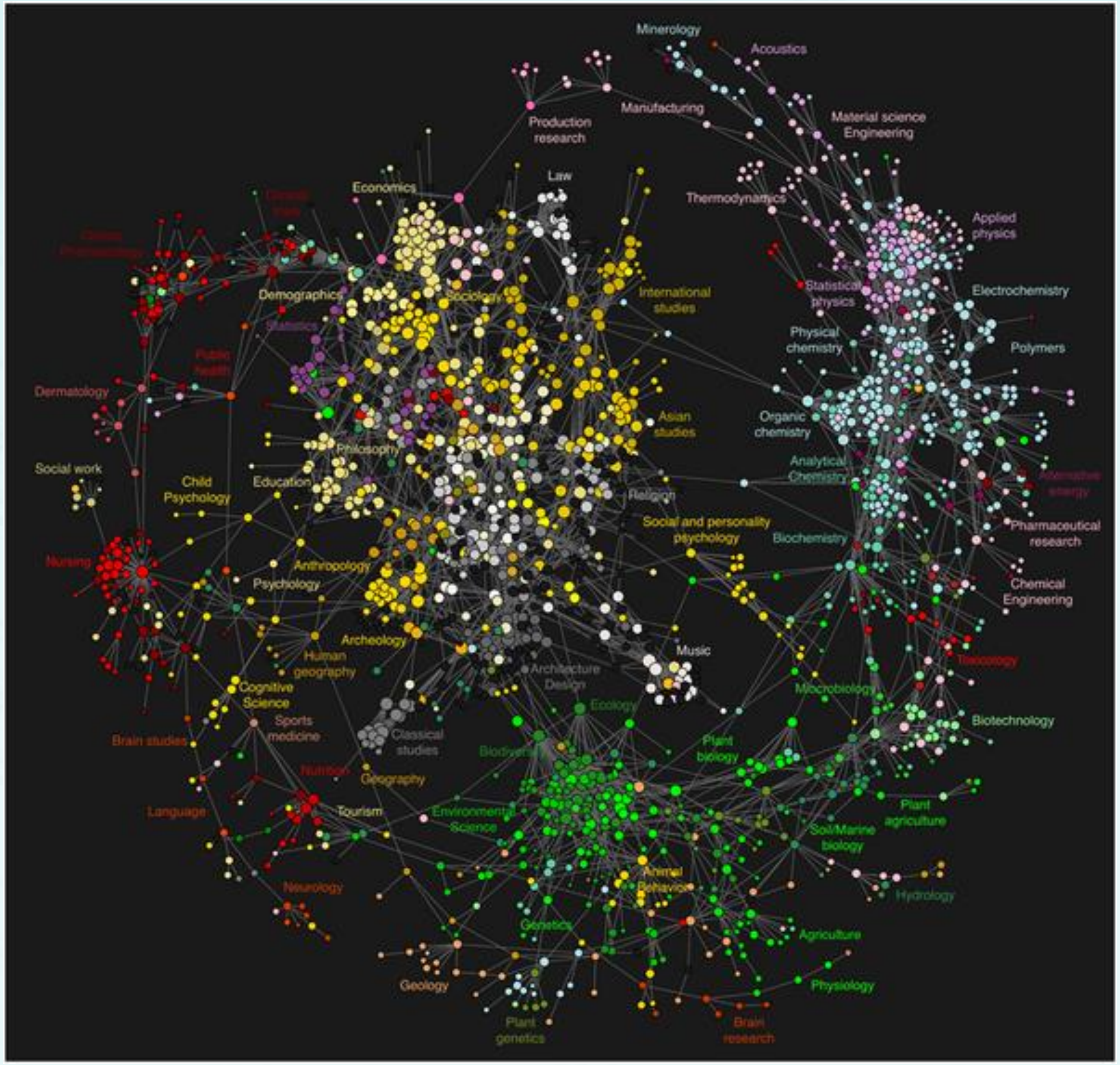
http://www.nejm.org/doi/full/10.1056/NEJMsa066082

http://www.youtube.com/watch?v=pJfq-o5nZQ4



the Opte Project: Visualization of the various routes through a portion of the Internet





Clickstream Data Yields High-Resolution Maps of Science.
http://www.plosone.org/article/info:doi/10.1371/journal.pone.0004803

"Visualizing Friendships" by Paul Butler – an intern at Facebook

Graph Applications

graph	vertex	edge
communication	telephone, computer	fiber optic cable
circuit	gate, register, processor	wire
mechanical	joint	rod, beam, spring
financial	stock, currency	transactions
transportation	street intersection, airport	highway, airway route
internet	class C network	connection
social relationship	person, actor	friendship, movie cast
neural network	neuron	synapse
protein network	protein	protein-protein interaction
molecule	atom	bond

Graph Terminology

Path: Sequence of vertices connected by edges.
Cycle: Path whose first and last vertices are the same.

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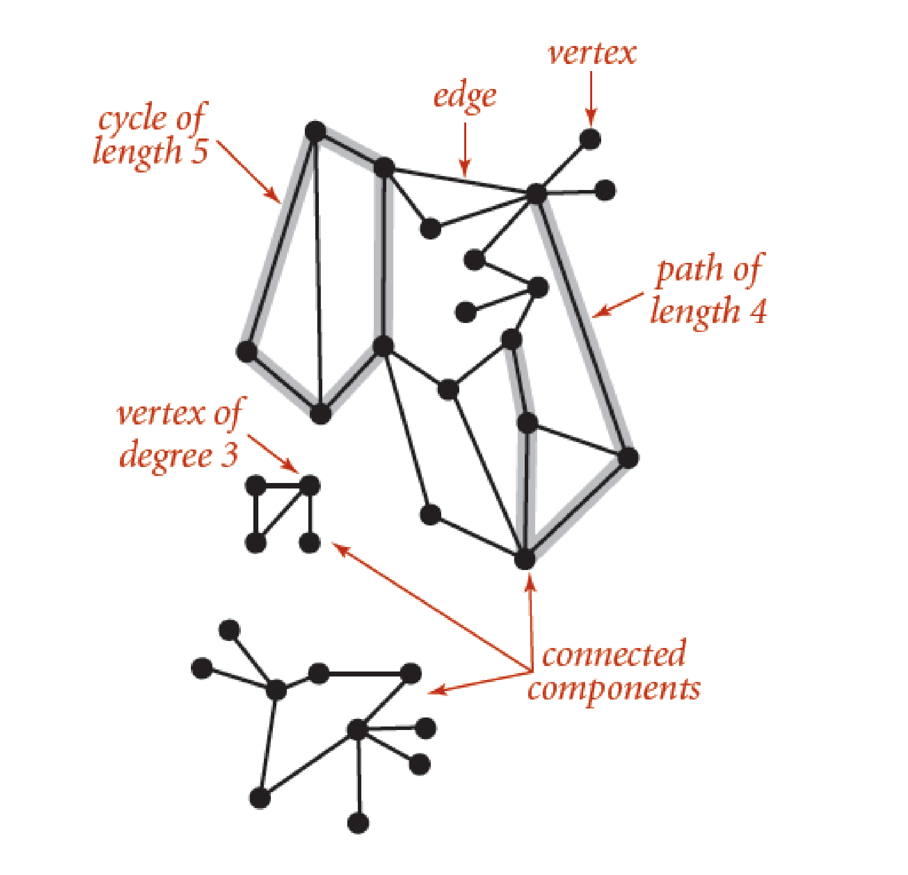

Some graph-processing problems

Path	Is there a path between *s* and *t* ?
Shortest Path	What is the shortest path between *s* and *t* ?

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Euler tour	Is there a cycle that uses each edge exactly once?
Hamilton tour	Is there a cycle that uses each vertex exactly once.

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MST	What is the best way to connect all of the vertices?
BiConnectivity	Is there a vertex whose removal disconnects the graph?

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Graph isomorphism	Do two adjacency lists represent the same graph?

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Challenge	Which of these problems are easy? difficult? intractable?

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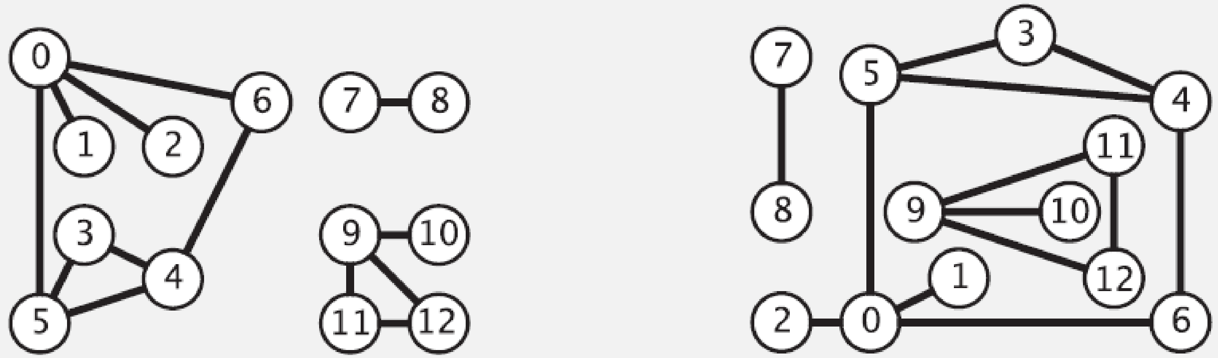
Graph Representation

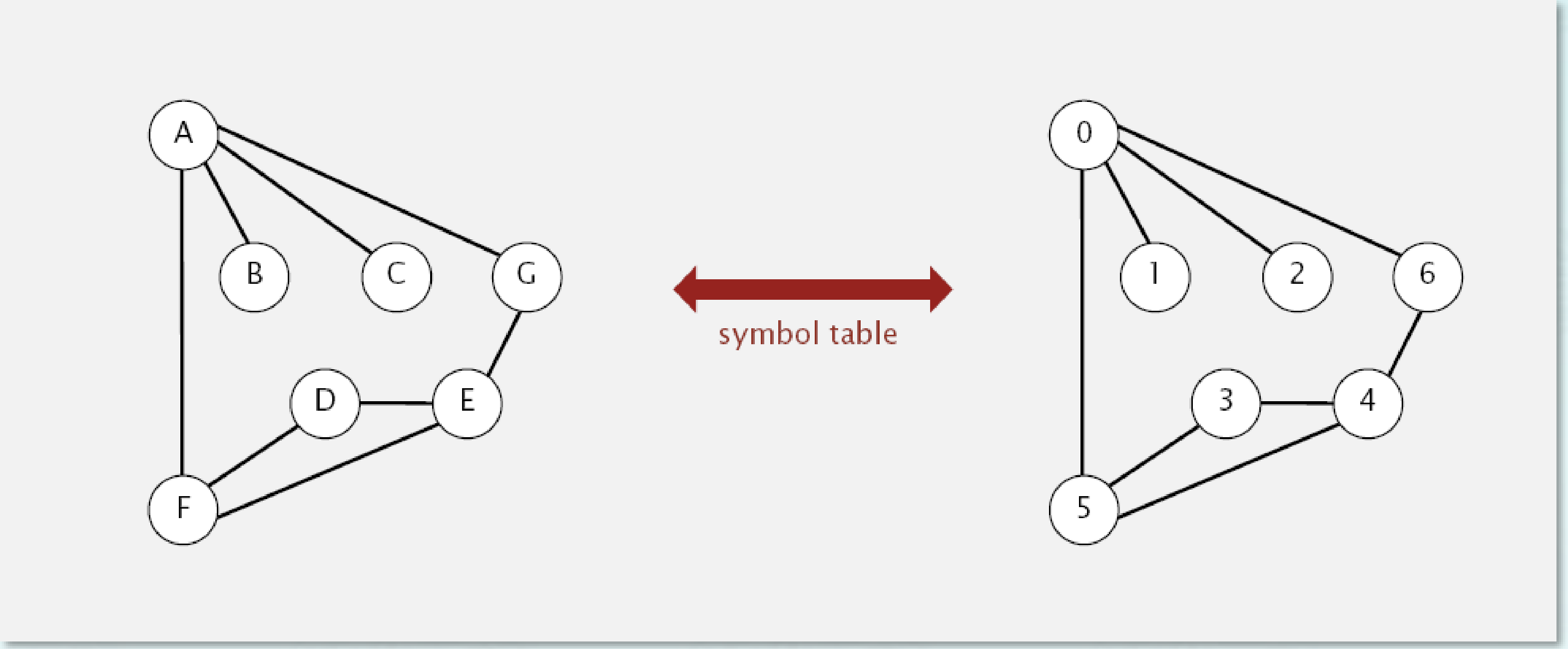
Graph drawing. Provides intuition about the structure of the graph.



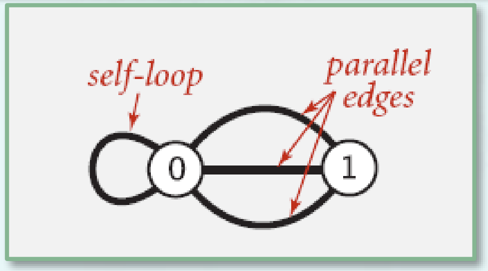
two drawings of the same graph

Vertex representation.

- We use integers between **0** and ***V* – 1**.
- Applications: convert between names and integers with symbol table.



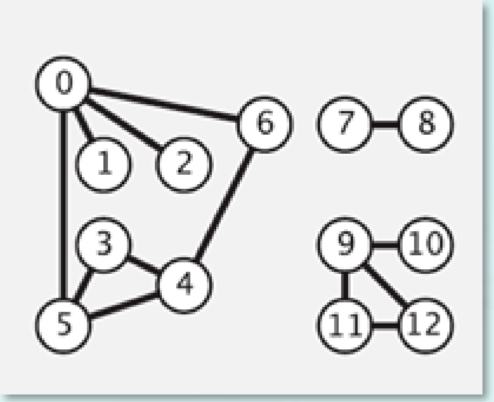
Anomalies.



Graph ADT in Java

public class	Graph	
	Graph(int V)	*create an empty graph with V vertices*
	Graph(char *fname)	*create a graph from input stream*
void	addEdge(int v, int w)	*add an edge v-w*
Iterable<Integer>	adjacent(int V)	*vertices adjacent to v*
int	V()	*number of vertices*
int	E()	*number of edges*
	toString()	*string representation*

Graph Input Format

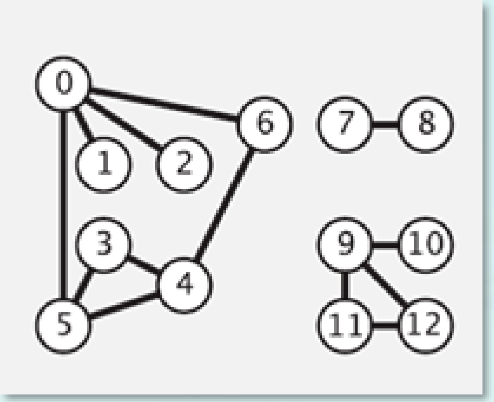


graph3.txt V

```
13
13
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

E

Graph Input Format



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11 12
9 10
0 6
7 8
9 11
5 3
```

E

graph.cpp

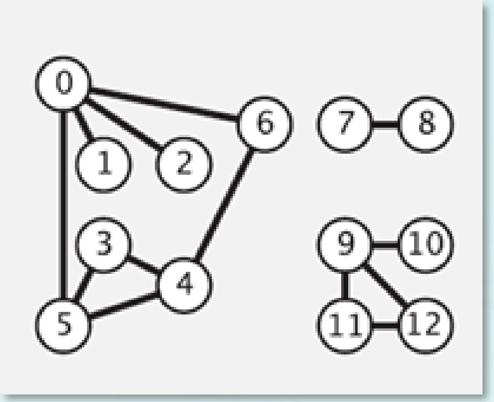
```
Graph g = graph_by_file(argv[1]);

for (int v = 0; v < V(g); ++v) {
  cout << "V[" << v << "]: ";
  for (gnode w = g->adj[v].next; w; w = w->next) {
    cout << w->item << " ";
    (w->next == nullptr) ? (cout << endl) : (cout << "-> ");
  }
}
```

read graph from input stream

print out edge list by vertices

Graph Input Format



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13
0 5
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9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

E

C:\GitHub\nowicx\Debug\graph.exe

```
Adjacency-list:
V[0]: 6 -> 2 -> 1 -> 5
V[1]: 0
V[2]: 0
V[3]: 5 -> 4
V[4]: 5 -> 6 -> 3
V[5]: 3 -> 4 -> 0
V[6]: 0 -> 4
V[7]: 8
V[8]: 7
V[9]: 11 -> 10 -> 12
V[10]: 9
V[11]: 9 -> 12
V[12]: 11 -> 9

 [0] ----------------[6]          [7]--------[8]
  | \------------[2]  |
  |  \--------[1]     |           [9]--------[10]
  |                  /             | \
  |    [3]       ---/              |  \
  |   /   \     /                  |   \
  |  /     \   /                  [11]----[12]
 [5]--------[4]

   vertex[0..12] =   0   1   2   3   4   5   6   7   8   9  10  11  12
    color[0..12] =   0   0   0   0   0   0   0   0   0   0   0   0   0
```

graph.cpp

```
Graph g = graph_by_file(argv[1]);

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    (w->next == nullptr) ? (cout << endl) : (cout << "-> ");
  }
}
```

Graph Coding

Compute the degree of V

```
int degree(graph g, int v) {
  if (!validVertex(g, v)) return -1;
  int deg = 0;
  for (gnode w = g->adj[v].next; w; w = w->next, deg++);
  return deg;
}
```

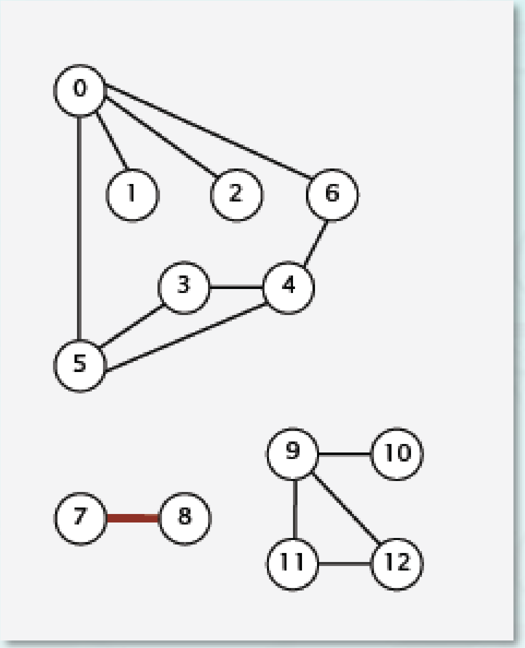
Compute maximum degree

```
int degree(graph g) {
  int max = 0;
  for (int v = 0; v < V(g); ++v) {
    int deg = degree(g, v);
    if (deg > max) max = deg;
  }
  return max;
}
```

Compute average degree

```
double degree_average(graph g) {
  int return 2.0 * E(g) / V(g);
}
```

Graph Coding – edge list



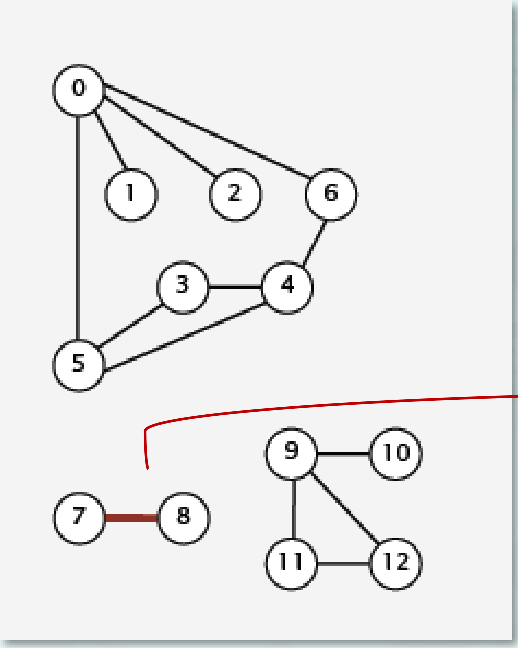
graph3.txt V

```
13
13   E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

1. Edge list
2. Adjacency matrix
3. Adjacency list

Graph Coding – edge list

1. **Maintain a list of the edges (linked list or array)**

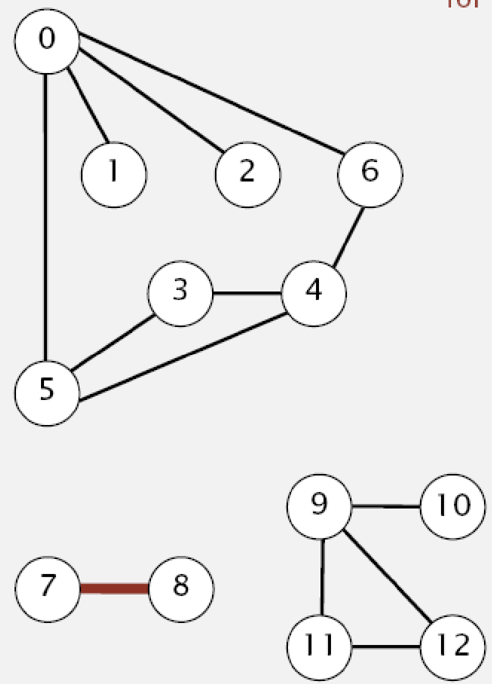


Edge list

```
0 1
0 2
0 5
0 6
3 4
3 5
4 5
4 6
7 8
9 10
9 11
9 12
11 12
```

Graph Coding – Adjacency-matrix 인접행렬

2. Maintain a two-dimensional V-by-V Boolean array;
   for each edge v-w in graph: adj[v][w] = adj[w][v] = true.



two entries
for each edge

Why two entries?

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1	1	0	0	1	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	1	0	0	0	0	0	0	0
4	0	0	0	1	0	1	1	0	0	0	0	0	0
5	1	0	0	1	1	0	0	0	0	0	0	0	0
6	1	0	0	0	1	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1
10	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	1
12	0	0	0	0	0	0	0	0	0	1	0	1	0

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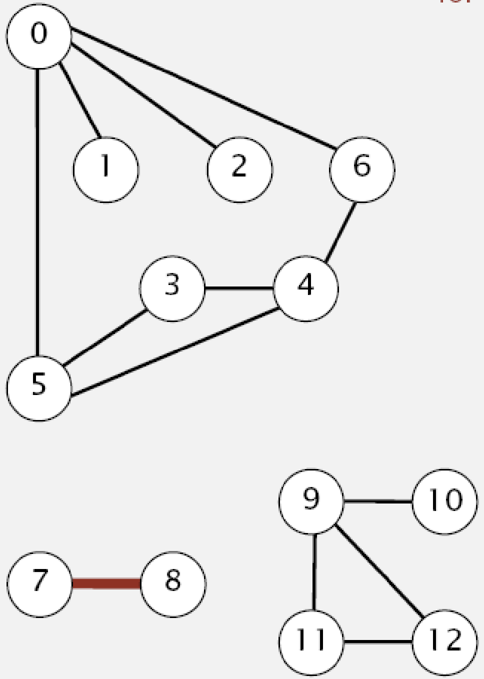


two entries for each edge

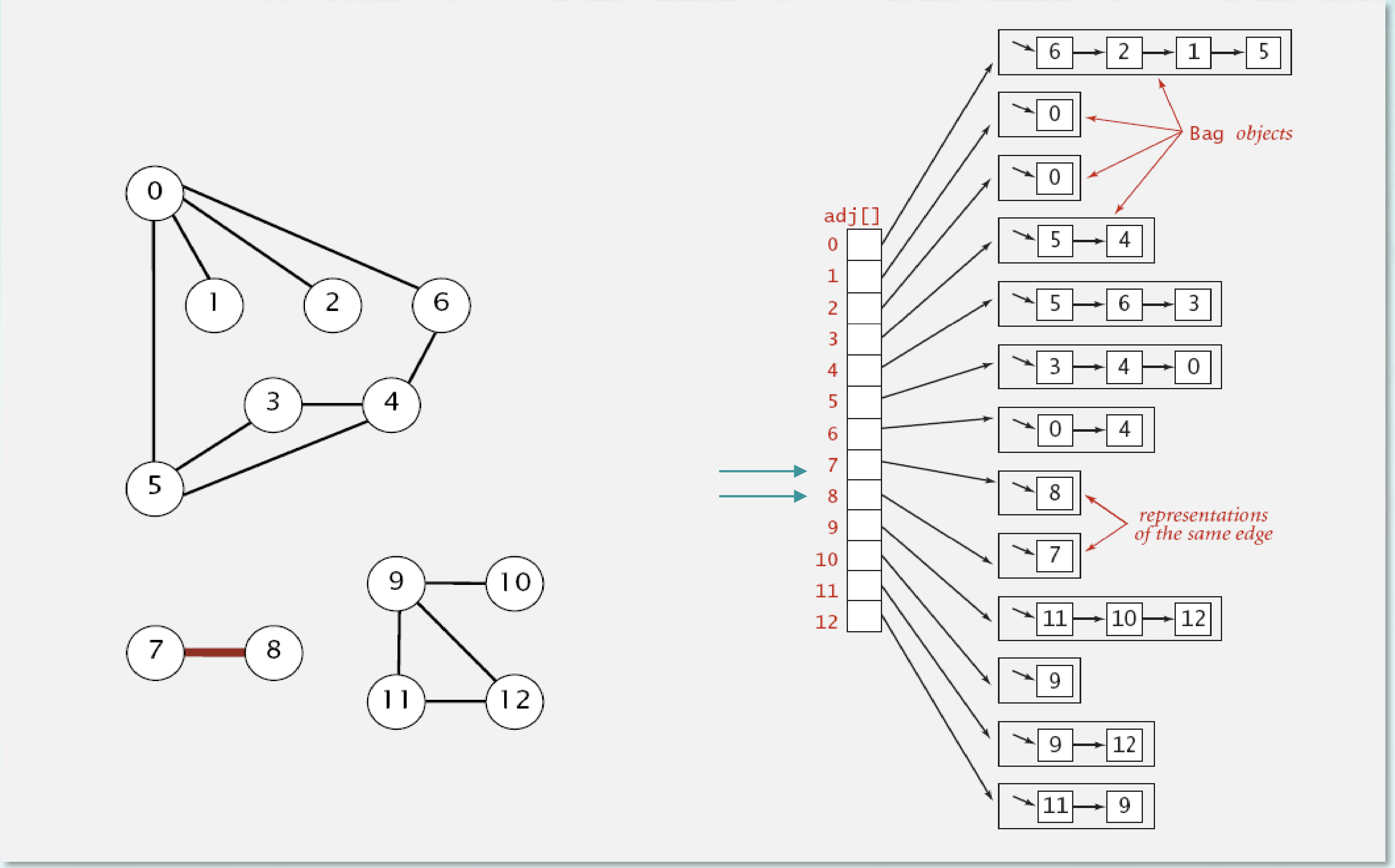
Since it is undirected edge (or graph)

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1	1	0	0	1	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	1	0	0	0	0	0	0	0
4	0	0	0	1	0	1	1	0	0	0	0	0	0
5	1	0	0	1	1	0	0	0	0	0	0	0	0
6	1	0	0	0	1	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1
10	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	1
12	0	0	0	0	0	0	0	0	0	1	0	1	0

Graph Coding – Adjacency list 인접리스트

3. Maintain vertex-index array of lists.

use Bag in Java.
use a linked list in C.



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Graph Coding – graph.h

```
// a structure to represent an adjacency list node
struct Gnode {
  int    item;
  Gnode* next;
  Gnode (int i = 0, Gnode *p = nullptr) {
    item = i;  next = p;
  }
  ~Gnode() {}
};

using gnode = Gnode *;
```

adjacency list nodes (using a linked list)

Graph Coding – graph.cpp

```
struct Graph {
  int V;                // number of vertices in the graph
  int E;                // number of edges in the graph
  gnode adj;            // an array of adjacency lists (or gnode pointers)

  Graph(int v = 0) { // constructs a graph with v vertices
    V = v;
    E = 0;

    // initialize each adjacency list as an empty list;
    for (int  i = 0; i < V; i++) {
                                   ← set each adj list nullptr

                                   ← unused; but may store the size of degree.
    }
  }
  ~Graph() {}
};
using graph = Graph *;
```

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    // initialize each adjacency list as an empty list;
    for (int  i = 0; i < V; i++) {
        g->adj[i].next = nullptr;
        g->adj[i].item = i;
    }
  }
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using graph = Graph *;
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set each adj list nullptr

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Graph Coding

```
// add an edge to an undirected graph

void addEdgeFromTo(graph g, int v, int w) {
  // add an edge from v to w.
  // A new vertex is added to the adjacency list of v.
  // The vertex is added at the beginning

  gnode node = new Gnode(w);
  g->adj[v].next = node;
  g->E++;
}

// add an edge to an undirected graph
void addEdge(graph g, int v, int w) {
  addEdgeFromTo(g, v, w); // add an edge from v to w.
  addEdgeFromTo(g, w, v); // if graph is undirected, add both
}
```

With a bug

instantiate a node w insert it
at **the front of** adjacency list[v]

add an edge for undirected graph

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  gnode node = new Gnode(w, g->adj[v].next);
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instantiate a node w insert it at **the front of** adjacency list[v]

add an edge for undirected graph

Graph Coding – Adjacency-matrix 인접행렬

In practice: Use adjacency-lists representation.

- Algorithms based on iterating over vertices adjacent to v.
- Real-world graphs tend to be sparse.

huge number of vertices,
small average vertex degree

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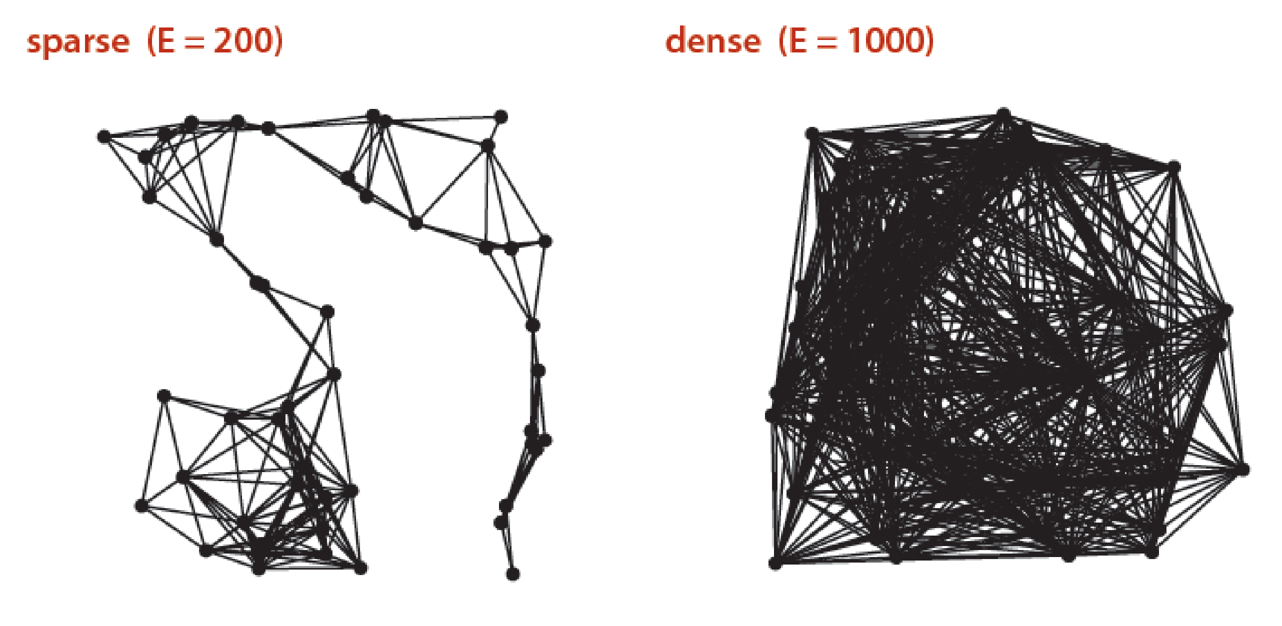

Two graphs (V = 50)

Graph Coding – Adjacency-matrix 인접행렬

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huge number of vertices,
small average vertex degree

representation	space	add edge	edge between v and w?	iterate over vertices adjacent to v?
list of edges	E	1	E	E
adjacency matrix	V^2	1		
adjacency lists	$E + V$	1		

Graph Coding – Adjacency-matrix 인접행렬

In practice: Use adjacency-lists representation.

- Algorithms based on iterating over vertices adjacent to v.
- Real-world graphs tend to be sparse.

huge number of vertices,
small average vertex degree

representation	space	add edge	edge between v and w?	iterate over vertices adjacent to v?
list of edges	E	1	E	E
adjacency matrix	V^2	1	1	V
adjacency lists	$E + V$	1	$degree(v)$	$degree(v)$

Graph

- **Introduction**
- **Graph API**
- Elementary Graph Operations
    - DFS: Depth first search
    - BFS: Breadth first search
    - CC: Connected components

Major references:

1. Fundamentals of Data Structures by Horowitz, Sahni, Anderson-Freed,
2. Algorithms 4th edition - Part 1 & Part 2 by Robert Sedgewick and Kevin Wayne
3. Wikipedia and many resources available from internet