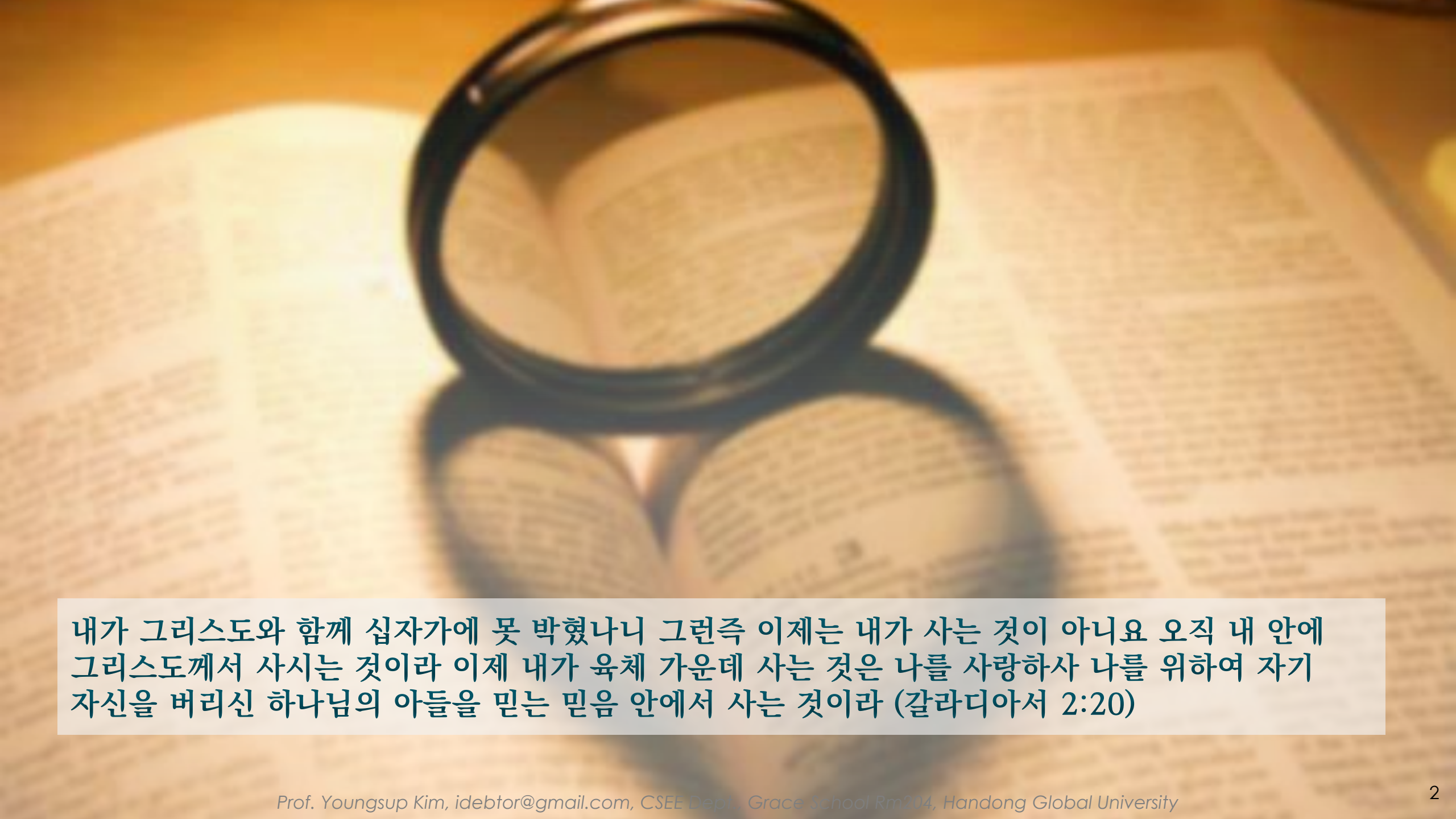


A pair of glasses with a dark frame and light-colored lenses is resting on a piece of white paper. The background is a soft, out-of-focus yellow and orange gradient.

Data Structures

Chapter 5: Heap and Priority Queue

1. Heap & Priority Queue
- 2. Heapsort**
3. Heap & PQ Coding



내가 그리스도와 함께 십자가에 못 박혔나니 그런즉 이제는 내가 사는 것이 아니요 오직 내 안에 그리스도께서 사시는 것이라 이제 내가 육체 가운데 사는 것은 나를 사랑하사 나를 위하여 자기 자신을 버리신 하나님의 아들을 믿는 믿음 안에서 사는 것이라 (갈라디아서 2:20)



Data Structures

Chapter 5: Heap and Priority Queue

1. Heap & Priority Queue
2. **Heapsort**
 - Heap Construction – Heapify
 - Heapsort
 - Time Complexity
3. Heap & PQ Coding

Heapsort

Basic plan for in-place sort

- **1st Pass:** Build maxheap with all **N** keys.
- **2nd Pass:** Repeatedly remove the maximum key.

unsorted	C	H	R	I	S	T	A	L	O	N	E
sorted	A	C	E	H	I	L	N	O	R	S	T
	1	2	3	4	5	6	7	8	9	10	11

Heapsort

Basic plan for in-place sort

- **1st Pass:** Build maxheap with all **N** keys.
- **2nd Pass:** Repeatedly remove the maximum key.

an array of **N** keys
in arbitrary order

1st Pass



build a maxheap
(in place)

2nd Pass



sorted in place

unsorted

C H R I S T A L O N E

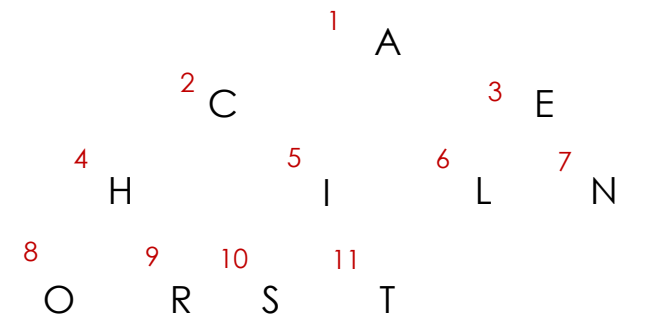
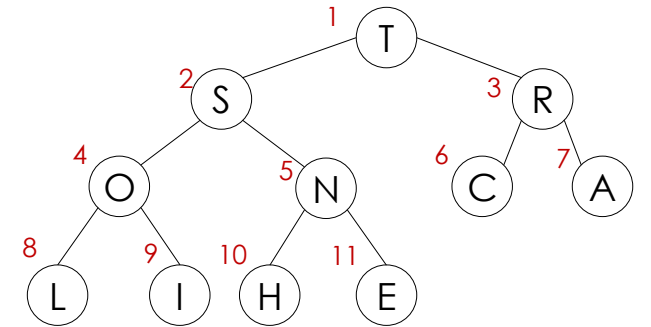
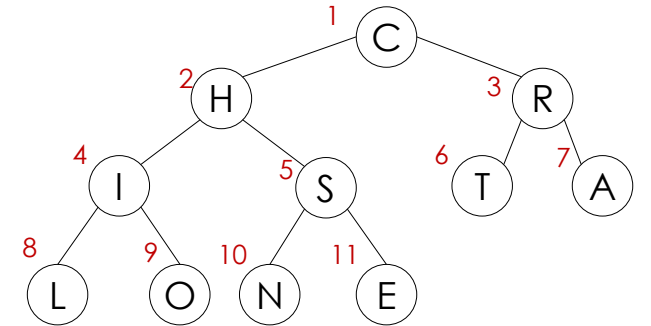
heap-ordered

C H R I S T A L O N E

sorted

A C E H I L N O R S T

1 2 3 4 5 6 7 8 9 10 11



Heapsort

Basic plan for in-place sort

- **1st Pass:** Build maxheap with all **N** keys.
- **2nd Pass:** Repeatedly remove the maximum key.

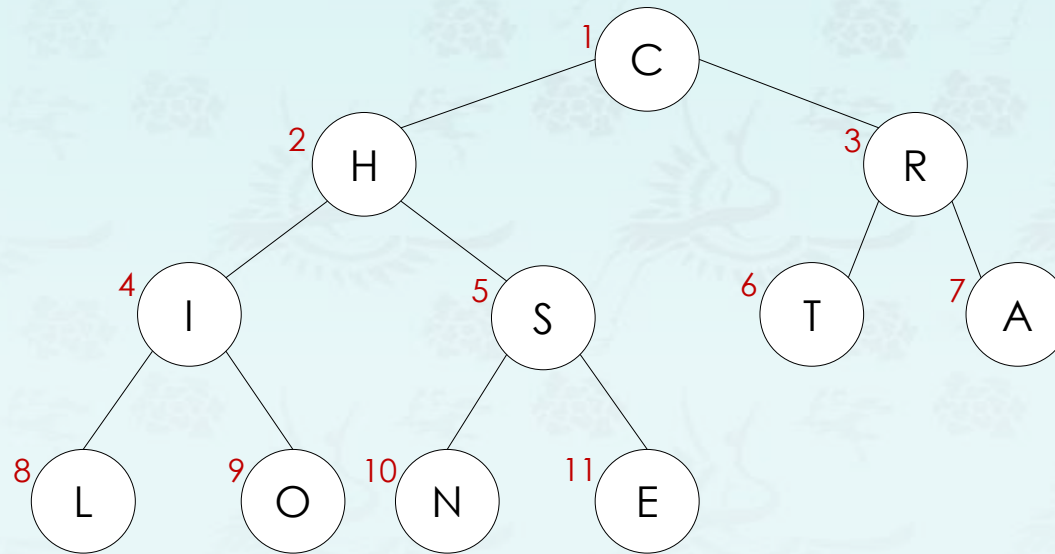
	C	H	R	I	S	T	A	L	O	N	E
0	1	2	3	4	5	6	7	8	9	10	11
											N

Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order



	C	H	R	I	S	T	A	L	O	N	E
0	1	2	3	4	5	6	7	8	9	10	11
											N

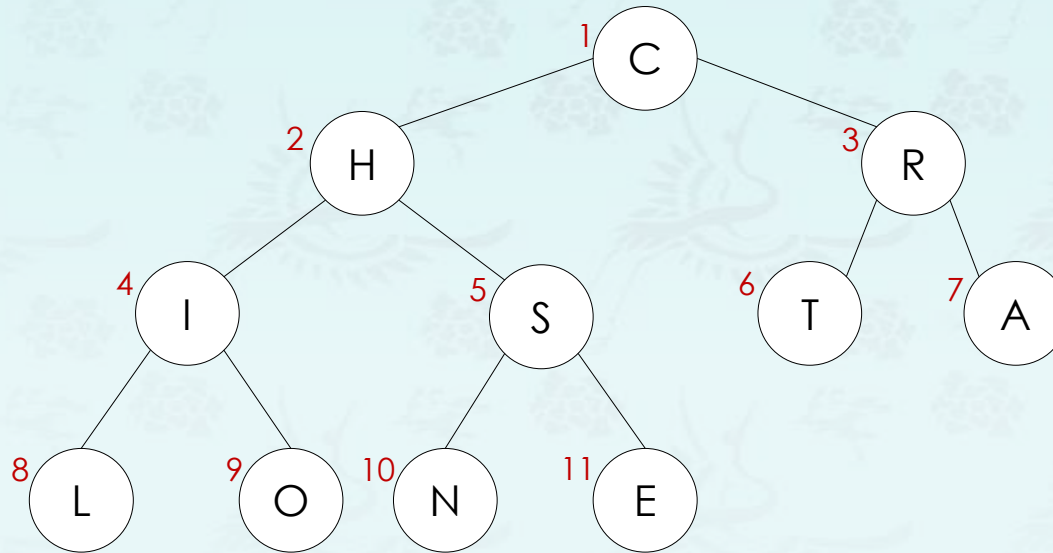
Heapsort

Where should we start from?

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order



	C	H	R	I	S	T	A	L	O	N	E
0	1	2	3	4	5	6	7	8	9	10	11
											N

Heapsort

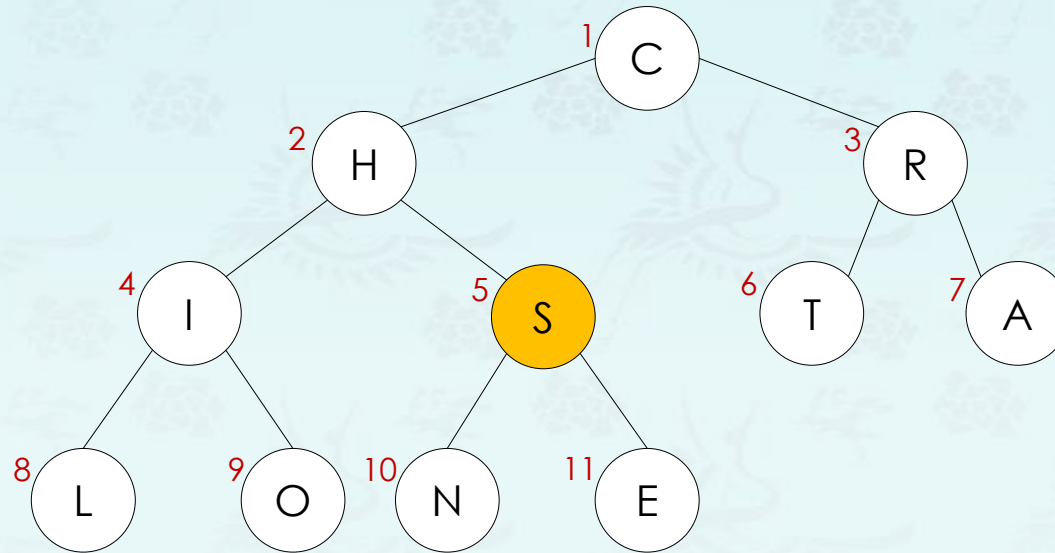
1st Pass: Heap construction (heapify)

Where should we start from?

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

- root, leaf, or any particular node?
- Since leaf nodes are already 1-node heap, go up and to the left node which is not a leaf which is called _____.
- How do you locate the **last internal node**?

array in arbitrary order



Heapsort

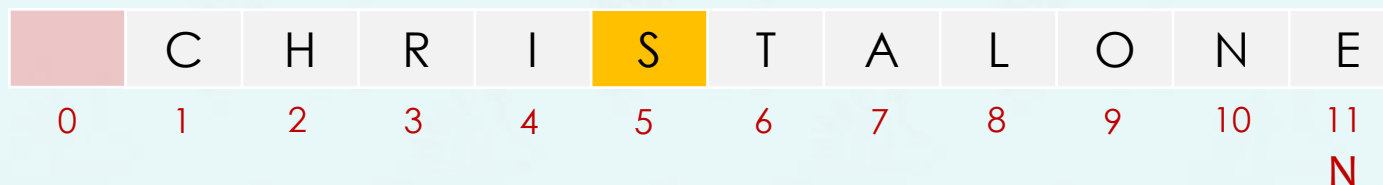
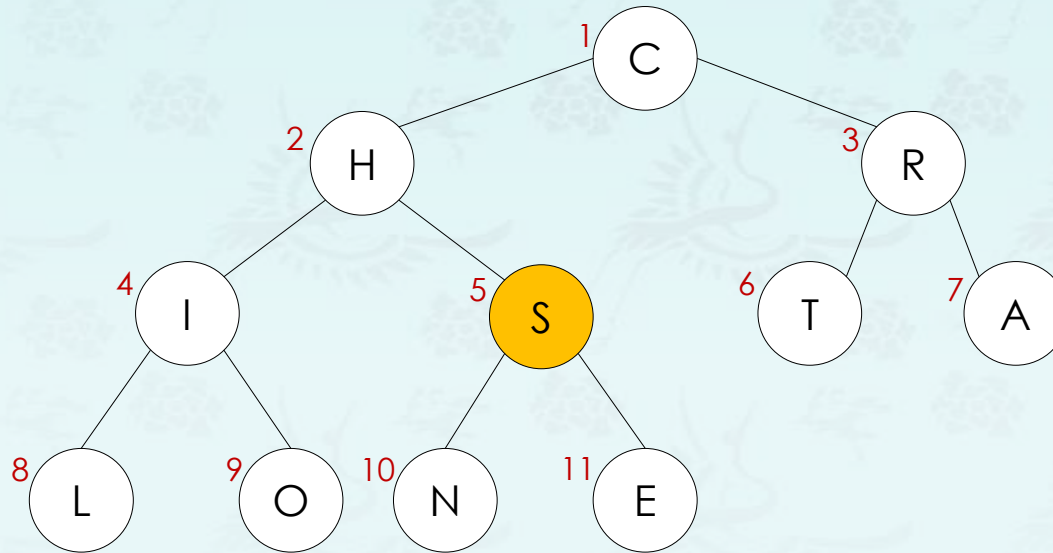
1st Pass: Heap construction (heapify)

Where should we start from?

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- Assume array entries are indexed from 1 to N.

- root, leaf, or any particular node?
- Since leaf nodes are already 1-node heap, go up and to the left node which is not a leaf which is called _____.
- How do you locate the **last internal node**?
 $\text{floor}(N / 2)$

array in arbitrary order



Heapsort

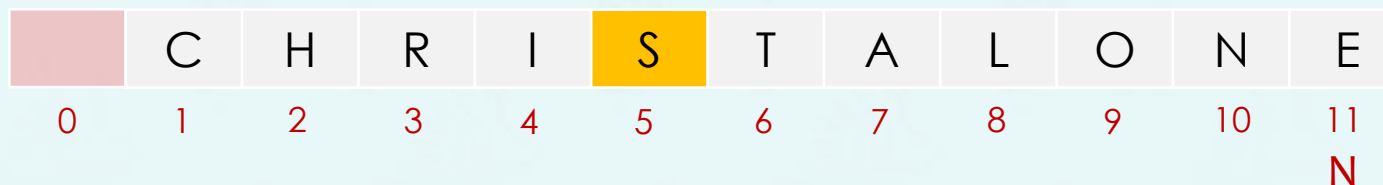
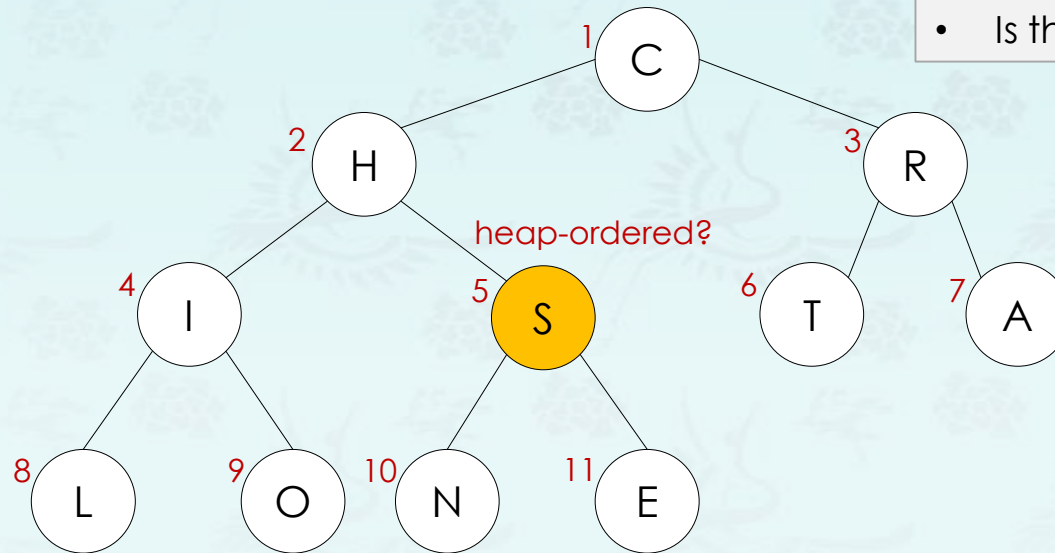
1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

Where should we start from?

- root, leaf, or any particular node?
- Since leaf nodes are already 1-node heap, go up and to the left node which is not a leaf which is called _____.
- How do you locate the **last internal node**? **$\text{floor}(N / 2)$**
- Is this 3-node heap at 5 heap-ordered?



Heapsort

1st Pass: Heap construction (heapify)

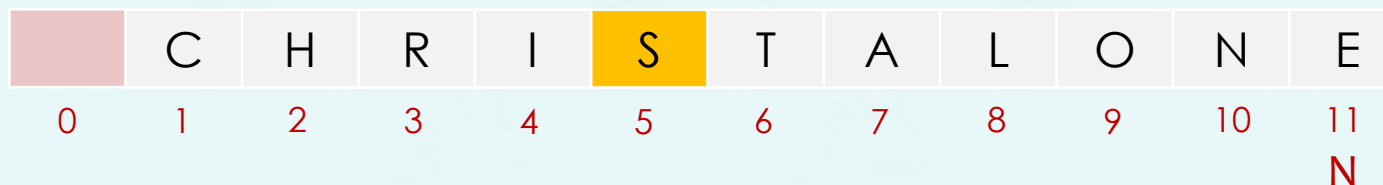
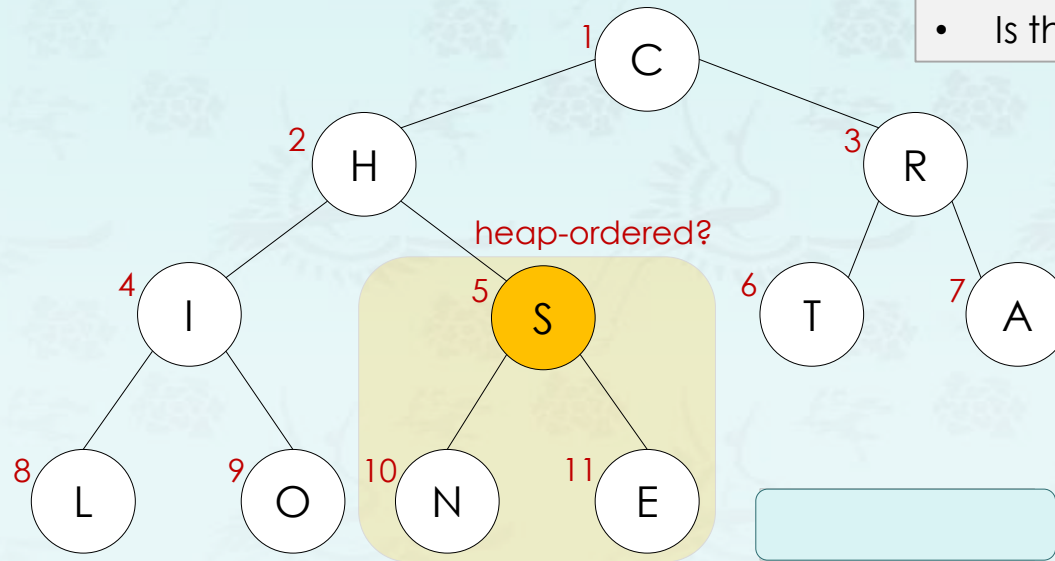
- Build max heap using bottom-up method.
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array in arbitrary order

Where should we start from?

- root, leaf, or any particular node?
- Since leaf nodes are already 1-node heap, go up and to the left node which is not a leaf which is called _____.
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- Is this 3-node heap at 5 heap-ordered?

Recall: $\text{root} \geq \max(\text{left}, \text{right})$



Heapsort

1st Pass: Heap construction (heapify)

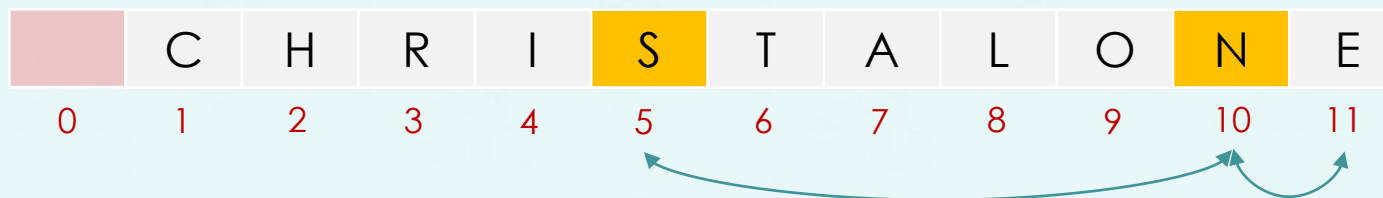
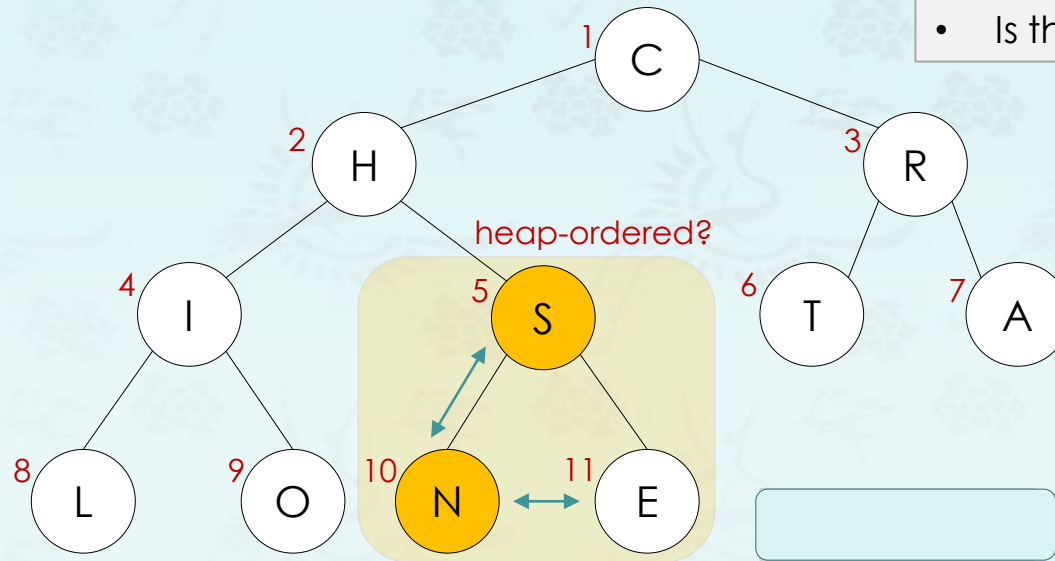
Where should we start from?

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

- root, leaf, or any particular node?
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- How do you locate the **last internal node**?
 $\text{floor}(N / 2)$
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Recall: $\text{root} \geq \max(\text{left}, \text{right})$



Heapsort

1st Pass: Heap construction (heapify)

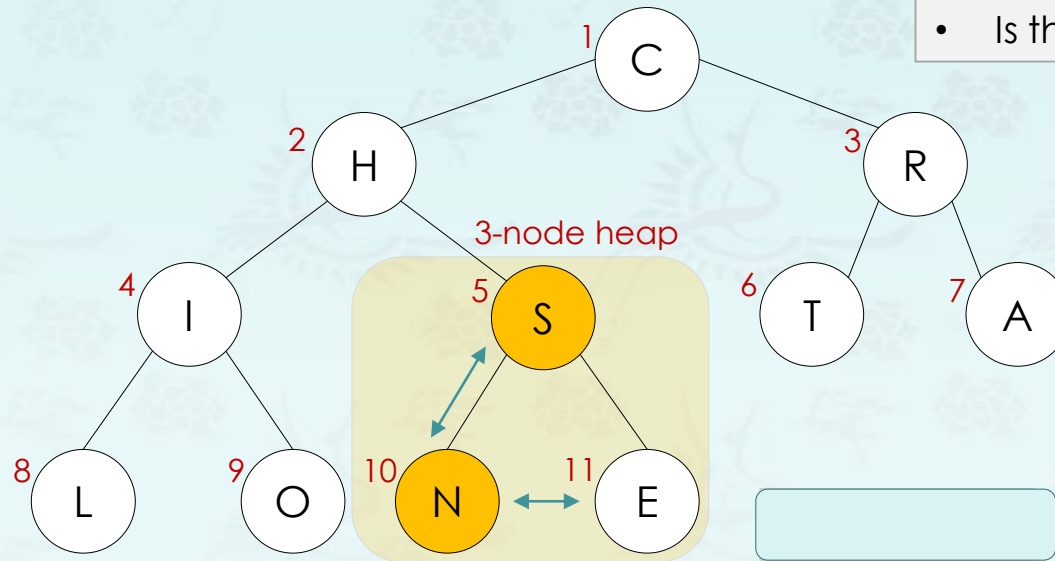
Where should we start from?

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

- root, leaf, or any particular node?
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 $\text{floor}(N / 2)$
- Is this 3-node heap at 5 heap-ordered?

Recall: $\text{root} \geq \max(\text{left}, \text{right})$



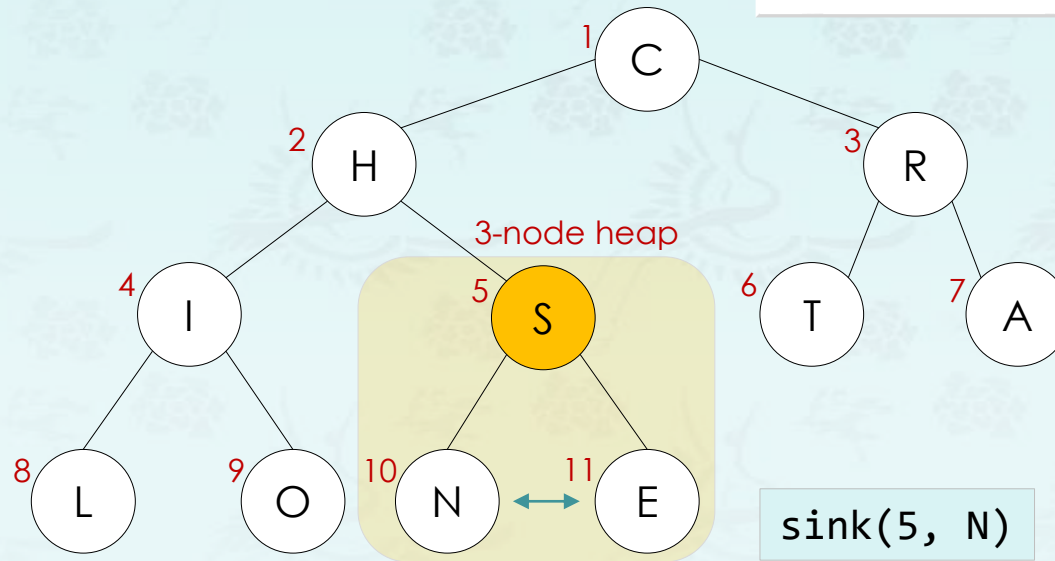
Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

```
void sink(heap h, int k) {  
    while (2 * k <= h->N) {  
        int j = 2 * k; k=5, j=10  
        if (j < h->N && less(h, j, j + 1)) j++;  
        if (!less(h, k, j)) break; k=5, j=10  
        swap(h, k, j);  
        k = j;  
    }  
}
```



Heapsort

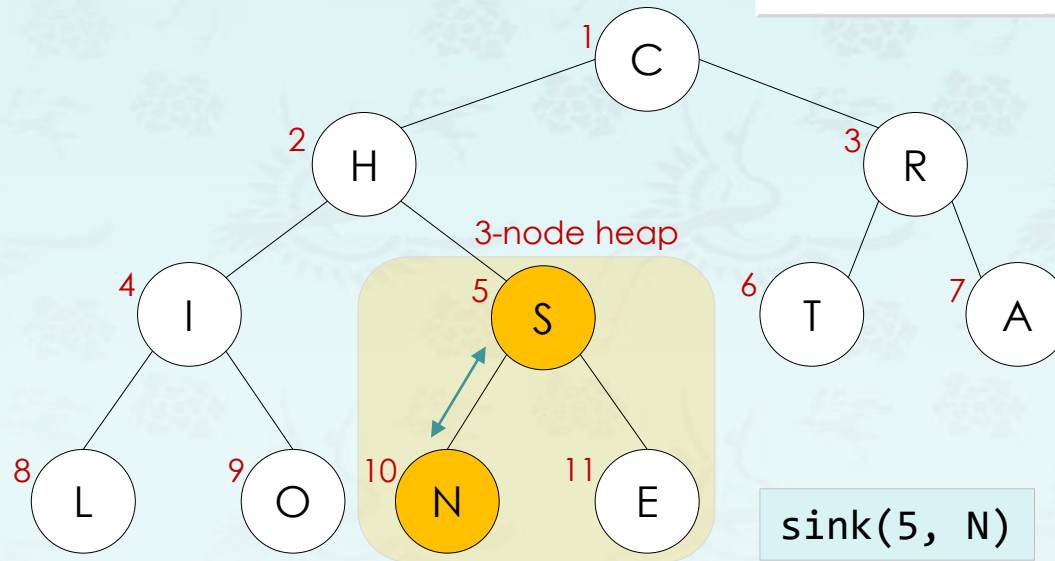
1st Pass: Heap construction (heapify)

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array in arbitrary order

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        if (!less(h, k, j)) break;  
        swap(h, k, j);  
        k = j;  
    }  
}
```

k=5, j=10
k=5, j=10
break

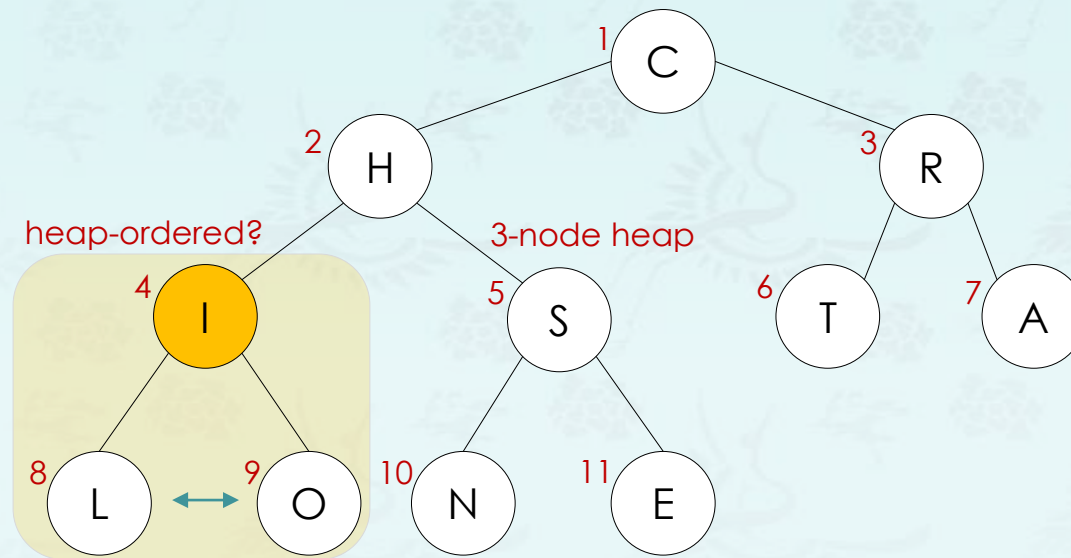


Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
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array in arbitrary order

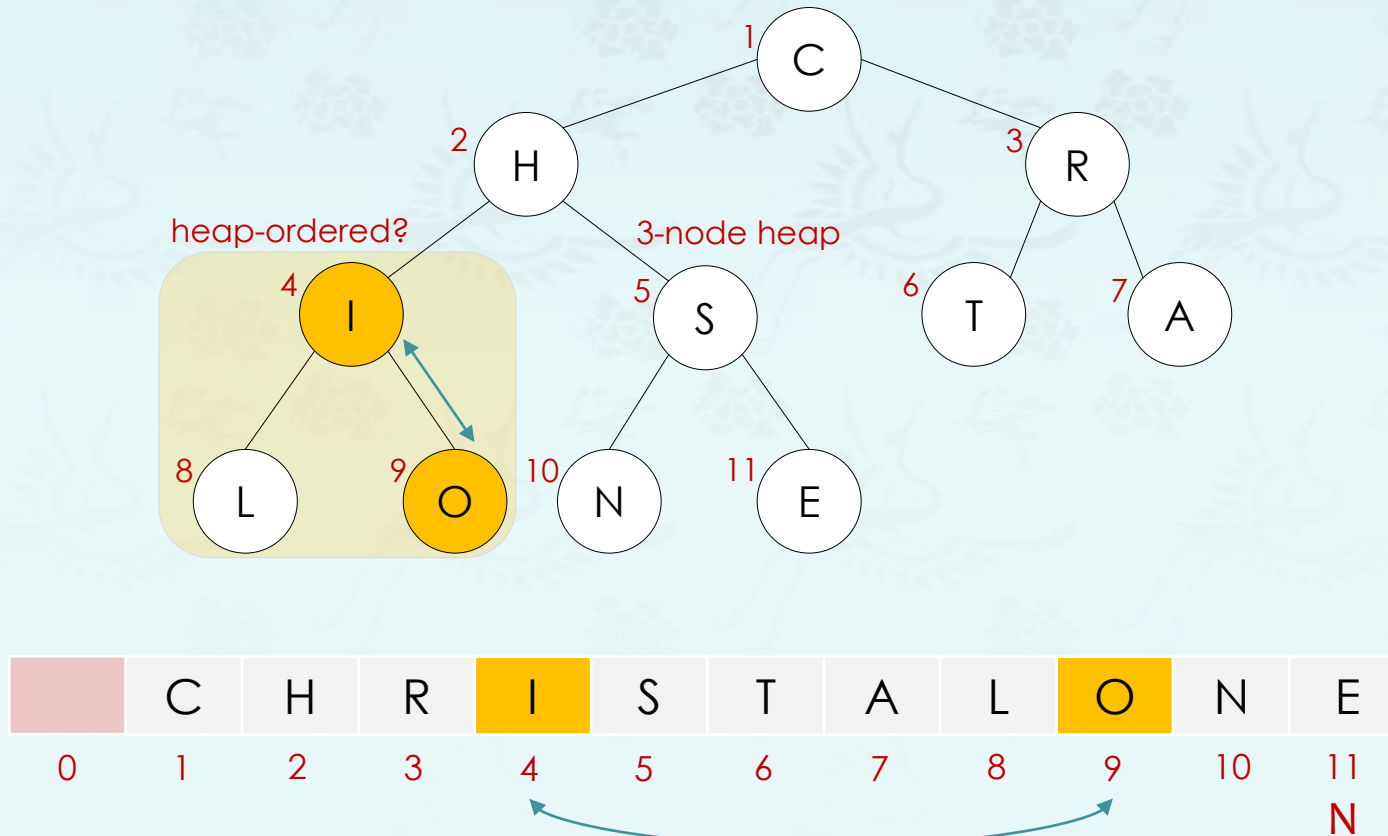


Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

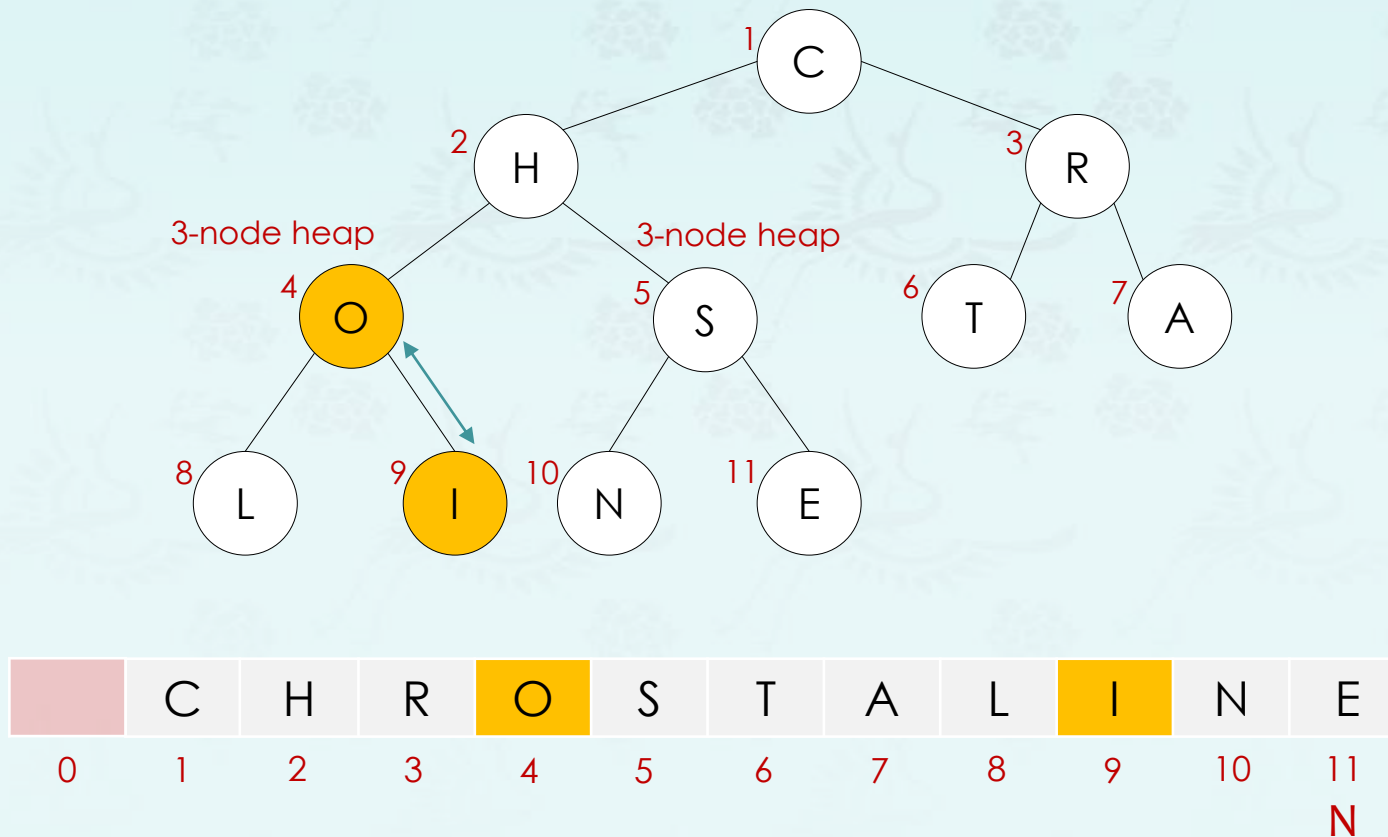


Heapsort

1st Pass: Heap construction (heapify)

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array in arbitrary order



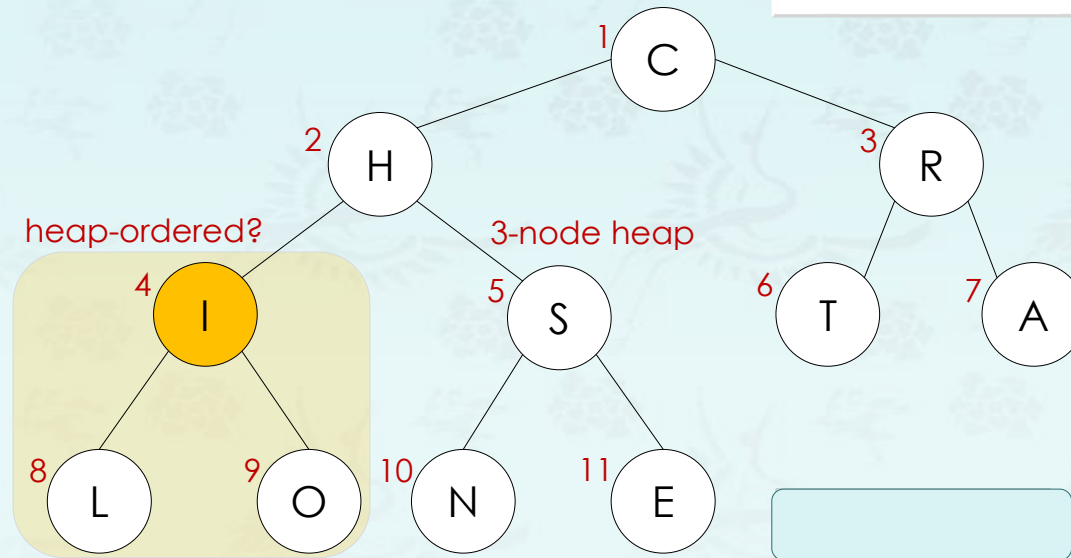
Heapsort

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array in arbitrary order

```
void sink(heap h, int k) {  
    while (2 * k <= h->N) {  
        int j = 2 * k;  
        if (j < h->N && less(h, j, j + 1)) j++;  
        if (!less(h, k, j)) break;  
        swap(h, k, j);  
        k = j;  
    }  
}
```



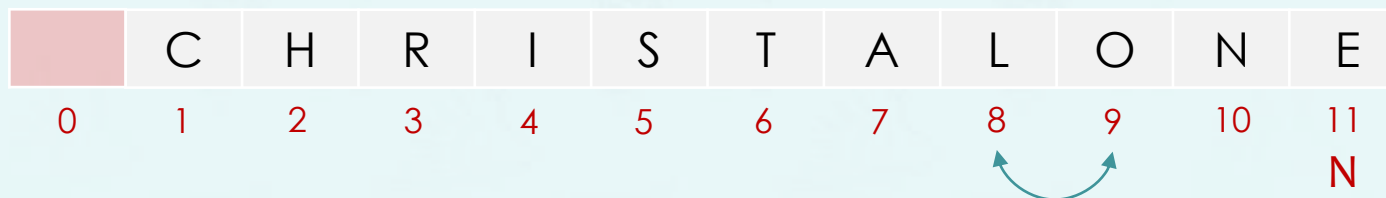
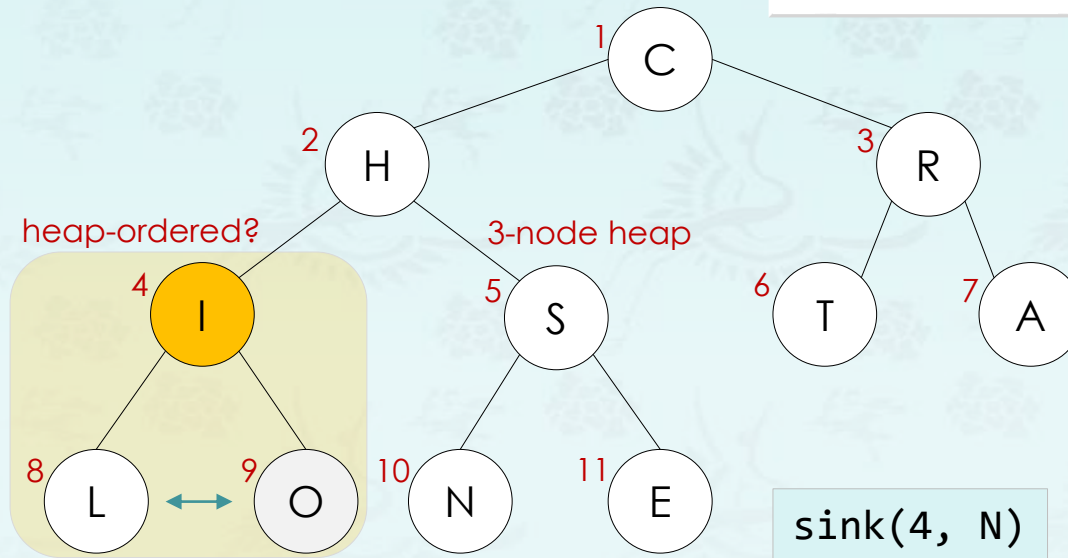
Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

```
void sink(heap h, int k) {  
    while (2 * k <= h->N) {  
        int j = 2 * k; k=4, j=8  
        if (j < h->N && less(h, j, j + 1)) j++;  
        if (!less(h, k, j)) break; k=4, j=9  
        swap(h, k, j);  
        k = j;  
    }  
}
```



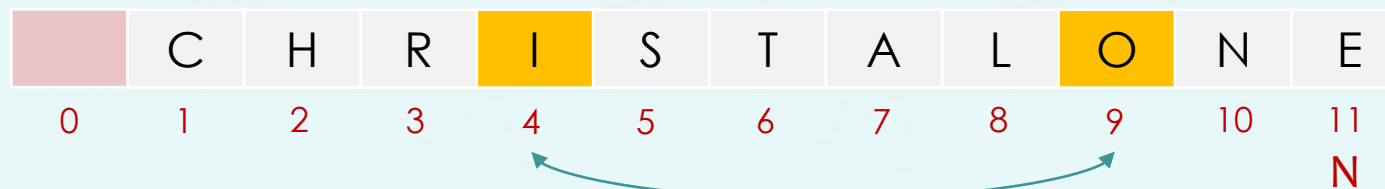
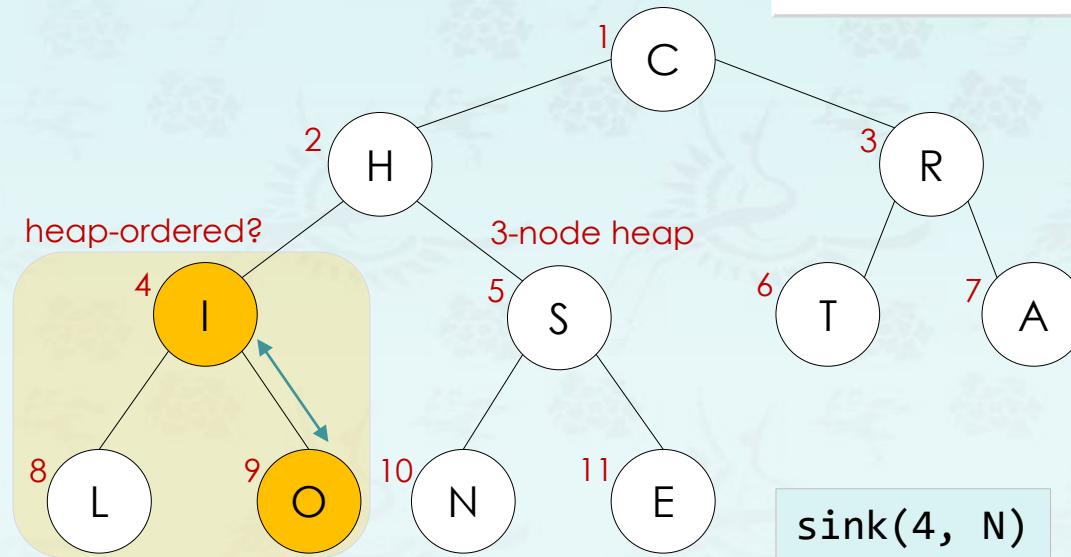
Heapsort

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array in arbitrary order

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        if (!less(h, k, j)) break; k=4, j=9  
        swap(h, k, j);  
        k = j;  
    }  
}
```



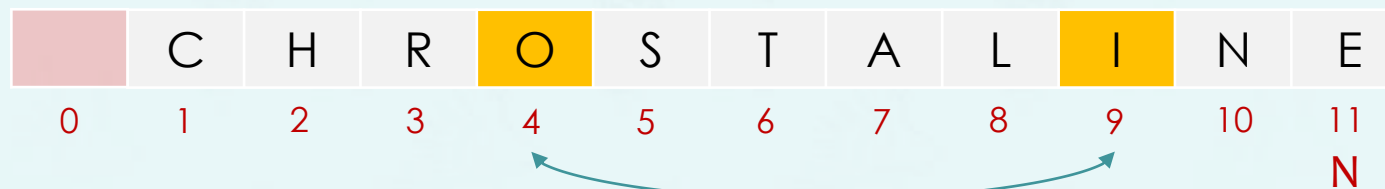
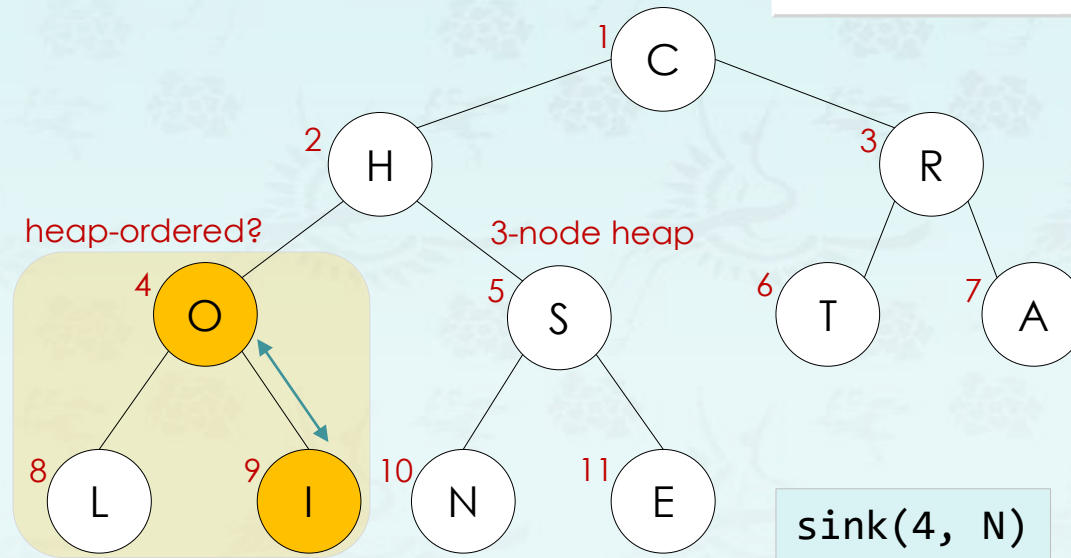
Heapsort

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array in arbitrary order

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        if (!less(h, k, j)) break; k=4, j=9  
        swap(h, k, j);  
        k = j;                   k=9  
    }  
    exit while()  
}
```



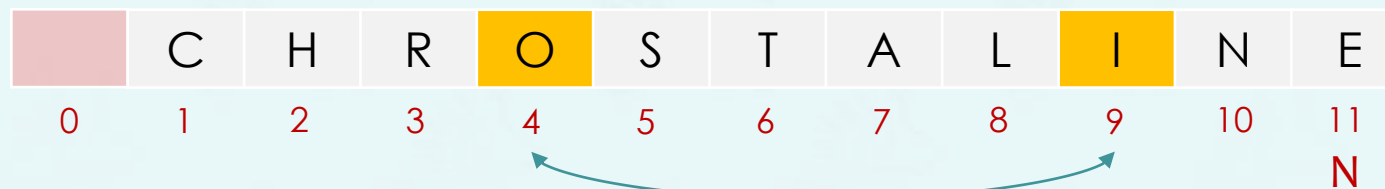
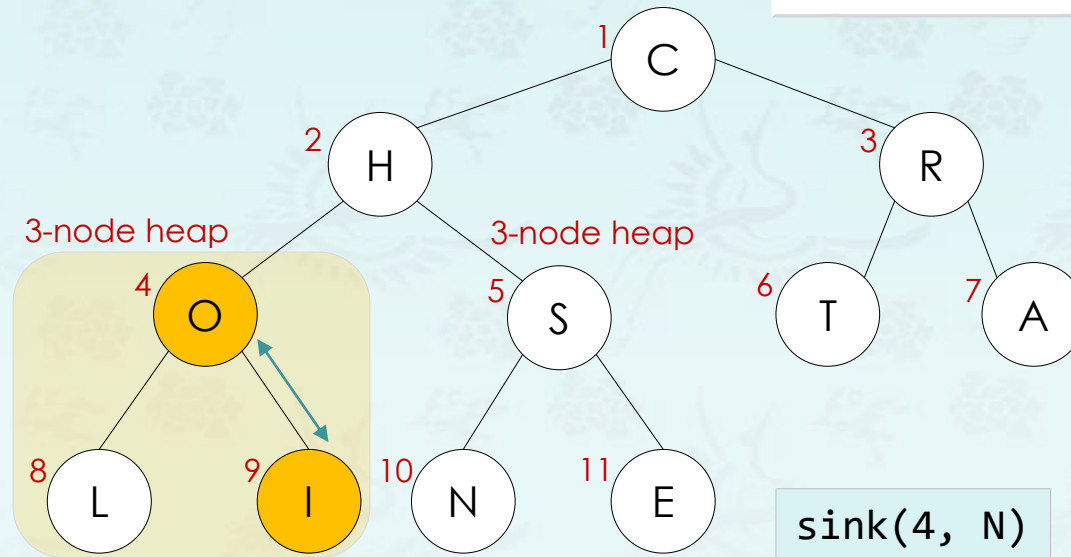
Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

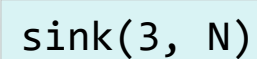
array in arbitrary order

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        if (j < h->N && less(h, j, j + 1)) j++;  
        if (!less(h, k, j)) break; k=4, j=9  
        swap(h, k, j);  
        k = j;                   k=9  
    }  
    exit while()  
}
```



1st Pass: Heap construction (heapify)

- array in arbitrary order



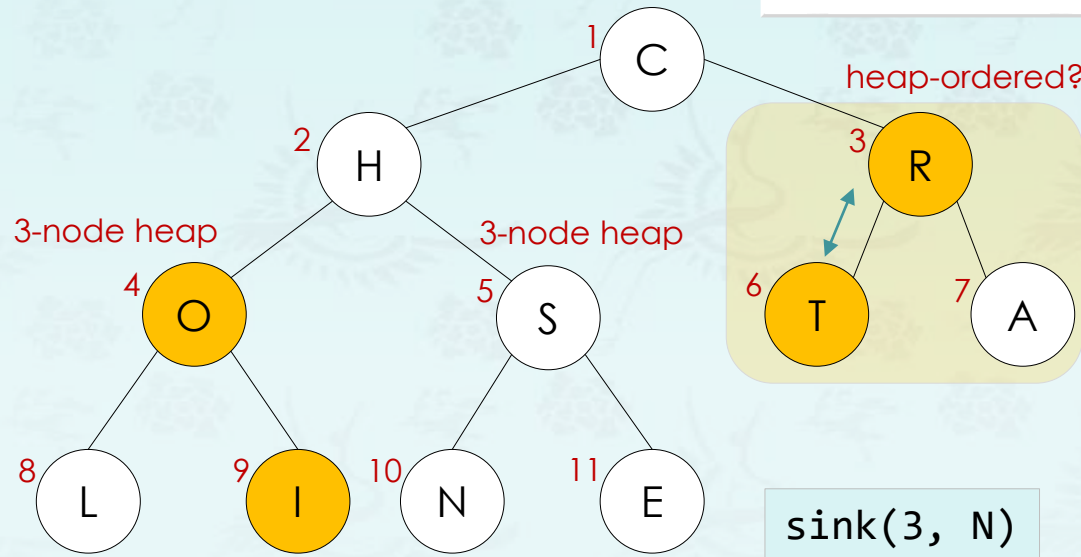
Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

```
void sink(heap h, int k) {  
    while (2 * k <= h->N) {  
        int j = 2 * k;           k=3, j=6  
        if (j < h->N && less(h, j, j + 1)) j++;  
        if (!less(h, k, j)) break; k=3, j=6  
        swap(h, k, j);           k=6  
        k = j;                   exit while()  
    }  
}
```



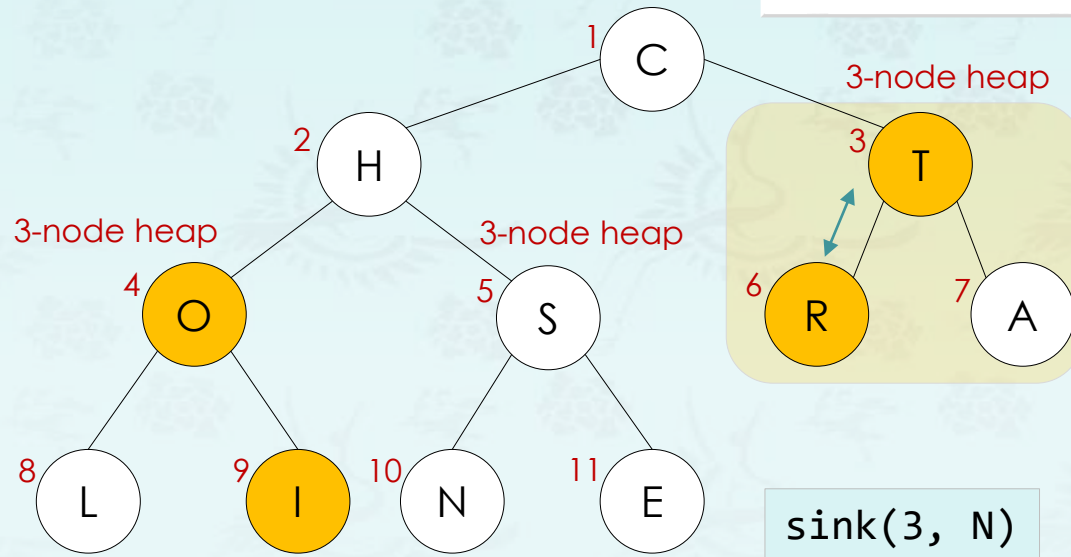
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        swap(h, k, j);           k=6  
        k = j;                   exit while()  
    }  
}
```



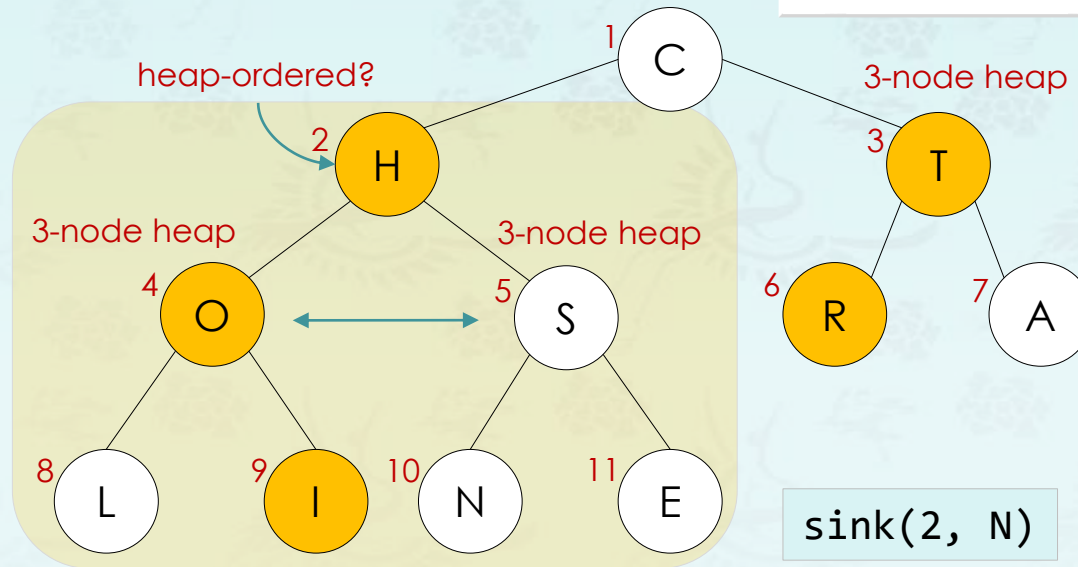
Heapsort

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array in arbitrary order

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        if (j < h->N && less(h, j, j + 1)) j++;  
        if (!less(h, k, j)) break; k=2, j=5  
        swap(h, k, j);           k=5  
        k = j;                   exit while()  
    }  
}
```



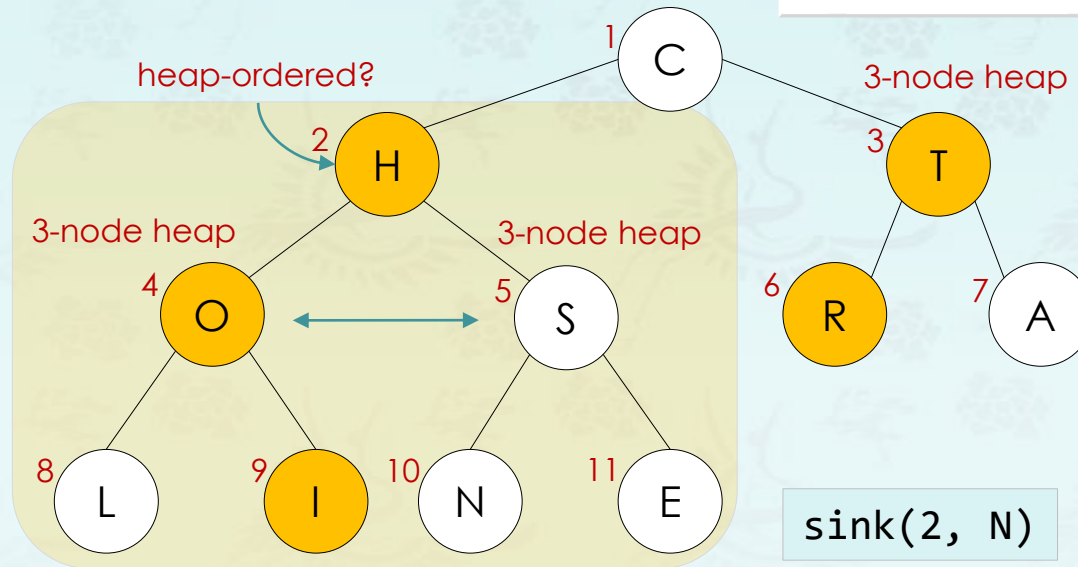
Heapsort

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        swap(h, k, j);           k=5  
        k = j;                   exit while()  
    }  
}
```



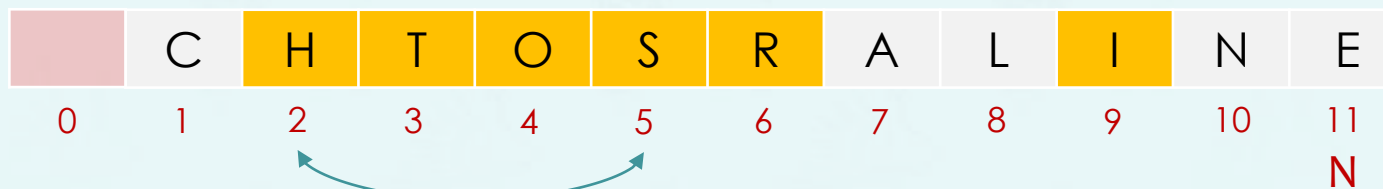
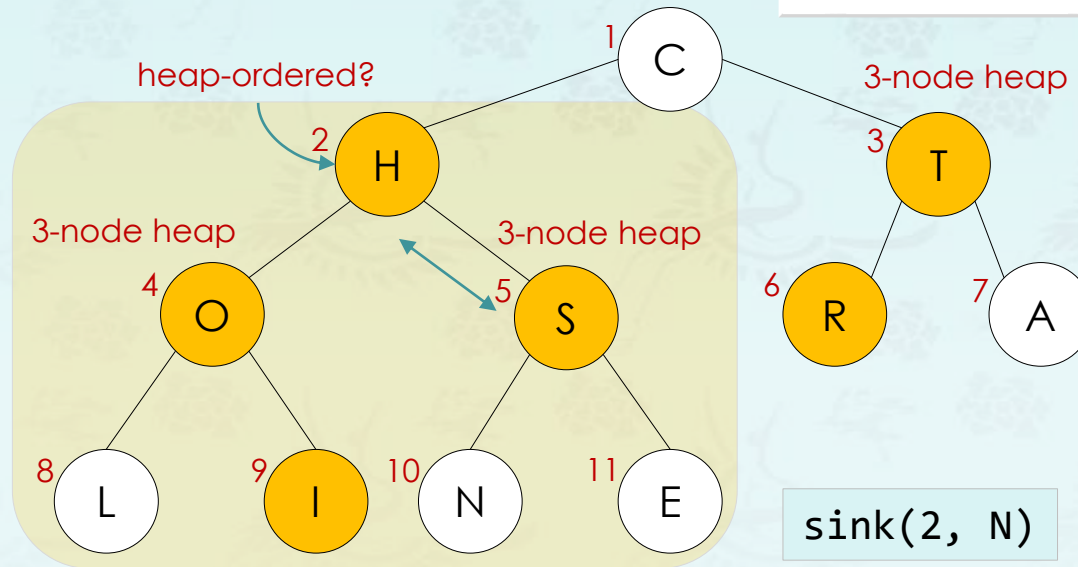
Heapsort

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array in arbitrary order

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        if (!less(h, k, j)) break; k=2, j=5  
        swap(h, k, j);           swap()  
        k = j;  
    }  
}
```



Heapsort

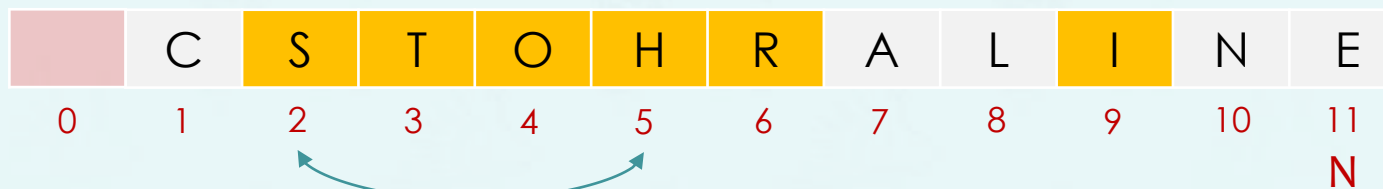
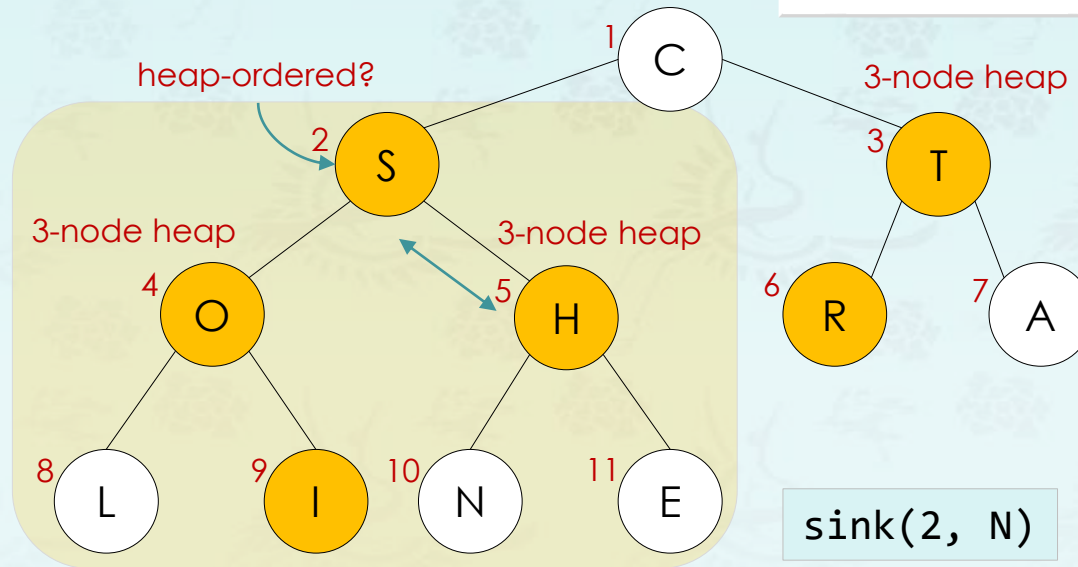
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        if (!less(h, k, j)) break;  
        swap(h, k, j);  
        k = j;  
    }  
}
```

k=2, j=4
k=2, j=5
swap()
k=5
while()
What's next?



Heapsort

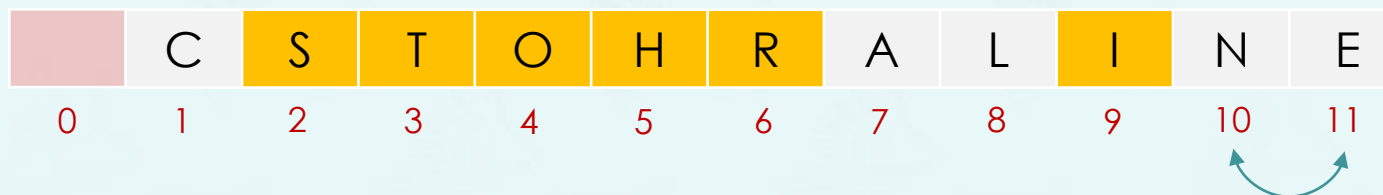
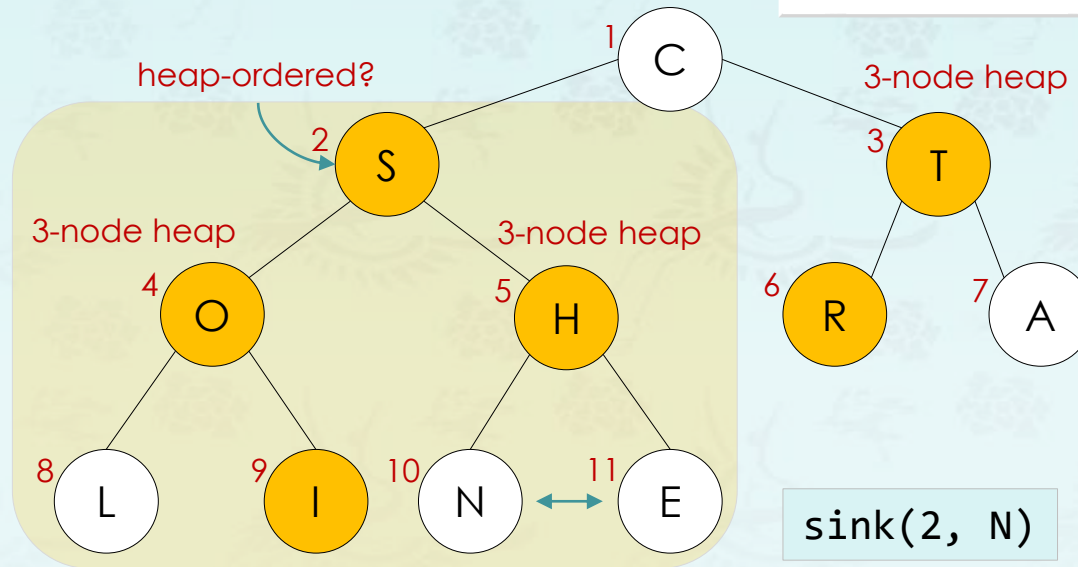
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array in arbitrary order

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        swap(h, k, j);  
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    }  
}
```

k=2, j=4
k=2, j=5
swap()
k=5
while()
What's next?



Heapsort

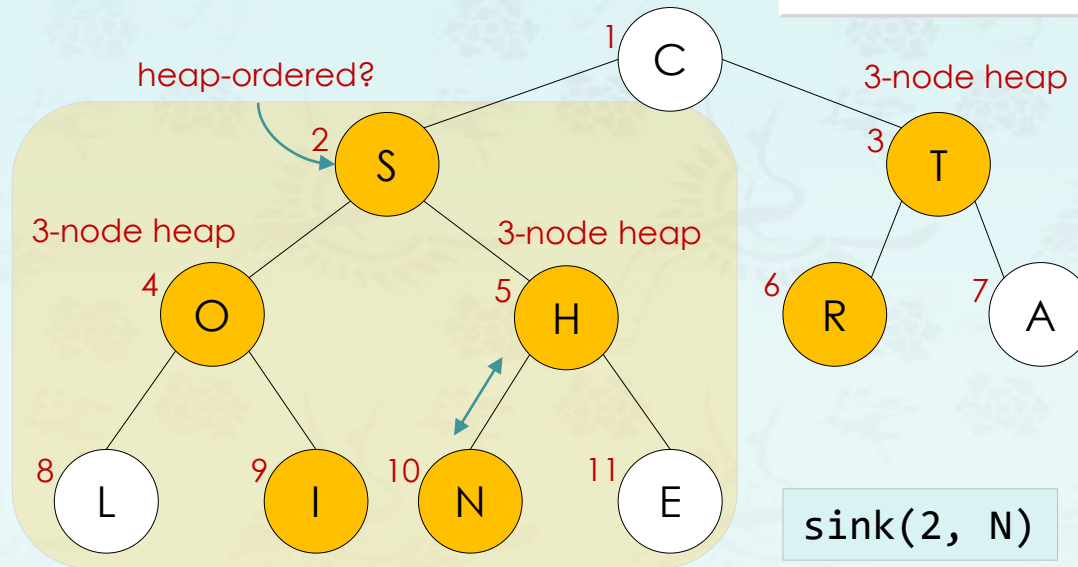
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- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

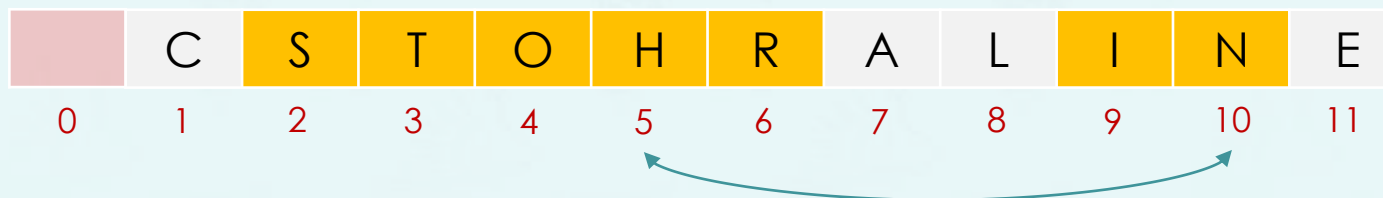
array in arbitrary order

```
void sink(heap h, int k) {  
    while (2 * k <= h->N) {  
        int j = 2 * k;  
        if (j < h->N && less(h, j, j + 1)) j++;  
        if (!less(h, k, j)) break;  
        swap(h, k, j);  
        k = j;  
    }  
}
```

k=2, j=4
k=2, j=5
swap()
k=5
while()
What's next?



k=5, j=10
k=5, j=10
swap()



Heapsort

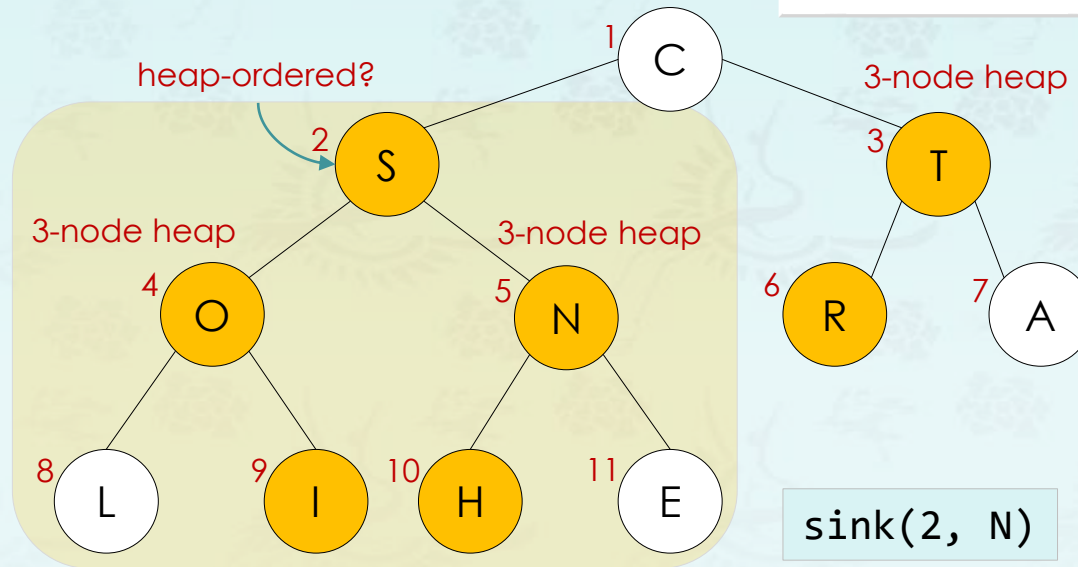
1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

```
void sink(heap h, int k) {  
    while (2 * k <= h->N) {  
        int j = 2 * k;  
        if (j < h->N && less(h, j, j + 1)) j++;  
        if (!less(h, k, j)) break;  
        swap(h, k, j);  
        k = j;  
    }  
}
```

k=2, j=4
k=2, j=5
swap()
k=5
while()
What's next?



k=5, j=10
k=5, j=10
swap()
k=10
while() exit

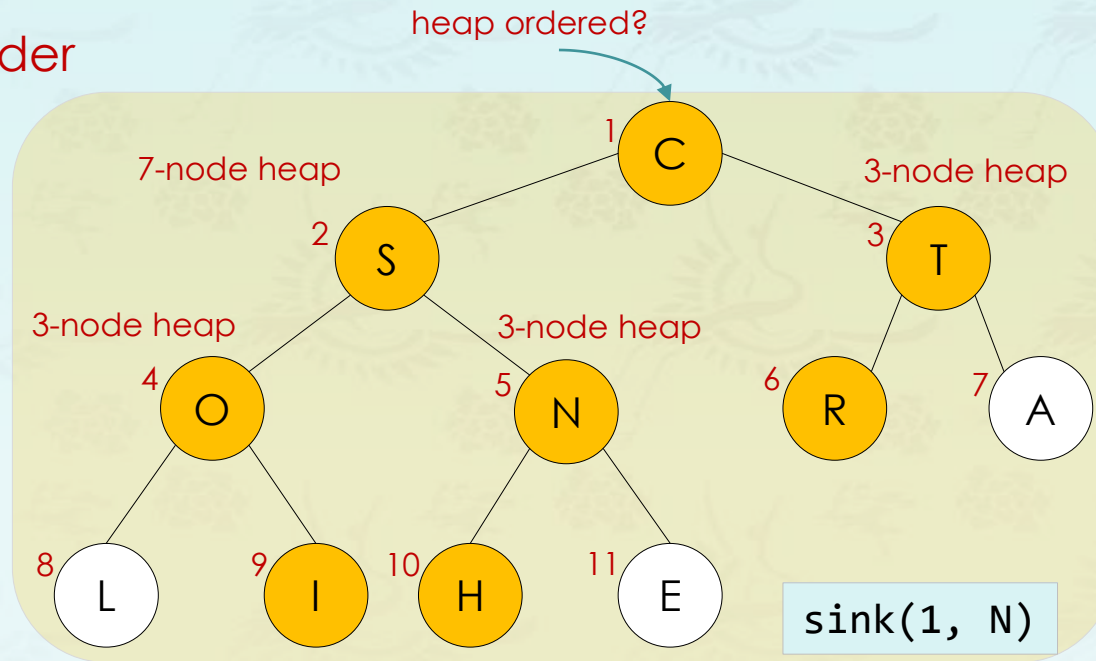


Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

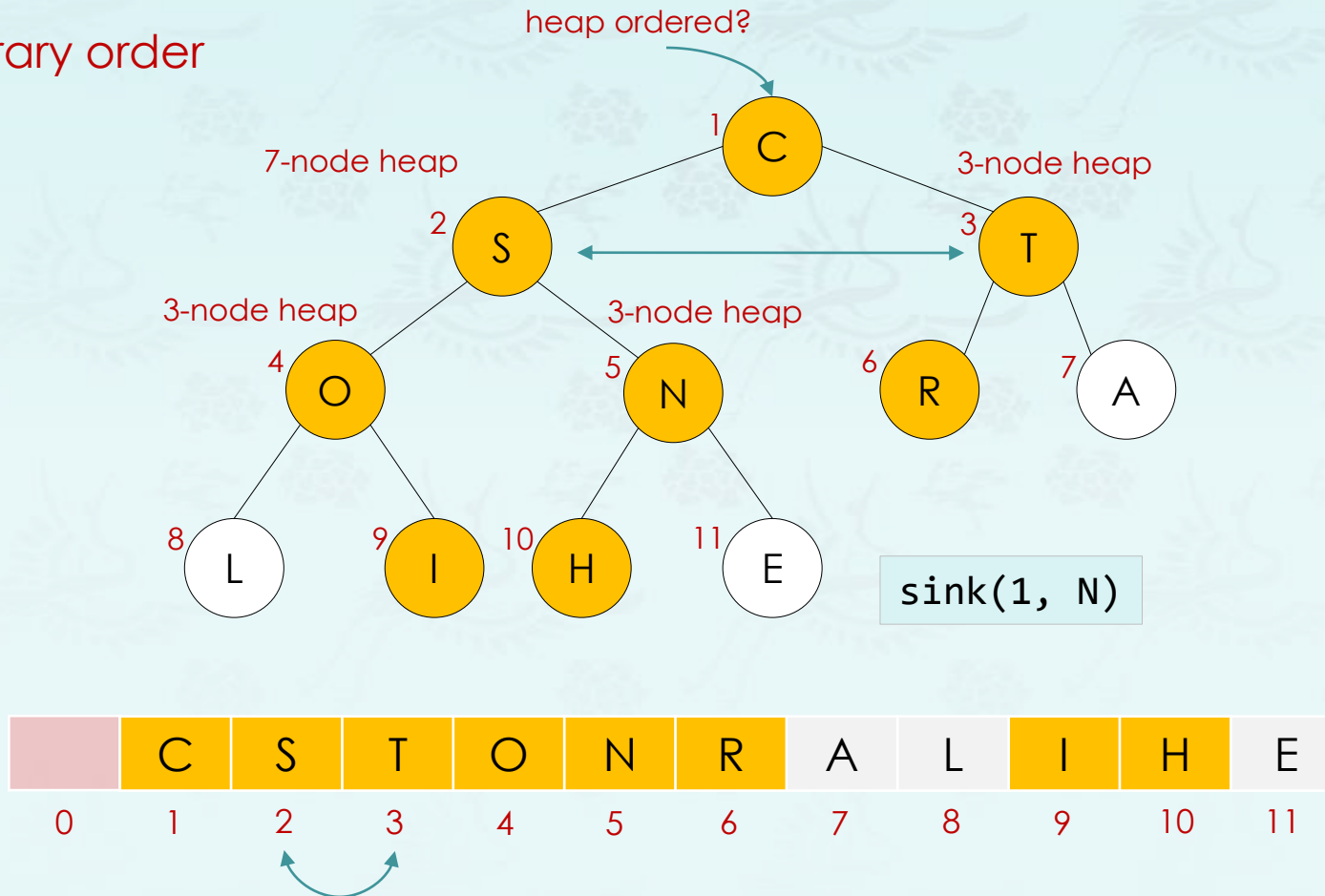


Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

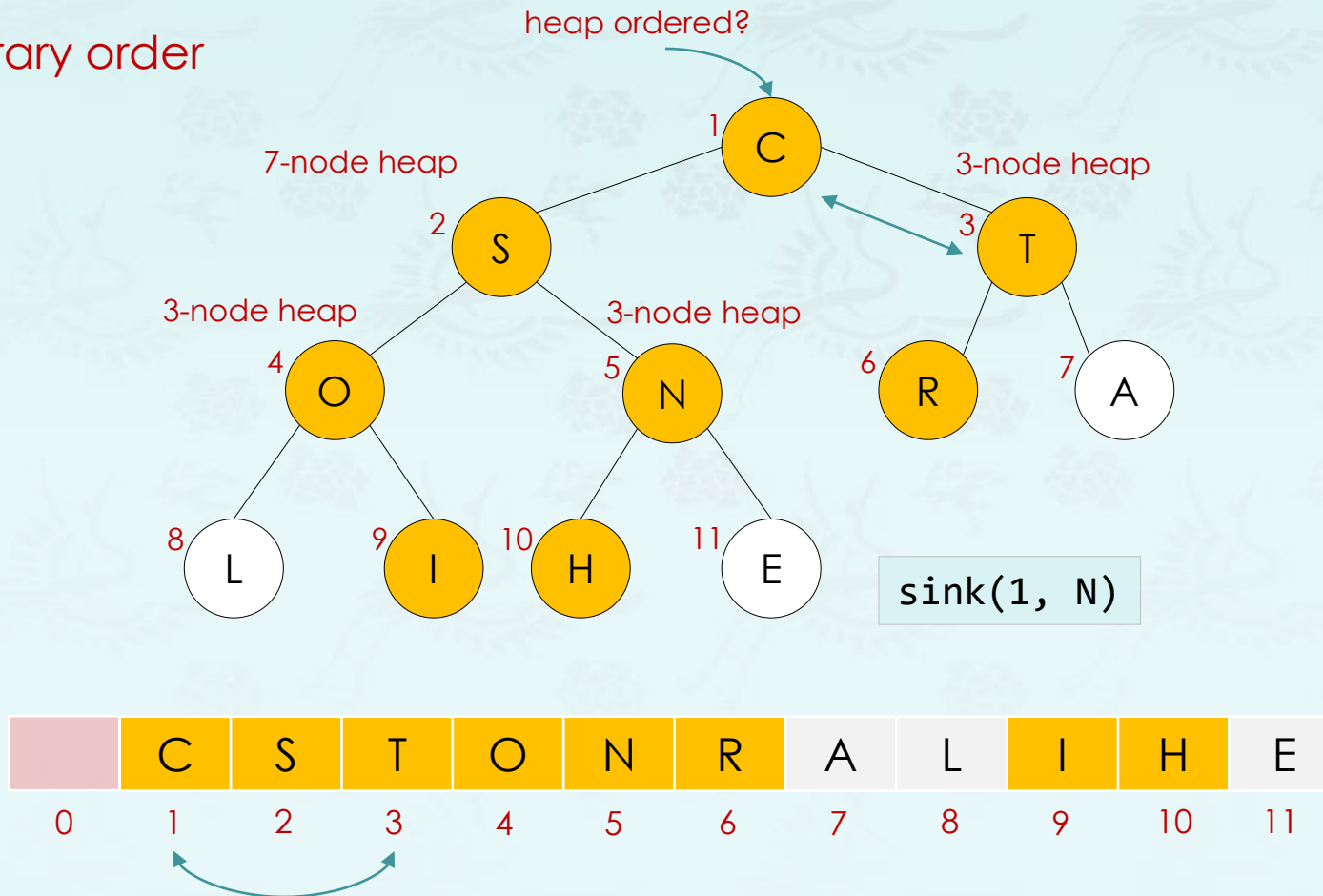


Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

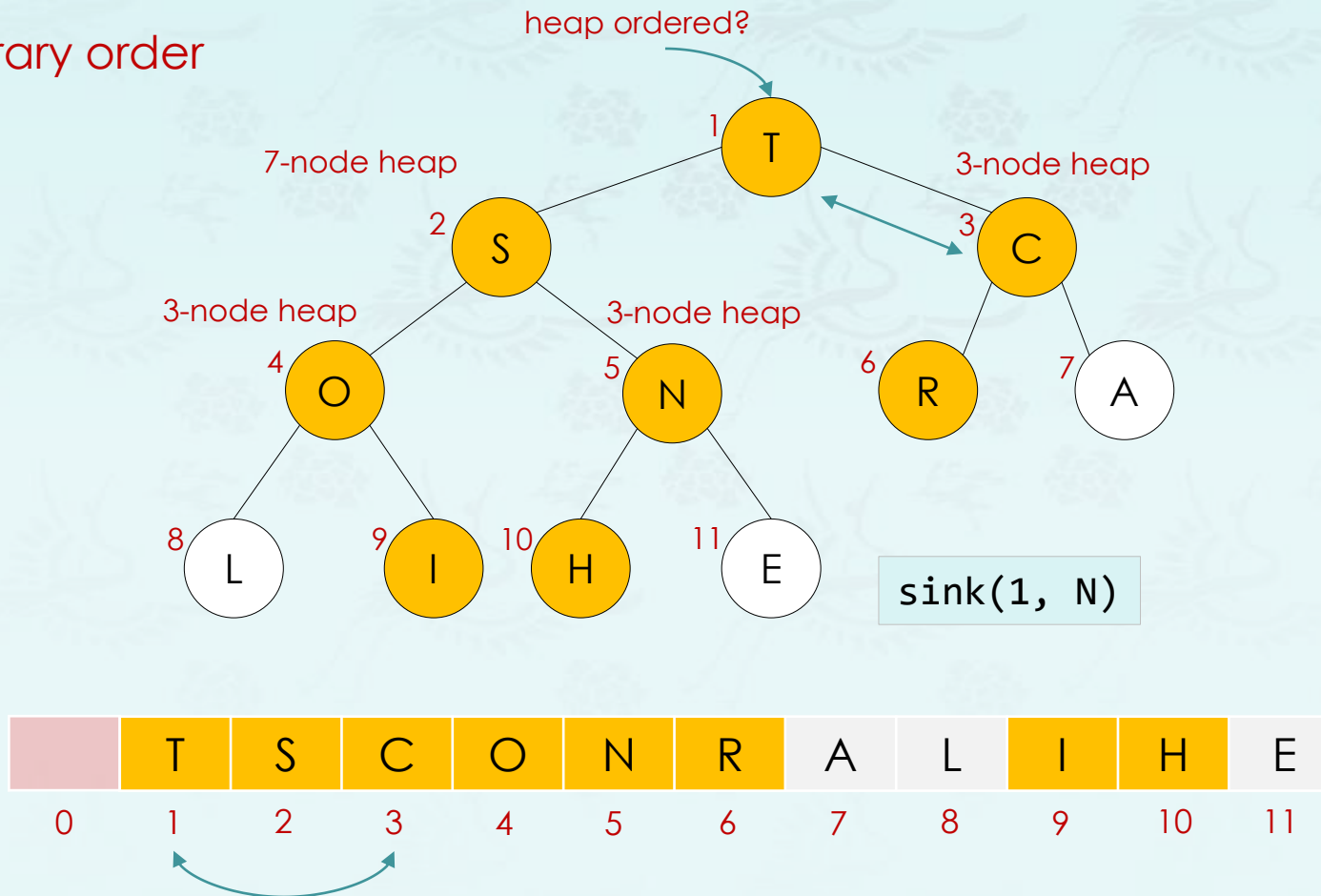


Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

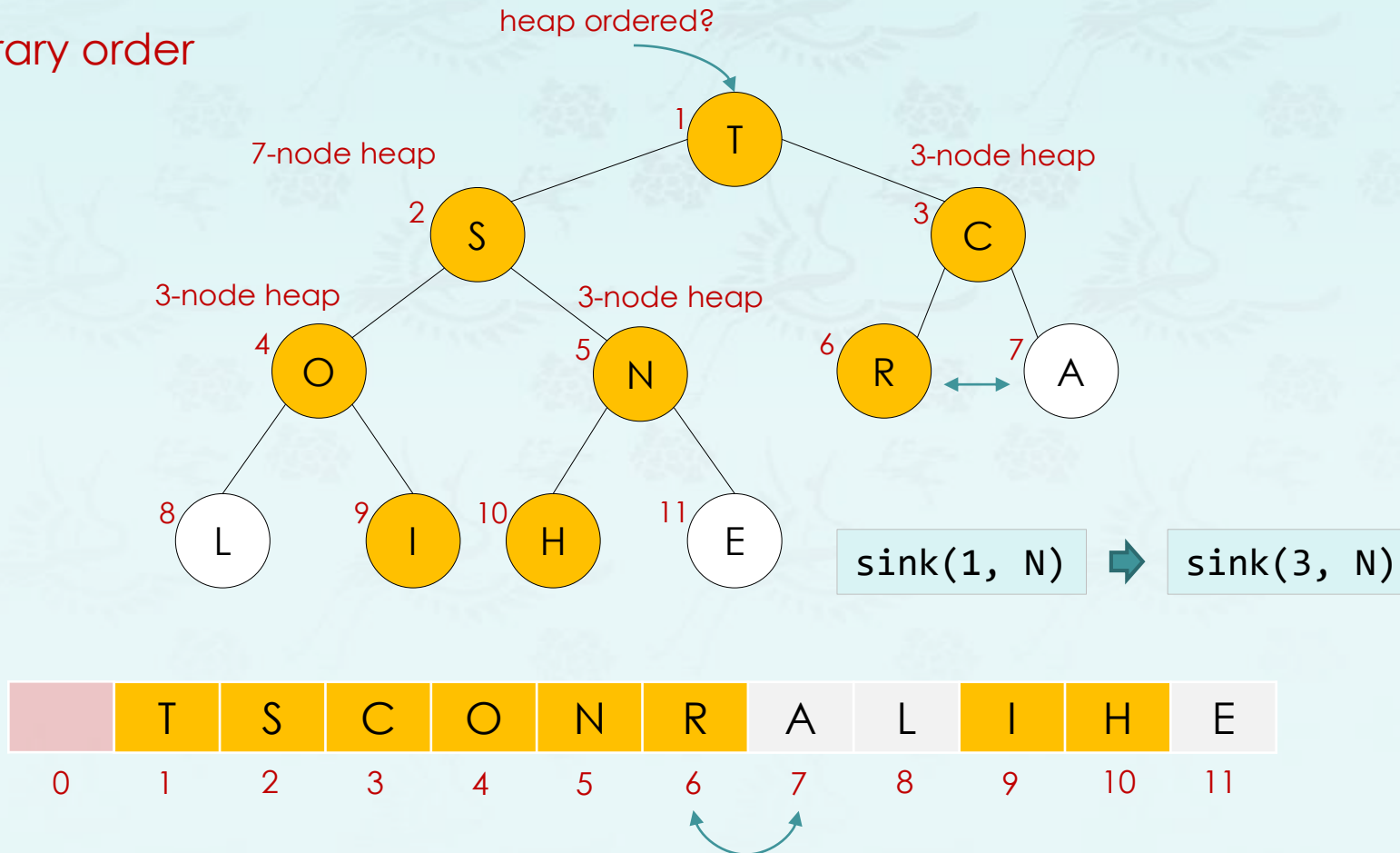


Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

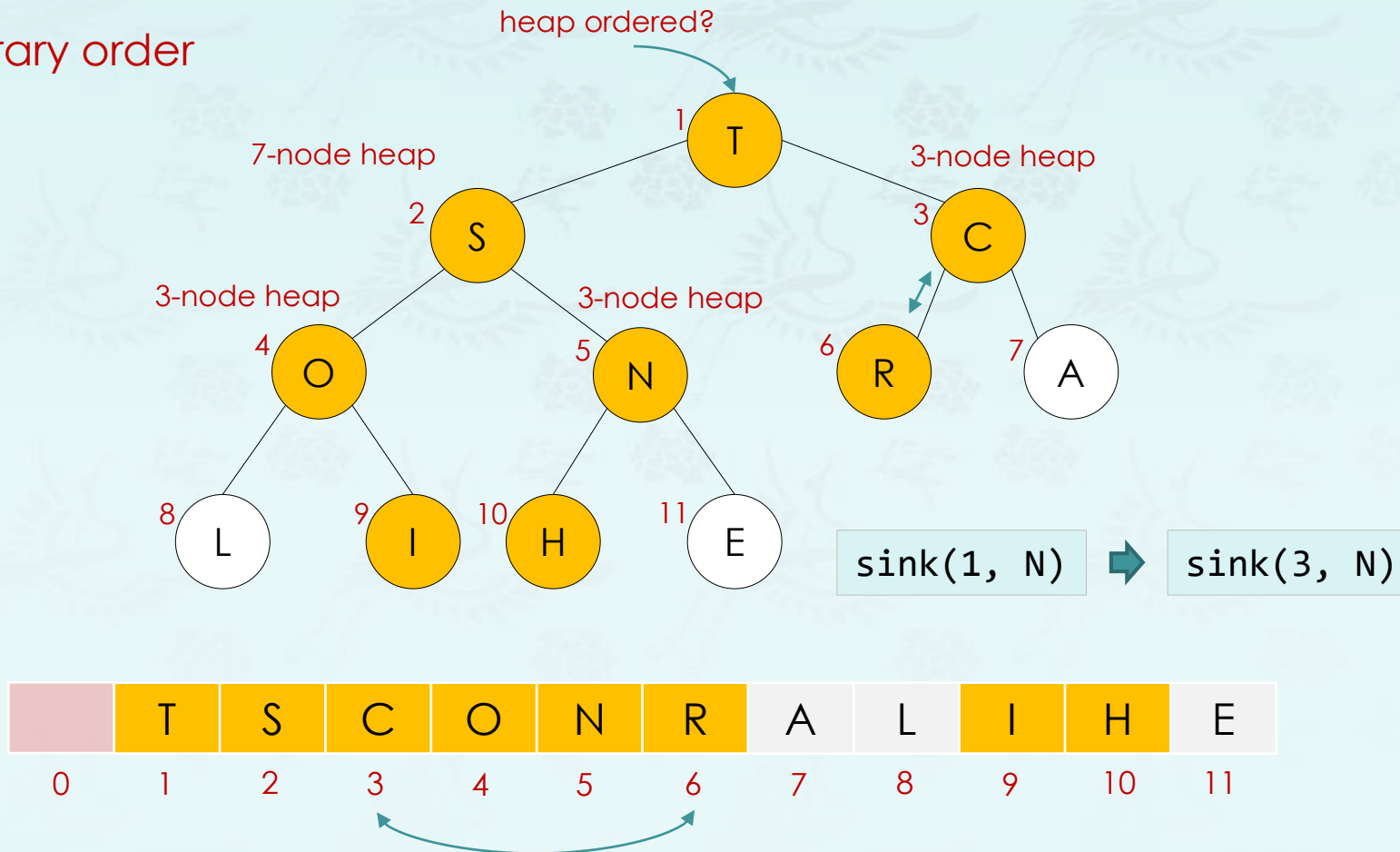


Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in arbitrary order

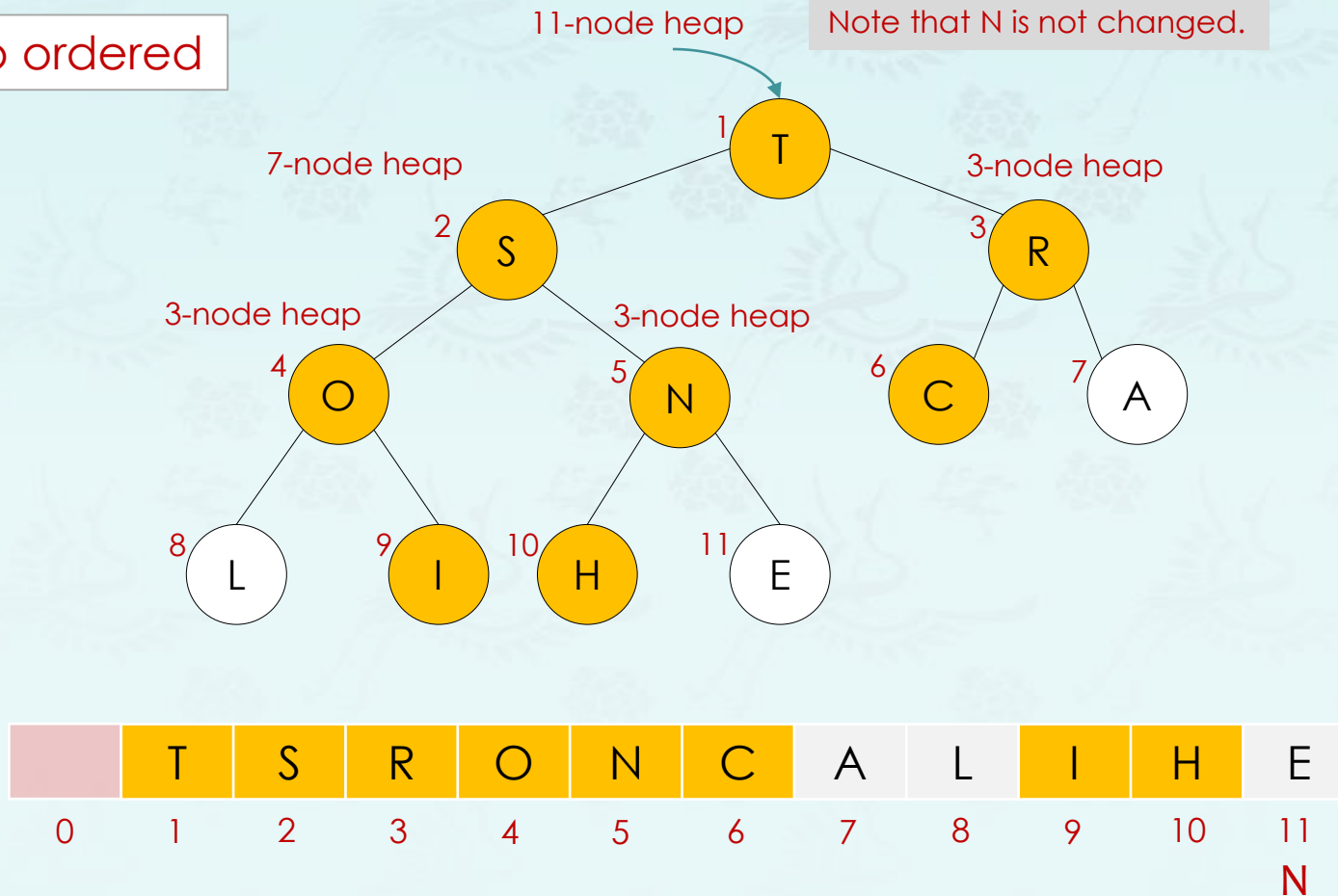


Heapsort

1st Pass: Heap construction (heapify)

- Build max heap using bottom-up method.
- Assume array entries are indexed from 1 to N.

array in heap ordered



Heapsort

Basic plan for in-place sort

- **1st Pass:** Build maxheap with all **N** keys.
- **2nd Pass:** Repeatedly remove the maximum key.

an array of **N** keys
in arbitrary order

1st Pass



build a maxheap
(in place)

2nd Pass



sorted in place

unsorted

C H R I S T A L O N E

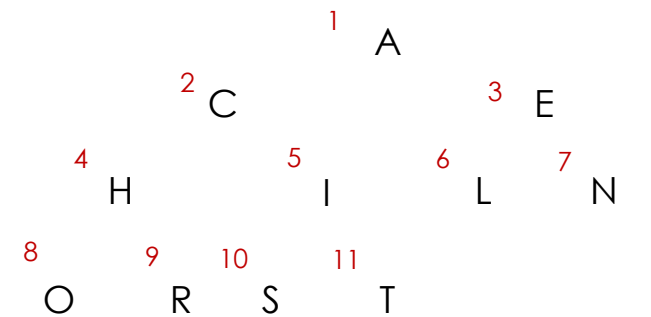
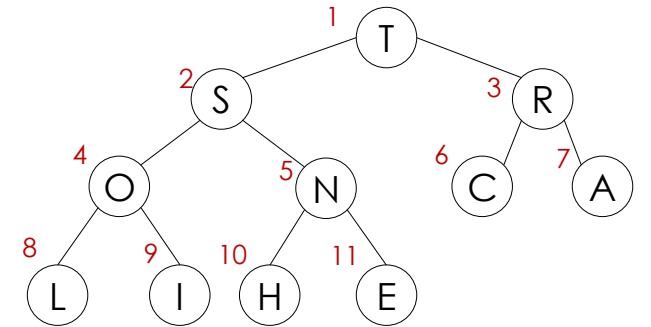
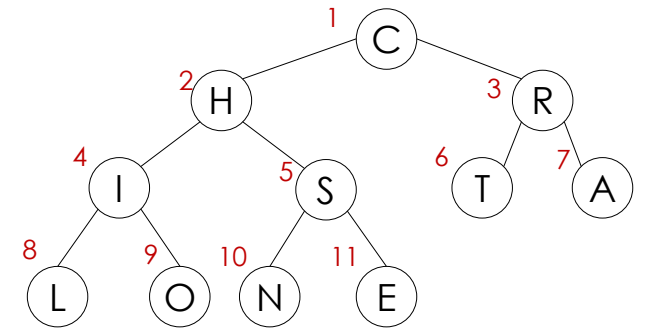
heap-ordered

C H R I S T A L O N E

sorted

A C E H I L N O R S T

1 2 3 4 5 6 7 8 9 10 11



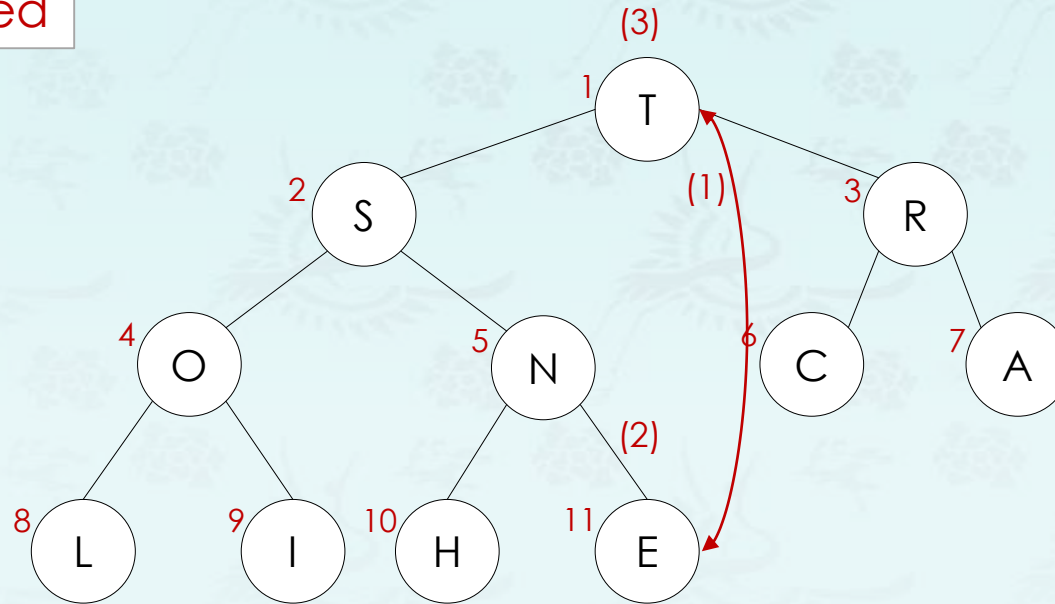
Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

- (1) Swap root (the max) and the last node.
- (2) Reduce N by one, but the value in memory.
- (3) Sink the new root to make it heap-ordered.

array in heap ordered



										(2)	N
	T	S	R	O	N	C	A	L	I	H	E
0	1	2	3	4	5	6	7	8	9	10	11

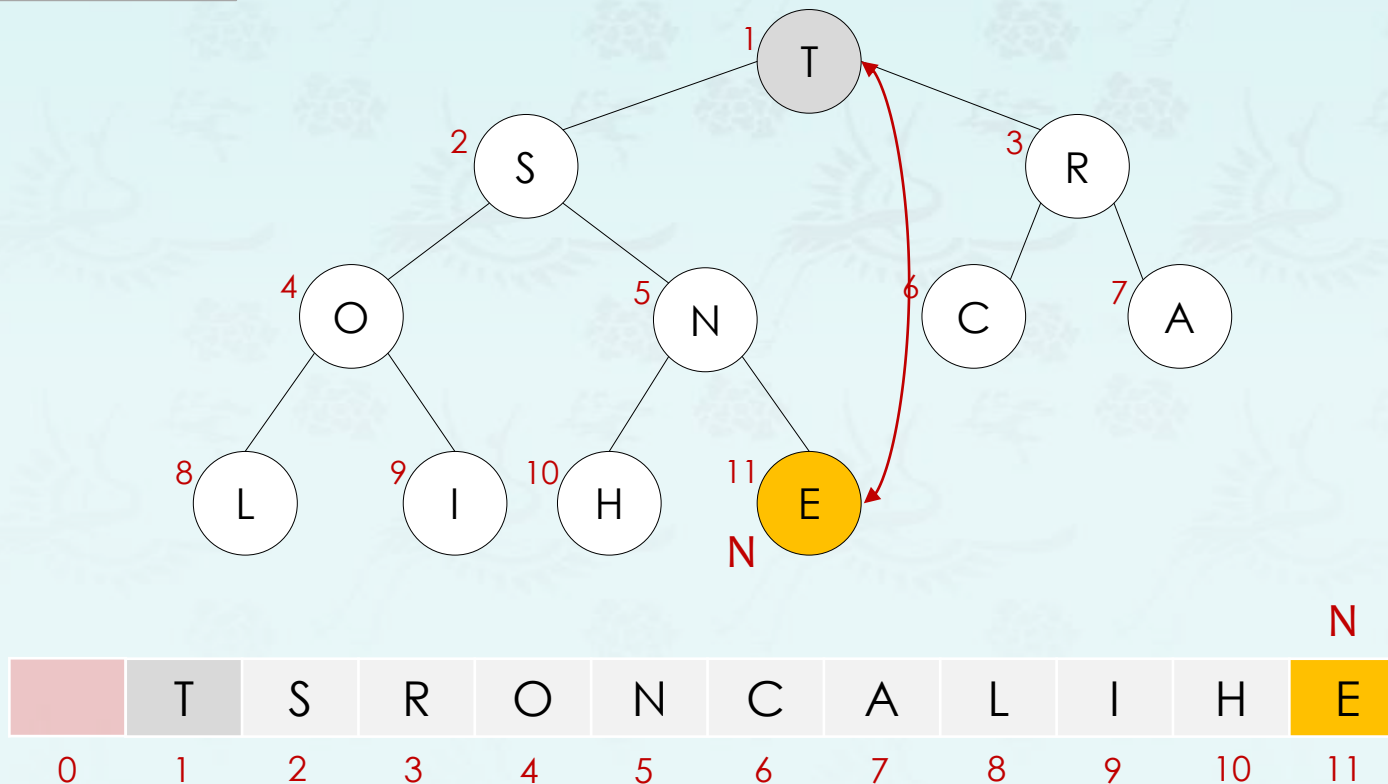
Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

- (1) Swap root (the max) and the last node.
- (2) Reduce N by one, but the value in memory.
- (3) Sink the new root to make it heap-ordered.

array in heap ordered



swap(1, N);

N=11

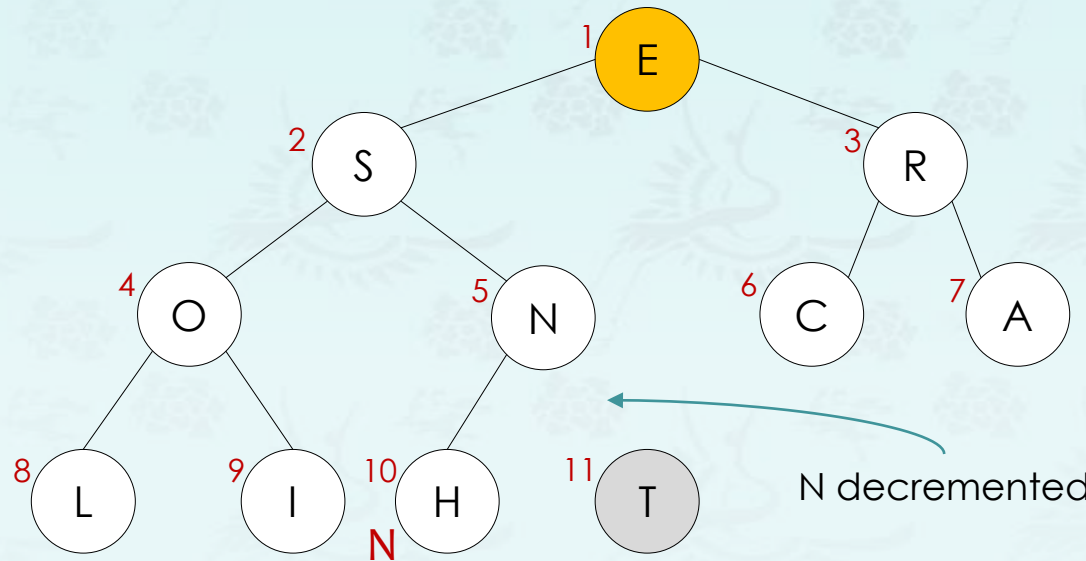
Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

- (1) Swap root (the max) and the last node.
- (2) Reduce N by one, but the value in memory.
- (3) Sink the new root to make it heap-ordered.

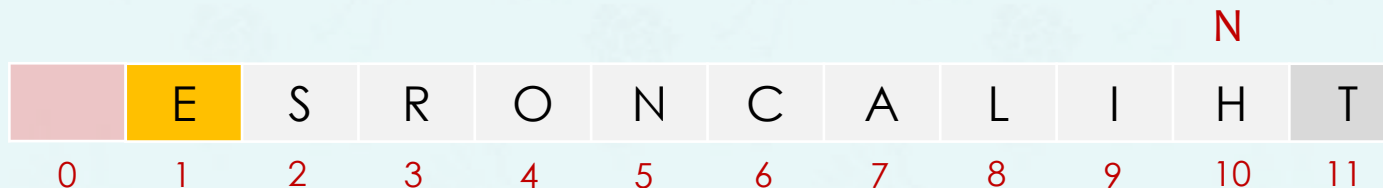
array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11

N=10

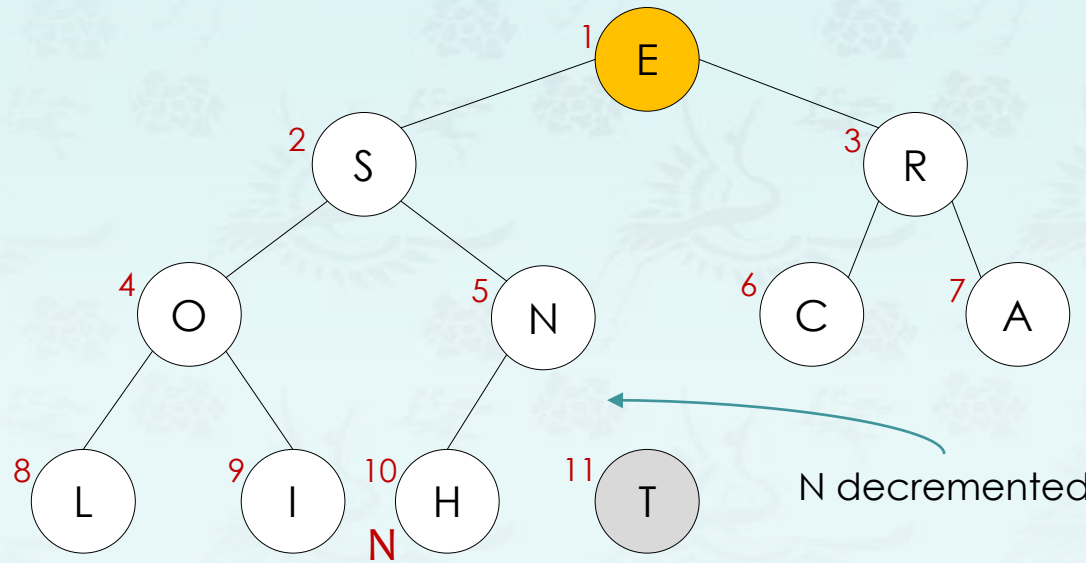


Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

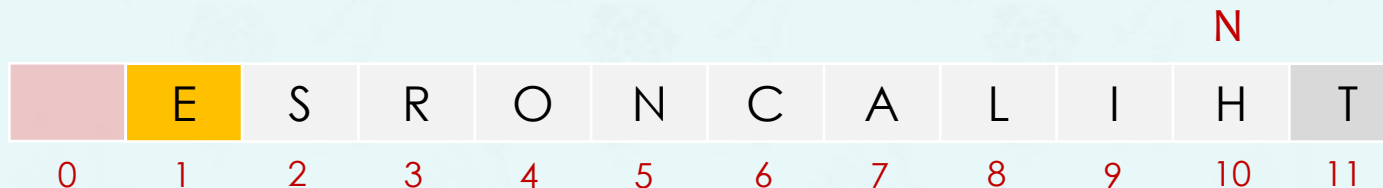
array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11

N=10

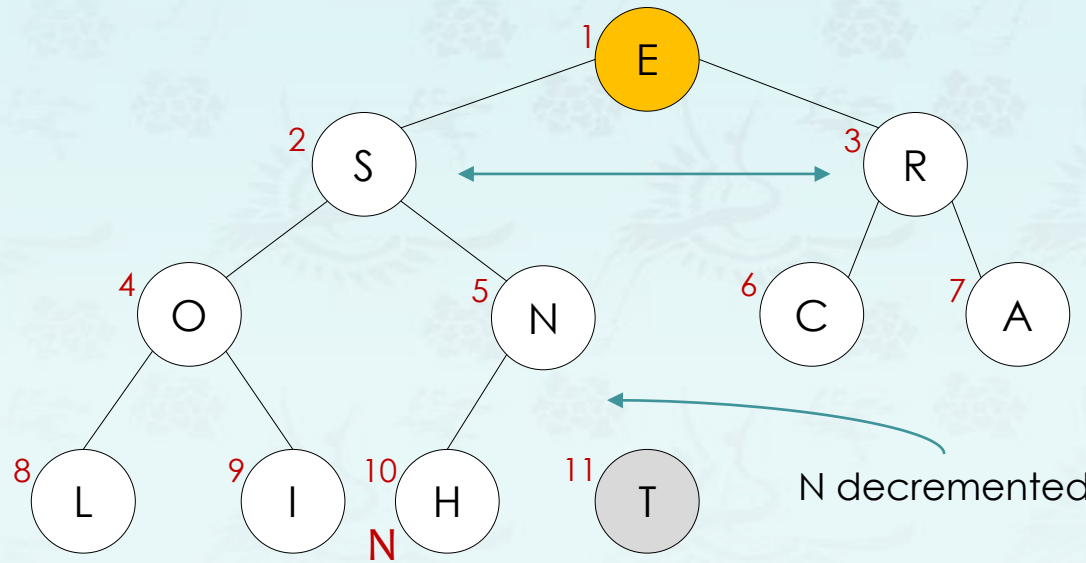


Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

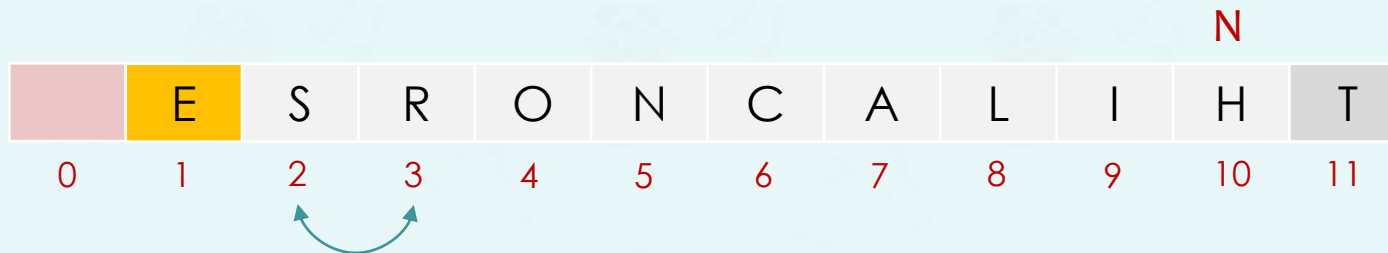
array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11

N=10

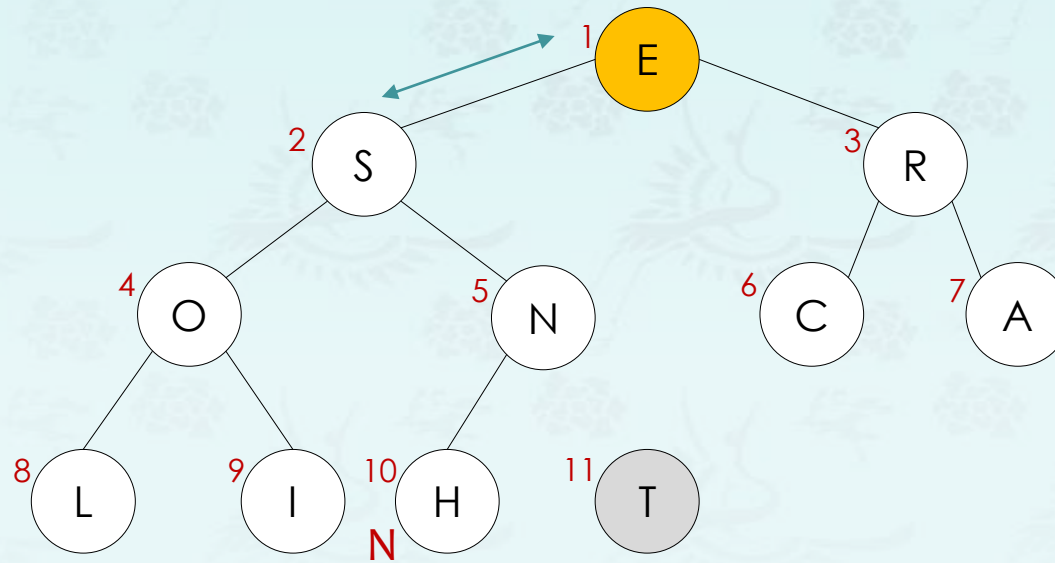


Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

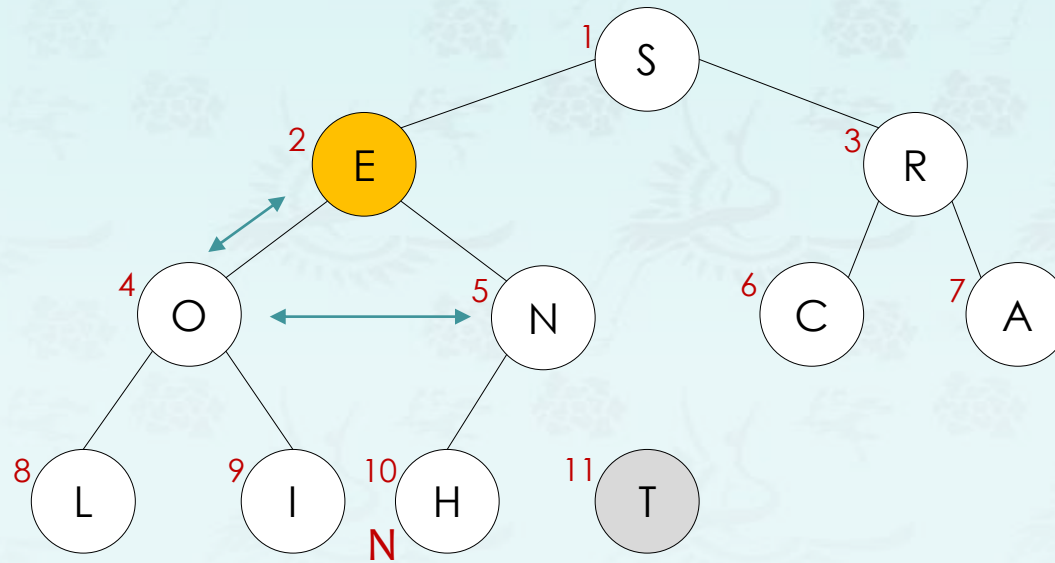


Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

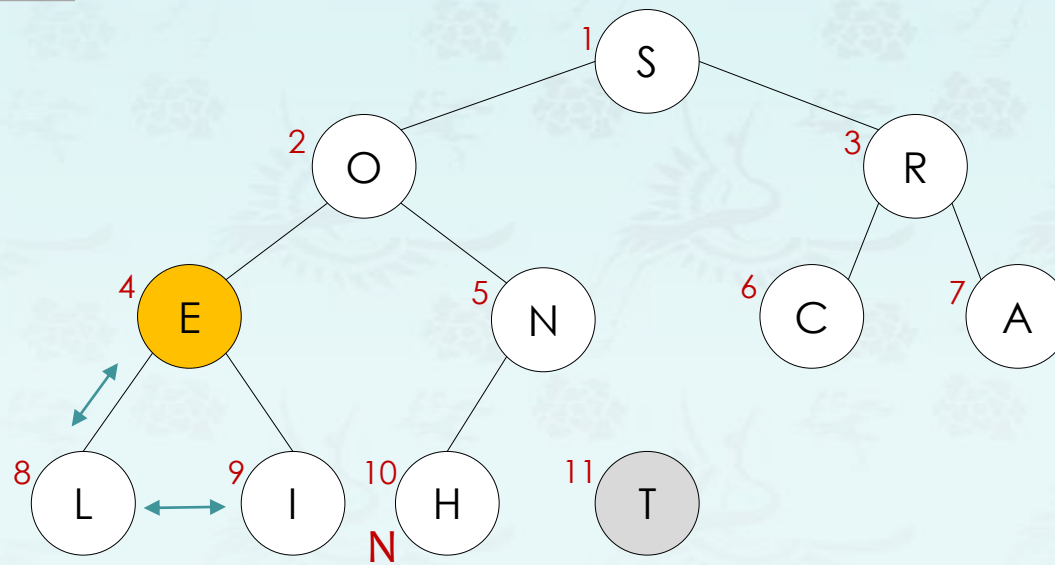


Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11

N=10

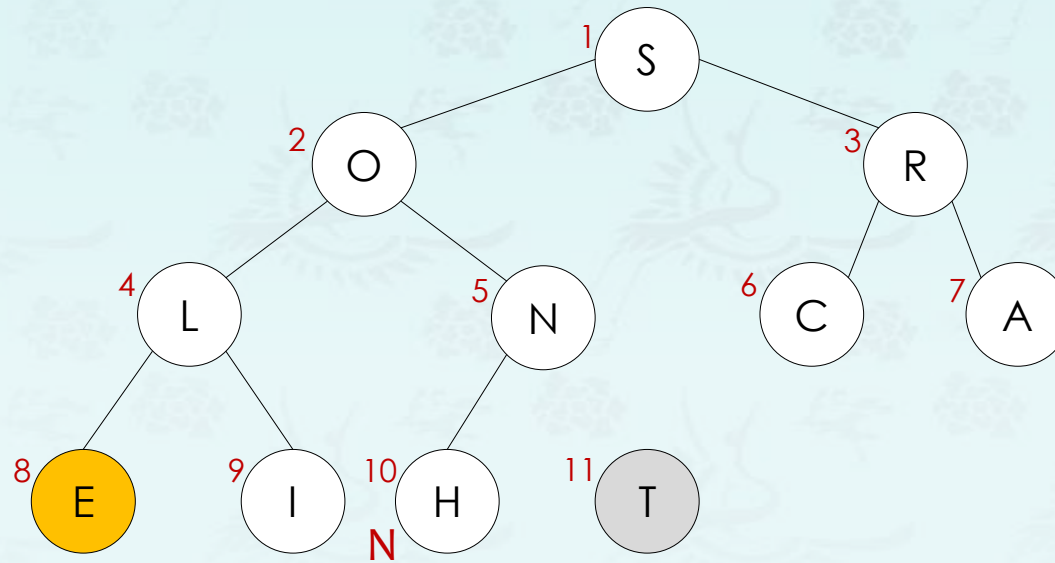


Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

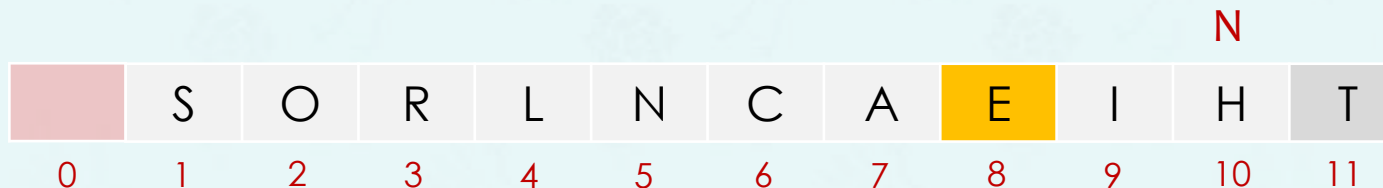
array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

What's next?

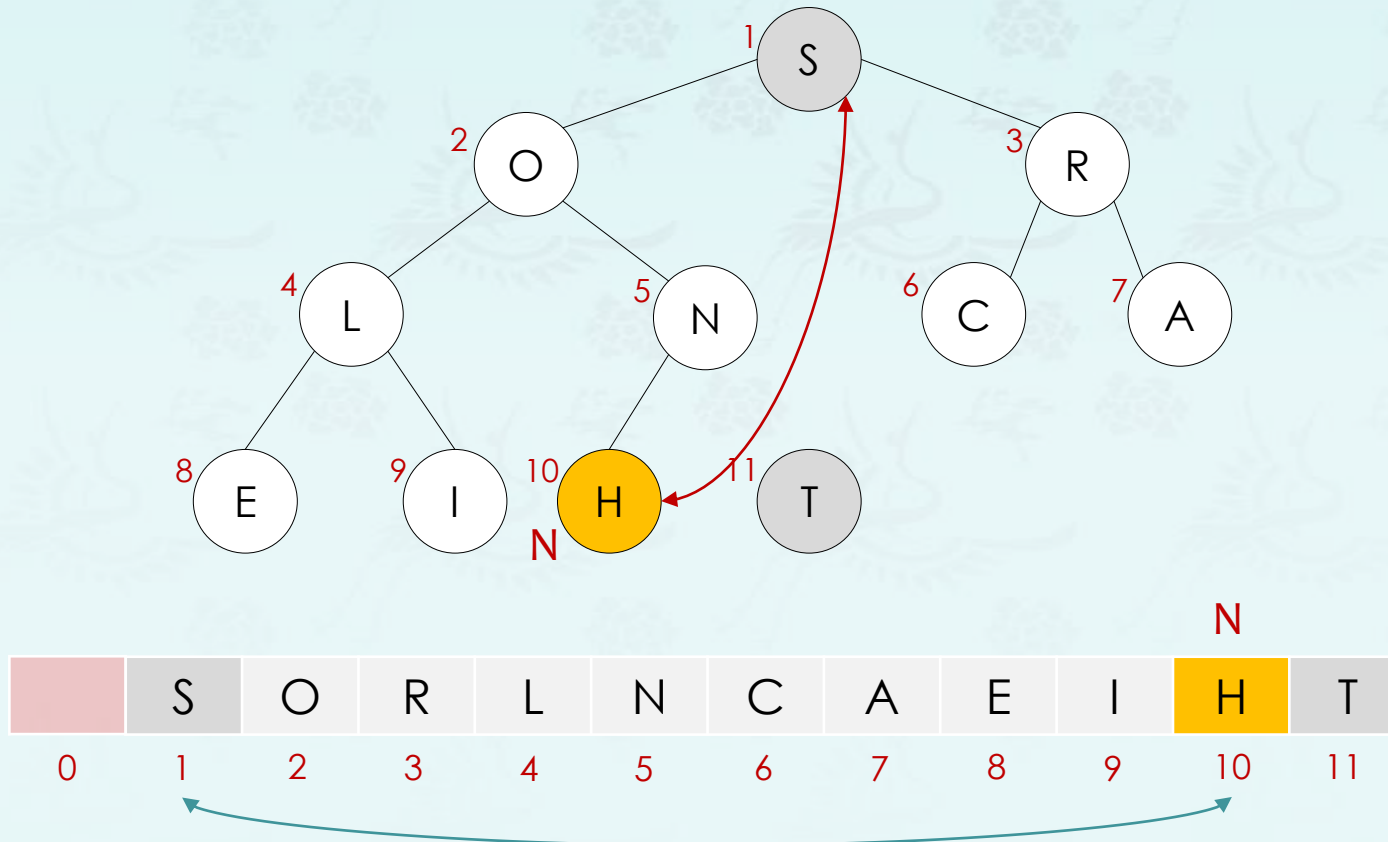


Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

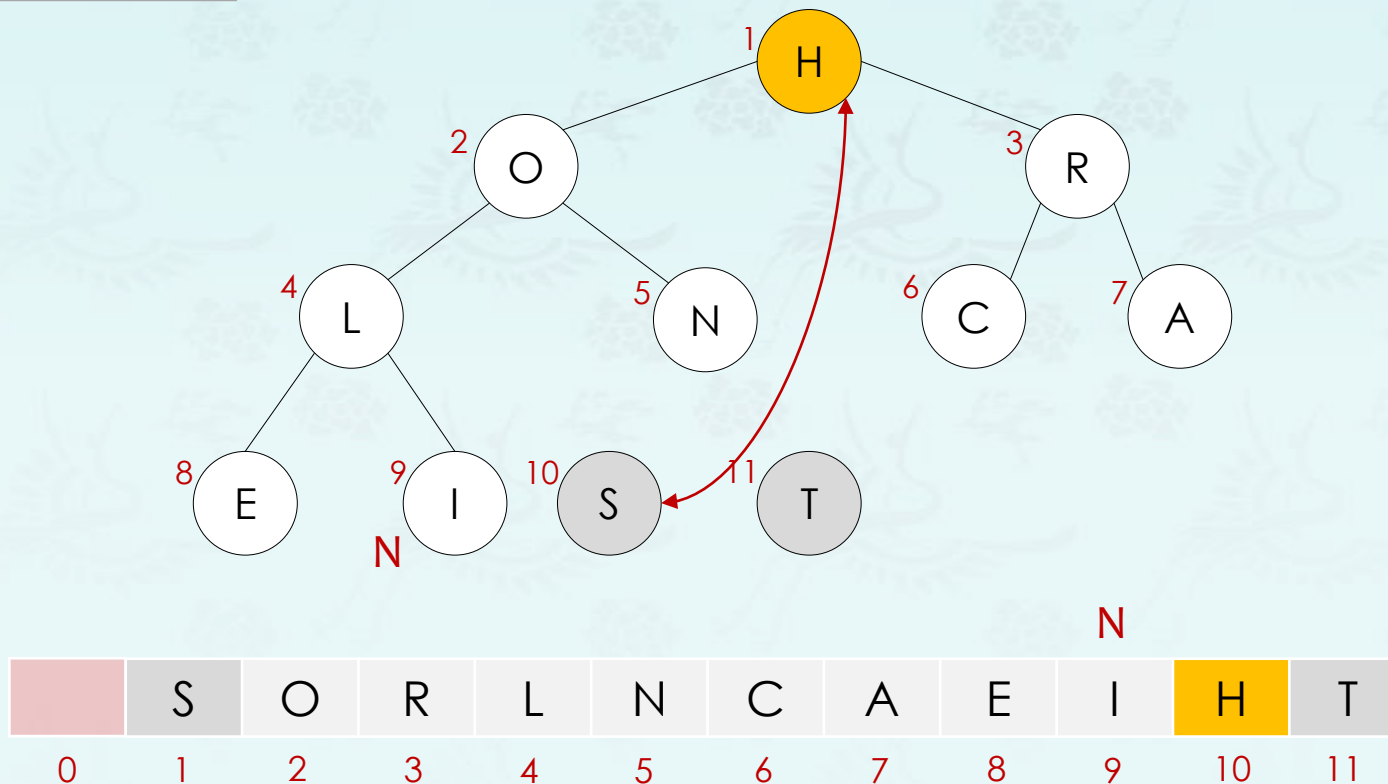
N=10
N=9

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

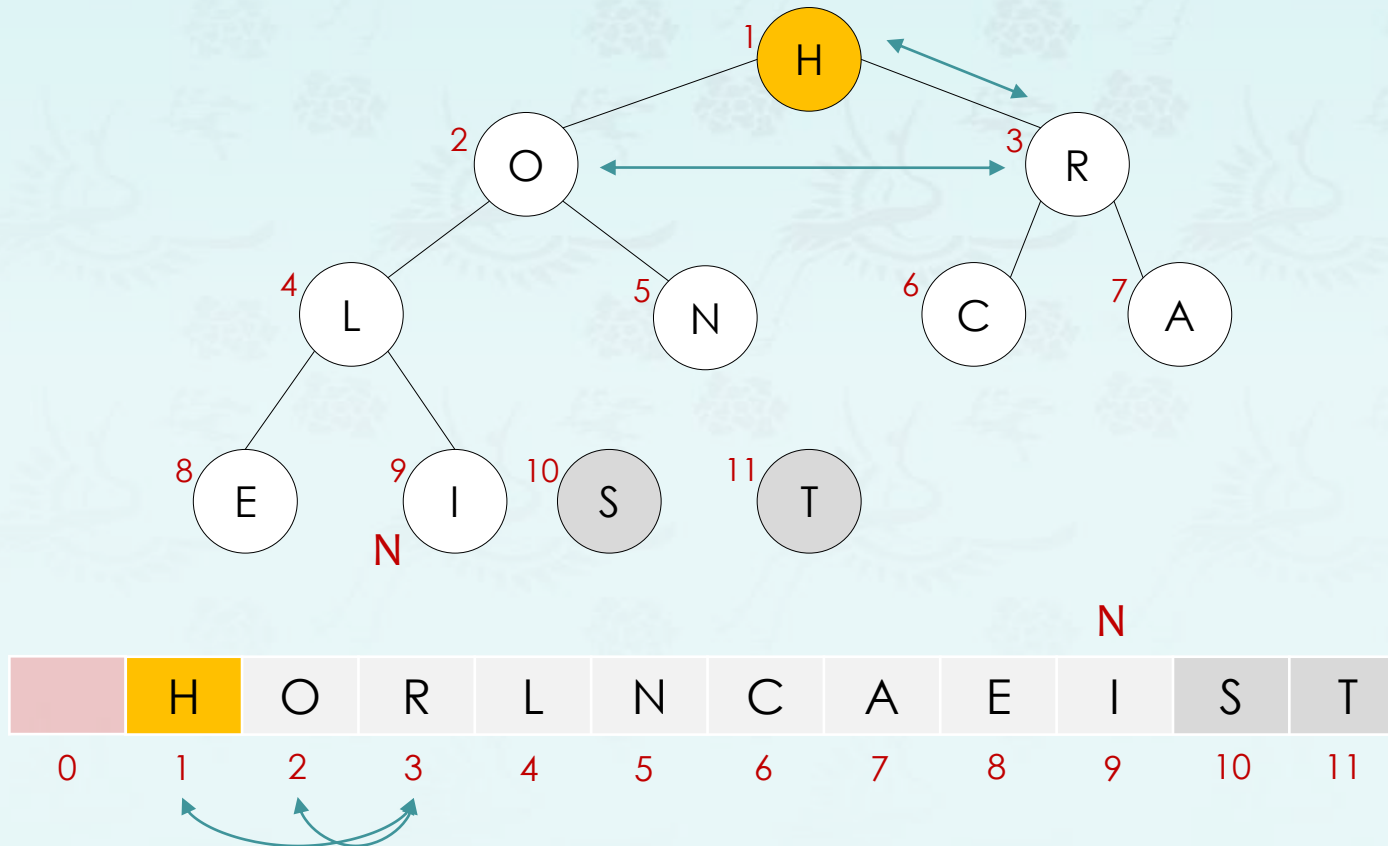
N=10
N=9

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

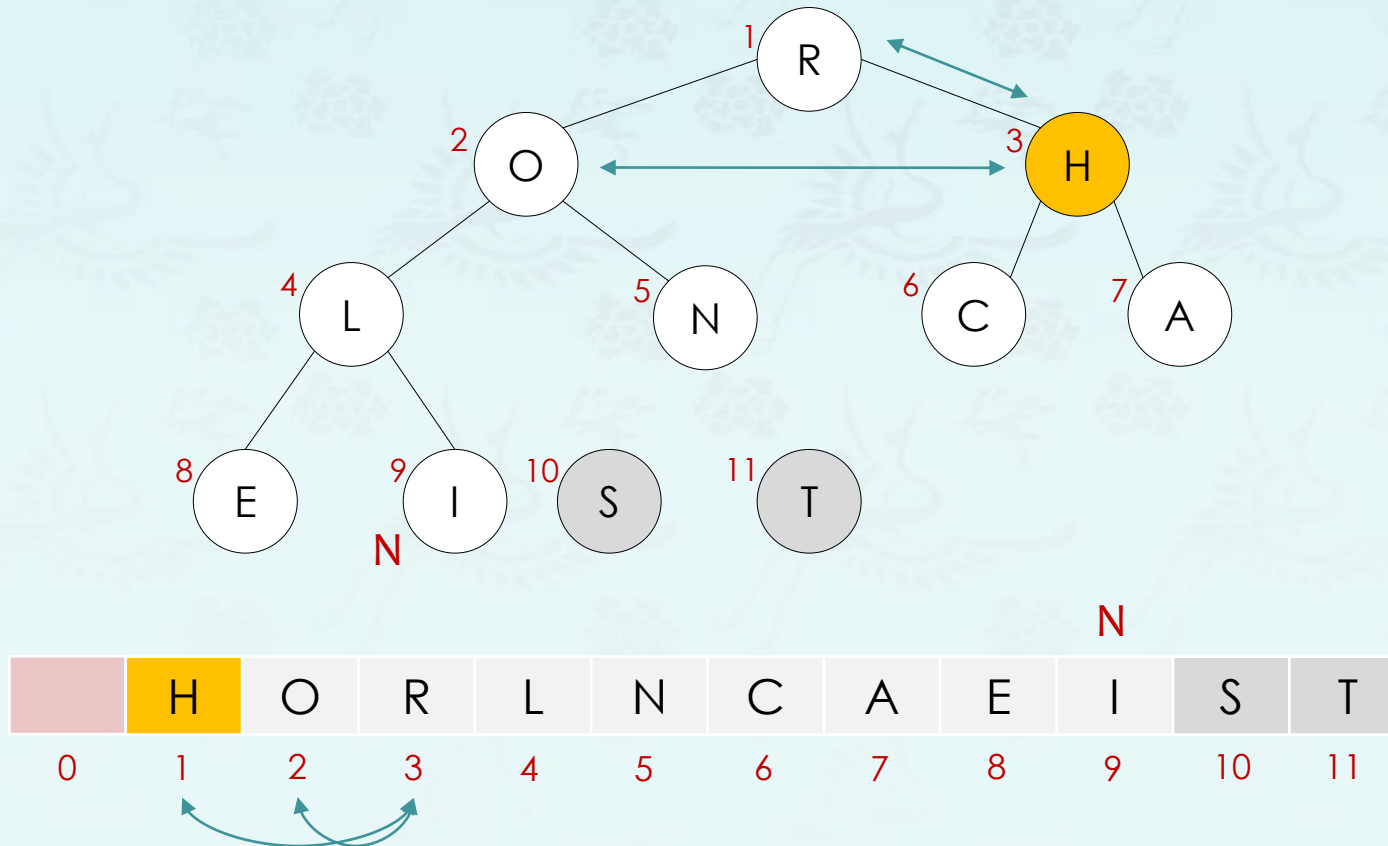
N=10
N=9

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

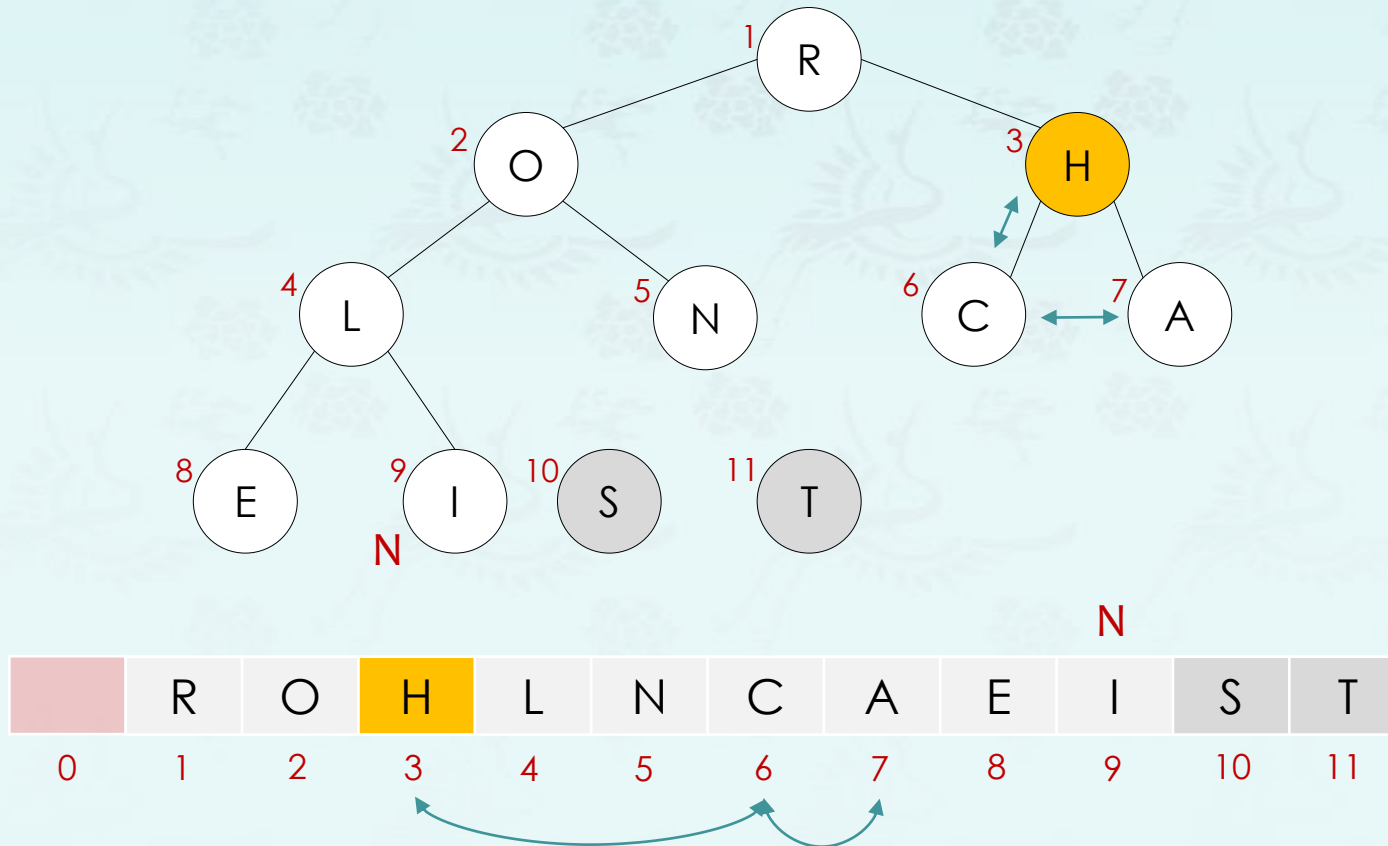
N=10
N=9

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

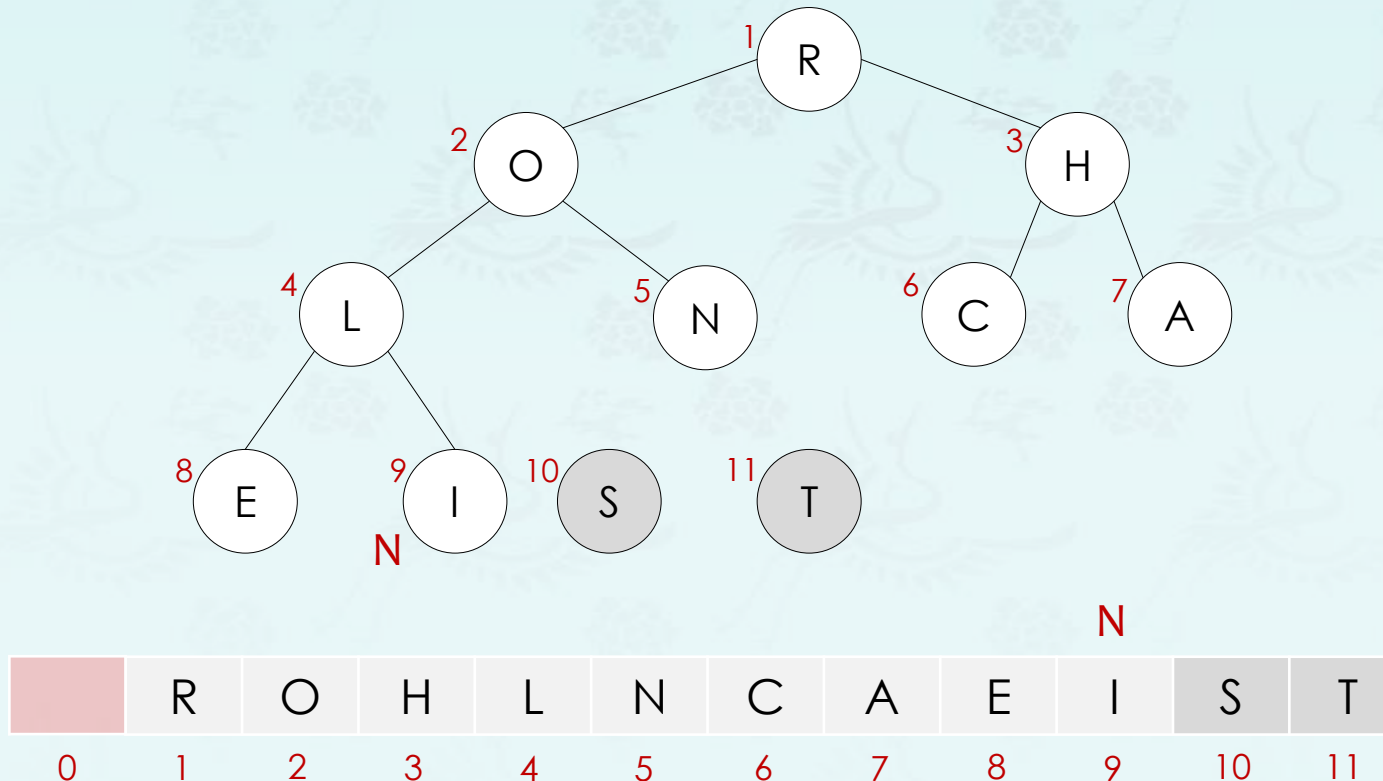
N=10
N=9

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

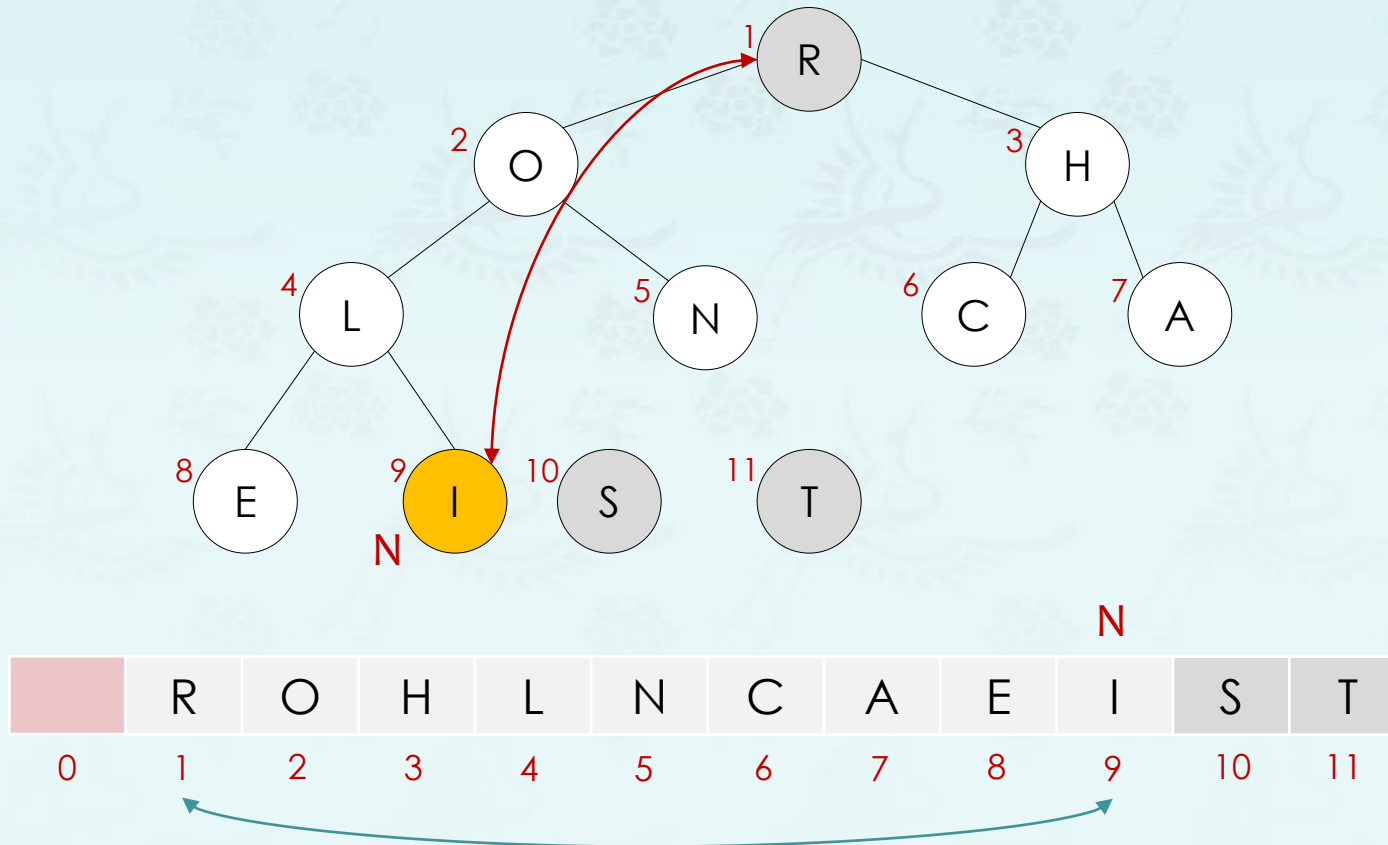
What's next?

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

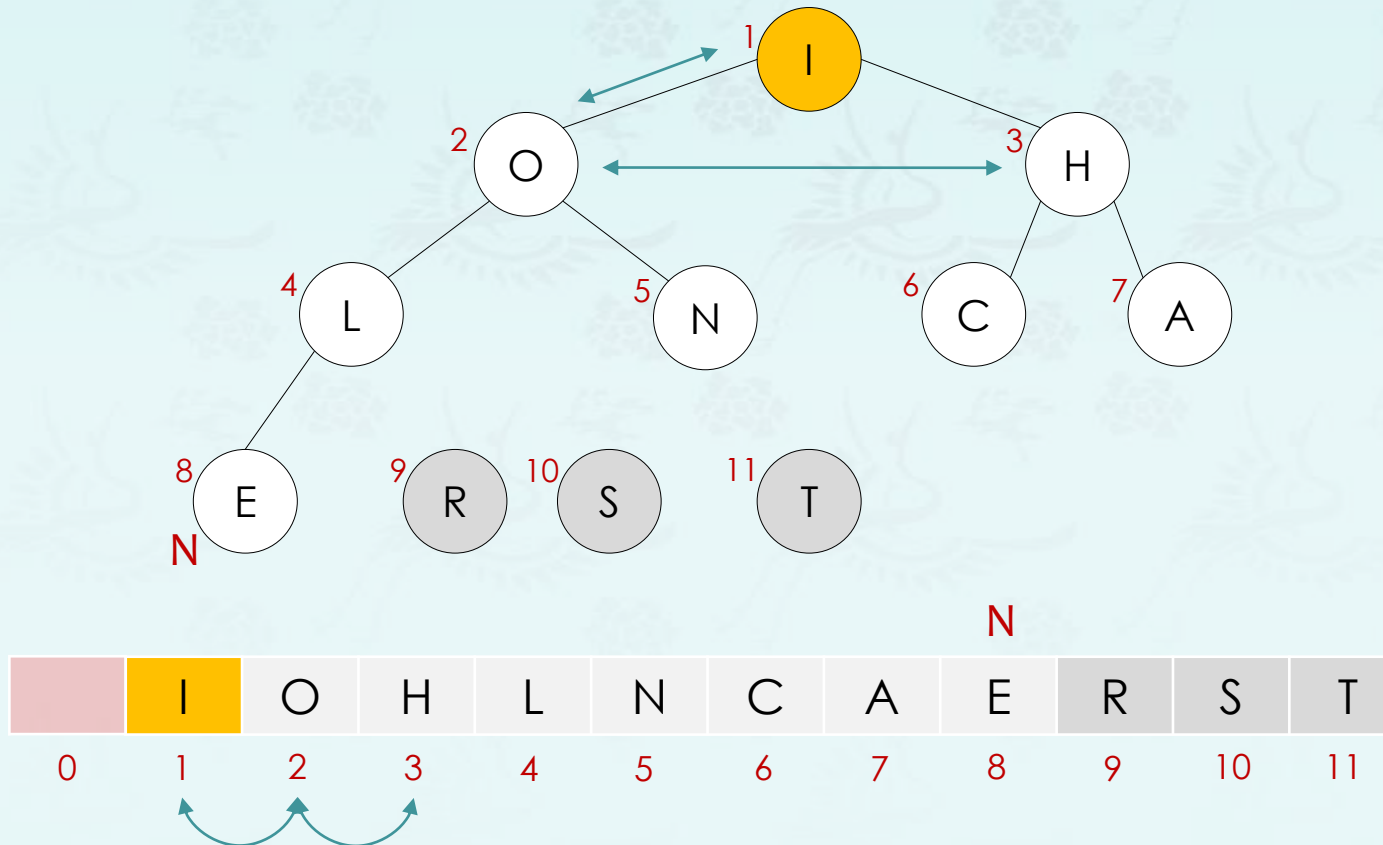
N=9
N=8

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

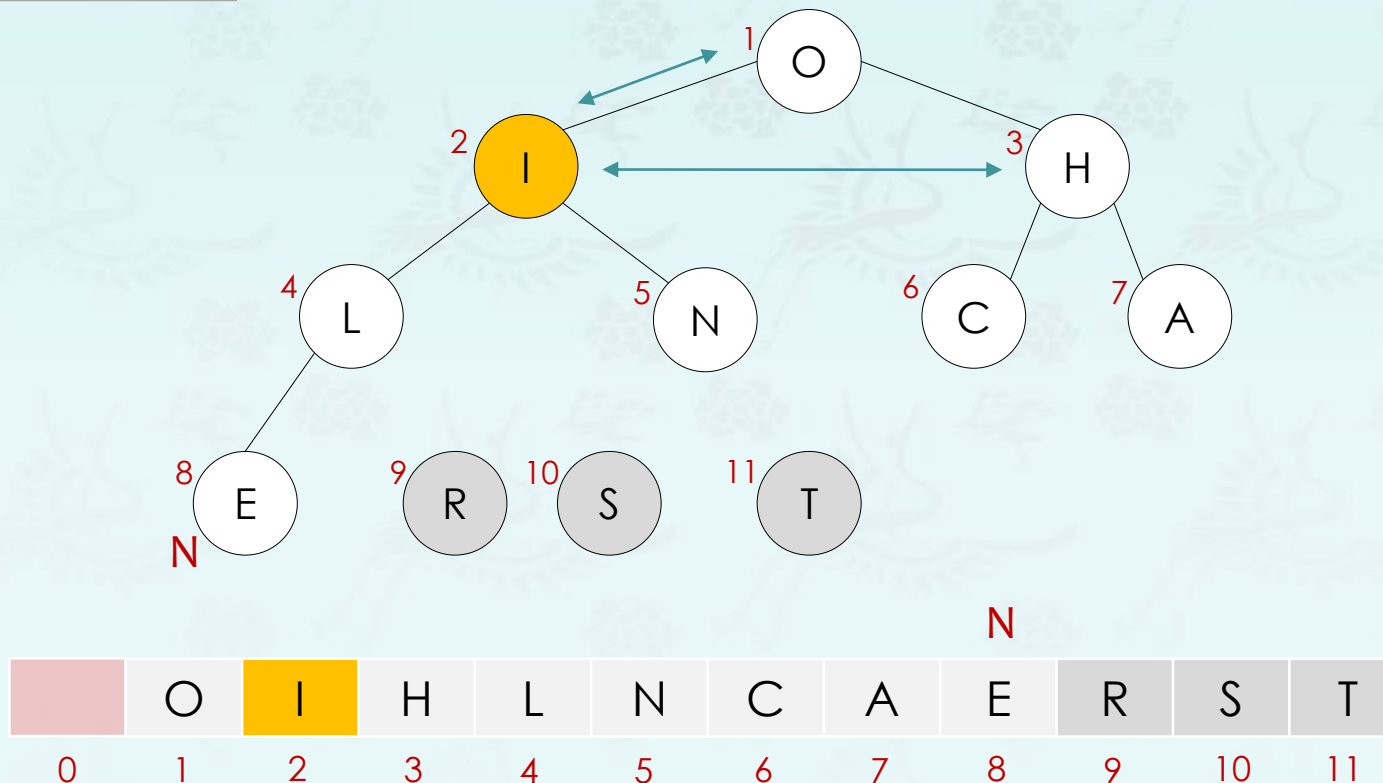
N=9
N=8

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

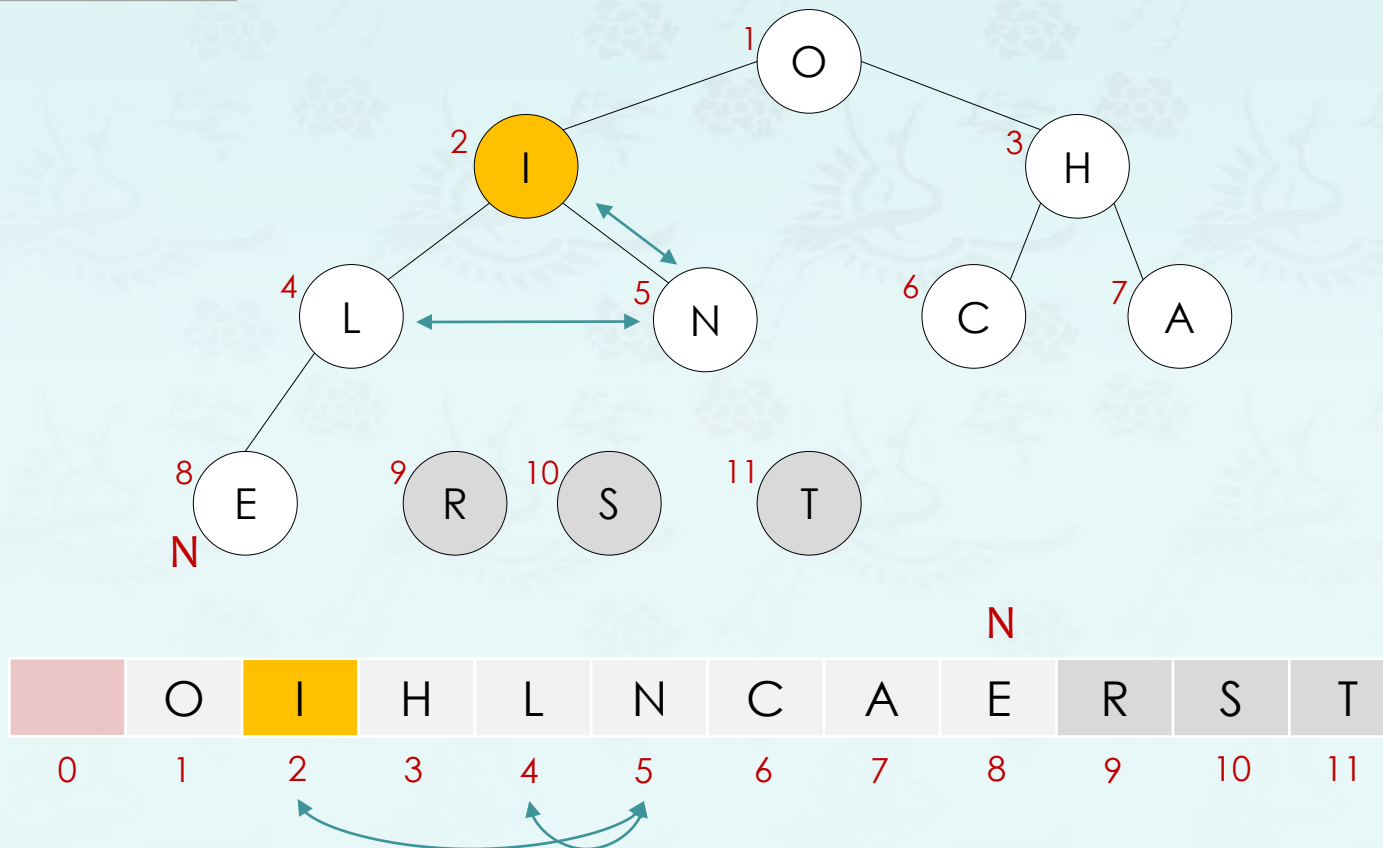
N=9
N=8

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

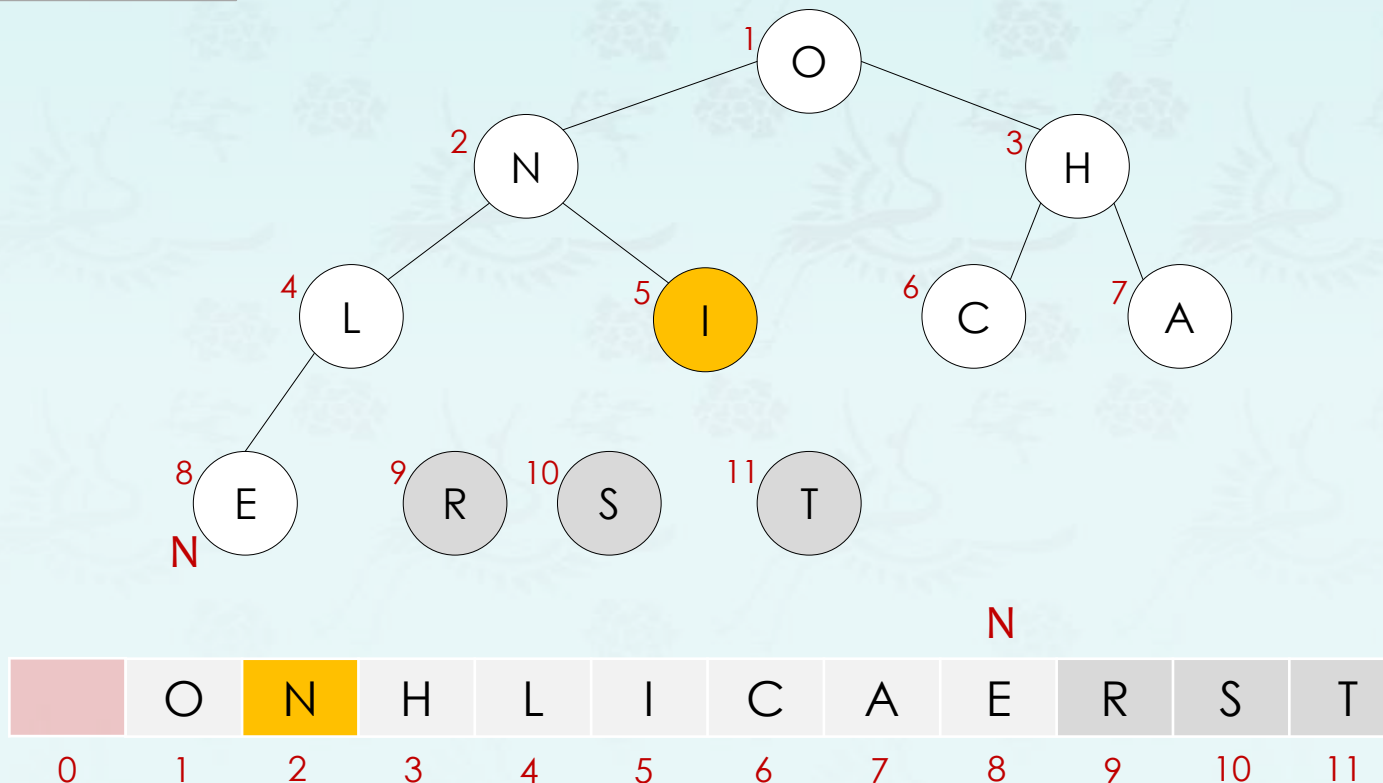
N=9
N=8

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

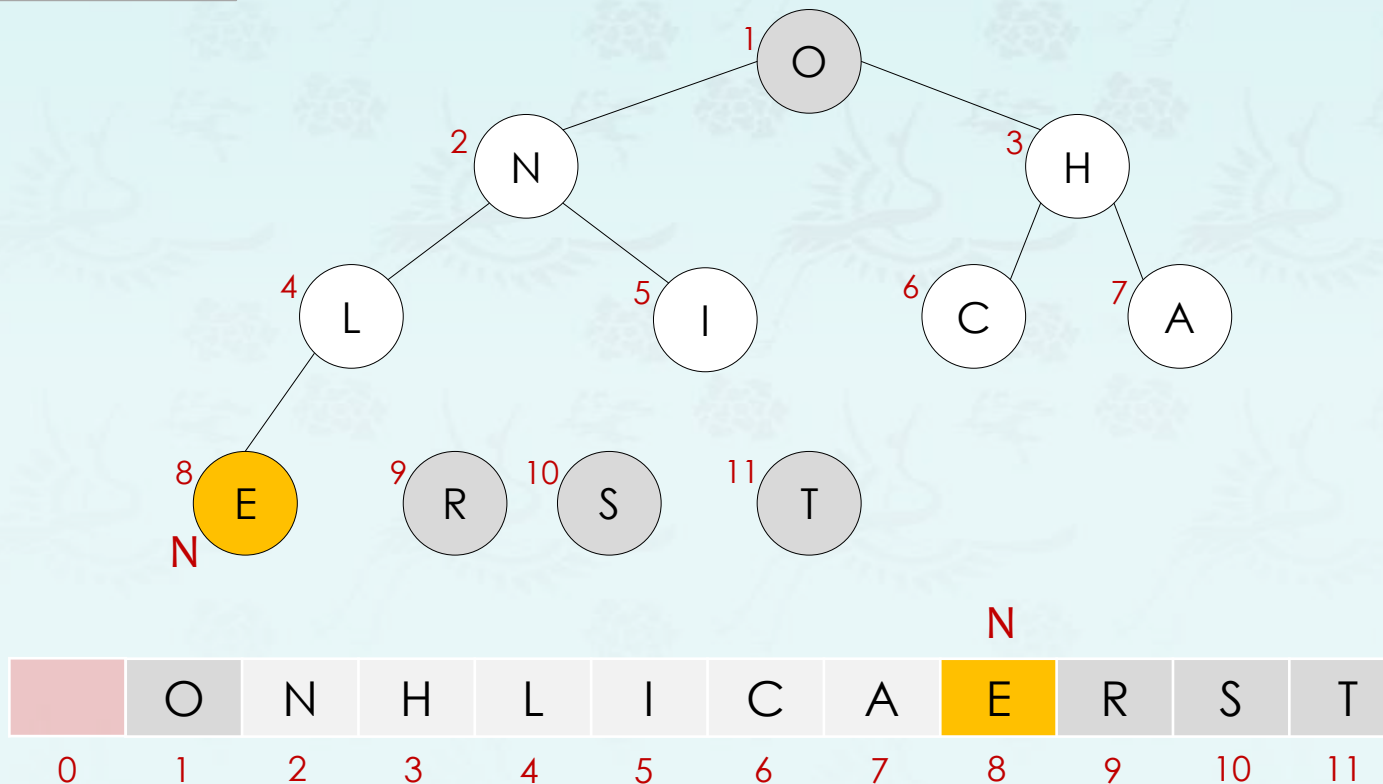
N=9
N=8

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

N=9
N=8

```
swap(1, N);  
sink(1, --N);
```

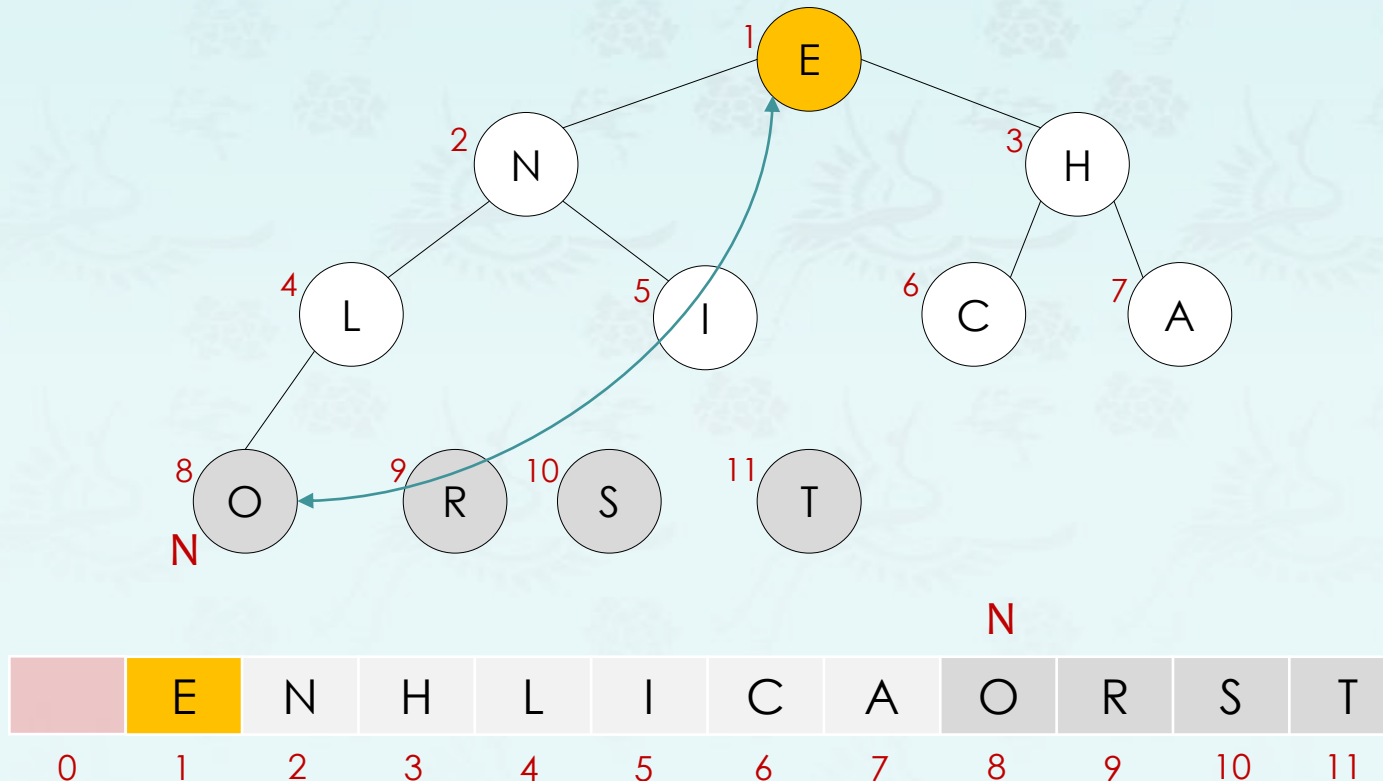
N=8
N=7

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

N=9
N=8

```
swap(1, N);  
sink(1, --N);
```

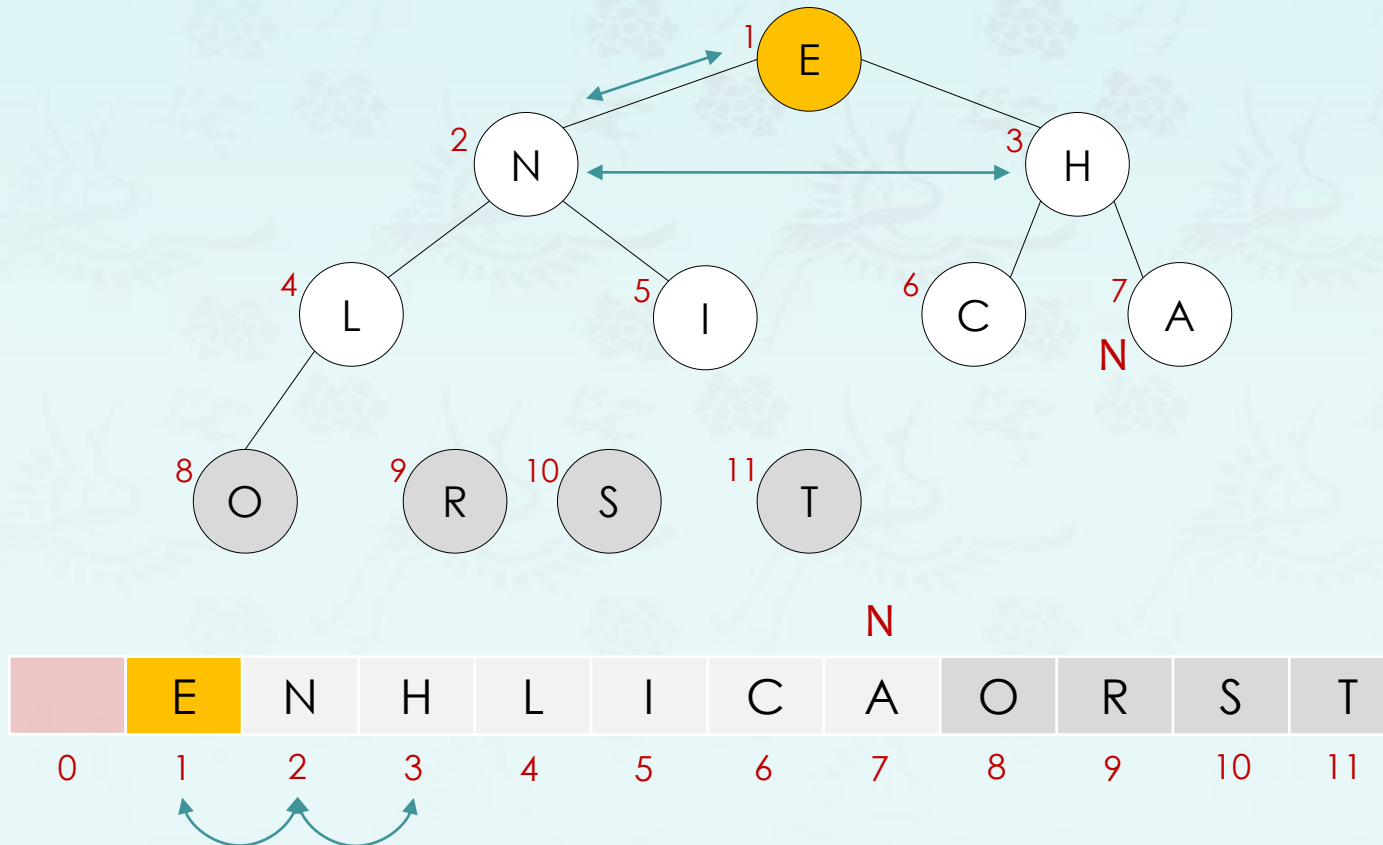
N=8
N=7

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

N=9
N=8

```
swap(1, N);  
sink(1, --N);
```

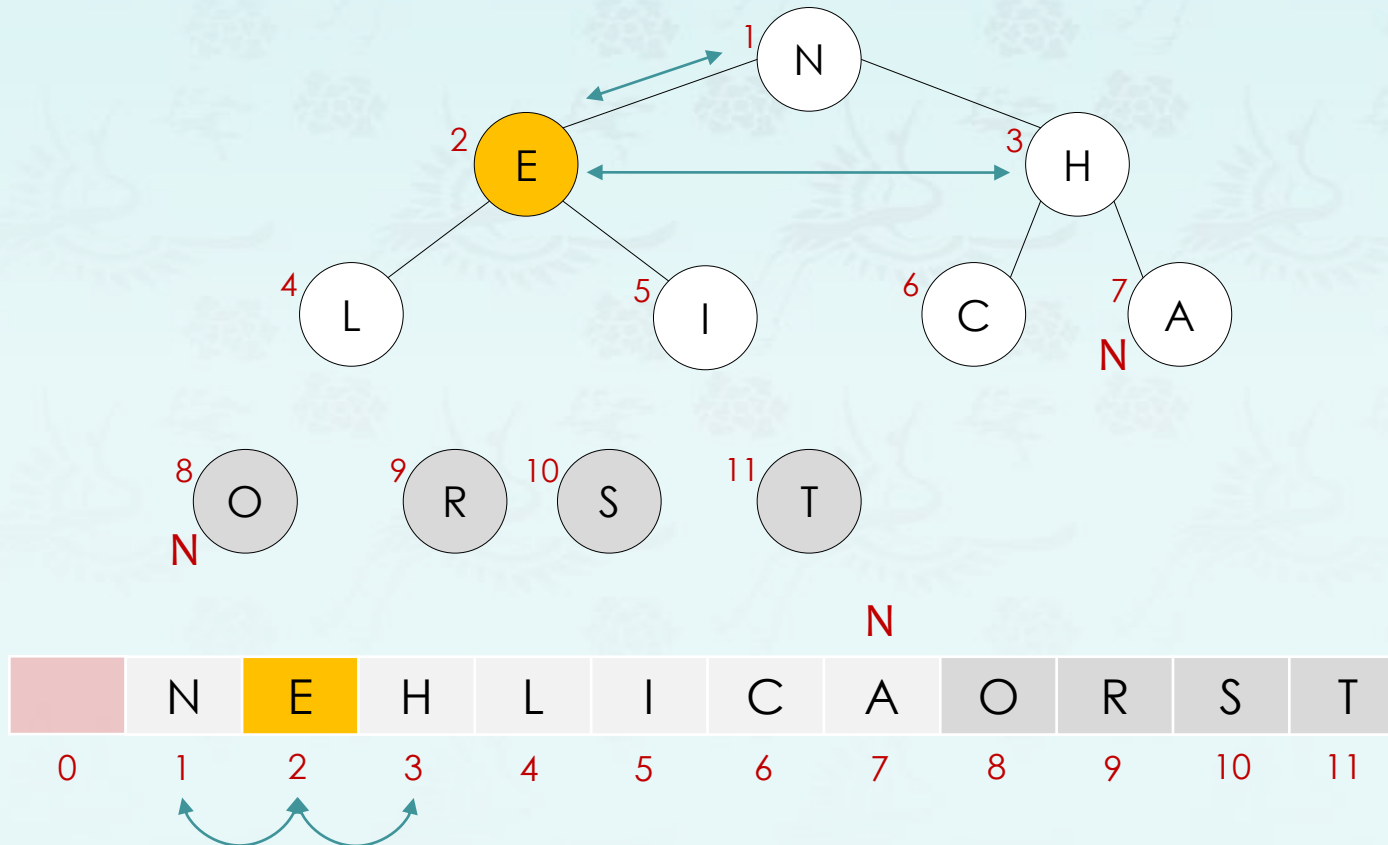
N=8
N=7

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

N=9
N=8

```
swap(1, N);  
sink(1, --N);
```

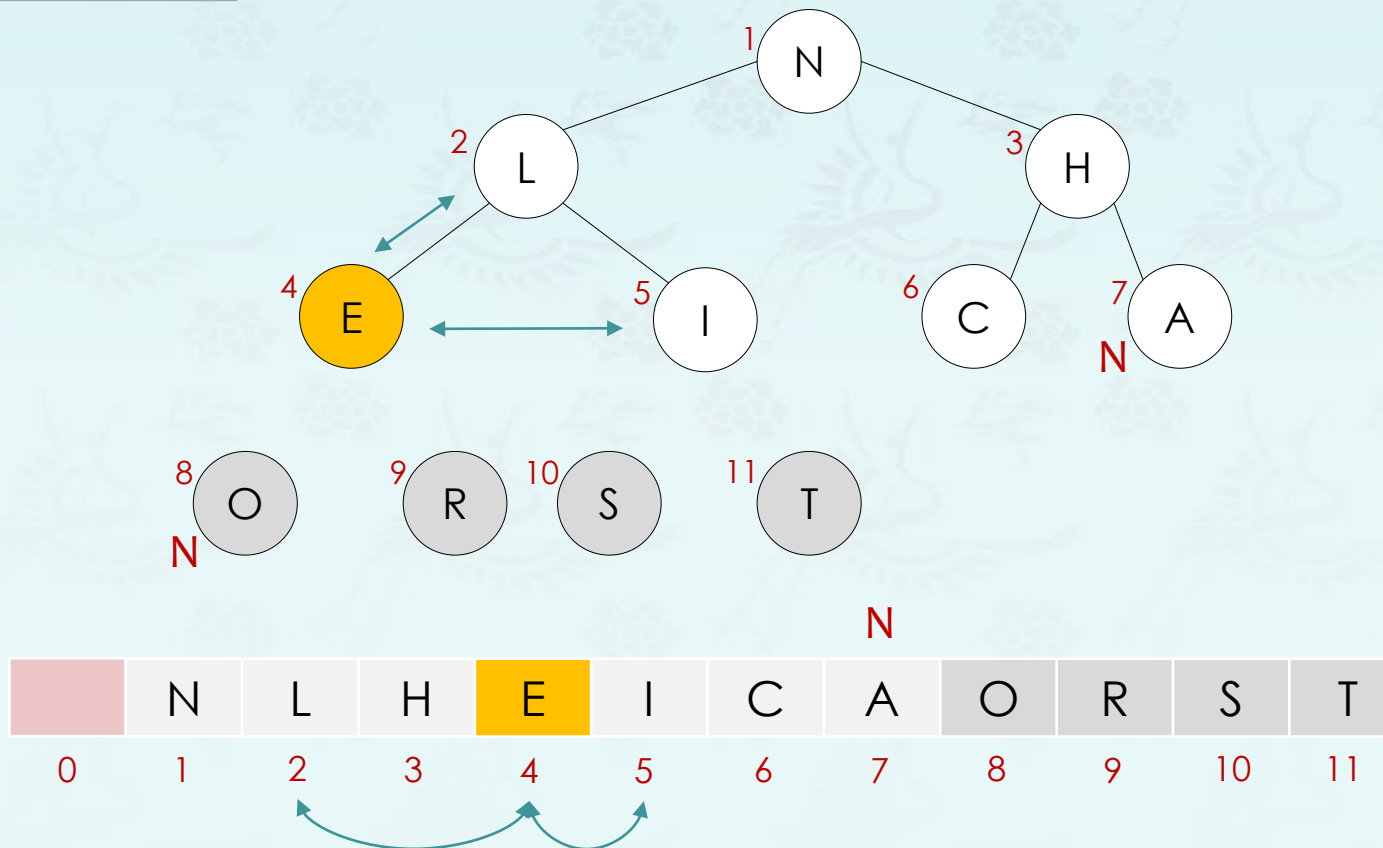
N=8
N=7

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

N=9
N=8

```
swap(1, N);  
sink(1, --N);
```

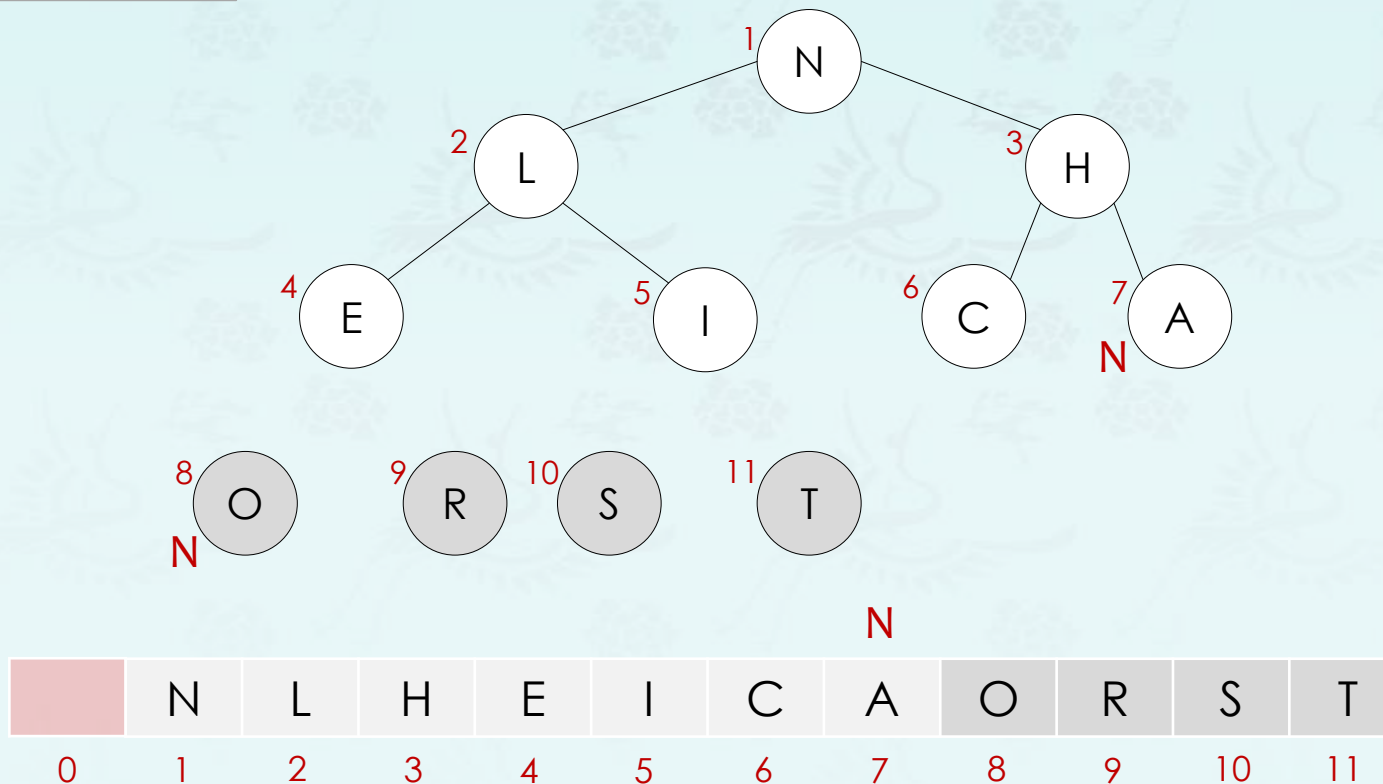
N=8
N=7

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out

array in heap ordered



```
swap(1, N);  
sink(1, --N);
```

N=11
N=10

```
swap(1, N);  
sink(1, --N);
```

N=10
N=9

```
swap(1, N);  
sink(1, --N);
```

N=9
N=8

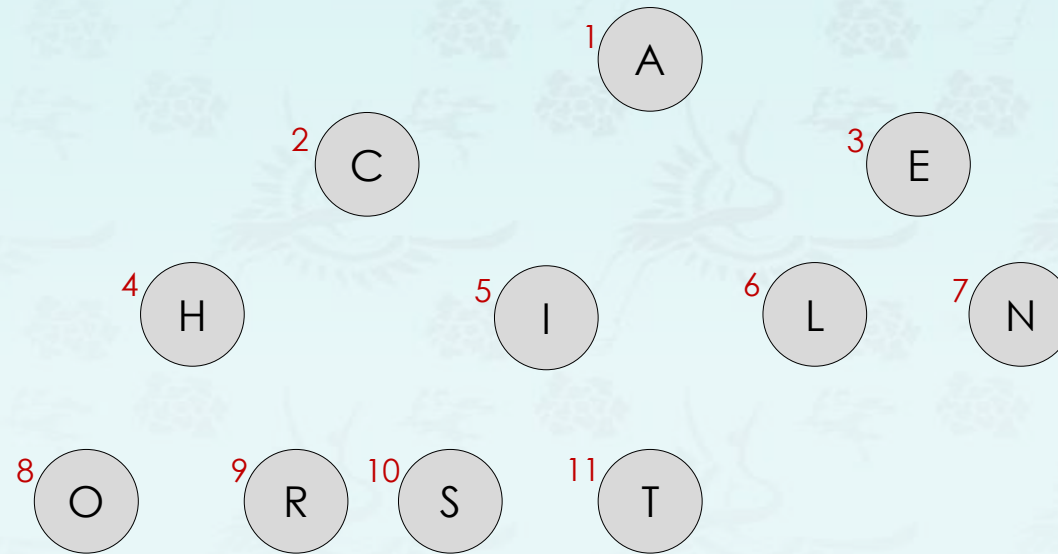
```
swap(1, N);  
sink(1, --N);
```

N=8
N=7

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out



```
swap(1, N); N=11  
sink(1, --N); N=10
```

```
swap(1, N); N=10  
sink(1, --N); N=9
```

```
swap(1, N); N=9  
sink(1, --N); N=8
```

```
swap(1, N); N=8  
sink(1, --N); N=7
```

```
swap(1, N); N=7  
swap(1, N); N=6  
swap(1, N); N=5  
swap(1, N); N=4  
swap(1, N); N=3  
swap(1, N); N=2  
swap(1, N); N=1
```

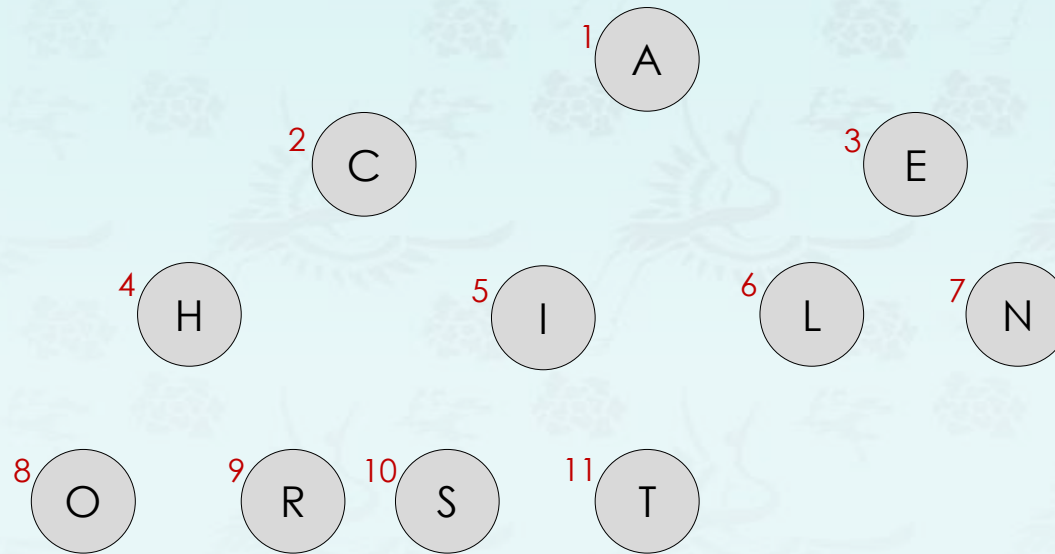
array sorted		A	C	E	H	I	L	N	O	R	S	T
	0	1	2	3	4	5	6	7	8	9	10	11

Heapsort

2nd Pass: Repeatedly remove the maximum key.

- Remove the maximum, one at a time.
- Leave them in array, instead of nulling out
- After sorted, do not forget resetting N.

```
while (N > 1) {  
    swap(a, 1, N);  
    sink(a, 1, --N);  
} // reset N = 11
```



```
swap(1, N);    N=11  
sink(1, --N);  N=10
```

```
swap(1, N);    N=10  
sink(1, --N);  N=9
```

```
swap(1, N);    N=9  
sink(1, --N);  N=8
```

```
swap(1, N);    N=8  
sink(1, --N);  N=7
```

```
swap(1, N);    N=7  
sink(1, --N);  N=6  
swap(1, N);    N=6  
sink(1, --N);  N=5  
swap(1, N);    N=5  
sink(1, --N);  N=4  
swap(1, N);    N=4  
sink(1, --N);  N=3  
swap(1, N);    N=3  
sink(1, --N);  N=2  
swap(1, N);    N=2  
sink(1, --N);  N=1
```

array sorted		A	C	E	H	I	L	N	O	R	S	T
	0	1	2	3	4	5	6	7	8	9	10	11

Heapsort tracing

```
Enter a word to sort: CHRISTALONE
Input String:[ CHRISTALONE ], N=11
Input  a[11]: C H R I S T A L O N E
ASCENDING:
```

```
1st pass(heapify -  $O(n)$ ) begins:
```

```
  N=11 k=5 C H R I S T A L O N E
  N=11 k=4 C H R O S T A L I N E
  N=11 k=3 C H T O S R A L I N E
  N=11 k=2 C S T O N R A L I H E
  N=11 k=1 T S R O N C A L I H E
```

```
HeapOrdered: T S R O N C A L I H E
```

```
2nd pass(swap and sink -  $O(n \log n)$ ) begins:
```

```
  N=10 k=1 S O R L N C A E I H
  N=9  k=1 R O H L N C A E I
  N=8  k=1 O N H L I C A E
  N=7  k=1 N L H E I C A
  N=6  k=1 L I H E A C
  N=5  k=1 I E H C A
  N=4  k=1 H E A C
  N=3  k=1 E C A
  N=2  k=1 C A
  N=1  k=1 A
```

```
a[11]: A C E H I L N O R S T
```

← printed in main()
← printed in main()

1st path
printed in sink()

← printed in heapSort()

2nd path
printed in sink()

← printed in main()

NOTE: This implementation does not sort the first element in the array.
NOTE: N=?? k=? lines are outputs at the end of each sink()

Binary heap operations time complexity with N items:

- Level of heap is $\lfloor \log_2 N \rfloor$
- insert: $O(\log N)$ for each insert
 - In practice, expect less
- delete: $O(\log N)$ // deleting root node in min/max heap
- decreaseKey: $O(\log N)$
- increaseKey: $O(\log N)$
- remove: $O(\log N)$ // removing a node in any location

- **Heapify():** $O(N)$
- **Heapsort():** $O(n \log n)$
- Because $O(N)$ heapify + $O(n \log n)$ remove nodes = $O(n \log n)$
- **Proof:**
 - <https://stackoverflow.com/questions/9755721/how-can-building-a-heap-be-on-time-complexity>
 - <https://www.growingwiththeweb.com/data-structures/binary-heap/build-heap-proof/>
 - <https://www.quora.com/How-is-the-time-complexity-of-building-a-heap-is-o-n>
 - <http://www.cs.umd.edu/~meesh/351/mount/lectures/lect14-heapsort-analysis-part.pdf>