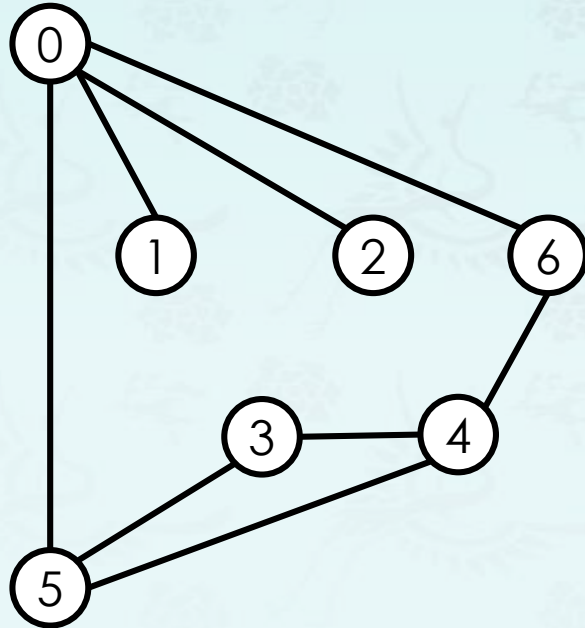# Graph

- Adjacency list processing
- Graph API - Implementation
  - **Cycle**
  - Bipartite

Major references:
1. Fundamentals of Data Structures by Horowitz, Sahni, Anderson-Freed,
2. Algorithms 4th edition - Part 1 & Part 2 by Robert Sedgewick and Kevin Wayne
3. Wikipedia and many resources available from internet

Adjacency list processing

Challenge: How to process adj[v] and its vertices:

V-E lists

Adjacency lists

adj[]

| 0 | 6 | 2 | 1 | 5 |

| 1 | 0 |

| 2 | 0 |

| 3 | 5 | 4 |

| 4 | 5 | 6 | 3 |

| 5 | 3 | 4 | 0 |

| 6 | 0 | 4 |

```
graph3.txt
13        ←      V
13        ←      E
0  5
4  3
0  1
9  12
6  4
5  4
0  2
11  12
9  10
0  6
7  8
9  11
5  3
```

Graph g

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Adjacency list processing

Challenge: How to process adj[v] and its vertices:

```
// print the adjacency list of graph
void print_adjlist(graph g) {

  cout << "\n\tAdjacency-list: \n";
  for (int v = 0; v < V(g); ++v) {
    cout << "\tV[" << v << "]: ";
    gnode w = g->adj[v].next;
    while (w) {
        ~~

    }
    cout << endl;
  }
}
```

Adjacency lists

adj[]

0	6 2 1 5
1	0
2	0
3	5 4
4	5 6 3
5	3 4 0
6	0 4

Adjacency list processing

Challenge: How to process adj[v] and its vertices:

```
// print the adjacency list of graph
void print_adjlist(graph g) {

  cout << "\n\tAdjacency-list: \n";
  for (int v = 0; v < V(g); v++) {
    cout << "\tV[" << v << "]: ";
    for (gnode w = g->adj[v].next; w; w = w->next) {

        ~~
    }
}
```

Adjacency lists

adj[]

0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	3	4	0	
6	0	4		

Cycle detection using depth-first search
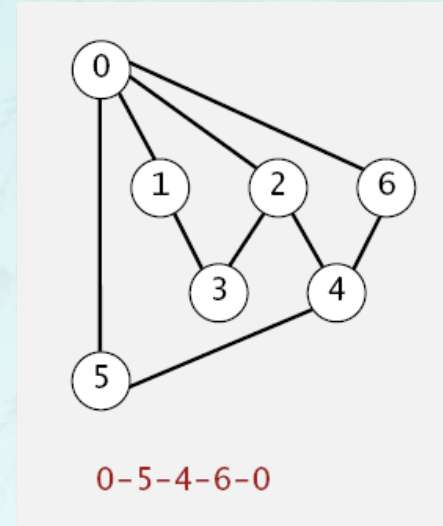
Problem: Find a cycle.

0-5-4-6-0

How difficult?

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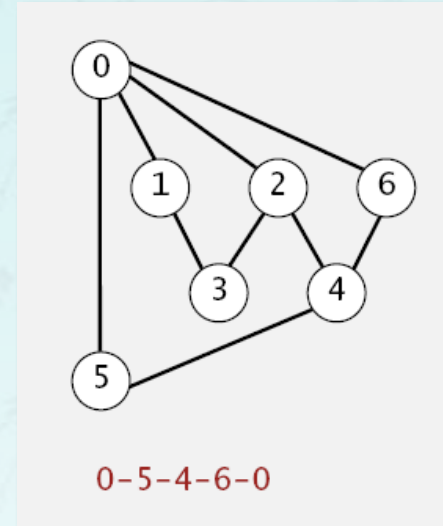
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Cycle detection using depth-first search

Problem: Find a cycle.

0-5-4-6-0

How difficult?
1. Any programmer could do it.
2. Typical diligent algorithms student could do it.
3. Hire an expert.
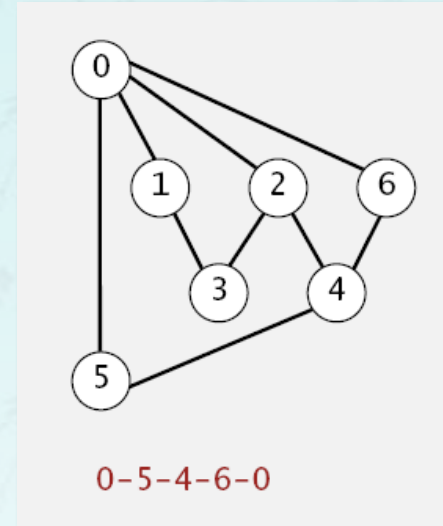4. Intractable.
5. No one knows.
6. Impossible.

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Cycle detection using depth-first search

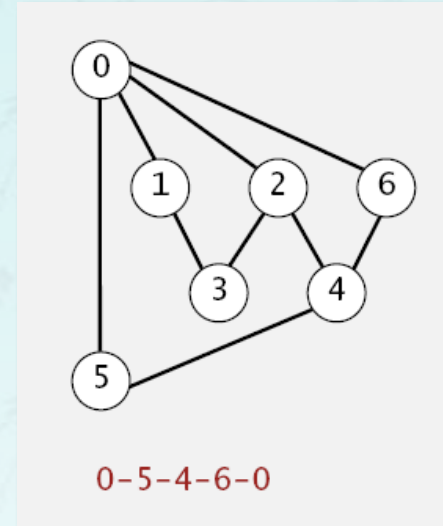**Problem: Find a cycle.**

0-5-4-6-0

How difficult?
1. Any programmer could do it.
2. Typical diligent algorithms student could do it.
3. Hire an expert.
4. Intractable.
5. No one knows.
6. Impossible.

simple DFS-based solution

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Cycle detection using depth-first search

Problem: Find a cycle.
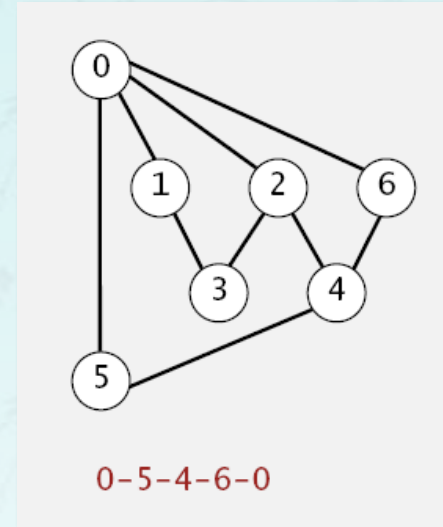

0-5-4-6-0

- A *cycle* is a path (with at least one edge) whose first and last vertices are the same.
- A *simple cycle* is a cycle with no repeated edges or vertices (except the *requisite* repetition of the first and last vertices).

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Cycle detection using depth-first search

Challenge - Cycle detection: Is a given graph cyclic?

Implementation: Use depth-first search to determine whether a graph has a cycle, and if so return one.
It takes time proportional to V + E in the worst case.

0-5-4-6-0

- A *cycle* is a path (with at least one edge) whose first and last vertices are the same.
- A *simple cycle* is a cycle with no repeated edges or vertices (except the requisite repetition of the first and last vertices).

Cycle detection **example** using depth-first search

To visit a vertex v:

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.

Graph g:

```
graph6.txt
5          ←——  V
5          ←——  E
0 1
0 3
1 2
2 3
3 4
```
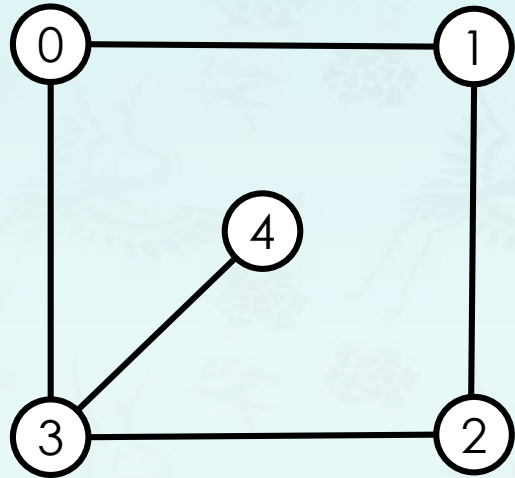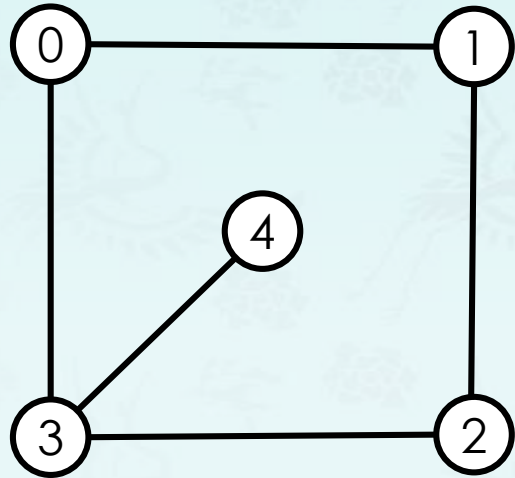
Challenge: build adjacency lists?

Cycle detection **example** using depth-first search

To visit a vertex v:
- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.

Adjacency lists

adj[]

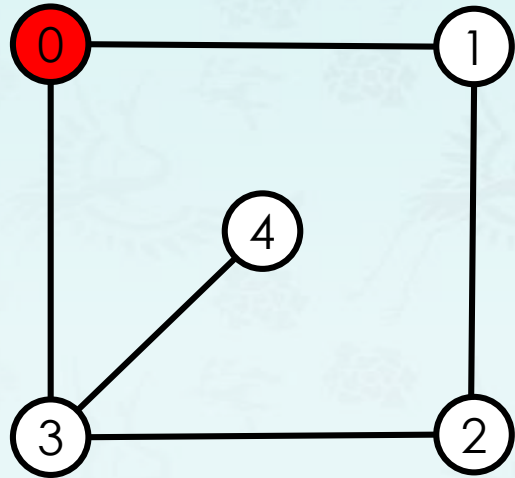|		
0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

graph6.txt
```
5        ← V
5        ← E
0 1
0 3
1 2
2 3
3 4
```

Graph g:

Cycle detection **example** using depth-first search

To visit a vertex v:
- Mark vertex v as visited.
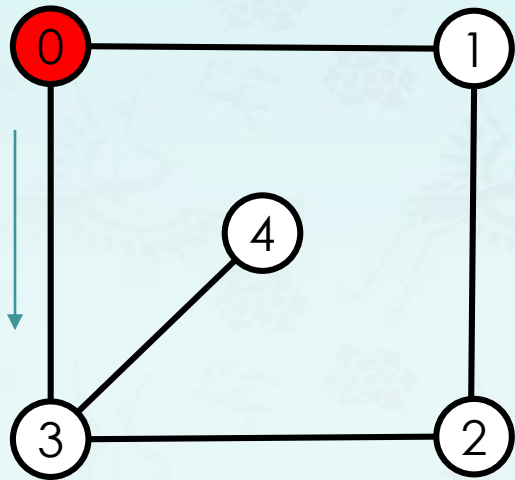- Recursively visit all unmarked vertices adjacent to v.

Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

3		1	

visit 0: check 3, **check 1**

DFS: 0

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Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3

| 3 | 1 |

visit 0: check 3, **check 1**

DFS: 0

Adjacency lists

adj[]

0	3		1	
1	2		0	
2	3		1	
3	4		2		0	
4	3	

| 4 | | 2 | | 0 | |
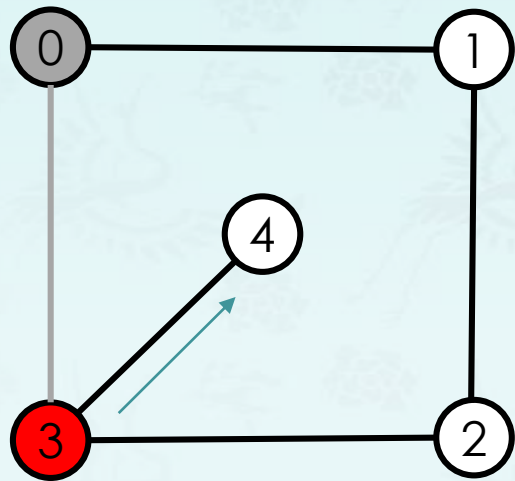
visit 3: check 4, **check 2, check 0**

DFS: 0 3

Adjacency lists

adj[]
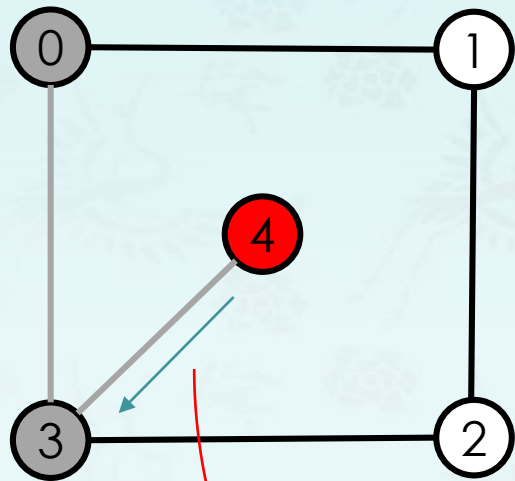
0	3		1	
1	2		0	
2	3		1	
3	4		2		0	
4	3			

This is not cycle.
When you look for a cycle,
disregard the back edge leading to the previous vertex.

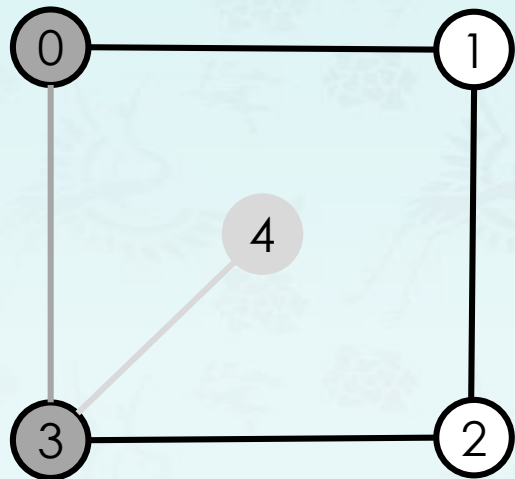| 3	

visit 4: check 3

DFS: 0 3 4

Adjacency lists

adj[]
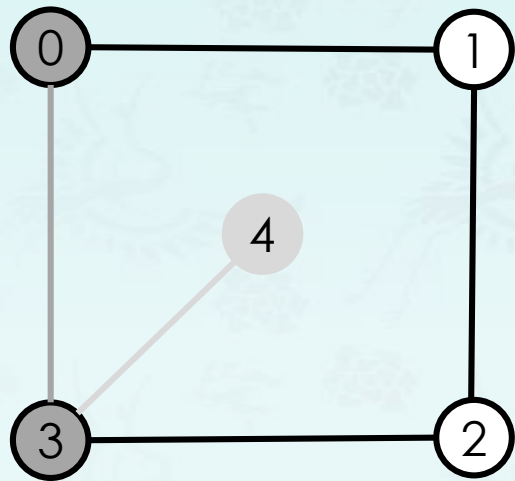
0	3		1		
1	2		0		
2	3		1		
3	4		2		0
4	3				

4 done

DFS: 0 3 4

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Adjacency lists

adj[]

0	3		1	
1	2		0	
2	3		1	
3	4		2		0	
4	3	

| 4 | | 2 | | 0 | |

visit 3: check 4, **check 2**, **check 0**

DFS: 0 3 4

Adjacency lists

adj[]

0	3	1	
1	2	0	
2	3	1	
3	4	2	0
4	3		

| 4 | 2 | 0 | |

visit 3: check 4, **check 2**, **check 0**

DFS: 0 3 4

Adjacency lists

adj[]

0	3	1	
1	2	0	
2	3	1	
3	4	2	0
4	3		

This is not cycle.

| 3 | | 1 | |

visit 2: **check 3,** check 1

DFS: 0 3 4 2

Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3

3 | 1 |

visit 2: **check 3,** check 1

DFS: 0 3 4 2

Adjacency lists

adj[]

0	3	1	
1	2	0	
2	3	1	
3	4	2	0
4	3		

| 3 | 1 |

visit 2: check 3, **check 1**

DFS: 0 3 4 2

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Cycle detection **example** using depth-first search

Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

This is not cycle.

| 2 | 0 | |

visit 1: check 2, **check 0**

DFS: 0 3 4 2 1

Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

| 2 | 0 | |

visit 1: check 2, **check 0**

DFS: 0 3 4 2 1

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Cycle is found!

Adjacency lists

adj[]

0	3		1	
1	2		0	
2	3		1	
3	4		2		0	
4	3	

0 → 1

4

3 → 2

| 2 | | 0 | |

visit 1: check 2, **check 0**

DFS: 0 3 4 2 1

Cycle detection **example** using depth-first search

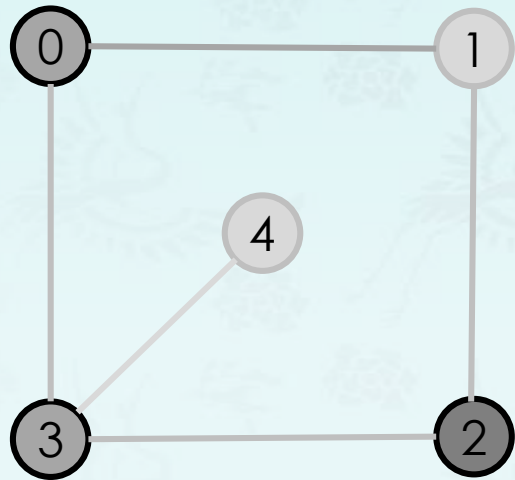Adjacency lists

adj[]			
0	3	1	
1	2	0	
2	3	1	
3	4	2	0
4	3		

1 done

DFS: 0 3 4 2 1

Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	
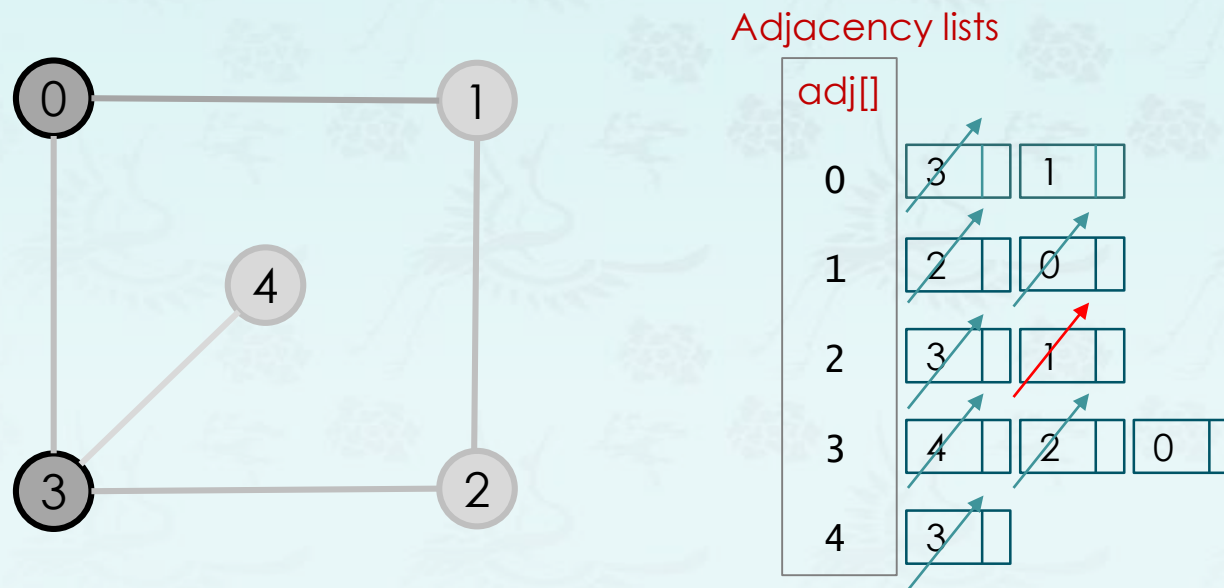
2 done   Once 1 done, 2 is done;
since it was recurred from "visit2: check 3, check 1"

DFS: 0 3 4 2 1

Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

visit 3: check 4, **check 2**, **check 0**

DFS: 0 3 4 2 1

Adjacency lists

adj[]
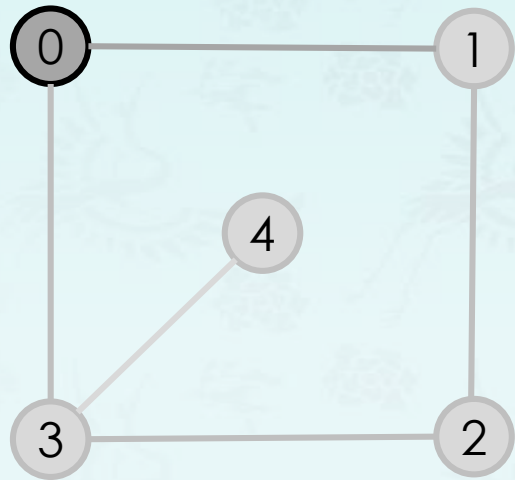
0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

3 done

DFS: 0 3 4 2 1
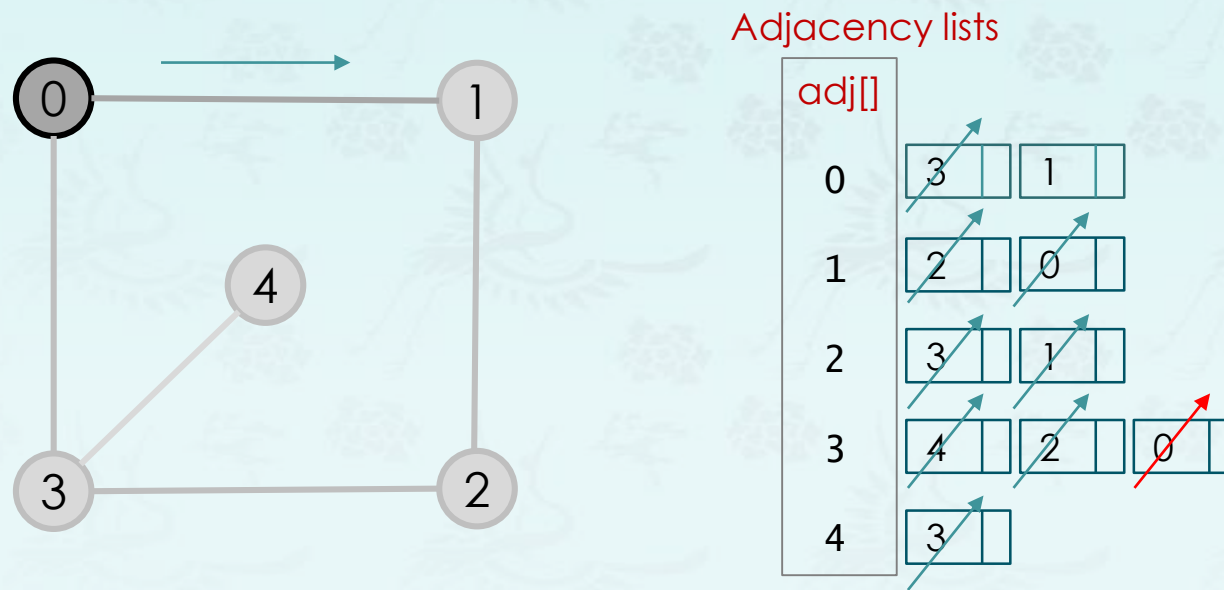
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Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

3		1	

visit 0: check 3, **check 1**

DFS: 0 3 4 2 1

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Cycle detection **example** using depth-first search

Adjacency lists

adj[]
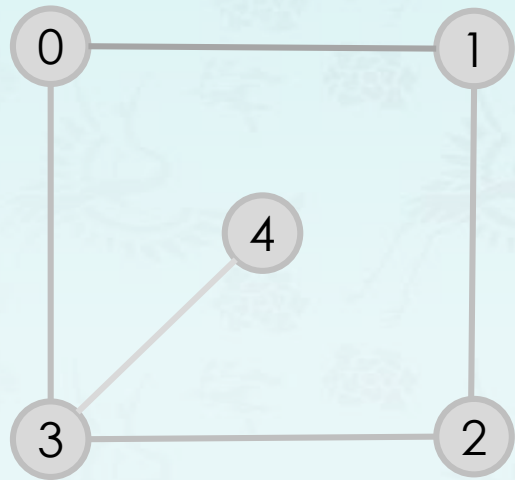
0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

0 done

DFS: 0 3 4 2 1

Cycle detection **example** using depth-first search

Adjacency lists

adj[]
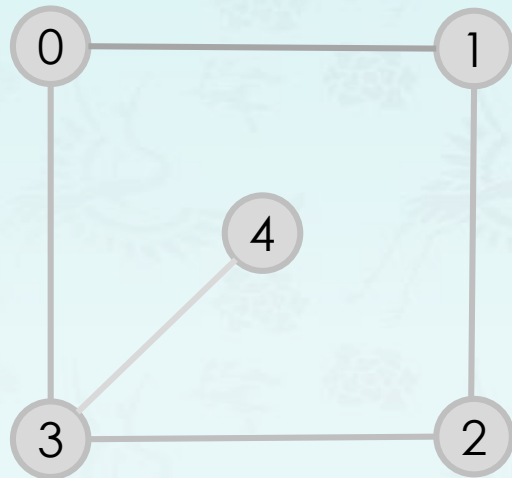
0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

visit(0)
check(3)
visit(3)
check(4)
visit(4)
check(3)
4 done
check(2)
visit(2)
check(3)
check(1)
visit(1)
check(2)
check(0)
1 done
2 done
check(0)
3 done
check(1)
0 done

DFS: 0 3 4 2 1

Cycle detection **example** using depth-first search

Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	
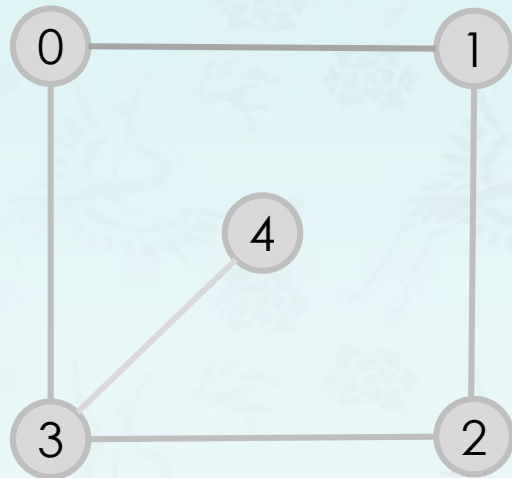
v	marked[]	parent[v]
0	T	−1
1	T	2
2	T	3
3	T	0
4	T	3

DFS: 0 3 4 2 1

Cycle detection **example** using depth-first search

Cycle is found:
- **push path**

Cycle is found!

Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	
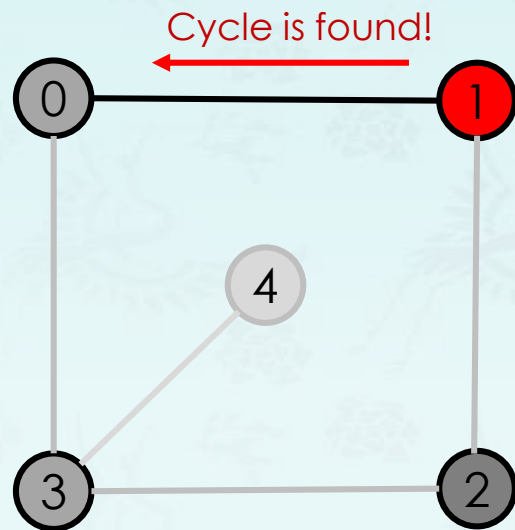
v	marked[]	parent[v]
0	T	–1
1	T	2
2	T	3
3	T	0
4	T	3

2	0	

visit 1: check 2, **check 0**

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Cycle detection **example** using depth-first search

Cycle is found: ← starting at itself
- **push path (1, 2, 3 or retrace back parent[] until you hit 0)**

Cycle is found!

Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

v	marked[]	parent[v]
0	T	–1
1	T	2
2	T	3
3	T	0
4	T	3

push to stack
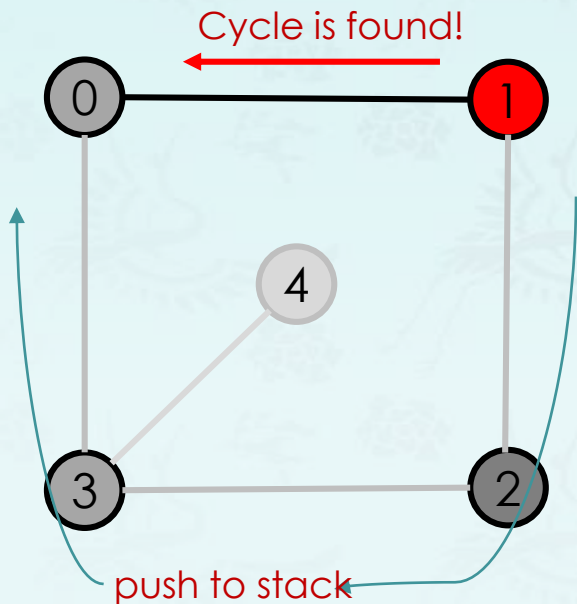
stack top

stack: 3, 2, 1

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Cycle detection **example** using depth-first search

Cycle is found:
- **push path (1, 2, 3 or retrace back parent[] until you hit 0)**
- **push 0**

Cycle is found!

Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

v	marked[]	parent[v]
0	T	–1
1	T	2
2	T	3
3	T	0
4	T	3

stack top

stack: 0, 3, 2, 1

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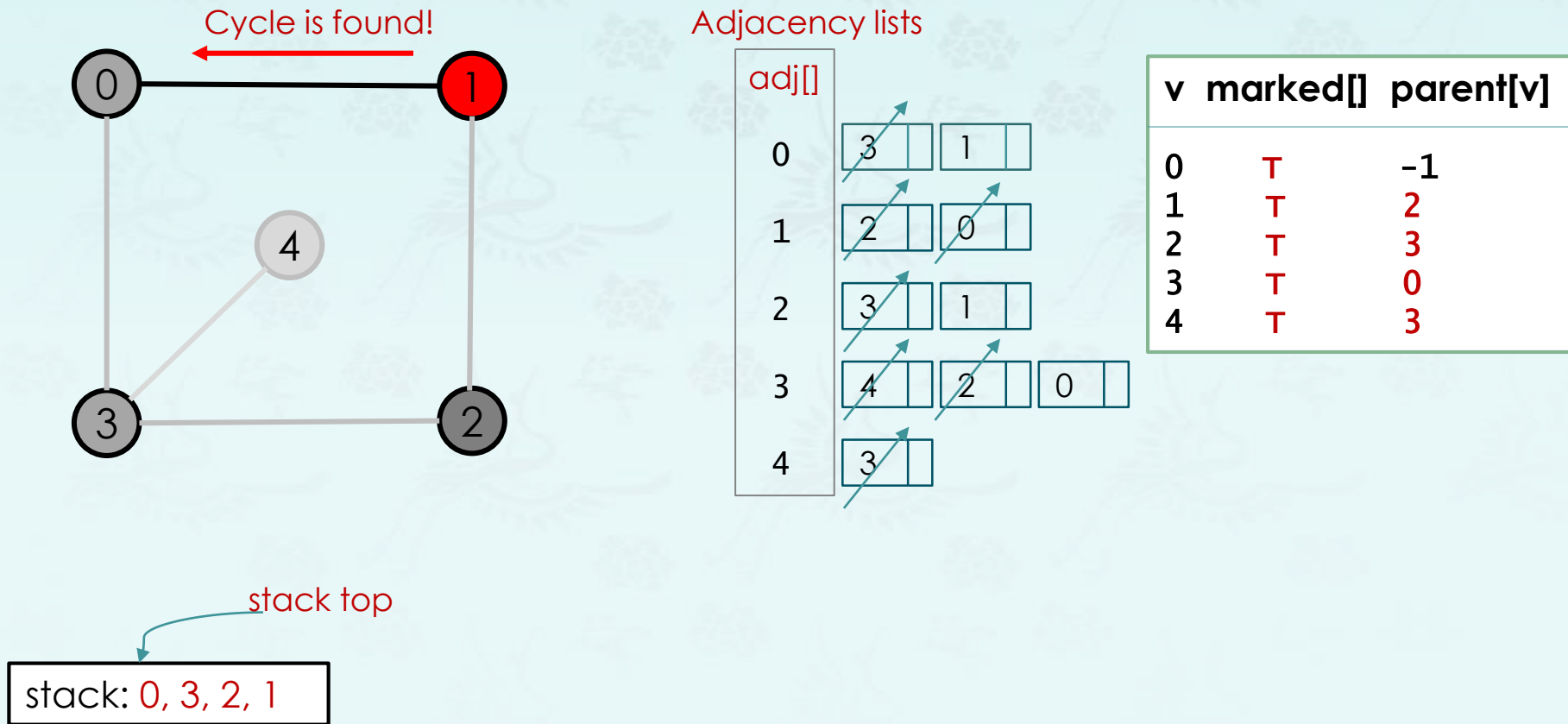
Cycle detection **example** using depth-first search

Cycle is found:
- **push path (1, 2, 3 or retrace back parent[] until you hit 0)**
- **push 0**
- **push 1 (to complete the cycle)**
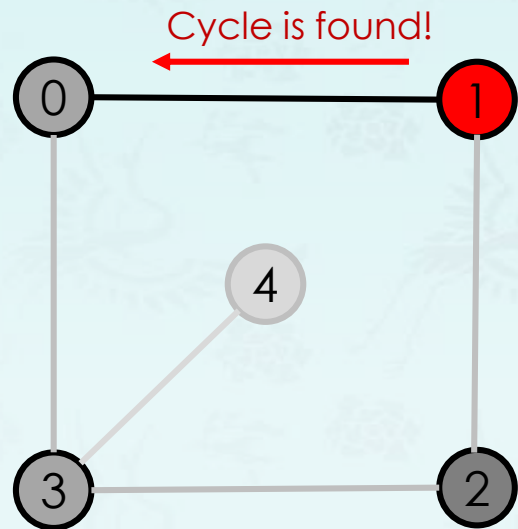
Cycle is found!

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Adjacency lists

adj[]

0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

v	marked[]	parent[v]
0	T	–1
1	T	2
2	T	3
3	T	0
4	T	3

stack top

stack: 0, 3, 2, 1   .... and add myself(1) to make a cycle   stack: 1, 0, 3, 2, 1

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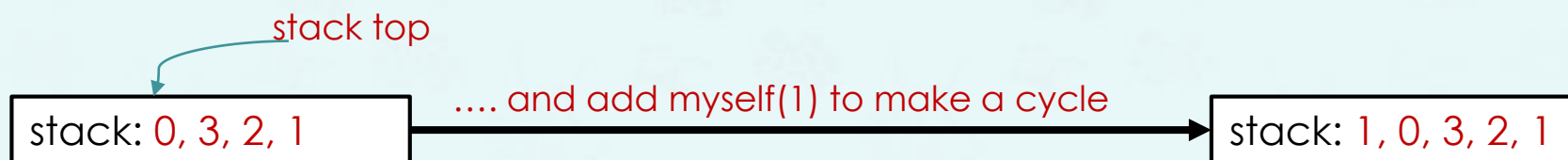
Cycle detection using DFS implementation

```
// finds a cycle in graph and returns a stack that has a list of vertices
// using DFS to find a cycle in the graph.
// The cycle() takes time proportional to V + E(in the worst case),
// where V is the number of vertices and E is the number of edges.

bool cyclic_at(graph g, int v, stack<int>& cy) {
  if (hasSelfLoop(g, cy) || hasParallelEdges(g, cy)) return true;

  for (int i = 0; i < V(g); i++) {
    g->marked[i] = false;
    g->parentDFS[i] = -1;
  }

  return DFScyclic(g, -1, v, cy); // u:vertex visited previously, v:visiting vertex
}
```
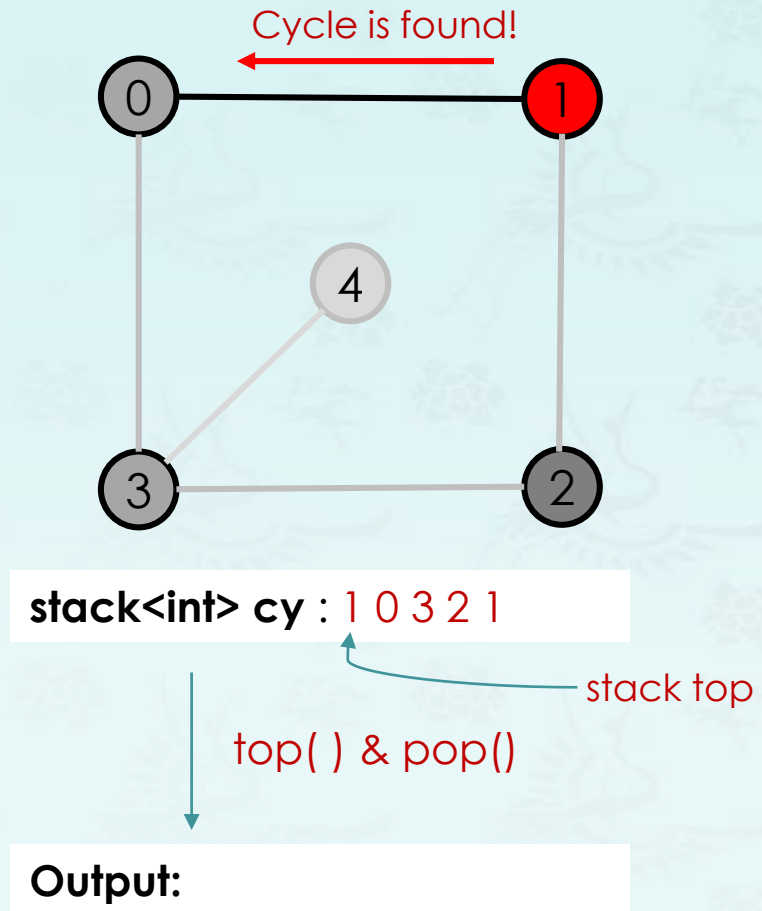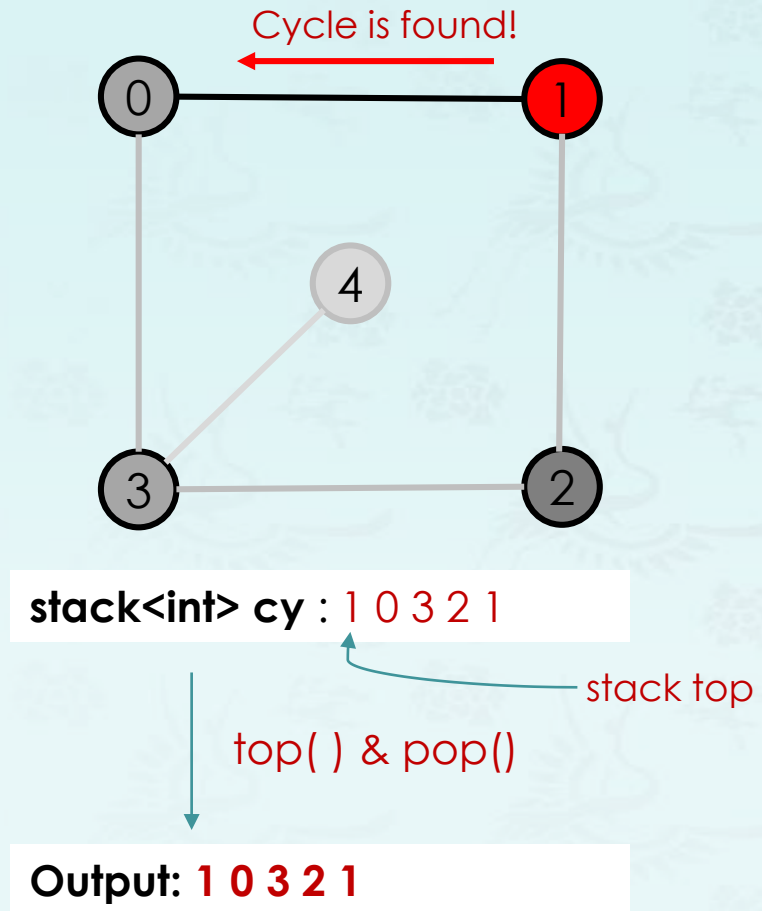
why? stay tuned.

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Cycle detection using DFS implementation

```
// Recursive DFS does the work
// g: the gragph, u: vertex visited previously, v: visiting vertex
bool DFScyclic(graph g, int u, int v, stack<int>& cy){
    g->marked[v] = true;            // visit vertex v
    for (gnode w = g->adj[v].next; w; w = w->next) {        // check all vertices in adj.list
        if (cy.size() > 0) return true;
        if (!g->marked[w->item]) {
            g->parent[w->item] = v;
            DFScyclic (g, v, w->item, cy);
        }
        // check for cycle (but disregard reverse of edge leading to v)
        else if (w->item != u) {
            // Now... a cycle is found
            // instantiate a stack
            // push all vertices that led us here or (1, 2, 3) - use for loop and g->parentDFS[]
            // push the last v or starting v of cycle (1, 2, 3, 0)
            // push the current visit v (1, 2, 3, 0, 1)
            return true;
        }
    }
    return false;
}
```

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Cycle detection using DFS implementation

Cycle is found!

stack<int> cy : 1 0 3 2 1

stack top

top( ) & pop()

Output:

Cycle detection using DFS implementation

Cycle is found!

stack<int> cy : 1 0 3 2 1

stack top

top( ) & pop()

Output: 1 0 3 2 1

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Graph-processing challenge 1 – Review

Problem: Is a graph bipartite (or bigraph)?

a set of graph vertices decomposed into two disjoint sets
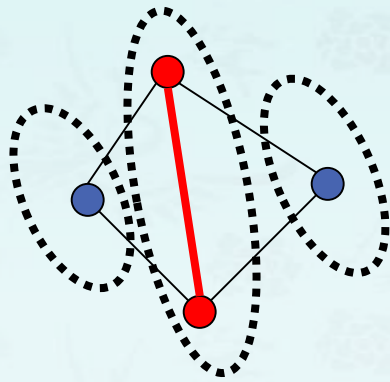such that no two graph vertices within the same set are adjacent.

How difficult?
- Any programmer could do it.
- Typical diligent algorithms student could do it.
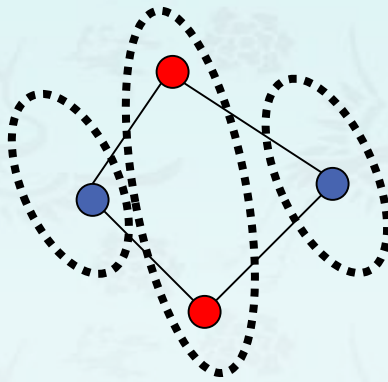- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

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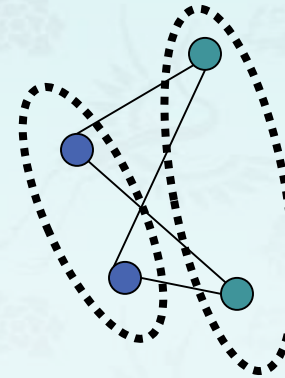**Problem:** Is a graph bipartite (or bigraph)?

a set of graph vertices decomposed into **two disjoint sets**
such that no two graph vertices within the same set are adjacent.
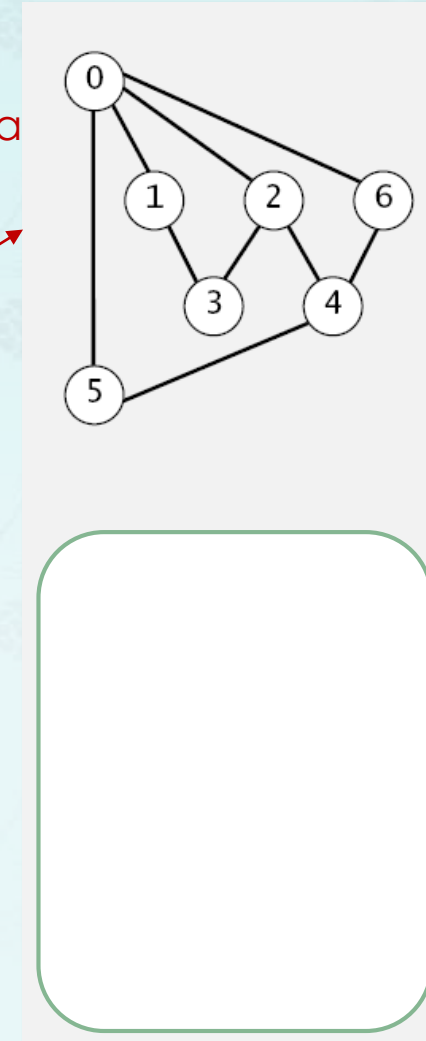

non bipartite   bipartite   bipartite

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Graph-processing challenge 1 – Review

Problem: Is a graph bipartite (or bigraph)?

a set of graph vertices decomposed into two disjoint sets
such that <u>no two graph </u>vertices within the same set are adja

a bigraph ?

How difficult?
- Any programmer could do it.
- Typical diligent algorithms student could do it.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

Graph-processing challenge 1 – Review
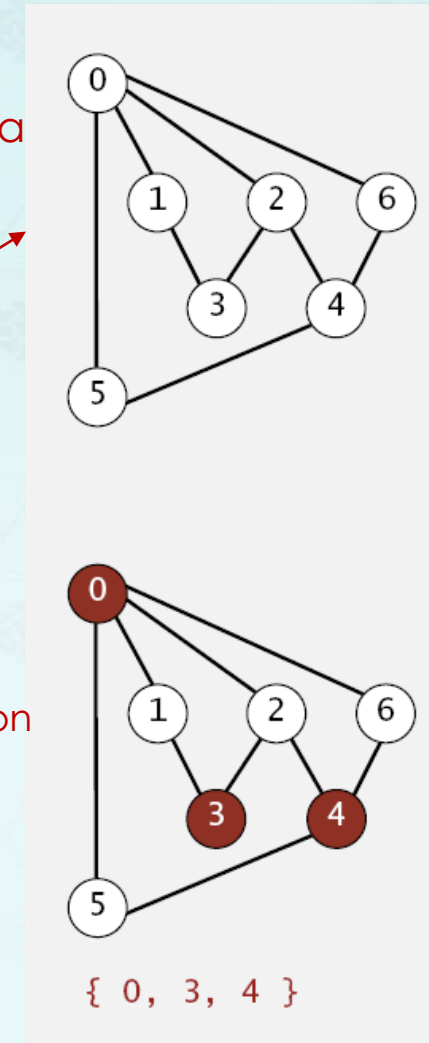
Problem: Is a graph bipartite (or bigraph)?

a set of graph vertices decomposed into two disjoint sets
such that <u>no two graph </u>vertices within the same set are adja
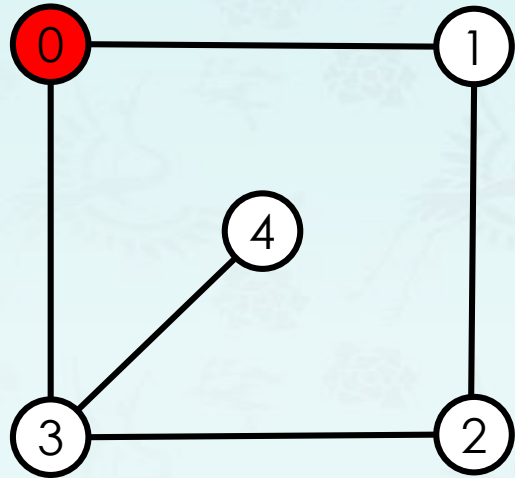
a bigraph ?

How difficult?
- Any programmer could do it.
- <u>Typical diligent algorithms student could do it.</u>
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

simple DFS or BFS-based solution

{ 0, 3, 4 }

Prof. Youngsup Kim, idebtor@gmail.com, Data Structures, CSEE Dept., Handong Global University

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Graph-processing challenge 1 – bigraph

Problem: Is a graph bipartite (or bigraph)?

Adjacency lists

adj[]

0	3	1	
1	2	0	
2	3	1	
3	4	2	0
4	3		

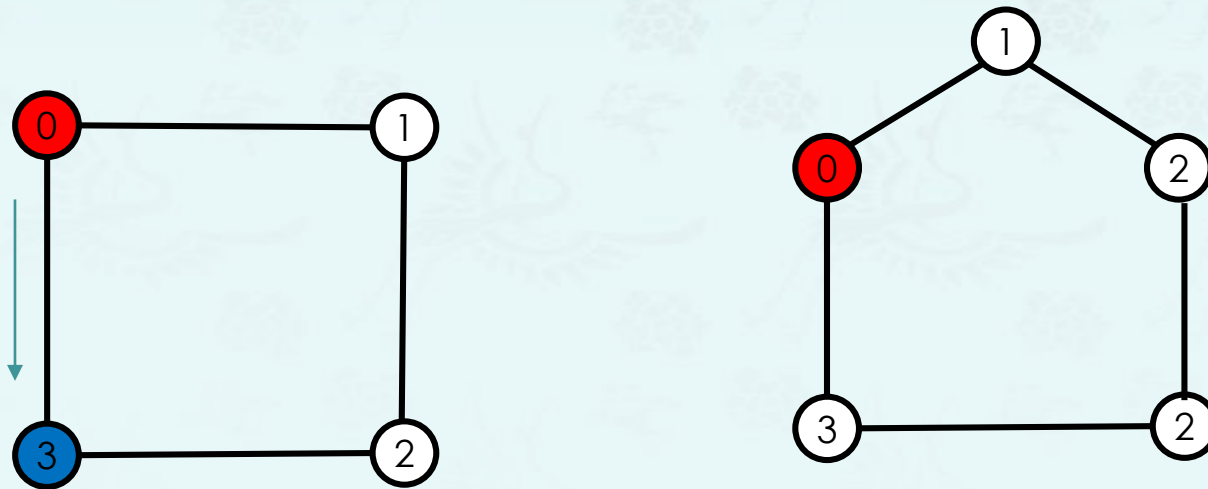| 3	1	

visit 0: check 3, **check 1**

Graph-processing challenge 1 – bigraph

Problem: Is a graph bipartite (or bigraph)?

Solution: Two-colorability
The vertices of a given graph can be assigned one of two colors in such a way that no edge connects vertices of the same color.

Solution: It is called two-colorability. graphBipartite() uses depth-first search to determine whether or not a graph has a bipartition; if so, return one; if not, return an odd-length cycle. It takes time proportional to V + E in the worst case.

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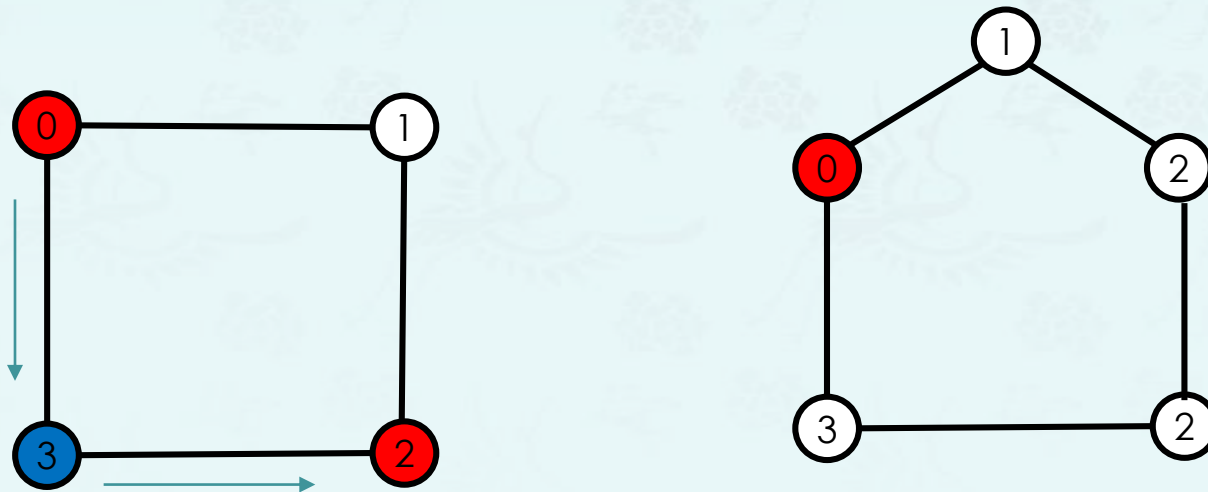
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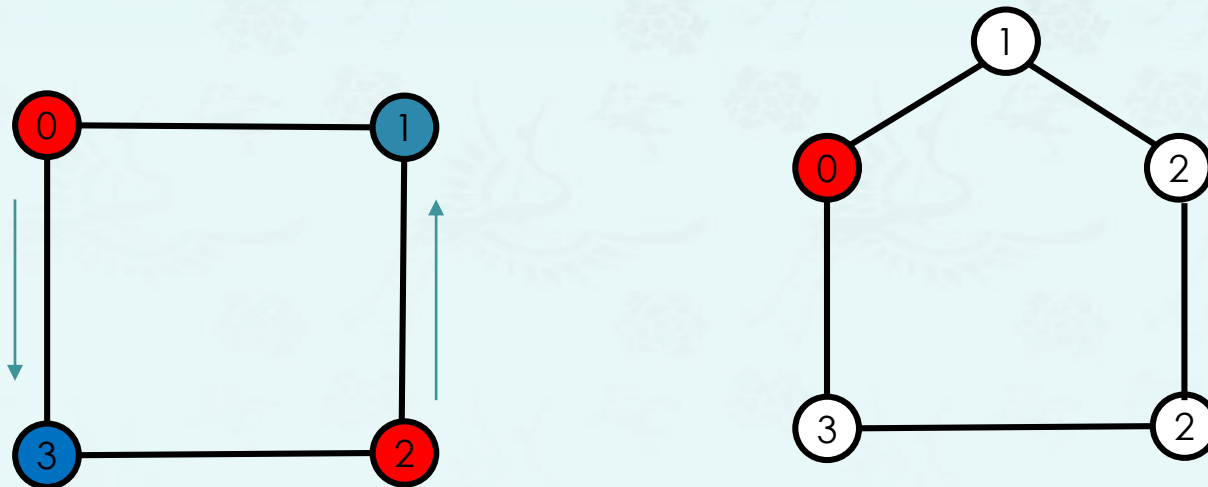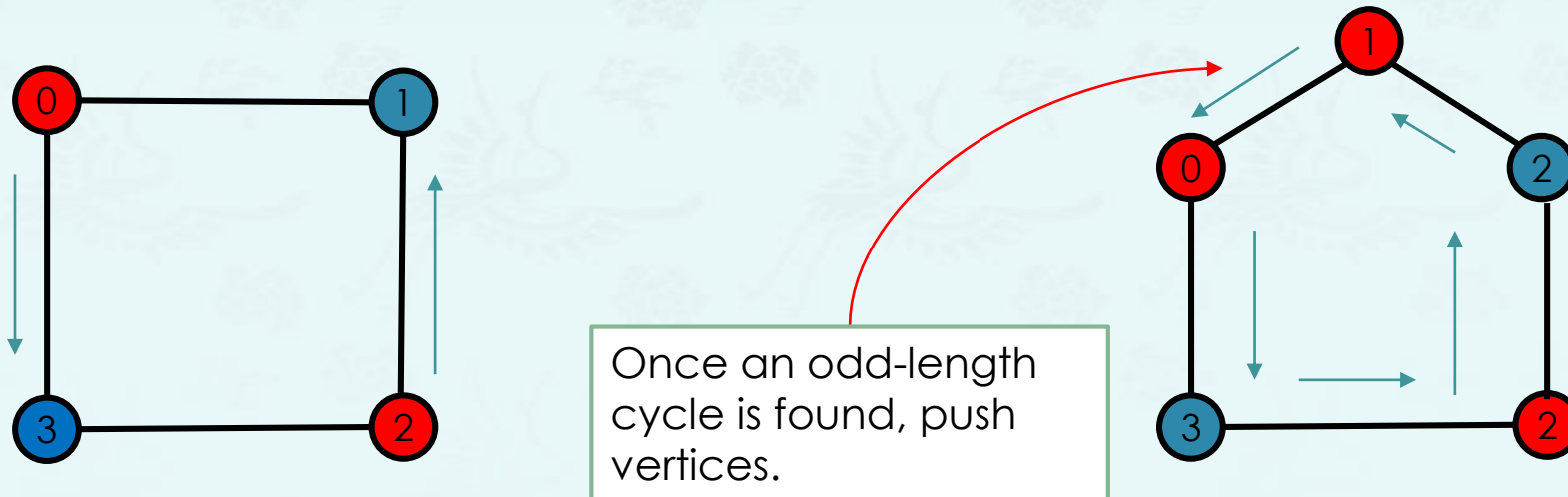
Prof. Youngsup Kim, idebtor@gmail.com, Data Structures, CSEE Dept., Handong Global University

Graph-processing challenge 1 – bigraph

Problem: Is a graph bipartite (or bigraph)?

Solution: Two-colorability
The vertices of a given graph can be assigned one of two colors in such a way that no edge connects vertices of the same color.

Solution: bipartite() uses depth-first search to determine whether a graph has a bipartition or not; if not, return an odd-length cycle.
It takes time proportional to V + E in the worst case.

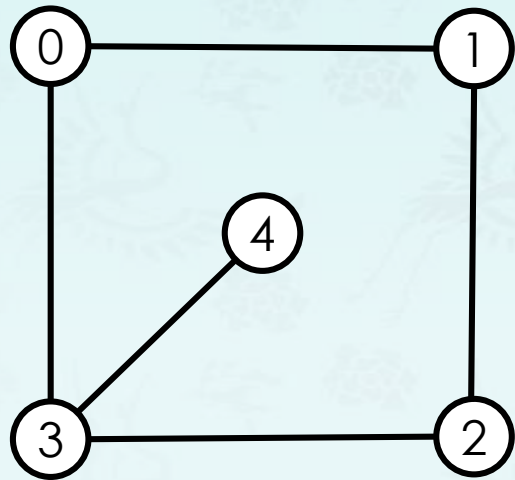Once an odd-length cycle is found, push vertices.

```
// determines whether or not an undirected graph is bigraph and
// finds either a bipartition or an odd length cycle.
// returns a stack with cyclic vertices pushed.
bool bigraph(graph g, stack<int>& cy) {
    if (empty(g)) return false;
    for (int i = 0; i < V(g); i++) {
        g->marked[i] = false;
        g->color[i] = BLACK;  // BLACK=0, WHITE=1
        g->parentDFS[i] = -1;   // needs info when backtrack the cycle.
    }
    cy = {};                              // clear stack
    for (int v = 0; v < V(g); v++) {
        if (!g->marked[v]) {
            if (!DFSbigraph(g, v, cy))
                return false;  // found an odd-length cycle
        }
    }
    return true;
}
```

Prof. Youngsup Kim, idebtor@gmail.com, Data Structures, CSEE Dept., Handong Global University

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Graph-processing challenge 1 – bigraph coding

```cpp
// Recursive DFS does the work
bool DFSbigraph(graph g, int v, stack<int>& cy) {
  g->marked[v] = true;
  for (gnode w = g->adj[v].next; w; w = w->next) {// short circuit if odd-length cycle found
    if (cy.size() > 0) return false;                // found 1st cycle
    if (!g->marked[w->item]) {                       // found uncolored vertex, so recur
        g->parentDFS[w->item] = v;                   // keep it to backtrack the cycle.
        g->color[w->item] = !g->color[v];        // flip the color
        DPRINT(cout << " " << v << " Color:" << g->color[v] << ",";);
        DPRINT(cout << " " << w->item << " Color:" << g->color[w->item] << endl;);
        DFSbigraph(g, w->item, cy);
    }   // if v-w create an odd-length cycle, find it (push vertices and push them)
    else if (g->color[w->item] == g->color[v]) {    // bipartite = false;
        // 1. instantiate a new stack and set it to g->cycle
        // 2. push w->item since first v = last v, duplicated
        // 3.  retrace g->parent[x] from v to w->item
        //        and push them to stack – need a for loop here.
        //  4. push w->item (to form a cycle)
        return false
    }
  }
  return true;
}
```

Prof. Youngsup Kim, idebtor@gmail.com, Data Structures, CSEE Dept., Handong Global University

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Graph-processing challenge 1 – bigraph two-colorability coding

Solution: for every v, the color of adj[v] is different from those of adj[v]'s list vertices, if it is bipartite.

Adjacency lists

adj[]

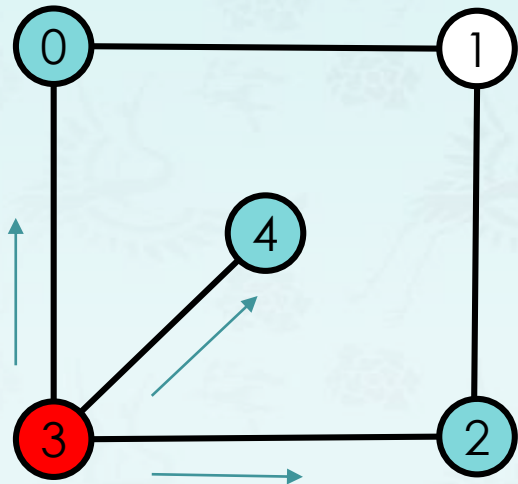| 0 | 3 | 1 | |
1	2	0	
2	3	1	
3	4	2	0
4	3		

myG.txt
```
5        V
5        E
0 1
0 3
1 2
2 3
3 4
```

Graph g:

Graph-processing challenge 1 – bigraph two-colorability coding

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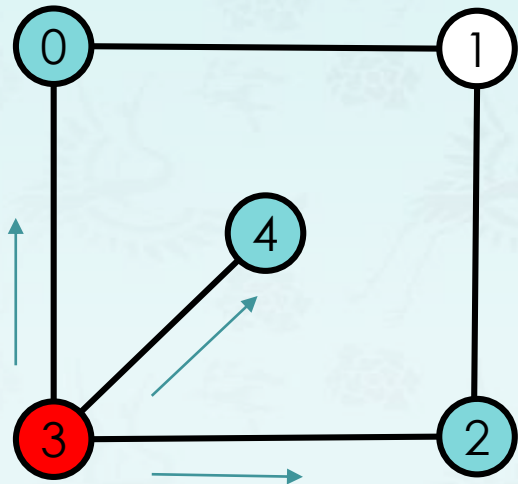|		
0	3	1
1	2	0
2	3	1
3	4	2	0
4	3	

```
myG.txt
5          V
5          E
0 1
0 3
1 2
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```

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Adjacency lists

adj[]

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1	2	0
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3	4	2	0
4	3	

myG.txt
```
5        ← V
5        ← E
0 1
0 3
1 2
2 3
3 4
```

v	marked[]	color[]
1	F	−1
2	F	−1
3	F	−1
4	F	−1
5	F	−1

Graph g: