习题三补充讲解(3)

[20] 证 (1) $k_0\beta_0 + k_1\beta_1 + \cdots + k_r\beta_r = (k_0 + k_1 + \cdots + k_r)\eta + k_1\alpha_1 + \cdots + k_r\alpha_r = \theta$

左乘 A 得 $(k_0 + k_1 + \cdots + k_r)A\eta + k_1A\alpha_1 + \cdots + k_rA\alpha_r = (k_0 + k_1 + \cdots + k_r)b = \theta$

又 $b \neq \theta$，故 $k_0 + k_1 + \cdots + k_r = 0$，于是 $k_1\alpha_1 + \cdots + k_r\alpha_r = \theta$，因 $\alpha_1, \cdots, \alpha_r$

为基础解系，故无关，从而 $k_1 = k_2 = \cdots = k_r = 0$，进而 $k_0 = 0$，

故 $\beta_0, \beta_1, \cdots, \beta_r$ 线性无关

(2) β 为 $Ax = b$ 的任意解，则 $\beta = \eta + k_1\alpha_1 + \cdots + k_r\alpha_r$，令 $k_0 = 1 - (k_1 + \cdots + k_r)$

则有 $k_0\beta_0 + k_1\beta_1 + \cdots + k_r\beta_r = (k_0 + \cdots + k_r)\eta + k_1\alpha_1 + \cdots + k_r\alpha_r = \eta + k_1\alpha_1 + \cdots + k_r\alpha_r = \beta$

而 $k_0 + k_1 + \cdots + k_r = 1$