习题二补充讲解(1)

8 解 $AA^T=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}=|A|E$，故$|A|^2=|A||A^T|=|AA^T|=||A|E|=|A|^3$.

又有 $|A|=a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}=a_{11}^2+a_{12}^2+a_{13}^2\geq a_{11}^2>0$，由$|A|^2=|A|^3$可得$|A|=1$

12 解 设$B=\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$有$AB=BA$，故

$$O=AB-BA=\begin{pmatrix} 0 & -b_{13} & -2b_{12}+3b_{13} \\ 2b_{31} & -b_{23}+2b_{32} & -2b_{22}+3b_{23}+2b_{33} \\ b_{21}-3b_{31} & b_{22}-3b_{32}-b_{33} & b_{23}-2b_{32} \end{pmatrix}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

于是有关系 $\begin{cases} b_{12}=b_{13}=b_{21}=b_{31}=0, \\ b_{23}=2b_{32} \\ b_{22}-3b_{32}-b_{33}=0 \end{cases}$ 故 $B=\begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 2b_{32} \\ 0 & b_{32} & b_{22}-3b_{32} \end{pmatrix}$, b_{11},b_{22},b_{32}为任意数

16 解 由$AA^T=E$可得$(A+E)A^T=E+A^T=(A+E)^T$，取行列式值

$|A+E||A|=|A+E||A^T|=|(A+E)A^T|=|(A+E)^T|=|A+E|$，

即$|A+E|(1-|A|)=0$. 因为$|A|<0$，故有 $|A+E|=0$.