习题四补充讲解(3)

25 证 $A^TA=E, B^TB=E$，则 $\left|A^T(A+B)B^T\right|=\left|B^T+A^T\right|=\left|(A+B)^T\right|=\left|A+B\right|$，

又 $\left|A^T(A+B)B^T\right|=\left|A^T\right|\left|A+B\right|\left|B^T\right|=\left|A\right|\left|B\right|\left|A+B\right|=-\left|B\right|^2\left|A+B\right|$

故 $\left|A+B\right|=-\left|B\right|^2\left|A+B\right|$，即 $(1+\left|B\right|^2)\left|A+B\right|=0$，故 $\left|A+B\right|=0$