



行列式计算拓展


行或列比例递减

$$\begin{vmatrix} a^2 & (a+1)^2 & (a+2)^2 & (a+3)^2 \\ b^2 & (b+1)^2 & (b+2)^2 & (b+3)^2 \\ c^2 & (c+1)^2 & (c+2)^2 & (c+3)^2 \\ d^2 & (d+1)^2 & (d+2)^2 & (d+3)^2 \end{vmatrix}$$



(自右向左减左边相邻列)

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ a_1 & a_1 + a_2 & a_1 + a_2 + a_3 & a_1 + a_2 + a_3 + a_4 \\ a_1 & 2a_1 + a_2 & 3a_1 + 2a_2 + a_3 & 4a_1 + 3a_2 + 2a_3 + a_4 \\ a_1 & 3a_1 + a_2 & 6a_1 + 3a_2 + a_3 & 10a_1 + 6a_2 + 3a_3 + a_4 \end{vmatrix}$$


(自下而上减上一行)

$$\begin{vmatrix} n & n-1 & n-2 & \cdots & 1 \\ -1 & x & 0 & \cdots & 0 \\ 0 & -1 & x & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & x \end{vmatrix}$$


(自左向右加左列的 x 倍)

$$\begin{vmatrix} 12345 & 12245 \\ 67813 & 67913 \end{vmatrix}$$


(第1列减第2列)

行或列全加

$$\begin{vmatrix} a_1 - b & a_2 & \cdots & a_n \\ a_1 & a_2 - b & \cdots & a_n \\ \vdots & \vdots & \ddots & \vdots \\ a_1 & a_2 & \cdots & a_n - b \end{vmatrix}$$

(每一列加到第一列)

$$\begin{vmatrix} x & y & x + y \\ y & x + y & x \\ x + y & x & y \end{vmatrix}$$

(每一列加到第一列)

$$\begin{vmatrix} 1 & 2 & \cdots & n \\ 2 & 3 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ n & 1 & \cdots & n-1 \end{vmatrix}$$

列全加
→

$$\begin{vmatrix} 1 & 2 & \cdots & n \\ 1 & 3 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & n-1 \end{vmatrix}$$

行递减
第1列展开

$$\begin{vmatrix} 1 & 1 & \cdots & 1-n \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1-n & 1 & 1 & 1 \end{vmatrix}_{n-1}$$

列全加
→

$$\begin{vmatrix} -1 & 1 & 1 & \cdots & 1-n \\ -1 & 1 & 1 & \ddots & 1 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -1 & 1-n & 1 & \cdots & 1 \\ -1 & 1 & 1 & \cdots & 1 \end{vmatrix}$$

配套相加

$$\begin{vmatrix} a & 0 & \cdots & 0 & b \\ 0 & a & \cdots & b & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & b & \cdots & a & 0 \\ b & 0 & \cdots & 0 & a \end{vmatrix}_{n=2k}$$

(下面一半：第 $2k+1-i$ 行加到第 i 行，
右面一半：第 $2k+1-i$ 列减去第 i 列)

化三角形消零

$$\begin{array}{c} \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \\ \left| \begin{array}{cccccc} 1 & 2 & 3 & \cdots & n-1 & n \\ -1 & 0 & 3 & \cdots & n-1 & n \\ -1 & -2 & 0 & \cdots & n-1 & n \\ -1 & -2 & -3 & \cdots & n-1 & n \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -2 & -3 & \cdots & -(n-1) & 0 \end{array} \right| \end{array}$$

(第1行加到各行)

$$\begin{array}{c} \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \\ \left| \begin{array}{cccccc} a_0 & 1 & 1 & \cdots & 1 \\ 1 & a_1 & 0 & \cdots & 0 \\ 1 & 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & a_n \end{array} \right| \end{array}$$

(若 $a_i \neq 0$, 各列的 $-1/a_i$ 倍加到第1列)
(若有 $a_i = 0$, 可按第1行展开)

$$\begin{array}{c} \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \\ \left| \begin{array}{ccccc} 1 & 2 & 3 & \cdots & n \\ 2 & 1 & 2 & \cdots & n-1 \\ 3 & 2 & 1 & \cdots & n-2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n & n-1 & n-2 & \cdots & 1 \end{array} \right| \xrightarrow{\text{行递减}} \left| \begin{array}{ccccc} 1 & 2 & 3 & \cdots & n \\ 1 & -1 & -1 & \cdots & -1 \\ 1 & 1 & -1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & -1 \end{array} \right| \xrightarrow{\text{最后列加到每一列}} \left| \begin{array}{ccccc} n+1 & n+2 & \cdots & n+n-1 & n \\ 0 & -2 & \cdots & -2 & -1 \\ 0 & 0 & \ddots & -2 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & -1 \end{array} \right| \end{array}$$

递推式

$$\begin{vmatrix} 1 & 1 & & \\ 1 & 1 & \ddots & \\ & \ddots & \ddots & 1 \\ & & 1 & 1 \end{vmatrix}$$

$$(D_n = D_{n-1} - D_{n-2} = -D_{n-3})$$

$$\begin{vmatrix} 2a & a^2 & & \\ 1 & 2a & \ddots & \\ & \ddots & \ddots & a^2 \\ & & 1 & 2a \end{vmatrix}$$

$$(D_n - aD_{n-1} = aD_{n-1} - a^2D_{n-2})$$

分裂行列式

$$\begin{vmatrix} 2a & a^2 & & \\ 1 & 2a & \ddots & \\ & \ddots & \ddots & a^2 \\ & & 1 & 2a \end{vmatrix}$$

(第一列分裂成 $a+a$ 和 $1+0$,
得 $D_n = a^n + aD_{n-1}$)

$$\begin{vmatrix} x_1 & a_1b_2 & \cdots & a_1b_n \\ a_2b_1 & x_2 & \cdots & a_2b_n \\ \vdots & \vdots & \ddots & \vdots \\ a_nb_1 & a_nb_2 & \cdots & x_n \end{vmatrix}$$

(最后列分裂成

$a_1b_n+0, a_2b_n+0, \dots, a_nb_n+(x_n-a_nb_n)$ 得

$$D_n = (x_1 - a_1b_1) \dots (x_{n-1} - a_{n-1}b_{n-1}) a_nb_n + (x_n - a_nb_n) D_{n-1}$$