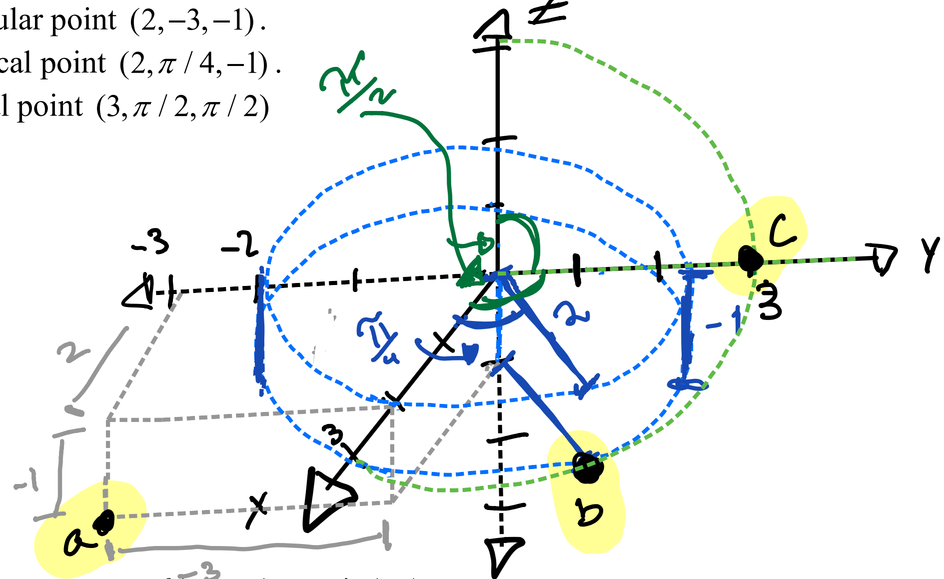
MTH 3400 **Test 1** Name: Georgia Smith

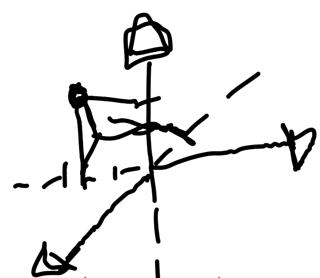
100 points total

1) (3 pts each) Plot the following points on the same axes:
   a. Rectangular point $(2,-3,-1)$.
   b. Cylindrical point $(2,\pi/4,-1)$.
   c. Spherical point $(3,\pi/2,\pi/2)$




2) (6 pts) Find the gradient of $f(x,y,z)=x\sin(yz)$.

$$\nabla f(x,y,z) = \left(\sin(yz),\ xz\cos(yz),\ xy\cos(yz)\right)$$



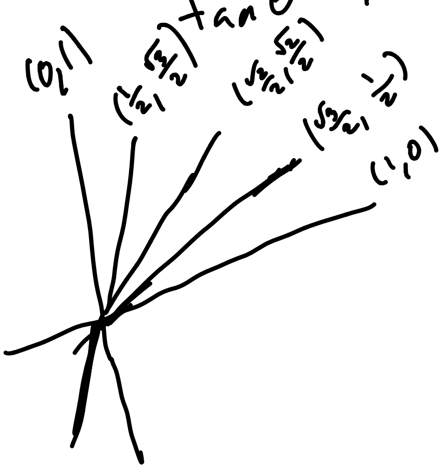
3) (6 pts) Find the cylindrical coordinates of the rectangular point $\left(-1,-\sqrt{3},2\right)$.

$\left(2, \frac{4\pi}{3}, 2\right)$

$Z = Z$

$r^2 = x^2 + y^2$

$\tan\theta = y/x$




$r^2 = (-1)^2 + (-\sqrt{3})^2$

$= 1 + 3$

$= 4$

$r = \sqrt{4} = 2$

$\tan\theta = \frac{-\sqrt{3}}{-1} = \frac{\sqrt{3}}{1}$

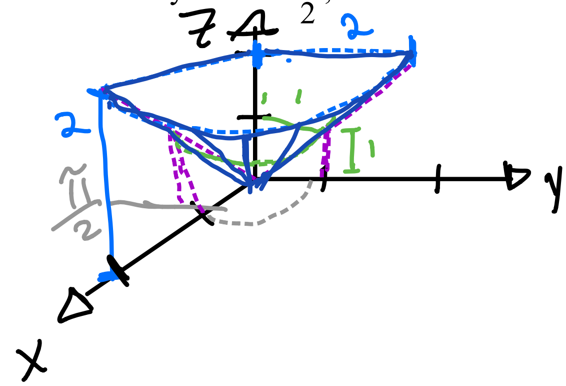
$\frac{4\pi}{3}$

$\theta = \cancel{\frac{\pi}{3}}$

$\theta = \frac{4\pi}{3}$

4) (6 pts) Sketch the solid described by $0 \le \theta \le \frac{\pi}{2}$, $r \le z \le 2$.

it is a 1/4 cone with base r=2 and height 2




5) (6 pts) Find the rectangular coordinates of the spherical point $(2, \pi/3, \pi/2)$. $(0, \sqrt{3}, 1)$

$z = \rho \cos\varphi$
$x = \rho \sin\varphi \cos\theta$
$y = \rho \sin\varphi \sin\theta$

$z = 2\cos\left(\frac{\pi}{3}\right) = 1$
$x = 2\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{2}\right) = 0$
$y = 2\sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{2}\right) = 2 \cdot \frac{\sqrt{3}}{2} \cdot 1 = \sqrt{3}$

6) (6 pts) Write in spherical coordinates: $x + 2y + 3z = 1$.

$\rho \sin\varphi \cos\theta + 2\rho \sin\varphi \sin\theta + 3\rho \cos\varphi = 1$

7) (2 pts each) Classify the following functions

$f: \mathbb{R} \to \mathbb{R}$ real valued function of 1 var
$\vec{g}: \mathbb{R} \to \mathbb{R}^2$ vector valued function of 1 var
$\vec{h}(x, y) = (x, y)$ vector valued function of 2 vars
$\vec{P}(\vec{x}) = x_1 x_2 \vec{\imath} + x_3 \vec{\jmath}$ vector valued function of 3 vars

$= (\cos(t), \sin(t), t)$

8) (8 pts) For $\vec{x}(t) = \cos(t)\,\vec{\imath} + \sin(t)\,\vec{\jmath} + t\vec{k}$ find the unit tangent vector and the curvature.

$\vec{x}'(t) = (-\sin(t), \cos(t), 1)$

$\|\vec{x}'(t)\| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$

$\vec{T} = \frac{1}{\sqrt{2}}(-\sin(t), \cos(t), 1)$

$\vec{T}' = \frac{1}{\sqrt{2}}(-\cos(t), -\sin(t), 0)$

$\kappa = \frac{1}{2}(-\cos(t), -\sin(t), 0)$

9) (3 pts) Give the equation to find the torsion of a path.

$$\frac{\vec{B}'(t)}{\|\vec{x}'(t)\|} = -\tau\vec{N}$$

10) (8 pts) Assuming the unit tangent vector for some path is $\vec{T}(t) = \left(-\frac{\sqrt{2}\sin t}{2}, \frac{\sqrt{2}\cos t}{2}, \frac{\sqrt{2}}{2}\right)$, find the principal normal vector and the binormal vector.

$\vec{T}' = \left(\frac{-\sqrt{2}\cos t}{2}, \frac{-\sqrt{2}\sin t}{2}, 0\right)$

$\vec{N} = \frac{-1}{2}(\cos t, \sin t, 0)$

$\|\vec{T}'\| = \sqrt{1/2\cos^2 t + 1/2\sin^2 t + 0}$

$= \sqrt{1/2} = 1/\sqrt{2} = \frac{\sqrt{2}}{2}$

$$\vec{B} = \begin{vmatrix} i & j & k \\ \frac{-\sqrt{2}\cos t}{2} & \frac{-\sqrt{2}\sin t}{2} & 0 \\ \frac{-2\cos t}{2} & \frac{-\sin t}{2} & 0 \end{vmatrix}$$

$= i(0) - j(0) + k\left(\frac{\sqrt{2}}{4}\cos t\sin t - \frac{\sqrt{2}}{4}\cos t\sin t\right)$

$\vec{B} = (0, 0, 0)$

11) (6 pts) Determine if the vector field $\vec{F}(\vec{x}) = (0, \cos(xz), -\sin(xy))$ is incompressible

$\nabla \cdot \vec{F} = 0 + 0 + 0 = 0$

$\therefore$ $\vec{F}$ is incompressible

12) (6 pts each) For the vector field $\vec{F}(x,y,z) = < z\cos x,\ z sin x, 2xyz >$ find the following

a. The divergence of the vector field

$$\nabla \cdot \vec{F} = -z\sin x + 0 + 2xy$$

b. The curl of the vector field

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z\cos x & z\sin x & 2xyz \end{vmatrix} = i(2xz - \sin x) - j(2yz - \cos x) + k(2\cos x - 0)$$

$$= \langle 2xz - \sin x,\ \cos x - 2yz,\ 2\cos x \rangle$$

13) For the vector field $\vec{F}(x,y) = (y,x)$ with x > 0

a. (5 pts) Sketch the vector field.

b. (3 pts) Sketch a sample flowline through the point $\vec{x}(0) = (1,0)$ .

c. (3 pts) What does your flowline look like?

d. (5 pts) Find the flowline through the point $\vec{x}(0) = (1,0)$. Hint: $\dfrac{dx/dt}{dy/dt} = \dfrac{dx}{dy}$.

y x

c) my flowline looks like a hyperbola

$$(x'(t), y'(t)) = (y(t), x(t))$$

$$\partial y = x(t)\,\partial t$$

$$\partial x = y(t)\,\partial t$$

$$\frac{\partial x}{\partial y} = \frac{y(t)}{x(t)}$$

$$x\,\partial x = y\,\partial y$$

$$\frac{x^2}{2} + c_1 = \frac{y^2}{2} + c_2$$

$$y^2 = x^2 + 2c_1 - 2c_2$$

$$y = \sqrt{x^2 + 2c_1 - 2c_2}$$

$$1 = \sqrt{2c_1 - 2c_2}$$

$$y = \sqrt{x^2 + 1}$$

$\vec{F}(x,y) = (y, x)$