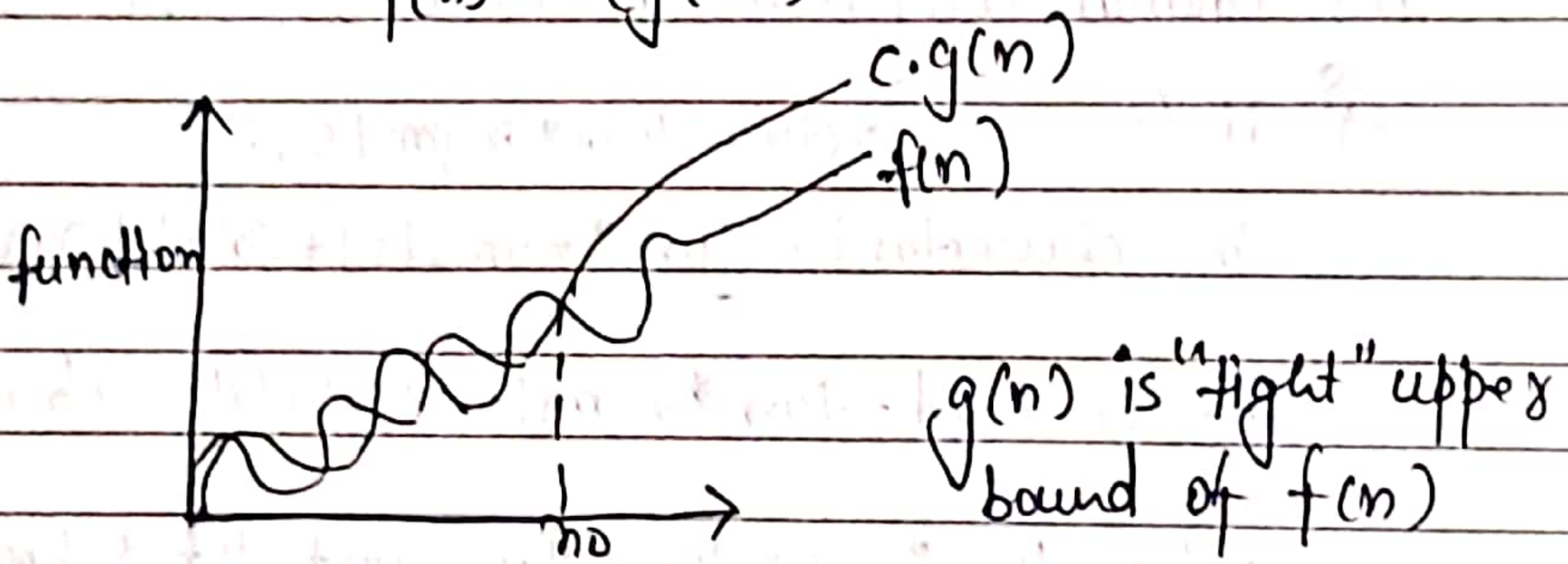
Ques-1: What do you mean by Asymptotic notations. Define different type of notations along with example.

Ans: Asymptotic Notations: Means tending to infinity.
They are used to used to tell the complexity when input is very large.

→ Different types of Asymptotic Notations:

1. Big oh (O) Notation:

$f(n) = O(g(n))$



$f(n) = O(g(n))$
iff $f(n) \leq c \cdot g(n)$
$\forall\ n \geq n_0$ and some constant, c

Example:

```
for(i=1; i<=n; i++)
    printf("*");  ——— O(1)
```

$\Rightarrow T(n) = O(n)$

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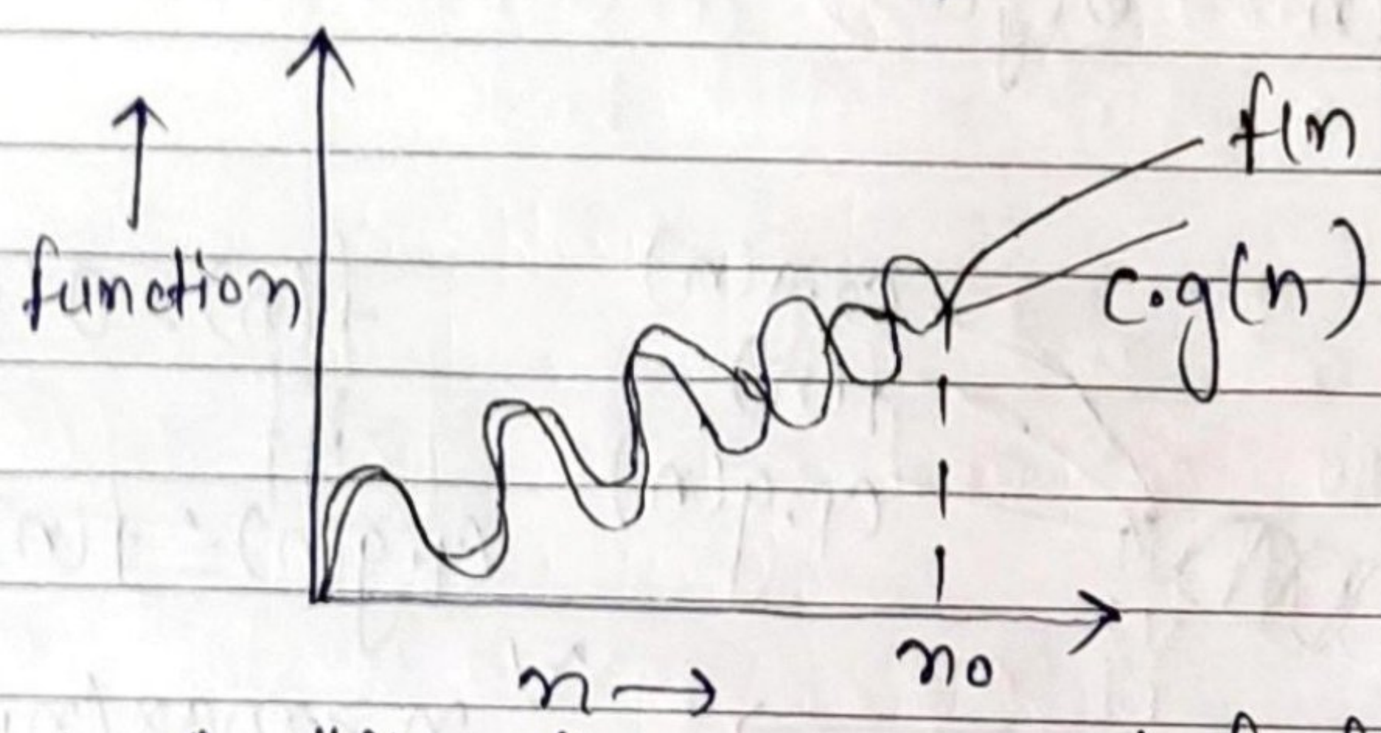
2. Big Omega (Ω):

$$f(n) = \Omega(g(n))$$



$g(n)$ is "tight" lower bound of $f(n)$

$$f(n) = \Omega(g(n))$$

iff

$$f(n) \geq c \cdot g(n)$$

$\forall\ n \geq n_0$, and some constant $c > 0$

Example:

$$f(n) = 2n^2 + 3n + 5, \quad g(n) = n^2$$

$$0 \leq c \cdot g(n) \leq f(n)$$

$$0 \leq c \cdot n^2 \leq 2n^2 + 3n + 5$$

$$c \leq 2 + \frac{3}{n} + \frac{5}{n^2}$$

On putting $n = \infty$, $\frac{3}{n} \to \infty$, $\frac{5}{n^2} \to \infty$

$$\Rightarrow c = 2$$

$$\Rightarrow 2n^2 \leq 2n^2 + 3n + 5$$

On putting $n = 1$

$$2 \leq 2 + 3 + 5$$

$$2 \leq 10 \quad \text{True.}$$

$$\Rightarrow \boxed{c = 2,\ n = n_0 = 1}$$

$$0 \leq 2n^2 \leq 2n^2 + 3n + 5$$

$$\therefore f(n) = \Omega(n^2)$$

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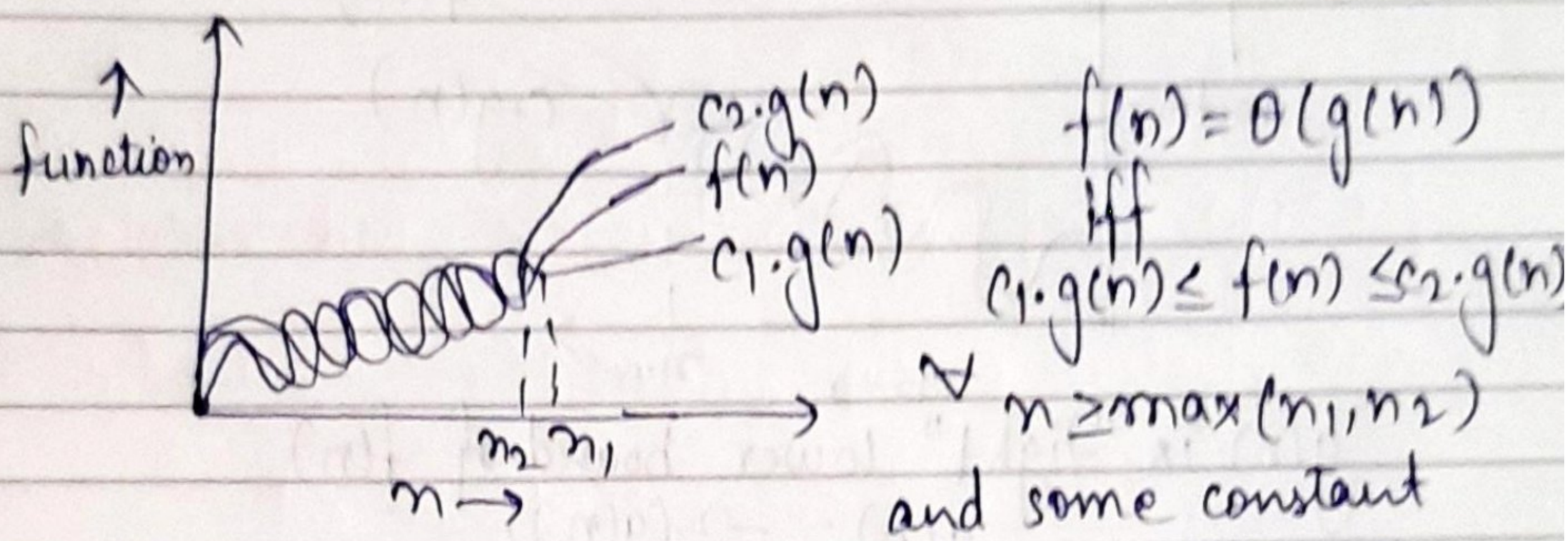
3. Big Theta (Θ):

$$f(n) = \Theta(g(n))$$



Example: $f(n) = 10\log_2 n + 4$, $g(n) = \log_2 n$

$$f(n) \leq c_2 \cdot g(n)$$

$$\Rightarrow 10\log_2 n + 4 \leq 10\log_2 n + \log_2 n$$

$$10\log_2 n + 4 \leq 11\log_2 n$$

$$c_2 = 11$$

$$\rightarrow 4 \leq 11\log_2 n - 10\log_2 n$$

$$4 \leq \log_2 n$$

$$16 \leq n$$

Here

$\forall$ $n \geq 16$

$n_2 = 16$

& $c_2 = 11$

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$f(n) \geq c_1 \cdot g(n)$

$\Rightarrow n \log_2 n + n \geq c_1 \log_2 n$

$c_1 = 1$; $n \geq 0$

$\Rightarrow n_0 = 1 \Rightarrow n_0 = \max(n_1, n_2) \Rightarrow n_0 = 16$

$\Rightarrow n \log_2 n \leq n \log_2 n^n + 4 \leq 11 \log_2 n$

$c_1 = 1$, $c_2 = 11$

$\Rightarrow \Theta(n \log_2 n)$

4. Small oh [o]:

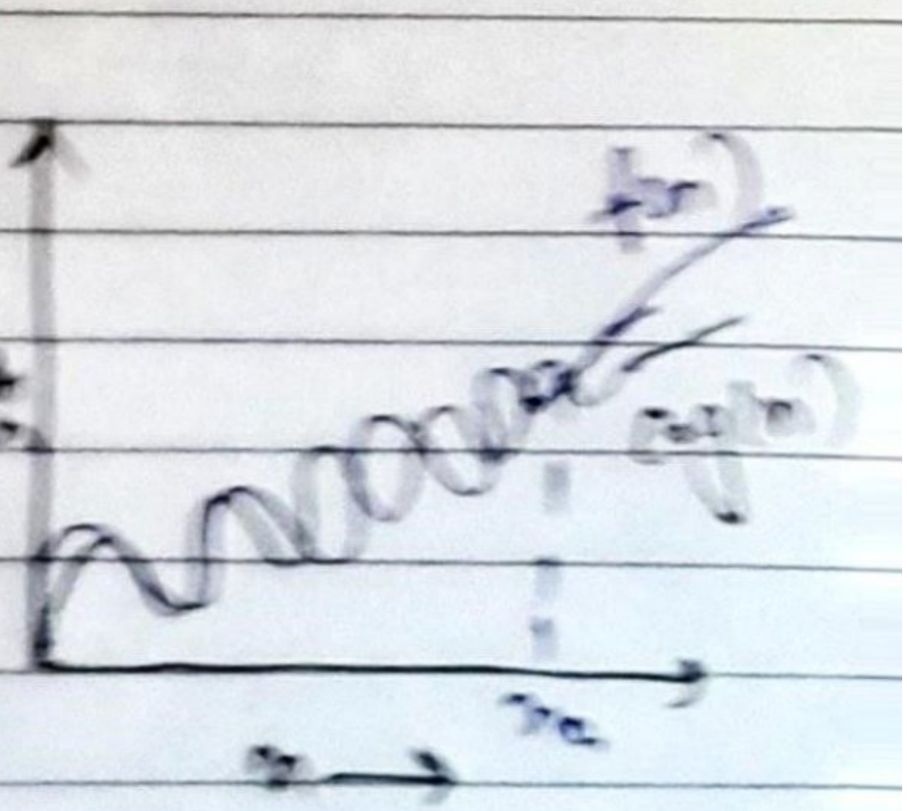
$f(n) = o(g(n))$

$g(n)$ is the upper bound of $f(n)$

$f(n) = o(g(n))$

iff $f(n) < c \cdot g(n)$

$\forall n \geq n_0$ and $\forall$ constant $c > 0$



5. Small omega [ω]:

$f(n) = \omega(g(n))$

$g(n)$ is lower bound of $f(n)$

$f(n) = \omega(g(n))$

when $f(n) > c g(n)$

$\forall n > n_0$

and $\forall c > 0$

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Ques-2: What should be the time complexity of:
for (i=1 ton) { i=i*2; }

Soln

values of i = 1, 2, 4, 8, 16, ... n
(K terms)

As this is a G.P with a = 1, r = 2

Now,

K^{th} term :- $t_K = ar^{K-1}$

$n = 1 \cdot 2^{K-1}$

$n = 2^{K-1}$

taking $\log_2$ on to the both sides.

$\Rightarrow \log_2 n = \log_2 2^{K-1}$

$\log_2 n = (K-1)\log_2 2$

$\log_2 n = K-1 \Rightarrow K = 1 + \log_2 n$ [$\because \log_2 2 = 1$]

$\therefore$ Time complexity $T(n) = O(K)$
$= O(1 + \log_2 n)$
$= O(\log_2 n)$.

Ques-3:

$T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

Soln.

$T(n) = 3T(n-1)$ ---- (1)

put $n = n-1$ in eq^n (1)

$T(n-1) = 3T(n-1-1)$

$T(n-1) = 3T(n-2)$ ---- (2)

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Put value of $T(n-1)$ from eq^n (2) in eq^n (1)

$$T(n) = 3[3T(n-2)]$$
$$T(n) = 9T(n-2) \quad \cdots (3)$$

Put $n = n-2$ in eq^n (1)

$$T(n-2) = 3T(n-3) \quad \cdots (4)$$

Put value of $T(n-2)$ in eq^n (3)

$$T(n) = 3[9T(n-3)]$$
$$T(n) = 27T(n-3) \quad \cdots (5)$$

On Generalising eq^n (5)

$$T(n) = 3^k T(n-k)$$

Put $n-k = 0$

$$\Rightarrow T(n) = 3^k T(0)$$
$$= 3^k \qquad (\because T(0) = 1)$$

$$\therefore T(n) = O(3^n)$$

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Ques-4. $T(n) = \begin{cases} 2T(n-1)-1 & \text{if } n>0, \text{ otherwise } 1 \end{cases}$

Sol^n.

$$T(n) = 2T(n-1)-1 \quad ---(1)$$

put $n = n-1$ in eq^n (1)

$$T(n-1) = 2T(n-1-1)-1$$

$$T(n-1) = 2T(n-2)-1 \quad ---(2)$$

put value of $T(n-1)$ from (2) in (1)

$$T(n) = 2[2T(n-2)-1]-1$$

$$T(n) = 4T(n-2)-2-1 \quad --(3)$$

put $n = n-2$ in eq^n (1)

$$T(n-2) = 2T(n-3)-1$$

put value of $T(n-2)$ in eq^n (3)

$$T(n) = 4[2T(n-3)-1]-2-1$$

$$T(n) = 8T(n-3)-4-2-1$$

On Generalising

$$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - \cdots - 1$$

put $n-k=0 \Rightarrow n=k$, $T(0)=1$ (Given)

$$T(n) = 2^n T(0) - 2^{n-1} - 2^{n-2} - \cdots - 1$$

$$= 2^n - \underbrace{[2^{n-1} + 2^{n-2} - \cdots + 1]}_{k \text{ terms}}$$

$\Rightarrow a = 2^{n-1}$, $r = 1/2$

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$$\text{Sum of GP} = \frac{2^{n-1}\left[1-\left(\frac{1}{2}\right)^{n-1}\right]}{1-1/2}$$

$$= 2^n - 2$$

$$\Rightarrow T(n) = 2^n - [2^n - 2] = 2$$

$$= O(2)$$

$$\boxed{T(n) = O(1)}$$

Ques-5 What should be the time complexity of-

```
int i=1, s=1;
while (s<=n) {
    i++; s=s+i;
    printf("#");
}
```

Soln:

i	s
1	1
2	3
3	6
4	10
5	15
⋮	⋮
n	n
k times	

$$S = \underbrace{1, 3, 6, 10, 15, \ldots n}_{k \text{ terms}}$$

k^{th} term, $t_k = t_{k-1} + k$

$$k = t_k - t_{k-1} \quad \text{---} (1)$$

$$\Rightarrow k = n - t_{k-1}$$

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loop runs k-times

Time complexity = $O(1+1+1+n-t_{n-1})$

but $t_{n-1} = C$ (constant)

∴ Time complexity = $O(3+n-C)$

$= O(n)$

Ques-7: Time complexity of

```
void function (int n)
{ int i,j,k count=0;
   for (i= n/2; i<=n; i++)
      for (j=1; j<=n; j=j*2)
        for (k=1, k<=n; k=k*2)
          count++
}
```

Sol^n

$i \to n/2, \frac{n+2}{2}, \frac{n+4}{2}, \frac{n+6}{2}, \dots$ upto n

$= \frac{n+0\times2}{2} + \frac{n+1\times2}{2} + \frac{n+2\times2}{2}, \dots$ upto n

General form.

$$t_k = \frac{n+k*2}{2}$$

total terms = k+1

$t_{k+1} = n$

$$\Rightarrow \frac{n+(k+1)*2}{2} = n$$

$n + 2k + 2 = 2n$

$2k = n-2$

$k = \frac{n}{2} - 1$

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i	j	k
$n/2$	$\log_2 n$ times	$(\log_2 n)^2$
$\frac{n+2}{2}$	$\log_2 n$ times	$(\log_2 n)^2$
1	1	
1	1	
1	1	
$\frac{n}{\left(\frac{n}{2}-1\right)}$ times	$\log_2 n$ times	$(\log_2 n)^2$

$$\Rightarrow \left(\frac{n}{2}-1\right)(\log_2 n)^2$$

$$= O\left(\frac{n}{2}\log_2^2 n - \log_2 n\right)$$

$$= O(n \log^2 n)$$

Que-6 Time complexity of-

```
void function(int n) {
    int i, count = 0;        — O(1)
    for (i=1; i*i<=n; i++)
        count++;             — O(1)
}
```

Soln.

i*i
1^2
2^2
3^2
4^2
1
1
1
n

$i*i = \underbrace{1^2, 2^2, 3^2, 4^2, \ldots n}_{k \text{ terms}}$

$\Rightarrow$ kth term; $t_k = k^2$

$k^2 = n$

$k = n^{1/2}$

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Time complexity $= O(1+1+1+n^{1/2})$

$= O(n^{1/2})$

$= O(\sqrt{n})$

Ques-8: Time complexity of

```
function(int n) {
    if (n==1) return;  —— O(1)
    for (i=1 to n) {  —— O(n)
        for(j=1 to n) {  —— O(n)
            printf("*");  —— O(1)
        }
    }
    function(n-3);
}
```

Sol^n, for function call

$\underbrace{n, n-3, n-6, n-9 \cdots\cdots 1}_{K \text{ terms}}$

AP with $d = -3$, $a = n$

$a_n = a + (n-1)d$

$1 = n + (K-1)(-3)$

$\frac{1-n}{(-3)} = K-1$

$K-1 = \frac{n-1}{3}$

$K = \frac{n-1+3}{3}$

$\boxed{K = \frac{n+2}{3}}$

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Hence 1 function have a recursive call $\frac{n+2}{3}$ times

$\Rightarrow$ Time Complexity $= \left(\frac{n+2}{3}\right)(n)(n)$

$= O(n^3)$

Ques-9 Time Complexity of

```
void function (int n) {
    for (i=1 to n) {
        for (j=1; j<=n; j=j+i)
            printf ("*");
    }
}
```

Sol^n

for $i=1 \rightarrow j = 1,2,3,4,\cdots\cdots n = n$

for $i=2 \rightarrow j = 1,3,5,7,\cdots\cdots n = n/2$

for $i=3 \rightarrow j = 1,4,7,\cdots\cdots n = n/3$

for $i=n \Rightarrow j = 1,\cdots\cdots,n = 1$

$$\Rightarrow \sum_{j=n}^{1} n + n/2 + n/3 + n/4 + \cdots + 1$$

$$\sum_{j=n}^{1} n\left[1 + 1/2 + 1/3 + \cdots\cdots + 1/n\right]$$

$$\sum_{j=n}^{1} n\log n$$

$$T(n) = [n\log n]$$

$$T(n) = O(n\log n)$$

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Qun-10 for the functions, n^k and c^n, what is the asymptotic notation relationship between these functions. Assume that $k>=1$ and $c>1$ are constants. find out the value of c and n_0 for which relation holds.

Sol^n:

As given n^k and c^n

relation b/w n^k and c^n is $\boxed{n^k = O(c^n)}$

as $n^k \leq ac^n$ $\forall$ $n \geq n_0$ for a constant $a>0$

for $n_0 = 1$

$c = 2$

$\Rightarrow 1^k \leq a2^1$

$\therefore \boxed{n_0 = 1, \text{ and } c = 2}$

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