

1. We have a fair coin and we keep throwing it.
 - a. What's the expected number of throws to get 2 heads in a row?
 - b. What's the expected number of throws to get n heads in a row?
 - c. What's the probability that HHH occurs before THH?
 - d. What's the probability that HTH occurs before TTH?
 - e. Could you give an approach that can find the expected number of throws to get any given sequence of heads and tails?
2. We throw a coin 100 times and they are all head. If we throw it for the 101st time, what's the expected probability of head for this throw? (Hint: the coin is not necessarily a fair coin)
3. When you are working on a linear regression and find that the X matrix is in ill-condition (i.e. the smallest eigen-value of $X'X$ is very close to zero) which might cause serious numerical problem when you take the inverse. What would you do to fix it and why do you think it can help?
4. Let X_1 and X_2 be two i.i.d random variables with uniform distribution between 0 and 1. $Y = \min(X_1, X_2)$ and $Z = \max(X_1, X_2)$.
 - a. What are the following statistics $E[Y]$, $E[Z]$, $\text{Corr}(Y, Z)$?
 - b. What if you have n ($n > 2$) i.i.d random variables with uniform distribution between 0 and 1. What are the expected values for max and min, and the correlation between the max value and the min value?
5. Let x be an integer between 1 and 10^{12} , what is the probability that cubic of x ends with 51?
6. Expectation of length of arc:
 - a. Randomly draw two dots on a unit circle (radius=1), what's the expectation of the length of the arc between them (always take the shorter one)?
 - b. Instead of a unit circle, now you have a unit sphere (radius=1), what's your answer?