Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

1.3 Example

2 Bode Plot

2.1 Introduction

2.2 Example

2.1. For an LTI system, the Bode plot for its gain defined as

$$G(s) = 20\log|H(s)| \tag{2.1.1}$$

is as illustrated in the Fig. 2.1. Find the corner frequencies ω_{01} and ω_{02} from the plot.

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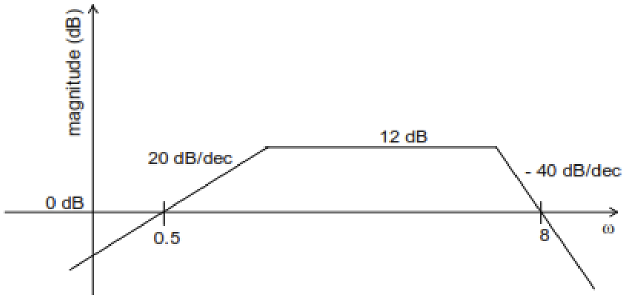

Fig. 2.1

Solution: The corner frequencies can be calculated as follows:

$$slope = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1}$$

Therefore for ω_{02},

$$-40 = \frac{0-12}{\log 8 - \log\omega_{02}}$$

$$\log 8 - \log\omega_{02} = \frac{12}{40}$$

$$\log\omega_{02} = \log 8 - \frac{12}{40}$$

$$\omega_{02} = 4$$

And for ω_{01},

$$20 = \frac{0-12}{\log 0.5 - \log\omega_{01}}$$

$$\log 0.5 - \log\omega_{01} = \frac{-12}{20}$$

$$\log\omega_{01} = \log 0.5 + \frac{12}{20}$$

$$\omega_{01} = 2$$

So, the corner frequencies are ω_{01}=2 and ω_{02} = 4.

2.2. Express the given bode plot as a piece-wise linear function.

Solution:

$$G(\omega) = \begin{cases} 20\log 2\omega & 0 < \omega \le 2 \\ 12 & 2 \le \omega \le 4 \\ -40\log\frac{\omega}{8} & \omega \ge 4 \end{cases} \tag{2.2.1}$$

2.3. Find the transfer function from the calculated frequencies.

Solution: By looking to the plot, we can say that since the initial slope is +20, there must be a zero at the origin. At ω_{01}, the change in slope is -20dB, so their exists one pole at this frequency.

At ω_{02}, the change in slope is -40dB, so their exists two pole at this frequency.

The denominators have the form,

$$(1+\frac{s}{\omega})$$

So, the denominator of the transfer function is

$$(1+\frac{s}{2})(1+\frac{s}{4})^2$$

Therefore, the transfer function is,

$$\frac{cs}{(1+\frac{s}{2})(1+\frac{s}{4})^2}$$

here c is some constant

2.4. Compare the above calculated transfer function with one of the options that best represents it.

$$(A)\frac{2s}{(1+0.5s)(1+0.25s)^2} \qquad (B)\frac{4(1+0.5s)}{s(1+0.25s)}$$

$$(C)\frac{2s}{(1+2s)(1+4s)} \qquad (D)\frac{4s}{(1+2s)(1+4s)^2}$$

Solution: From the above given options, we can see that option (A) best represents our transfer function.

$$\frac{2s}{(1+0.5s)(1+0.25s)^2}$$

2.5. Verify the above transfer function by plotting the bode plot.

Solution: Refer Fig. 2.5 for the bode plot:

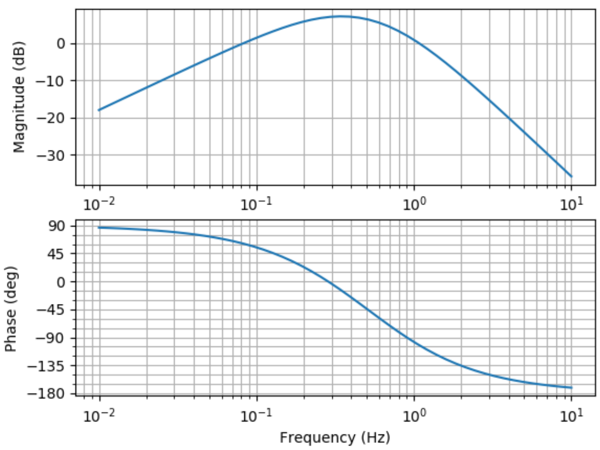

Fig. 2.5: Plot of G(s)

The plot was plotted using the following code:

```
import numpy as np
import control.matlab as ml
import matplotlib.pyplot as plt

# If using termux
import subprocess
import shlex
#end if

# num is the numerator of the trasfer
    function which is (2s)
# dem is the denominator of the transfer
    function which is (0.25s + 1)(0.25s + 1)
    (0.5s + 1)
num = np.array([2, 0])
den = np.polymul(np.array([0.5, 1]), np.array
    ([0.25, 1]))
den = np.polymul(den, np.array([0.25, 1]))

# Generating the transfer function
g = ml.tf(num, den)
print(”The transfer function is: ”, g)
print(”The poles of the above function are ”,
    ml.pole(g))
print(”The zeros of the above function are ”,
    ml.zero(g))

# Generating the bode plot as well as plotting
    it
mag, phase, w = ml.bode(g)

# If using termux
plt.savefig(”./figs/ep18btech11016_plot.pdf”)
plt.savefig(”./figs/ep18btech11016_plot.eps”)
subprocess.run(shlex.split(”termux−open ./figs
    /ep18btech11016_plot.pdf”))
# else
plt.show()
```

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