Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

1.3 Example

2 Bode Plot

2.1 Introduction

2.2 Example

2.1. For an LTI system, the Bode plot for its gain defined as

$$G(s) = 20 \log |H(s)| \tag{2.1.1}$$

is as illustrated in the Fig. 2.1. Find the corner frequencies ω_{01} and ω_{02} from the plot.

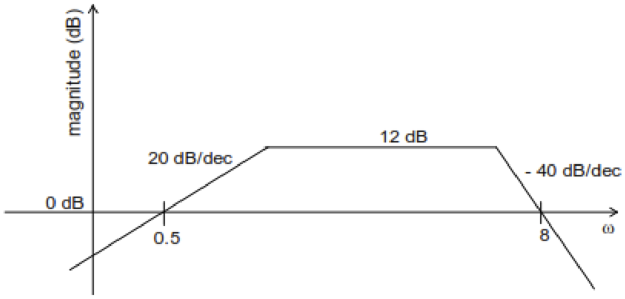

Fig. 2.1

Solution: The corner frequencies can be calculated as follows:

$$slope = \frac{M_2 - M_1}{\log \omega_2 - \log \omega_1}$$

Therefore for ω_{02},

$$-40 = \frac{0 - 12}{\log 8 - \log \omega_{02}}$$

$$\log 8 - \log \omega_{02} = \frac{12}{40}$$

$$\log \omega_{02} = \log 8 - \frac{12}{40}$$

$$\omega_{02} = 4$$

And for ω_{01},

$$20 = \frac{0 - 12}{\log 0.5 - \log \omega_{01}}$$

$$\log 0.5 - \log \omega_{01} = \frac{-12}{20}$$

$$\log \omega_{01} = \log 0.5 + \frac{12}{20}$$

$$\omega_{01} = 2$$

So, the corner frequencies are ω_{01}=2 and ω_{02} = 4.

2.2. Find the transfer function from the calculated frequencies.
Solution: By looking to the plot, we can say that since the initial slope is +20, there must be a zero at the origin. At ω_{01}, the change in slope is -20dB, so their exists one pole at this frequency.
At ω_{02}, the change in slope is -40dB, so their exists two pole at this frequency.
The denominators have the form,

$$(1 + \frac{s}{\omega})$$

So, the denominator of the transfer function is

$$(1 + \frac{s}{2})(1 + \frac{s}{4})^2$$

Therefore, the transfer function is,

$$\frac{cs}{(1 + \frac{s}{2})(1 + \frac{s}{4})^2}$$

here c is some constant

2.3. Compare the above calculated transfer function with one of the options that best represents it.

$$(A)\frac{2s}{(1 + 0.5s)(1 + 0.25s)^2} \quad (B)\frac{4(1 + 0.5s)}{s(1 + 0.25s)}$$

$$(C)\frac{2s}{(1 + 2s)(1 + 4s)} \quad (D)\frac{4s}{(1 + 2s)(1 + 4s)^2}$$

Solution: From the above given options, we can see that option (A) best represents our transfer function.

$$\frac{2s}{(1 + 0.5s)(1 + 0.25s)^2}$$

2.4. Verify the above transfer function by plotting the bode plot.
Solution: Refer Fig. 2.4 for the bode plot:

The plot was plotted using the following code:

```
import numpy as np
import control.matlab as ml
import matplotlib.pyplot as plt

# If using termux
import subprocess
import shlex
#end if
```

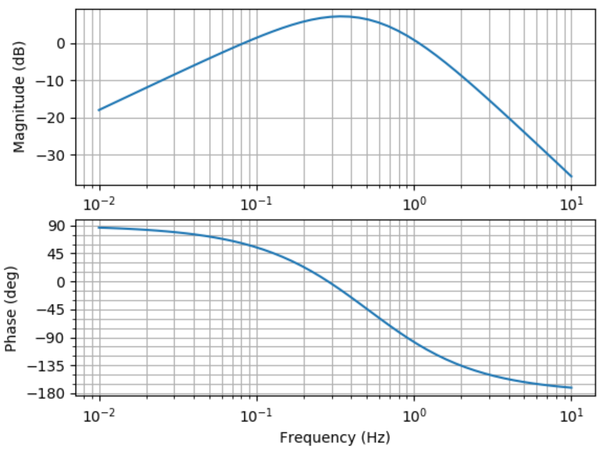

Fig. 2.4: Plot of G(s)

```
# num is the numerator of the trasfer
     function which is (2s)
# dem is the denominator of the transfer
     function which is (0.25s + 1)(0.25s + 1)
     (0.5s + 1)
num = np.array([2, 0])
den = np.polymul(np.array([0.5, 1]), np.array
     ([0.25, 1]))
den = np.polymul(den, np.array([0.25, 1]))

# Generating the transfer function
g = ml.tf(num, den)
print("The transfer function is: ", g)
print("The poles of the above function are ",
     ml.pole(g))
print("The zeros of the above function are ",
     ml.zero(g))

# Generating the bode plot as well as plotting
      it
mag, phase, w = ml.bode(g)

# If using termux
plt.savefig("./figs/ep18btech11016_plot.pdf")
plt.savefig("./figs/ep18btech11016_plot.eps")
subprocess.run(shlex.split("termux-open ./figs
     /ep18btech11016_plot.pdf"))
# else
plt.show()
```

2.3 Phase

3 Second order System

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4.3 Stability

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