# Control Systems

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Contents



***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

## 1 Feedback Voltage Amplifier: Series-Shunt

## 2 Feedback Current Amplifier: Shunt-Series

### 2.1 Ideal Case

### 2.2 Practical Case

## 3 Feedback Current Amplifier: Example

3.1. The feedback current amplifier in Fig. 3.2.1 utilizes an op amp with an input differential resistance $R_{id}$, an open-loop gain $\mu$ , and an output resistance $r_o$. The output current $I_o$ that is delivered to the load resistance $R_L$ is sensed by the feedback network composed of the two resistances $R_M$ and $R_F$ and a fraction $I_f$, is fed back to the amplifier input node. Find expressions for $G = \frac{I_o}{I_i}$, $H = \frac{I_f}{I_o}$ and $T = \frac{I_o}{I_s}$, assuming that the feedback causes the voltage at the input node to be near ground. If the loop gain is large, what does the closed-loop current gain become? State precisely the condition under which this is obtained. For $\mu$ = $10^4$ V/V, $R_{id}$ = 1 MΩ, $r_o$ = 100 Ω, $R_L$ = 10 kΩ , $R_M$ = 100 Ω , and $R_F$ = 10 kΩ, find G, H, and T.
**Solution:** Follow the below sub-questions for solution.

3.2. Fig. 3.2.1 shows a feedback current amplifier. Draw the equivalent control system.
**Solution:** See fig 3.2.2

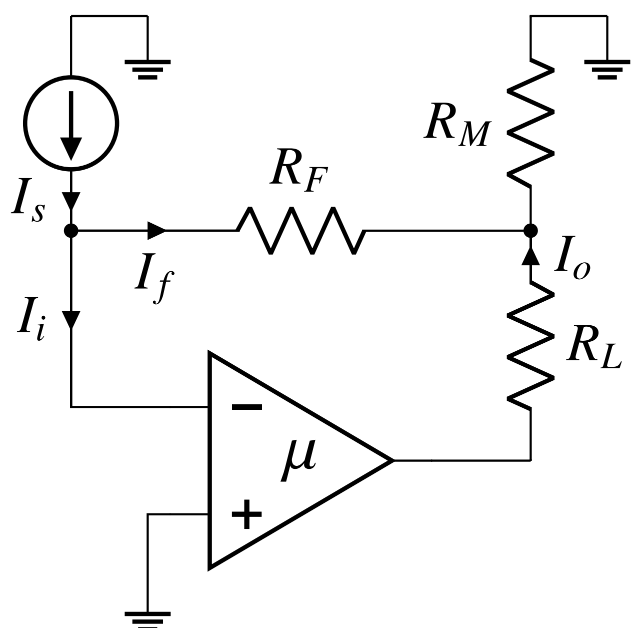


Fig. 3.2.1

3.3. Refer table 3.3 for the parameters and draw the small signal equivalent model of the fig 3.2.1
**Solution:** See fig 3.3

3.4. Given G (open-loop gain), H (feedback gain)

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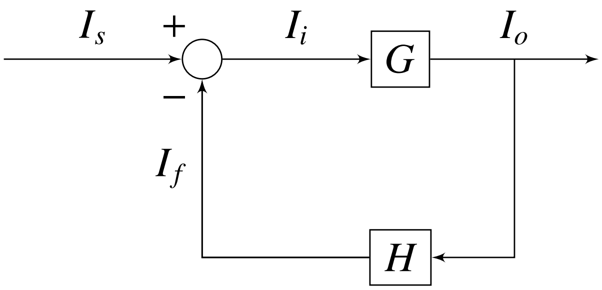


Fig. 3.2.2

| Component | Description |
|---|---|
| $R_{id}$ | Input Resistance of Op Amp |
| $R_{out}$ | Output Resistance of Op Amp |
| $I_s$ | Input Current |
| $I_o$ | Output Current |
| $R_M, R_F$ | Feedback Resistances |
| $R_L$ | Load Resistance |

TABLE 3.3

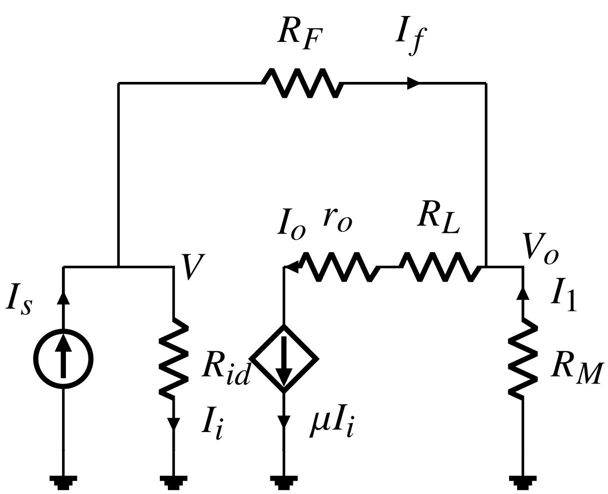


Fig. 3.3

and T (closed-loop gain) as

$$G = \frac{I_o}{I_i} \tag{3.4.1}$$

$$H = \frac{I_f}{I_o} \tag{3.4.2}$$

$$T = \frac{I_o}{I_f} \tag{3.4.3}$$

Find G and H as a function of the resistances.
**Solution:** Refer fig. 3.3,

We get,

$$I_o = \mu I_i \tag{3.4.4}$$

$$\implies G = \frac{I_o}{I_i} = \mu \tag{3.4.5}$$

And,

$$I_i = I_s - I_f \tag{3.4.6}$$

Using eq. 3.4.5, we get,

$$I_o = \mu\left(I_s - I_f\right) \tag{3.4.7}$$

Assuming virtual ground at $V_1$, we get,

$$V_o = -I_f R_F \tag{3.4.8}$$

Therefore,

$$I_1 = -\frac{V_o}{R_M} \tag{3.4.9}$$

$$\implies I_1 = I_f \frac{R_F}{R_M} \tag{3.4.10}$$

The output current is also expressed as,

$$I_o = I_f + I_1 \tag{3.4.11}$$

$$\implies \frac{I_o}{1 + \frac{R_F}{R_M}} = I_f \tag{3.4.12}$$

Now substituting $I_f$ in eq. 3.4.7 we get,

$$I_o = \mu\left(I_s - \frac{I_o}{1 + \frac{R_F}{R_M}}\right) \tag{3.4.13}$$

(3.4.14)

$$\implies T = \frac{I_o}{I_s} = \frac{\mu}{1 + \frac{\mu}{1 + \frac{R_F}{R_M}}} \tag{3.4.15}$$

Therefore,

$$\implies H = \frac{I_f}{I_o} = \frac{1}{1 + \frac{R_F}{R_M}} \tag{3.4.16}$$

3.5. What will be closed-loop gain(T) if $\mu \to \infty$
**Solution:** From eq. 3.4.15 we get,

$$T = \frac{\mu}{1 + \frac{\mu}{1 + \frac{R_F}{R_M}}} \tag{3.5.1}$$

$$T = \frac{1}{\frac{1}{\mu} + \frac{1}{1 + \frac{R_F}{R_M}}} \tag{3.5.2}$$

Applying the limit, we get,

$$\implies T = 1 + \frac{R_F}{R_M} \tag{3.5.3}$$

3.6. Refer table 3.6 and find G, H and T

| **Component** | **Value** |
|---|---|
| $\mu$ | $10^4$ |
| $R_{id}$ | $1\ M\Omega$ |
| $r_o$ | $100\ \Omega$ |
| $R_L$ | $10\ \text{k}\Omega$ |
| $R_M$ | $100\ \Omega$ |
| $R_F$ | $10\ \text{k}\Omega$ |

TABLE 3.6

**Solution:** Using eqs. 3.4.5, 3.4.16 and 3.4.15
We get,

$$G = \mu = 10^4 \tag{3.6.1}$$

$$H = \frac{1}{1 + \frac{R_F}{R_M}} = 9.9 \times 10^{-3} \tag{3.6.2}$$

$$T = \frac{\mu}{1 + \frac{\mu}{1 + \frac{R_F}{R_M}}} = 100 \tag{3.6.3}$$

3.7. Tabulate your results.
**Solution:** Refer table 3.7,

| **Gain** | **Value** |
|---|---|
| G | $10^4$ |
| H | $9.9 \times 10^{-3}$ |
| T | 100 |

TABLE 3.7

## 4 Feedback Transconductance Amplifier: Series-Series