

Control Systems

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Abstract—The objective of this manual is to introduce control system design at an elementary level.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/ketan/codes>

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1 FREQUENCY RESPONSE ANALYSIS

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2 STABILITY IN FREQUENCY DOMAIN

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3 DESIGN IN FREQUENCY DOMAIN

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3.1.	For a unity feedback system given below refer,	
3.5.1,	with	

$$G(s) = \frac{K}{s(s+5)(s+11)} \quad (3.1.1)$$

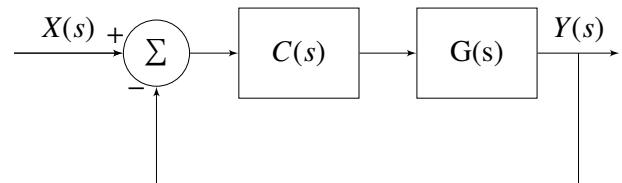


Fig. 3.5.1

Find the value of the gain, K , of the uncompensated system operating at 30% overshoot.

Solution:

The damping ratio ζ is given by:

$$\zeta = -\frac{\ln \frac{\%OS}{100}}{\sqrt{\pi^2 + \ln \frac{\%OS}{100}^2}} \quad (3.1.2)$$

Therefore, solving the above equation with %OS = 30, we get,

$$\zeta = 0.358 \quad (3.1.3)$$

Further, we need to find the point on the root locus which crosses the 0.358 damping ratio line.

Let this point be $-\sigma_d + j\omega_d$, where σ_d is the exponential damping frequency and ω_d is the damped frequency of oscillation.

And the relation between σ_d and ω_d is given by,

$$\omega_d = \sigma \tan(\arccos \zeta) \quad (3.1.4)$$

where,

$$\sigma_d = \zeta \omega_n \quad (3.1.5)$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \quad (3.1.6)$$

ω_n is the natural frequency.

Now, by solving the below equation and equating the real and imaginary parts to zero,

$$1 + G(-\sigma_d + j\omega_d) = 0 \quad (3.1.7)$$

We get,

$$\sigma_d = 1.464 \quad (3.1.8)$$

$$\omega_d = 3.818 \quad (3.1.9)$$

$$\text{Gain, } K = 218.6 \quad (3.1.10)$$

3.2. Evaluate the performance in terms of peak time and settling time as well as find K_v of the uncompensated system.

Solution:

The peak time, T_p is given by,

$$T_p = \frac{\pi}{\omega_d} \quad (3.2.1)$$

And, settling time is given by,

$$T_s = \frac{4}{\sigma_d} \quad (3.2.2)$$

So, we get,

$$T_p = 0.823 \quad (3.2.3)$$

$$T_s = 2.732 \quad (3.2.4)$$

$$K_v = \lim_{s \rightarrow 0} sG(s) \quad (3.2.5)$$

$$\lim_{s \rightarrow 0} \left(\frac{K}{(s+5)(s+11)} \right) = \frac{218.6}{(5)(11)} \quad (3.2.6)$$

$$\Rightarrow K_v = 3.975 \quad (3.2.7)$$

3.3. Design a lag-lead compensator to:

- Decrease the peak time by a factor of 2
- Decrease the percent overshoot by a factor of 2
- Improve the steady state error by a factor of 30

Solution:

Lead Design: Using the required specifications, we can calculate the damping ratio and the natural frequency, Using eq. 3.1.2, we get,

$$\zeta = 0.517 \quad (3.3.1)$$

And,

$$\omega_d = \frac{\pi}{T_p} = \omega_n \sqrt{1 - \zeta^2} = 7.634. \quad (3.3.2)$$

Hence, $\omega_n = 8.919$.

Thus, the desired pole is located at,

$$-\zeta \omega_n + j\omega_n \sqrt{1 - \zeta^2} = -4.61 + j7.634 \quad (3.3.3)$$

Let us now assume a lead compensator zero at -5. The summation of the system's original poles and lead compensator zero to the design point is -171.2° . Thus, the compensator pole must contribute $171.2^\circ - 180^\circ = -8.8^\circ$.

Refer figure 3.5.2 for clarification.

$$\tan(8.8^\circ) = \frac{7.634}{p_c - 4.61} \quad (3.3.4)$$

Hence, $p_c = 53.92$.

Therefore, the compensated open-loop transfer

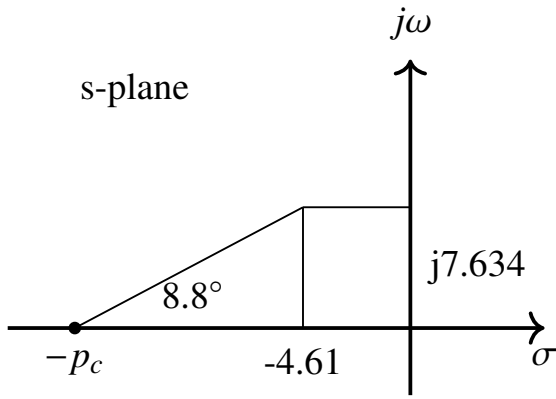


Fig. 3.5.2

function is,

$$\frac{K}{s(s+11)(s+53.92)} \quad (3.3.5)$$

Evaluating the gain for this function at the desired pole, we get $K = 4430$.

Lag Design:

The lead compensated $K_v = 7.469$.

We need an improvement over the lead compensated system of,

$$\frac{(30)(3.975)}{7.469} = \frac{119.25}{7.469} \quad (3.3.6)$$

$$\Rightarrow = 15.97 \quad (3.3.7)$$

$$K_v = \lim_{s \rightarrow 0} sG(s) \quad (3.3.8)$$

Choose p_c (compensator pole) = 0.001, we get z_c (compensator zero) = 0.001597. Thus, the compensator is given by,

$$G_{lag}(s) = \frac{s + 0.01597}{s + 0.001} \quad (3.3.9)$$

So, the final compensated open loop transfer function is,

$$C(s)G(s) = \frac{4430(s + 0.01597)}{s(s + 11)(s + 53.92)(s + 0.001)} \quad (3.3.10)$$

3.4. Plot the graph after adding a lag-lead compensator.

Solution:

See Fig. 3.5.3 generated by

codes/ep18btech11016/ep18btech11016.py

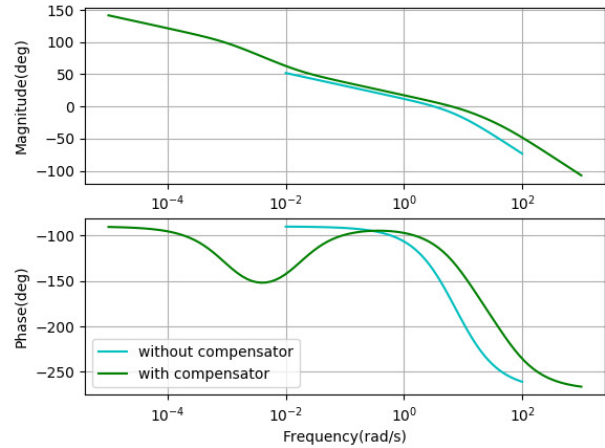


Fig. 3.5.3

3.6

3.7 Lead Compensator

3.8

3.9

3.10

4 PID CONTROLLER DESIGN

4.1 PD

4.2 PID