

Control Systems

G V V Sharma*

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

1.3 Example

2 BODE PLOT

2.1 Introduction

2.2 Example

Question: Consider the following asymptotic Bode magnitude plot (ω is in rad/s).

Which of the following transfer function is best represented by the above Bode magnitude plot?

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

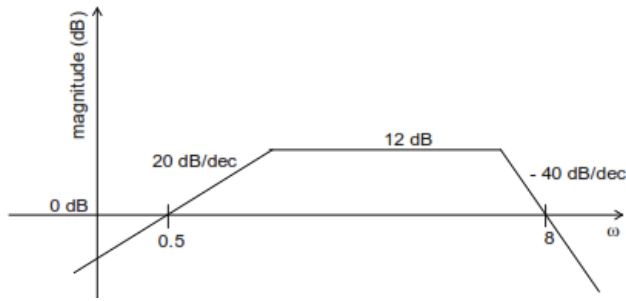


Fig. 2.0: Plot of G(s)

$$(A) \frac{2s}{(1 + 0.5s)(1 + 0.25s)^2} \quad (B) \frac{4(1 + 0.5s)}{s(1 + 0.25s)}$$

$$(C) \frac{2s}{(1 + 2s)(1 + 4s)} \quad (D) \frac{4s}{(1 + 2s)(1 + 4s)^2}$$

By looking to the plot, we can say that since the initial slope is +20, there must be a zero at the origin. Let the corner frequencies of the plot be ω_{01} and ω_{02} . They can be calculated as follows:

$$\text{slope} = \frac{M_2 - M_1}{\log \omega_2 - \log \omega_1}$$

Therefore for ω_{02} ,

$$-40 = \frac{0 - 12}{\log 8 - \log \omega_{02}}$$

$$\log 8 - \log \omega_{02} = \frac{12}{40}$$

$$\log \omega_{02} = \log 8 - \frac{12}{40}$$

$$\omega_{02} = 4$$

And for ω_{01} ,

$$20 = \frac{0 - 12}{\log 0.5 - \log \omega_{01}}$$

$$\log 0.5 - \log \omega_{01} = \frac{-12}{20}$$

$$\log \omega_{01} = \log 0.5 + \frac{12}{20}$$

$$\omega_{01} = 2$$

So, the corner frequencies are $\omega_{01}=2$ and $\omega_{02} = 4$. At ω_{01} , the change in slope is -20dB, so there exists one pole at this frequency and at ω_{02} , the change in slope is -40dB, so there exists two poles at this frequency.

The denominators have the form $(1 + \frac{s}{\omega})$
So, the denominator of the transfer function is
 $(1 + \frac{s}{2})(1 + \frac{s}{4})^2$
Therefore, the transfer function is,

$$\frac{cs}{(1 + \frac{s}{2})(1 + \frac{s}{4})^2}$$

here c is some constant

The answer is therefore option (A)

$$\frac{2s}{(1 + 0.5s)(1 + 0.25s)^2}$$

We will now plot the bode plot of the given transfer function to verify it. The bode plot is:

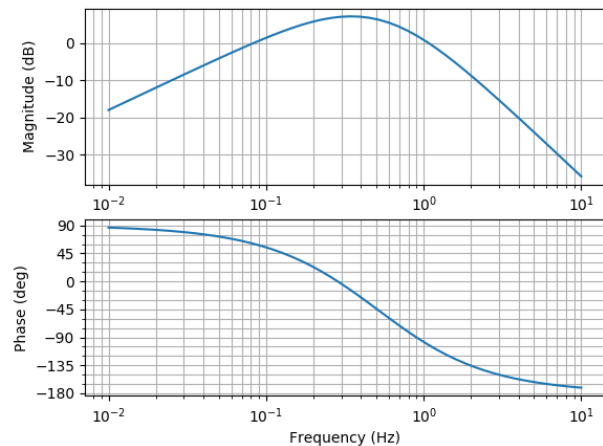


Fig. 2.0: Plot of G(s)

The plot was plotted using the following code:

```
import numpy as np
import control.matlab as ml
import matplotlib.pyplot as plt

num = np.array([2, 0])
den = np.polymul(np.array([0.5, 1]), np.array([0.25, 1]))
den = np.polymul(den, np.array([0.25, 1]))

g = ml.tf(num, den)
print(g)
print(ml.pole(g))
print(ml.zero(g))
```

```
mag, phase, w = ml.bode(g)
plt.show()
```

2.3 Phase

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Example

3.3 Settling Time

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

4.5 Example

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

5.5 Example

5.6 Example

5.7 Example

6 NYQUIST PLOT

6.1 Introduction

6.2 Example

7 COMPENSATORS

7.1 Phase Lead

7.2 Lag Lead

7.3 Example

8 GAIN MARGIN

8.1 Introduction

8.2 Example

8.3 Example

9 PHASE MARGIN

9.1 Introduction

9.2 Example

10 OSCILLATOR

10.1 Introduction

10.2 Example

11 ROOT LOCUS

11.1 Introduction