

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/control/codes
```

1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

1.3 Example

2 BODE PLOT

2.1 Introduction

2.2 Example

2.1. For an LTI system, the Bode plot for its gain defined as

$$G(s) = 20 \log |H(s)| \quad (2.1.1)$$

is as illustrated in the Fig. 2.1. Find the corner frequencies ω_{01} and ω_{02} from the plot.

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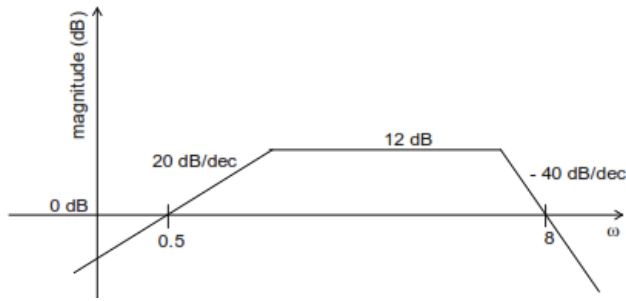


Fig. 2.1

Solution: The corner frequencies can be calculated as follows:

$$\text{slope} = \frac{M_2 - M_1}{\log \omega_2 - \log \omega_1}$$

Therefore for ω_{02} ,

$$-40 = \frac{0 - 12}{\log 8 - \log \omega_{02}}$$

$$\log 8 - \log \omega_{02} = \frac{12}{40}$$

$$\log \omega_{02} = \log 8 - \frac{12}{40}$$

$$\omega_{02} = 4$$

And for ω_{01} ,

$$20 = \frac{0 - 12}{\log 0.5 - \log \omega_{01}}$$

$$\log 0.5 - \log \omega_{01} = \frac{-12}{20}$$

$$\log \omega_{01} = \log 0.5 + \frac{12}{20}$$

$$\omega_{01} = 2$$

So, the corner frequencies are $\omega_{01}=2$ and $\omega_{02} = 4$.

2.2. Find the transfer function from the calculated frequencies.

Solution: By looking to the plot, we can say

that since the initial slope is +20, there must be a zero at the origin.

At ω_{01} , the change in slope is -20dB, so there exists one pole at this frequency.

At ω_{02} , the change in slope is -40dB, so there exists two poles at this frequency.

The denominators have the form $(1 + \frac{s}{\omega})$

So, the denominator of the transfer function is

$$(1 + \frac{s}{2})(1 + \frac{s}{4})^2$$

Therefore, the transfer function is,

$$\frac{cs}{(1 + \frac{s}{2})(1 + \frac{s}{4})^2}$$

here c is some constant

2.3. Compare the above calculated transfer function with one of the options that best represents it.

$$(A) \frac{2s}{(1 + 0.5s)(1 + 0.25s)^2} \quad (B) \frac{4(1 + 0.5s)}{s(1 + 0.25s)}$$

$$(C) \frac{2s}{(1 + 2s)(1 + 4s)} \quad (D) \frac{4s}{(1 + 2s)(1 + 4s)^2}$$

Solution: From the above given option, we can see that option (A) best represents our transfer function.

$$\frac{2s}{(1 + 0.5s)(1 + 0.25s)^2}$$

2.4. Verify the above plot by plotting the transfer function.

Solution: The bode plot is Fig. 2.4:

The plot was plotted using the following code:

```
import numpy as np
import control.matlab as ml
import matplotlib.pyplot as plt

# If using termux
import subprocess
import shlex
#end if

# num is the numerator of the transfer
function which is (2s)
```

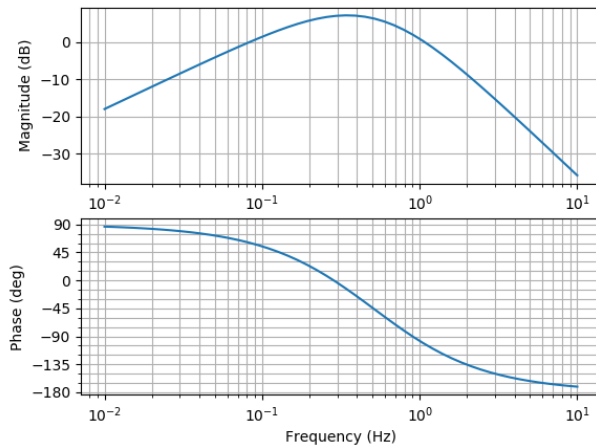


Fig. 2.4: Plot of $G(s)$

```
# dem is the denominator of the transfer
# function which is (0.25s + 1)(0.25s + 1)
# (0.5s + 1)
num = np.array([2, 0])
den = np.polymul(np.array([0.5, 1]), np.array
([0.25, 1]))
den = np.polymul(den, np.array([0.25, 1]))

# Generating the transfer function
g = ml.tf(num, den)
print("The transfer function is: ", g)
print("The poles of the above function are ",
      ml.pole(g))
print("The zeros of the above function are ",
      ml.zero(g))

# Generating the bode plot as well as plotting
# it
mag, phase, w = ml.bode(g)

# If using termux
plt.savefig("../figs/ep18btech11016_plot.pdf")
plt.savefig("../figs/ep18btech11016_plot.eps")
subprocess.run(shlex.split("termux-open ./figs
/ep18btech11016_plot.pdf"))
# else
plt.show()
```

2.3 Phase

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Example

3.3 Settling Time

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

4.5 Example

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

5.5 Example

5.6 Example

5.7 Example

6 NYQUIST PLOT

6.1 Introduction

6.2 Example

7 COMPENSATORS

7.1 Phase Lead

7.2 Lag Lead

7.3 Example

8 GAIN MARGIN

8.1 Introduction

8.2 Example

8.3 Example

9 PHASE MARGIN

9.1 Introduction

9.2 Example

10 OSCILLATOR

10.1 Introduction

10.2 Example

11 ROOT LOCUS

11.1 Introduction