Control Systems

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***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

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1 Frequency Response Analysis

1.1 Polar Plot

1.2 Direct and Inverse Polar Plot

1.3 Bode Plot

1.4 Transient Response from Bode Plot

2 Stability in Frequency Domain

2.1 Nyquist Criterion

2.2 Nyquist Criterion

2.3

2.4 Nyquist and Routh-Hurwitz

2.5 Nyquist

2.6 Nyquist

3 Design in Frequency Domain

3.1

3.2

3.3 Lag Compensator

3.4 Lead Compensator

3.5 Compensator for Overshoot

3.1. For a unity feedback system given below refer, 3.5.1, with

$$G(s) = \frac{K}{s(s+5)(s+11)} \tag{3.1.1}$$

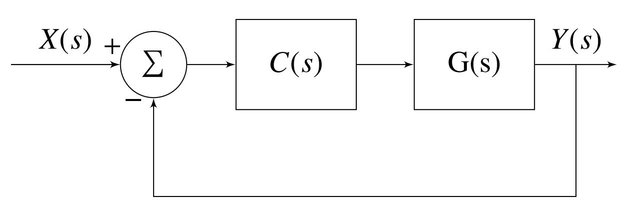

Fig. 3.5.1

Find the value of the gain, K, of the uncompensated system operating at 30% overshoot.

Solution:
The damping ratio ζ is given by:

$$\zeta = -\frac{\ln \frac{\%OS}{100}}{\sqrt{\pi^2 + \ln \frac{\%OS}{100}^2}} \tag{3.1.2}$$

Therefore, solving the above equation with %OS = 30, we get,

$$\zeta = 0.358 \tag{3.1.3}$$

Further, we need to find the point on the root locus which crosses the 0.358 damping ratio line.
Let this point be $-\sigma_d + j\omega_d$, where σ_d is the exponential damping frequency and ω_d is the damped frequency of oscillation.
And the relation between σ_d and ω_d is given by,

$$\omega_d = \sigma \tan(\arccos \zeta)) \tag{3.1.4}$$

where,

$$\sigma_d = \zeta\omega_n \tag{3.1.5}$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2} \tag{3.1.6}$$

ω_n is the natural frequency.

Now, by solving the below equation and equating the real and imaginary parts to zero,

$$1 + G(-\sigma_d + j\omega_d) = 0 \tag{3.1.7}$$

We get,

$$\sigma_d = 1.464 \tag{3.1.8}$$

$$\omega_d = 3.818 \tag{3.1.9}$$

$$Gain, K = 218.6 \tag{3.1.10}$$

3.2. Evaluate the performance in terms of peak time and settling time as well as find K_v of the uncompensated system.

Solution:
The peak time, T_p is given by,

$$T_p = \frac{\pi}{\omega_d} \tag{3.2.1}$$

And, settling time is given by,

$$T_s = \frac{4}{\sigma_d} \tag{3.2.2}$$

So, we get,

$$T_p = 0.823 \tag{3.2.3}$$

$$T_s = 2.732 \tag{3.2.4}$$

$$K_v = \lim_{s\to 0} sG(s) \tag{3.2.5}$$

$$\lim_{s\to 0}\left(\frac{K}{(s+5)(s+11)}\right) = \frac{218.6}{(5)(11)} \tag{3.2.6}$$

$$\implies K_v = 3.975 \tag{3.2.7}$$

3.3. Design a lag-lead compensator to:
   a) Decrease the peak time by a factor of 2
   b) Decrease the percent overshoot by a factor of 2
   c) Improve the steady state error by a factor of 30

Solution:
Lead Design: Using the required specifications, we can calculate the damping ratio and the natural frequency, Using eq. 3.1.2, we get,

$$\zeta = 0.517 \tag{3.3.1}$$

And,

$$\omega_d = \frac{\pi}{T_p} = \omega_n \sqrt{1-\zeta^2} = 7.634. \tag{3.3.2}$$

Hence, $\omega_n = 8.919$.
Thus, the desired pole is located at,

$$-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} = -4.61 + j7.634 \tag{3.3.3}$$

Let us now assume a lead compensator zero at -5. The summation of the system's original poles and lead compensator zero to the design point is -171.2. Thus, the compensator pole must contribute 171.2° - 180° = -8.8°.
Refer figure 3.5.2 for clarification.

$$\tan(8.8°) = \frac{7.634}{p_c - 4.61} \tag{3.3.4}$$

Hence, $p_c = 53.92$.
Therefore, the compensated open-loop transfer

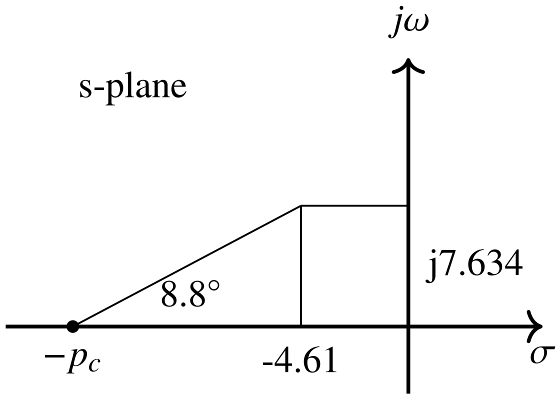

Fig. 3.5.2

function is,

$$\frac{K}{s(s+11)(s+53.92)} \tag{3.3.5}$$

Evaluating the gain for this function at the desired pole, we get $K = 4430$.

Lag Design:
The lead compensated $K_v = 7.469$.
We need an improvement over the lead compensated system of,

$$\frac{(30)(3.975)}{7.469} = \frac{119.25}{7.469} \tag{3.3.6}$$

$$\implies = 15.97 \tag{3.3.7}$$

$$K_v = \lim_{s \to 0} sG(s) \tag{3.3.8}$$

Choose p_c (compensator pole) = 0.001, we get z_c (compensator zero) = 0.001597. Thus, the compensator is given by,

$$G_{lag}(s) = \frac{s+0.01597}{s+0.001} \tag{3.3.9}$$

So, the final compensated open loop transfer function is,

$$C(s)G(s) = \frac{4430(s+0.01597)}{s(s+11)(s+53.92)(s+0.001)} \tag{3.3.10}$$

3.4. Plot the graph after adding a lag-lead compensator.
Solution:
See Fig. 3.5.3 generated by

```
codes/ep18btech11016/ep18btech11016.py
```

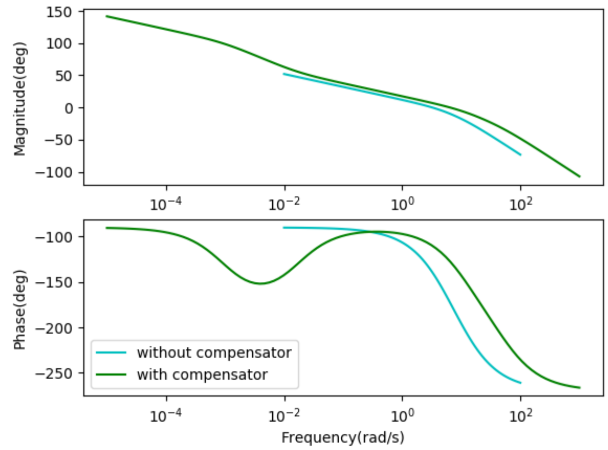

Fig. 3.5.3

3.6

3.7 Lead Compensator

3.8

3.9

3.10

4 PID Controller Design

4.1 PD

4.2 PID