教育部教改計畫
開發課程模組

IR Drop Analysis Concepts

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Outline

- Introduction
- Modified Nodal Analysis
- Solving Systems of Linear Equations
  - LU decomposition
  - LUP decomposition
  - Inversion and Multiplication
- Summary

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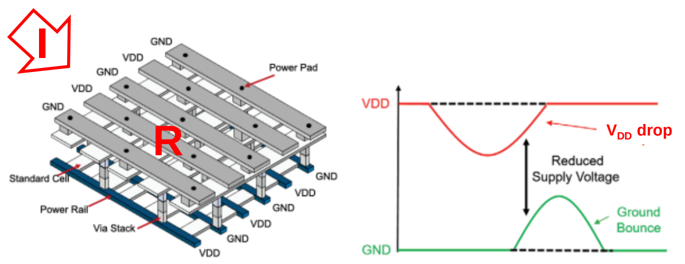
What is IR Drop?

- Non-ideal voltage change when logic gates switching
  - Excess current *(I)* and power network resistance *(R)*
- Two types:
  - V_{DD} drop
  - Ground bounce



(a) Power ground network. (b) Dynamic IR-drop.

X.Huang et al " Dynamic IR-Drop ECO Optimization by Cell Movement with Current Waveform Staggering and Machine Learning Guidance, "2020 ICCAD.

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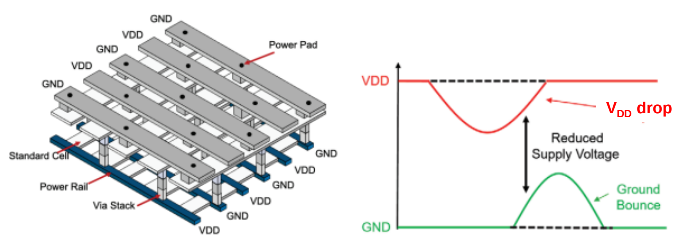
Why is IR Drop Important?

- High IR Drop causes
  - Extra circuit delay: slower than expected
  - Test yield loss: good circuits fail testing



(a) Power ground network. (b) Dynamic IR-drop.

X.Huang et al " Dynamic IR-Drop ECO Optimization by Cell Movement with Current Waveform Staggering and Machine Learning Guidance, "2020 ICCAD.

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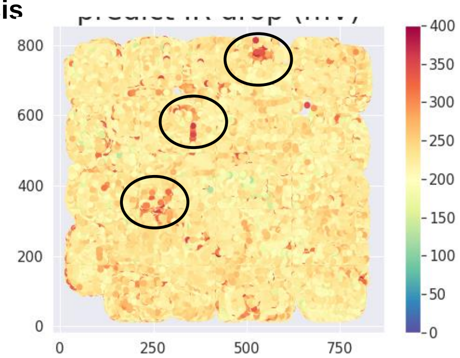
IR Drop Map

- IR Drop ***Hotspots***
    - Locations where IR drop is serious
- How to find hotspots?
    - ***IR Drop Analysis***
- Categories of IR drop analysis
    - ***Vectorless***
        - No input vectors
    - ***Vector-based***



J. Chen et al " Vector-based Dynamic IR-drop Prediction Using Machine Learning, " 2022 ASP-DAC.

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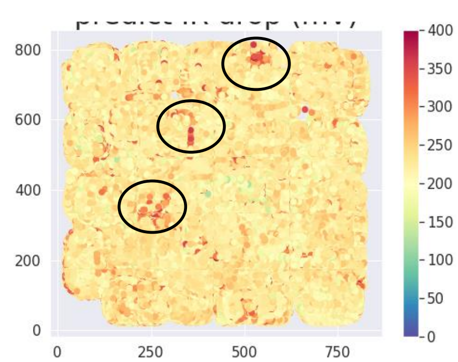
How to Fix IR Drop Hotspots?

- Many solutions (not in this lecture)
- ***Engineering Change Order*** (ECO)
    - 1. cell move
    - 2. cell downsizing
    - 3. add decouping capacitors



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Modified Nodal Analysis (MNA)

- Suppose *n* nodes, *m* voltage sources
- Five steps:

- 1. Select a *reference node* (usually ground)
  - name remaining *n*-1 nodes
  - Also label currents through each current source
- 2. Assign name to current through each of *m* voltage source
  - current flows from positive node to negative node
- 3. Apply Kirchoff's current law to each node
  - currents out of node to be positive
- 4. Write an equation for each voltage source
- 5. Solve system of *linear equations*
  - *n*+*m* variables

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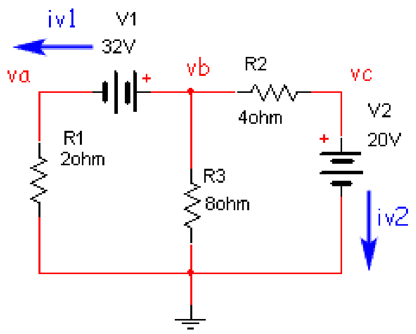
MNA: example 1

- 3 nodes and 2 voltage sources : (n=3, m=2)
- Step 2, 3



$$\frac{v_a}{R_1} - i_{v1} = 0 \qquad \textbf{a}$$

$$i_{v1} + \frac{v_b}{R_3} + \frac{v_b - v_c}{R_2} = 0 \qquad \textbf{b}$$

$$i_{v2} + \frac{v_c - v_b}{R_2} = 0 \qquad \textbf{c}$$

$$v_b - v_a = V1$$

$$v_c = V2$$

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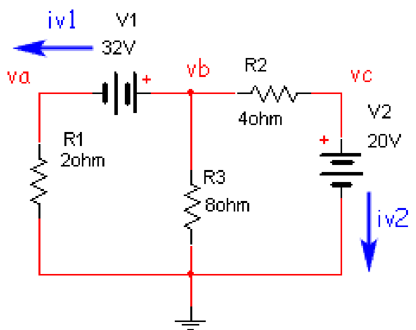
MNA: example 1 (2)

- Step 4 in matrix form



$$\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & -1 & 0 \\ 0 & \frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & 1 & 0 \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \\ i_{v1} \\ i_{v2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V1 \\ V2 \end{bmatrix}$$

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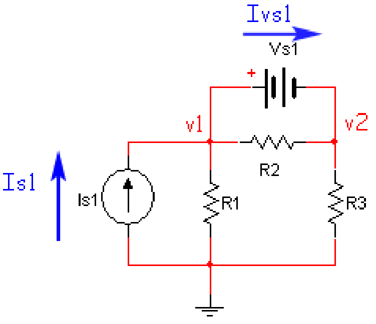
MNA: example 2 (1)

- Independent current source I_{s1}
- Steps 2, 3



$$-I_{s1} + \frac{v_1}{R_1} + I_{Vs1} + \frac{v_1 - v_2}{R_2} = 0 \qquad \text{(node 1)}$$

$$-I_{Vs1} + \frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} = 0 \qquad \text{(node 2)}$$

$$v_1 - v_2 = V_{s1} \qquad (V_{s1})$$

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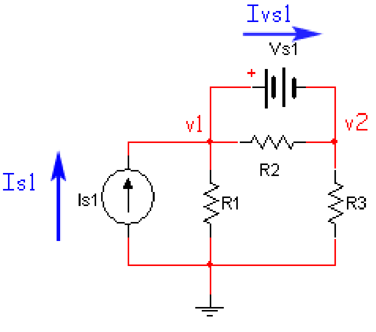
MNA: example 2 (2)

- in matrix form
- Solve *Ax=b*



$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} & 1 \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3} & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ I_{Vs1} \end{bmatrix} = \begin{bmatrix} I_{s1} \\ 0 \\ V_{s1} \end{bmatrix}$$

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System of Linear Equations

- *n* linear equations
  - a_{ij}, b_i are constants, x_{ij} are variables to be solved

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
&\vdots \\
a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n
\end{aligned}
$$

- In matrix form

$$
\begin{pmatrix} a_{11} & a_{21} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}
$$

$$Ax = b$$

$$x = A^{-1}b$$

Problem of Inversion

- Assume A is *non-singular*,
  - *Rank* of *A* = number of variables (Theorem D.1*)

Introduction to algorithms, 3ed. MIT Press

- In practice, $x = A^{-1}b$ suffers from *numerical instability*
  - Small *round-off error* results in large difference in x
    - $1 \div 3 \text{ x } 3 \neq 1$
- Example:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1.001 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad x = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1001 & -1000 \\ -1000 & 1000 \end{bmatrix} \approx \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \text{x} 10^3 \qquad x = A^{-1}b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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Idea of LU Decomposition

- LU decomposition of matrix A

$$A = LU$$

  - L is *unit lower triangular matrix* (*i.e.* all ones on diagonal)
  - U is *upper triangular matrix*

$$Ax = b$$

$$LUx = b, \quad \text{let } Ux = y$$

- Solve y by *forward substitution*

$$Ly = b$$

- Then solve x by *backward substitution*

$$Ux = y$$

No Division, Numerically Stable

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LU Example

$$A=\begin{bmatrix}5&6&3\\3&5&4\\1&2&0\end{bmatrix}\quad L=\begin{bmatrix}1&0&0\\0.6&1&0\\0.2&0.571&1\end{bmatrix}\quad U=\begin{bmatrix}5&6&3\\0&1.4&2.2\\0&0&-1.856\end{bmatrix}\quad b=\begin{bmatrix}10.3\\12.5\\0.1\end{bmatrix}$$

$$Ly=\begin{bmatrix}1&0&0\\0.6&1&0\\0.2&0.571&1\end{bmatrix}\begin{bmatrix}y1\\y2\\y3\end{bmatrix}=b=\begin{bmatrix}10.3\\12.5\\0.1\end{bmatrix}\xrightarrow{\text{Forward substitution}}y=\begin{bmatrix}y1\\y2\\y3\end{bmatrix}=\begin{bmatrix}10.3\\6.32\\-5.569\end{bmatrix}\Downarrow$$

$$Ux=\begin{bmatrix}5&6&3\\0&1.4&2.2\\0&0&-1.856\end{bmatrix}\begin{bmatrix}x1\\x2\\x3\end{bmatrix}=y=\begin{bmatrix}10.3\\6.32\\-5.569\end{bmatrix}\xrightarrow{\text{Backward substitution}}x=\begin{bmatrix}x1\\x2\\x3\end{bmatrix}=\begin{bmatrix}0.5\\-0.2\\3.0\end{bmatrix}\Uparrow$$

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LU Decomposition

- ***A* = *LU***

$$A=\left(\begin{array}{c|ccc}a_{11}&a_{12}&\cdots&a_{1n}\\\hline a_{21}&a_{22}&\cdots&a_{2n}\\\vdots&\vdots&\ddots&\vdots\\a_{n1}&a_{n2}&\cdots&a_{nn}\end{array}\right)=\begin{pmatrix}a_{11}&w^T\\v&A'\end{pmatrix}$$

- ***v*= column *n-1* vector**
- ***A'* = (*n*-1) x (*n*-1) submatrix**
- **w^T = row *n-1* vector**

$$A=\begin{pmatrix}a_{11}&w^T\\v&A'\end{pmatrix}$$
$$=\begin{pmatrix}1&0\\v/a_{11}&I_{n-1}\end{pmatrix}\begin{pmatrix}a_{11}&w^T\\0&A'-vw^T/a_{11}\end{pmatrix}$$
$$=\begin{pmatrix}1&0\\v/a_{11}&I_{n-1}\end{pmatrix}\begin{pmatrix}a_{11}&w^T\\0&L'U'\end{pmatrix}$$
$$=\begin{pmatrix}1&0\\v/a_{11}&L'\end{pmatrix}\begin{pmatrix}a_{11}&w^T\\0&U'\end{pmatrix}$$
$$=LU$$

- **Recursively decompose *A***
  - ***$A'-vw_T/a_{11}$ = L'U'***
    - ***L'* = unit lower triangular matrix**
    - ***U'* = upper triangular matrix**

$A'-vw^T/a_{11}$ is called *schur component* of A with respect to a_{11}

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LU Example

- **Fig 28.1**

$$
\begin{array}{cccc}
2 & 3 & 1 & 5 \\
6 & 13 & 5 & 19 \\
2 & 19 & 10 & 23 \\
4 & 10 & 11 & 31
\end{array}
\qquad
\begin{array}{c|ccc}
\textcircled{2} & 3 & 1 & 5 \\
\hline
3 & 4 & 2 & 4 \\
1 & 16 & 9 & 18 \\
2 & 4 & 9 & 21
\end{array}
\qquad
\begin{array}{cc|cc}
2 & 3 & 1 & 5 \\
3 & \textcircled{4} & 2 & 4 \\
\hline
1 & 4 & 1 & 2 \\
2 & 1 & 7 & 17
\end{array}
\qquad
\begin{array}{ccc|c}
2 & 3 & 1 & 5 \\
3 & 4 & 2 & 4 \\
1 & 4 & \textcircled{1} & 2 \\
\hline
2 & 1 & 7 & 3
\end{array}
$$

(a) (b) (c) (d)

$$
\underset{A}{\begin{pmatrix} 2 & 3 & 1 & 5 \\ 6 & 13 & 5 & 19 \\ 2 & 19 & 10 & 23 \\ 4 & 10 & 11 & 31 \end{pmatrix}}
=
\underset{L}{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 2 & 1 & 7 & 1 \end{pmatrix}}
\underset{U}{\begin{pmatrix} 2 & 3 & 1 & 5 \\ 0 & 4 & 2 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{pmatrix}}
$$

$$
\begin{aligned}
A &= \begin{pmatrix} a_{11} & w^T \\ v & A' \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ v/a_{11} & I_{n-1} \end{pmatrix} \begin{pmatrix} a_{11} & w^T \\ 0 & A' - vw^T/a_{11} \end{pmatrix}
\end{aligned}
$$

LU Algorithm

- **Three-level nested loops. Complexity = $\Theta(n^3)$**

```
LU-DECOMPOSITION(A)
n = A.rows
let L and U be new n × n matrices
initialize U with 0s below the diagonal
initialize L with 1s on the diagonal and 0s above the diagonal
for k = 1 to n
    u_kk = a_kk
    for i = k + 1 to n
        l_ik = a_ik / u_kk        // l_ik holds v_i
        u_ki = a_ki               // u_ki holds w_i^T
    for i = k + 1 to n
        for j = k + 1 to n
            a_ij = a_ij − l_ik u_kj
return L and U
```

$$
\begin{aligned}
A &= \begin{pmatrix} a_{11} & w^T \\ v & A' \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ v/a_{11} & I_{n-1} \end{pmatrix} \begin{pmatrix} a_{11} & w^T \\ 0 & A' - vw^T/a_{11} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ v/a_{11} & I_{n-1} \end{pmatrix} \begin{pmatrix} a_{11} & w^T \\ 0 & L'U' \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ v/a_{11} & L' \end{pmatrix} \begin{pmatrix} a_{11} & w^T \\ 0 & U' \end{pmatrix} \\
&= LU
\end{aligned}
$$

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Permutation Matrix

- What if $a_{11} = 0$ or very small? Round-off error can still occur!
  - Exchange rows so that we *pivot* on large value
- Suppose exchange row 1 and row *k*
  - Multiply *A* by a *permutation matrix P*

$$PA = \begin{pmatrix} a_{k1} & a_{k2} & \cdots & a_{kn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{11} & a_{12} & \ddots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

- Permutation matrix has exactly one 1 in each row or column
  - Example: P exchange row 2 and row 3

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

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Idea of LUP Decomposition

- LUP decomposition of matrix A

$$PA = LU$$

  - *P* is permutation matrix

$$PAx = Pb$$

$$LUx = Pb, \quad \text{let } Ux = y$$

- Solve y by *forward substitution*

$$Ly = Pb$$

- Then solve x by *backward substitution*

$$Ux = y$$

Similar to LU, Just Permutated

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LUP Example

- PA swaps row 1 and row 3 of A

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 4 \\ 5 & 6 & 3 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 0.6 & 1 & 0 \\ 0.2 & 0.571 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 5 & 6 & 3 \\ 0 & 1.4 & 2.2 \\ 0 & 0 & -1.856 \end{bmatrix}$$

$$b = \begin{bmatrix} 0.1 \\ 12.5 \\ 10.3 \end{bmatrix} \quad Pb = \begin{bmatrix} 10.3 \\ 12.5 \\ 0.1 \end{bmatrix}$$

$$Ly = \begin{bmatrix} 1 & 0 & 0 \\ 0.6 & 1 & 0 \\ 0.2 & 0.571 & 1 \end{bmatrix} \begin{bmatrix} y1 \\ y2 \\ y3 \end{bmatrix} = Pb = \begin{bmatrix} 10.3 \\ 12.5 \\ 0.1 \end{bmatrix} \xrightarrow{\text{Forward sub.}} y = \begin{bmatrix} y1 \\ y2 \\ y3 \end{bmatrix} = \begin{bmatrix} 10.3 \\ 6.32 \\ -5.569 \end{bmatrix}$$

$$Ux = \begin{bmatrix} 5 & 6 & 3 \\ 0 & 1.4 & 2.2 \\ 0 & 0 & -1.856 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = y = \begin{bmatrix} 10.3 \\ 6.32 \\ -5.569 \end{bmatrix} \xrightarrow{\text{backward sub.}} x = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.2 \\ 3.0 \end{bmatrix}$$

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LUP Decomposition (1)

- Exchange row 1 and row *k*, a_{k1} is largest of first column
  - Multiplied by permutation matrix Q

$$QA = \begin{pmatrix} a_{k1} & a_{k2} & \cdots & a_{kn} \\ \vdots & \vdots & \cdots & \vdots \\ a_{11} & a_{12} & \ddots & a_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

$$QA = \begin{pmatrix} a_{k1} & w^T \\ v & A' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ v/a_{k1} & I_{n-1} \end{pmatrix} \begin{pmatrix} a_{k1} & w^T \\ 0 & A' - vw^T/a_{k1} \end{pmatrix}$$

- Similarly
  - v = $(n\text{-}1)$ column vector
  - a_{11} replaced a_{k1}
  - A' = $(n\text{-}1)$ x $(n\text{-}1)$ submatrix
  - w^T = $(n\text{-}1)$ row vector (a_{k2}, … a_{kn})

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LUP Decomposition (2)

- *P* is n x n permutation matrix
- *P'* is (n-1) x (n-1) submatrix

$$P = \begin{pmatrix} 1 & 0 \\ 0 & P' \end{pmatrix} Q$$

product of two *P* matrices is still a *P* matrix

- Multiplied by *P'*
  - v/a_{k1}
  - $A'\text{-}vw^T/a_{k1}$

$$\begin{aligned} PA &= \begin{pmatrix} 1 & 0 \\ 0 & P' \end{pmatrix} QA \\ &= \begin{pmatrix} 1 & 0 \\ 0 & P' \end{pmatrix} \begin{pmatrix} 1 & 0 \\ v/a_{k1} & I_{n-1} \end{pmatrix} \begin{pmatrix} a_{k1} & w^T \\ 0 & A' - vw^T/a_{k1} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ P'v/a_{k1} & P' \end{pmatrix} \begin{pmatrix} a_{k1} & w^T \\ 0 & A' - vw^T/a_{k1} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ P'v/a_{k1} & I_{n-1} \end{pmatrix} \begin{pmatrix} a_{k1} & w^T \\ 0 & P'(A' - vw^T/a_{k1}) \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ P'v/a_{k1} & I_{n-1} \end{pmatrix} \begin{pmatrix} a_{k1} & w^T \\ 0 & L'U' \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ P'v/a_{k1} & L' \end{pmatrix} \begin{pmatrix} a_{k1} & w^T \\ 0 & U' \end{pmatrix} \\ &= LU \end{aligned}$$

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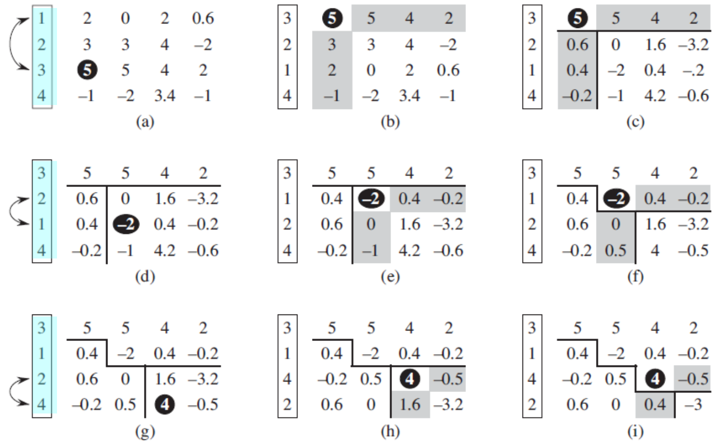
LUP Example

- **Fig 28.2:** ***Pivot*** **on the *largest element* in k_{th} column**



$$\underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}}_{P} \underbrace{\begin{pmatrix} 2 & 0 & 2 & 0.6 \\ 3 & 3 & 4 & -2 \\ 5 & 5 & 4 & 2 \\ -1 & -2 & 3.4 & -1 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.4 & 1 & 0 & 0 \\ -0.2 & 0.5 & 1 & 0 \\ 0.6 & 0 & 0.4 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 5 & 5 & 4 & 2 \\ 0 & -2 & 0.4 & -0.2 \\ 0 & 0 & 4 & -0.5 \\ 0 & 0 & 0 & -3 \end{pmatrix}}_{U}$$

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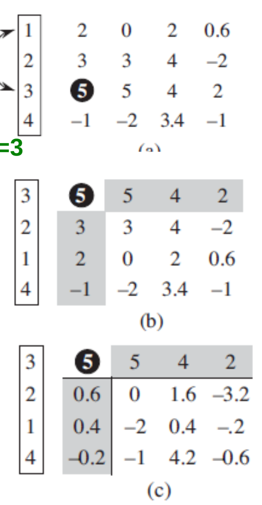
LUP Algorithm

- ***π[i]=j*** **means P_{ij}=1**

π



k=1, *k'*=3

```
LUP-DECOMPOSITION(A)
n = A.rows
let π[1..n] be a new array
for i = 1 to n                         Initial P = diagonal
    π[i] = i
for k = 1 to n
    p = 0
    for i = k to n                     Choose largest element
        if |a_ik| > p                  in column k
            p = |a_ik|
            k' = i
    if p == 0
        error "singular matrix"
    exchange π[k] with π[k']           Exchange rows k, k'
    for i = 1 to n
        exchange a_ki with a_k'i
    for i = k + 1 to n
v/a_k1  a_ik = a_ik / a_kk
        for j = k + 1 to n
A'-vw^T/a_k1  a_ij = a_ij - a_ik a_kj
```

LUP-solve

- **$b_{\pi[i]}$ = i_{th} element in *Pb***
- **Two-level nested loop**
  - **Complexity=$\Theta(n^2)$**

$$Ly = \begin{bmatrix} 1 & 0 & 0 \\ 0.6 & 1 & 0 \\ 0.2 & 0.571 & 1 \end{bmatrix} \begin{bmatrix} y1 \\ y2 \\ y3 \end{bmatrix} = Pb = \begin{bmatrix} 10.3 \\ 12.5 \\ 0.1 \end{bmatrix}$$

$$Ux = \begin{bmatrix} 5 & 6 & 3 \\ 0 & 1.4 & 2.2 \\ 0 & 0 & -1.856 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = y = \begin{bmatrix} 10.3 \\ 6.32 \\ -5.569 \end{bmatrix}$$

```
LUP-SOLVE(L, U, π, b)
  n = L.rows
  let x be a new vector of length n
  for i = 1 to n
        y_i = b_π[i] − Σ_{j=1}^{i−1} l_ij y_j
  for i = n downto 1
        x_i = (y_i − Σ_{j=i+1}^{n} u_ij x_j) / u_ii
  return x
```

forward sub.

backward sub.

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How to Find A^{-1} Using LU?

- Find X = A^{-1}, such that I is n x n *identity matrix*

$$AX = I$$

- Already know how to solve a column of X

$$Ax = b \qquad AX_i = e_i$$

  - unit vector e_i is i_{th} column of I
  - X_i is i_{th} column of X
- So we can solve whole matrix X column by column
- Example:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1.001 \end{bmatrix} \qquad e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 1001 \\ -1000 \end{bmatrix} \quad X_2 = \begin{bmatrix} -1000 \\ 1000 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 1001 & -1000 \\ -1000 & 1000 \end{bmatrix}$$

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Complexity Analysis

- LUP decomposition $A = LU$
  - $\Theta(n^3)$
- For i = 1 to n
  - LUP-solve: compute each i_{th} column of X
  - Each LUP-solve is $\Theta(n^2)$
- Totally, $\Theta(n^3)$

$$\boxed{\text{Matrix Inversion Using LU is } \Theta(n^3)}$$

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Matrix Multiplication

- C = AB
- Direct implementation $\Theta(n^3)$
  - Strassen's method $\Theta(n^{\lg 7})$ *but slow in practice

```
SQUARE-MATRIX-MULTIPLY(A,B)
  n = A.rows
  let C be a new n × n matrix
   for i = 1 to n
     for j = 1 to n
         c_ij = 0
         for k = 1 to n
               c_ij = c_ij + a_ik · b_kj
  return C
```

- **Theorem 28.1 and 28.2*** Introduction to algorithms, 3ed. MIT Press

  Matrix inversion is equivalent to matrix multiplication

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Summary

- Gaussian Elimination: first systematic way to solve linear system
  - $\Theta(n^3)$
- Matrix multiplication = Matrix inversion
  - $\Theta(n^3)$
  - Strassen's method $\Theta(n^{\lg 7})$
- LUP decomposition
  - $\Theta(n^3)$

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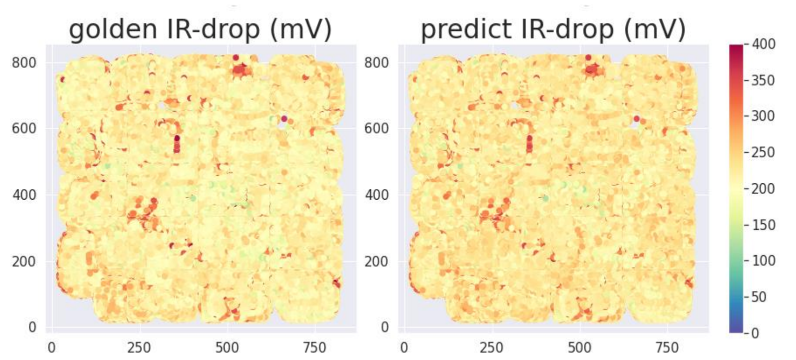
Traditional IR Drop Analysis is Slow

- What can we do?
  - Use Machine Learning to speed up!



J. Chen et al " Vector-based Dynamic IR-drop Prediction Using Machine Learning, " 2022 ASP-DAC.

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Conclusion

- *IR drop analysis* very important EDA tool
  - Find IR drop hotspots
- *Modified Nodal Analysis (MNA)*
  - Very practical way to analyze large circuits
- Solving Systems of Linear Equations
  - *Gauss Elimination*
  - *LUP decomposition*
  - All $\Theta(n^3)$ Very slow!
- What can we do better?
  - Machine learning

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