EN1060 Signals and Systems: Discrete-Time Fourier Series

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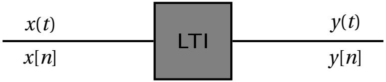
  - DT periodic signals → DT Fourier series
  - DT aperiodic signals → DT Fourier transform

Philosophy



Decompose the input as

$$x = a_1\phi_1 + a_2\phi_2 + \cdots \quad \text{linear combination of basic inputs}$$

Then

$$y = a_1\psi_1 + a_2\psi_2 + \cdots \quad \text{linear combination of corresponding outputs}$$

Choose $\phi_k(t)$ or $\phi_k[n]$ such that

- Broad class of signals can be constructed, and
- Response to ϕ_ks easy to compute.

Continuous-Time:

$\phi_k(t) = e^{j\omega_k t}$:

$$e^{j\omega_k t} \longrightarrow H(\omega_k) e^{j\omega_k t} \quad \text{(a scaled-version of the input)}$$

"Discrete-Time":

Continuous-Time:
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"Discrete-Time": $\phi_k[n] = e^{j\omega_k n}$

$$e^{j\omega_k n} \longrightarrow \underset{\substack{\uparrow \\ \text{eigenfunction}}}{e^{j\omega_k n}} \underbrace{\sum_{r=-\infty}^{\infty} h[r] e^{-j\omega_k r}}_{\text{eigenvalue}}$$

$x[n]$ periodic
period N
fundamental frequency $\omega_0 = \dfrac{2\pi}{N}$
$e^{jk\omega_0 n} = e^{j(k+N)\omega_0 n}$

$$x[n] = \sum_k a_k e^{jk\omega_0 n}, \quad k = 0, 1, 2, \ldots, N-1.$$

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}.$$

N equations in N unknowns.

$$a_k = \frac{1}{N} \sum_{<N>} x[n] e^{-jk\omega_0 n}.$$

Continuous-Time

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$$

Discrete-Time Fourier Series

Synthesis

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}.$$

Analysis

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk(2\pi/N)n}.$$

Periodicity

$x[n]$	periodic in n	true for CT
$e^{jk\omega_0 n}$	periodic in n	true for CT
$e^{jk\omega_0 n}$	periodic in k	not true for CT
a_k	periodic in k	not true for CT

Convergence

Continuous-time:

- $x(t)$ square-integrable OR
- Dirichlet condition

Discrete-time

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}.$$

$$\hat{x}[n] = \sum_{p \text{ terms}} a_k e^{jk\omega_0 n}.$$

$p = N$

$$\hat{x}[n] \equiv x[n].$$

There is no issue of convergence in DT.

Determine and sketch the DTFT of

$$x[n] = 1 + \sin \omega_0 n + 3 \cos \omega_0 n + \cos\left(2\omega_0 n + \frac{\pi}{2}\right).$$

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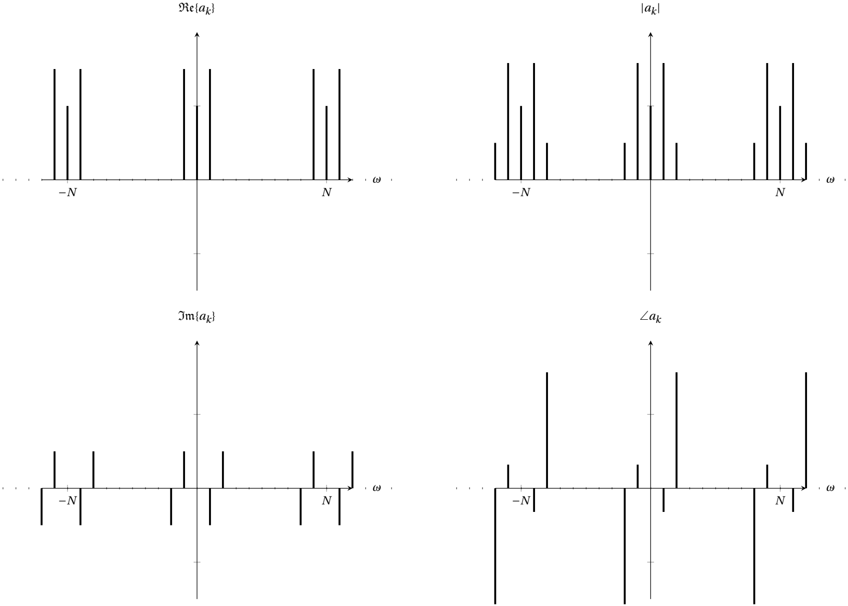
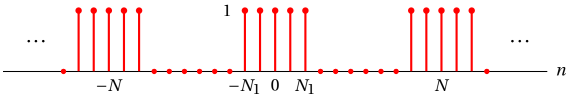


$\Re\{a_k\}$
$|a_k|$
$\Im\{a_k\}$
$\angle a_k$
ω
$-N$
N

Determine and sketch the DTFT of $x[n]$ of which is shown in the figure.



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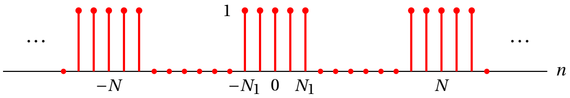
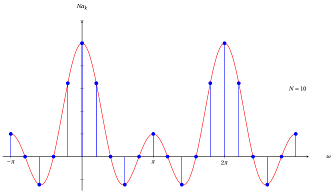
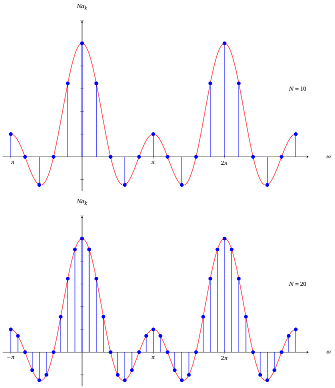
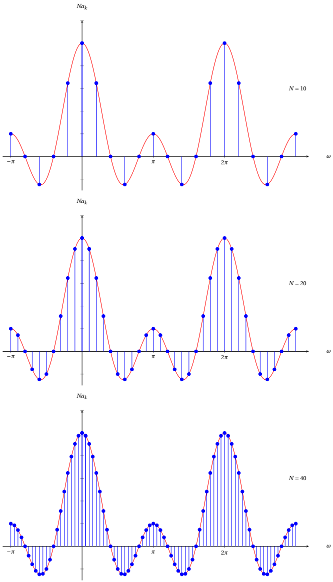




Na_k
$N = 10$
ω
$-\pi$
π
2π



Na_k
$N = 10$
ω
$-\pi$
π
2π
Na_k
$N = 20$
ω
$-\pi$
π
2π



Na_k
ω
$-\pi$
π
2π
$N = 10$
Na_k
ω
$-\pi$
π
2π
$N = 20$
Na_k
ω
$-\pi$
π
2π
$N = 40$