Department of Electronic and Telecommunication Engineering
University of Moratuwa
Sri Lanka

EN1060 Signals and Systems: Tutorial 06 Discreet-Time Fourier Transfrom *

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1. Write the synthesis and analysis equation for the discrete-time Fourier transform.
2. Consider the signal
$$x[n] = a_n u[n], \; |a| < 1.$$
   (a) Express its DTFT $X\left(e^{j\omega}\right)$.
   (b) Sketch the magnitude and phase of $X\left(e^{j\omega}\right)$ when $a > 0$.
   (c) Sketch the magnitude and phase of $X\left(e^{j\omega}\right)$ when $a < 0$.
3. Express the DTFT of
$$x[n] = a^{|n|}, \; |a| < 1,$$
   and sketch.
4. Express the DTFT of
$$x[n] = \begin{cases} 1, & |n| \leq N_1 \\ 0, & |n| > N_1 \end{cases}$$
   and sketch it for $N_1 = 2$.
5. Express and sketch the DTFT of $x[n] = \cos \omega_0 n$.
6. Express and sketch the DTFT of the discrete-time periodic impulse train $x[n] = \sum_{k=-\infty}^{+\infty} \delta[n - kN]$.
7. Compute the Fourier transformof each of the following signals:
   (a) $x[n] = u[n-2] - u[n-1]$
   (b) $x[n] = \left(\frac{1}{2}\right)^{-n} u[-n-1]$
   (c) $x[n] = \left(\frac{1}{3}\right)^{|n|} u[-n-2]$

*All the questions are from Oppenheim *et al.* chapter 4.

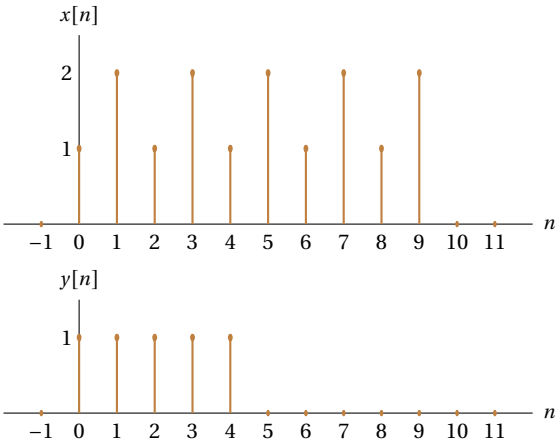

Figure 1: Sequences

(d) $x[n] = \begin{cases} n, & -3 \le n \le 3 \\ 0, & \text{otherwise.} \end{cases}$

8. The following are the Fourier transforms of discrete-time signals. Determine the signal corresponding to each transform.

   (a) $X\left(e^{j\omega}\right) = \begin{cases} 1, & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le |\omega| \le \pi,\ 0 \le |\omega| < \frac{\pi}{4} \end{cases}$

   (b) $X\left(e^{j\omega}\right) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-j3\omega} + e^{-j10\omega}$

   (c) $X\left(e^{j\omega}\right) = e^{-j\omega/2}$ for $-\pi \le \omega \le \pi$

   (d) $X\left(e^{j\omega}\right) = \frac{e^{-j\omega} - \frac{1}{5}}{1 - \frac{1}{5}e^{-j\omega}}$

9. Show that

$$X\left(e^{j(\omega+2\pi)}\right) = X\left(e^{j\omega}\right).$$

   Is the continuous-time Fourier transform always periodic as the DTFT?

10. Fig. 1 shows a sequence $x[n]$ adn $y[n]$.

    (a) Express $x[n]$ using subsampled versions of $y[n]$.

    (b) Express $Y\left(e^{j\omega}\right)$.

    (c) Hence, express $X\left(e^{j\omega}\right)$.

11. Let $X\left(e^{j\omega}\right)$ denote the Fourier transformof the signal shown in Fig. 2. Perform the following calculations wihtout explicitly evaluating $X\left(e^{j\omega}\right)$:

    (a) Evaluate $X\left(e^{j0}\right)$

    (b) Find $\angle X\left(e^{j\omega}\right)$

    (c) Evaluate $\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) d\omega$

    (d) Find $X\left(e^{j\pi}\right)$

    (e) Determine and sketch the signal whose Fourier transformis $\Re\mathfrak{e}\{x(\omega)\}$

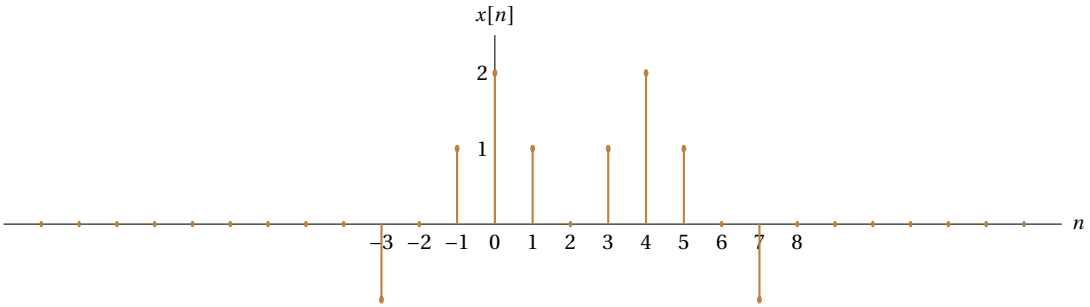

Figure 2: Sequences

(f) Evaluate $\int_{-\pi}^{\pi} \left|X\left(e^{j\omega}\right)\right|^2 d\omega$

(g) Evaluate $\int_{-\pi}^{\pi} \left|\frac{dX\left(e^{j\omega}\right)}{d\omega}\right|^2 d\omega$