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EN1060 SIGNALS AND SYSTEMS: TUTORIAL 02 [*]

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1. A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 8$. The non-zero Fourier series coefficients for $x(t)$ are specified as

$$a_1 = a_{-1}^* = j, \quad a_5 = a_{-5}^* = 2.$$

Express $x(t)$ in the form

$$x(t) = \sum_{k=0}^{\infty} \cos(\omega_k t + \phi_k).$$

2. Determine the Fourier series representation of the following signals:

(a) Each $x(t)$ illustrated in Figure 1.

(b) $x(t)$ periodic with period 2 and

$$x(t) = e^{-t} \quad \text{for} \quad -1 < t < 1.$$

(c) $x(t)$ periodic with period 4 and

$$x(t) = \begin{cases} \sin \pi t, & 0 \le t \le 2, \\ 0, & 2 < t \le 4. \end{cases}$$

3. In each of the following, we specify the Fourier series coefficients of a continuous-time signal that is periodic with period 4. Determine the signal $x(t)$ in each case.

(a) $a_k = \begin{cases} 0, & k = 0, \\ (j)^k \frac{\sin k\pi/4}{k\pi}, & \text{otherwise.} \end{cases}$

(b) $a_k = (-1)^k \frac{\sin k\pi/8}{2k\pi}$

(c) $a_k = \begin{cases} jk, & |k| < 3, \\ 0, & \text{otherwise.} \end{cases}$

(d) $a_k = \begin{cases} 1, & k \text{ even}, \\ 2, & k \text{ odd}. \end{cases}$

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[*]All the questions are from Oppenheim *et al.* chapter 3.

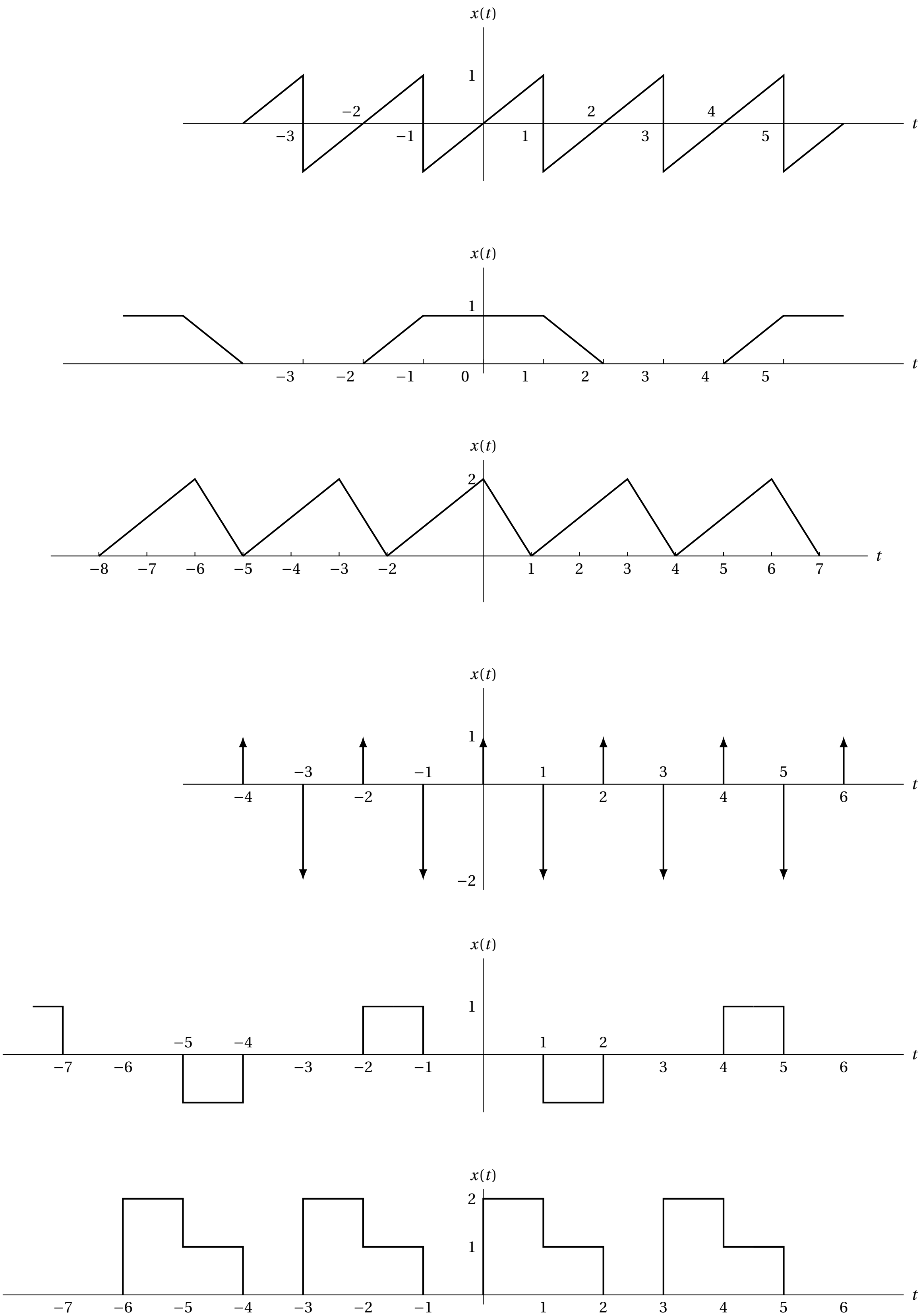

Figure 1: Figure Q02

4. Let

$$x(t) = \begin{cases} t, & 0 \le t \le 1, \\ 2-t, & 1 \le t \le 2, \end{cases}$$

be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k.

(a) Determine the value of a_0.

(b) Determine the Fourier series representation of $dx(t)/dt$.

(c) Use this result and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.

5. Consider the following three continuous-time signals with a fundamental period of $T = 1/2$:

$$\begin{aligned} x(t) &= \cos(4\pi t), \\ y(t) &= \sin(4\pi t), \\ z(t) &= x(t)y(t). \end{aligned}$$

(a) Determine the Fourier series coefficients of $x(t)$.

(b) Determine the Fourier series coefficients of $y(t)$.

(c) Use these results along with the multiplication property of the continuous-time Fourier series to determine the Fourier series coefficients of $z(t)$.

(d) Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric from, and compare the result with that of part 5c.

6. Let $x(t)$ be a periodic signal whose Fourier series coefficients are

$$a_k = \begin{cases} 2, & k = 0, \\ j\left(\frac{1}{2}\right)^{|k|}, & \text{otherwise.} \end{cases}$$

Use Fourier series properties to answer the Following questions:

(a) Is $x(t)$ real?

(b) Is $x(t)$ even?

(c) Is $dx(t)/dt$ even?

7. Let $x(t)$ be a periodic signal with fundamental period T and Fourier series coefficients a_k. Derive the Fourier series coefficients of each of the following signals in terms of a_k:

(a) $x(t - t_0) + x(t + t_0)$

(b) $\mathfrak{Ev}\{x(t)\}$

(c) $\mathfrak{Re}\{x(t)\}$

(d) $\dfrac{d^2x(t)}{dt^2}$

(e) $x(3t-1)$ [for this part, first determine the period of $x(3t-1)$]

8. Suppose we are given the following information about a continuous-time periodic signal with period 3 and Fourier coefficients a_k.

(a) $a_k = a_{k+2}$.

(b) $a_k = a_{-k}$.

(c) $\int_{-0.5}^{0.5} x(t)dt = 1$.

(d) $\int_{0.5}^{1.5} x(t)dt = 2$.

Determine $x(t)$.

9. Let $x(t)$ be a real-valued signal with fundamental period T and Fourier series coefficients a_k.

   (a) Show that $a_k = a_{-k}^*$ and a_0 must be real.

   (b) Show that if $x(t)$ is even, then its Fourier series coefficients must be real and even.

   (c) Show that if $x(t)$ is odd, then its Fourier series coefficients are imaginary and odd and $a_0 = 0$.

   (d) Show that the Fourier series coefficients of the even part of $x(t)$ are equal to $\mathfrak{Re}\{a_k\}$.

   (e) Show that the Fourier series coefficients of the odd part of $x(t)$ are equal to $j\mathfrak{Im}\{a_k\}$.

10. Let $x(t)$ be a real periodic signal with Fourier series representation given in the sine-cosine form

$$x(t) = a_0 + 2\sum_{k=1}^{\infty}[B_k \cos k\omega_0 t - C_k \sin k\omega_0 t]. \tag{1}$$

   (a) Find the exponential Fourier series representation of the even and odd parts of $x(t)$, that is, find the coefficients α_k and β_k in terms of the coefficients in eq. 1 so that

$$\begin{aligned} \mathfrak{Ev}\{x(t)\} &= \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega_0 t}, \\ \mathfrak{Od}\{x(t)\} &= \sum_{k=-\infty}^{\infty} \beta_k e^{jk\omega_0 t}. \end{aligned}$$

   (b) What is the relationship between α_k and α_{-k}? What is the relationship between β_k and β_{-k}?