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EN1060 Signals and Systems: Tutorial 07 LaplaceTransfrom [*]

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1. (a) Express the Laplace transformof a general signal $x(t)$.
   (b) Express the Fourier transformof the signal in terms of its Fourier transform.

2. (a) Show that the Laplace transform of

$$x(t) = e^{-at}u(t)$$

   is

$$X(s) = \frac{1}{s+a}, \quad \Re\mathfrak{e}\{s\} > \Re\mathfrak{e}\{-a\}.$$

   (b) Deduce the Fourier transformof $x(t)$.
   (c) Deduce the Laplace transformof the unit step funciton.
   (d) Determine the inverse Laplace transform of

$$X(s) = \frac{7s+17}{(s+2)(s+3)}, \quad \Re\mathfrak{e}\{s\} > -2.$$

3. Find the Laplace transformof

$$x(t) = -e^{-at}u(-t).$$

4. Consider the following information of a particular LTI system:

$$\begin{aligned} X(s) &= \frac{s+2}{s-2}, \\ x(t) &= 0, \ \ t > 0, \\ y(t) &= -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t). \end{aligned}$$

   (a) Determine $H(s)$ and its region of convergence.
   (b) Determine $h(t)$.

[*] All the questions are from Oppenheim *et al.* chapter 4.

5. A causal LTI system is described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = x(t).$$

Suppose that the system is at initial rest.

   (a) Find the system function $H(s)$.
   (b) Find the Laplace transform of the output $Y(s)$ if the input is $x(t) = \alpha u(t)$.
   (c) Find the output $y(t)$.

6. Consider the continuous-time linear time-invariant system for which the input $x(t)$ and the output $y(t)$ are related by the differential equation

$$\frac{d^2y(t)}{dt^2} - 2\frac{dy(t)}{dt} - 3y(t) = x(t).$$

Assume initial rest.

   (a) Express the transfer function $H(s)$ (the Laplace transform of the impulse response of the system $h(t)$) as a ratio of two polynomials.
   (b) Sketch the pole-zero pattern of $H(s)$.
   (c) Determine $h(t)$ if the system is neither stable nor causal.

7. Obtain the

   (a) bilateral Laplace transform$X(s)$, and
   (b) unilateral Laplace transform$\mathcal{X}(s)$

   of

$$x(t) = e^{-1(t+1)}u(t+1).$$

8. Consider the LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.

   (a) Deetermine the Laplace transformof $x(t)$ and $h(t)$.
   (b) Using the convolution property, determine the Laplace transform$Y(s)$ of the output $y(t)$.
   (c) From the Laplace transformof $y(t)$ as obtained in 8b , determine $y(t)$.
   (d) Verify the result in 8b by explicitly convolving $x(t)$ and $y(t)$.

9. Consider a continuous-time LTI system for which the input $x(t)$ adn output $y(t)$ are related by the differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t). \tag{1}$$

Let $X(s)$ and $Y(s)$ denote the Laplace transforms of $x(t)$ and $y(t)$, respectively, and $H(s)$ denote hte Laplace transform of $h(t)$, the system impulse response.

   (a) Determine $H(s)$ as a ratio of two polynomials in s. Sketch the pole-zero pattern of $H(s)$.
   (b) Determine the $h(t)$ for each of the following cases:
      i. The system is stable.
      ii. The system is causal.
      iii. The system is neither stable nor causal.