EN1060 Signals and Systems: Discrete-Time Fourier Series

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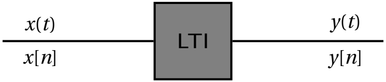
Introduction

- Now, we have studied Fourier series and Fourier transform for CT signals.
- In this lesson we will develop a similar tool for discrete time.
- Specifically, we consider the representation of discrete-time signals through a decomposition as a linear combination of complex exponentials.
    - DT periodic signals $\rightarrow$ DT Fourier series
    - DT aperiodic signals $\rightarrow$ DT Fourier transform

Philosophy



Decompose the input as

$$x = a_1\phi_1 + a_2\phi_2 + \cdots \quad \text{linear combination of basic inputs}$$

Then

$$y = a_1\psi_1 + a_2\psi_2 + \cdots \quad \text{linear combination of corresponding outputs}$$

Choose $\phi_k(t)$ or $\phi_k[n]$ such that

- Broad class of signals can be constructed, and
- Response to ϕ_ks easy to compute.

Continuous-Time:
$\phi_k(t) = e^{j\omega_k t}$:

$$e^{j\omega_k t} \longrightarrow H(\omega_k) e^{j\omega_k t} \quad \text{(a scaled-version of the input)}$$

"Discrete-Time":

Continuous-Time

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt$$

Discrete-Time Fourier Series

Example

Consider the signal

1. When is this signal periodic?
2. If it is periodic, what are discrete-time Fourier series coefficients?

Fourier Coefficients for $x[n] = \sin(2\pi/N)n$ for $N = 5$

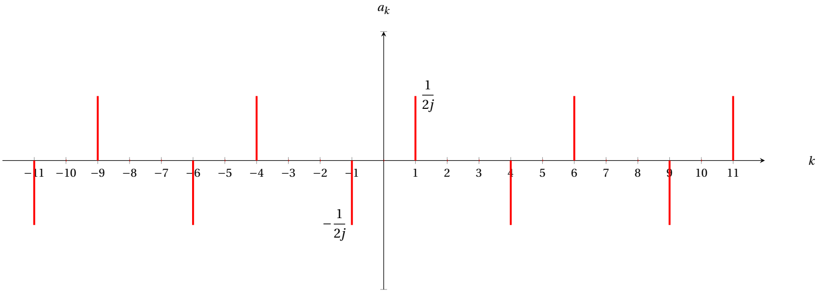
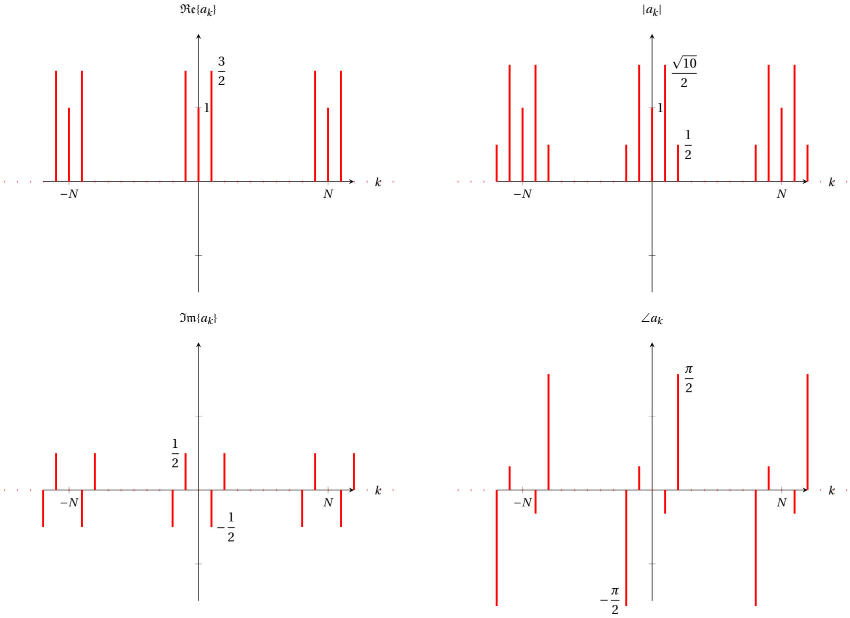


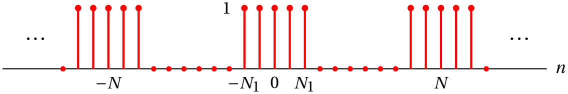
Figure: Fourier coefficients for $x[n] = \sin(2\pi/5)n$.

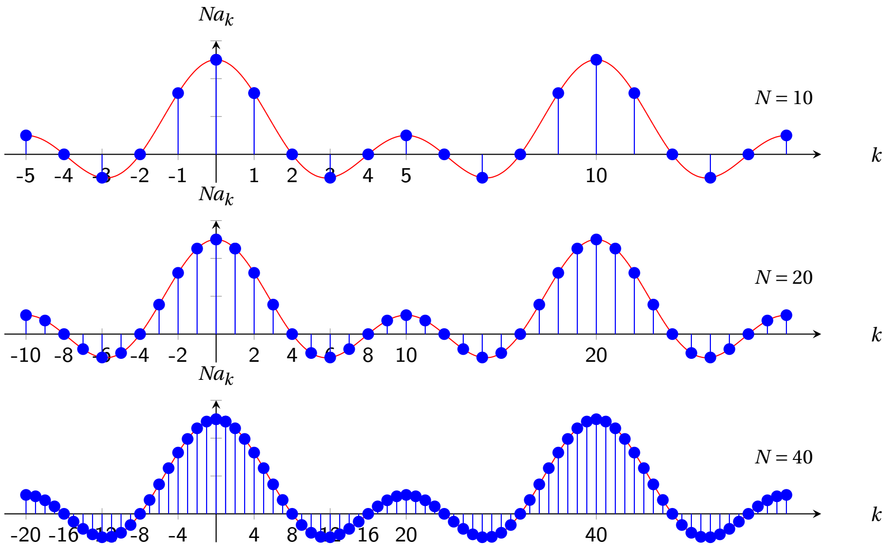
Determine and sketch the DTFS of

$$x[n] = 1 + \sin \omega_0 n + 3 \cos \omega_0 n + \cos \left(2\omega_0 n + \frac{\pi}{2} \right).$$


$\Re\mathfrak{e}\{a_k\}$
$\frac{3}{2}$
1
$-N$
N
k
$|a_k|$
$\frac{\sqrt{10}}{2}$
1
$\frac{1}{2}$
$-N$
N
k
$\Im\mathfrak{m}\{a_k\}$
$\frac{1}{2}$
$-\frac{1}{2}$
$-N$
N
k
$\angle a_k$
$\frac{\pi}{2}$
$-\frac{\pi}{2}$
$-N$
N
k

Determine and sketch the DTFS of $x[n]$ of which is shown in the figure.



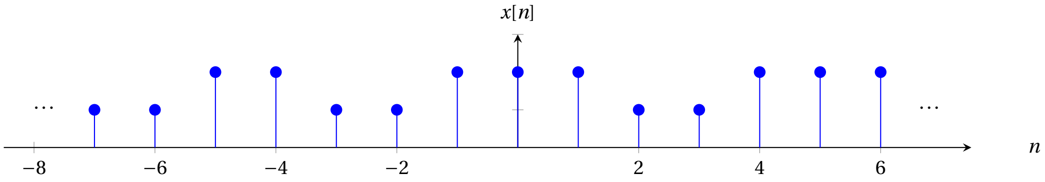


Na_k
$N = 10$
k
-5 -4 -3 -2 -1 1 2 3 4 5 10
Na_k
$N = 20$
k
-10 -8 -6 -4 -2 2 4 6 8 10 20
Na_k
$N = 40$
k
-20 -16 -12 -8 -4 4 8 12 16 20 40

Section 1

Properties of Discrete-Time Fourier Series

Find the Fourier series coefficients a_k of $x[n]$.



Suppose that we are given the following facts about a sequence $x[n]$:

1. $x[n]$ is periodic with period $n = 6$.
2. $\sum_{n=0}^{5} x[n] = 2$.
3. $\sum_{n=2}^{7} (-1)^n x[n] = 1$.
4. $x[n]$ has the minimum power per period among the set of signals satisfying the proceeding three conditions.

Determine the sequence $x[n]$.