

# EN1060 Signals and Systems: Signals

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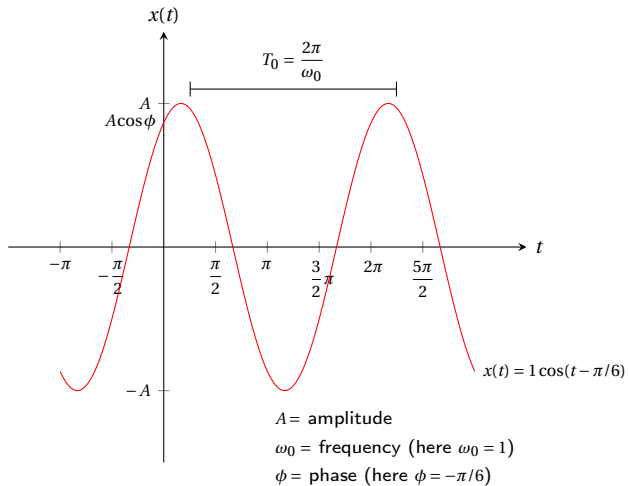
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## Section 1

### Signals

# Continuous-Time Sinusoidal Signal

$$x(t) = A \cos(\omega_0 t + \phi). \quad (1)$$



Sinusoidal signal is **periodic**.

A periodic continuous-time signal  $x(t)$  has the property that there is a positive value  $T$  for which

$$x(t) = x(t + T) \quad (2)$$

for all values of  $t$ . Under an appropriate time-shift the signal repeats itself. In this case we say that  $x(t)$  is periodic with period  $T$ .

**Fundamental period  $T_0$**  = smallest positive value of  $T$  for which 2 holds.

A signal that is not periodic is referred to as aperiodic.

E.g.: Consider  $A\cos(\omega_0 t + \phi)$

$$\begin{aligned} A\cos(\omega_0 t + \phi) &= A\cos(\omega_0(t + T) + \phi) \quad \text{here } \omega_0 T = 2\pi m \quad \text{an integer multiple of } 2\pi \\ &= A\cos(\omega_0 t + \phi) \end{aligned}$$

$$T = \frac{2\pi m}{\omega_0} \Rightarrow \text{fundamental period } T_0 = \frac{2\pi}{\omega_0}.$$

A time-shift in a CT sinusoid is equivalent to a phase shift.

E.g.: Show that a time-shift of a sinusoid is equal to a phase shift.

Phase of a Sinusoidal:  $\phi = 0$

Phase of a Sinusoidal:  $\phi = -\pi/2$

$$x[n] = A\cos(\omega_0 n + \phi) \text{ with } \phi = 0$$



$$x[n] = A\cos(\omega_0 n + \phi) \text{ with } \phi = -\pi/2$$

# Phase Change and Time Shift in DT

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Does a phase change always correspond to a time shift in discrete-time signals?

All continuous-time sinusoids are periodic. However, discrete-time sinusoids are not necessarily so.

$$x[n] = x[n + N], \quad \text{smallest integer } N \text{ is the fundamental period.} \quad (3)$$

$$\begin{aligned}x(t) &= Ce^{a(t+t_0)}, \quad C \text{ and } a \text{ are real numbers} \\ &= Ce^{at_0} e^{at}.\end{aligned}$$

$$x[n] = Ce^{\beta n} = C\alpha^n, \quad C \text{ and } \alpha \text{ are real numbers}$$





$$x(t) = Ce^{at} \quad C \text{ and } a \text{ are complex numbers.}$$

$$C = |C|e^{j\theta}$$

$$a = r + j\omega_0$$

$$x(t) = |C|e^{j\theta} e^{(r+j\omega_0)t}$$

$$= |C|e^{rt} e^{j(\omega_0 t + \theta)}$$

$$= |C|e^{rt} [\cos(\omega_0 t + \theta) + j \sin(\omega_0 t + \theta)]$$





$$x[n] = C\alpha^n, \quad C \text{ and } \alpha \text{ are complex numbers.} \quad (4)$$

$$C = |C|e^{j\theta} \quad (5)$$

$$\alpha = |\alpha|e^{j\omega_0} \quad (6)$$

$$x[n] = |C|e^{j\theta} \left( |\alpha|e^{j\omega_0} \right)^n \quad (7)$$

$$= |C||\alpha|^n \cos(\omega_0 n + \theta) + j|C||\alpha|^n \sin(\omega_0 n + \theta) \quad (8)$$

$$(9)$$

Comments:

- When  $|\alpha| = 1$ : sinusoidal real and imaginary parts.
- $e^{j\omega_0 n}$  may or may not be periodic depending on the value of  $\omega_0$ .
- Sinusoidal, exponential, step, and impulse signal form the cornerstones for signals and systems analysis.



$$u[n] = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0. \end{cases} \quad (10)$$

## Discrete-Time Unit Impulse (Unit Sample) $\delta[n]$

$$u[n] = \begin{cases} 1, & n = 0, \\ 0, & n \neq 0. \end{cases} \quad (11)$$

Unit impulse is the first backward difference of the unit step sequence.

$$\delta[n] = u[n] - u[n-1]. \quad (12)$$

The unit step sequence is the running sum of the unit impulse.

$$u[n] = \sum_{m=-\infty}^n \delta[m]. \quad (13)$$

The unit step sequence is a superposition of delayed unit impulses.

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]. \quad (14)$$



## Continuous-Time Unit Step Function $u(t)$

$$u(t) = \begin{cases} 0, & t < 0, \\ 1, & t > 0. \end{cases} \quad (15)$$

## Continuous-Time Unit Impulse Function $\delta(t)$

$$\delta(t) = \frac{du(t)}{dt}. \quad (16)$$

## CT Unit Step Function and Unit Impulse Function

$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau. \quad (17)$$

The total energy over a time interval  $t_1 \leq t \leq t_2$  in a continuous-time signal  $x(t)$  is

$$\int_{t_1}^{t_2} |x(t)|^2 dt$$

The total energy over a time interval  $n_1 \leq n \leq n_2$  in a discrete-time signal  $x[n]$  is

$$\sum_{n=n_1}^{n_2} |x[n]|^2$$

Total energy over an infinite interval in a CT signal:

$$E_{\infty} \triangleq \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_{-\infty}^{+\infty} |x(t)|^2 dt. \quad (18)$$

Total energy over an infinite interval in a DT signal:

$$E_{\infty} \triangleq \lim_{N \rightarrow \infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2. \quad (19)$$

Note that this integral and may not converge for some signals. Such signals have infinite energy, while signals with  $E_\infty < \infty$  have finite energy.

Time-averaged power over an infinite interval in a CT signal:

$$P_{\infty} \triangleq \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt. \quad (20)$$

Total energy in a DT signal:

$$P_{\infty} \triangleq \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2. \quad (21)$$

With these definitions, we can identify three important classes of signals:

- ① Energy signals: Signals with finite total energy  $E_{\infty} < \infty$ . These have zero average power.
- ② Power signals: Signals with finite average power  $0 < P_{\infty} < \infty$ . As  $P_{\infty} > 0$ ,  $E_{\infty} = \infty$ .
- ③ Signals with neither  $E_{\infty}$  nor  $P_{\infty}$  are finite.