## **Syntax**

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Type identifiers
                                 ::= C, G, T, S, \dots
                                 ::= T \mid dynamic \mid Object \mid Null \mid Type \mid num \mid bool
Types \tau, \sigma
                                           \mid \overrightarrow{\tau} \rightarrow \sigma \mid C < \overrightarrow{\tau} >
Term identifiers
                                 ::= a, b, x, y, m, n, \dots
Primops (\phi)
                                 ::= +, - \dots || \dots
lhs (l)
                                 := x \mid e.m
                                 ::= x \mid i \mid \mathrm{tt} \mid \mathrm{ff} \mid \mathrm{null} \mid \mathrm{this} \mid \mathrm{super} \mid (\overrightarrow{x:\tau}) \Rightarrow e \mid \mathrm{new} \, C < \overrightarrow{\tau} > (\overrightarrow{e})
Expressions e
                                            |\phi(\overrightarrow{e})| e(\overrightarrow{e})| e.m| l = e
                                           \mid e \text{ as } \tau \mid e \text{ is } \tau
Declaration (vd)
                                 ::= \mathbf{var} \ x = e \mid \mathbf{var} \ x : \tau = e \mid \tau \ f(\overrightarrow{x} : \overrightarrow{\tau}) \ b
Statements (s)
                                 ::= vd \mid e \mid if (E1) then b_1 else b_2 \mid return e \mid s; s
Blocks (b)
                                 ::= class C < \overrightarrow{T} >  extends G < \overrightarrow{S} > \{\overrightarrow{vd}\}
Class decl (cd)
Toplevel decl (td) ::= vd \mid cd
                                 := let \overrightarrow{td} in b
Program (P)
         Class signature (Sig) ::= class C < \overrightarrow{T} > extends G < \overrightarrow{S} > \{ \overrightarrow{x} : \overrightarrow{\tau} \}
                                                := \epsilon \mid \Delta, X <: \tau
         Type context (\Delta)
         Class hierarchy (\Phi)
                                              := \epsilon \mid \Phi, C : Sig
```

## Subtyping

$$\overline{\Phi,\Delta \vdash \tau <: \mathbf{dynamic}} \qquad \overline{\Phi,\Delta \vdash \tau <: \mathbf{Object}}$$

$$\overline{\Phi,\Delta \vdash \mathbf{bottom} <: \tau} \qquad \overline{\Phi,\Delta \vdash \tau <: \mathbf{Object}}$$

$$\underline{\Delta = \Delta'[S <: \sigma] \quad \Phi,\Delta \vdash \sigma <: \tau} \quad \overline{\Phi,\Delta \vdash \tau <: \tau}$$

$$\underline{\Phi,\Delta \vdash \sigma_i <: \tau_i \quad i \in 0, \dots, n \quad \Phi,\Delta \vdash \tau_r <: \sigma_r} \quad \overline{\Phi,\Delta \vdash \tau_0, \dots, \tau_n \to \tau_r <: \sigma_0, \dots, \sigma_n \to \sigma_r}$$

$$\underline{\Phi,\Delta \vdash \tau_i <: \sigma_i \quad i \in 0, \dots, n} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >}$$

$$\underline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n > <: C < \sigma_0, \dots, \sigma_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n >} \quad \overline{\Phi,\Delta \vdash C < \tau_0, \dots, \tau_n >} \quad \overline{\Phi,\Delta \vdash$$