Assignment 05

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In [1]:
```
import numpy as np
import pandas as pd
import datetime
import netCDF4
import xarray as xr
import matplotlib as mpl
import matplotlib.pyplot as plt
%matplotlib inline
from scipy import integrate
from numpy import exp
import math
from matplotlib.ticker import (MultipleLocator, AutoMinorLocator)
```

1. Modeling of carbon cycle

In [2]:
```
CO2_data = pd.read_csv("Global Fossil-Fuel CO2 Emissions.csv")
```

In [3]:
```
#Mauna Loa 1986-2004数据
CO2_Mauna_1 = pd.read_csv("Mauna Loa CO2 annual mean data.csv")
```

1.1

In [4]:
```
#model_1是方程1-2的计算
def model_1(z, t):
    x,y = z

    #k1,k2常数
    k1 = 105/740
    k2 = 102/900

    #c is the rate of production of CO2 by fossil-fuel burning
    a = CO2_data.loc[CO2_data['Year']== int(t)]['Total'].values
    c= a[0]/1000

    #偏微分方程组1-2
    dN1dt = -k1 * x + k2 * y + c
    dN2dt = k1 * x - k2 * y
    return [dN1dt,dN2dt]

#初始值
z0=[740,900]

#时间1986-2005
t = np.arange(1985,2005)

#integrate.odeint求解
f = integrate.odeint(model_1, z0, t)

#1987 - 2004年大气CO2浓度
f[2:,0]/2.13
```

Out[4]:
```
array([349.77850999, 351.04809333, 352.39543217, 353.77221098,
       355.15238517, 356.54913362, 357.89747141, 359.22957711,
       360.60060843, 362.01846517, 363.47468504, 364.96005426,
       366.41277228, 367.82628922, 369.30342386, 370.83766208,
       372.37960382, 374.08459257])
```

1.2

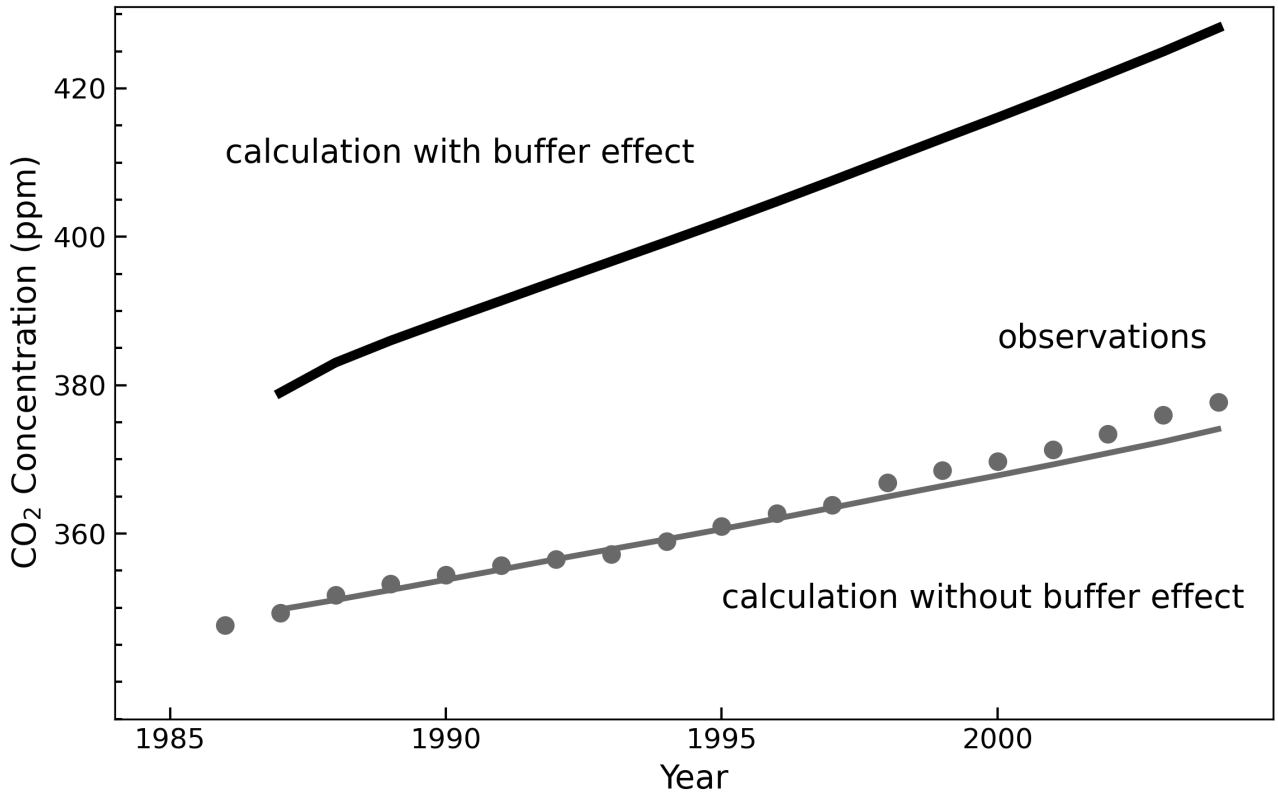
In [5]:
```
#model_2是方程3-4的计算
def model_2(z, t):
    x,y = z

    #k1,k2,N20常数
    k1 = 105/740
    k2 = 102/900
    N20 = 821

    #q is buffer factor
    q= 3.69 + 0.0186 * x/2.13 - 1.8*(1e-6) * x * x/2.13/2.13
```

```
    #c is the rate of production of CO2 by fossil-fuel burning
    a = CO2_data.loc[CO2_data['Year']== int(t)]['Total'].values
    c= a[0]/1000

    #偏微分方程组1-2
    dN1dt = -k1 * x + k2 * (N20 + q*(y - N20)) + c
    dN2dt = k1 * x - k2 * (N20 + q*(y - N20))
    return [dN1dt,dN2dt]

#初始值
z0=[740,900]

#时间1986-2005
t = np.arange(1985,2005)

#integrate.odeint求解
g = integrate.odeint(model_2, z0, t)

#1987 - 2004年大气CO2浓度
g[2:,0]/2.13
```

Out[5]:
```
array([378.98728653, 383.04052679, 386.0146096 , 388.7240129 ,
       391.37593291, 394.04500045, 396.68064862, 399.31457628,
       402.00075647, 404.7491574 , 407.55444184, 410.40988303,
       413.25358304, 416.07485387, 418.97394383, 421.94757915,
       424.94943065, 428.13941829])
```

1.3

In [6]:
```
#绘图
fig,axm = plt.subplots(1,1,figsize = [8,5],dpi=300)

#分别将观测数据，以及公式1-2和3-4计算得到的数据绘制成图
t = np.arange(1987,2005)
axm.plot(CO2_Mauna_1['Year'],CO2_Mauna_1['PPM'],'o',color='#696969',markersize=7)
axm.plot(t, f[2:,0]/2.13,lw=2.5,color='#696969')
axm.plot(t, g[2:,0]/2.13,lw=4.1,color='k')

#设置y轴格式
axm.set_ylim(335,431)
axm.set_yticks([360,380,400,420])
axm.yaxis.set_minor_locator(MultipleLocator(5))

#设置x轴格式
axm.set_xlim(1984,2005)
axm.set_xticks([1985,1990,1995,2000])

#设置刻度轴样式
axm.tick_params(axis="both", which="major", direction="in", width=1, length=5,labelsize=12)
axm.tick_params(axis="both", which="minor", direction="in", width=1, length=3)

#x,y下标
axm.set_xlabel('Year',fontsize=14)
axm.set_ylabel('CO$_{2}$ Concentration (ppm)',fontsize=14)

#文本框
axm.text(2000,385,'observations',fontsize=14)
axm.text(1986,410,'calculation with buffer effect',fontsize=14)
axm.text(1995,350,'calculation without buffer effect',fontsize=14)

plt.tight_layout()
plt.show()
```



Bonus

In [7]:
```
#观测数据由1750-1960的Historical CO2 Records from the Law Dome+1890-2000的Mauna Loa CO2 records数据组成
CO2_250years = pd.read_csv("CO2 for 250 years.csv")
CO2_250years
```

Out[7]:

	Year	CO2
0	1750	277.00
1	1755	277.20
2	1760	277.60
3	1765	278.00
4	1770	278.60
...	...	...
79	1996	362.74
80	1997	363.88
81	1998	366.84
82	1999	368.54
83	2000	369.71

84 rows × 2 columns

In [8]:
```
#land_use数据在https://cdiac.ess-dive.lbl.gov/trends/landuse/houghton/houghton.html网站下载
land_use = pd.read_csv("Carbon to the Atmosphere from Land-Use Change.csv")
```

In [9]:
```
#model_3是方程5-13的计算（当The fertilization factor =0.38）
def model_3(z, t):
    a,s,d,g,h,j,p = z

    #k1,k2,N20常数
    k12 = 60/615
    k21 = 60/842
    k23 = 9/842
    k24 = 43/842
    k32 = 52/9744
    k34 = 162/9744
    k43 = 205/26280
    k45 = 0.2/26280
    k51 = 0.2/90000000
```

```
    k67 = 62/731
    k71 = 62/1328
    N20 = 821

    #q is buffer factor
    q= 3.69 + 0.0186 * a/2.13 - 1.8*(10**-6) * a * a/2.13/2.13


    #c is the rate of production of CO2 by fossil-fuel burning
    o = CO2_data.loc[CO2_data['Year']== int(t)]['Total'].values
    c= o[0]/1000

    #L is the emission rate to the atmosphere by changes in land use
    l = land_use.loc[land_use['Year']== int(t)]['Global'].values
    L =l[0]

    # f is the net primary productivity
    f = 62*(1+0.38*(np.log(a/615)))

    #偏微分方程组1-2
    dN1dt = -k12*a+k21*(N20 + q*(s - N20))+c-f+L+k51*h+k71*p
    dN2dt = k12*a-k21*(N20 + q*(s - N20))-k23*s+k32*d-k24*s
    dN3dt = k23*s-k32*d-k34*d+k43*g
    dN4dt = k34*d-k43*g+k24*s-k45*g
    dN5dt = k45*g-k51*h
    dN6dt = f-k67*j-2*L
    dN7dt = k67*j-k71*p+L
    return [dN1dt,dN2dt,dN3dt,dN4dt,dN5dt,dN6dt,dN7dt]

#初始值
z0=[615,842,9744,26280,90000000,731,1238]

#时间1986-2005
t = np.arange(1751,2001)

#integrate.odeint求解
r = integrate.odeint(model_3, z0, t,)

#1987 - 2004年大气CO2浓度
r[0:,0]/2.13
```

Out[9]:
```
array([288.73239437, 290.98993005, 291.42659229, 291.09647968,
       290.47774969, 289.7833784 , 289.10577273, 288.48289614,
       287.92831767, 287.44470897, 287.03006128, 286.68041907,
       286.3911534 , 286.15750437, 285.97485371, 285.83878887,
       285.74517301, 285.69015907, 285.67015506, 285.68186114,
       285.7222271 , 285.78883146, 285.87860385, 285.9891603 ,
       286.11831843, 286.26408363, 286.42461154, 286.59821481,
       286.78338409, 286.97872612, 287.1829706 , 287.39541338,
       287.614532  , 287.83941946, 288.0692344 , 288.30321842,
       288.5406916 , 288.78104259, 289.02372761, 289.26826199,
       289.51421011, 289.76163672, 290.00971762, 290.25815731,
       290.50672074, 290.75516423, 291.00330214, 291.25138997,
       291.49880533, 291.74544721, 291.99164395, 292.23682719,
       292.48182051, 292.72516683, 292.96731268, 293.20823385,
       293.44835043, 293.68714583, 293.92462085, 294.16077374,
       294.39560347, 294.6295576 , 294.86215936, 295.09343004,
       295.32339273, 295.55249428, 295.7807251 , 296.00807779,
       296.23409984, 296.45883417, 296.68229841, 296.90453632,
       297.12601992, 297.34672589, 297.56620117, 297.78493631,
       298.00247904, 298.21930354, 298.43496842, 298.64950805,
       298.86560793, 299.07996004, 299.29314345, 299.50564428,
       299.7170187 , 299.92773895, 300.13913894, 300.34932241,
       300.55880733, 300.76760101, 300.97616196, 301.1840195 ,
       301.39162646, 301.59852474, 301.80516401, 302.01241073,
       302.21842313, 302.42459268, 302.62998404, 302.83551891,
       303.04159116, 303.24168492, 303.4660676 , 303.68698191,
       303.90829057, 304.12605645, 304.34552115, 304.56470702,
       304.78350503, 305.00372112, 305.22635586, 305.45194616,
       305.64804484, 305.84500767, 306.04287547, 306.24112254,
       306.43760632, 306.63391432, 306.82875918, 307.02309656,
       307.21633968, 307.42020246, 307.6677505 , 307.92004315,
       308.16645915, 308.41866091, 308.67119335, 308.92397125,
       309.17658314, 309.43421654, 309.70151688, 309.98265487,
       310.24248526, 310.50694023, 310.76940937, 311.02922531,
       311.28672861, 311.54493833, 311.81136243, 312.07097118,
       312.33735297, 312.60269922, 312.86906627, 313.12824749,
       313.3960263 , 313.6702958 , 313.94518491, 314.22518371,
       314.51083574, 314.80932896, 315.1130774 , 315.44750725,
       315.78163074, 316.14398108, 316.51220909, 316.90086884,
       317.31340472, 317.75348759, 318.17024792, 318.59612622,
       319.02899484, 319.43173765, 319.82708364, 320.22681401,
       320.56877537, 320.89329228, 321.24037671, 321.60416176,
       321.95132393, 322.23558479, 322.57420359, 322.86937829,
       323.17681067, 323.53880951, 323.89146259, 324.24388138,
       324.59563236, 324.99426667, 325.38667667, 325.81935828,
       326.23655343, 326.60149145, 326.87764508, 327.17390234,
       327.4974593 , 327.83864867, 328.22274546, 328.62231511,
       328.98369988, 329.36071726, 329.77226977, 330.18176667,
       330.59549022, 331.02050819, 331.43582524, 331.74514424,
       332.12774597, 332.58819038, 333.07559333, 333.53396823,
       334.08466688, 334.78287805, 335.4854976 , 336.18982136,
       336.91328028, 337.71987764, 338.59133194, 339.49349756,
       340.42107783, 341.32288415, 342.24590725, 343.18301622,
       344.13993157, 345.14231869, 346.19309725, 347.28066448,
       348.42194275, 349.57874397, 350.74820167, 351.98137921,
       353.27912301, 354.53775576, 355.82126044, 357.16616739,
       358.47631329, 359.73461793, 361.11267477, 362.52704583,
       363.93008029, 365.41261847, 366.8131331 , 368.12067168,
       369.47901975, 370.82789478, 372.26008034, 373.74618862,
       375.28662303, 376.86633305, 378.52276186, 380.2026309 ,
       381.87591844, 383.58890227, 385.19340806, 386.76473676,
       388.35581639, 389.97354327, 391.60665837, 393.24366618,
       394.839639  , 396.36994564])
```

In [10]:
```python
#model_3是方程5-13的计算（当The fertilization factor =0.5）
def model_3(z, t):
    a,s,d,g,h,j,p = z

    #k1,k2,N20常数
    k12 = 60/615
    k21 = 60/842
    k23 = 9/842
    k24 = 43/842
    k32 = 52/9744
    k34 = 162/9744
    k43 = 205/26280
    k45 = 0.2/26280
    k51 = 0.2/90000000
    k67 = 62/731
    k71 = 62/1328
    N20 = 821
```

```
    #q is buffer factor
    q= 3.69 + 0.0186 * a/2.13 - 1.8*(10**-6) * a * a/2.13/2.13


    #c is the rate of production of CO2 by fossil-fuel burning
    o = CO2_data.loc[CO2_data['Year']== int(t)]['Total'].values
    c= o[0]/1000

    #L is the emission rate to the atmosphere by changes in land use
    l = land_use.loc[land_use['Year']== int(t)]['Global'].values
    L =l[0]

    # f is the net primary productivity
    f = 62*(1+0.5*(np.log(a/615)))

    #偏微分方程组1-2
    dN1dt = -k12*a+k21*(N20 + q*(s - N20))+c-f+L+k51*h+k71*p
    dN2dt = k12*a-k21*(N20 + q*(s - N20))-k23*s+k32*d-k24*s
    dN3dt = k23*s-k32*d-k34*d+k43*g
    dN4dt = k34*d-k43*g+k24*s-k45*g
    dN5dt = k45*g-k51*h
    dN6dt = f-k67*j-2*L
    dN7dt = k67*j-k71*p+L
    return [dN1dt,dN2dt,dN3dt,dN4dt,dN5dt,dN6dt,dN7dt]

#初始值
z0=[615,842,9744,26280,90000000,731,1238]

#时间1986-2005
t = np.arange(1751,2001)

#integrate.odeint求解
v = integrate.odeint(model_3, z0, t,)

#1987 - 2004年大气CO2浓度
v[0:,0]/2.13
```

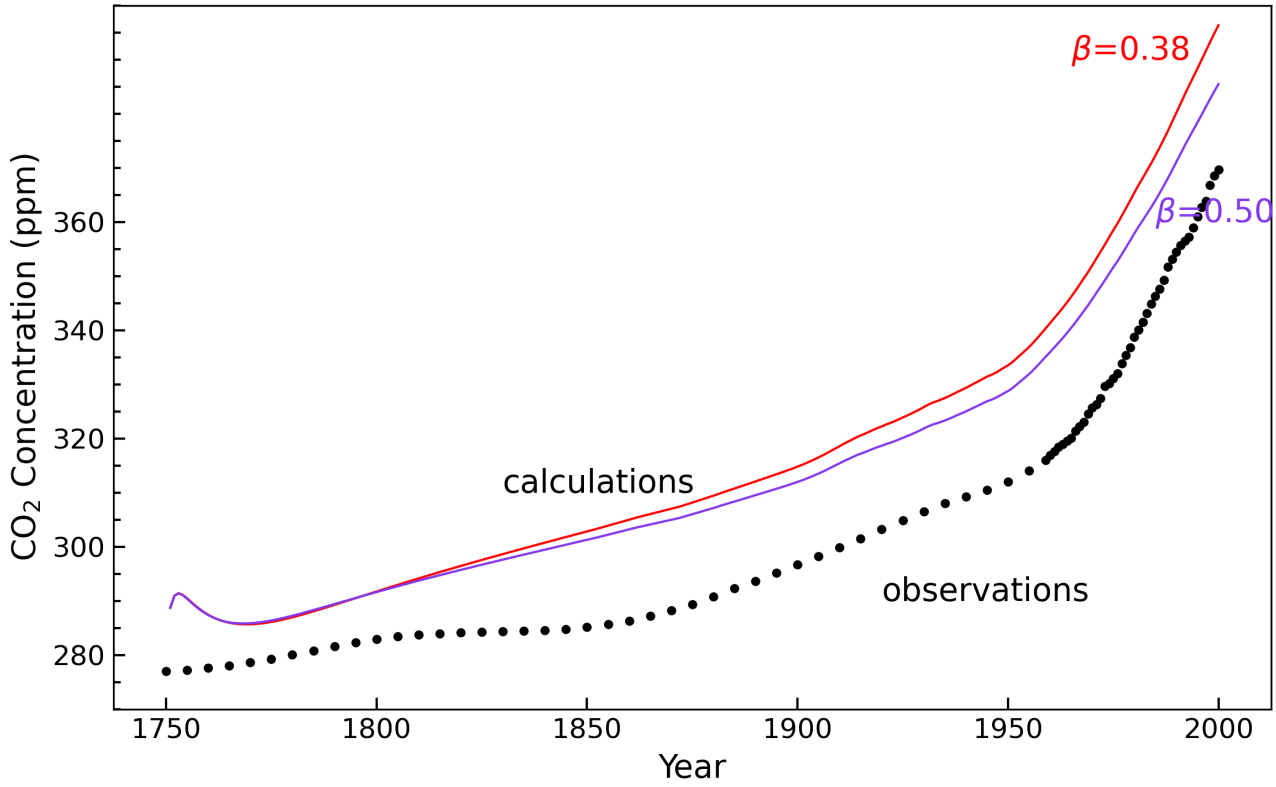
Out[10]:

```
array([288.73239437, 290.9743773 , 291.38349856, 291.02843613,
       290.3923209 , 289.68919475, 289.01098516, 288.39463581,
       287.85252093, 287.38620104, 286.99257934, 286.66671959,
       286.40310504, 286.19626122, 286.0408252 , 285.93180624,
       285.8645929 , 285.83488815, 285.83873566, 285.87253542,
       285.93298665, 286.01750833, 286.12286873, 286.24660353,
       286.38645849, 286.54038908, 286.70656287, 286.88332303,
       287.06917177, 287.26278637, 287.4629662 , 287.66909173,
       287.87974481, 288.09410074, 288.31142513, 288.53106302,
       288.75245056, 288.97507793, 289.19852057, 289.42239467,
       289.64637915, 289.87062644, 290.09443732, 290.31761925,
       290.54000113, 290.76144518, 290.98185616, 291.20156865,
       291.42003432, 291.63721265, 291.85350945, 292.06841394,
       292.28281099, 292.49528748, 292.70636351, 292.91605284,
       293.12480591, 293.33216456, 293.53815311, 293.74279842,
       293.94614968, 294.14867206, 294.34991173, 294.54991433,
       294.74872249, 294.94680057, 295.14414601, 295.34075616,
       295.53621527, 295.73055979, 295.92382833, 296.11605365,
       296.30771886, 296.49880675, 296.68886864, 296.87840576,
       297.06695715, 297.2550071 , 297.44210531, 297.62829011,
       297.81623893, 298.00264334, 298.1880793 , 298.3730349 ,
       298.55706282, 298.7406439 , 298.92508137, 299.10849102,
       299.29138156, 299.47377423, 299.65611623, 299.83792924,
       300.01965312, 300.200825  , 300.38190085, 300.5637581 ,
       300.74450738, 300.92557317, 301.10600699, 301.28671777,
       301.46808394, 301.64364313, 301.84352769, 302.03986331,
       302.23655743, 302.42967254, 302.6244835 , 302.81901303,
       303.01313702, 303.20867762, 303.40660051, 303.60743837,
       303.77890224, 303.9514689 , 304.12519329, 304.2994992 ,
       304.47224801, 304.64502519, 304.81653978, 304.98778233,
       305.15811369, 305.33919589, 305.5637848 , 305.79269877,
       306.01536787, 306.24348336, 306.4715826 , 306.69961645,
       306.92718915, 307.15947135, 307.4010741 , 307.65605481,
       307.88935189, 308.1270233 , 308.36247501, 308.5950823 ,
       308.82521106, 309.05592555, 309.29470126, 309.52649744,
       309.76493919, 310.00219961, 310.2403568 , 310.47125205,
       310.71066971, 310.95643613, 311.20266021, 311.45380499,
       311.71039211, 311.97950822, 312.25352578, 312.55768674,
       312.86090403, 313.19158622, 313.52728697, 313.88246341,
       314.26042372, 314.66463097, 315.04428986, 315.4320191 ,
       315.82567935, 316.18840062, 316.54316615, 316.90181381,
       317.20254629, 317.48609411, 317.79242986, 318.11546447,
       318.42187759, 318.66588689, 318.96475238, 319.22057425,
       319.48922416, 319.81258698, 320.12653414, 320.44025299,
       320.75331839, 321.11303848, 321.46610778, 321.85888739,
       322.23553768, 322.55973592, 322.79576155, 323.05277911,
       323.33765211, 323.6404255 , 323.98604029, 324.3467934 ,
       324.66918133, 325.0071547 , 325.37935218, 325.7490297 ,
       326.1225378 , 326.50687006, 326.88106851, 327.14953542,
       327.49165708, 327.91112224, 328.35654592, 328.77205139,
       329.27872042, 329.93047862, 330.58367877, 331.23581251,
       331.90441717, 332.65309973, 333.46308405, 334.29990591,
       335.15831576, 335.98742155, 336.83448795, 337.69244813,
       338.56707737, 339.48393488, 340.44568498, 341.44055422,
       342.48529779, 343.54167995, 344.60696284, 345.73212659,
       346.91764595, 348.06008192, 349.22381268, 350.44518257,
       351.6282356 , 352.75663898, 354.00181567, 355.27977182,
       356.54302805, 357.88238767, 359.13664844, 360.2960439 ,
       361.50474213, 362.70235432, 363.98141028, 365.31201917,
       366.69424132, 368.11282016, 369.60483875, 371.11680178,
       372.61894719, 374.15763204, 375.58537059, 376.9783911 ,
       378.38980993, 379.82637577, 381.27678416, 382.72962305,
       384.14028013, 385.48481352])
```

In [11]:

```
fig,ax = plt.subplots(1,1,figsize = [8,5],dpi=300)

#观测值数据
ax.plot(CO2_250years['Year'],CO2_250years['CO2'],'ko',markersize=3)

#计算数据
t = np.arange(1751,2001)
ax.plot(t, r[0:,0]/2.13 ,'red',lw=1) #The fertilization factor = 0.38
ax.plot(t, v[0:,0]/2.13, '#8338ec',lw=1) #The fertilization factor = 0.5

#x,y下标
ax.set_xlabel('Year',fontsize=14)
ax.set_ylabel('CO$_{2}$ Concentration (ppm)',fontsize=14)

#坐标轴设置
ax.tick_params(axis="both", which="major", direction="in", width=1, length=5,labelsize=12)
ax.tick_params(axis="both", which="minor", direction="in", width=1, length=3)
ax.set_ylim(270,400)
```

```
ax.set_yticks([280,300,320,340,360])
ax.yaxis.set_minor_locator(MultipleLocator(5))

#文本框
ax.text(1920,290,'observations',fontsize=14)
ax.text(1830,310,'calculations',fontsize=14)
ax.text(1965,390,r'$\beta$'+'=0.38',color = 'red',fontsize=14)
ax.text(1985,360,r'$\beta$'+'=0.50',color = '#8338ec',fontsize=14)

plt.tight_layout()
plt.show()
```



我所计算拟合出来的结果与文献以及真实值有所差距，这可能与参数、数据或方法有关。但自己也检查思考了许久都未发现问题所在，因此将最后的结果成图。