Digital Data Transmission
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Design and Analysis of a QAM Modulation System:

1. Introduction:

In this report, we aim to analyze the situation where we transmit X = 253045 equiprobable bits, within 1 second using a shift-orthonormal QAM signal with symbol period T = 32µs and average bit energy $E_b = 1$.

2. Modulation Scheme definition

In digital data transmission, a constellation is a graphical representation that illustrates the modulation schemes used in communication systems, where each point in the diagram corresponds to a specific symbol. In our case, we are using a QAM modulation which is a technique to convey two independent signals defined as follows:

$$A \sum_{i=1}^{n} (B_i \phi_1(t - iT) + C_i \phi_2(t - iT)) \quad (I)$$

3. Determining The Constellation Size

Firstly, in order to design a QAM modulation system is essential to compute the constellation size M as it will condition the entire network.

Considering our scenario, we will find the constellation size with applicable parameters to define the system properly.

M can be computed in terms of the information bits k knowing that $k = log_2(M) \Rightarrow M = 2^k$.

Nonetheless, to calculate k we will need the symbol rate R_s as they are related to the following formula: $k = \left\lceil \frac{transmitted\ bits}{R_s} \right\rceil \quad (II)$.

Realizing the procedure we need to follow we will put our hands to work to find the values of each parameter.

Given the symbol period, $T_s = 32\mu s$, we can compute R_s as its inverse. This value will state the number of symbols transmitted per second.

$R_s = \frac{1}{32\mu s} = 31250 symbols/sec$.

Using the formula of the information bits mentioned before (II) and knowing that we want to transmit 253045 bits, we will compute the value of k.

$k = \left\lceil \frac{253045}{31250} \right\rceil = \lceil 8,1 \rceil = 9$ bits/symbol. Consequently $M = 2^9 = 512$. With this last computation, we can state that the size of the constellation is 512, resulting in a 512-QAM.

4. Constellation design:

We will design a suitable constellation, taking into account QAM's definition already stated in section 1. As said previously, we can represent the QAM graphically, specifically in a (rectangular/square) grid. However, to do that, we need to know the values of the coefficients B_i *and* C_i to locate them accordingly.

Ideally, we would like to have a square grid, so the values should be scattered as: $C_{values} = \sqrt{M}$, $B_{values} = \sqrt{M}$.

Nevertheless, as $\sqrt{M} = \sqrt{512} \simeq 22,62$ (is not a perfect square), I decided to approximate the number of points per axis to the closest perfect squares (up and below), leading to 32 and 16 points along the real and complex axis, respectively.

Now that we know the number of rows and columns, we will distribute them in an equidistant way with a step size d.

5. Energy:

Considering that QAM by definition is shift-orthonormal (meaning that its signals must be unit energy) and taking into account equation (I) we will compute the required energy starting from the symbol energy formula:

$$E_s = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \sum_{-\infty}^{+\infty} |B_i + C_i j|^2 = E_i = |B_i + C_i|^2 \Rightarrow$$

as bits are equiprobable, the average per symbol energy is

$$\frac{1}{M} \sum_{i=1}^{16} \sum_{r=1}^{32} |B_r + C_i|^2.$$

In our specific case, as all the parameters depend on a step size d it will affect the equation as shown:

$$\frac{1}{M} \sum_{i=1}^{16} \sum_{r=1}^{32} |B_r d + C_i d|^2 \Rightarrow \frac{1}{M} d^2 \sum_{i=1}^{16} \sum_{r=1}^{32} |B_r + C_i|^2 \quad (III).$$

Bearing in mind we can relate the energy of the bit and the energy of the symbols as follows: $E_b = \frac{1}{log_2 M} E_s$.

We can estimate the energy of the symbol knowing that $E_b = 1$ $\Rightarrow \frac{1}{log_2 512} E_s = 1 \Rightarrow \frac{1}{9} E_s = 1 \Rightarrow E_s = 9$

Considering this condition we can equal the equation (IV) to 9 to find at which distance will be the points of our 512-QAM placed.

6. Computation of the step size d:

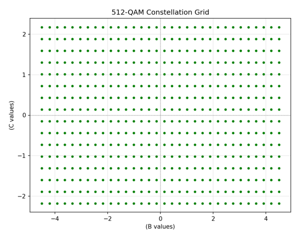
To be efficient, I built a Python code using ChatGPT to know the value of the energy per symbol in terms of the step size.

After doing the applicable calculations we end up obtaining that

$$E_s = 106,5d^2 = 9 \Rightarrow d = \sqrt{\frac{9}{106,5}} = \frac{\sqrt{426}}{71} \simeq$$
$$\simeq 0.29\ u.m$$

7. Graphical Representation:

Being aware of the step size I continued with the same .py to bring this framework to life using Matplotlib.




```
Computed C_values: [-2.175 -1.885 -1.595 -1.305 -1.015 -0.725 -0.435
 -0.145  0.145  0.435
  0.725  1.015  1.305  1.595  1.885  2.175]
Computed B_values: [-4.495 -4.205 -3.915 -3.625 -3.335 -3.045 -2.755
 -2.465 -2.175 -1.885
 -1.595 -1.305 -1.015 -0.725 -0.435 -0.145  0.145  0.435  0.725  1.0
15
  1.305  1.595  1.885  2.175  2.465  2.755  3.045  3.335  3.625  3.9
15
  4.205  4.495]
```

These are the C_i and B_i values respectively, taking into account that the points are centered at 0 with a step_size of 0,29.

8. Bandwidth computations:

The Nyquist theorem states that the minimum bandwidth required to transmit a signal without inter-symbol interference is equal to Therefore, in QAM it satisfies that $W \geq \frac{1}{T_s}$.

The equality only holds in ideal conditions if not a roll-off factor is introduced thus increasing the minimum bandwidth required.

In our specific case, the value which by the bandwidth is lower bounded can be computed as follows: $\frac{1}{32\mu s} \Rightarrow W \geq 31.25kHz$.

Conclusion:

This report designed a 512-QAM modulation system transmitting 253045 equiprobable bits in one second. Key parameters were computed and validated through simulations ensuring reliability.

Appendix:

The approach adopted for this report was mainly understanding and redacting the design and analysis of a QAM modulation system. The main information to accomplish this assignment was mostly taken from the internet (from which I took the definition of QAM modulation system and other key parameters), AI resources (which helped me structure the report and helped me know the procedure I needed to follow) and the lecture notes and seminars (which help me with some doubts arisen and especially with the formulas and computations). After understanding the assignment and doing the applicable computations on a Python .py made by me and helped by chatGPT I started with the writing of the assignment.