Data Structures (in C++)

- Stacks -

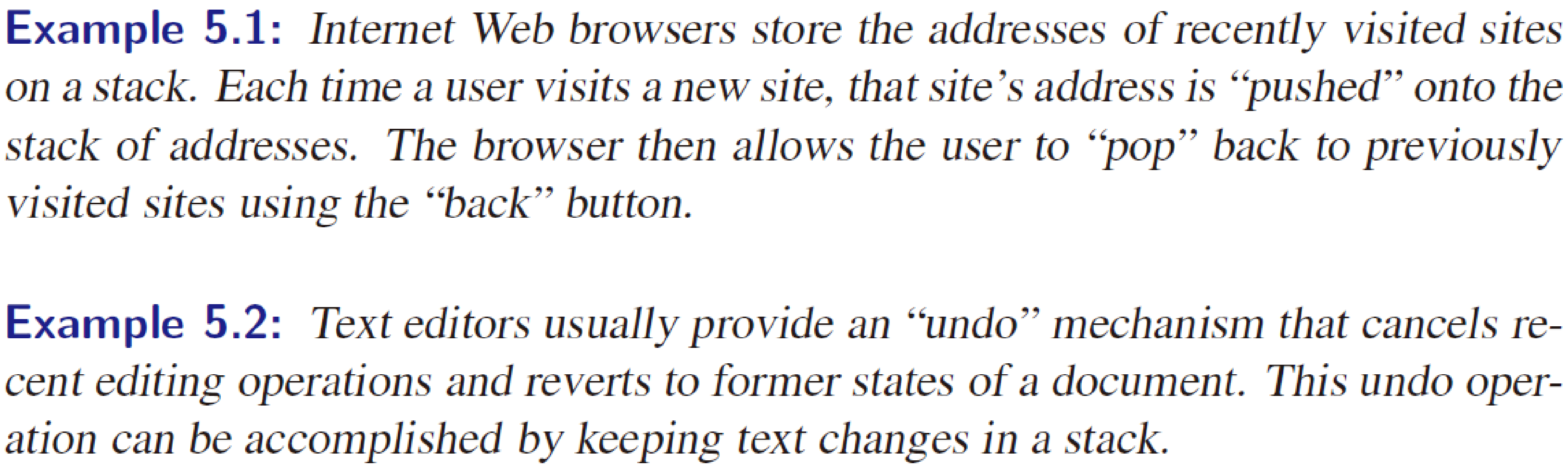
부산대학교 정보·의생명공학대학

정보컴퓨터공학부

Stacks

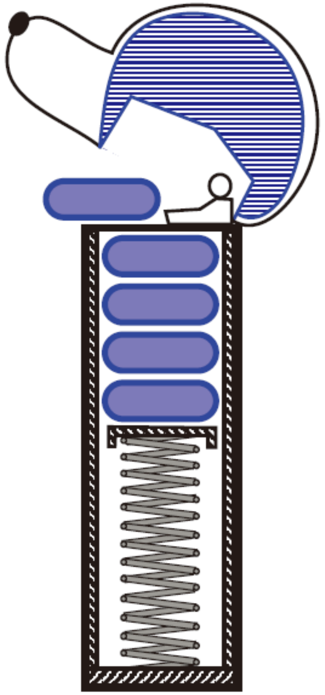
Stacks

- **Stack**
  - A container of objects that are inserted and removed according to the **last-in first-out (LIFO)** principle
  - Objects can be inserted into a stack at any time
  - The most recently inserted (*i.e.*, the last) object can be removed from the stack



Example 5.1: *Internet Web browsers store the addresses of recently visited sites on a stack. Each time a user visits a new site, that site's address is "pushed" onto the stack of addresses. The browser then allows the user to "pop" back to previously visited sites using the "back" button.*

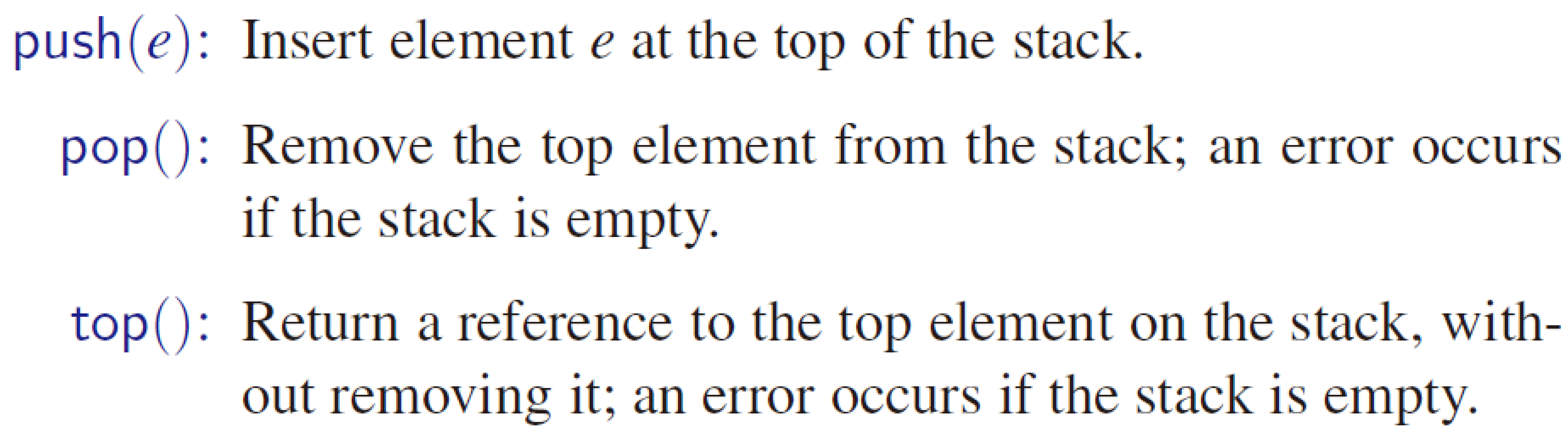
Example 5.2: *Text editors usually provide an "undo" mechanism that cancels recent editing operations and reverts to former states of a document. This undo operation can be accomplished by keeping text changes in a stack.*



A dispenser schematic

Stack ADT

- A stack is an ADT that supports the following operations:

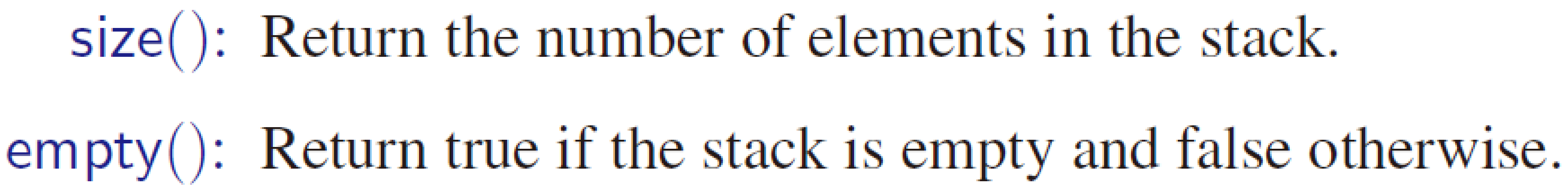


push(e): Insert element e at the top of the stack.

pop(): Remove the top element from the stack; an error occurs if the stack is empty.

top(): Return a reference to the top element on the stack, without removing it; an error occurs if the stack is empty.

- Additional utility functions:



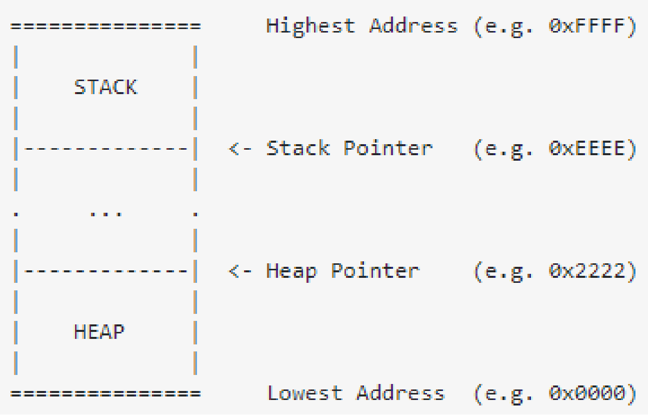
size(): Return the number of elements in the stack.

empty(): Return true if the stack is empty and false otherwise.

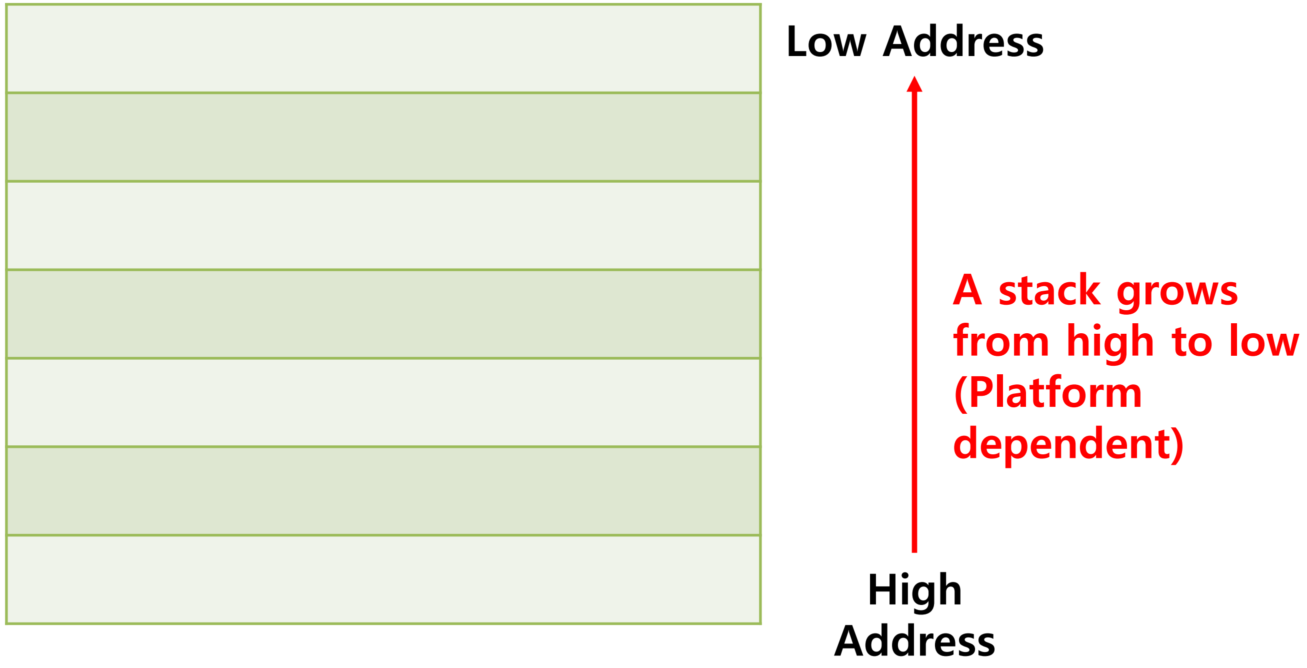
Stack Example

Operation	Output	Stack Contents
push(5)	–	(5)
push(3)	–	(5,3)
pop()	–	(5)
push(7)	–	(5,7)
pop()	–	(5)
top()	5	(5)
pop()	–	()
pop()	"error"	()
top()	"error"	()
empty()	true	()
push(9)	–	(9)
push(7)	–	(9,7)
push(3)	–	(9,7,3)
push(5)	–	(9,7,3,5)
size()	4	(9,7,3,5)
pop()	–	(9,7,3)
push(8)	–	(9,7,3,8)
pop()	–	(9,7,3)
top()	3	(9,7,3)

https://stackoverflow.com/a/4560763



```
===============     Highest Address (e.g. 0xFFFF)
|             |
|    STACK    |
|             |
|-------------|  <- Stack Pointer   (e.g. 0xEEEE)
|             |
.     ...     .
|             |
|-------------|  <- Heap Pointer    (e.g. 0x2222)
|             |
|    HEAP     |
|             |
===============     Lowest Address  (e.g. 0x0000)
```



Low Address

A stack grows from high to low (Platform dependent)

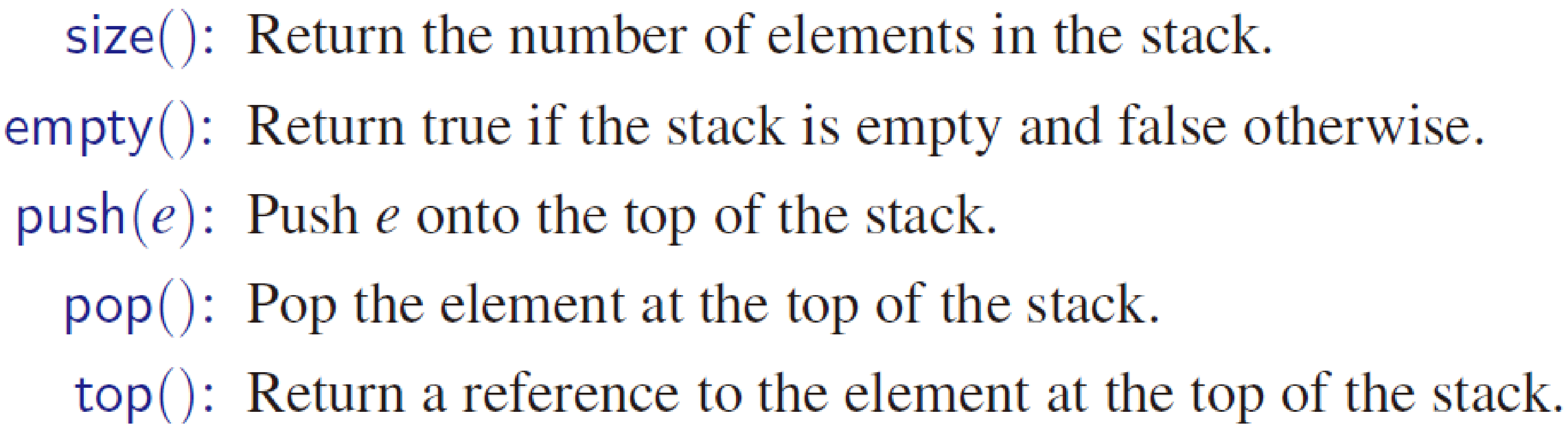
High Address

The STL Stack

- The STL stack implementation is based on the STL deque, vector, or list class

```
#include <stack>
using std::stack;                    // make stack accessible
stack<int> myStack;                  // a stack of integers
```

Stack's base type



size(): Return the number of elements in the stack.

empty(): Return true if the stack is empty and false otherwise.

push(*e*): Push *e* onto the top of the stack.

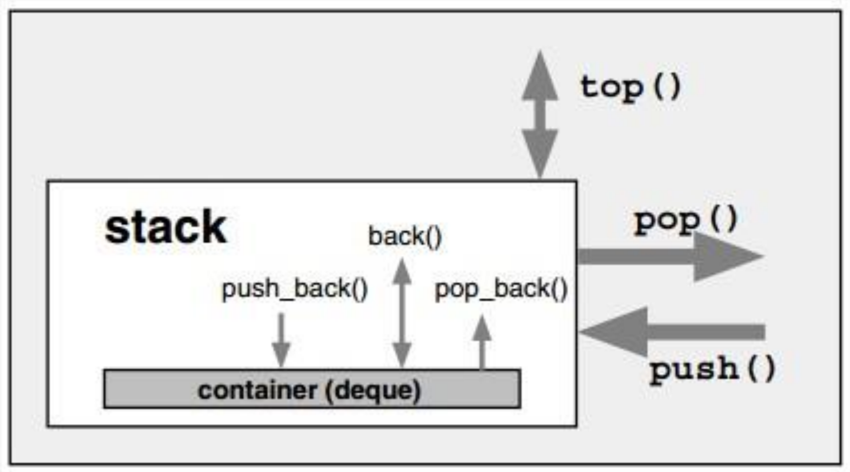
pop(): Pop the element at the top of the stack.

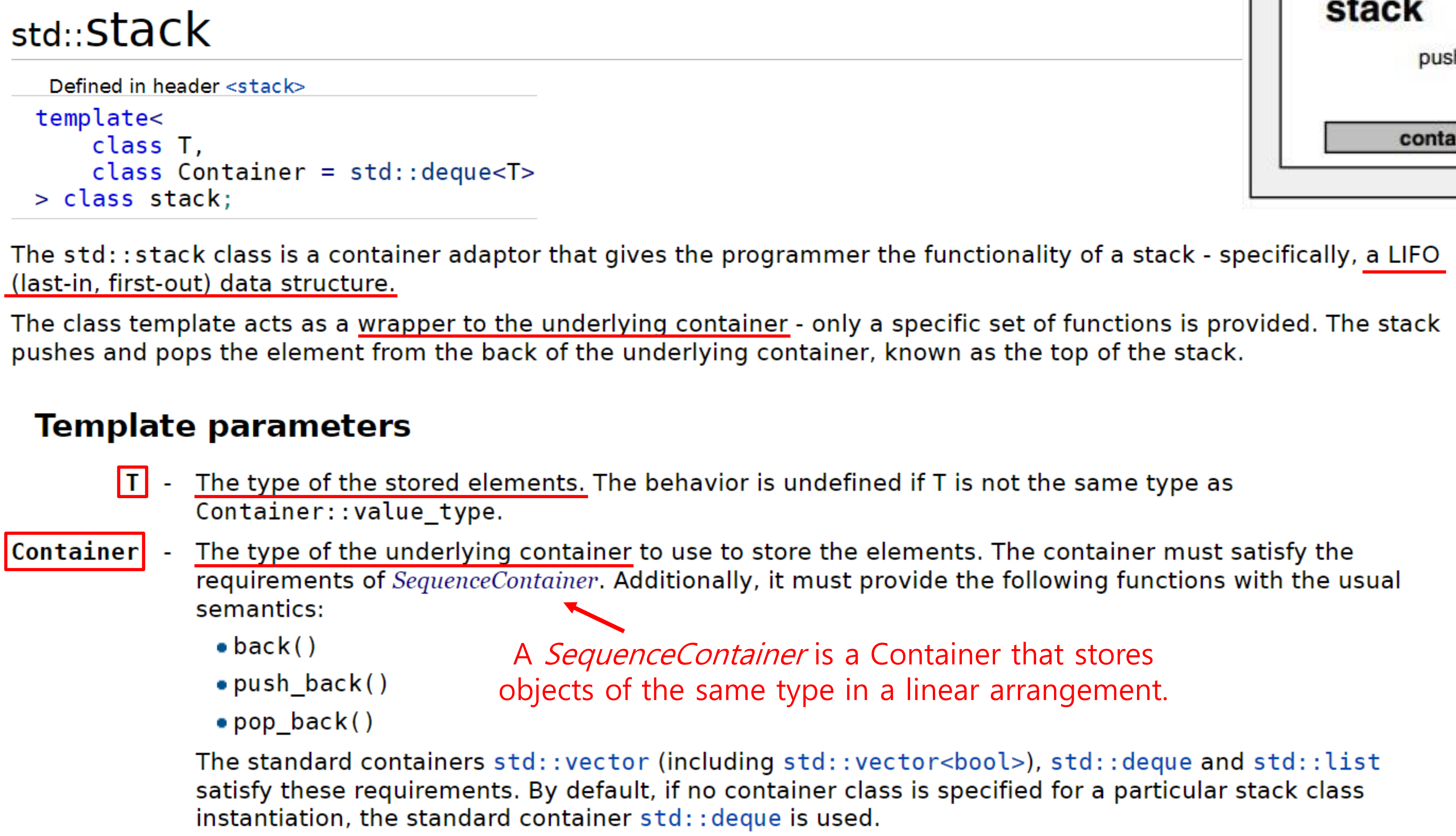
top(): Return a reference to the element at the top of the stack.

- Applying *top()* or *pop()* to an empty stack is undefined

The STL Stack

- **The STL Stack Reference Manual**





std::stack

Defined in header <stack>

```
template<
    class T,
    class Container = std::deque<T>
> class stack;
```

The `std::stack` class is a container adaptor that gives the programmer the functionality of a stack - specifically, a LIFO (last-in, first-out) data structure.

The class template acts as a wrapper to the underlying container - only a specific set of functions is provided. The stack pushes and pops the element from the back of the underlying container, known as the top of the stack.

Template parameters

T - The type of the stored elements. The behavior is undefined if T is not the same type as `Container::value_type`.

Container - The type of the underlying container to use to store the elements. The container must satisfy the requirements of *SequenceContainer*. Additionally, it must provide the following functions with the usual semantics:

- `back()`
- `push_back()`
- `pop_back()`

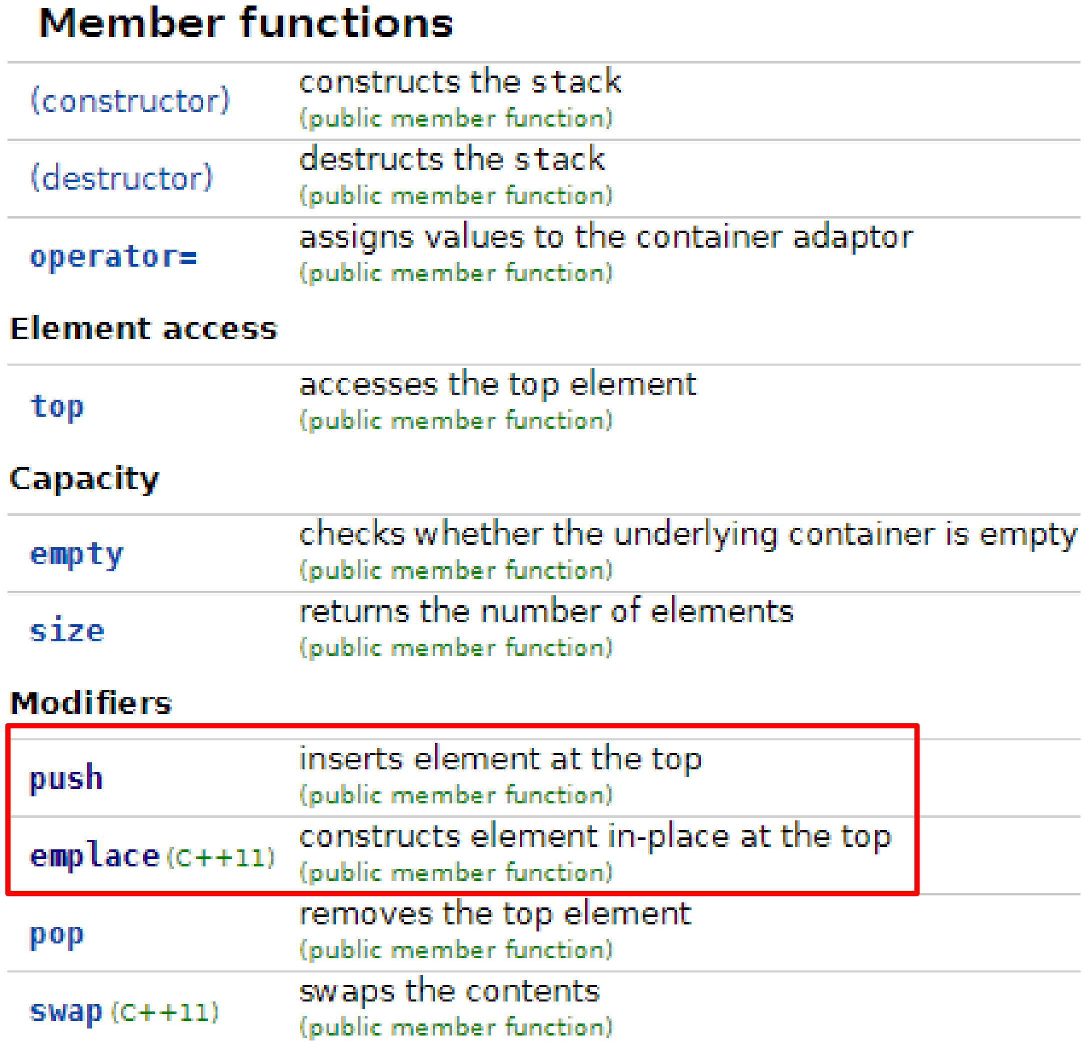
The standard containers `std::vector` (including `std::vector<bool>`), `std::deque` and `std::list` satisfy these requirements. By default, if no container class is specified for a particular stack class instantiation, the standard container `std::deque` is used.

A *SequenceContainer* is a Container that stores objects of the same type in a linear arrangement.

https://en.cppreference.com/w/cpp/container/stack

The STL Stack

- **The STL Stack Reference Manual**



Member functions

(constructor)	constructs the stack (public member function)
(destructor)	destructs the stack (public member function)
`operator=`	assigns values to the container adaptor (public member function)

Element access

`top`	accesses the top element (public member function)

Capacity

`empty`	checks whether the underlying container is empty (public member function)
`size`	returns the number of elements (public member function)

Modifiers

`push`	inserts element at the top (public member function)
`emplace` (C++11)	constructs element in-place at the top (public member function)
`pop`	removes the top element (public member function)
`swap` (C++11)	swaps the contents (public member function)

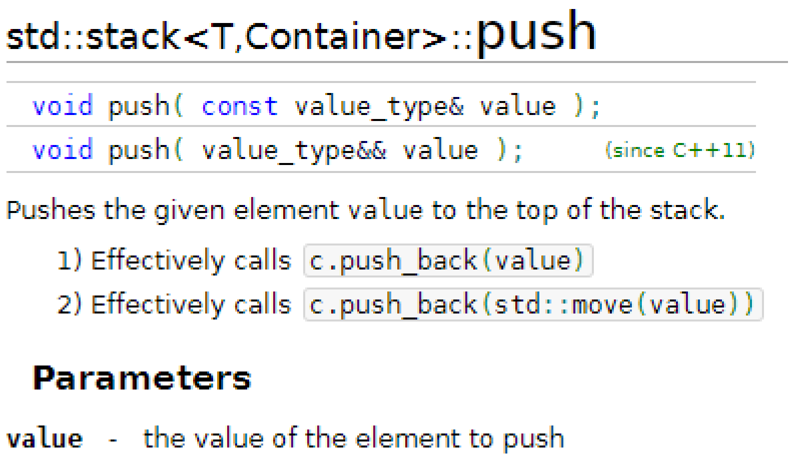
https://en.cppreference.com/w/cpp/container/stack

The STL Stack

- The container gets one more element appended (*i.e.*, the same result)

- ***push()***
  - Takes an existing element and **copy** it to append
  - Takes exactly one argument

- ***emplace()***
  - The element to be pushed is constructed **in-place**
  - Takes arguments for the constructor of the element



std::stack<T,Container>::push

```
void push( const value_type& value );
void push( value_type&& value );            (since C++11)
```

Pushes the given element value to the top of the stack.

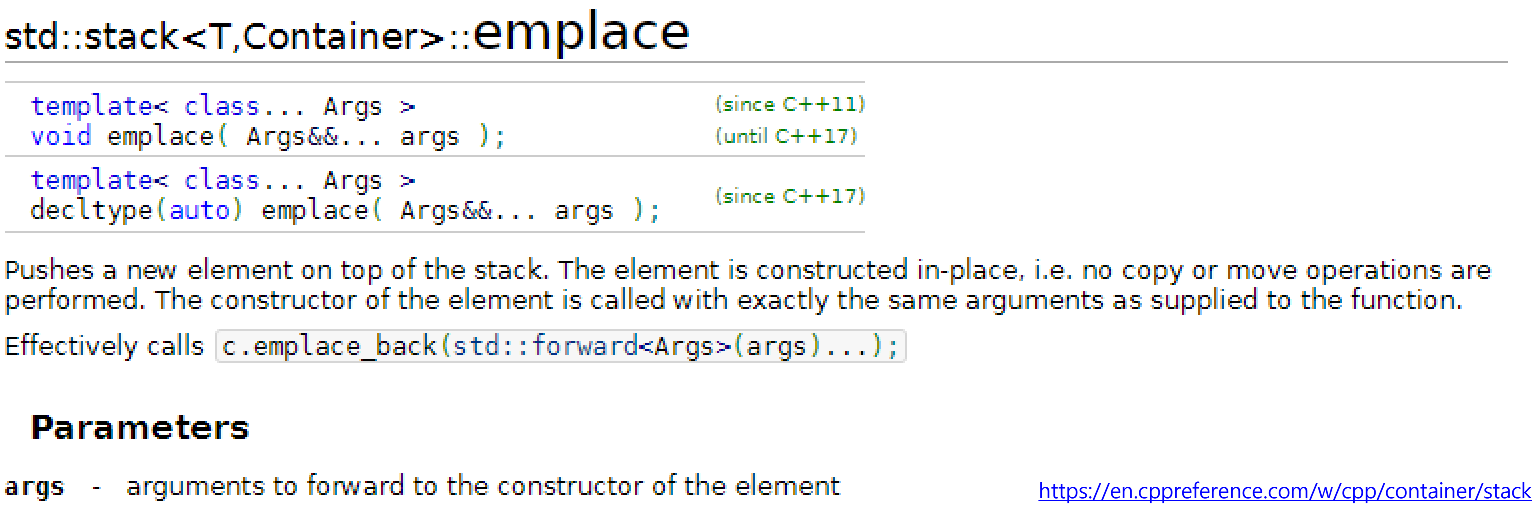
1) Effectively calls `c.push_back(value)`

2) Effectively calls `c.push_back(std::move(value))`

Parameters

value - the value of the element to push

Think about a class with a costly constructor...



std::stack<T,Container>::emplace

```
template< class... Args >                   (since C++11)
void emplace( Args&&... args );             (until C++17)
template< class... Args >
decltype(auto) emplace( Args&&... args );   (since C++17)
```

Pushes a new element on top of the stack. The element is constructed in-place, i.e. no copy or move operations are performed. The constructor of the element is called with exactly the same arguments as supplied to the function.

Effectively calls `c.emplace_back(std::forward<Args>(args)...);`

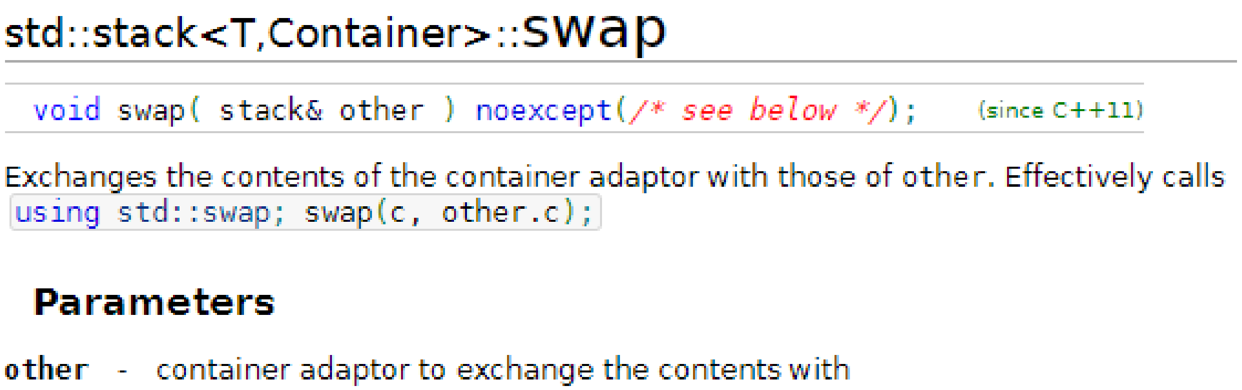
Parameters

args - arguments to forward to the constructor of the element

https://en.cppreference.com/w/cpp/container/stack

The STL Stack

- ***swap()***
  - Exchanges the contents of two containers



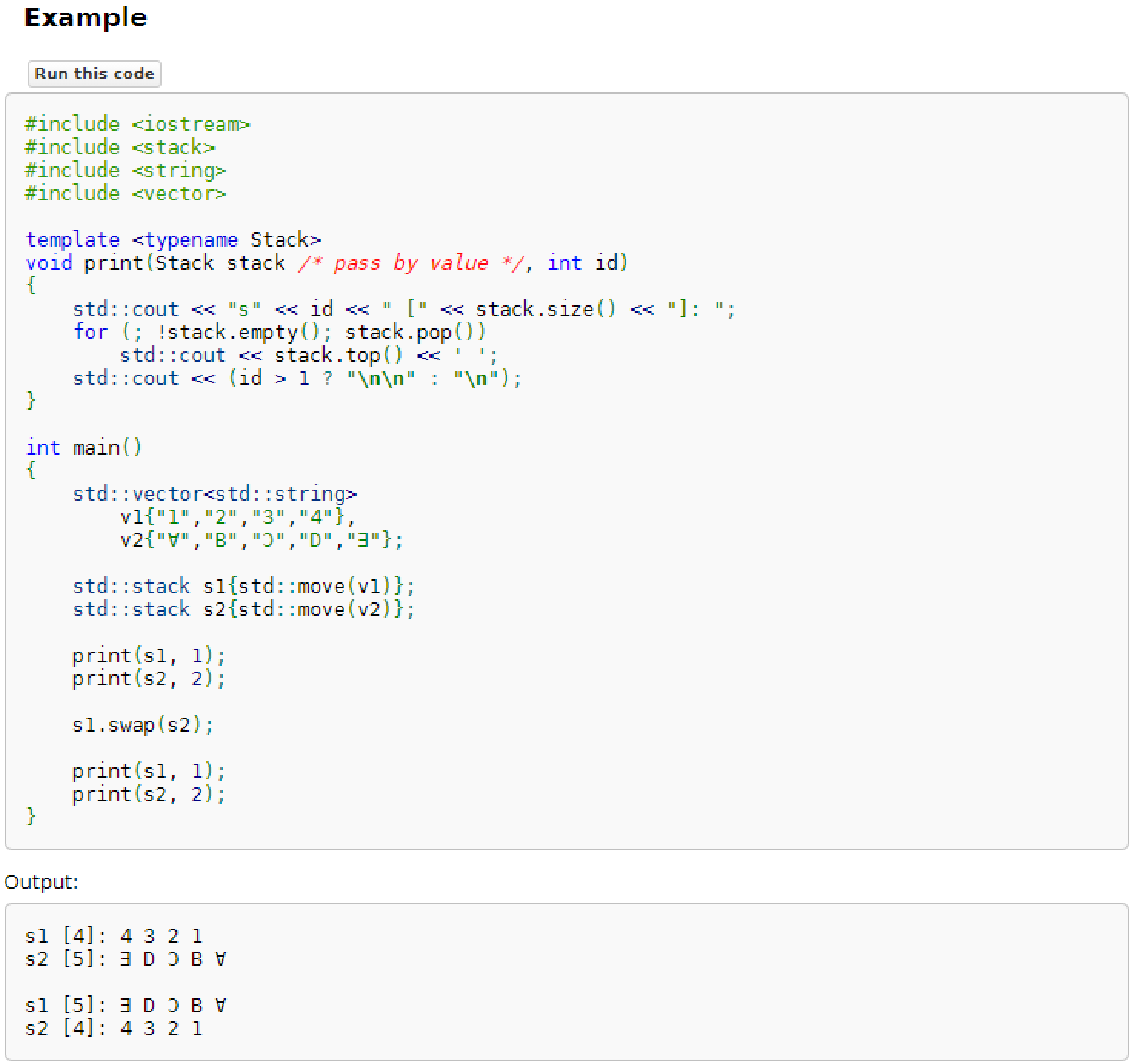
std::stack<T,Container>::swap

```
void swap( stack& other ) noexcept(/* see below */);        (since C++11)
```

Exchanges the contents of the container adaptor with those of other. Effectively calls `using std::swap; swap(c, other.c);`

Parameters

other - container adaptor to exchange the contents with



Example

```
#include <iostream>
#include <stack>
#include <string>
#include <vector>

template <typename Stack>
void print(Stack stack /* pass by value */, int id)
{
    std::cout << "s" << id << " [" << stack.size() << "]: ";
    for (; !stack.empty(); stack.pop())
        std::cout << stack.top() << ' ';
    std::cout << (id > 1 ? "\n\n" : "\n");
}

int main()
{
    std::vector<std::string>
        v1{"1","2","3","4"},
        v2{"Ɐ","B","Ɔ","D","Ǝ"};

    std::stack s1{std::move(v1)};
    std::stack s2{std::move(v2)};

    print(s1, 1);
    print(s2, 2);

    s1.swap(s2);

    print(s1, 1);
    print(s2, 2);
}
```

Output:

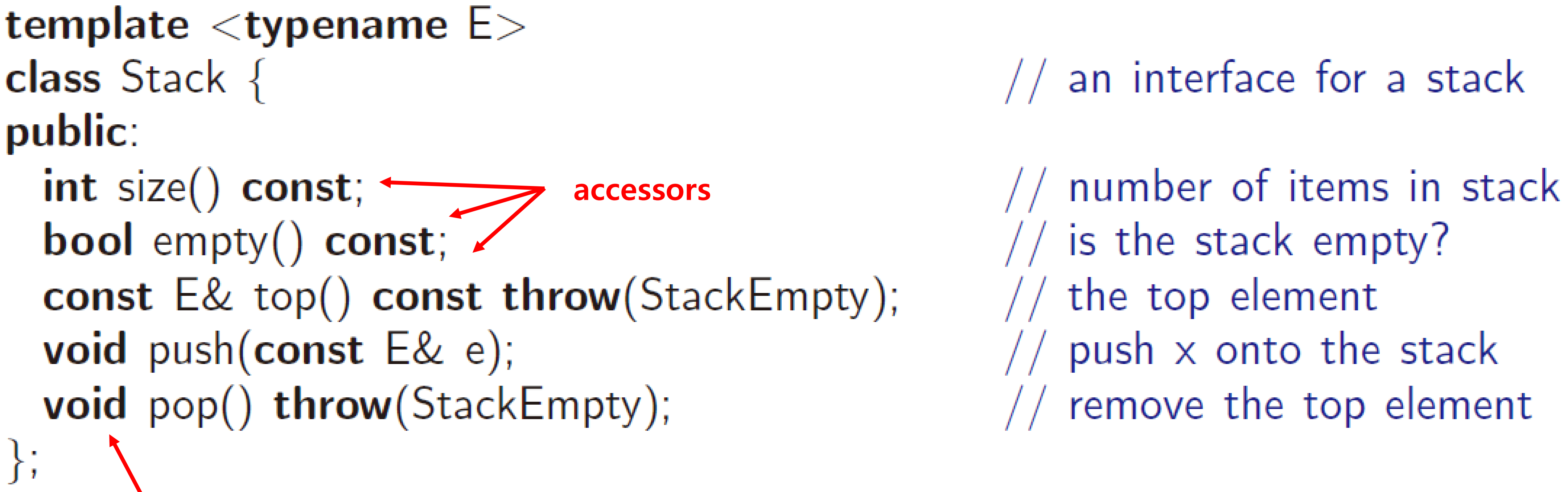
```
s1 [4]: 4 3 2 1
s2 [5]: Ǝ D Ɔ B Ɐ

s1 [5]: Ǝ D Ɔ B Ɐ
s2 [4]: 4 3 2 1
```

https://en.cppreference.com/w/cpp/container/stack

C++ Stack Interface

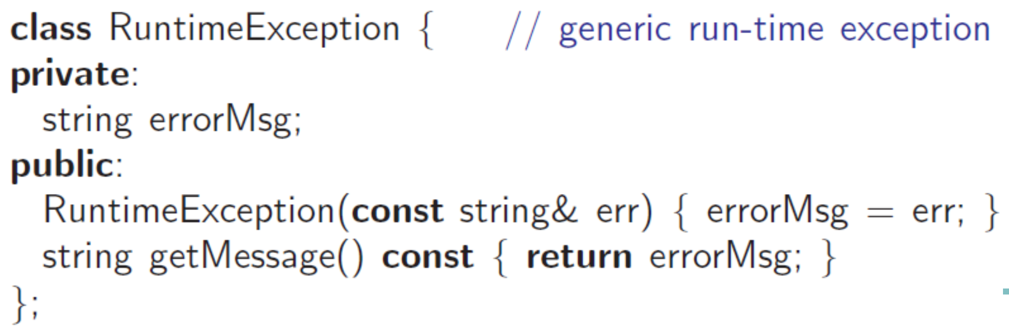
- **An Informal Stack Interface**



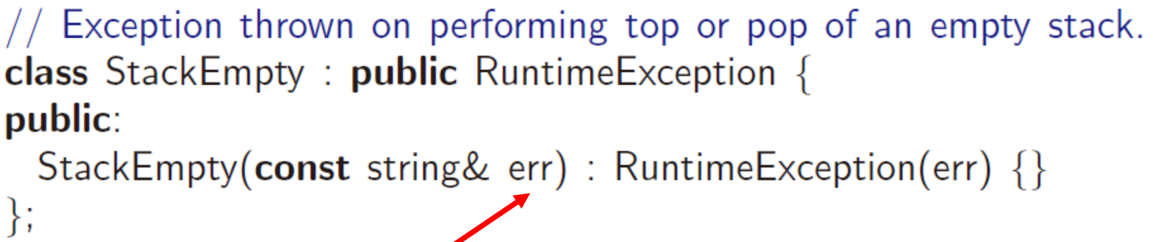
```
template <typename E>
class Stack {                                        // an interface for a stack
public:
  int size() const;                                  // number of items in stack
  bool empty() const;                                // is the stack empty?
  const E& top() const throw(StackEmpty);            // the top element
  void push(const E& e);                             // push x onto the stack
  void pop() throw(StackEmpty);                      // remove the top element
};
```

accessors

no return for pop



```
class RuntimeException {       // generic run-time exception
private:
  string errorMsg;
public:
  RuntimeException(const string& err) { errorMsg = err; }
  string getMessage() const { return errorMsg; }
};
```



```
// Exception thrown on performing top or pop of an empty stack.
class StackEmpty : public RuntimeException {
public:
  StackEmpty(const string& err) : RuntimeException(err) {}
};
```

error message

Stack Implementation: Array-Based

- The stack consists of:
  - an *N*-element array *S*
  - an integer variable *t* indicating the top element in *S*
  - *t* is initialized to -1 to denote the empty stack
- Each function executes a constant number of statements
  - Arithmetic operation
  - Comparison
  - Indexing
  - Assignment
- Can be a good option when we have a good estimate on the number of items

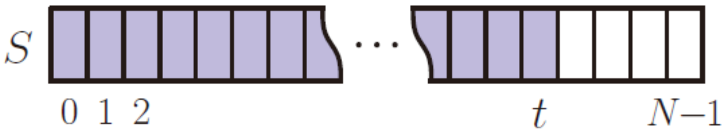


Figure 5.2: Realization of a stack by means of an array S. The top element in the stack is stored in the cell $S[t]$.

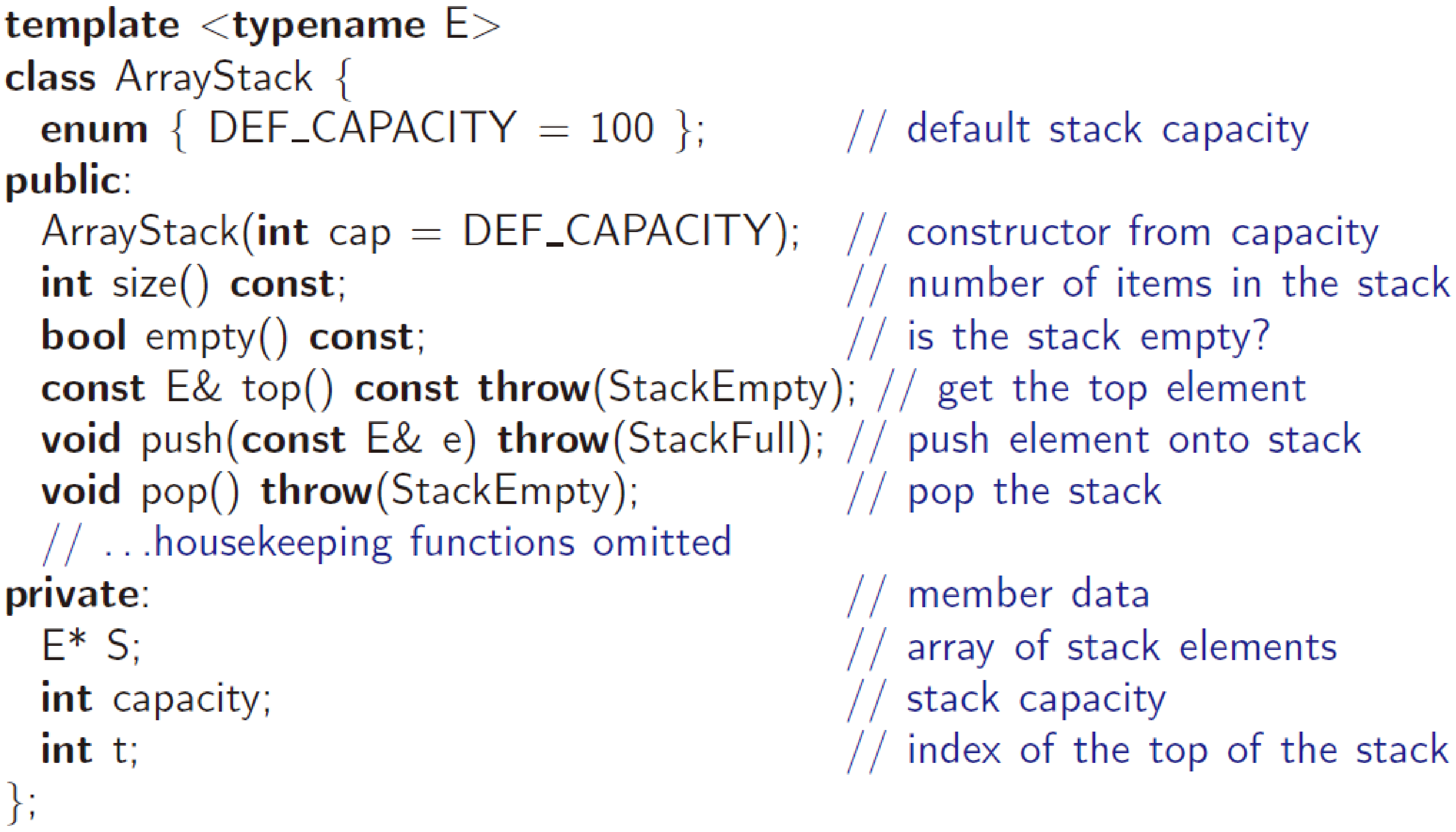
```
Algorithm size():
    return $t + 1$
Algorithm empty():
    return $(t < 0)$
Algorithm top():
    if empty() then
        throw StackEmpty exception
    return $S[t]$
Algorithm push($e$):
    if size() $= N$ then
        throw StackFull exception
    $t \leftarrow t + 1$
    $S[t] \leftarrow e$
Algorithm pop():
    if empty() then
        throw StackEmpty exception
    $t \leftarrow t - 1$
```

Specific exception to array-based stack implementation

Operation	*Time*
size	$O(1)$
empty	$O(1)$
top	$O(1)$
push	$O(1)$
pop	$O(1)$

Stack Implementation: Array-Based

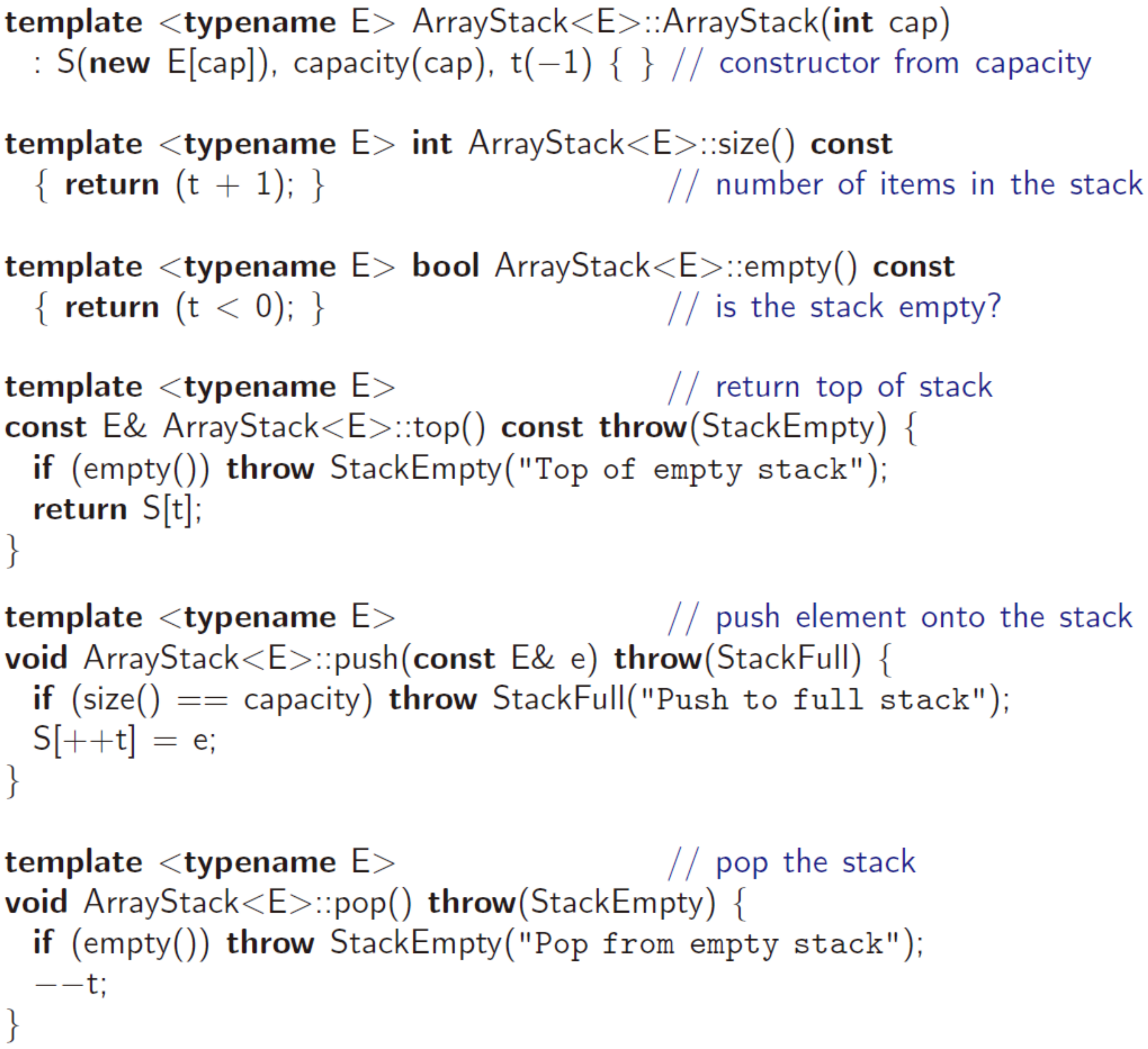
- **C++ Implementation**



```
template <typename E>
class ArrayStack {
  enum { DEF_CAPACITY = 100 };           // default stack capacity
public:
  ArrayStack(int cap = DEF_CAPACITY);    // constructor from capacity
  int size() const;                      // number of items in the stack
  bool empty() const;                    // is the stack empty?
  const E& top() const throw(StackEmpty); // get the top element
  void push(const E& e) throw(StackFull); // push element onto stack
  void pop() throw(StackEmpty);          // pop the stack
  // ...housekeeping functions omitted
private:                                 // member data
  E* S;                                  // array of stack elements
  int capacity;                          // stack capacity
  int t;                                 // index of the top of the stack
};
```

Stack Implementation: Array-Based

- **C++ Implementation**



```
template <typename E> ArrayStack<E>::ArrayStack(int cap)
  : S(new E[cap]), capacity(cap), t(-1) { } // constructor from capacity

template <typename E> int ArrayStack<E>::size() const
  { return (t + 1); }                        // number of items in the stack

template <typename E> bool ArrayStack<E>::empty() const
  { return (t < 0); }                        // is the stack empty?

template <typename E>                        // return top of stack
const E& ArrayStack<E>::top() const throw(StackEmpty) {
  if (empty()) throw StackEmpty("Top of empty stack");
  return S[t];
}

template <typename E>                        // push element onto the stack
void ArrayStack<E>::push(const E& e) throw(StackFull) {
  if (size() == capacity) throw StackFull("Push to full stack");
  S[++t] = e;
}

template <typename E>                        // pop the stack
void ArrayStack<E>::pop() throw(StackEmpty) {
  if (empty()) throw StackEmpty("Pop from empty stack");
  --t;
}
```

Stack Implementation: Array-Based

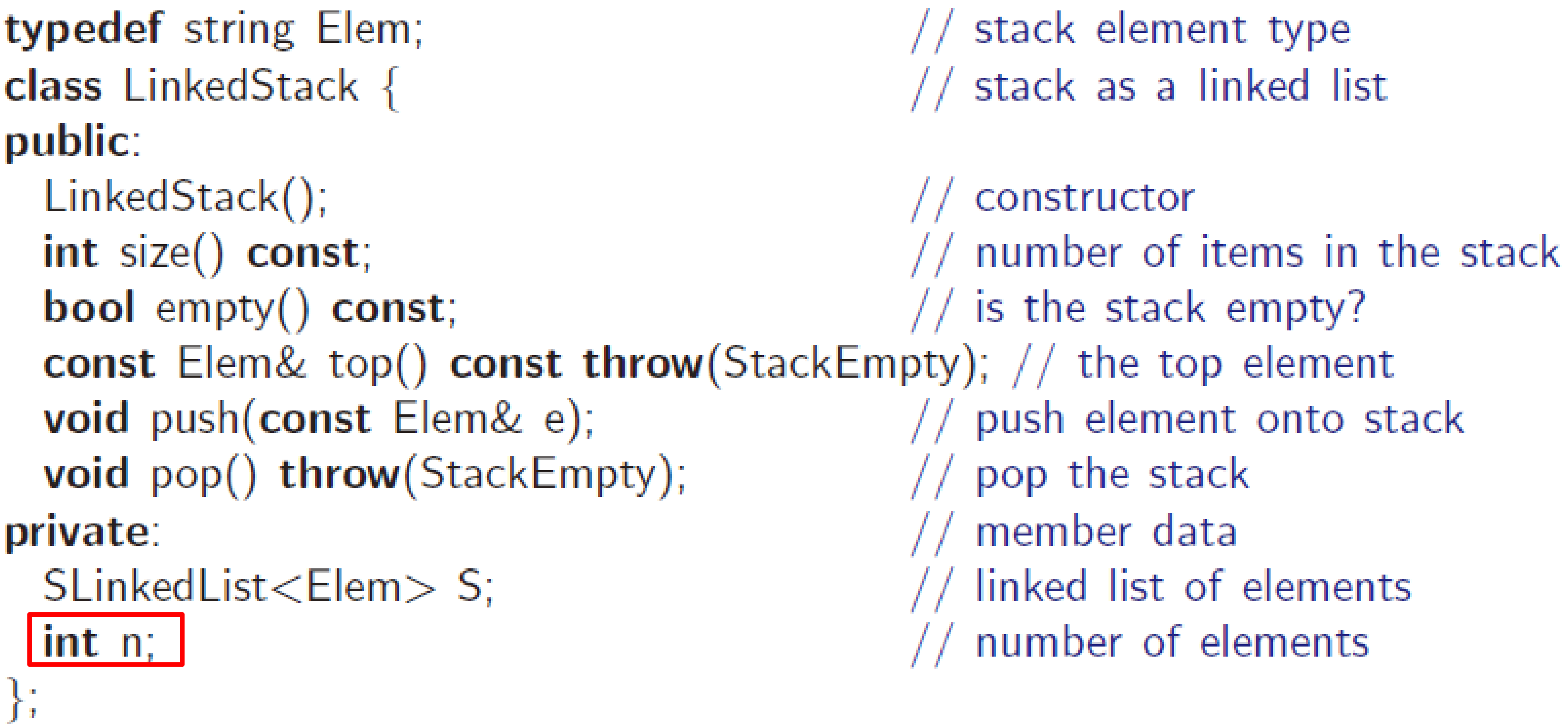
- **Example Output**

```
ArrayStack<int> A;                        // A = [], size = 0
A.push(7);                                // A = [7*], size = 1
A.push(13);                               // A = [7, 13*], size = 2
cout << A.top() << endl; A.pop();         // A = [7*], outputs: 13
A.push(9);                                // A = [7, 9*], size = 2
cout << A.top() << endl;                  // A = [7, 9*], outputs: 9
cout << A.top() << endl; A.pop();         // A = [7*], outputs: 9
ArrayStack<string> B(10);                 // B = [], size = 0
B.push("Bob");                            // B = [Bob*], size = 1
B.push("Alice");                          // B = [Bob, Alice*], size = 2
cout << B.top() << endl; B.pop();         // B = [Bob*], outputs: Alice
B.push("Eve");                            // B = [Bob, Eve*], size = 2
```


The top of the stack is indicated by an asterisk ("*").

Stack Implementation: Linked List

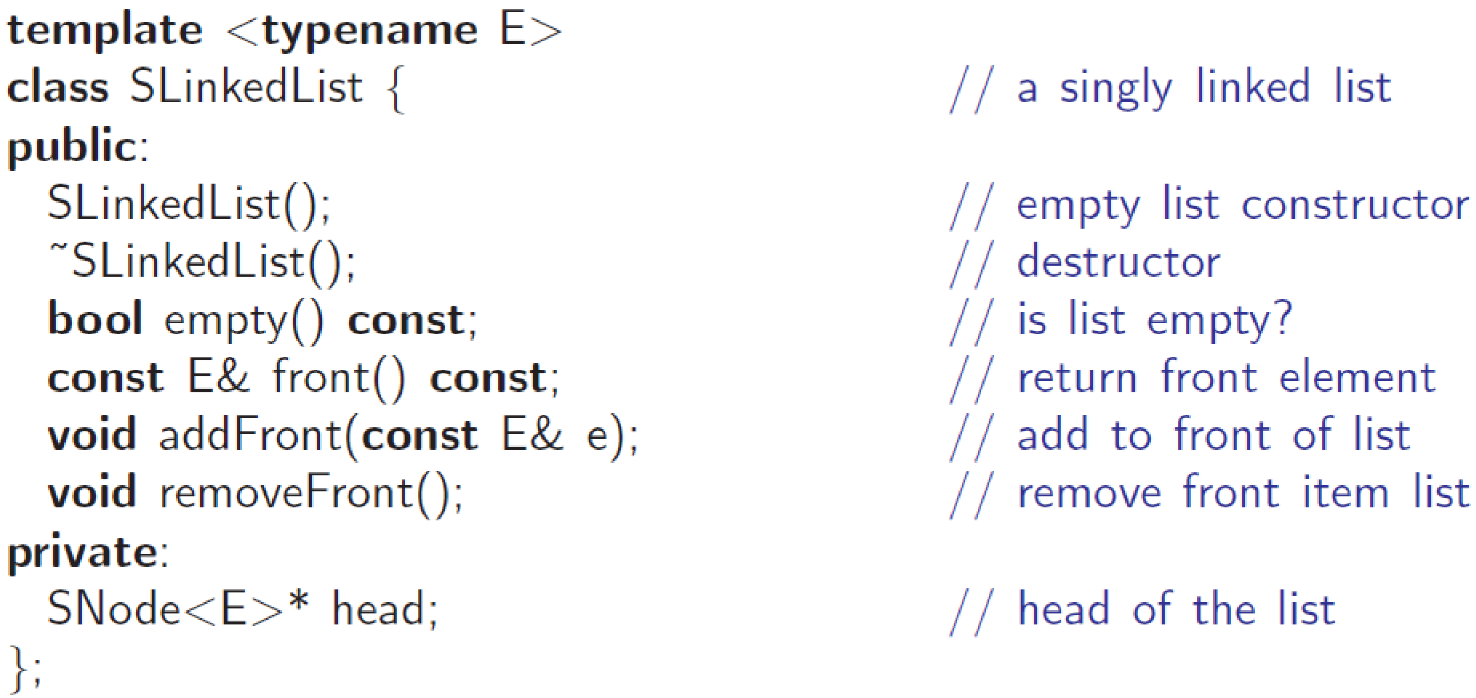
- **C++ Implementation**



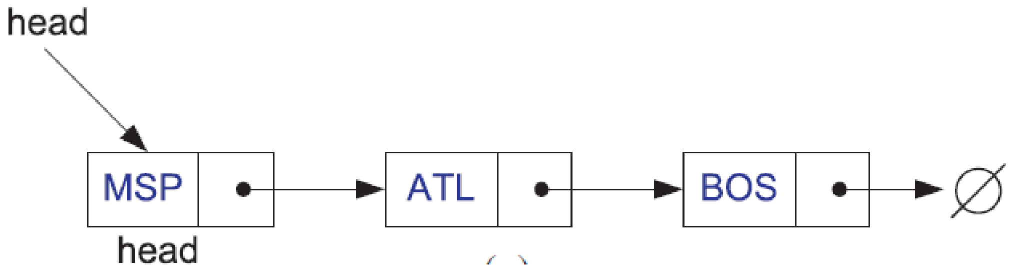
```
typedef string Elem;                              // stack element type
class LinkedStack {                               // stack as a linked list
public:
  LinkedStack();                                  // constructor
  int size() const;                               // number of items in the stack
  bool empty() const;                             // is the stack empty?
  const Elem& top() const throw(StackEmpty); // the top element
  void push(const Elem& e);                       // push element onto stack
  void pop() throw(StackEmpty);                   // pop the stack
private:                                          // member data
  SLinkedList<Elem> S;                            // linked list of elements
  int n;                                          // number of elements
};
```

Stack Implementation: Linked List

- **C++ Implementation (Singly Linked List)**



```
template <typename E>
class SLinkedList {                          // a singly linked list
public:
  SLinkedList();                             // empty list constructor
  ~SLinkedList();                            // destructor
  bool empty() const;                        // is list empty?
  const E& front() const;                    // return front element
  void addFront(const E& e);                 // add to front of list
  void removeFront();                        // remove front item list
private:
  SNode<E>* head;                            // head of the list
};
```

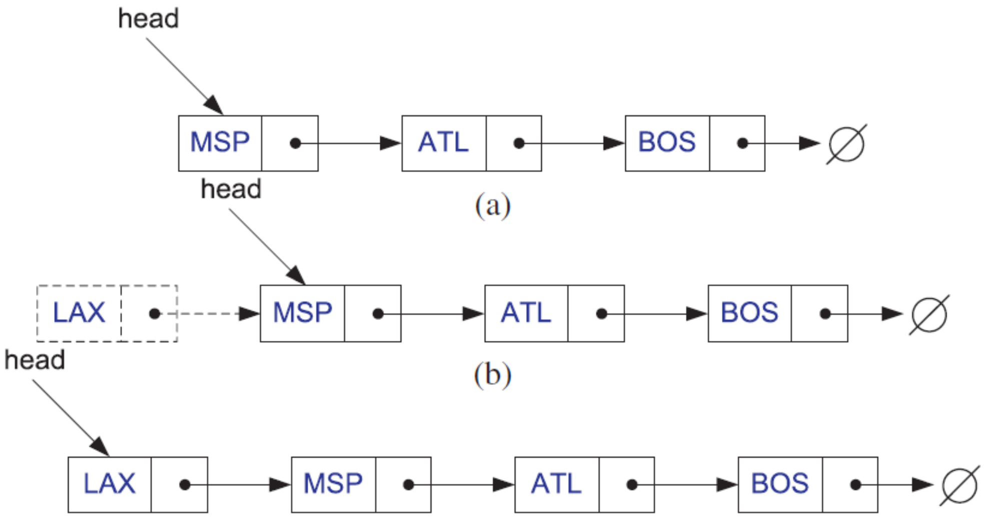


Stack Implementation: Linked List

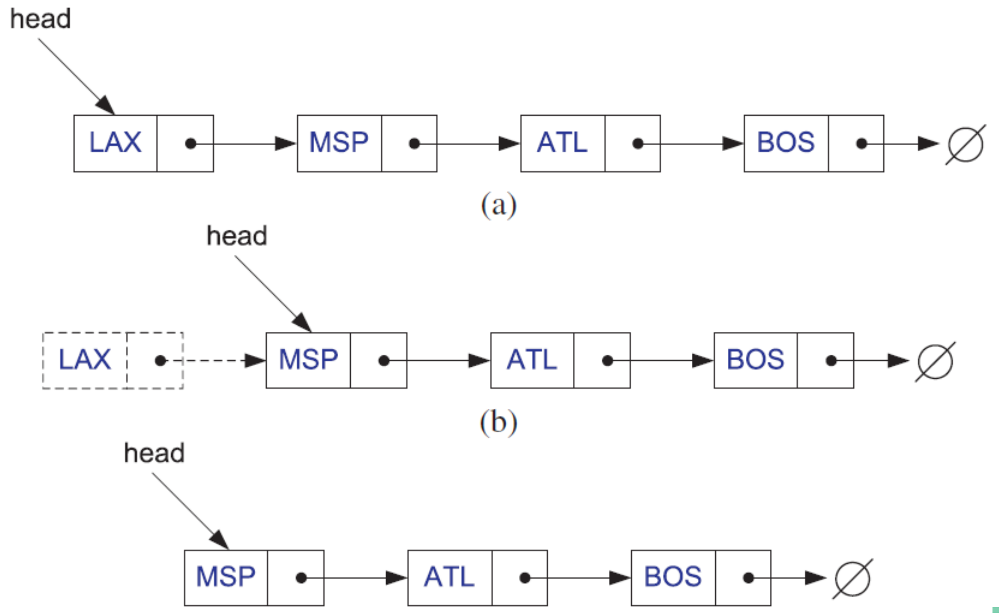
- **C++ Implementation (Singly Linked List)**

```
template <typename E>
void SLinkedList<E>::addFront(const E& e) {  // add to front of list
  SNode<E>* v = new SNode<E>;                 // create new node
  v->elem = e;                                // store data
  v->next = head;                             // head now follows v
  head = v;                                   // v is now the head
}

template <typename E>
void SLinkedList<E>::removeFront() {          // remove front item
  SNode<E>* old = head;                       // save current head
  head = old->next;                           // skip over old head
  delete old;                                 // delete the old head
}
```



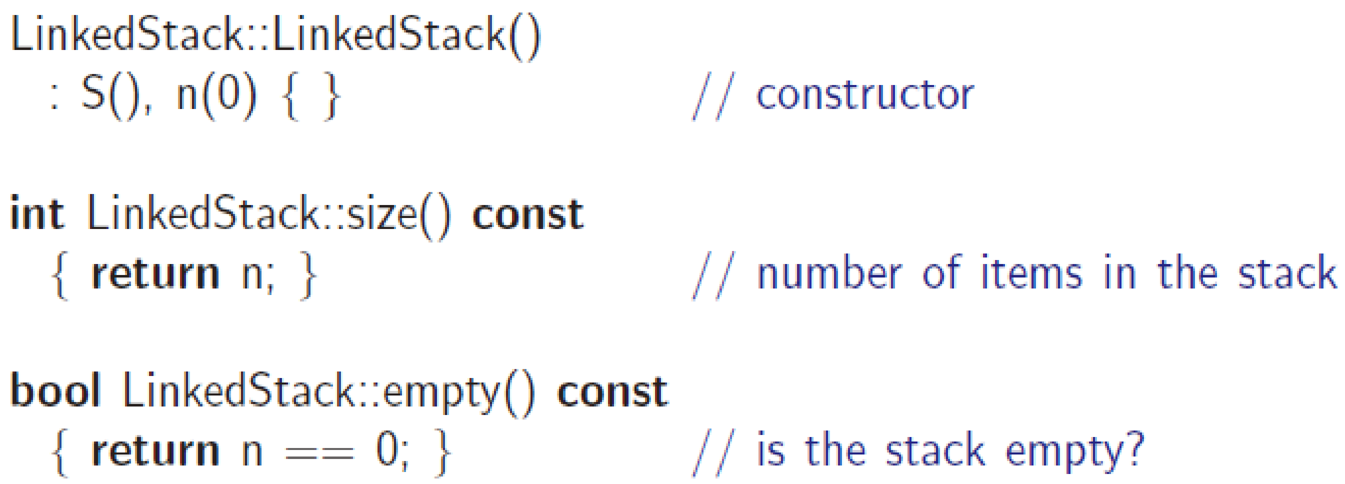
addFront



removeFront

Stack Implementation: Linked List

- **C++ Implementation**



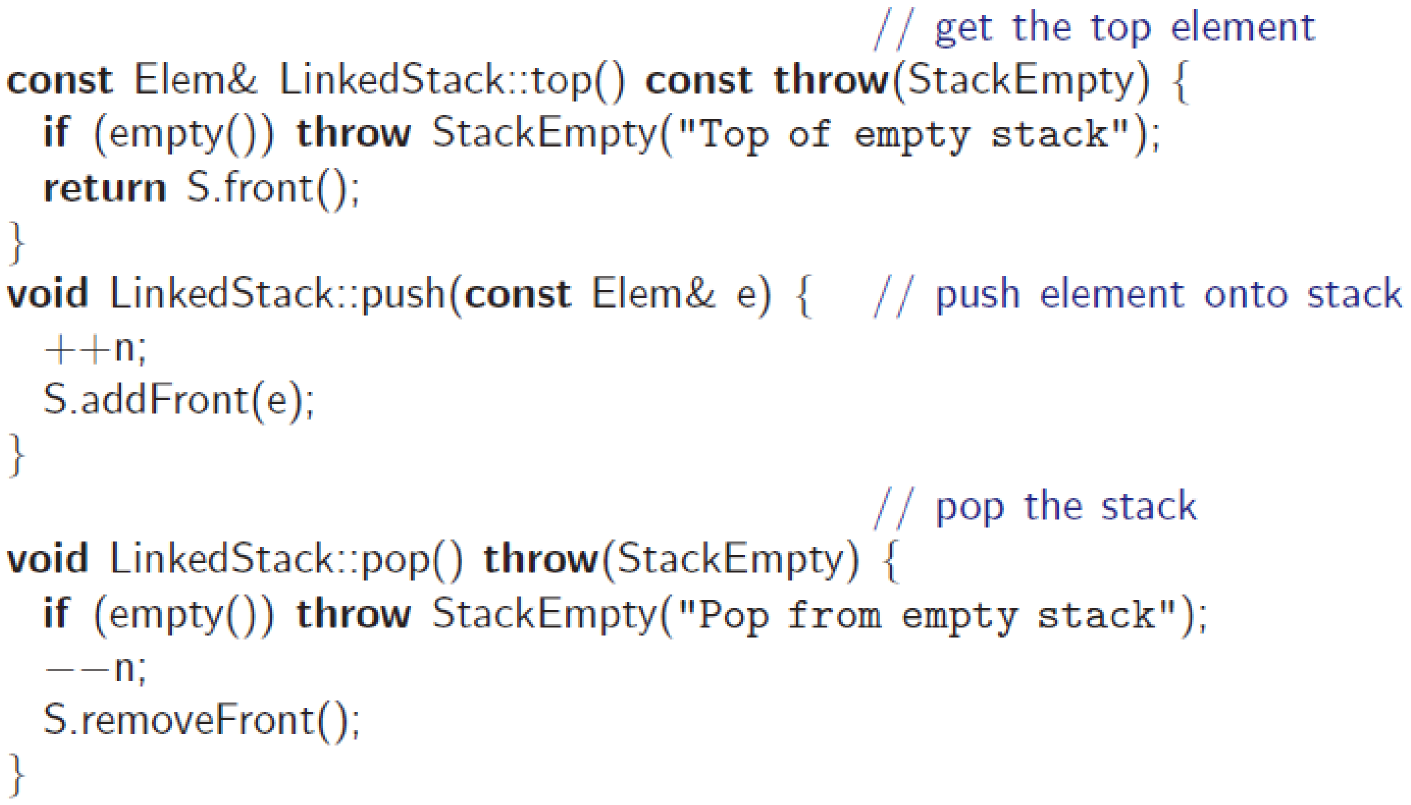
```
LinkedStack::LinkedStack()
  : S(), n(0) { }                    // constructor

int LinkedStack::size() const
  { return n; }                      // number of items in the stack

bool LinkedStack::empty() const
  { return n == 0; }                 // is the stack empty?
```

Stack Implementation: Linked List

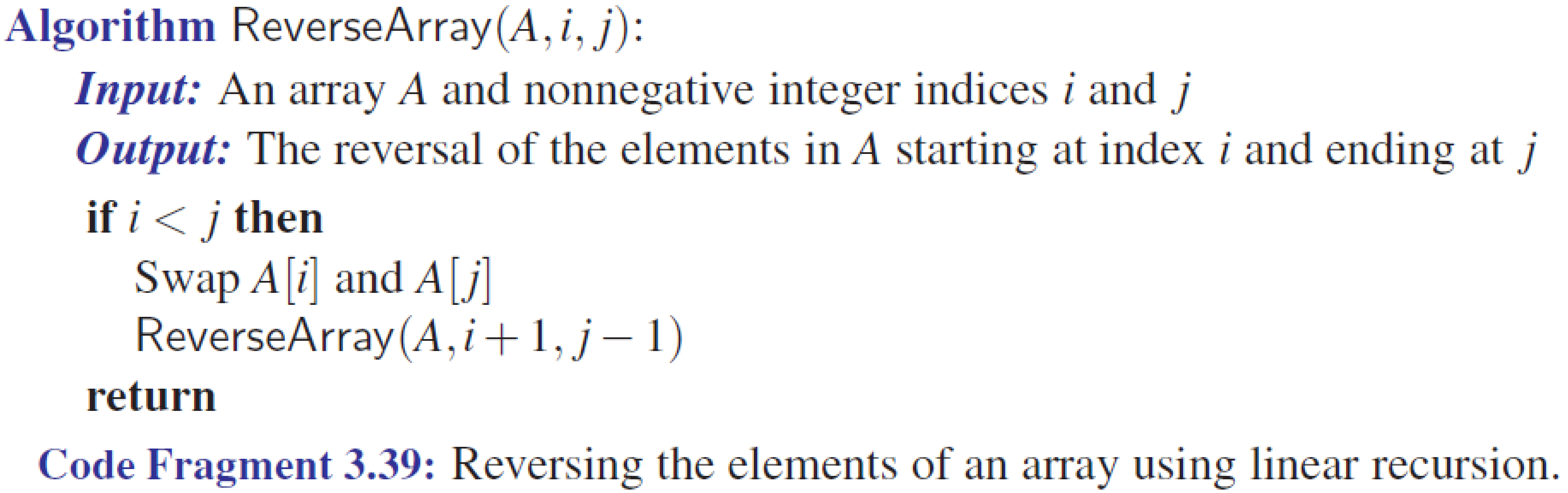
- **C++ Implementation**



```
                                                    // get the top element
const Elem& LinkedStack::top() const throw(StackEmpty) {
  if (empty()) throw StackEmpty("Top of empty stack");
  return S.front();
}
void LinkedStack::push(const Elem& e) {             // push element onto stack
  ++n;
  S.addFront(e);
}
                                                    // pop the stack
void LinkedStack::pop() throw(StackEmpty) {
  if (empty()) throw StackEmpty("Pop from empty stack");
  −−n;
  S.removeFront();
}
```

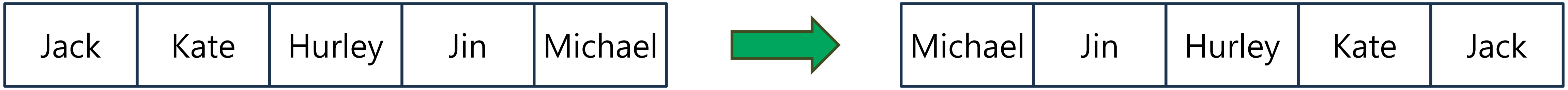
Stack Applications: Reversing an Array

- Swapping the first and last elements and then recursively reversing the remaining elements in the array.



```
Algorithm ReverseArray($A, i, j$):
    Input: An array $A$ and nonnegative integer indices $i$ and $j$
    Output: The reversal of the elements in $A$ starting at index $i$ and ending at $j$
    if $i < j$ then
        Swap $A[i]$ and $A[j]$
        ReverseArray($A, i+1, j-1$)
    return
```

Code Fragment 3.39: Reversing the elements of an array using linear recursion.

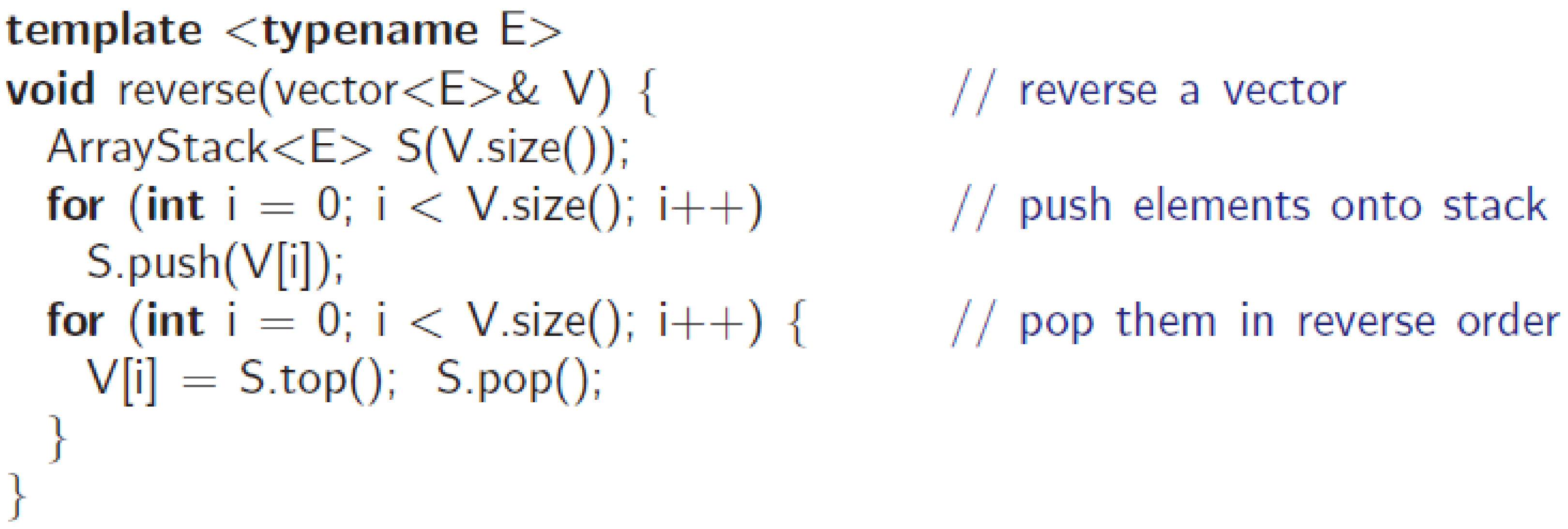


Jack	Kate	Hurley	Jin	Michael

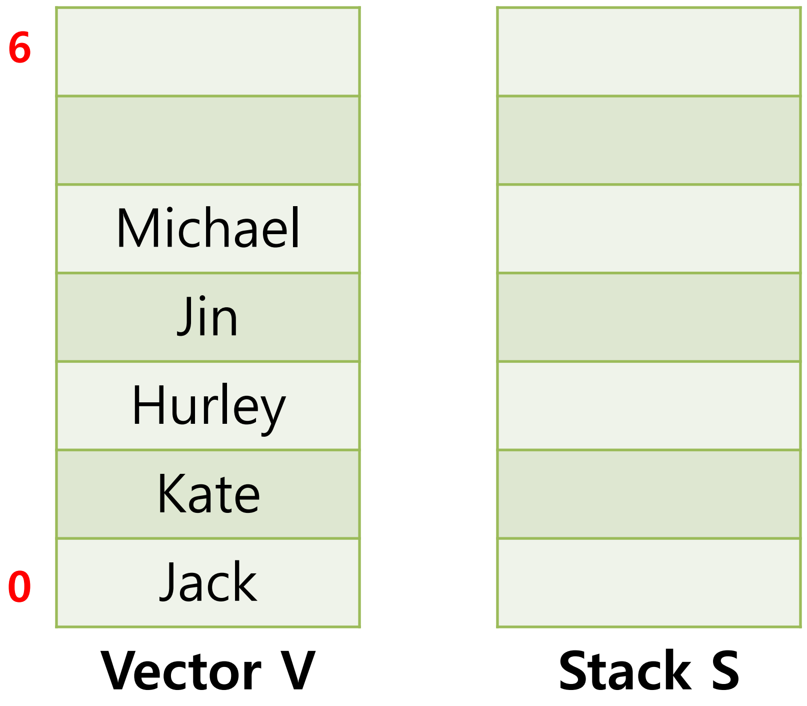
Michael	Jin	Hurley	Kate	Jack

Stack Applications

- **Reversing a Vector Using a Stack**
  - Push all the elements into a stack
  - Fill the vector again by popping the elements off of the stack



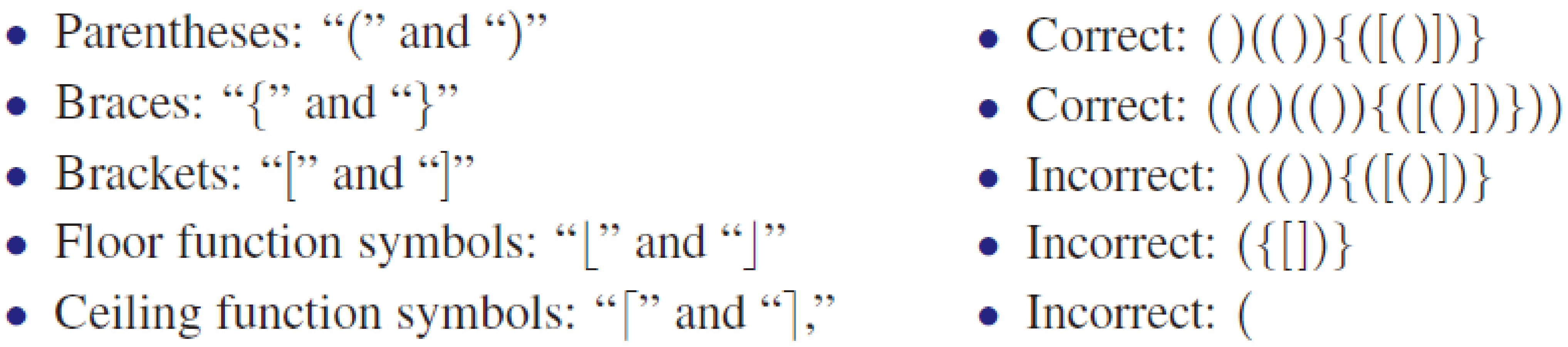
```
template <typename E>
void reverse(vector<E>& V) {                    // reverse a vector
  ArrayStack<E> S(V.size());
  for (int i = 0; i < V.size(); i++)            // push elements onto stack
    S.push(V[i]);
  for (int i = 0; i < V.size(); i++) {          // pop them in reverse order
    V[i] = S.top();  S.pop();
  }
}
```



Stack Applications

Matching Parentheses

- Matching parentheses and grouping symbols
- Each opening symbol must match with its corresponding closing symbol



- Parentheses: “(” and “)”
- Braces: “{” and “}”
- Brackets: “[” and “]”
- Floor function symbols: “⌊” and “⌋”
- Ceiling function symbols: “⌈” and “⌉,”

- Correct: ()(()){([()])}
- Correct: ((()(()){([()])}))
- Incorrect:)(()){([()])}
- Incorrect: ({[])}
- Incorrect: (

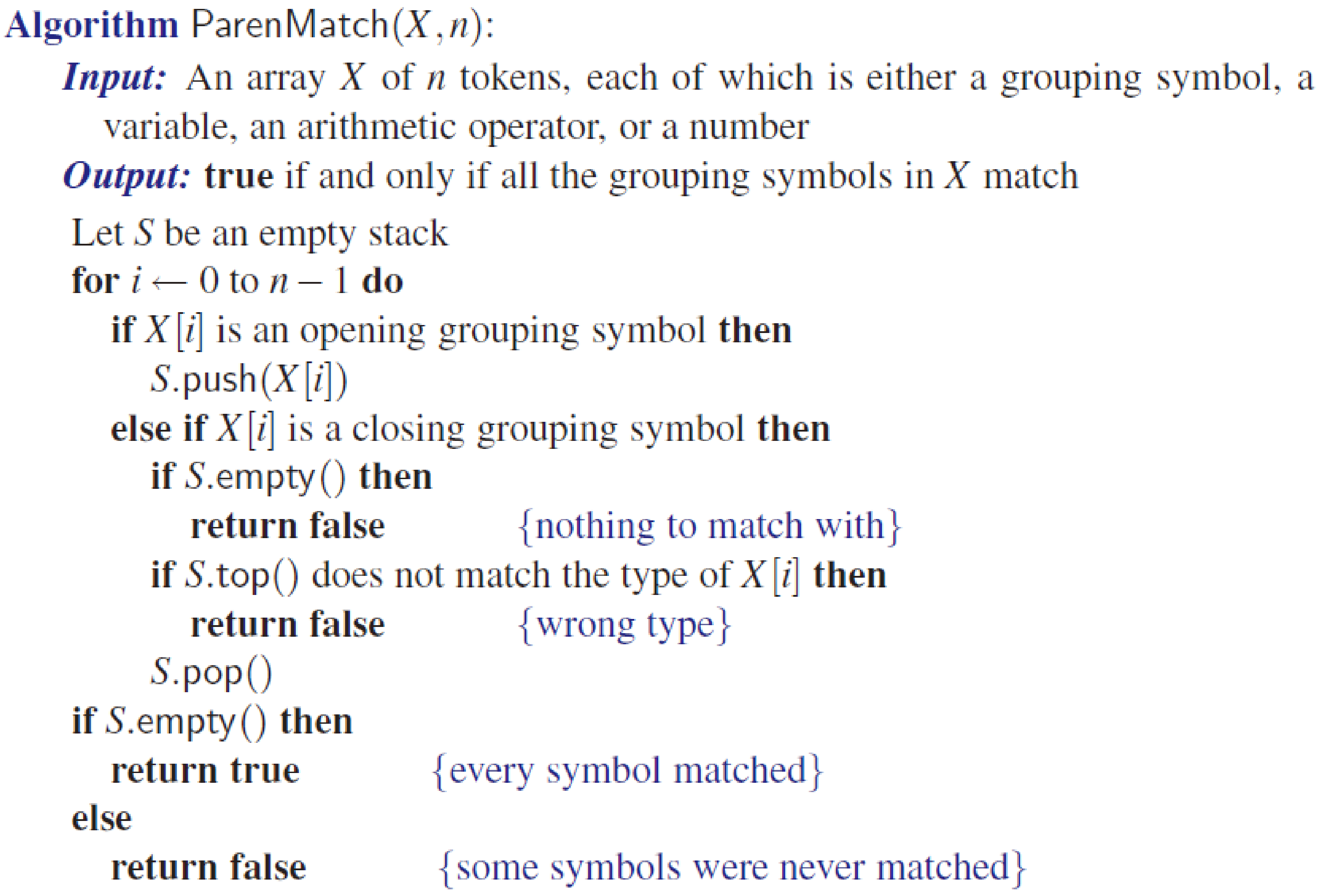
Stack Applications

- **An Algorithm for Parentheses Matching**
  - Suppose we are given a sequence $X = x_0x_1x_2 \ldots x_{n-1}$
  - Each x_i is a token that can be:
    - A grouping symbol
    - A variable name
    - An arithmetic operator
    - A number
  - Push a token when we encounter an opening symbol
  - Pop the top token when we encounter a closing symbol and check the correctness
  - The symbols in X match if the stack is empty after the whole sequence processing

$O(n)$

Stack Applications

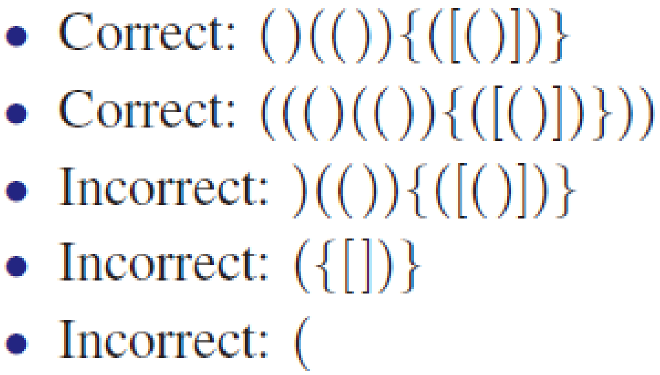
- **An Algorithm for Parentheses Matching**



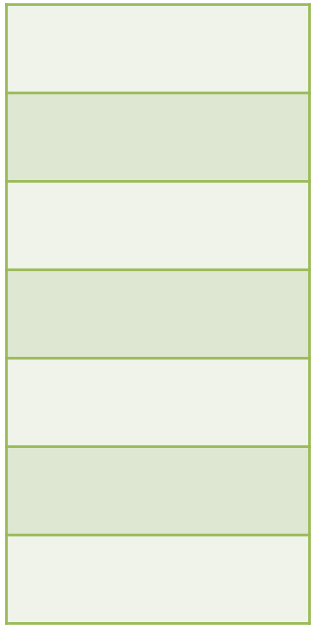
```
Algorithm ParenMatch($X, n$):
    Input: An array $X$ of $n$ tokens, each of which is either a grouping symbol, a
      variable, an arithmetic operator, or a number
    Output: true if and only if all the grouping symbols in $X$ match
    Let $S$ be an empty stack
    for $i \leftarrow 0$ to $n-1$ do
      if $X[i]$ is an opening grouping symbol then
        $S$.push($X[i]$)
      else if $X[i]$ is a closing grouping symbol then
        if $S$.empty() then
          return false          {nothing to match with}
        if $S$.top() does not match the type of $X[i]$ then
          return false          {wrong type}
        $S$.pop()
    if $S$.empty() then
      return true          {every symbol matched}
    else
      return false          {some symbols were never matched}
```

Stack Applications

- **Parentheses Matching Examples**



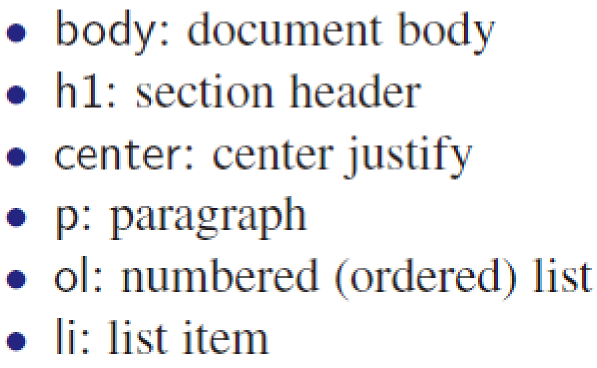
- Correct: ()(()){([()])}
- Correct: ((()(()){([()])}))
- Incorrect:)(()){([()])}
- Incorrect: ({[])}
- Incorrect: (



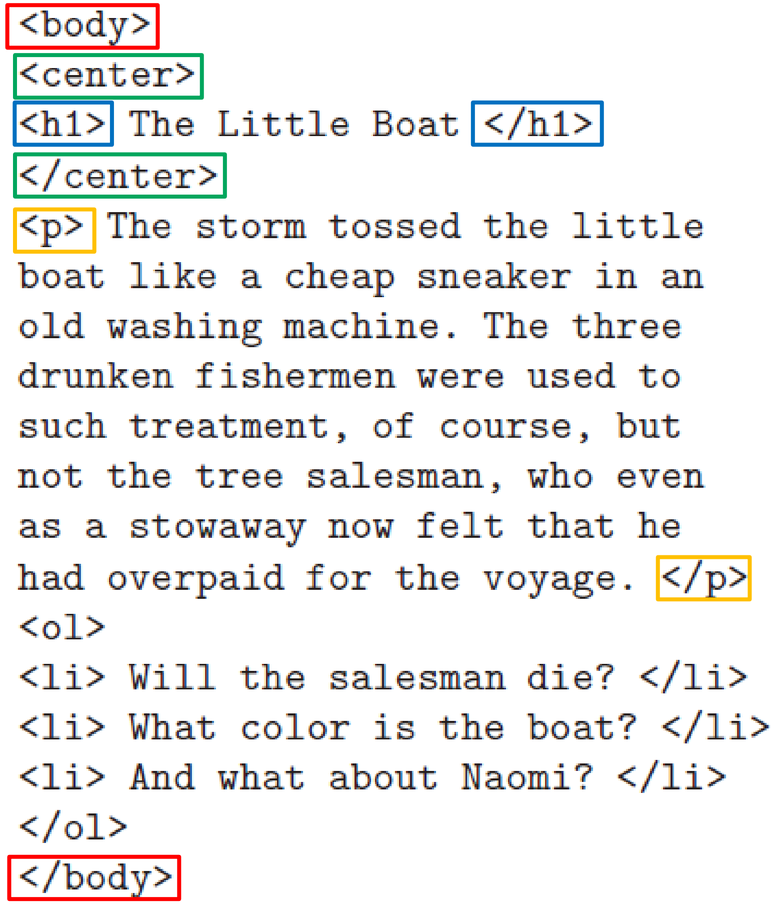
Stack

Stack Applications

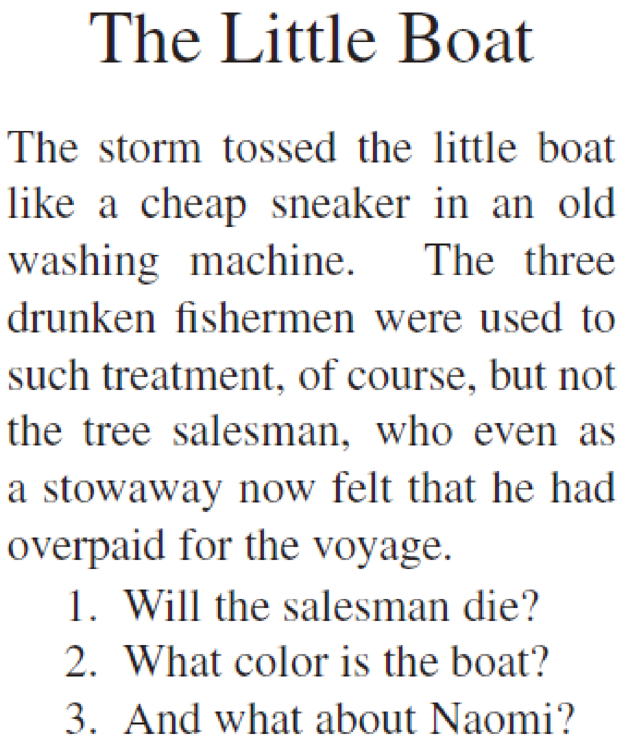
- **Matching Tags in an HTML Document**
  - HTML: HyperText Markup Language
  - An HTML tag consists of opening and closing tags
    - Opening: <name>
    - Closing: </name>



- body: document body
- h1: section header
- center: center justify
- p: paragraph
- ol: numbered (ordered) list
- li: list item



```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even
as a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```



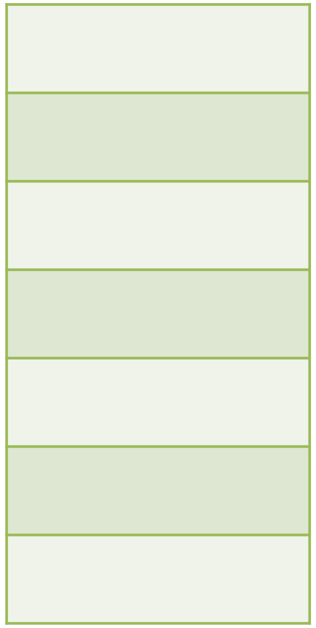
The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

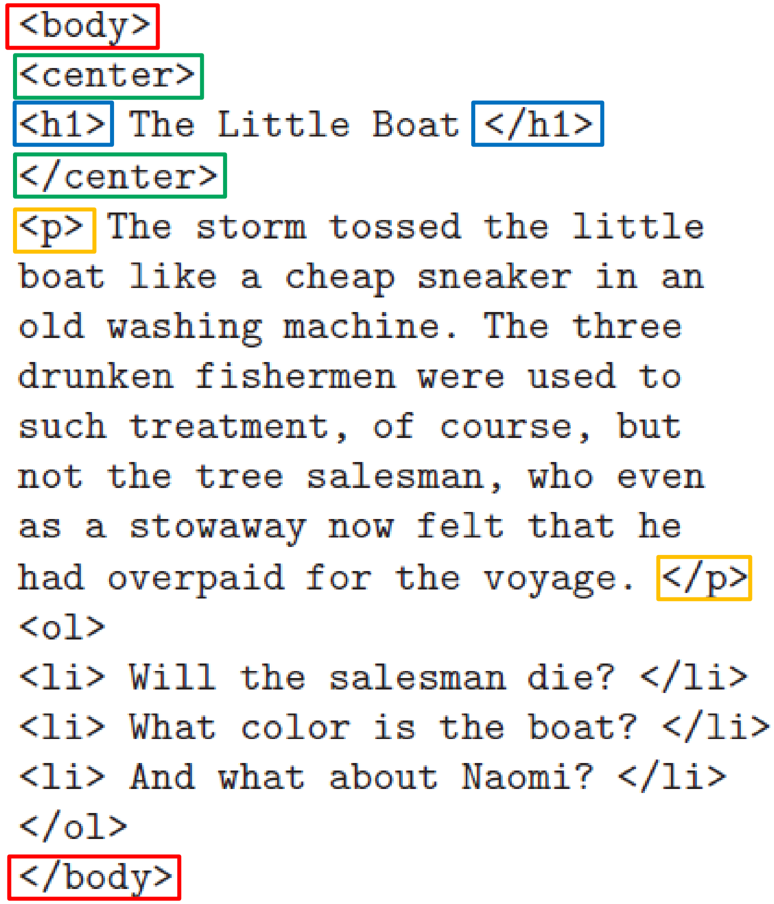
1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

Stack Applications

- **Matching Tags in an HTML Document**
  - Push each opening tag on a stack
  - When we encounter a closing tag
    - pop the stack and verify that the two tags match.



Stack



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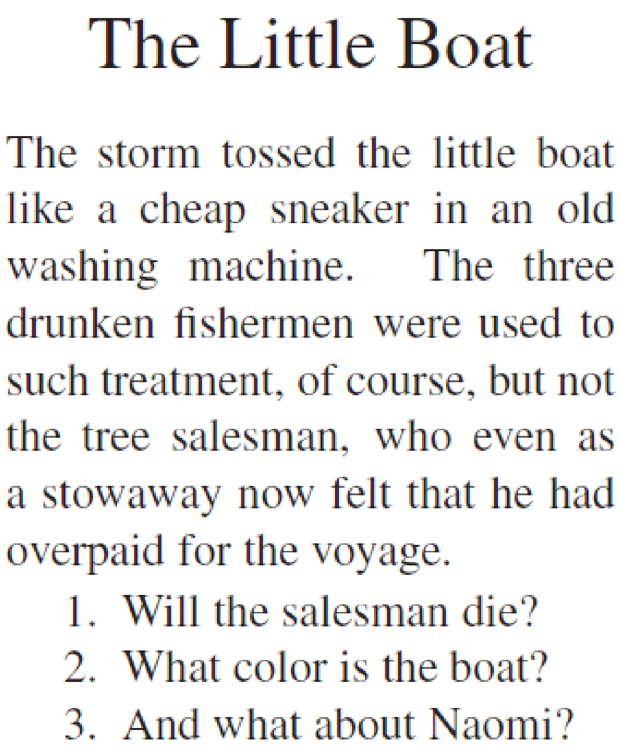
</ol>
</body>
```



The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

Stack Applications

- **Matching Tags in an HTML Document**
  - getHTMLTags()
    - Read the input line by line
    - Extract tags and stores them in a vector

```
vector<string> getHtmlTags() {                     // store tags in a vector
  vector<string> tags;                             // vector of html tags
  while (cin) {                                    // read until end of file
    string line;
    getline(cin, line);                            // input a full line of text
    int pos = 0;                                   // current scan position
    int ts = line.find("<", pos);                  // possible tag start
    while (ts != string::npos) {                   // repeat until end of string
      int te = line.find(">", ts+1);               // scan for tag end
      tags.push_back(line.substr(ts, te-ts+1));    // append tag to the vector
      pos = te + 1;                                // advance our position
      ts = line.find("<", pos);
    }
  }
  return tags;                                     // re
}
```

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
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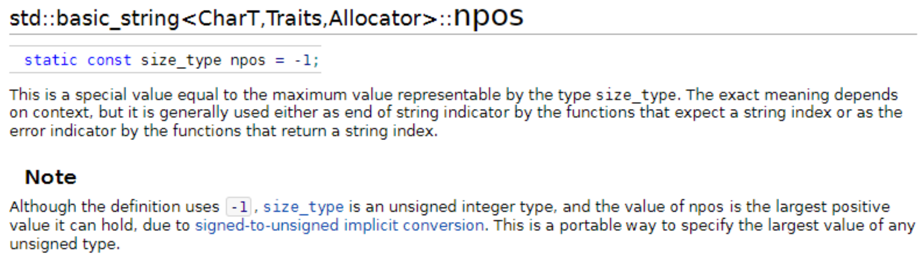
```



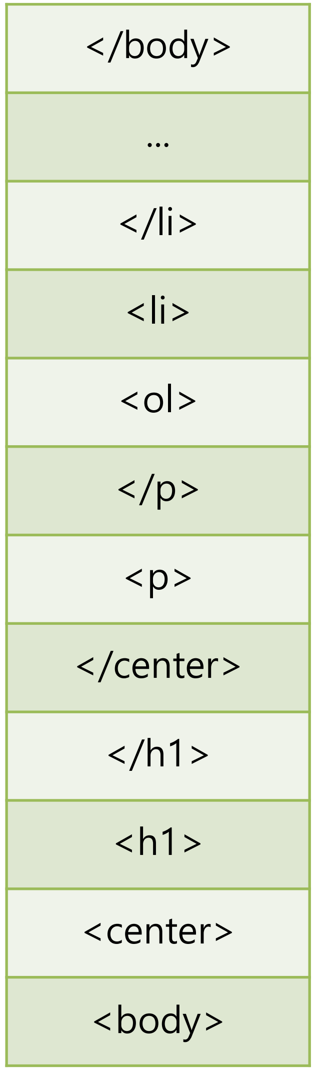
std::basic_string<CharT,Traits,Allocator>::npos

```
static const size_type npos = -1;
```

This is a special value equal to the maximum value representable by the type size_type. The exact meaning depends on context, but it is generally used either as end of string indicator by the functions that expect a string index or as the error indicator by the functions that return a string index.

Note

Although the definition uses -1, size_type is an unsigned integer type, and the value of npos is the largest positive value it can hold, due to signed-to-unsigned implicit conversion. This is a portable way to specify the largest value of any unsigned type.



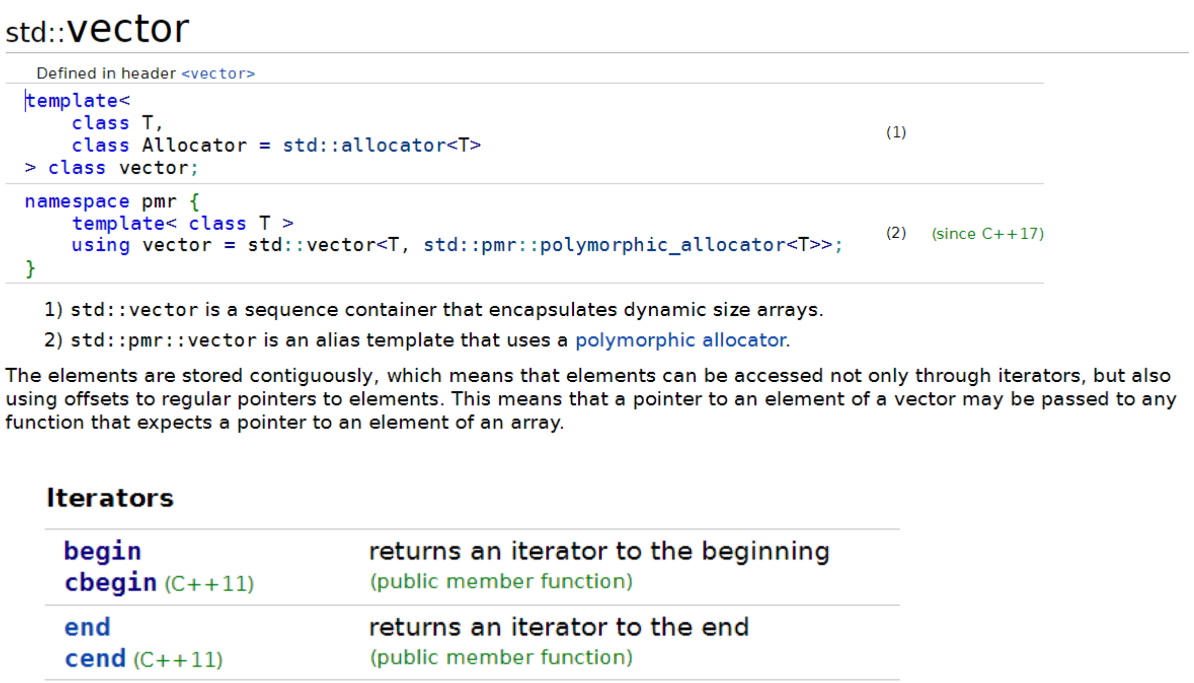
Vector

https://en.cppreference.com/w/cpp/string/basic_string/npos

Stack Applications

- **Matching Tags in an HTML Document**

```
                                            // check for matching tags
bool isHtmlMatched(const vector<string>& tags) {
  LinkedStack S;                            // stack for opening tags
  typedef vector<string>::const_iterator Iter;// iterator type
                                            // iterate through vector
  for (Iter p = tags.begin(); p != tags.end(); ++p) {
    if (p->at(1) != ’/’)                    // opening tag?
      S.push(*p);                           // push it on the stack
    else {                                  // else must be closing tag
      if (S.empty()) return false;          // nothing to match - failure
      string open = S.top().substr(1);      // opening tag excluding '<'
      string close = p->substr(2);          // closing tag excluding '</'
      if (open.compare(close) != 0) return false; // fail to match
      else S.pop();                         // pop matched element
    }
  }
  if (S.empty()) return true;               // everything matched - good
  else return false;                        // some unmatched - bad
}
```



std::vector

Defined in header `<vector>`

```
template<
    class T,
    class Allocator = std::allocator<T>
> class vector;                                              (1)
```

```
namespace pmr {
    template< class T >
    using vector = std::vector<T, std::pmr::polymorphic_allocator<T>>;   (2) (since C++17)
}
```

1) `std::vector` is a sequence container that encapsulates dynamic size arrays.

2) `std::pmr::vector` is an alias template that uses a polymorphic allocator.

The elements are stored contiguously, which means that elements can be accessed not only through iterators, but also using offsets to regular pointers to elements. This means that a pointer to an element of a vector may be passed to any function that expects a pointer to an element of an array.

Iterators

begin **cbegin** (C++11)	returns an iterator to the beginning (public member function)
end **cend** (C++11)	returns an iterator to the end (public member function)