JAWABAN MATRIKULASI

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Dokumen ini berisi jawaban dari latihan soal Matrikulasi Dasar Institut Teknologi Kalimantan tahun 2024. Adapun pembuatan dokumen ini sebagai bahan percobaan saya untuk mempelajari bahasa LaTex yang cukup berguna. Berhubung saat ini sedang diadakan Matrikulasi Dasar di Institut Teknologi Kalimantan tahun 2024, maka saya akan mencoba menjawab setiap soal latihan yang ada dengan menjawabnya melalui LaTex. Seluruh latihan soal yang berada di dokumen ini diambil berdasarkan latihan soal dari Matrikulasi Dasar Institut Teknologi Kalimantan tahun 2024. Sedikit catatan, beberapa dari hasil jawaban yang ada, ada yang disederhanakan, namun ada juga yang tidak disederhanakan, dan kebanyakan jawaban yang ada tidak disederhanakan :). Saya mohon maaf jika ada jawaban yang salah ataupun kurang tepat.

Contents

1 Latihan soal untuk Operasi Bilangan Real.

1. Diberikan daftar bilangan real berikut.

$$\left\{ 0, -10, 50, \frac{22}{7}, 0.538, \sqrt{7}, 1.23, -\frac{1}{3}, \sqrt[3]{2}, \pi, 11, \frac{13}{15}, \sqrt{16}, 3.14 \right\}$$

Klasifikasikan bilangan tersebut yang mana merupakan:

- (a) Bilangan Bulat
- (b) Bilangan Asli
- (c) Bilangan Rasional
- (d) Bilangan Irrasional

JAWABAN:

- (a) Yang Bilangan Bulat adalah:

$$\left\{ 0, -10, 50, 11, \sqrt{16} \rightarrow (\text{karena} \sqrt{16} = 4 \text{ dan } 4 \text{ adalah bilangan bulat}) \right\}$$

- (b) Yang Bilangan Asli adalah:

$$\left\{ 50, 11, \sqrt{16} \rightarrow (\text{karena} \sqrt{16} = 4 \text{ dan } 4 \text{ adalah bilangan bulat positif}) \right\}$$

- (c) Yang Bilangan Rasional adalah:

$$\left\{ \frac{22}{7}, 0.538, 1.23, -\frac{1}{3}, \frac{13}{15}, 3.14 \right\}$$

- (d) Yang Bilangan Irrasional adalah:

$$\left\{ \sqrt{7}, \sqrt[3]{2}, \pi \right\}$$

2. Operasikan hasil bilangan bulat berikut:

- (a) $3 + 5 \cdot (-1)$
- (b) $1 - 3 \cdot 7$
- (c) $-1 \cdot (-2 + 3)$
- (d) $(-2 + 3) \cdot 7$

JAWABAN:

- (a) $3 + 5 \cdot (-1)$
  $\rightarrow 3 + (-5)$
  $\rightarrow 3 - 5$
  $\rightarrow -2$

- (b) $1 - 3 \cdot 7$
  $\rightarrow 1 - 21$
  $\rightarrow -20$

- (c) $-1 \cdot (-2 + 3)$
  $\rightarrow 2 + (-3)$
  $\rightarrow 2 - 3$
  $\rightarrow -1$

- (d) $(-2 + 3) \cdot 7$
  $\rightarrow -14 + 21$
  $\rightarrow 7$

3. Operasikan dan sederhanakan pecahan berikut:

(a) $\dfrac{1}{7} + \dfrac{4}{15}$

(b) $\dfrac{1}{4} + \dfrac{1}{5}$

(c) $\dfrac{2}{3} - \dfrac{3}{5}$

(d) $\dfrac{5}{3} - \dfrac{1}{6}$

(e) $1 + \dfrac{2}{3}$

(f) $2 - \dfrac{1}{7}$

(g) $1 + \dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{4}$

(h) $\left(\dfrac{1}{2} - \dfrac{1}{3}\right) \cdot \left(\dfrac{1}{2} + \dfrac{1}{3}\right)$

(i) $\dfrac{2}{\frac{2}{3}} - \dfrac{\frac{2}{3}}{2}$

(j) $\dfrac{2}{3} \cdot \dfrac{4}{7}$

(k) $\dfrac{3}{5} \div \dfrac{7}{6}$

(l) $\dfrac{1}{2} + \dfrac{1}{3} \cdot \dfrac{1}{5}$

JAWABAN:

(a) $\dfrac{1}{7} + \dfrac{4}{15}$

$\rightarrow \dfrac{15 + 28}{105}$

$\rightarrow \dfrac{43}{105}$

(b) $\dfrac{1}{4} + \dfrac{1}{5}$

$\rightarrow \dfrac{5 + 4}{20}$

$\rightarrow \dfrac{9}{20}$

(c) $\dfrac{2}{3} - \dfrac{3}{5}$

$\rightarrow \dfrac{10 - 9}{15}$

$\rightarrow \dfrac{1}{15}$

(d) $\dfrac{5}{3} - \dfrac{1}{6}$

$\rightarrow \dfrac{30 - 3}{18}$

$\rightarrow \dfrac{27}{18} = \dfrac{3}{2}$

(e) $1 + \dfrac{2}{3}$

$\rightarrow \dfrac{1}{1} + \dfrac{2}{3}$

$\rightarrow \dfrac{3 + 2}{3}$

$\rightarrow \dfrac{5}{3}$

(f) $2 - \dfrac{1}{7}$

$\rightarrow \dfrac{2}{1} - \dfrac{1}{7}$

$\rightarrow \dfrac{14 - 1}{7}$

$\rightarrow \dfrac{13}{7}$

(g) $1 + \dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{4}$

$\rightarrow \left(\dfrac{1}{1} + \dfrac{1}{2}\right) - \dfrac{1}{3} + \dfrac{1}{4}$

$\rightarrow \left(\dfrac{2 + 1}{2}\right) - \left(\dfrac{4 + 3}{12}\right)$

$\rightarrow \dfrac{3}{2} - \dfrac{7}{12}$

$\rightarrow \dfrac{36 - 7}{12}$

$\rightarrow \dfrac{22}{24} = \dfrac{11}{12}$

(h) $\left(\dfrac{1}{2} - \dfrac{1}{3}\right) \cdot \left(\dfrac{1}{2} + \dfrac{1}{3}\right)$

$\rightarrow \left(\dfrac{3 - 2}{6}\right) - \left(\dfrac{3 + 2}{6}\right)$

$\rightarrow \dfrac{1}{6} \cdot \dfrac{5}{6} = \dfrac{5}{36}$

(i) $\dfrac{2}{\frac{2}{3}} - \dfrac{\frac{2}{3}}{2}$

$\rightarrow \left(\dfrac{2}{1} \div \dfrac{2}{3}\right) - \left(\dfrac{2}{3} \div \dfrac{2}{1}\right)$

$\rightarrow \left(\dfrac{2}{1} \cdot \dfrac{3}{2}\right) - \left(\dfrac{2}{3} \cdot \dfrac{1}{2}\right)$

$\rightarrow \dfrac{6}{2} - \dfrac{2}{6}$

$\rightarrow \dfrac{36 - 4}{12} = \dfrac{32}{12} = \dfrac{8}{3}$

(j) $\dfrac{2}{3} \cdot \dfrac{4}{7}$

$\rightarrow \dfrac{8}{21}$

(k) $\dfrac{3}{5} \div \dfrac{7}{6}$

$\rightarrow \dfrac{3}{5} \cdot \dfrac{6}{7}$

$\rightarrow \dfrac{18}{35}$

(l) $\dfrac{1}{2} + \dfrac{1}{3} \cdot \dfrac{1}{5}$

$\rightarrow \dfrac{1}{2} + \left(\dfrac{1}{3} \cdot \dfrac{1}{5}\right)$

$\rightarrow \dfrac{1}{2} + \dfrac{1}{15}$

$\rightarrow \dfrac{15 + 2}{30}$

$\rightarrow \dfrac{17}{30}$

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4. Tentukan hasil dari perkalian polinom berikut:

(a) $3 \cdot (2x - 7)$

(b) $(x - 1) \cdot (x + 1)$

(c) $(x + 3) \cdot (-x + 2)$

(d) $(2x - 3) \cdot (x + 3)$

(e) $(x^2 - 1) \cdot (x + 7)$

(f) $(x^2 + 2) \cdot (x^2 + x + 1)$

JAWABAN:

(a) $3 \cdot (2x - 7)$

$\rightarrow 3 \cdot 2x - 3 \cdot 7$

$\rightarrow 6x - 21$

(b) $(x - 1) \cdot (x + 1)$

$\rightarrow x \cdot x + x \cdot 1 + -1 \cdot x + -1 \cdot 1$

$\rightarrow x^2 + x - x - 1$

$\rightarrow x^2 - 1$

(c) $(x + 3) \cdot (-x + 2)$

$\rightarrow x \cdot -x + x \cdot 2 + 3 \cdot -x + 3 \cdot 2$

$\rightarrow -x^2 + 2x - 3x + 6$

$\rightarrow -x^2 - x + 6$

$\rightarrow -(-x^2 - x + 6)$

$\rightarrow x^2 + x - 6$

(d) $(2x - 3) \cdot (x + 3)$

$\rightarrow 2x \cdot x + 2x \cdot 3 + -3 \cdot x + -3 \cdot 3$

$\rightarrow 2x^2 + 6x - 3x - 9$

$\rightarrow 2x^2 + 3x - 9$

(e) $(x^2 - 1) \cdot (x + 7)$

$\rightarrow x^2 \cdot x + x^2 \cdot 7 + -1 \cdot x + -1 \cdot 7$

$\rightarrow x^3 + 7x^2 - x - 7$

(f) $(x^2 + 2) \cdot (x^2 + x + 1)$

$\rightarrow x^2 \cdot x^2 + x^2 \cdot x + x^2 \cdot 1 + 2 \cdot x^2 + 2 \cdot x + 2 \cdot 1$

$\rightarrow x^4 + x^3 + x^2 + 2x^2 + 2x + 2$

$\rightarrow x^4 + x^3 + 3x^2 + 2x + 2$

2 Latihan Soal untuk Persamaan Linear dan Kuadratik.

1. Selesaikan Persamaan Linear Berikut:

(a) $2x + 7 = 31$

(b) $5x - 3 = 4$

(c) $\frac{1}{2}x - 1 = 9$

(d) $3 + \frac{1}{3}x = 5$

(e) $-7w = 15 - 2w$

(f) $5t - 13 = 12 - 5t$

(g) $\frac{1}{2}y - 2 = \frac{1}{3}y$

(h) $\frac{z}{5} = \frac{3}{10}z + 7$

(i) $2(1 - x) = 3(1 + 2x) + 5$

(j) $\frac{2}{3}y + \frac{1}{2}(y - 3) = \frac{y + 1}{4}$

(k) $x - \frac{1}{3}x - \frac{1}{2}x - 5 = 0$

(l) $2x - \frac{x}{2} + \frac{x + 1}{4} = 6x$

JAWABAN:

(a) $2x + 7 = 31$

$\rightarrow 2x + 7 - 7 = 31 - 7$

$\rightarrow 2x = 24$

$\rightarrow \frac{2x}{2} = \frac{24}{2}$

$\rightarrow x = 12$

(b) $5x - 3 = 4$

$\rightarrow 5x - 3 + 3 = 4 + 3$

$\rightarrow 5x = 7$

$\rightarrow \frac{5x}{5} = \frac{7}{5}$

$\rightarrow x = \frac{7}{5}$

(c) $\frac{1}{2}x - 1 = 9$

$\rightarrow \frac{1}{2}x - 1 + 1 = 9 + 1$

$\rightarrow \frac{1}{2}x = 10$

$\rightarrow \frac{\frac{1}{2}x}{\frac{1}{2}} = \frac{10}{\frac{1}{2}}$

$\rightarrow x = \frac{10}{1} \cdot \frac{2}{1} = 20$

(d) $3 + \frac{1}{3}x = 5$

$\rightarrow 3 + \frac{1}{3}x - 3 = 5 - 3$

$\rightarrow \frac{1}{3}x = 2$

$\rightarrow \frac{\frac{1}{3}x}{\frac{1}{3}} = \frac{2}{\frac{1}{3}}$

$\rightarrow x = \frac{2}{1} \cdot \frac{3}{1} = 6$

(e) $-7w = 15 - 2w$

$\rightarrow -7w + 2w = 15 - 2w + 2w$

$\rightarrow -5w = 15$

$\rightarrow \frac{-5w}{5} = \frac{15}{5}$

$\rightarrow -(-w) = -(3)$

$\rightarrow w = -3$

(f) $5t - 13 = 12 - 5t$

$\rightarrow 5t - 13 + 5t = 12 - 5t + 5t$

$\rightarrow 10t - 13 = 12$

$\rightarrow 10t - 13 + 13 = 12 + 13$

$\rightarrow 10t = 25$

$\rightarrow \frac{10t}{10} = \frac{25}{10}$

$\rightarrow t = \frac{25}{10} = \frac{5}{2}$

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(g) $\dfrac{1}{2}y - 2 = \dfrac{1}{3}y$

$\rightarrow \dfrac{1}{2}y - \dfrac{1}{3}y - 2 = \dfrac{1}{3}y - \dfrac{1}{3}y$

$\rightarrow \dfrac{1}{6}y - 2 = 0$

$\rightarrow \dfrac{1}{6}y - 2 + 2 = 0 + 2$

$\rightarrow \dfrac{1}{6}y = 2$

$\rightarrow \dfrac{\frac{1}{6}y}{\frac{1}{6}} = \dfrac{2}{\frac{1}{6}}$

$\rightarrow y = \dfrac{2}{1} \cdot \dfrac{6}{1} = 12$

(h) $\dfrac{z}{5} = \dfrac{3}{10}z + 7$

$\rightarrow \dfrac{\frac{z}{5}}{\frac{3}{10}} = \dfrac{\frac{3}{10}z}{\frac{3}{10}} + \dfrac{7}{\frac{3}{10}}$

$\rightarrow \dfrac{z}{5} \cdot \dfrac{10}{3} = z + \dfrac{7}{1} \cdot \dfrac{10}{3}$

$\rightarrow \dfrac{10z}{15} = z + \dfrac{70}{3}$

$\rightarrow \dfrac{10z}{15} - z = z - z + \dfrac{70}{3}$

$\rightarrow \dfrac{10z}{15} - \dfrac{z}{1} = \dfrac{70}{3}$

$\rightarrow \dfrac{10z - 15z}{15} = \dfrac{70}{3}$

$\rightarrow -\dfrac{5}{15}z = \dfrac{70}{3}$

$\rightarrow -\dfrac{1}{3}z = \dfrac{70}{3}$

$\rightarrow \dfrac{-\frac{1}{3}z}{-\frac{1}{3}} = \dfrac{\frac{70}{3}}{-\frac{1}{3}}$

$\rightarrow z = \dfrac{70}{3} \cdot -\dfrac{3}{1} = -\dfrac{210}{3} = -70$

$\rightarrow z = -70$

(i) $2(1 - x) = 3(1 + 2x) + 5$

$\rightarrow 2 - 2x = 3 + 6x + 5$

$\rightarrow 2 - 2x = 6x + 8$

$\rightarrow 2 - 2x - 2 = 6x + 8 - 2$

$\rightarrow -2x = 6x + 6$

$\rightarrow \dfrac{-2x}{2} = \dfrac{6x}{2} + \dfrac{6}{2}$

$\rightarrow -x = 3x + 3$

$\rightarrow -x + x = 3x + 3 + x$

$\rightarrow 4x + 3 = 0$

$\rightarrow 4x + 3 - 3 = 0 - 3$

$\rightarrow 4x = -3$

$\rightarrow \dfrac{4x}{4} = \dfrac{-3}{4}$

$\rightarrow x = -\dfrac{3}{4}$

(j) $\dfrac{2}{3}y + \dfrac{1}{2}(y - 3) = \dfrac{y + 1}{4}$

$\rightarrow \dfrac{2}{3}y + \dfrac{1}{2}y - \dfrac{3}{2} = \dfrac{y + 1}{4}$

$\rightarrow \dfrac{4 + 3}{6}y - \dfrac{3}{2} = \dfrac{y}{4} + \dfrac{1}{4}$

$\rightarrow \dfrac{7}{6}y - \dfrac{3}{2} + \dfrac{3}{2} = \dfrac{y}{4} + \dfrac{1}{4} + \dfrac{3}{2}$

$\rightarrow \dfrac{7}{6}y = \dfrac{y}{4} + \dfrac{2 + 12}{8}$

$\rightarrow \dfrac{7}{6}y = \dfrac{y}{4} + \dfrac{14}{8}$

$\rightarrow \dfrac{7}{6}y = \dfrac{y}{4} + \dfrac{7}{4}$

$\rightarrow \dfrac{7}{6}y - \dfrac{7}{6}y = \dfrac{y}{4} - \dfrac{7y}{6} + \dfrac{7}{4}$

$\rightarrow \dfrac{6y - 28y}{24} + \dfrac{7}{4} = 0$

$\rightarrow -\dfrac{22}{24}y = -\dfrac{7}{4}$

$\rightarrow -\dfrac{11}{12}y = -\dfrac{7}{4}$

$\rightarrow \dfrac{-\frac{11}{12}y}{-\frac{11}{12}} = \dfrac{-\frac{7}{4}}{-\frac{11}{12}}$

$\rightarrow y = -\dfrac{7}{4} \cdot -\dfrac{12}{11} = \dfrac{84}{44}$

$\rightarrow y = \dfrac{84}{44} = \dfrac{42}{22} = \dfrac{21}{11}$

(k) $\quad x - \dfrac{1}{3}x - \dfrac{1}{2}x - 5 = 0$

$\quad \rightarrow \left(\dfrac{x}{1} - \dfrac{x}{3} \right) - \dfrac{1}{2}x - 5 = 0$

$\quad \rightarrow \left(\dfrac{3x - x}{3} \right) - \dfrac{1}{2}x - 5 = 0$

$\quad \rightarrow \dfrac{2}{3}x - \dfrac{1}{2}x - 5 = 0$

$\quad \rightarrow \left(\dfrac{2x}{3} - \dfrac{x}{2} \right) - 5 = 0$

$\quad \rightarrow \left(\dfrac{4x - 3x}{6} \right) - 5 = 0$

$\quad \rightarrow \dfrac{1}{6}x - 5 = 0$

$\quad \rightarrow \dfrac{1}{6}x - 5 + 5 = 0 + 5$

$\quad \rightarrow \dfrac{1}{6}x = 5$

$\quad \rightarrow \dfrac{\frac{1}{6}x}{\frac{1}{6}} = \dfrac{5}{\frac{1}{6}}$

$\quad \rightarrow x = \dfrac{5}{1} \cdot \dfrac{6}{1} = \dfrac{30}{1}$

$\quad \rightarrow x = 30$

(l) $\quad 2x - \dfrac{x}{2} + \dfrac{x + 1}{4} = 6x$

$\quad \rightarrow \dfrac{2x}{1} - \dfrac{x}{2} + \dfrac{x}{4} + \dfrac{1}{4} = 6x$

$\quad \rightarrow \left(\dfrac{2x}{1} - \dfrac{x}{2} \right) + \dfrac{x}{4} + \dfrac{1}{4} = 6x$

$\quad \rightarrow \left(\dfrac{4x - x}{2} \right) + \dfrac{x}{4} + \dfrac{1}{4} = 6x$

$\quad \rightarrow \dfrac{3}{2}x + \dfrac{x}{4} + \dfrac{1}{4} = 6x$

$\quad \rightarrow \left(\dfrac{3x}{2} + \dfrac{x}{4} \right) + \dfrac{1}{4} = 6x$

$\quad \rightarrow \left(\dfrac{12x + 2x}{8} \right) + \dfrac{1}{4} = 6x$

$\quad \rightarrow \dfrac{14}{8}x + \dfrac{1}{4} = 6x$

$\quad \rightarrow \dfrac{14}{8}x + \dfrac{1}{4} - \dfrac{1}{4} = 6x - \dfrac{1}{4}$

$\quad \rightarrow \dfrac{14}{8}x = 6x - \dfrac{1}{4}$

$\quad \rightarrow \dfrac{14}{8}x - 6x = 6x - \dfrac{1}{4} - 6x$

$\quad \rightarrow \dfrac{14x}{8} - \dfrac{6x}{1} = -\dfrac{1}{4}$

$\quad \rightarrow \dfrac{14x - 48x}{8} = -\dfrac{1}{4}$

$\quad \rightarrow -\dfrac{34}{8}x = -\dfrac{1}{4}$

$\quad \rightarrow \dfrac{-\frac{34}{8}x}{-\frac{34}{8}} = \dfrac{-\frac{1}{4}}{-\frac{34}{8}}$

$\quad \rightarrow x = -\dfrac{1}{4} \cdot -\dfrac{8}{34} = \dfrac{8}{136}$

$\quad \rightarrow x = \dfrac{8}{136} = \dfrac{1}{17}$

2. Selesaikan Persamaan untuk variabel yang diinginkan:

(a) $PV = nRT$ untuk R.

(b) $F = G\dfrac{mM}{r^2}$ untuk m.

(c) $P = 2\ell + 2w$ untuk ℓ.

(d) $a^2 x + (a-1) = (a+1)x$ untuk x.

(e) $V = \dfrac{1}{3}\pi r^2 h$ untuk r.

(f) $a^2 + b^2 = c^2$ untuk b.

JAWABAN:

(a) $PV = nRT$ untuk R.
$\rightarrow PV = nT \quad [R] \text{ (Isolasi } R)$
$\rightarrow \dfrac{PV}{nT} = \dfrac{nT}{nT} \quad [R]$
$\rightarrow \dfrac{PV}{nT} = 1 \quad [R]$
$\rightarrow \dfrac{PV}{nT} = 1 \cdot R$
$\rightarrow 1 \cdot R = \dfrac{PV}{nT}$
$\rightarrow R = \dfrac{PV}{nT}$

(b) $F = G\dfrac{mM}{r^2}$ untuk m.
$\rightarrow F = G\dfrac{M}{r^2} \quad [m] \text{ (Isolasi } m)$
$\rightarrow F = \dfrac{GM}{r^2} \quad [m]$
$\rightarrow F \cdot \dfrac{r^2}{GM} = \dfrac{GM}{r^2} \cdot \dfrac{r^2}{GM} \quad [m]$
$\rightarrow \dfrac{Fr^2}{GM} = 1 \cdot m$
$\rightarrow m = \dfrac{Fr^2}{GM}$

(c) $P = 2\ell + 2w$ untuk ℓ.
$\rightarrow P = 2w \quad [+2\ell] \text{ (Isolasi } 2\ell)$
$\rightarrow P - 2w = 2w - 2w \quad [+2\ell]$
$\rightarrow P - 2w = 0 \quad [+2\ell]$
$\rightarrow P - 2w = 0 + 2\ell$
$\rightarrow P - 2w = 2\ell$
$\rightarrow 2\ell = P - 2w$
$\rightarrow \dfrac{2\ell}{2} = \dfrac{P - 2w}{2}$
$\rightarrow 1 \cdot \ell = \dfrac{P - 2w}{2}$
$\rightarrow \ell = \dfrac{P - 2w}{2}$

(d) $a^2 x + (a-1) = (a+1)x$ untuk x.
$\rightarrow a^2 x - a^2 x + (a-1) = (a+1)x - a^2 x$
$\rightarrow 0 + (a-1) = ax + x - a^2 x$
$\rightarrow (a-1) = -a^2 x + ax + x$
$\rightarrow (a-1) = -a^2 + a + 1 \quad [x] \text{ (Isolasi } x)$
$\rightarrow \dfrac{(a-1)}{-a^2 + a + 1} = \dfrac{-a^2 + a + 1}{-a^2 + a + 1} \quad [x]$
$\rightarrow \dfrac{(a-1)}{-a^2 + a + 1} = 1 \quad [x]$
$\rightarrow \dfrac{(a-1)}{-a^2 + a + 1} = 1 \cdot x$
$\rightarrow \dfrac{(a-1)}{-a^2 + a + 1} = x$
$\rightarrow x = \dfrac{(a-1)}{-a^2 + a + 1}$

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(e) $V = \dfrac{1}{3}\pi r^2 h$ untuk r.

$\rightarrow V = \dfrac{1}{3}\pi h \quad [r^2]$ (Isolasi r^2)

$\rightarrow V = \dfrac{1 \cdot \pi \cdot h}{3} \quad [r^2]$

$\rightarrow V = \dfrac{\pi h}{3} \quad [r^2]$

$\rightarrow V \cdot \dfrac{3}{\pi h} = \dfrac{\pi h}{3} \cdot \dfrac{3}{\pi h} \quad [r^2]$

$\rightarrow \dfrac{3V}{\pi h} = 1.r^2$

$\rightarrow 1.r^2 = \dfrac{3V}{\pi h}$

$\rightarrow r^2 = \dfrac{3V}{\pi h}$

$\rightarrow \sqrt{r^2} = \sqrt{\dfrac{3V}{\pi h}}$

$\rightarrow r = \sqrt{\dfrac{3V}{\pi h}}$

(f) $a^2 + b^2 = c^2$ untuk b.

$\rightarrow a^2 = c^2 \quad [+b^2]$ (Isolasi b^2)

$\rightarrow a^2 - a^2 = c^2 - a^2 \quad [+b^2]$

$\rightarrow c^2 - a^2 = 0 \quad [+b^2]$

$\rightarrow c^2 - a^2 = 0 + b^2$

$\rightarrow 0 + b^2 = c^2 - a^2$

$\rightarrow b^2 = c^2 - a^2$

$\rightarrow \sqrt{b^2} = \sqrt{c^2 - a^2}$

$\rightarrow b = \sqrt{c^2 - a^2}$

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3. Selesaikan persamaan kuadratik berikut dengan cara memfaktorkan:

(a) $x^2 + x - 12 = 0$

(b) $x^2 + 3x - 4 = 0$

(c) $x^2 - 7x + 10 = 0$

(d) $x^2 + 8x + 12 = 0$

(e) $4x^2 - 4x - 15 = 0$

(f) $2y^2 + 7y + 3 = 0$

(g) $3x^2 + 5x = 2$

(h) $2x^2 = 8$

JAWABAN:

(a) $x^2 + x - 12 = 0$
$\to a = 1, b = 1, c = -12$
$\to$ (2 bilangan yang jika $[+]$ = b
    dan jika $[\times]$ = c.)
$\to (x + 4) \mid (x - 3)$
$\to x + 4 = 0 \mid x - 3 = 0$
$\to x + 4 - 4 = 0 - 4 \mid x - 3 + 3 = 0 + 3$
$\to x_1 = -4 \mid x_2 = 3$

(b) $x^2 + 3x - 4 = 0$
$\to a = 1, b = 3, c = -4$
$\to$ (2 bilangan yang jika $[+]$ = b
    dan jika $[\times]$ = c.)
$\to (x - 1) \mid (x + 4)$
$\to x - 1 = 0 \mid x + 4 = 0$
$\to x - 1 + 1 = 0 + 1 \mid x + 4 - 4 = 0 - 4$
$\to x_1 = 1 \mid x_2 = -4$

(c) $x^2 - 7x + 10 = 0$
$\to a = 1, b = -7, c = 10$
$\to$ (2 bilangan yang jika $[+]$ = b
    dan jika $[\times]$ = c.)
$\to (x - 2) \mid (x - 5)$
$\to x - 2 = 0 \mid x - 5 = 0$
$\to x - 2 + 2 = 0 + 2 \mid x - 5 + 5 = 0 + 5$
$\to x_1 = 2 \mid x_2 = 5$

(d) $x^2 + 8x + 12 = 0$
$\to a = 1, b = 8, c = 12$
$\to$ (2 bilangan yang jika $[+]$ = b
    dan jika $[\times]$ = c.)
$\to (x + 2) \mid (x + 6)$
$\to x + 2 = 0 \mid x + 6 = 0$
$\to x + 2 - 2 = 0 - 2 \mid x + 6 - 6 = 0 - 6$
$\to x_1 = -2 \mid x_2 = -6$

(e) $4x^2 - 4x - 15 = 0$
$\to x^2 - 4x - 15 \cdot 4 = 0$
$\to x^2 - 4x - 60 = 0$
$\to a = 1, b = -4, c = -60$
$\to$ (2 bilangan yang jika $[+]$ = b
    dan jika $[\times]$ = c.)
$\to (x + 6) \mid (x - 10)$
$\to x + 6 = 0 \mid x - 10 = 0$
$\to x + 6 - 6 = 0 - 6 \mid x - 10 + 10 = 0 + 10$
$\to x_1 = -6 \mid x_2 = 10$

(f) $2y^2 + 7y + 3 = 0$
$\to y^2 + 7y + 3 \cdot 2 = 0$
$\to y^2 + 7y + 6 = 0$
$\to a = 1, b = 7, c = 6$
$\to (x + 1) \mid (x + 6)$
$\to x + 1 = 0 \mid x + 6 = 0$
$\to x + 1 - 1 = 0 - 1 \mid x + 6 - 6 = 0 - 6$
$\to x_1 = -1 \mid x_2 = -6$

(g) $3x^2 + 5x = 2$

$\rightarrow 3x^2 + 5x - 2 = 2 - 2$

$\rightarrow 3x^2 + 5x - 2 = 0$

$\rightarrow x^2 + 5x - 2 \cdot 3 = 0$

$\rightarrow x^2 + 5x - 6 = 0$

$\rightarrow a = 1, b = 5, c = -6$

$\rightarrow (x - 1) \mid (x + 6)$

$\rightarrow x - 1 = 0 \mid x + 6 = 0$

$\rightarrow x - 1 + 1 = 0 + 1 \mid x + 6 - 6 = 0 - 6$

$\rightarrow x_1 = 1 \mid x_2 = -6$

(h) $2x^2 = 8$

$\rightarrow \dfrac{2x^2}{2} = \dfrac{8}{2}$

$\rightarrow x^2 = 4$

$\rightarrow \sqrt{x^2} = \sqrt{4}$

$\rightarrow x = \sqrt{4}$

$\rightarrow x = \pm 2$

$\rightarrow x_1 = 2 \mid x_2 = -2$

4. Selesaikan persamaan kuadratik berikut dengan cara melengkapi kuadrat sempurna:

(a) $x^2 + 2x - 5 = 0$

(b) $x^2 - 4x + 2 = 0$

(c) $x^2 - 6x - 11 = 0$

(d) $x^2 + 3x - \dfrac{7}{4} = 0$

(e) $2x^2 + 8x + 1 = 0$

(f) $3x^2 - 6x - 1 = 0$

(g) $4x^2 - x = 0$

(h) $x^2 = \dfrac{3}{4}x - \dfrac{1}{8}$

JAWABAN:

(a) $x^2 + 2x - 5 = 0$

$\rightarrow a = 1, b = 2, c = -5$

$\rightarrow \left(x + \dfrac{2}{2}\right)^2 - \dfrac{2^2}{4} + (-5)$

$\rightarrow (x + 1)^2 - 1 - 5$

$\rightarrow (x + 1)^2 - 6$

$\rightarrow (x + 1)^2 - 6 = 0$

$\rightarrow (x + 1)^2 - 6 + 6 = 0 + 6$

$\rightarrow (x + 1)^2 = 6$

$\rightarrow \sqrt{(x + 1)^2} = \sqrt{6}$

$\rightarrow x + 1 = \pm\sqrt{6}$

$\rightarrow x + 1 - 1 = -1 \pm \sqrt{6}$

$\rightarrow x = -1 \pm \sqrt{6}$

$\rightarrow x_1 = -1 + \sqrt{6} \mid x_2 = -1 - \sqrt{6}$

(c) $x^2 - 6x - 11 = 0$

$\rightarrow a = 1, b = -6, c = -11$

$\rightarrow \left(x + \dfrac{-6}{2}\right)^2 - \dfrac{-6^2}{4} + (-11)$

$\rightarrow (x - 3)^2 - 9 - 11$

$\rightarrow (x - 3)^2 - 20$

$\rightarrow (x - 3)^2 - 20 = 0$

$\rightarrow (x - 3)^2 - 20 + 20 = 0 + 20$

$\rightarrow (x - 3)^2 = 20$

$\rightarrow \sqrt{(x - 3)^2} = \sqrt{20}$

$\rightarrow x - 3 = \pm\sqrt{20}$

$\rightarrow x - 3 + 3 = +3 \pm \sqrt{20}$

$\rightarrow x = 3 \pm \sqrt{20}$

$\rightarrow x_1 = 3 + \sqrt{20} \mid x_2 = 3 - \sqrt{20}$

(b) $x^2 - 4x + 2 = 0$

$\rightarrow a = 1, b = -4, c = 2$

$\rightarrow \left(x + \dfrac{-4}{2}\right)^2 - \dfrac{-4^2}{4} + 2$

$\rightarrow (x - 2)^2 - 4 + 2$

$\rightarrow (x - 2)^2 - 2$

$\rightarrow (x - 2)^2 - 2 = 0$

$\rightarrow (x - 2)^2 - 2 + 2 = 0 + 2$

$\rightarrow (x - 2)^2 = 2$

$\rightarrow \sqrt{(x - 2)^2} = \sqrt{2}$

$\rightarrow x - 2 = \pm\sqrt{2}$

$\rightarrow x - 2 + 2 = +2 \pm \sqrt{2}$

$\rightarrow x = 2 \pm \sqrt{2}$

$\rightarrow x_1 = 2 + \sqrt{2} \mid x_2 = 2 - \sqrt{2}$

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(d) $x^2 + 3x - \dfrac{7}{4} = 0$

$\rightarrow a = 1, b = 3, c = -\dfrac{7}{4}$

$\rightarrow \left(x + \dfrac{3}{2}\right)^2 - \dfrac{3^2}{4} + \left(-\dfrac{7}{4}\right)$

$\rightarrow \left(x + \dfrac{3}{2}\right)^2 - \dfrac{9}{4} - \dfrac{7}{4}$

$\rightarrow \left(x + \dfrac{3}{2}\right)^2 - \dfrac{16}{4}$

$\rightarrow \left(x + \dfrac{3}{2}\right)^2 - 4$

$\rightarrow \left(x + \dfrac{3}{2}\right)^2 - 4 = 0$

$\rightarrow \left(x + \dfrac{3}{2}\right)^2 - 4 + 4 = 0 + 4$

$\rightarrow \left(x + \dfrac{3}{2}\right)^2 = 4$

$\rightarrow \sqrt{\left(x + \dfrac{3}{2}\right)^2} = \sqrt{4}$

$\rightarrow x + \dfrac{3}{2} = \pm\sqrt{4}$

$\rightarrow x + \dfrac{3}{2} - \dfrac{3}{2} = -\dfrac{3}{2} \pm 2$

$\rightarrow x = -\dfrac{3}{2} \pm \dfrac{2}{1}$

$\rightarrow x_1 = -\dfrac{3}{2} + \dfrac{2}{1} = \dfrac{1}{2} \mid x_2 = -\dfrac{3}{2} - \dfrac{2}{1} = -\dfrac{7}{2}$

(e) $2x^2 + 8x + 1 = 0$

$\rightarrow a = 2, b = 8, c = 1$

$\rightarrow 2\left(x^2 + \dfrac{8}{2}x\right) + 1$

$\rightarrow 2(x^2 + 4x) + 1 = 0$

$\rightarrow 2\left[\left(x + \dfrac{4}{2}\right)^2 - \dfrac{4^2}{4}\right] + 1$

$\rightarrow 2\left[(x + 2)^2 - 4\right] + 1$

$\rightarrow 2(x + 2)^2 - 8 + 1$

$\rightarrow 2(x + 2)^2 - 7 = 0$

$\rightarrow 2(x + 2)^2 - 7 + 7 = 0 + 7$

$\rightarrow 2(x + 2)^2 = 7$

$\rightarrow \dfrac{\cancel{2}(x + 2)^2}{\cancel{2}} = \dfrac{7}{2}$

$\rightarrow (x + 2)^2 = \dfrac{7}{2}$

$\rightarrow \sqrt{(x + 2)^2} = \pm\sqrt{\dfrac{7}{2}}$

$\rightarrow x + 2 = \pm\sqrt{\dfrac{7}{2}}$

$\rightarrow x + 2 - 2 = -2 \pm \sqrt{\dfrac{7}{2}}$

$\rightarrow x = -2 \pm \sqrt{\dfrac{7}{2}}$

$\rightarrow x_1 = -2 + \sqrt{\dfrac{7}{2}} \mid x_2 = -2 - \sqrt{\dfrac{7}{2}}$

(f) $3x^2 - 6x - 1 = 0$

$\rightarrow a = 3, b = -6, c = -1$

$\rightarrow 3\left(x^2 + \dfrac{-6}{3}x\right) - 1$

$\rightarrow 3(x^2 - 2x) - 1 = 0$

$\rightarrow 3\left[\left(x + \dfrac{(-2)}{2}\right)^2 - \dfrac{(-2)^2}{4}\right] - 1$

$\rightarrow 3\left[(x-1)^2 - 1\right] - 1$

$\rightarrow 3(x-1)^2 - 3 - 1$

$\rightarrow 3(x-1)^2 - 4 = 0$

$\rightarrow 3(x-1)^2 - 4 + 4 = 0 + 4$

$\rightarrow 3(x-1)^2 = 4$

$\rightarrow \dfrac{\cancel{3}(x-1)^2}{\cancel{3}} = \dfrac{4}{3}$

$\rightarrow \sqrt{(x-1)^2} = \pm\sqrt{\dfrac{4}{3}}$

$\rightarrow x - 1 = \pm\sqrt{\dfrac{4}{3}}$

$\rightarrow x - 1 + 1 = +1 \pm \sqrt{\dfrac{4}{3}}$

$\rightarrow x = 1 \pm \sqrt{\dfrac{4}{3}}$

$\rightarrow x_1 = 1 + \sqrt{\dfrac{4}{3}} \mid x_2 = 1 - \sqrt{\dfrac{4}{3}}$

(g) $4x^2 - x = 0$

$\rightarrow a = 4, b = -1, c = 0$

$\rightarrow 4\left(x^2 + \dfrac{-1}{4}x\right)$

$\rightarrow 4\left(x^2 - \dfrac{1}{4}x\right) = 0$

$\rightarrow 4\left[\left(x - \dfrac{1}{8}\right)^2 - \dfrac{1^2}{8^2}\right]$

$\rightarrow 4\left[\left(x - \dfrac{1}{8}\right)^2 - \dfrac{1}{64}\right]$

$\rightarrow 4\left(x - \dfrac{1}{8}\right)^2 - \dfrac{4}{64}$

$\rightarrow 4\left(x - \dfrac{1}{8}\right)^2 - \dfrac{4}{64} = 0$

$\rightarrow 4\left(x - \dfrac{1}{8}\right)^2 - \dfrac{4}{64} + \dfrac{4}{64} = 0 + \dfrac{4}{64}$

$\rightarrow 4\left(x - \dfrac{1}{8}\right)^2 = \dfrac{4}{64}$

$\rightarrow \dfrac{\cancel{4}\left(x - \frac{1}{8}\right)^2}{\cancel{4}} = \dfrac{\frac{4}{64}}{4}$

$\rightarrow \left(x - \dfrac{1}{8}\right)^2 = \dfrac{4}{256}$

$\rightarrow \sqrt{\left(x - \dfrac{1}{8}\right)^2} = \pm\sqrt{\dfrac{4}{256}}$

$\rightarrow x - \dfrac{1}{8} = \pm\sqrt{\dfrac{4}{256}}$

$\rightarrow x - \dfrac{1}{8} + \dfrac{1}{8} = +\dfrac{1}{8} \pm \sqrt{\dfrac{4}{256}}$

$\rightarrow x = \dfrac{1}{8} \pm \sqrt{\dfrac{4}{256}}$

$\rightarrow x = \dfrac{1}{8} \pm \dfrac{2}{16}$

$\rightarrow x_1 = \dfrac{1}{8} + \dfrac{2}{16} \mid x_2 = \dfrac{1}{8} - \dfrac{2}{16}$

$\rightarrow x_1 = \dfrac{4}{16} \mid x_2 = 0$

(h) $\quad x^2 = \dfrac{3}{4}x - \dfrac{1}{8}$

$\rightarrow x^2 - \dfrac{3}{4}x + \dfrac{1}{8} = \dfrac{3}{4}x - \dfrac{1}{8} - \dfrac{3}{4}x + \dfrac{1}{8}$

$\rightarrow x^2 - \dfrac{3}{4}x + \dfrac{1}{8} = 0$

$\rightarrow a = 1, b = -\dfrac{3}{4}, c = \dfrac{1}{8}$

$\rightarrow \left(x - \dfrac{3}{8}\right)^2 - \dfrac{(-3)^2}{64} + \dfrac{1}{8}$

$\rightarrow \left(x - \dfrac{3}{8}\right)^2 - \dfrac{9}{64} + \dfrac{1}{8}$

$\rightarrow \left(x - \dfrac{3}{8}\right)^2 - \dfrac{1}{64} = 0$

$\rightarrow \left(x - \dfrac{3}{8}\right)^2 - \dfrac{1}{64} + \dfrac{1}{64} = 0 + \dfrac{1}{64}$

$\rightarrow \left(x - \dfrac{3}{8}\right)^2 = \dfrac{1}{64}$

$\rightarrow \sqrt{\left(x - \dfrac{3}{8}\right)^2} = \pm\sqrt{\dfrac{1}{64}}$

$\rightarrow x - \dfrac{3}{8} = \pm\sqrt{\dfrac{1}{64}}$

$\rightarrow x - \dfrac{3}{8} + \dfrac{3}{8} = +\dfrac{3}{8} \pm \sqrt{\dfrac{1}{64}}$

$\rightarrow x = \dfrac{3}{8} \pm \sqrt{\dfrac{1}{64}}$

$\rightarrow x = \dfrac{3}{8} \pm \dfrac{1}{8}$

$\rightarrow x_1 = \dfrac{3}{8} + \dfrac{1}{8} \mid x_2 = \dfrac{3}{8} - \dfrac{1}{8}$

$\rightarrow x_1 = \dfrac{4}{8} \mid x_2 = \dfrac{2}{8}$

5. Tentukan seluruh solusi (jika ada) dari persamaan kuadratik berikut dengan rumus persamaan kuadratik:

(a) $x^2 - 2x - 15 = 0$

(b) $x^2 + 5x - 6 = 0$

(c) $x^2 - 7x + 10 = 0$

(d) $x^2 + 30x + 200 = 0$

(e) $2x^2 + x - 3 = 0$

(f) $3x^2 + 7x + 4 = 0$

(g) $3x^2 + 6x - 5 = 0$

(h) $x^2 - 6x + 1 = 0$

JAWABAN:

(a) $x^2 - 2x - 15 = 0$

$\rightarrow a = 1, b = -2, c = -15$

$\rightarrow \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot -15}}{2 \cdot 1}$

$\rightarrow \dfrac{2 \pm \sqrt{(4 - (-60))}}{2}$

$\rightarrow \dfrac{2 \pm \sqrt{64}}{2}$

$\rightarrow \dfrac{2 \pm 8}{2}$

$\rightarrow x_1 = \dfrac{2 + 8}{2} \mid x_2 = \dfrac{2 - 8}{2}$

$\rightarrow x_1 = \dfrac{10}{2} \mid x_2 = \dfrac{-6}{2}$

$\rightarrow x_1 = 5 \mid x_2 = -3$

(b) $x^2 + 5x - 6 = 0$

$\rightarrow a = 1, b = 5, c = -6$

$\rightarrow \dfrac{-(5) \pm \sqrt{5^2 - 4 \cdot 1 \cdot -6}}{2 \cdot 1}$

$\rightarrow \dfrac{-5 \pm \sqrt{(25 - (-24))}}{2}$

$\rightarrow \dfrac{-5 \pm \sqrt{49}}{2}$

$\rightarrow \dfrac{-5 \pm 7}{2}$

$\rightarrow x_1 = \dfrac{-5 + 7}{2} \mid x_2 = \dfrac{-5 - 7}{2}$

$\rightarrow x_1 = \dfrac{2}{2} \mid x_2 = \dfrac{-12}{2}$

$\rightarrow x_1 = 1 \mid x_2 = -6$

(c) $x^2 - 7x + 10 = 0$

$\rightarrow a = 1, b = -7, c = 10$

$\rightarrow \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1}$

$\rightarrow \dfrac{7 \pm \sqrt{(49 - 40)}}{2}$

$\rightarrow \dfrac{7 \pm \sqrt{9}}{2}$

$\rightarrow \dfrac{7 \pm 3}{2}$

$\rightarrow x_1 = \dfrac{7 + 3}{2} \mid x_2 = \dfrac{7 - 3}{2}$

$\rightarrow x_1 = \dfrac{10}{2} \mid x_2 = \dfrac{4}{2}$

$\rightarrow x_1 = 5 \mid x_2 = 2$

(d) $x^2 + 30x + 200 = 0$

$\rightarrow a = 1, b = 30, c = 200$

$\rightarrow \dfrac{-(30) \pm \sqrt{30^2 - 4 \cdot 1 \cdot 200}}{2 \cdot 1}$

$\rightarrow \dfrac{-30 \pm \sqrt{(900 - 800)}}{2}$

$\rightarrow \dfrac{-30 \pm \sqrt{100}}{2}$

$\rightarrow \dfrac{-30 \pm 10}{2}$

$\rightarrow x_1 = \dfrac{-30 + 10}{2} \mid x_2 = \dfrac{-30 - 10}{2}$

$\rightarrow x_1 = \dfrac{-20}{2} \mid x_2 = \dfrac{-40}{2}$

$\rightarrow x_1 = -10 \mid x_2 = -20$

(e) $2x^2 + x - 3 = 0$

$\rightarrow a = 2, b = 1, c = -3$

$\rightarrow \dfrac{-(1) \pm \sqrt{(1)^2 - 4 \cdot 2 \cdot -3}}{2 \cdot 2}$

$\rightarrow \dfrac{-1 \pm \sqrt{(1 - (-24))}}{4}$

$\rightarrow \dfrac{-1 \pm \sqrt{25}}{4}$

$\rightarrow \dfrac{-1 \pm 5}{2}$

$\rightarrow x_1 = \dfrac{-1 + 5}{2} \mid x_2 = \dfrac{-1 - 5}{2}$

$\rightarrow x_1 = \dfrac{4}{2} \mid x_2 = \dfrac{-6}{2}$

$\rightarrow x_1 = 2 \mid x_2 = -3$

(f) $3x^2 + 7x + 4 = 0$

$\rightarrow a = 3, b = 7, c = 4$

$\rightarrow \dfrac{-(7) \pm \sqrt{7^2 - 4 \cdot 3 \cdot 4}}{2 \cdot 3}$

$\rightarrow \dfrac{-7 \pm \sqrt{(49 - 48)}}{6}$

$\rightarrow \dfrac{-7 \pm \sqrt{1}}{6}$

$\rightarrow \dfrac{-7 \pm 1}{6}$

$\rightarrow x_1 = \dfrac{-7 + 1}{6} \mid x_2 = \dfrac{-7 - 1}{6}$

$\rightarrow x_1 = \dfrac{-6}{2} \mid x_2 = \dfrac{-8}{6}$

$\rightarrow x_1 = -3 \mid x_2 = -\dfrac{8}{6}$

(g) $3x^2 + 6x - 5 = 0$

$\rightarrow a = 3, b = 6, c = -5$

$\rightarrow \dfrac{-(6) \pm \sqrt{6^2 - 4 \cdot 3 \cdot -5}}{2 \cdot 3}$

$\rightarrow \dfrac{-6 \pm \sqrt{(36 - (-60))}}{6}$

$\rightarrow \dfrac{-6 \pm \sqrt{96}}{6}$

$\rightarrow \dfrac{-6 \pm \sqrt{96}}{6}$

$\rightarrow x_1 = \dfrac{-6 + \sqrt{96}}{6} \mid x_2 = \dfrac{-6 - \sqrt{96}}{6}$

(h) $x^2 - 6x + 1 = 0$

$\rightarrow a = 1, b = -6, c = 1$

$\rightarrow \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$

$\rightarrow \dfrac{6 \pm \sqrt{(36 - 4)}}{2}$

$\rightarrow \dfrac{6 \pm \sqrt{32}}{2}$

$\rightarrow \dfrac{6 \pm \sqrt{32}}{2}$

$\rightarrow x_1 = \dfrac{6 + \sqrt{32}}{2} \mid x_2 = \dfrac{6 - \sqrt{32}}{2}$

6. Gunakan Diskriminan untuk mengetahui berapa banyak solusi real dari persamaan kuadrat berikut:

(a) $x^2 - 6x + 1 = 0$

(b) $3x^2 = 6x - 9$

(c) $x^2 + 2.2x + 1.21 = 0$

(d) $x^2 + 2.21x + 1.21 = 0$

JAWABAN:

(a) $x^2 - 6x + 1 = 0$
$\rightarrow a = 1, b = -6, c = 1$
$\rightarrow D = (-6)^2 - 4 \cdot 1 \cdot 1$
$\rightarrow D = 36 - 4$
$\rightarrow D = 32$
$\rightarrow$ Ada 2 solusi real, karena $D > 0$.

(b) $3x^2 = 6x - 9$
$\rightarrow 3x^2 - 6x + 9 = 6x - 9 - 6x + 9$
$\rightarrow 3x^2 - 6x + 9 = 0$
$\rightarrow a = 3, b = -6, c = 9$
$\rightarrow D = (-6)^2 - 4 \cdot 3 \cdot 9$
$\rightarrow D = 36 - 108$
$\rightarrow D = -72$
$\rightarrow$ Tidak ada solusi real, karena $D < 0$.

(c) $x^2 + 2.2x + 1.21 = 0$
$\rightarrow a = 1, b = 2.2, c = 1.21$
$\rightarrow D = (2.2)^2 - 4 \cdot 1 \cdot 1.21$
$\rightarrow D = 4.84 - 4.84$
$\rightarrow D = 0$
$\rightarrow$ Hanya ada 1 solusi real, karena $D = 0$.

(d) $x^2 + 2.21x + 1.21 = 0$
$\rightarrow a = 1, b = 2.21, c = 1.21$
$\rightarrow D = (2.21)^2 - 4 \cdot 1 \cdot 1.21$
$\rightarrow D = 4.8841 - 4.84$
$\rightarrow D = 0.0441$
$\rightarrow$ Ada 2 solusi real, karena $D > 0$.

7. Suatu bola dilemparkan sehingga setelah t detik dilemparkan, ketinggian bola atas permukaan tanah h mengikuti persamaan.

$$h = -16t^2 + 288$$

Tentukan kapan bola mencapai tanah.

JAWABAN:

$\rightarrow$ Jadikan $h = 0$ meter, ketika bola menyentuh tanah.

$\rightarrow 0 = -16t^2 + 288$

$\rightarrow -16t^2 + 288 = 0$

$\rightarrow -16t^2 + 288 - 288 = 0 - 288$

$\rightarrow -16t^2 = -288$

$\rightarrow \dfrac{\cancel{-16}t^2}{\cancel{-16}} = \dfrac{-288}{-16}$

$\rightarrow 1.t^2 = 18$

$\rightarrow t^2 = 18$

$\rightarrow \sqrt{t^2} = \sqrt{18}$

$\rightarrow t = 4.242640687$ detik

$\rightarrow t \approx 4.2$ detik

$\rightarrow$ Jadi bola menyentuh tanah ketika $t = 4.2$ detik.

8. Suatu bola dilemparkan sehingga setelah t detik dilemparkan, ketinggian bola atas permukaan tanah h mengikuti persamaan.

$$h = -16t^2 + 40t$$

Tentukan.

(a) Kapan bola mencapai ketinggian 24 meter dari permukaan tanah.

(b) Kapan bola mencapai ketinggian 48 meter dari permukaan tanah.

(c) Kapan bola mencapai tanah.

JAWABAN:

(a) t berapa bola mencapai ketinggian $h = 24$ meter.

$\rightarrow$ Jadikan $h = 24$ meter, dari atas permukaan tanah.

$\rightarrow 24 = -16t^2 + 40t$

$\rightarrow -16t^2 + 40t = 24$

$\rightarrow -16t^2 + 40t - 24 = 24 - 24$

$\rightarrow -16t^2 + 40t - 24 = 0$

$\rightarrow a = -16, b = 40, c = -24$

$\rightarrow \dfrac{-(40) \pm \sqrt{40^2 - 4 \cdot -16 \cdot -24}}{2 \cdot -16}$

$\rightarrow \dfrac{-40 \pm \sqrt{1600 - 1536}}{-32}$

$\rightarrow \dfrac{-40 \pm \sqrt{64}}{-32}$

$\rightarrow \dfrac{-40 \pm 8}{-32}$

$\rightarrow \dfrac{-40 \pm 8}{-32}$

$\rightarrow t_1 = \dfrac{-40 + 8}{-32} \mid t_2 = \dfrac{-40 - 8}{-32}$

$\rightarrow t_1 = \dfrac{-32}{-32} \mid t_2 = \dfrac{-48}{-32}$

$\rightarrow t_1 = 1 \mid t_2 = \dfrac{3}{2} = 1.5$

$\rightarrow$ Jadi bola mencapai ketinggian 24 meter ketika $t = 1$ detik atau $t = 1.5$ detik.

(b) t berapa bola mencapai ketinggian $h = 48$ meter.

$\rightarrow$ Jadikan $h = 24$ meter, dari atas permukaan tanah.

$\rightarrow 48 = -16t^2 + 40t$

$\rightarrow -16t^2 + 40t = 48$

$\rightarrow -16t^2 + 40t - 48 = 48 - 48$

$\rightarrow -16t^2 + 40t - 48 = 0$

$\rightarrow a = -16, b = 40, c = -40$

$\rightarrow \dfrac{-(40) \pm \sqrt{40^2 - 4 \cdot -16 \cdot -48}}{2 \cdot -16}$

$\rightarrow \dfrac{-40 \pm \sqrt{1600 - 3072}}{-32}$

$\rightarrow \dfrac{-40 \pm \sqrt{-1472}}{-32}$

$\rightarrow$ Bola tidak akan mencapai ketinggian 48 meter karena tidak ada solusi real, karena $D < 0$.

(c) t berapa bola mencapai ketinggian $h = 0$ meter.

$\rightarrow$ Jadikan $h = 0$ meter, ketika bola menyentuh tanah.

$\rightarrow 0 = -16t^2 + 40t$

$\rightarrow -16t^2 + 40t = 0$

$\rightarrow t(-16t + 40) = 0$

$\rightarrow t = 0 \mid -16t + 40 = 0$

$\rightarrow t = 0 \mid -16t + 40 - 40 = 0 - 40$

$\rightarrow t = 0 \mid -16t = -40$

$\rightarrow t = 0 \mid \dfrac{-16t}{-16} = \dfrac{-40}{-16}$

$\rightarrow t = 0 \mid t = 2.5$

$\rightarrow$ Bola akan menyentuh tanah ketika $t = 0$ detik atau $t = 2.5$ detik.

9. Populasi ikan F dalam suatu danau mengikuti persamaan.

$$F = 1000(30 + 17t - t^2)$$

Dengan F menyatakan banyak ikan dalam waktu t dimana t dihitung dalam tahun sejak 1 Januari 2002 (ketika populasi pertama kali diestimasi)

(a) Tentukan populasi ikan saat tanggal 1 Januari 2002.

(b) Tentukan waktu ketika populasi ikan sama dengan populasi awalnya.

(c) Estimasi waktu populasi ikan pada danau tersebut akan punah.

JAWABAN:

(a) Populasi ikan pada tanggal 1 Januari 2002.
$\rightarrow$ Karena tanggal 1 Januari 2002 adalah waktu awal estimasi (t_0), maka $t_0 = 0$.
$\rightarrow$ Kita tinggal subtitusikan $t = 0$ pada persamaannya.
$\rightarrow F = 1000(30 + 17t - t^2)$
$\rightarrow F = 1000(30 + 17 \cdot (0) - (0)^2)$
$\rightarrow F = 1000(30 + 0 - 0)$
$\rightarrow F = 1000(30)$
$\rightarrow F = 1000 \cdot 30$
$\rightarrow F = 30000$
$\rightarrow$ Jadi populasi ikan pada tanggal 1 Januari 2024 adalah 30000

(b) Mencari waktu kapan saat populasi ikan (F) sama dengan 30000, yaitu populasi awal ketika $t_0 = 0$.
$\rightarrow$ Kita subtitusikan $F = 30000$ pada persamaannya.
$\rightarrow 30000 = 1000(30 + 17t - t^2)$
$\rightarrow 30000 = 30000 + 17000t - 1000t^2$
$\rightarrow 30000 + 17000t - 1000t^2 = 30000$
$\rightarrow -1000t^2 + 17000t + 30000 = 30000$
$\rightarrow \dfrac{-1000t^2 + 17000t + 30000}{1000} = \dfrac{30000}{1000}$
$\rightarrow -1t^2 + 17t + 30 = 30$
$\rightarrow -t^2 + 17t + 30 - 30 = 30 - 30$
$\rightarrow -t^2 + 17t = 0$
$\rightarrow t(-t + 17) = 0$
$\rightarrow t = 0 \mid -t + 17 = 0$
$\rightarrow t = 0 \mid -t + 17 - 17 = 0 - 17$
$\rightarrow t = 0 \mid -t = -17$
$\rightarrow t = 0 \mid \dfrac{-t}{-1} = \dfrac{-17}{-1}$
$\rightarrow t_1 = 0 \mid t_2 = 17$

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$\rightarrow$ Kita ambil $t_2 = 17$ tahun, maka waktu pada saat populasi ikan (F) sama dengan 30000 adalah 1 Januari 2002 + 17 tahun = 1 Januari 2019.

(b) Mencari waktu kapan saat populasi ikan (F) sama dengan 0.

$\rightarrow$ Kita subtitusikan $F = 0$ pada persamaannya.

$\rightarrow 0 = 1000(30 + 17t - t^2)$

$\rightarrow 1000(30 + 17t - t^2) = 0$

$\rightarrow 30000 + 17000t - 1000t^2 = 0$

$\rightarrow -1000t^2 + 17000t + 30000 = 0$

$\rightarrow \dfrac{-1000t^2 + 17000t + 30000}{1000} = \dfrac{0}{1000}$

$\rightarrow -1t^2 + 17t + 30 = 0$

$\rightarrow -(-t^2 + 17t + 30) = 0$

$\rightarrow t^2 - 17t - 30 = 0$

$\rightarrow a = 1, b = -17, c = -30$

$\rightarrow \dfrac{-(-17) \pm \sqrt{(-17)^2 - 4 \cdot 1 \cdot -30}}{2 \cdot 1}$

$\rightarrow \dfrac{17 \pm \sqrt{(289 - (-120))}}{2}$

$\rightarrow \dfrac{17 \pm \sqrt{289 + 120}}{2}$

$\rightarrow \dfrac{17 \pm \sqrt{409}}{2}$

$\rightarrow \dfrac{17 + \sqrt{409}}{2} \mid \dfrac{17 - \sqrt{409}}{2}$

$\rightarrow t_1 = \dfrac{17 + \sqrt{409}}{2} \mid t_2 = \dfrac{17 - \sqrt{409}}{2}$

$\rightarrow t_1 = 18.61187421 \mid t_2 = -1.611874208$

$\rightarrow t_1 = (18.61187421 \approx 19) \mid t_2 = (-1.611874208 \approx -2)$

$\rightarrow t_1 = 19 \mid t_2 = -2$

$\rightarrow$ Kita ambil $t_1 = 19$ tahun, karena t_2 bernilai negatif dan waktu tidak mungkin negatif, maka waktu pada saat populasi ikan (F) sama dengan 0 adalah 1 Januari 2002 + 19 tahun = 1 Januari 2021.

10. Suatu persegi panjang mempunyai luas 150 m^2 dengan panjangnya adalah $25m$. Tentukan lebar dari persegi panjang tersebut.

JAWABAN:

(10) rumus luas persegi panjang adalah $L = p \times \ell$.

$\to 150m^2 = 25m \times \ell$

$\to 150m^2 = 25m \quad [\ell]$ (isolasi ℓ)

$\to \dfrac{150m^2}{25m} = \dfrac{25m}{25m} \quad [\ell]$

$\to 6m = 1 \quad [\ell]$

$\to 6m = 1 \cdot \ell$

$\to 6m = \ell$

$\to \ell = 6m$

11. Suatu peternak mempunyai lahan berbentuk persegi panjang yang dipagari oleh 200 meter pagar rotan. Tentukan lebar dan panjang lahan peternak tersebut, jika luas lahannya adalah $2400 \ m^2$.

JAWABAN:

(11) rumus luas persegi panjang adalah $L = p \times \ell$ dan rumus keliling adalah $K = 2p + 2\ell$.

$\rightarrow 200m = 2p + 2\ell$

$\rightarrow 200m - 2\ell = 2p + 2\ell - 2\ell$

$\rightarrow 200m - 2\ell = 2p$

$\rightarrow 2p = 200m - 2\ell$

$\rightarrow \dfrac{2p}{2} = \dfrac{200m - 2}{2}\ell$

$\rightarrow p = 100m - 1\ell$

$\rightarrow p = 100m - \ell$

$\rightarrow L = p \times \ell$

$\rightarrow 2400m^2 = (100m - \ell) \times \ell$

$\rightarrow 2400m^2 - 2400m^2 = 100\ell \, m - \ell^2 - 2400m^2$

$\rightarrow 0 = 100\ell \, m - \ell^2 - 2400m^2$

$\rightarrow 100\ell \, m - \ell^2 - 2400m^2 = 0$

$\rightarrow -\ell^2 + 100\ell \, m - 2400m^2 = 0$

$\rightarrow -(-\ell^2 + 100\ell \, m - 2400m^2) = 0$

$\rightarrow \ell^2 - 100\ell \, m + 2400m^2 = 0$

$\rightarrow a = 1, b = -100, c = 2400$

$\rightarrow (\ell - 40) \mid (\ell - 60)$

$\rightarrow \ell - 40 = 0 \mid \ell - 60 = 0$

$\rightarrow \ell - 40 + 40 = 0 + 40 \mid \ell - 60 + 60 = 0 + 60$

$\rightarrow \ell_1 = 40 \mid \ell_2 = 60$ (Pilih salah satu ℓ saja, saya memilih $\ell_1 = 40$ meter).

$\rightarrow p = 100m - 40m = 60$ meter.

$\rightarrow$ Jadi panjang persegi panjang $= 60$ meter dan lebar persegi panjang $= 40$ meter.

12. Suatu perusahaan sewa mobil memberikan beban biaya sewa mobil 150 ribu rupiah per hari dengan tambahan 15 ribu rupiah per kilometer. Jika Pratama menyewa mobil pada penyewaan tersebut, dan membayar 495 ribu rupiah. Tentukan jarak yang ditempuh oleh Pratama.

JAWABAN:

(12) Karena yang ditanya jarak (kilometer),maka kilometer kita jadikan variabel x.

$\rightarrow$ Kita tulis ulang apa yang ada di soal ke dalam persamaan.

$\rightarrow 150.000 + 15.000x = 495.000$

$\rightarrow 150.000 + 15.000x - 150.000 = 495.000 - 150.000$

$\rightarrow 15.000x = 345.000$

$\rightarrow \dfrac{15.000x}{15.000} = \dfrac{345.000}{15000}$

$\rightarrow 1 \cdot x = 23$

$\rightarrow x = 23$

$\rightarrow$ Jadi jarak yang ditempuh Pratama adalah 23 kilometer.

13. Tahun ini Adam 4 kali dari umur anaknya. Jika 6 tahun kemudian, umur Adam 3 kali umur anaknya. Tentukan umur anak Adam tahun ini.

JAWABAN:

(13) Karena yang ditanya umur anak adam, maka umur anak adam kita jadikan variabel x.

$\rightarrow$ Kita tulis ulang apa yang ada di soal ke dalam persamaan.

$\rightarrow 4x + 6 = 3(x + 6)$

$\rightarrow 4x + 6 = 3x + 18$

$\rightarrow 4x - 3x + 6 = 3x - 3x + 18$

$\rightarrow x + 6 = 18$

$\rightarrow x + 6 - 6 = 18 - 6$

$\rightarrow x = 12$

$\rightarrow$ Jadi umur anak Adam adalah 12 tahun.

3 Latihan Soal Bentuk Eksponen dan Akar.

1. Ubah bentuk akar berikut menjadi bentuk pangkat!

(a) $\sqrt{3}$

(b) $\dfrac{1}{\sqrt{6}}$

(c) $\sqrt[3]{7^2}$

(d) $\sqrt[3]{2^4}$

(e) $\dfrac{1}{\sqrt[3]{11}}$

(f) $\sqrt[5]{5^3}$

JAWABAN:

(a) $\sqrt{3}$

$\to 3^{1/2}$

(b) $\dfrac{1}{\sqrt{6}}$

$\to \sqrt{6} = 6^{1/2}$

$\to \dfrac{1}{6^{1/2}} = 6^{-1/2}$

$\to 6^{-1/2}$

(c) $\sqrt[3]{7^2}$

$\to 7^{2/3}$

(d) $\sqrt[3]{2^4}$

$\to 2^{4/3}$

(e) $\dfrac{1}{\sqrt[3]{11}}$

$\to \sqrt[3]{11} = 11^{1/3}$

$\to \dfrac{1}{11^{1/3}} = 11^{-1/3}$

$\to 11^{-1/3}$

(f) $\sqrt[5]{5^3}$

$\to 5^{3/5}$

2. Ubah bentuk pangkat berikut menjadi bentuk akar!

(a) $3^{1/3}$

(b) $2^{2/3}$

(c) $5^{1/3}$

(d) $7^{3/2}$

(e) $5^{-4/3}$

(f) $2^{-5/2}$

JAWABAN:

(a) $3^{1/3}$

$\rightarrow \sqrt[3]{3}$

(b) $2^{2/3}$

$\rightarrow \sqrt[3]{2^2}$

(c) $5^{1/3}$

$\rightarrow \sqrt[3]{5}$

(d) $7^{3/2}$

$\rightarrow \sqrt[2]{7^3}$

(e) $5^{-4/3}$

$\rightarrow \sqrt[3]{5^{-4}} = \dfrac{1}{\sqrt[3]{5^4}}$

$\rightarrow \dfrac{1}{\sqrt[3]{5^4}}$

(f) $2^{-5/2}$

$\rightarrow \sqrt[2]{2^{-5}} = \dfrac{1}{\sqrt[2]{2^5}}$

$\rightarrow \dfrac{1}{\sqrt[2]{2^5}}$

3. Hitung dan sederhanakan operasi pangkat berikut!

(a) $\quad -3^2$

(b) $\quad (-3)^2$

(c) $\quad \left(\dfrac{1}{3}\right)^4 (-3)^2$

(d) $\quad 5^4 \cdot 5^{-2}$

(e) $\quad \dfrac{10^7}{10^4}$

(f) $\quad \dfrac{3}{3^{-2}}$

(g) $\quad \dfrac{2^{-3}}{3^0}$

(h) $\quad \left(\dfrac{1}{3}\right)^{-2}$

(i) $\quad \left(-\dfrac{2}{3}\right)^{-3}$

(j) $\quad \left(\dfrac{3}{2}\right)^{-2} \cdot \dfrac{9}{16}$

(k) $\quad \left(\dfrac{1}{2}\right)^4 \cdot \dfrac{5^{-2}}{2}$

JAWABAN:

(a) $\quad -3^2$

$\rightarrow -3 \cdot -3 = 9$

$\rightarrow 9$

(b) $\quad (-3)^2$

$\rightarrow (-3)^2 = -3^2$

$\rightarrow -3 \cdot -3 = 9$

$\rightarrow 9$

(c) $\quad \left(\dfrac{1}{3}\right)^4 (-3)^2$

$\rightarrow \left(\dfrac{1^4}{3^4}\right)(9)$

$\rightarrow \dfrac{1}{81} \cdot 9 = \dfrac{9}{81}$

$\rightarrow \dfrac{9}{81}$

(d) $\quad 5^4 \cdot 5^{-2}$

$\rightarrow 5^{(4+(-2))} = 5^2$

$\rightarrow 5^2$

(e) $\quad \dfrac{10^7}{10^4}$

$\rightarrow 10^{(7-4)} = 10^3$

$\rightarrow 10^3$

(f) $\quad \dfrac{3}{3^{-2}}$

$\rightarrow \dfrac{3}{\frac{1}{3^2}} = \dfrac{3}{\frac{1}{9}}$

$\rightarrow \dfrac{3}{1} \div \dfrac{1}{9} = \dfrac{3}{1} \cdot \dfrac{9}{1}$

$\rightarrow \dfrac{27}{1} = 27$

$\rightarrow 27$

(g) $\quad \dfrac{2^{-3}}{3^0}$

$\rightarrow \dfrac{\frac{1}{2^3}}{3^0} = \dfrac{\frac{1}{8}}{1}$

$\rightarrow \dfrac{1}{8} \div \dfrac{1}{1} = \dfrac{1}{8} \cdot \dfrac{1}{1}$

$\rightarrow \dfrac{1}{8}$

(h) $\quad \left(\dfrac{1}{3}\right)^{-2}$

$\rightarrow \dfrac{1}{\frac{1^2}{3^2}} = \dfrac{1}{\frac{1}{9}}$

$\rightarrow \dfrac{1}{1} \div \dfrac{1}{9} = \dfrac{1}{1} \cdot \dfrac{9}{1}$

$\rightarrow \dfrac{9}{1} = 9$

$\rightarrow 9$

(i) $\quad \left(-\dfrac{2}{3}\right)^{-3}$

$\rightarrow \dfrac{1}{-\frac{2^3}{3^3}} = \dfrac{1}{-\frac{8}{27}}$

$\rightarrow \dfrac{1}{1} \div -\dfrac{8}{27} = \dfrac{1}{1} \cdot -\dfrac{27}{8}$

$\rightarrow -\dfrac{27}{8}$

(j) $\quad \left(\dfrac{3}{2}\right)^{-2} \cdot \dfrac{9}{16}$

$\rightarrow \dfrac{1}{\frac{3^2}{2^2}} \cdot \dfrac{9}{16}$

$\rightarrow \dfrac{1}{\frac{9}{4}} \cdot \dfrac{9}{16}$

$\rightarrow \dfrac{4}{9} \cdot \dfrac{9}{16} = \dfrac{36}{144}$

$\rightarrow \dfrac{36}{144}$

(k) $\quad \left(\dfrac{1}{2}\right)^4 \cdot \dfrac{5^{-2}}{2}$

$\rightarrow \dfrac{1^4}{2^4} \cdot \dfrac{1}{\frac{5^2}{2^2}}$

$\rightarrow \dfrac{1}{16} \cdot \dfrac{4}{25} = \dfrac{4}{400}$

$\rightarrow \dfrac{4}{400}$

4. Hitung dan sederhanakan operasi akar berikut!

(a) $\sqrt{16}$

(b) $\sqrt[4]{16}$

(c) $\sqrt[4]{\dfrac{1}{16}}$

(d) $\sqrt{64}$

(e) $\sqrt[3]{-64}$

(f) $\sqrt[5]{-32}$

(g) $\sqrt{\dfrac{4}{9}}$

(h) $\sqrt[4]{256}$

(i) $\sqrt[6]{\dfrac{1}{64}}$

(j) $\sqrt{7} \cdot \sqrt{28}$

(k) $\dfrac{\sqrt{48}}{\sqrt{3}}$

(l) $\sqrt[4]{54} \cdot \sqrt[4]{24}$

JAWABAN:

(a) $\sqrt{16}$

$\rightarrow \sqrt{16} = 4^2$

$\rightarrow 4$

(b) $\sqrt[4]{16}$

$\rightarrow \sqrt[4]{16} = 2^4$

$\rightarrow 2$

(c) $\sqrt[4]{\dfrac{1}{16}}$

$\rightarrow \sqrt[4]{\dfrac{1}{16}} = \dfrac{\sqrt[4]{1}}{\sqrt[4]{16}}$

$\rightarrow \dfrac{\sqrt[4]{1}}{\sqrt[4]{16}} = \dfrac{1^4}{2^4}$

$\rightarrow \dfrac{1}{2}$

(d) $\sqrt{64}$

$\rightarrow \sqrt{16} = 8^2$

$\rightarrow 8$

(e) $\sqrt[3]{-64}$

$\rightarrow \sqrt[3]{-64} = -4^3$

$\rightarrow -4$

(f) $\sqrt[5]{-32}$

$\rightarrow \sqrt[5]{-32} = -2^5$

$\rightarrow -2$

(g) $\sqrt{\dfrac{4}{9}}$

$\rightarrow \sqrt{\dfrac{4}{9}} = \dfrac{\sqrt{4}}{\sqrt{9}}$

$\rightarrow \dfrac{2^2}{3^2} = \dfrac{2}{3}$

$\rightarrow \dfrac{2}{3}$

(h) $\sqrt[4]{256}$

$\rightarrow \sqrt[4]{256} = 4^4$

$\rightarrow 4$

(i) $\sqrt[6]{\dfrac{1}{64}}$

$\rightarrow \sqrt[6]{\dfrac{1}{64}} = \dfrac{\sqrt[6]{1}}{\sqrt[6]{64}}$

$\rightarrow \dfrac{\sqrt[6]{1}}{\sqrt[6]{64}} = \dfrac{1^6}{2^6}$

$\rightarrow \dfrac{1}{2}$

(j) $\sqrt{7} \cdot \sqrt{28}$

$\rightarrow \sqrt{196} = 14^2$

$\rightarrow 14$

(k) $\dfrac{\sqrt{48}}{\sqrt{3}}$

$\rightarrow \dfrac{\sqrt{48}}{\sqrt{3}} = \dfrac{\sqrt{16} \cdot \sqrt{3}}{\sqrt{3}}$

$\rightarrow \dfrac{4\sqrt{3}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{4\sqrt{9}}{\sqrt{9}}$

$\rightarrow \dfrac{4 \cdot 3}{3} = \dfrac{12}{3}$

$\rightarrow 4$

(l) $\sqrt[4]{54} \cdot \sqrt[4]{24}$

$\rightarrow \sqrt[4]{54} \cdot \sqrt[4]{24} = \sqrt[4]{54 \cdot 24}$

$\rightarrow \sqrt[4]{1296} = 6^4$

$\rightarrow 6$

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5. Hitung dan sederhanakan bentuk pangkat rasional berikut!

(a) $\left(\dfrac{4}{9}\right)^{-1/2}$

(b) $(-32)^{2/5}$

(c) $-32^{2/5}$

(d) $1024^{-0.1}$

(e) $\left(-\dfrac{27}{8}\right)^{2/3}$

(f) $\left(\dfrac{25}{64}\right)^{-3/2}$

JAWABAN:

(a) $\left(\dfrac{4}{9}\right)^{-1/2}$

$\to \dfrac{1}{(\frac{4}{9})^{1/2}} = \dfrac{1}{\sqrt{\frac{4}{9}}}$

$\to \sqrt{\dfrac{9}{4}} = \dfrac{\sqrt{9}}{\sqrt{4}}$

$\to \dfrac{3^2}{2^2} = \dfrac{3}{2}$

$\to \dfrac{3}{2}$

(b) $(-32)^{2/5}$

$\to (-2^5)^{2/5} = -2^{10/5}$

$\to -2^2 = 4$

$\to 4$

(c) $-32^{2/5}$

$\to \sqrt[5]{-32^2} = \sqrt[5]{-1024}$

$\to \sqrt[5]{-1024} = -4^5$

$\to -4$

(d) $1024^{-0.1}$

$\to 1024^{-0.1} = 1024^{-1/10}$

$\to (2^{10})^{-1/10} = 2^{-10/10}$

$\to 2^{-1} = \dfrac{1}{2}$

$\to \dfrac{1}{2}$

(e) $\left(-\dfrac{27}{8}\right)^{2/3}$

$\to \sqrt[3]{\left(\dfrac{-27}{8}\right)^2} = \dfrac{\sqrt[3]{(-27)^2}}{\sqrt[3]{(8)^2}}$

$\to \dfrac{\sqrt[3]{729}}{\sqrt[3]{64}} = \dfrac{9^3}{4^3}$

$\to \dfrac{9}{4}$

(f) $\left(\dfrac{25}{64}\right)^{-3/2}$

$\to \left(\dfrac{64}{25}\right)^{3/2} = \sqrt[2]{\left(\dfrac{64}{25}\right)^3}$

$\to \dfrac{\sqrt[2]{(64)^3}}{\sqrt[2]{(25)^3}} = \dfrac{\sqrt[2]{262144}}{\sqrt[2]{15625}}$

$\to \dfrac{512^2}{125^2} = \dfrac{512}{125}$

$\to \dfrac{512}{125}$

6. Hitung nilai berikut ketika $x = 3, y = 4,$ dan $z = -1$

(a) $\sqrt{x^2 + y^2}$

(b) $\sqrt[4]{x^3 + 14y + 2z}$

(c) $(9x)^{2/3} + (2y)^{2/3} + z^{2/3}$

(d) $(xy)^{2z}$

JAWABAN:

(a) $\sqrt{x^2 + y^2}$
$\rightarrow x = 3, y = 4$
$\rightarrow \sqrt{3^2 + 4^2}$
$\rightarrow \sqrt{9 + 16}$
$\rightarrow \sqrt{25}$
$\rightarrow 5$

(b) $\sqrt[4]{x^3 + 14y + 2z}$
$\rightarrow x = 3, y = 4, z = -1$
$\rightarrow \sqrt[4]{3^3 + 14(4) + 2(-1)}$
$\rightarrow \sqrt[4]{27 + 56 + (-2)}$
$\rightarrow \sqrt[4]{83 - 2}$
$\rightarrow \sqrt[4]{81} = 3^4$
$\rightarrow 3$

(c) $(9x)^{2/3} + (2y)^{2/3} + z^{2/3}$
$\rightarrow x = 3, y = 4, z = -1$
$\rightarrow (9(3))^{2/3} + (2(4))^{2/3} + (-1)^{2/3}$
$\rightarrow (27)^{2/3} + (8)^{2/3} + (-1)^{2/3}$
$\rightarrow \sqrt[3]{27^2} + \sqrt[3]{8^2} + \sqrt[3]{(-1)^2}$
$\rightarrow \sqrt[3]{729} + \sqrt[3]{64} + \sqrt[3]{1}$
$\rightarrow 9^3 + 4^3 + 1^3$
$\rightarrow 9 + 4 + 1 = 14$
$\rightarrow 14$

(d) $(xy)^{2z}$
$\rightarrow x = 3, y = 4, z = -1$
$\rightarrow ((3)(4))^{2(-1)}$
$\rightarrow (12)^{-2}$
$\rightarrow \left(\frac{1}{12}\right)^2 = \frac{1^2}{12^2}$
$\rightarrow \frac{1}{144}$

7. Sederhanakan bentuk-bentuk pangkat berikut

(a) $x^8 \cdot x^2$

(b) $(3y^2)(4y^5)$

(c) $x^2 \cdot x^{-6}$

(d) $x^{-5} \cdot x^3$

(e) $w^{-2} \cdot w^{-4} \cdot w^6$

(f) $z^5 \cdot z^{-3} \cdot z^{-4}$

(g) $\dfrac{z^2 \cdot z^4}{z^3 \cdot z^{-1}}$

(h) $(2y^2)^3$

(i) $(8x)^2$

(j) $(a^2a^4)^3$

(k) $\left(\dfrac{a^2}{4}\right)^3$

(l) $(3z)^2(6z^2)^{-3}$

(m) $(2z^2)^{-5} \cdot z^{10}$

(n) $(2a^3 \cdot a^2)^4$

(o) $\left(\dfrac{3x^4}{4x^2}\right)^2$

JAWABAN:

(a) $x^8 \cdot x^2$
$\rightarrow x^{8+2} = x^{10}$
$\rightarrow x^{10}$

(b) $(3y^2)(4y^5)$
$\rightarrow (3 \cdot 4)y^{5+2} = (12)y^7$
$\rightarrow 12y^7$

(c) $x^2 \cdot x^{-6}$
$\rightarrow x^{2+(-6)} = x^{-4}$
$\rightarrow x^{-4}$

(d) $x^{-5} \cdot x^3$
$\rightarrow x^{-5+(3)} = x^{-2}$
$\rightarrow x^{-2}$

(e) $w^{-2} \cdot w^{-4} \cdot w^6$
$\rightarrow w^{(-2)+(-4)+6} = w^0$
$\rightarrow 1$

(f) $z^5 \cdot z^{-3} \cdot z^{-4}$
$\rightarrow z^{5+(-3)+(-4)} = z^{-2}$
$\rightarrow z^{-2}$

(g) $\dfrac{z^2 \cdot z^4}{z^3 \cdot z^{-1}}$
$\rightarrow \dfrac{z^{2+4}}{z^{3+(-1)}}$
$\rightarrow \dfrac{z^6}{z^2} = z^{6-2} = z^4$
$\rightarrow z^4$

(h) $(2y^2)^3$
$\rightarrow (2y^{(2\cdot3)}) = 2y^6$
$\rightarrow 2y^6$

(i) $(8x)^2$
$\rightarrow (8^2x^{1\cdot2}) = 64x^2$
$\rightarrow 64x^2$

(j) $(a^2a^4)^3$
$\rightarrow a^{(2\cdot3)}a^{(4\cdot3)} = (a^6a^{12})$
$\rightarrow a^{6+12} = a^{18}$
$\rightarrow a^{18}$

(k) $\left(\dfrac{a^2}{4}\right)^3$
$\rightarrow \left(\dfrac{a^{2\cdot3}}{4^3}\right) = \dfrac{a^6}{4^3}$
$\rightarrow \dfrac{a^6}{64}$

(l) $(3z)^2(6z^2)^{-3}$
$\rightarrow (3^2z^2)(6^{-3}z^{2\cdot-3})$
$\rightarrow (3^2z^2)(6^{-3}z^{-6})$
$\rightarrow (9z^2)\left(\dfrac{1}{6^3}z^{-6}\right)$
$\rightarrow (9z^2)\left(\dfrac{1}{216}z^{-6}\right)$
$\rightarrow \left(9 \cdot \dfrac{1}{216}\right)z^{2+(-6)}$
$\rightarrow \dfrac{9}{216}z^{-4}$

(m) $(2z^2)^{-5} \cdot z^{10}$
$\rightarrow (2^{-5}z^{2\cdot-5}) \cdot z^{10}$
$\rightarrow \left(\dfrac{1}{2^5}z^{-10}\right) \cdot z^{10}$
$\rightarrow \left(\dfrac{1}{32} \cdot z^{10}z^{-10+10}\right)$
$\rightarrow \left(\dfrac{1}{32}z^{10}z^0\right)$
$\rightarrow \left(\dfrac{1}{32}z^{10} \cdot 1\right)$
$\rightarrow \dfrac{z^{10}}{32}$

(n) $(2a^3 \cdot a^2)^4$
$\rightarrow (2a^{3+2})^4$
$\rightarrow (2a^5)^4$
$\rightarrow (2^4a^{5\cdot4})$
$\rightarrow 16a^{20}$

(o) $\left(\dfrac{3x^4}{4x^2}\right)^2$
$\rightarrow \left(\dfrac{3}{4}(x^{4-2})\right)^2 = \left(\dfrac{3}{4}x^2\right)^2$
$\rightarrow \left(\dfrac{3x^2}{4}\right)^2 = \left(\dfrac{3x^{2\cdot2}}{4^2}\right)$
$\rightarrow \dfrac{3x^4}{16}$

8. Sederhanakn bentuk berikut menjadi bentuk $K \cdot x^a \cdot y^b$ dengan K konstanta.

(a) $(4x^3y^4) \cdot (3x^5y)$

(b) $(3x^2y^3) \cdot (3^{-2}x^3y^{-2})$

(c) $\left(\dfrac{x}{y^3}\right)^{-1}$

(d) $\left(\dfrac{x}{y^3}\right)^{-1}$

(e) $(x^{3/4}y^{2/3}) \cdot (xy^2)$

(f) $(4x^2y^3)^{1/2} \cdot (9x^2y^4)^{1/3}$

(g) $\dfrac{xy^{1/3}}{x^{-1/3}y}$

(h) $(x^{-5}y^{1/3})^{-3/5}$

(i) $(4x^6y^8)^{3/2}$

(k) $\left(\dfrac{x^8y^{-4}}{16y^{4/3}}\right)^{-1/4}$

(k) $\left(\dfrac{-8x^{3/2}}{x^3y^6}\right)^{-1/3}$

JAWABAN:

(a) $(4x^3y^4) \cdot (3x^5y)$
$\to (4 \cdot 3x^{3+5}y^{4+1}) = 12x^8y^5$
$\to 12x^8y^5$

(b) $(3x^2y^3) \cdot (3^{-2}x^3y^{-2})$
$\to (3x^2y^3) \cdot (\dfrac{1}{3^2}x^3y^{-2})$
$\to (3x^2y^3) \cdot (\dfrac{1}{9}x^3y^{-2})$
$\to \left(3 \cdot \dfrac{1}{3}x^{2+3}y^{3+(-2)}\right) = \dfrac{3}{9} \cdot x^5 \cdot y^1$
$\to \dfrac{3}{9} \cdot x^5 \cdot y$

(c) $\left(\dfrac{x}{y^3}\right)^{-1}$
$\to \dfrac{1}{\left(\dfrac{x}{y^3}\right)^1} = \dfrac{y^3}{x}$
$\to \dfrac{y^3}{x} = y^3 \cdot x^{-1}$
$\to 1 \cdot x^{-1} \cdot y^3$

(d) $\left(\dfrac{x}{y^3}\right)^{-1}$
$\to \dfrac{1}{\left(\dfrac{x}{y^3}\right)^1} = \dfrac{y^3}{x}$
$\to \dfrac{y^3}{x} = y^3 \cdot x^{-1}$
$\to 1 \cdot x^{-1} \cdot y^3$

(e) $(x^{3/4}y^{2/3}) \cdot (xy^2)$
$\to (x^{(3/4)+1}y^{(2/3)+2}) = x^{7/4} \cdot y^{8/3}$
$\to 1 \cdot x^{7/4} \cdot y^{8/3}$

(f) $(4x^2y^3)^{1/2} \cdot (9x^2y^4)^{1/3}$
$\to (4^{1/2}x^1y^{3/2}) \cdot (9^{1/3}x^{2/3}y^{4/3})$
$\to (\sqrt[2]{4}x^1y^{3/2}) \cdot ((3^2)^{1/3}x^{2/3}y^{4/3})$
$\to (2x^1y^{3/2}) \cdot (3^{2/3}x^{2/3}y^{4/3})$
$\to (2 \cdot 3^{2/3} \cdot x^{1+(2/3)} \cdot y^{(3/2)+(4/3)})$
$\to 2 \cdot 3^{2/3} \cdot x^{5/3} \cdot y^{17/6}$

(g) $\dfrac{xy^{1/3}}{x^{-1/3}y}$
$\to (x^{1-(-1/3)}y^{(1/3)-1}) = x^{4/3} \cdot y^{-2/3}$
$\to 1 \cdot x^{4/3} \cdot y^{-2/3}$

(h) $(x^{-5}y^{1/3})^{-3/5}$
$\to (x^{-5 \cdot (-3/5)}y^{(1/3) \cdot (-3/5)}) = x^3 \cdot y^{-3/15}$
$\to 1 \cdot x^3 \cdot y^{-3/15}$

(i) $(4x^6y^8)^{3/2}$

$\rightarrow (4^{3/2}x^{6\cdot(3/2)}y^{8\cdot(3/2)}) = 4^{3/2} \cdot x^8 \cdot y^{12}$

$\rightarrow 4^{3/2} \cdot x^8 \cdot y^{12}$

(j) $\left(\dfrac{x^8y^{-4}}{16y^{4/3}}\right)^{-1/4}$

$\rightarrow \left(\dfrac{16y^{4/3}}{x^8y^{-4}}\right)^{1/4} = \left(\dfrac{16^{1/4}y^{(4/3)\cdot(1/4)}}{x^{8\cdot(1/4)}y^{-4\cdot(1/4)}}\right)$

$\rightarrow \dfrac{16^{1/4}y^{4/12}}{x^2y^{-1}} = \dfrac{\sqrt[4]{16}y^{4/12}}{x^2y^{-1}}$

$\rightarrow \dfrac{\sqrt[4]{16}y^{4/12}}{x^2y^{-1}} = \dfrac{2y^{4/12}}{x^2y^{-1}}$

$\rightarrow 2 \cdot x^{-2} \cdot y^{(4/12)-(-1)} = 2 \cdot x^{-2} \cdot y^{16/12}$

$\rightarrow 2 \cdot x^{-2} \cdot y^{16/12}$

(k) $\left(\dfrac{-8x^{3/2}}{x^3y^6}\right)^{-1/3}$

$\rightarrow \left(\dfrac{x^3y^6}{-8x^{3/2}}\right)^{1/3} = \left(\dfrac{x^{3\cdot(1/3)}y^{6\cdot(1/3)}}{-8^{1/3}x^{(3/2)\cdot(1/3)}}\right)$

$\rightarrow \dfrac{x^1y^2}{\sqrt[3]{-8}x^{3/6}} = \dfrac{x^1y^2}{-2x^{3/6}}$

$\rightarrow -2^{-1} \cdot x^{1-(3/6)} \cdot y^2 = -2^{-1} \cdot x^{1/2} \cdot y^2$

$\rightarrow -\dfrac{1}{2} \cdot x^{1/2} \cdot y^2$
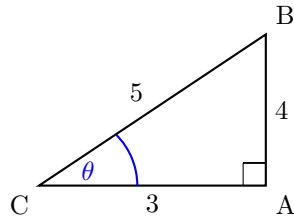
9. Sederhanakan bentuk akar berikut:

(a) $\sqrt[4]{x^4}$

(b) $\sqrt[5]{x^{10}}$

(c) $\sqrt[4]{16x^8}$

(d) $\sqrt[3]{x^3 y^6}$

(e) $\sqrt[3]{x^2 y} \cdot \sqrt[3]{64x^4 y}$

(f) $\sqrt[6]{x^5} \cdot \sqrt[3]{x^2}$

(g) $\dfrac{\sqrt[4]{x^7}}{\sqrt[4]{x^3}}$

(j) $\dfrac{\sqrt[3]{8x^2}}{\sqrt{x}}$

(i) $\sqrt[3]{y\sqrt{y}}$

(j) $\sqrt[3]{\dfrac{54x^2 y^4}{2x^5 y}}$

JAWABAN:

(a) $\sqrt[4]{x^4}$
$\rightarrow x^{4/4} = x^1$
$\rightarrow x$

(b) $\sqrt[5]{x^{10}}$
$\rightarrow x^{10/5} = x^2$
$\rightarrow x^2$

(c) $\sqrt[4]{16x^8}$
$\rightarrow (\sqrt[4]{16}x^{8/4}) = 2x^2$
$\rightarrow 2x^2$

(d) $\sqrt[3]{x^3 y^6}$
$\rightarrow (x^{3/3} y^{6/3}) = x^1 y^2$
$\rightarrow xy^2$

(e) $\sqrt[3]{x^2 y} \cdot \sqrt[3]{64x^4 y}$
$\rightarrow \sqrt[3]{64x^{(2+4)} y^{(1+1)}}$
$\rightarrow \sqrt[3]{64x^6 y^2} = \sqrt[3]{64} x^{6/3} y^{2/3}$
$\rightarrow 4x^2 y^{2/3}$

(f) $\sqrt[6]{x^5} \cdot \sqrt[3]{x^2}$
$\rightarrow x^{5/6} \cdot x^{2/3} = x^{(5/6)+(2/3)}$
$\rightarrow x^{9/6}$

(g) $\dfrac{\sqrt[4]{x^7}}{\sqrt[4]{x^3}}$
$\rightarrow x^{7/4} \div x^{3/4} = x^{(7/4)-(3-4)}$
$\rightarrow x^{1/4}$

(j) $\dfrac{\sqrt[3]{8x^2}}{\sqrt{x}}$
$\rightarrow \dfrac{8^{1/3} x^{2/3}}{x^{1/2}} = \sqrt[3]{8} x^{(2/3)-(1/2)}$
$\rightarrow 2x^{1/6}$

(i) $\sqrt[3]{y\sqrt{y}}$
$\rightarrow \sqrt[3]{y \cdot y^{1/2}} = \sqrt[3]{y^{1+(1/2)}}$
$\rightarrow \sqrt[3]{y^{3/2}} = y^{3/(3\cdot2)} = y^{3/6}$
$\rightarrow y^{3/6}$

(j) $\sqrt[3]{\dfrac{54x^2 y^4}{2x^5 y}}$
$\rightarrow \sqrt[3]{\dfrac{54}{2}} \mid x^{2-5} y^{4-1} = \sqrt[3]{27} x^{-3} y^3$
$\rightarrow 3x^{-3} y^3$

4 Latihan Soal Trigonometri.

1. Perhatikan gambar segitiga siku-siku berikut:

(a) $\sin \theta$ (b) $\sin \theta$ (c) $\tan \theta$ (d) $\sec \theta$ (e) $\csc \theta$ (f) $\cot \theta$

JAWABAN:

(a) $\sin \theta$

$\rightarrow \sin \theta = \dfrac{\text{depan}}{\text{miring}}$

$\rightarrow \sin \theta = \dfrac{4}{5}$

(b) $\cos \theta$

$\rightarrow \cos \theta = \dfrac{\text{samping}}{\text{miring}}$

$\rightarrow \cos \theta = \dfrac{3}{5}$

(c) $\tan \theta$

$\rightarrow \tan \theta = \dfrac{\text{depan}}{\text{samping}}$

$\rightarrow \tan \theta = \dfrac{4}{3}$

(d) $\sec \theta$

$\rightarrow \sec \theta = \cos^{-1} \theta$

$\rightarrow \sec \theta = \dfrac{\text{miring}}{\text{samping}}$

$\rightarrow \sec \theta = \dfrac{5}{3}$

(e) $\csc \theta$

$\rightarrow \csc \theta = \sin^{-1} \theta$

$\rightarrow \csc \theta = \dfrac{\text{miring}}{\text{depan}}$

$\rightarrow \csc \theta = \dfrac{5}{4}$

(f) $\cot \theta$
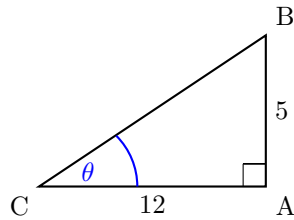
$\rightarrow \cot \theta = \tan^{-1} \theta$

$\rightarrow \cot \theta = \dfrac{\text{samping}}{\text{depan}}$

$\rightarrow \cot \theta = \dfrac{3}{4}$

2. Perhatikan gambar segitiga siku-siku berikut:

(a) $\sin \theta$ (b) $\cos \theta$ (c) $\tan \theta$ (d) $\sec \theta$ (e) $\csc \theta$ (f) $\cot \theta$

JAWABAN:

$$(\text{Mencari sisi miringnya terlebih dahulu})$$
$$\rightarrow \sqrt{5^2 + 12^2} = \sqrt{169}$$
$$\rightarrow \sqrt{169} = 13 (\text{sisi miring})$$

(a) $\sin \theta$

$\rightarrow \sin \theta = \dfrac{\text{depan}}{\text{miring}}$

$\rightarrow \sin \theta = \dfrac{5}{13}$

(c) $\tan \theta$

$\rightarrow \tan \theta = \dfrac{\text{depan}}{\text{samping}}$

$\rightarrow \tan \theta = \dfrac{5}{12}$

(e) $\csc \theta$

$\rightarrow \csc \theta = \sin^{-1} \theta$

$\rightarrow \csc \theta = \dfrac{\text{miring}}{\text{depan}}$

$\rightarrow \csc \theta = \dfrac{13}{5}$

(b) $\cos \theta$

$\rightarrow \cos \theta = \dfrac{\text{samping}}{\text{miring}}$

$\rightarrow \cos \theta = \dfrac{12}{13}$
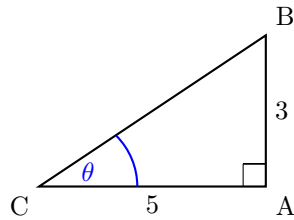
(d) $\sec \theta$

$\rightarrow \sec \theta = \cos^{-1} \theta$

$\rightarrow \sec \theta = \dfrac{\text{miring}}{\text{samping}}$

$\rightarrow \sec \theta = \dfrac{13}{12}$

(f) $\cot \theta$

$\rightarrow \tan \theta = \tan^{-1} \theta$

$\rightarrow \tan \theta = \dfrac{\text{samping}}{\text{depan}}$

$\rightarrow \tan \theta = \dfrac{12}{5}$

3. Perhatikan gambar segitiga siku-siku berikut:

(a) $\sin \theta$ (b) $\cos \theta$ (c) $\tan \theta$ (d) $\sec \theta$ (e) $\csc \theta$ (f) $\cot \theta$

JAWABAN:

$$\text{(Mencari sisi miringnya terlebih dahulu)}$$
$$\rightarrow \sqrt{3^2 + 5^2} = \sqrt{34}$$
$$\rightarrow \sqrt{34}(\text{sisi miring})$$

(a) $\sin \theta$

$\rightarrow \sin \theta = \dfrac{\text{depan}}{\text{miring}}$

$\rightarrow \sin \theta = \dfrac{3}{\sqrt{34}}$

$\rightarrow \dfrac{3}{\sqrt{34}} \cdot \dfrac{\sqrt{34}}{\sqrt{34}} = \dfrac{3\sqrt{34}}{34}$

$\rightarrow \dfrac{3}{34}\sqrt{34}$

(b) $\cos \theta$

$\rightarrow \cos \theta = \dfrac{\text{samping}}{\text{miring}}$

$\rightarrow \cos \theta = \dfrac{5}{\sqrt{34}}$

$\rightarrow \dfrac{5}{\sqrt{34}} \cdot \dfrac{\sqrt{34}}{\sqrt{34}} = \dfrac{5\sqrt{34}}{34}$

$\rightarrow \dfrac{5}{34}\sqrt{34}$

(c) $\tan \theta$

$\rightarrow \tan \theta = \dfrac{\text{depan}}{\text{samping}}$

$\rightarrow \tan \theta = \dfrac{3}{5}$

(d) $\sec \theta$

$\rightarrow \sec \theta = \cos^{-1} \theta$

$\rightarrow \sec \theta = \dfrac{\text{miring}}{\text{samping}}$

$\rightarrow \sec \theta = \dfrac{\sqrt{34}}{5}$

(e) $\csc \theta$

$\rightarrow \csc \theta = \sin^{-1} \theta$

$\rightarrow \csc \theta = \dfrac{\text{miring}}{\text{depan}}$

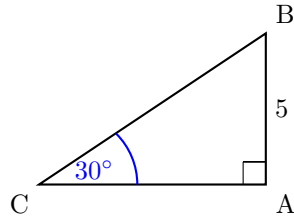$\rightarrow \csc \theta = \dfrac{\sqrt{34}}{3}$

(f) $\cot \theta$

$\rightarrow \tan \theta = \tan^{-1} \theta$

$\rightarrow \tan \theta = \dfrac{\text{samping}}{\text{depan}}$

$\rightarrow \tan \theta = \dfrac{5}{3}$

4. Perhatikan gambar segitiga siku-siku berikut:

(a) $\sin \theta$ (b) $\cos \theta$ (c) $\tan \theta$ (d) $\sec \theta$ (e) $\csc \theta$ (f) $\cot \theta$

JAWABAN:

(Mencari sisi samping)
$$\rightarrow \tan 30° = \frac{\text{depan}}{\text{samping}}$$
$$\rightarrow \frac{1}{\sqrt{3}} = \frac{5}{\text{samping}}$$
$$\rightarrow 1 \cdot \text{samping} = 5 \cdot \sqrt{3}$$
$$\rightarrow \text{samping} = \frac{5\sqrt{3}}{1}$$
$$\rightarrow \text{samping} = 5\sqrt{3}(\text{sisi samping})$$

(Mencari sisi miring)
$$\rightarrow \sqrt{5^2 + (5\sqrt{3})^2} = \sqrt{25 + 75}$$
$$\rightarrow \sqrt{100} = 10(\text{sisi miring})$$

(a) $\sin \theta$

$$\rightarrow \sin \theta = \frac{\text{depan}}{\text{miring}}$$
$$\rightarrow \sin \theta = \frac{5}{10}$$

(b) $\cos \theta$

$$\rightarrow \cos \theta = \frac{\text{samping}}{\text{miring}}$$
$$\rightarrow \cos \theta = \frac{5\sqrt{3}}{10}$$
$$\rightarrow \cos \theta = \frac{5}{10}\sqrt{3}$$

(c) $\tan \theta$

$$\rightarrow \tan \theta = \frac{\text{depan}}{\text{samping}}$$
$$\rightarrow \tan \theta = \frac{5}{5\sqrt{3}}$$
$$\rightarrow \tan \theta = \frac{5}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$\rightarrow \tan \theta = \frac{5\sqrt{3}}{15} = \frac{5}{15}\sqrt{3}$$

(d) $\sec \theta$

$$\rightarrow \sec \theta = \cos^{-1} \theta$$
$$\rightarrow \sec \theta = \frac{\text{miring}}{\text{samping}}$$
$$\rightarrow \sec \theta = \frac{10}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$\rightarrow \sec \theta = \frac{10\sqrt{3}}{15} = \frac{10}{15}\sqrt{3}$$

(e) $\csc \theta$

$$\rightarrow \csc \theta = \sin^{-1} \theta$$
$$\rightarrow \csc \theta = \frac{\text{miring}}{\text{depan}}$$
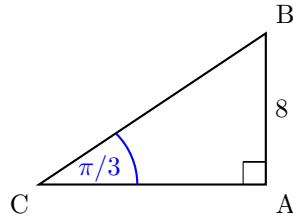$$\rightarrow \csc \theta = \frac{10}{5} = 2$$

(f) $\cot \theta$

$$\rightarrow \tan \theta = \tan^{-1} \theta$$
$$\rightarrow \tan \theta = \frac{\text{samping}}{\text{depan}}$$
$$\rightarrow \tan \theta = \frac{5\sqrt{3}}{5} = \sqrt{3}$$

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5. Perhatikan gambar segitiga siku-siku berikut:

(a) $\sin \theta$ (b) $\cos \theta$ (c) $\tan \theta$ (d) $\sec \theta$ (e) $\csc \theta$ (f) $\cot \theta$

JAWABAN:

(Mengubah dari radian ke derajat)
$$\rightarrow \frac{\pi}{3} = \frac{180}{3}$$
$$\rightarrow 60°$$

(Mencari sisi samping)
$$\rightarrow \tan 60° = \frac{\text{depan}}{\text{samping}}$$
$$\rightarrow \frac{\sqrt{3}}{1} = \frac{8}{\text{samping}}$$
$$\rightarrow \sqrt{3} \cdot \text{samping} = 8 \cdot 1$$
$$\rightarrow \text{samping} = \frac{8}{\sqrt{3}}$$
$$\rightarrow \frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$$
$$\rightarrow \text{samping} = \frac{8}{3}\sqrt{3} (\text{sisi samping})$$

(Mencari sisi miring)
$$\rightarrow \sqrt{8^2 + (\frac{8}{3}\sqrt{3})^2} = \sqrt{64 + \frac{192}{9}}$$
$$\rightarrow \sqrt{\frac{768}{9}} = \frac{16\sqrt{3}}{3} = \frac{16}{3}\sqrt{3} (\text{sisi miring})$$

(a) $\sin \theta$

$$\rightarrow \sin \theta = \frac{\text{depan}}{\text{miring}}$$
$$\rightarrow \sin \theta = \frac{8}{\frac{16}{3}\sqrt{3}}$$
$$\rightarrow \sin \theta = \frac{24}{16\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$\rightarrow \sin \theta = \frac{24\sqrt{3}}{48}$$
$$\rightarrow \sin \theta = \frac{24}{48}\sqrt{3}$$

(b) $\cos \theta$

$$\rightarrow \cos \theta = \frac{\text{samping}}{\text{miring}}$$
$$\rightarrow \cos \theta = \frac{\frac{8}{3}\sqrt{3}}{\frac{16}{3}\sqrt{3}}$$
$$\rightarrow \cos \theta = \frac{24\sqrt{3}}{48\sqrt{3}}$$
$$\rightarrow \cos \theta = \frac{24}{48}$$

(c) $\tan \theta$

$$\rightarrow \tan \theta = \frac{\text{depan}}{\text{samping}}$$
$$\rightarrow \tan \theta = \frac{8}{\frac{8}{3}\sqrt{3}}$$
$$\rightarrow \tan \theta = \frac{24}{8\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$\rightarrow \tan \theta = \frac{24\sqrt{3}}{24}$$
$$\rightarrow \tan \theta = \sqrt{3}$$

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(d) $\sec \theta$

$\rightarrow \sec \theta = \cos^{-1} \theta$

$\rightarrow \sec \theta = \dfrac{\text{miring}}{\text{samping}}$

$\rightarrow \sec \theta = \dfrac{\frac{16}{3}\sqrt{3}}{\frac{8}{3}\sqrt{3}}$

$\rightarrow \sec \theta = \dfrac{48\sqrt{3}}{24\sqrt{3}}$

$\rightarrow \sec \theta = 2$

(e) $\csc \theta$

$\rightarrow \csc \theta = \sin^{-1} \theta$

$\rightarrow \csc \theta = \dfrac{\text{miring}}{\text{depan}}$

$\rightarrow \csc \theta = \dfrac{\frac{16}{3}\sqrt{3}}{8}$

$\rightarrow \csc \theta = \dfrac{16\sqrt{3}}{24}$

$\rightarrow \csc \theta = \dfrac{16}{24}\sqrt{3}$

(f) $\cot \theta$

$\rightarrow \tan \theta = \tan^{-1} \theta$

$\rightarrow \tan \theta = \dfrac{\text{samping}}{\text{depan}}$

$\rightarrow \tan \theta = \dfrac{\frac{8}{3}\sqrt{3}}{8}$

$\rightarrow \tan \theta = \dfrac{8\sqrt{3}}{24}$

$\rightarrow \tan \theta = \dfrac{8}{24}\sqrt{3}$

6. Jika $\sin \alpha = \frac{2}{3}$, tentukan hasil dari

(a) $\cos \alpha$ (b) $\tan \alpha$ (c) $\sec \alpha$ (d) $\cot \alpha$ (e) $\csc \alpha$

JAWABAN:

$$(\text{Mencari sisi samping})$$
$$\rightarrow \sin \alpha = \frac{2 \ (\text{depan})}{3 \ (\text{miring})}$$
$$\rightarrow \sqrt{3^2 - 2^2} = \sqrt{5} \ (\text{samping})$$

(a) $\cos \alpha$

$$\rightarrow \cos \theta = \frac{\text{samping}}{\text{miring}}$$
$$\rightarrow \cos \theta = \frac{\sqrt{5}}{3}$$

(b) $\tan \alpha$

$$\rightarrow \tan \theta = \frac{\text{depan}}{\text{samping}}$$
$$\rightarrow \tan \theta = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$
$$\rightarrow \tan \theta = \frac{2\sqrt{5}}{5}$$
$$\rightarrow \tan \theta = \frac{2}{5}\sqrt{5}$$

(c) $\sec \alpha$

$$\rightarrow \sec \theta = \cos^{-1} \theta$$
$$\rightarrow \sec \theta = \frac{\text{miring}}{\text{samping}}$$
$$\rightarrow \sec \theta = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$
$$\rightarrow \sec \theta = \frac{3\sqrt{5}}{5}$$
$$\rightarrow \sec \theta = \frac{3}{5}\sqrt{5}$$

(d) $\cot \alpha$

$$\rightarrow \tan \theta = \tan^{-1} \theta$$
$$\rightarrow \tan \theta = \frac{\text{samping}}{\text{depan}}$$
$$\rightarrow \tan \theta = \frac{\sqrt{5}}{2}$$

(e) $\csc \alpha$

$$\rightarrow \csc \theta = \sin^{-1} \theta$$
$$\rightarrow \csc \theta = \frac{\text{miring}}{\text{depan}}$$
$$\rightarrow \csc \theta = \frac{3}{2}$$

7. Jika $\cos \alpha = \frac{7}{25}$, tentukan hasil dari

(a) $\sin \alpha$ (b) $\tan \alpha$ (c) $\sec \alpha$ (d) $\cot \alpha$ (e) $\csc \alpha$

JAWABAN:

$$\text{(Mencari sisi depan)}$$
$$\rightarrow \cos \alpha = \frac{7 \text{ (samping)}}{25 \text{ (miring)}}$$
$$\rightarrow \sqrt{25^2 - 7^2} = \sqrt{576} = 24 \text{ (depan)}$$

(a) $\sin \alpha$

$$\rightarrow \sin \theta = \frac{\text{depan}}{\text{miring}}$$
$$\rightarrow \sin \theta = \frac{24}{25}$$

(c) $\sec \alpha$

$$\rightarrow \sec \theta = \cos^{-1} \theta$$
$$\rightarrow \sec \theta = \frac{\text{miring}}{\text{samping}}$$
$$\rightarrow \sec \theta = \frac{25}{7}$$

(e) $\csc \alpha$

$$\rightarrow \csc \theta = \sin^{-1} \theta$$
$$\rightarrow \csc \theta = \frac{\text{miring}}{\text{depan}}$$
$$\rightarrow \csc \theta = \frac{25}{24}$$

(d) $\cot \alpha$

$$\rightarrow \tan \theta = \tan^{-1} \theta$$
$$\rightarrow \tan \theta = \frac{\text{samping}}{\text{depan}}$$
$$\rightarrow \tan \theta = \frac{7}{24}$$

(b) $\tan \alpha$

$$\rightarrow \tan \theta = \frac{\text{depan}}{\text{samping}}$$
$$\rightarrow \tan \theta = \frac{24}{7}$$

8. Jika $\tan \alpha = \frac{3}{5}$, tentukan hasil dari

(a) $\sin \alpha$ (b) $\cos \alpha$ (c) $\sec \alpha$ (d) $\cot \alpha$ (e) $\csc \alpha$

JAWABAN:

$$\text{(Mencari sisi miring)}$$
$$\rightarrow \tan \alpha = \frac{3 \text{ (depan)}}{5 \text{ (samping)}}$$
$$\rightarrow \sqrt{3^2 + 5^2} = \sqrt{34} \text{ (miring)}$$

(a) $\sin \alpha$

$$\rightarrow \sin \theta = \frac{\text{depan}}{\text{miring}}$$
$$\rightarrow \sin \theta = \frac{3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}}$$
$$\rightarrow \sin \theta = \frac{3\sqrt{34}}{34}$$
$$\rightarrow \sin \theta = \frac{3}{34}\sqrt{34}$$

(b) $\cos \alpha$

$$\rightarrow \tan \theta = \frac{\text{samping}}{\text{miring}}$$
$$\rightarrow \tan \theta = \frac{5}{\sqrt{34} \cdot \frac{\sqrt{34}}{\sqrt{34}}}$$
$$\rightarrow \tan \theta = \frac{5\sqrt{34}}{34}$$
$$\rightarrow \tan \theta = \frac{5}{34}\sqrt{34}$$

(c) $\sec \alpha$

$$\rightarrow \sec \theta = \cos^{-1} \theta$$
$$\rightarrow \sec \theta = \frac{\text{miring}}{\text{samping}}$$
$$\rightarrow \sec \theta = \frac{\sqrt{34}}{5}$$

(d) $\cot \alpha$

$$\rightarrow \tan \theta = \tan^{-1} \theta$$
$$\rightarrow \tan \theta = \frac{\text{samping}}{\text{depan}}$$
$$\rightarrow \tan \theta = \frac{5}{3}$$

(e) $\csc \alpha$

$$\rightarrow \csc \theta = \sin^{-1} \theta$$
$$\rightarrow \csc \theta = \frac{\text{miring}}{\text{depan}}$$
$$\rightarrow \csc \theta = \frac{\sqrt{34}}{3}$$

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9. Tentukan hasil dari nilai geometri berikut:

(a) $\sin\left(\dfrac{\pi}{4}\right)$

(b) $\cos\left(\dfrac{2\pi}{3}\right)$

(c) $\sin\left(\dfrac{3\pi}{2}\right)$

(d) $\tan\left(\dfrac{3\pi}{4}\right)$

(e) $\cos\left(\dfrac{7\pi}{4}\right)$

(f) $\tan\left(\dfrac{5\pi}{3}\right)$

JAWABAN:

(a) $\sin\left(\dfrac{\pi}{4}\right)$

$\rightarrow \sin\left(\dfrac{1\cdot 180°}{4}\right) = \sin\left(\dfrac{180°}{4}\right)$

$\rightarrow \sin(45°)[\text{Kdr I}] = \dfrac{1}{2}\sqrt{2}$

$\rightarrow \dfrac{1}{2}\sqrt{2}$

(b) $\cos\left(\dfrac{2\pi}{3}\right)$

$\rightarrow \cos\left(\dfrac{2\cdot 180°}{3}\right) = \cos\left(\dfrac{360°}{3}\right)$

$\rightarrow \cos(120°)[\text{Kdr II}] = -\cos(180° - 120°)$

$\rightarrow \cos(120°)[\text{Kdr II}] = -\cos(60°)$

$\rightarrow -\cos(60°) = -\dfrac{1}{2}$

$\rightarrow -\dfrac{1}{2}$

(c) $\sin\left(\dfrac{3\pi}{2}\right)$

$\rightarrow \sin\left(\dfrac{3\cdot 180°}{2}\right) = \sin\left(\dfrac{540°}{2}\right)$

$\rightarrow \sin(270°)[\text{Kdr IV}] = -\sin(360° - 270°)$

$\rightarrow \sin(270°)[\text{Kdr IV}] = -\sin(90°)$

$\rightarrow -\sin(90°) = -1$

$\rightarrow -1$

(d) $\tan\left(\dfrac{3\pi}{4}\right)$

$\rightarrow \tan\left(\dfrac{3\cdot 180°}{4}\right) = \tan\left(\dfrac{540°}{4}\right)$

$\rightarrow \tan(135°)[\text{Kdr II}] = -\tan(180° - 135°)$

$\rightarrow \tan(135°)[\text{Kdr II}] = -\tan(45°)$

$\rightarrow -\tan(45°) = -1$

$\rightarrow -1$

(e) $\cos\left(\dfrac{7\pi}{4}\right)$

$\rightarrow \cos\left(\dfrac{7\cdot 180°}{4}\right) = \cos\left(\dfrac{1260°}{4}\right)$

$\rightarrow \cos(315°)[\text{Kdr IV}] = \cos(360° - 315°)$

$\rightarrow \cos(315°)[\text{Kdr IV}] = \cos(45°)$

$\rightarrow \cos(45°) = \dfrac{1}{2}\sqrt{2}$

$\rightarrow \dfrac{1}{2}\sqrt{2}$

(f) $\tan\left(\dfrac{5\pi}{3}\right)$

$\rightarrow \tan\left(\dfrac{5\cdot 180°}{3}\right) = \tan\left(\dfrac{900°}{3}\right)$

$\rightarrow \tan(300°)[\text{Kdr IV}] = \tan(360° - 300°)$

$\rightarrow \tan(300°)[\text{Kdr IV}] = \tan(60°)$

$\rightarrow \tan(60°) = \sqrt{3}$

$\rightarrow \sqrt{3}$

10. Tentukan hasil dari nilai geometri berikut:

(a) $\sin (75°)$ (c) $\tan (75°)$ (e) $\cos (75°)$

JAWABAN:

(a) $\sin (75°)$

$\rightarrow \sin (75°) = \sin (45° + 30°)$

$\rightarrow \sin (45° + 30°) = \sin (45°) \cdot \cos (30°) + \cos (45°) \cdot \sin (30°)$

$\rightarrow \dfrac{1}{2}\sqrt{2} \cdot \dfrac{1}{2}\sqrt{3} + \dfrac{1}{2}\sqrt{2} \cdot \dfrac{1}{2}$

$\rightarrow \dfrac{1}{4}\sqrt{6} + \dfrac{1}{4}\sqrt{2}$

$\rightarrow \dfrac{\sqrt{6} + \sqrt{2}}{4}$

(b) $\tan (75°)$

$\rightarrow \tan (75°) = \tan (45° + 30°)$

$\rightarrow \tan (45° + 30°) = \dfrac{\tan(45°) + \tan(30°)}{1 - \tan(45°) \cdot \tan(30°)}$

$\rightarrow \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \dfrac{1 + \left(\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}\right)}$

$\rightarrow \dfrac{1 + \frac{1}{3}\sqrt{3}}{1 - \frac{1}{3}\sqrt{3}} = \dfrac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \dfrac{3 + \sqrt{3}}{3 - \sqrt{3}}$

$\rightarrow \dfrac{3 + \sqrt{3}}{3 - \sqrt{3}} \cdot \dfrac{3 + \sqrt{3}}{3 + \sqrt{3}} = \dfrac{9 + 3\sqrt{3} + 3\sqrt{3} + \sqrt{9}}{9 + 3\sqrt{3} - 3\sqrt{3} - \sqrt{9}} = \dfrac{9 + 3 + 6\sqrt{3}}{9 - 3} = \dfrac{12 + 6\sqrt{3}}{6}$

$\rightarrow \dfrac{\cancel{12} + \cancel{6}\sqrt{3}}{\cancel{6}} \div \dfrac{\cancel{6}}{\cancel{6}} = \dfrac{2 + 1\sqrt{3}}{1} = 2 + \sqrt{3}$

$\rightarrow 2 + \sqrt{3}$

(c) $\cos (75°)$

$\rightarrow \cos (75°) = \cos (45° + 30°)$

$\rightarrow \cos (45° + 30°) = \cos (45°) \cdot \cos (30°) - \sin (45°) \cdot \sin (30°)$

$\rightarrow \dfrac{1}{2}\sqrt{2} \cdot \dfrac{1}{2}\sqrt{3} - \dfrac{1}{2}\sqrt{2} \cdot \dfrac{1}{2}$

$\rightarrow \dfrac{1}{4}\sqrt{6} - \dfrac{1}{4}\sqrt{2}$

$\rightarrow \dfrac{\sqrt{6} - \sqrt{2}}{4}$

REFERENSI

Modul Matrikulasi Institut Teknologi Kalimantan 2024...(I)

ChatGPT(OpenAI)..(II)

Figure 1: Meowing skibidi gyattt +1000000 Aura