ECE 105 - Physics of Electrical Engineering 1

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Forces and Motion

Force is a vector, and therefore includes direction. For any vector a, $-a$ has the same magnitude but opposite direction.

Coordinate Systems

Given $\vec{AB}$ we can find A or B's position based on the position of the other one

$$\vec{O_B} = \vec{O_A} + \vec{AB}$$

for any $\vec{O_x}$ is the location of x relative to the origin.

Components

We can break any vector into **components** by finding the angle between it and the plane we want to model it off of.

Example: for $\vec{A} = 6@20°$, we can find it's components with relation to the standard x-y plane with

$$\vec{A_y} = \vec{A}\cos 20°$$
$$\vec{A_x} = \vec{A}\sin 20°$$

Constant Acceleration

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$
$$\vec{a} = \frac{\vec{v_f} - \vec{v_i}}{\Delta t}$$
$$\vec{a}\Delta t = \vec{v_f} - \vec{v_i}$$
$$\vec{v_f} = \vec{v_i} + \vec{a}\Delta t$$

For the position vector $\vec{d}, \vec{d_f} = \vec{d_i} + \vec{v_i}\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$

$\vec{v_f}^2 = \vec{v_i}^2 + 2\vec{a}\big(\Delta\vec{d}\big)$

Relative Motion

For any three objects a, b, and c

$$\vec{v_{ca}} = \vec{v_{cb}} + \vec{v_{ba}}$$

read "the velocity of c with respect to a is equal to the velocity of c with respect to b *plus* the velocity of b with respect to a.

Circular Motion

$a_c = \frac{v^2}{r}$
$\theta \approx \frac{\triangle x}{r}$ for very close points.
For circle with center O and radius r_0, r_1... connected to object on circumferance with velocity $\vec{v_0}, \vec{v_1}$... tangent to circumferance, $\vec{a} = \frac{\vec{v_1} - \vec{v_0}}{t}$. For arc length between object (at different times t_0, t_1...) s, $\theta = \frac{s}{r}$. For small $\theta \ll 1, |\vec{v_1} - \vec{v_0}| = \theta\vec{v}$.
$|\vec{a}| = \frac{\vec{v}\theta}{t}$
$\vec{a}$ is perpendicular to $\vec{v}$
$\vec{v} = \frac{r_1 - r_0}{t} = \frac{r\theta}{t}$
$\therefore \vec{a} = \frac{\vec{v}\theta}{r\frac{\theta}{\vec{v}}} = \frac{\vec{v}^2}{r}$

$$\begin{aligned} s &= r\theta \\ \frac{ds}{dt} = v &= r\frac{d\theta}{dt} \\ &= r\omega \\ \omega &= \frac{d\theta}{dt} \end{aligned}$$

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \frac{(r\omega)^2}{r} \\ &= r\omega^2 \end{aligned}$$

Types of Forces

A force is a push or pull interaction between two objects, reponsible for changing motion.

Springs

When unstretched, no **spring forces** exist. When a string is pushed from equilibrium, its spring force pushes back toward equilibrium.

$$F_s = -k\Delta x$$

Tension

The **tension force** pulls an object toward a rope and a rope toward an object. Ropes can never push.

Normal

The **normal force** ”pushes back” against other objects via molecular electromagnetism. It is always perpendicular to the surface for any surface-to-surface contact. Technically, it is a type of spring force.

Friction

Friction is the interaction between an object and a surface. It is a real force which acts opposite the direction of sliding, and is always tangent to surface.

$f \propto N$ is an experimental fact. $f = \mu N$, where μ is the coefficient of friction. μ is dependant on the type of objects and must be determined experimentally.

Kinetic friction is when objects are sliding relative to each other and **static friction** is when objects are not yet sliding

$$f_s \leq \mu_s N$$

Example: A 50kg person is in a 1000kg elevator at rest. When the elevator begins to rise, the person notices her weight is 600N. How far does the elevator move in 3s?

$$
\begin{aligned}
\Sigma\vec{F} &= m\vec{a} \\
\vec{a} &= \frac{\vec{F}_n - mg}{m} \\
&= \frac{600 - 50g}{50} \\
&= 2.2\mathrm{m/s}^2
\end{aligned}
$$

$$
\begin{aligned}
d &= \vec{v_i}t + \frac{1}{2}\vec{a}t^2 \\
&= 0 + \frac{1}{2}(2.2)9 \\
&= 9.9\mathrm{m}
\end{aligned}
$$

Energy

An object can be said to have a total **energy** equal to the sum of the various forms of energy it may posess.

Kinetic Energy

The **kinetic energy** of an object is determined by its mass and velocity

$$K = \frac{mv^2}{2}$$

For any object with a changing velocity

$$
\begin{aligned}
v_f^2 &= v_i^2 + 2ad \\
\frac{1}{2}mv_f^2 &= \frac{1}{2}mv_i^2 + mad \\
K_f &= K_i + \Sigma\vec{F}d \\
\Delta K &= \Sigma\vec{F}d
\end{aligned}
$$

Potential Gravitational Energy

Potential gravitational energy is a measure of stored energy of an object based on its height. It is essentially non-sensical to determine an object's ”absolute” potential gravitational energy, thus we often simply solve for the difference in energy.

For a distance h_f above a reference height h_i

$$U_g = mg(h_f - h_i)$$

thus if an object moves from h_i to h_f

$$\begin{aligned}\Delta U_g &= U_{gf} - U_{gi} \\ &= mgh_f - mgh_i \\ &= mg\Delta h\end{aligned}$$

Spring Energy

A **spring's energy** is based on its spring constant k and how far it is compressed from its equilibrium point

$$U_s = \frac{kx^2}{2}$$

Collisions

If a **collision** is isolated, then energy is conserved. Elastic collisions also conserve energy. For all real or inelastic collisions, energy is lost.

Work

Just as energy is a way of keeping track of motion, **work** is a mechanical means for transfering energy equal to the applied force multiplied by the distance it operates along

$$dW = \vec{F}\vec{ds}$$

It can be used to compute the change in energy of a system between two states, as the total work done by non-conservative forces (ie friction) will be equal to the work done by conservative forces (ie gravity, springs, motion)

For a system involving friction, motion, gravity, and a spring, we have

$$\Delta E_{th} = \Delta K + \Delta U_g + \Delta U_s$$

or, if we compute the value of the thermal work done by friction as energy (using $U_f = \mu N d$, where d is the distance during which the object undergoes friction), we get

$$0 = \Delta K + \Delta U_g + \Delta U_s + \Delta U_f$$

Rotation (of a non-deformable, rigid bodied object)

For any point on an object in **circular rotation**

$$\omega = \frac{d\theta}{dt}$$
$$s = r\theta$$
$$v = \frac{ds}{dt} = \frac{rd\theta}{dt} = r\omega$$
$$a_r = \frac{v^2}{r} = r\omega^2$$
$$a_t = \frac{dv}{dt} = \frac{rd\omega}{dt} = r\alpha$$

where ω is the angular frequency, s is the arc length of a circle, and α is the angular acceleration

Centre of Mass

For a uniform mass distribution, the **centre of mass** is in the geometric centre. Otherwise

$$x_{centre} = \frac{m_1 x_1 + ... + m_n x_n}{m_1 + ... + m_n}$$

Gravity acts as if all the mass is located at the centre of mass.

Rotational Energy

$$E_k = \frac{mv^2}{2} = \frac{mr^2\omega^2}{2} = \frac{I\omega^2}{2}$$

where I is the moment of inertia.

Moment of Inertia

$$I = m_1 x_1^2 + ... + m_n x_n^2$$

For a thin rod of length L and uniform mass m,

$$I = \frac{mL^2}{12}$$

For a filled ring of radius r and uniform mass m (regardless of length, ie. cylinders)

$$I = \frac{mr^2}{2}$$

For a hollow ring of radius r and uniform mass m (regardless of length, ie. hollow cylinders)

$$I = mr^2$$

For a filled sphere of radius r and uniform mass m

$$I = \frac{2mr^2}{5}$$

For a hollow sphere of radius r and uniform mass m

$$I = \frac{2mr^2}{3}$$

To find a "new" moment of inertia, where h is the distance to the new pivot

$$I = I_0 + mh^2$$

Torque

Torque is a measure of how much a given applied force "wants" to rotate an object, where r is the direction from an object to its pivot and θ is the angle between r and the applied force F

$$\tau = rFsin\theta$$

Static Equilibrium

Any object in **static equilibrium** undergoes no motion at all.

$$\Sigma F = \Sigma\tau = 0$$

Rotational Dynamics

$$\Sigma\tau = I\alpha$$

Oscillations

As **oscillation** is a periodic motion about an equilibrium position.

Simple Harmonic Motion

Any object in **simple harmonic motion** follows a sinusoidal shape in terms of its distance from its equilibrium position. Note:

$$\omega = 2\pi f$$

$$\begin{aligned} x &= A\cos(\omega t + \phi) \\ v &= -A\omega \sin(\omega t + \phi) \end{aligned}$$

As you can see

$$v_{\text{max}} = A\omega$$

Oscillation Dynamics

$$\begin{aligned} x(t) &= A\cos(\omega t + \phi) \\ v(t) &= -Aw \sin(\omega t + \phi) \\ a(t) &= -A\omega^2 \cos(\omega t + \phi) \\ a &= -\omega^2 x \\ a_{max} &= -A\omega^2 \end{aligned}$$

Simple Harmonic Motion Dynamics

$$\begin{aligned} x &= A\cos(\omega t + \phi) \\ \omega &= 2\pi f = \frac{2\pi}{T} \\ v &= -A\omega \sin(\omega t + \phi) \\ a &= -A\omega^2 \cos(\omega t + \phi) \\ \frac{\mathrm{d}^2 x}{\mathrm{dt}^2} &= -\omega^2 x \end{aligned}$$

Therefore, if

$$a = -Cx$$

we know that a solution of x is

$$x = A\cos(\sqrt{C}t + \phi)$$

For an ideal spring where

$$Fs = -kx$$

by dividng by mass we get

$$\omega = \sqrt{\frac{k}{m}}$$

Simple Pendulum

For a **simple pendulum**

$$a_r = \frac{v^2}{l}$$

and

$$a_t = \alpha l$$

$$\begin{aligned} mg\sin\theta &= m\alpha l \\ \alpha &= \frac{g}{l}\sin\theta \\ \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} &= \alpha \end{aligned}$$

If $\theta << 1$ then $\sin\theta = \theta$

Physical Pendula

For a **physical pendulum**, the centre of mass is the location where gravity acts, and thus we have

$$\begin{aligned} \Sigma\tau &= I\alpha \\ mgx\sin\theta &= I\alpha \\ \theta'' &= \frac{mgx}{I}\sin\theta \\ &\approx \frac{mgx}{I}\theta \\ \omega &= \sqrt{\frac{mgx}{I}} \end{aligned}$$

This tends to

$$\omega = \sqrt{n\frac{g}{l}}$$

where n is some real number.

Energy Conservation

For any object in simple harmonic motion, **energy must be conserved**. Thus we have

$$\begin{aligned} E &= \frac{1}{2} I \omega_{max}^2 \\ &= mgh \end{aligned}$$

Waves

Waves are physical areas of increased or decreased energy, which travel in simple harmonic motion. They can be visuallised on a horizontal line with regular "humps".

Wave Propogation

The **propogation** of a wave is the direction in which it travels. The form of the wave does not change as it propogates. A wave can be modelled by the equation

$$y = f(x \pm vt)$$

travelling tot he left/right (plus/minus) where $y = f(x)$ is the equation of the wave.

We can find the transverse travelling wave by assuming the shape is preserved.

Harmonic Waves

Harmonic waves have a sinsusoidal form.

For a stationary harmonic wave, we have

$$\begin{aligned} y = f(x) &= A \sin(2\pi \frac{x \pm vt}{\lambda} + \phi) \\ &= A \sin(\frac{2\pi x}{\lambda} - \omega t + \phi) \\ &= A \sin(kx - \omega t + \phi) \end{aligned}$$

where

$$k = \frac{2\pi}{\lambda}$$

Thus the speed of any particle on the wave (in the up/down direction, where x is constant) is

$$v = -A\omega \sin(kx - \omega t + \phi)$$

Waves on a String

The velocity of a **wave on a string** is based solely on the properties of the string. We define the mass density as $\mu = \frac{m}{l}$ so that

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension in the string.

Laws of Superposition

For $y_1 = f_1(x_1 t)$ and $y_2 = f_2(x_2 t)$

$$y(x,t) = f_1(x_1,t) + f_2(x_2,t)$$

The **superposition** of two waves is **constructive** if it results in a larger amplitude and **destructive** if it results in a small amplitude.

Consider two harmonic waves which are identical except for a phase shift travelling in the same direction on the same string.