全谱的原理与算法

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摘 要

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目 录

1 基本原理

1.1 旋转机械轴振动描述

转子系统振动的可描述为简谐运动 [1,2]

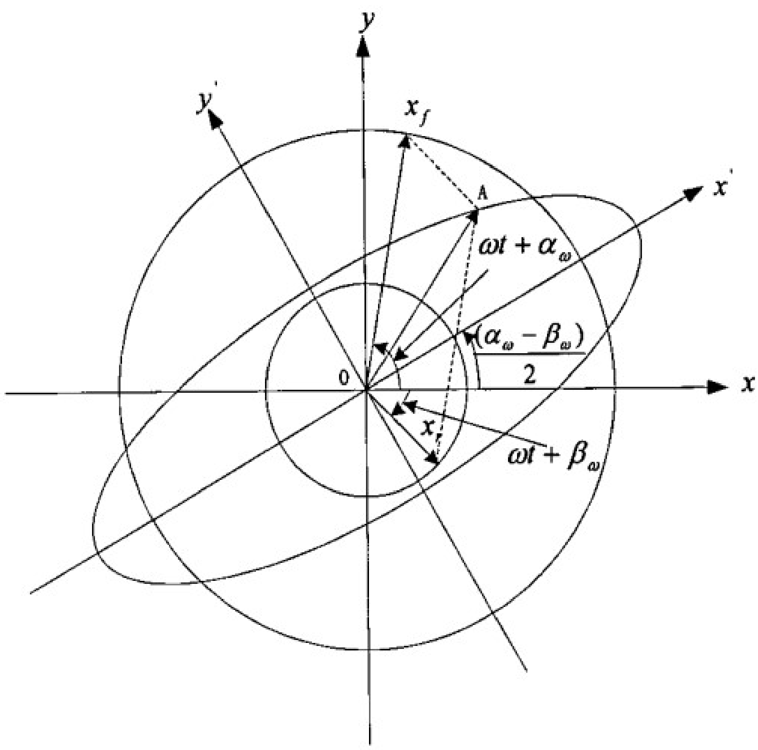


图 1: 简谐运动的合成

首先关注工频振动，在 x,y 方向的运动上可以描述为

$$\begin{cases} x = x_0 \cos(\omega t + \phi_x) \\ y = y_0 \sin(\omega t + \phi_y) \end{cases} \tag{1}$$

其中，x, y 为转子中心在 x_0, y_0 方向的运动幅值，ϕ_x, ϕ_y 为初始相位角；ω 为转子运动的角速度；t 为时间。

$$\begin{cases} x = x_c \cos\omega t - x_s \sin\omega t \\ y = y_c \cos\omega t - y_s \sin\omega t \end{cases} \tag{2}$$

其中

$$
\begin{cases}
x_c = x_0 \cos\phi_x, x_s = x_0 \sin\phi_x \\
y_c = y_0 \cos\phi_y, y_s = y_0 \sin\phi_y \\
x_0 = \sqrt{x_c^2 + x_s^2}, y_0 = \sqrt{y_c^2 + y_s^2}
\end{cases}
\tag{3}
$$

$$
\phi_x = \arctan\frac{x_s}{x_c}, \phi_y = \arctan\frac{y_s}{y_c}
$$

证明：由三角公式

$$
x = x_0 \cos(\omega t + \phi_x) = x_0 \cos\omega t \cos\phi_x - x_0 \sin\omega t \sin\phi_x
$$

$$
y = y_0 \sin(\omega t + \phi_y) = y_0 \sin\omega t \cos\phi_y + y_0 \cos\omega t \sin\phi_y
$$

得证。运动表达为如图所示

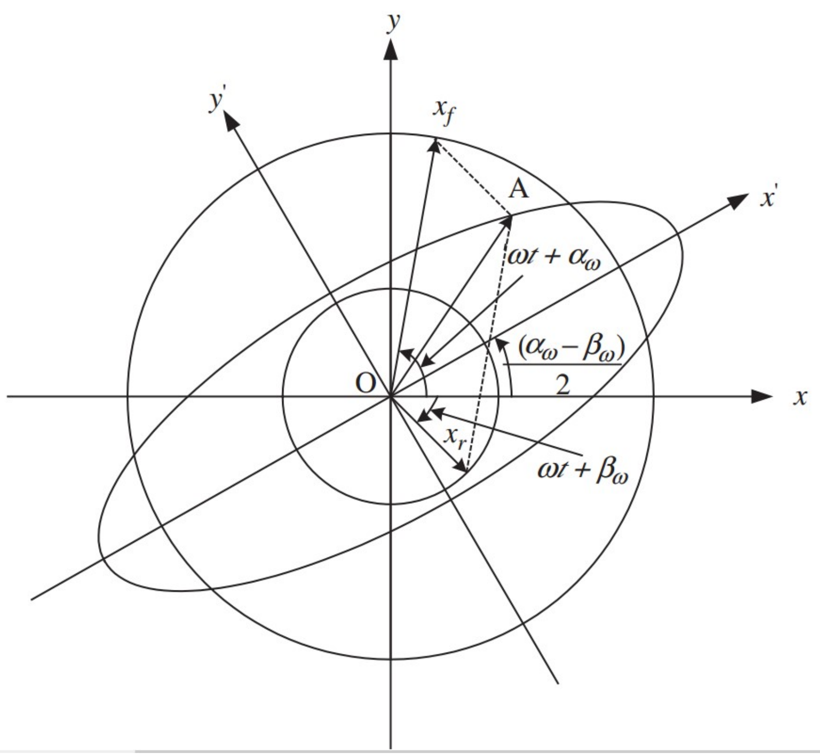


图 2: 简谐运动的合成

$$
\begin{cases}
x = |x_f| \cos(\omega t + \alpha_\omega) + |x_r| \cos(\omega t + \beta_\omega) \\
y = |x_f| \sin(\omega t + \alpha_\omega) - |x_r| \sin(\omega t + \beta_\omega)
\end{cases}
\tag{4}
$$

其中

$$
\begin{cases}
|x_f|\cos\alpha_\omega + |x_r|\cos\beta_\omega = x_c \\
|x_f|\sin\alpha_\omega + |x_r|\sin\beta_\omega = x_s \\
|x_f|\sin\alpha_\omega - |x_r|\sin\beta_\omega = y_c \\
-|x_f|\cos\alpha_\omega + |x_r|\cos\beta_\omega = y_s
\end{cases}
\tag{5}
$$

$$
\begin{cases}
\tan\alpha_\omega = \frac{x_s+y_c}{x_c-y_s} \\
\tan\beta_\omega = \frac{x_s-y_c}{x_c+y_s} \\
|x_f| = \frac{1}{2}\sqrt{(x_c-y_s)^2+(x_s+y_c)^2} \\
|x_r| = \frac{1}{2}\sqrt{(x_c+y_s)^2+(x_s-y_c)^2}
\end{cases}
\tag{6}
$$

注意：此处假定,x 和 y 有一样的表达形式。β_ω 为顺时针旋转的角。与后文用复数表示方法在形式上不同。但结果是一致的。

$$
\begin{cases}
x = x_0\cos(\omega t+\phi_x) \\
y = y_0\cos(\omega t+\phi_y)
\end{cases}
\tag{7}
$$

证明：

$$
\begin{aligned}
x &= |x_f|\cos(\omega t+\alpha_\omega) + |x_r|\cos(\omega t+\beta_\omega) \\
&= |x_f|\cos\omega t\cos\alpha_\omega - |x_f|\sin\omega t\sin\alpha_\omega + |x_r|\cos\omega t\cos\beta_\omega - |x_r|\sin\omega t\sin\beta_\omega \\
&= (|x_f|\cos\alpha_\omega + |x_r|\cos\beta_\omega)\cos\omega t - (|x_f|\sin\alpha_\omega + |x_r|\sin\beta_\omega)\sin\omega t \\
y &= |x_f|\sin(\omega t+\alpha_\omega) - |x_r|\sin(\omega t+\beta_\omega) \\
&= |x_f|\sin\omega t\cos\alpha_\omega + |x_f|\cos\omega t\sin\alpha_\omega - |x_r|\sin\omega t\cos\beta_\omega - |x_r|\cos\omega t\sin\beta_\omega \\
&= (|x_f|\sin\alpha_\omega - |x_r|\sin\beta_\omega)\cos\omega t - (-|x_f|\cos\alpha_\omega + |x_r|\cos\beta_\omega)\sin\omega t
\end{aligned}
$$

$$
\begin{aligned}
\tan\alpha_\omega &= \frac{\sin\alpha_\omega}{\cos\alpha_\omega} \\
&= \frac{x_s+y_c}{x_c-y_s} \\
&= \frac{|x_f|\sin\alpha_\omega + |x_r|\sin\beta_\omega + |x_f|\sin\alpha_\omega - |x_r|\sin\beta_\omega}{|x_f|\cos\alpha_\omega + |x_r|\cos\beta_\omega - (-|x_f|\cos\alpha_\omega + |x_r|\cos\beta_\omega)} \\
&= \frac{2|x_f|\sin\alpha_\omega}{2|x_f|\cos\alpha_\omega}
\end{aligned}
$$

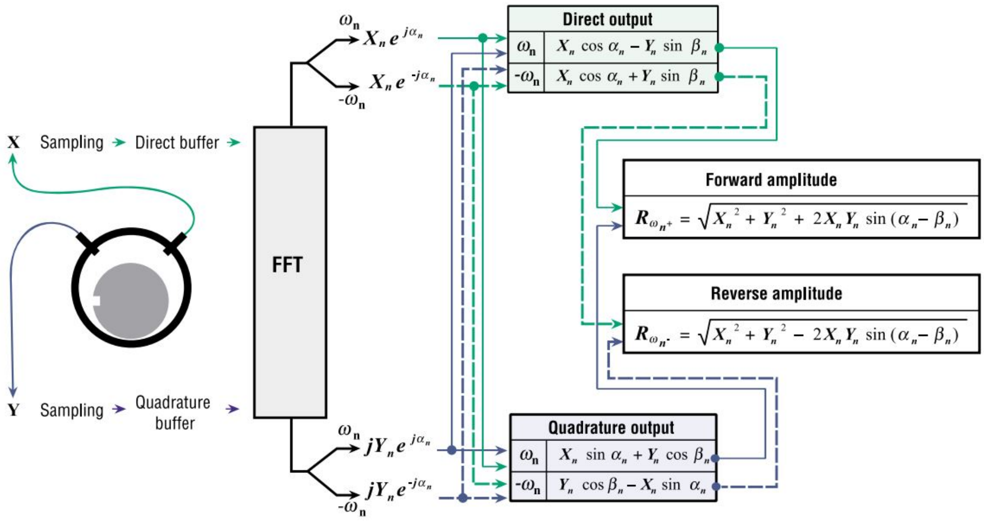


图 3: 文献 [4] 中给出了全谱的求解方法

同理可得

$$\tan\beta_\omega = \frac{x_s - y_c}{x_c + y_s}$$

$$\begin{aligned}
|x_f| &= \frac{1}{2}\sqrt{(x_c - y_s)^2 + (x_s + y_c)^2} \\
&= \frac{1}{2}\sqrt{(2\,|x_f|\cos\alpha_\omega)^2 + (2\,|x_f|\sin\alpha_\omega)^2} \\
&= \frac{1}{2}\sqrt{(2\,|x_f|)^2}
\end{aligned}$$

同理可得

$$|x_r| = \frac{1}{2}\sqrt{(x_c + y_s)^2 + (x_s - y_c)^2}$$

Paul Goldman 等人在文献 [4] 中给出了全谱的求解方法，但未进行推导和证明。

根据前文结论可以进行相关证明。此处 $R_{\omega_{n^+}}$ 即为 $|x_f|$，$R_{\omega_{n^-}}$ 即为 $|x_r|$。

$$\begin{aligned}
|x_f| &= \frac{1}{2}\sqrt{(x_c - y_s)^2 + (x_s + y_c)^2} \\
&= \frac{1}{2}\sqrt{x_c^2 + y_s^2 - 2x_c y_s + x_s^2 + y_c^2 + 2x_s y_c} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 - 2x_c y_s + 2x_s y_c} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 + 2x_0 \cos\phi_x y_0 \cos\phi_y + 2x_0 \sin\phi_x y_0 \sin\phi_y} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 + 2x_0 y_0 \cos(\phi_x - \phi_y)}
\end{aligned}$$

$$\begin{aligned}
|x_r| &= \frac{1}{2}\sqrt{(x_c + y_s)^2 + (x_s - y_c)^2} \\
&= \frac{1}{2}\sqrt{x_c^2 + y_s^2 + 2x_c y_s + x_s^2 + y_c^2 - 2x_s y_c} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 + 2x_c y_s - 2x_s y_c} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 - 2x_0 \cos\phi_x y_0 \cos\phi_y - 2x_0 \sin\phi_x y_0 \sin\phi_y} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 - 2x_0 y_0 \cos(\phi_x - \phi_y)}
\end{aligned}$$

该结论与原论文中结论存在偏差。需要进一步确认。经过分析发现，结果与原来不一致是因为 y 的形式，若 x y 都采用余弦形式表达，则为一致结论。而推论一开始将 y 用正弦形式表达。导致不一致。

证明如下：

$$\begin{aligned}
|x_f| &= \frac{1}{2}\sqrt{(x_c - y_s)^2 + (x_s + y_c)^2} \\
&= \frac{1}{2}\sqrt{x_c^2 + y_s^2 - 2x_c y_s + x_s^2 + y_c^2 + 2x_s y_c} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 - 2x_c y_s + 2x_s y_c} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 + 2x_0 \sin\phi_x y_0 \cos\phi_y - 2x_0 \cos\phi_x y_0 \sin\phi_y} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 + 2x_0 y_0 \sin(\phi_x - \phi_y)}
\end{aligned}$$

$$
\begin{aligned}
|x_r| &= \frac{1}{2}\sqrt{(x_c+y_s)^2+(x_s-y_c)^2} \\
&= \frac{1}{2}\sqrt{x_c^2+y_s^2+2x_cy_s+x_s^2+y_c^2-2x_sy_c} \\
&= \frac{1}{2}\sqrt{x_0^2+y_0^2+2x_cy_s-2x_sy_c} \\
&= \frac{1}{2}\sqrt{x_0^2+y_0^2-2x_0\sin\phi_x y_0\cos\phi_y+2x_0\cos\phi_x y_0\sin\phi_y} \\
&= \frac{1}{2}\sqrt{x_0^2+y_0^2-2x_0y_0\sin(\phi_x-\phi_y)}
\end{aligned}
$$

以下证明也在原 x 与 y 不一致假设下进行。

1.2 进动方向

这也能印证，进动方向的结论，当 $\cos(\phi_x-\phi_y)$ 大于 0, 即 $|x_f|>|x_r|$ 为正进动。否则为反进动。关于进动方向的另一种证明。由于椭圆轨迹上的点与圆心连线的夹角变化方向反映了轨迹的进动方向。当夹角增加时，为正进动，否则为反进动。设夹角为 θ，则有

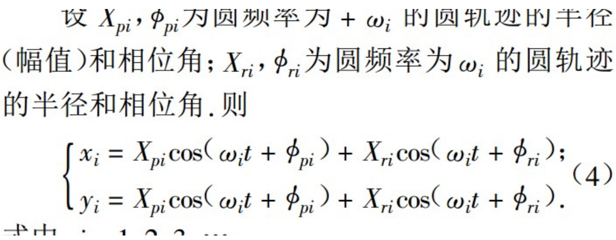
$$
\begin{aligned}
\tan(\theta) = f(t) &= \frac{y}{x} \\
&= \frac{y_0\sin(\omega t+\phi_y)}{x_0\cos(\omega t+\phi_x)}
\end{aligned}
$$

对 $f(t)$ 求导，以判别其单调性。

$$
\begin{aligned}
f'(t) &= \frac{x_0y_0\cos(\omega t+\phi_y)\cos(\omega t+\phi_x)+x_0y_0\sin(\omega t+\phi_y)\sin(\omega t+\phi_x)}{(x_0\cos(\omega t+\phi_x))^2} \\
&= \frac{x_0y_0\cos(\phi_x-\phi_y)}{(x_0\cos(\omega t+\phi_x))^2}
\end{aligned}
$$

又 $x_0, y_0>0$，因此函数单调性仅与 $\cos(\phi_x-\phi_y)$ 有关。当 $\cos(\phi_x-\phi_y)>0$，$\tan(\theta)$ 为单调增，为正进动，否则为反进动。与前面的结论相吻合。

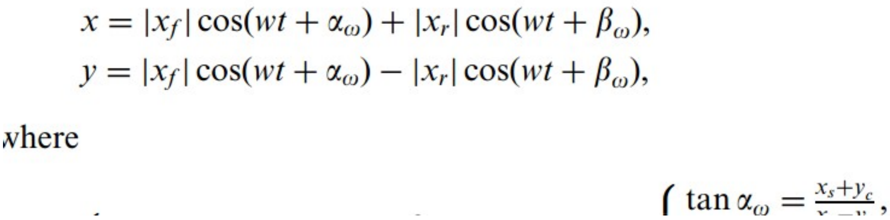
因此文献 [3, 5, 6] 中的表述均存在错误。以下为文献 [6] 中的写法。以下为文献 [3] 中的写法。



设 X_{pi}，ϕ_{pi} 为圆频率为 $+\omega_i$ 的圆轨迹的半径（幅值）和相位角；X_{ri}，ϕ_{ri} 为圆频率为 ω_i 的圆轨迹的半径和相位角. 则

$$\begin{cases} x_i = X_{pi}\cos(\omega_i t + \phi_{pi}) + X_{ri}\cos(\omega_i t + \phi_{ri}); \\ y_i = X_{pi}\cos(\omega_i t + \phi_{pi}) + X_{ri}\cos(\omega_i t + \phi_{ri}). \end{cases} \quad (4)$$

图 4: 文献 [6] 中的表达



$$x = |x_f|\cos(wt + \alpha_\omega) + |x_r|\cos(wt + \beta_\omega),$$
$$y = |x_f|\cos(wt + \alpha_\omega) - |x_r|\cos(wt + \beta_\omega),$$

where

图 5: 文献 [3] 中的表达

2 几个重要的关系式

2.1 进动圆与信号的关系

进一步推导可以得到如下关系

$$|x_f|\,|x_r|\sin(\alpha_\omega - \beta_\omega) = \frac{x_0 y_0}{2}\sin(\phi_y - \phi_x) \tag{8}$$

证明如下：

$$\begin{cases} \tan\alpha_\omega = \frac{\sin\alpha_\omega}{\cos\alpha_\omega} = \frac{x_s+y_c}{x_c-y_s} = \frac{x_0\sin\phi_x+y_0\sin\phi_y}{x_0\cos\phi_x+y_0\cos\phi_y} \\ \tan\beta_\omega = \frac{\sin\beta_\omega}{\cos\beta_\omega} = \frac{x_s-y_c}{x_c+y_s} = \frac{x_0\sin\phi_x-y_0\sin\phi_y}{x_0\cos\phi_x-y_0\cos\phi_y} \end{cases}$$

$$\begin{cases} \frac{\sin\alpha_\omega}{\cos\alpha_\omega} = \frac{x_0\sin\phi_x+y_0\sin\phi_y}{x_0\cos\phi_x+y_0\cos\phi_y} \\ \frac{\sin\beta_\omega}{\cos\beta_\omega} = \frac{x_0\sin\phi_x-y_0\sin\phi_y}{x_0\cos\phi_x-y_0\cos\phi_y} \end{cases}$$

等式两边分别相减

$$\frac{\sin\alpha_\omega\cos\beta_\omega-\sin\beta_\omega\cos\alpha_\omega}{\cos\alpha_\omega\cos\beta_\omega} = \frac{(x_0\sin\phi_x+y_0\sin\phi_y)(x_0\cos\phi_x-y_0\cos\phi_y)-(x_0\sin\phi_x-y_0\sin\phi_y)(x_0\cos\phi_x+y_0\cos\phi_y)}{x_0^2\cos^2\phi_x-y_0^2\cos^2\phi_y}$$

进一步化简

$$\frac{\sin\left(\alpha_\omega-\beta_\omega\right)}{\cos\alpha_\omega\cos\beta_\omega}=\frac{\left(-x_0\sin\phi_x y_0\cos\phi_y+y_0\sin\phi_y x_0\cos\phi_x\right)-\left(x_0\sin\phi_x y_0\cos\phi_y-y_0\sin\phi_y x_0\cos\phi_x\right)}{x_0^2{\cos}^2\phi_x-y_0^2{\cos}^2\phi_y}$$

$$\begin{aligned}\frac{\sin\left(\alpha_\omega-\beta_\omega\right)}{\cos\alpha_\omega\cos\beta_\omega}&=\frac{-2x_0y_0\left(\sin\phi_x\cos\phi_y-\sin\phi_y\cos\phi_x\right)}{x_0^2{\cos}^2\phi_x-y_0^2{\cos}^2\phi_y}\\&=\frac{-2x_0y_0\left(\sin\phi_x\cos\phi_y-\sin\phi_y\cos\phi_x\right)}{\left(\left|x_f\right|\cos\alpha_\omega+\left|x_r\right|\cos\beta_\omega\right)^2-\left(-\left|x_f\right|\cos\alpha_\omega+\left|x_r\right|\cos\beta_\omega\right)}\\&=\frac{-2x_0y_0\sin\left(\phi_x-\phi_y\right)}{4\left|x_f\right|\left|x_r\right|\cos\alpha_\omega\cos\beta_\omega}\end{aligned}$$

有

$$\left|x_f\right|\left|x_r\right|\sin\left(\alpha_\omega-\beta_\omega\right)=\frac{x_0y_0}{2}\sin\left(\phi_y-\phi_x\right)$$

得证。

2.2 进动圆半径与信号幅值之间的关系

$$\begin{cases}x_0^2=\left|x_f\right|^2+\left|x_r\right|^2+2\left|x_f\right|\left|x_r\right|\cos\left(\alpha_\omega-\beta_\omega\right)\\y_0^2=\left|x_f\right|^2+\left|x_r\right|^2-2\left|x_f\right|\left|x_r\right|\cos\left(\alpha_\omega-\beta_\omega\right)\\4\left|x_f\right|^2=x_0^2+y_0^2+2x_0y_0\cos\left(\phi_x-\phi_y\right)\\4\left|x_r\right|^2=x_0^2+y_0^2-2x_0y_0\cos\left(\phi_x-\phi_y\right)\end{cases}\tag{9}$$

变换可得

$$\begin{cases}\left|x_f\right|^2+\left|x_r\right|^2=\frac{x_0^2+y_0^2}{2}\\\frac{x_0^2-y_0^2}{4}=\left|x_f\right|\left|x_r\right|\cos\left(\alpha_\omega-\beta_\omega\right)\\\left|x_f\right|^2-\left|x_r\right|^2=x_0y_0\cos\left(\phi_x-\phi_y\right)\end{cases}\tag{10}$$

证明

$$
\begin{aligned}
|x_f| &= \frac{1}{2}\sqrt{\left(x_c - y_s\right)^2 + \left(x_s + y_c\right)^2} \\
&= \frac{1}{2}\sqrt{x_c^2 + y_s^2 - 2x_c y_s + x_s^2 + y_c^2 + 2x_s y_c} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 - 2x_c y_s + 2x_s y_c} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 + 2x_0 \cos\phi_x y_0 \cos\phi_y + 2x_0 \sin\phi_x y_0 \sin\phi_y} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 + 2x_0 y_0 \cos\left(\phi_x - \phi_y\right)} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 - 2\left(|x_f| \cos\alpha_\omega + |x_r| \cos\beta_\omega\right)\left(-|x_f| \cos\alpha_\omega + |x_r| \cos\beta_\omega\right) + 2\left(|x_f| \sin\alpha_\omega + |x_r| \sin\beta_\omega\right)} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 + 2\left(\left(|x_f| \cos\alpha_\omega\right)^2 - \left(|x_r| \cos\beta_\omega\right)^2\right) + 2\left(\left(|x_f| \sin\alpha_\omega\right)^2 - \left(|x_r| \sin\beta_\omega\right)^2\right)} \\
&= \frac{1}{2}\sqrt{x_0^2 + y_0^2 + |x_f|^2 - |x_r|^2}
\end{aligned}
$$

$$
\begin{aligned}
x_0^2 &= x_0^2 \cos\phi_x + x_0^2 \sin\phi_x = x_c^2 + x_s^2 \\
&= \left(|x_f| \cos\alpha_\omega + |x_r| \cos\beta_\omega\right)^2 + \left(|x_f| \sin\alpha_\omega + |x_r| \sin\beta_\omega\right)^2 \\
&= |x_f|^2 + |x_r|^2 + 2|x_f||x_r|\left(\cos\alpha_\omega \cos\beta_\omega + \sin\alpha_\omega \sin\beta_\omega\right) \\
&= |x_f|^2 + |x_r|^2 + 2|x_f||x_r| \cos\left(\alpha_\omega - \beta_\omega\right)
\end{aligned}
$$

$$
\begin{aligned}
y_0^2 &= y_0^2 \cos\phi_y + y_0^2 \sin\phi_y = y_c^2 + y_s^2 \\
&= \left(|x_f| \sin\alpha_\omega - |x_r| \sin\beta_\omega\right)^2 + \left(-|x_f| \cos\alpha_\omega + |x_r| \cos\beta_\omega\right)^2 \\
&= |x_f|^2 + |x_r|^2 - 2|x_f||x_r|\left(\cos\alpha_\omega \cos\beta_\omega + \sin\alpha_\omega \sin\beta_\omega\right) \\
&= |x_f|^2 + |x_r|^2 - 2|x_f||x_r| \cos\left(\alpha_\omega - \beta_\omega\right)
\end{aligned}
$$

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